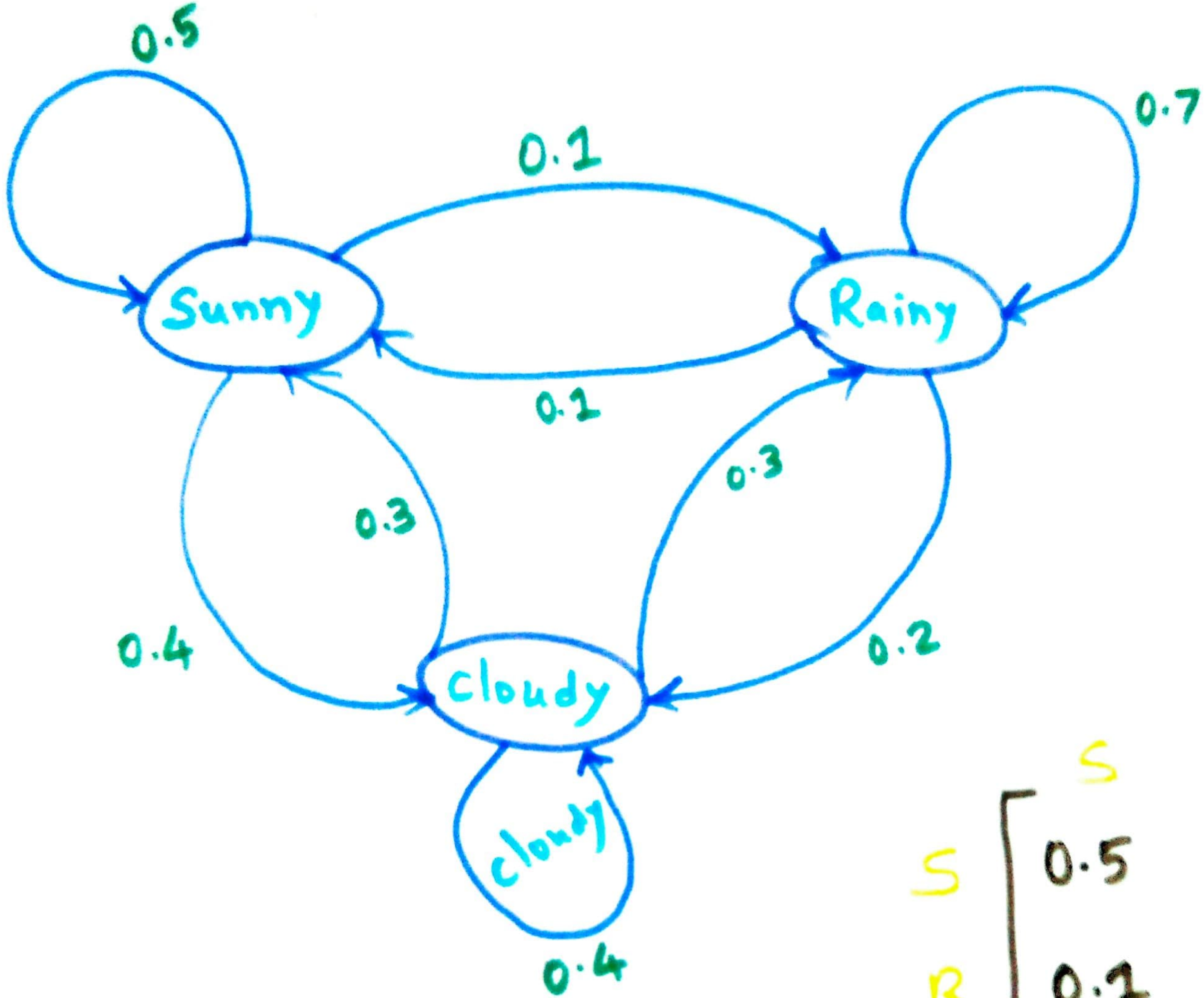
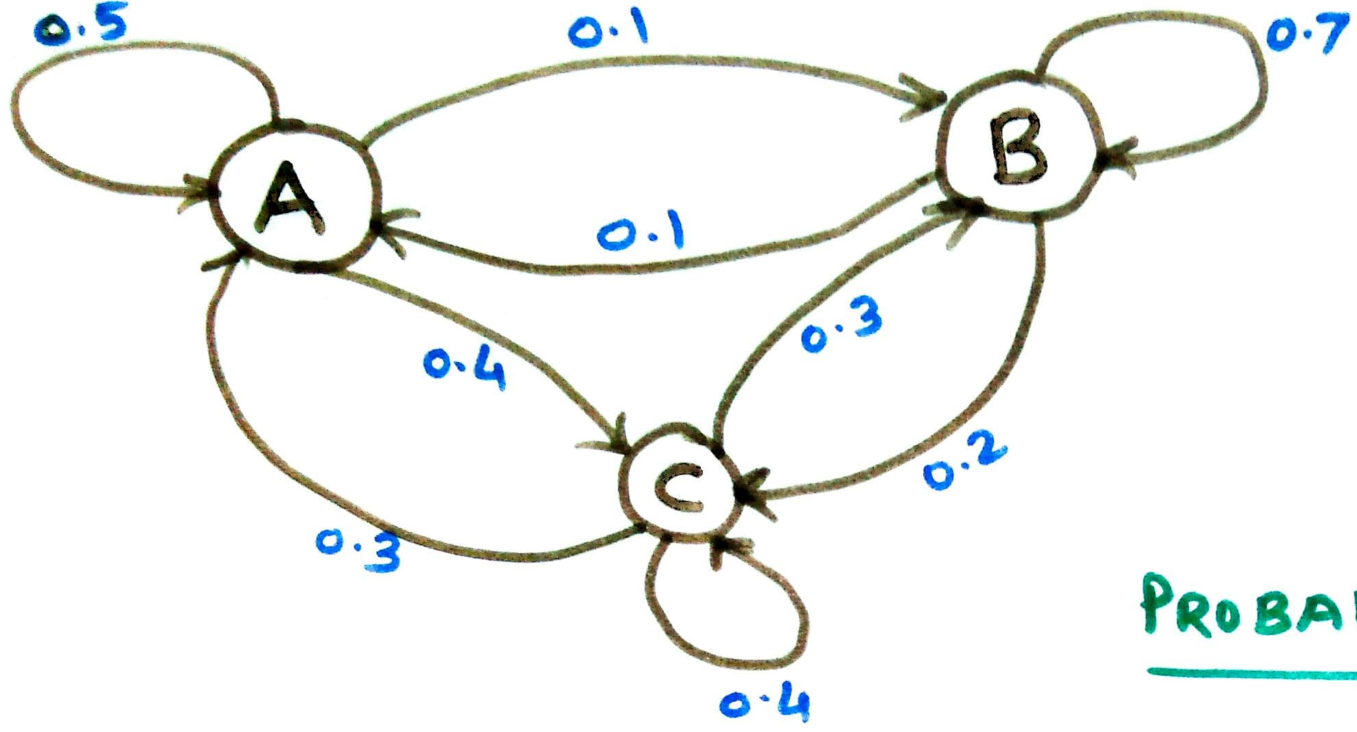


$$
\begin{array}{c}
 \\ S \\ R \\ C
\end{array}
\begin{array}{c}
\begin{array}{ccc} S & R & C \end{array} \\
\begin{bmatrix}
0.5 & 0.1 & 0.4 \\
0.1 & 0.7 & 0.2 \\
0.3 & 0.3 & 0.4
\end{bmatrix}
\end{array}
$$

1. Compute probability of having Rainy weather tomorrow if it is cloudy today.

2. What is the probability of a specific state sequence : Rainy - Sunny - cloudy - Sunny - Sunny ?

3. State the probability of reaching to state 'Sunny' in the third transition, if the first transition/state is 'Cloudy'.



PROBABILITY OF A SEQUENCE

$$p(ABC)$$

$$= p(A) \cdot p(B|A) \cdot p(C|B)$$

$$= (1)(0.1)(0.2)$$

$$= 0.02$$

Let transition probability matrix

$$T = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

$P_0 \leftarrow$ Initial probability vector

$P_0 = [1 \quad 0 \quad 0]$

$P_0 = [0.3 \quad 0.6 \quad 0.1]$

It represents probability distribution.

$$P_1 = P_0 \cdot T = [1 \quad 0 \quad 0] \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} = [0.5 \quad 0.1 \quad 0.4]$$

To continue, what will be the probability vector for next transition?

$$P_2 = P_1 \cdot T$$

$$= \begin{bmatrix} 0.5 & 0.1 & 0.4 \end{bmatrix} \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.1 & 0.7 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.38 & 0.24 & 0.38 \end{bmatrix}$$

In general,

$$P_1 = P_0 \cdot T$$

$$P_2 = P_1 \cdot T$$

$$= (P_0 \cdot T) \cdot T = P_0 \cdot T^2$$

$$P_3 = P_0 \cdot T^3$$

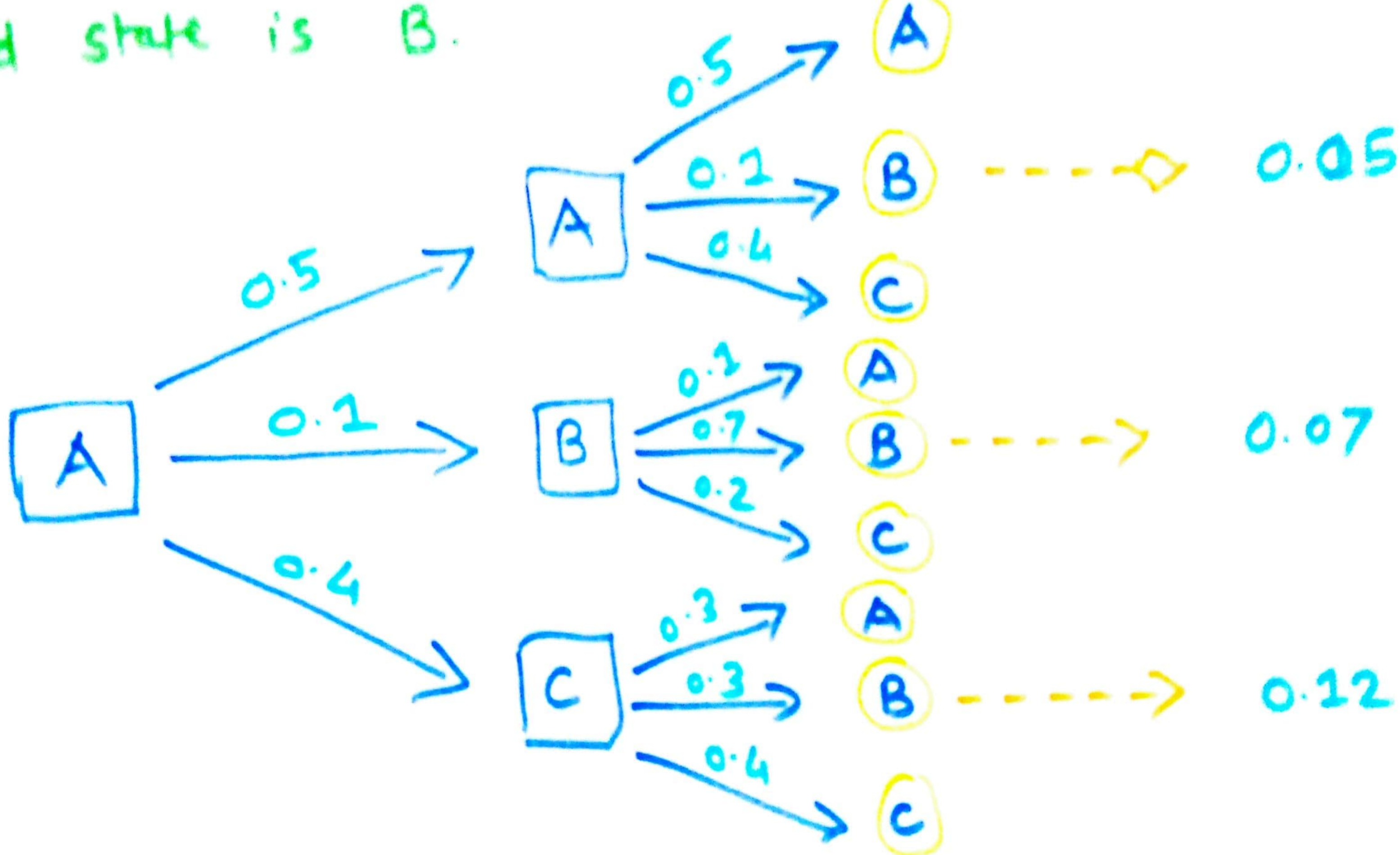
$$\vdots$$

$$\boxed{P_n = P_0 \cdot T^n}$$

The probability vector after n transitions is given by P_n.

PROBABILITY TREE DIAGRAM

Compute probability if first state is A and third state is B.



Final probability $= 0.05 + 0.07 + 0.12 = 0.24$

STEADY STATE & EQUILIBRIUM

If for any system $P_K = P_{K-1}$ then the system is said to be in steady state.

ABSORBING STATE