4. Show that if a problem is in PSPACE-complete, then it is in NP-hard.

Solution:

let problem P be in PSPACE-complete, this means that:

1. P is in PSPACE
2. For every problem X in PSPACE, X $\leq_p$ P.

We know that SAT is in PSPACE, since we can try all the possible truth assignments which will take space 2^n where n is the number of variables we have in the truth statement (also discussed in the lectures).

Now since SAT is in PSPACE, then we can say SAT $\leq_p$ P.

We know that a problem is in NP-hard if every problem in NP poly-time reduces to it.

SAT is in NP-Complete (from the lectures) and we proved above that SAT $\leq_p$ P. Now we can say that for all α in NP, $\alpha \leq_p$ SAT $\leq_p$ P. So for all α in NP, $\alpha \leq_p$ P which by the definition of NP-hard means that P is NP-hard.

We have proven that for any problem P in PSPACE-complete, P is also in NP-hard.