3. Prove that following problem (called the strong matching problem) is NP-complete.
Input: A graph G and an integer k.
Question: Are there k edges in G that form a matching and are joined by no other edges of G?
Solution:
To prove that this problem is NP-complete:
let this problem be y.
1. I will first prove that this problem is in NP, so (y $\in$ NP)
If we are given a certificate, so k edges that form a strong matching, we can verify this certificate in polynomial time. We can simply make a list of all the neighbours of the 2k vertices incident to the edges in our set and return TRUE of non of the 2k vertices are in the neighbour list and FALSE if any of the 2k vertices are in the neighbour list.
2. We know from the lectures that the Independent Set problem is NP-complete (slide 34 of Intractability-II), so let the Independent Set problem be x $\in$ NP-complete.
3. I will reduce the Independent Set problem to this problem, so (x $\leq_{\mathrm{p}}$ y).
I will be showing that if we have an oracle that solve problem y then we can also solve problem x.
If we are given a set S of k edges that create a strong matching on graph G we can build a Independent set of size k from the vertices incident to the edges in set S. We know that the vertices incident to the edges in set S are not connected to each other, so we can create a set M of independent vertices by selecting only one vertex from each of the edges in set S. Set M will have k vertices that are independent from each other, essentially a k-independent set.
So if our graph has a k-strong matching set, it also has a k-independent set.
Now assume that we have an Oracle that return YES if the graph G has k edges that are "strongly matched" and No if not. From the result of this Oracle we can tell if the graph has a k-independent set. So essentially by solving problem y we have also solved problem x, meaning that (x $\leq_{\mathrm{p}}$ y).

From the steps above I have proven that the problem is NP-complete.