

Q3

Question's data:

Max Revenue

W --> max weight that the crate can carry

V --> volume of crate

$\{(w_i, v_i, c_i), \dots\}$ -->

w_i = amount that we have of item i (kg)

v_i = density of item i (kg/liter)

c_i = price of item i (dollar/kg)

** I'll assume V is in liters.

My solution:

Let x_i be the weight (in kg) that we take from each item i .

Objective Function:

We want to maximize the revenue, so

Max $z = x_1 * c_1 + x_2 * c_2 + \dots + x_n * c_n$

This means that we are taking x_1 kilograms of item 1, x_2 kilograms of item 2, ..., and the revenue for item 1 will be $x_1 * c_1$ \$.

Constraints:

Now we have two weight constraints that we need to deal with:

1. The max weight that the crates can carry:

To deal with this, the sum of the amount of the products we take has to be $\leq W$. So

$$x_1 + x_2 + \dots + x_n \leq W$$

This means that we have to take x_i kilograms of item i , then the sum of x_i should be $\leq W$.

2. The second weight constraint is from the amount of products we have, for each item i we have w_i kilograms, so x_i can't be more than w_i for every i .

$$x_1 \leq w_1$$

$$x_2 \leq w_2$$

....

$$x_n \leq w_n$$

The other constrain that we have is related to the volume of the crate. The crate has a volume of V liters, and we have the density of our products, this means that the sum of the volumes of the items that we choose has to be $\leq V$, so they fit into the crate. v_i is the density of item i (kg/liters), so x_i/v_i will be the volume of item i that we choose.

$$x_1/v_1 + x_2/v_2 + \dots + x_n/v_n \leq V$$

Finally the $x_i \geq 0$, for all i .

Final LP:

Let x_i be the amount that we take from item i (in kilograms).

Max

$$z = x_1 * c_1 + x_2 * c_2 + \dots + x_n * c_n$$

Subject to:

$$x_1 + x_2 + \dots + x_n \leq W$$

$$x_i \leq w_i$$

$$x_1/v_1 + x_2/v_2 + \dots + x_n/v_n \leq V$$

$$x_i \geq 0$$

Where i goes from 1 to n