1. Either prove that the following problem is NP-complete, or show that it belongs to P:

Input: A CNF ϕ.

Question: Is there a truth assignment that satisfies none of the clauses in ϕ.

Solution:

This problem is in P.

To have a truth assignment that satisfies none of the clauses in ϕ, all the literal in each clause need to be false.

So as a brute force algorithm that solves this problem in polynomial time, we can start from the first clause and set the literals one by one is a way that makes the clause false (set the values so that all the literals in the clause are false). As we go ahead thought the clauses, if we reach any literal that has already been set to TRUE then we know that we have a clause that has a true value and we can return NO, otherwise when all the clauses are processed we return YES.

References: Prof. Hamed Hatami's website.