
3. Suppose we are given a flow network $G = (V, E)$ in which every edge has capacity 1. Describe and analyze an algorithm to identify one edge in G such that after deleting that edge, the value of the maximum (s, t) -flow in the remaining graph decreases by one.

Solution:

Run any algorithm to find the max flow of the flow network (let's say you run the Shortest Augmented Path algorithm), use BFS to find the min-cut, let's call that cut C . We know that the capacity of the min-cut = the value of the max flow. Since C is a min cut if you remove one edge that crosses the cut (from the source side to the sink side) then we will be decreasing the min-cut value by 1 and essentially decreasing the max-flow by 1 as well.

The complexity of the algorithm is $O(\text{max-flow algorithm}) + O(\text{BFS}) = O(mn^2) + O(m+n) = O(mn^2)$ the running time will probably change with other max-flow algorithms.