Q1

I couldn’t find any correlation between the thigs taught in class and this problem, so I used some online resources to try to figure this problem out:

1. http://www.ams.jhu.edu/~castello/625.414/Handouts/FractionalProg.pdf
2. https://en.wikipedia.org/wiki/Linear-fractional_programming

Summery of my solution:
I'll use the Charnes-Cooper transformation to write the fractional LP as a normal LP. Them I will solve the normal LP and use the results to find the solution for the fractional LP.

Q1.

$$\max \quad \frac{x_1 + 2x_2 + \cdots + nx_n}{x_1 + \cdots + x_n}$$

s/t $\quad x_1 + \cdots + x_n > 0$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j = \begin{cases} a_{11} x_1 + \cdots + a_{1n} x_n \geq 0 \\ \vdots \qquad\qquad \vdots \\ a_{m1} x_1 + \cdots + a_{mn} x_n \geq 0 \end{cases}$$

$$x_1, x_2, \ldots, x_n \geq 0$$

this is a linear - fractional problem where we have

$$\max \quad \frac{c^T x + \alpha}{d^T x + \beta} \qquad s/t \quad Ax \leq b$$

$$c = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} \quad d = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & - & - & a_{1n} \\ \vdots & \vdots & & & \vdots \\ a_{m1} & a_{m2} & - & - & a_{mn} \end{bmatrix} \qquad \alpha, \beta = 0$$

Now I'll use the charnes - cooper transformation to get the normal LP for it:

$$\text{let } t = \frac{1}{d^t x + \beta} = \frac{1}{x_1 + x_2 + \cdots + x_n}$$

$$\text{let } y = t \cdot x = \begin{bmatrix} \frac{x_1}{x_1 + x_2 + \cdots + x_n} \\ \vdots \\ \frac{x_n}{x_1 + x_2 + \cdots x_n} \end{bmatrix}$$

we will have:

$$\max\ c^T y + \alpha t =$$

$$\frac{1 x_1}{x_1 + x_2 + \cdots + x_n} + \cdots\cdots + \frac{n x_n}{x_1 + x_2 + \cdots + x_n}$$

s/t

$$Ay \leq bt \Rightarrow$$

$$Ay = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \cdots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} \frac{x_1}{x_1+x_2+\cdots+x_n} \\ \vdots \\ \frac{x_n}{x_1+x_2+\cdots+x_n} \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{x_1}{x_1+x_2+\cdots+x_n} + \cdots + \frac{x_n}{x_1+x_2+\cdots+x_n} \leq 0$$

$$\sum_{j=1}^{n} \frac{a_{ij} x_j}{x_1+x_2+\cdots+x_n} \leq 0$$ i goes from $1 \rightarrow m$

also:

$$d^T y + \beta t = 1 \Rightarrow$$

$$\frac{x_1}{x_1+x_2+\cdots+x_n} + \frac{x_2}{x_1+x_2+\cdots+x_n} + \cdots + \frac{x_n}{x_1+x_2+\cdots+x_n} = 1$$

$$\frac{1}{x_1+x_2+\cdots+x_n} \geq 0$$

So I have turned the fractional LP to a normal LP as see below

and the soultion for y, t yeilds the solution for the original problem as $x = \frac{1}{t} y$

$$\max \quad \frac{1\,x_1}{x_1 + x_2 + \cdots + x_n} + \cdots\cdots + \frac{n\,x_n}{x_1 + x_2 + \cdots + x_n}$$

S/t

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} \frac{x_1}{x_1 + x_2 + \cdots + x_n} \\ \vdots \\ \frac{x_n}{x_1 + x_2 + \cdots + x_n} \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{x_1}{x_1 + x_2 + \cdots + x_n} + \cdots + \frac{x_n}{x_1 + x_2 + \cdots + x_n} = 1$$

$$\frac{1}{x_1 + x_2 + \cdots + x_n} \geq 0$$

Now that I have the normal LP of the

fractional LP, I will replace the

$$\frac{x_i}{x_1 + \cdots + x_n} \quad \text{with} \quad t_i$$

Solve the LP for y, t

and then use the solution to find the

optimal solution of the main LP by

using $x = \frac{1}{t} y$