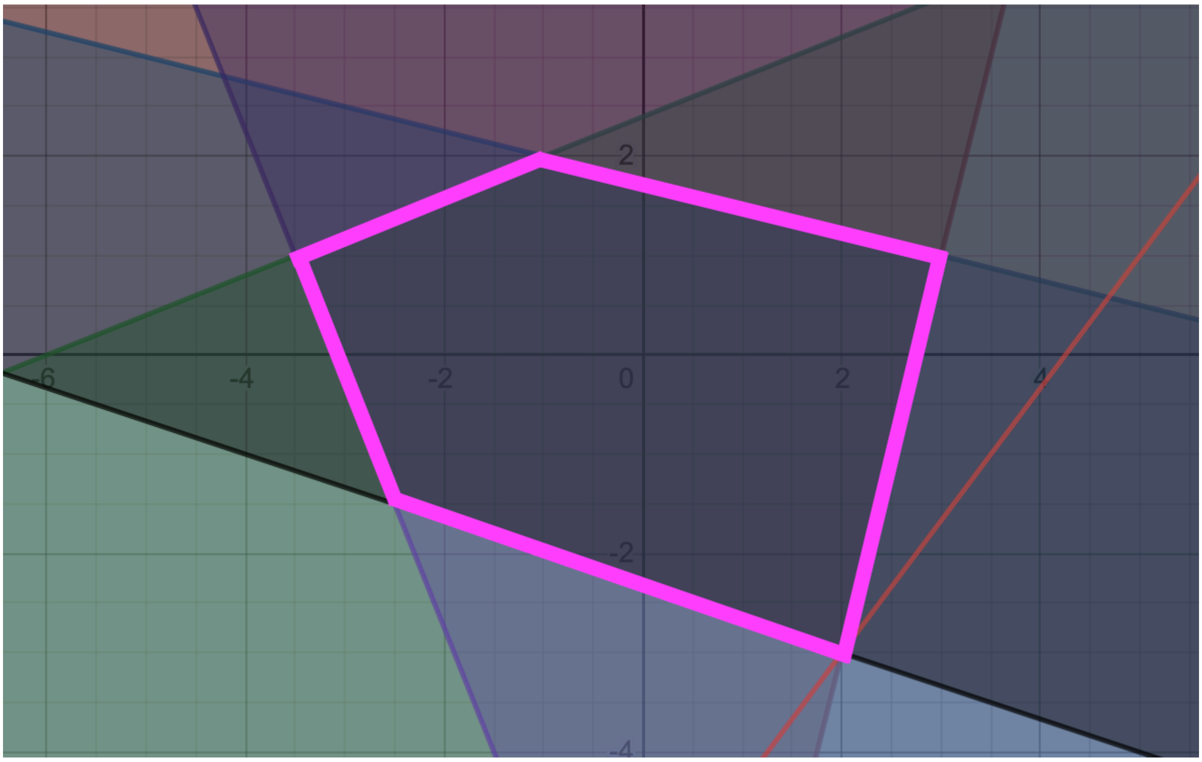
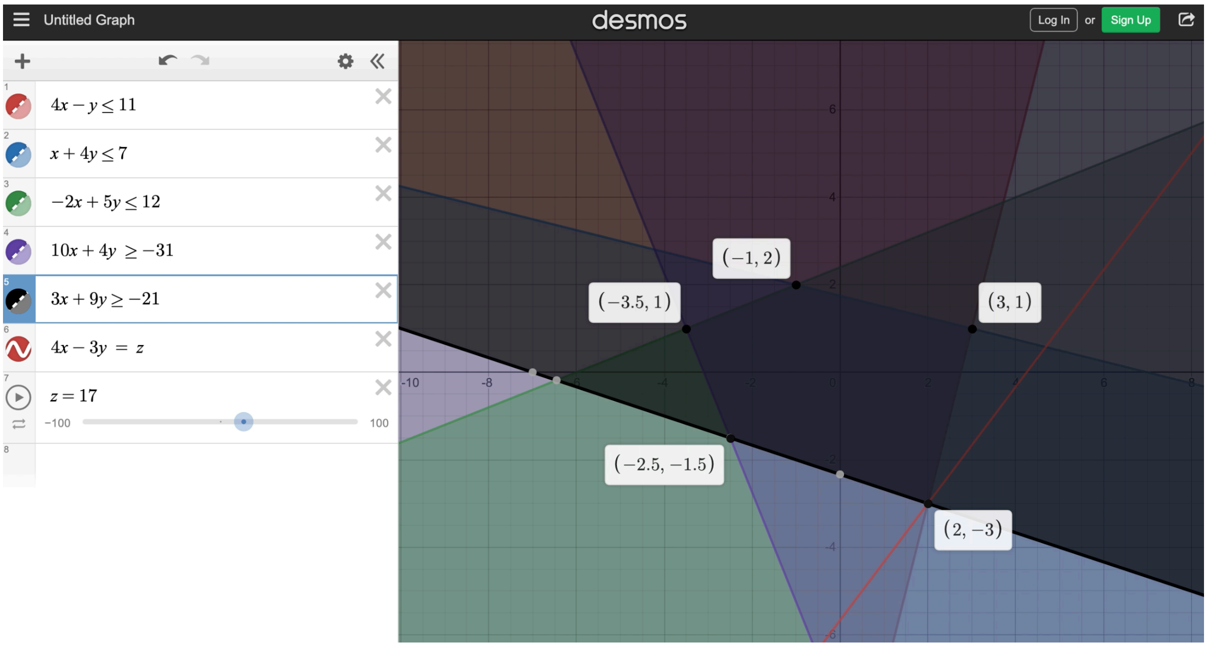
Q2

Q2.1:

The area bounded by pink lines is the feasible region and the vertices are indicated on the second picture.





2 -

b) max $4x-3y$

s/t $4x-y \leqslant 11$

$x+4y \leqslant 7$

$-2x+5y \leqslant 12$

$10x+4y \geqslant -31 \quad = \quad -10x-4y \leqslant 31$

$3x+9y \geqslant -21 \quad = \quad -3x-9y \leqslant 21$

x, y unristricted

from the lectures:

(P) max $c^T x$

s/t $Ax \leqslant b$

$x \geqslant 0$

(D) min $y^T b$

s/t $A^T y \geqslant c$

$y \geqslant 0$

in this lp $A = \begin{bmatrix} 4 & -1 \\ 1 & 4 \\ -2 & 5 \\ -10 & -4 \\ -3 & -9 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$

$c^T = [4 \ -3] \Rightarrow c = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ $b = \begin{bmatrix} 11 \\ 7 \\ 12 \\ 31 \\ 21 \end{bmatrix}$

let $$y = \begin{bmatrix} B \\ G \\ D \\ E \\ F \end{bmatrix} \Rightarrow y^T = [B \ G \ D \ E \ F]$$

(D) $$\min \ y^T b \Rightarrow \min \ [B \ G \ D \ E \ F] \begin{bmatrix} 11 \\ 7 \\ 12 \\ 31 \\ 21 \end{bmatrix}$$

$$\Rightarrow \min \ 11B + 7G + 12D + 31E + 21F$$

s/t $$A^T y = c \Rightarrow$$

$$\begin{bmatrix} 4 & 1 & -2 & -10 & -3 \\ -1 & 4 & 5 & -4 & -9 \end{bmatrix} \begin{bmatrix} B \\ G \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{cases} 4B + G - 2D - 10E - 3F = 4 \\ -B + 4G + 5D - 4E - 9F = -3 \end{cases}$$

$$y \geq 0 \Rightarrow B, G, D, E, F \geq 0$$

(D)

min $11B + 7G + 12D + 31E + 21F$

s/t

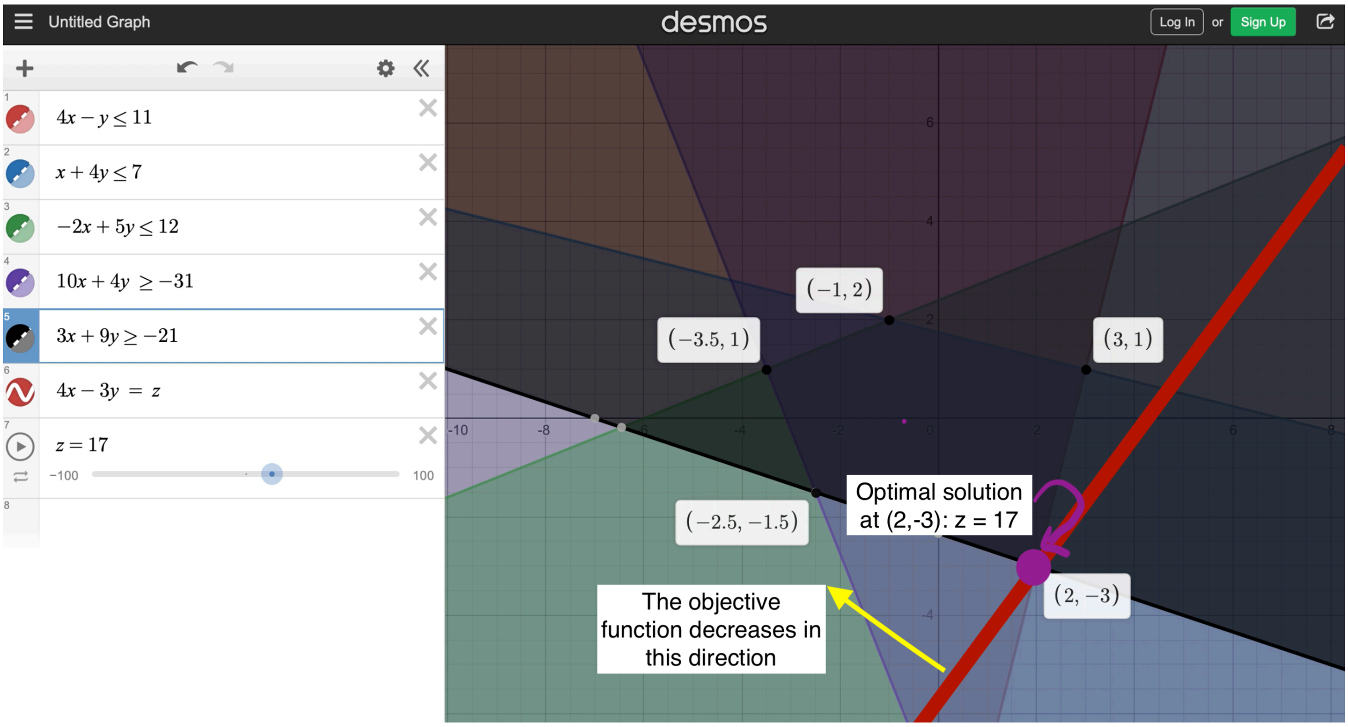
$$\begin{cases} 4B + G - 2D - 10E - 3F = 4 \\ -B + 4G + 5D - 4E - 9F = -3 \end{cases}$$

$$B, G, D, E, F \geq 0$$

↳ the 5 variables of the dual problem

Q2.3:

From the first part of the question we know that the area graph covered by the 5 vertices below is the feasible region and the red line is the objective function. The objective function reaches an max solution at the point (2,-3) and has the max value of 17. Now I'll try to find an optimal solution for the dual problem.



Solving the dual

max

$$-11x_1 - 7x_2 - 12x_3 - 31x_4 - 21x_5$$

s/t

$$4x_1 + 1x_2 - 2x_3 - 10x_4 - 3x_5 = 4$$

$$x_1 - 4x_2 - 5x_3 + 4x_4 + 9x_5 = 3$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

No obvious initial basis, using 2phase simplex

phase I :

max

$$w = -x_6 - x_7 = 5x_1 - 3x_2 - 7x_3 - 6x_4 + 6x_5 - 7$$

s/t $\quad 4x_1 + x_2 - 2x_3 - 10x_4 - 3x_5 + x_6 = 4$

$$x_1 - 4x_2 - 5x_3 + 4x_4 + 9x_5 + x_7 = 3$$

$$x_i \geq 0 \quad i \in [1, 7]$$

①

w	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
1	-5	3	7	6	-6	0	0	-7
0	4	1	-2	-10	-3	1	0	4
0	1	-4	-5	4	9	0	1	3

$B = \{x_6, x_7\}$

$N = \{x_1, x_2, x_3, x_4, x_5\}$

pivot at x_5, R_2

$$\begin{cases} R_1 \leftarrow R_1 + R_2/3 \\ R_2 \leftarrow R_2/9 \\ R_0 \leftarrow R_0 + R_2 \cdot 2/3 \end{cases}$$

②

w	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
1	$-13/3$	$1/3$	$1/3$	$26/3$	0	0	$2/3$	-5
0	$13/3$	$-1/3$	$-1/3$	$-26/3$	0	1	$1/3$	5
0	$1/9$	$-4/9$	$-5/9$	$4/9$	1	0	$1/9$	$1/3$

$B = \{x_5, x_6\}$ $N = \{x_1, x_2, x_3, x_4, x_7\}$

pivot at x_1, R_1

w	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
1	0	0	0	0	0	1	1	0
0	1	$-1/13$	$-11/13$	-2	0	$3/13$	$1/13$	$15/13$
0	0	$-17/39$	$-6/13$	$2/3$	1	$-1/39$	$4/39$	$8/39$

done with phase I

phase II: $B = \{x_1, x_5\}$ $N = \{x_2, x_3, x_4\}$

$Z = 17x_2 + 31x_3 + 39x_4 - 17$

Z	x_1	x_2	x_3	x_4	x_5	
1	0	-17	-31	-39	0	17
0	1	$-1/13$	$-11/13$	-2	0	$15/13$
0	0	$-17/39$	$-6/13$	$2/3$	1	$8/39$

the optimal solution of the dual is also $\underline{\underline{17}}$

Strong duality holds since the

primal and dual have the same

optimal solution $= 17$