2. Prove that the following problem is NP-complete.
Input: A CNF ϕ.
Question: Does ϕ have a truth assignment that assigns True to exactly half the terms in each clause?

Solution:
To prove that this problem is NP-complete:
let this problem be y.
1. I will first prove that this problem is in NP, so (y $\in$ NP)
If we are given a certificate, so a truth statement, we can verify this certificate in polynomial time. The certificate take the truth statement and check if it assigns true to only half of the terms in the clause. This can be done in linear time since we just need to check what literals we have in each clause.
2. We know from the lectures that the 3-SAT is NP-complete (slide 34 of Intractability-II), so let the 3-SAT be x $\in$ NP-complete.
3. I will reduce the 3-SAT problem to this problem, so (x $\leq_{\mathrm{p}}$ y).
Imagine an oracle that takes in a CNF ϕ and return YES if there is a possible truth statement that satisfies exactly half the literals in each clause of ϕ and NO if there is no possible truth statement that does so.
Now, consider α as an input to 3-SAT. We will construct a ϕ 6-CNF (CNF with 6 literals) by adding 3 literals to each clause of α.
Let $c_i = t_1 \vee t_2 \vee t_3$ to be the i'th clause of α. To built ϕ on α, we will introduce a new shared literal z and add it to all the clauses, we will also add individual literals a_{i1} and a_{i2} as well all the clauses. So we will have $m_i = t_1 \vee t_2 \vee t_3 \vee a_{i1} \vee a_{i2} \vee z$ as the i'th clause of ϕ.
Now imagine if we have a truth statement that satisfies α, then for ϕ we can set z to false and depending on how many of t_1, t_2, and t_3 are true we can set a_{i1} and a_{i2} to false in a way that for each clause of m_i of ϕ only half the literals are satisfied.

$$\alpha \text{ is satisfiable} \Rightarrow \phi \text{ is a YES Oracle input}$$

Also lets imagine that we have a ϕ that satisfies the oracle, so there is a truth statement that assigns true to exactly half the literals in each clause of ϕ. Now if z is true we will negate all the literals in ϕ to make z false (this will not effect the result of the Oracle, because by negating all the literals in all the clauses we will still have 3 TRUE and 3 FALSE literals in each clause). Now that we have a FALSE z, and we also know that half the literals in each clause are TRUE we can be sure that at least one of t_1, t_2, or t_3 will be true, thus we can be sure that α is satisfiable.

$$\alpha \text{ is satisfiable} \Leftarrow \phi \text{ is a YES Oracle input}$$

Now from the two proofs able we have:

$$\alpha \text{ is satisfiable} \iff \phi \text{ is a YES Oracle input}$$

Now we know that from the result we get from the Oracle for ϕ we can also deduce whether α is satisfiable or not, proving $\mathrm{x} \equiv_{\mathrm{p}} \mathrm{y}$ thus proving $\mathrm{x} \leq_{\mathrm{p}} \mathrm{y}$.

From the steps above I have proven that the problem given is in NP-complete.
References: Prof. Hamed Hatami's website.