
5. Prove that the following problem belongs to P:
Input: A graph G.
Question: Does G have an independent set of size 100.

Solution:

Since the k that we have is a constant we essentially have a k-Independent Set problem, k is 100 here, and this problem can be solved in polynomial time. I have provided a simple (brute force) algorithm below.

Let n be the number of vertices in the graph. We will check if we have an independent set for every possible way of choosing a subset of 100 vertices from the graph. We have $\binom{n}{100} \leq n^{100}$ possible ways of choosing a subset of size 100 from the vertices. Checking if the set is independent depends of how we represent the graph, but worst case we will iterate through all the 100 vertices of the set and see if they have any neighbours in $O(1)$. As you can see this algorithm is $O(n^{100})$ and polynomial.