Q3.

Part I:

yes, this point is a feasible point, since:

$x_1 + x_2 + x_3 + x_4 = 10 + 0 + 10 + 20 = 40$ ✓

$2x_1 + x_2 - x_3 - x_5 = 20 + 0 - 10 - 0 = 10$ ✓

$-x_2 + x_3 - x_6 = 0 + 10 + 0 = 10$ ✓

$x_i \geq 0$ ✓

Since the point given matches all the constraints, it is a feasible point.

Q5.

part2: I'll try to figure out the table of this problem at this point and continue solving it with that.

max $Z = x_1 + 3x_2 + x_3$

s/t

$$x_1 + x_2 + x_3 + x_4 = 40$$

$$2x_1 + x_2 - x_3 - x_5 = 10$$

$$-x_2 + x_3 - x_6 = 10$$

$B = \{1, 3, 4\}$ $N = \{2, 5, 6\}$

$x_1 = 10$

$x_3 = 10$

$x_4 = 20$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \end{bmatrix}$$

$$A_B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad A_B^{-1} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & -3/2 \end{bmatrix}$$

$$A_N = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} \qquad x_N = \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix}$$

$$c_B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad c_N = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$b = \begin{bmatrix} 40 \\ 10 \\ 10 \end{bmatrix}$$

$$c_B^T = [1 \ 1 \ 0] \qquad c_N^T = [3 \ 0 \ 0]$$

$$A_B^{-1} A_N = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & -3/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 & -1/2 \\ -1 & 0 & -1 \\ 2 & 1/2 & 3/2 \end{bmatrix}$$

From the slides, to find the simplex table for a give basis:

$$Z = (c_N^T - c_B^T A_B^{-1} A_N) x_N + c_B^T A_B^{-1} b$$

$$= \left([3 \ 0 \ 0] - [1 \ 1 \ 0] \begin{bmatrix} 0 & -1/2 & -1/2 \\ -1 & 0 & -1 \\ 2 & 1/2 & 3/2 \end{bmatrix} \right) \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix} + [1 \ 1 \ 0] \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & -3/2 \end{bmatrix} b$$

$$= \left([3 \ 0 \ 0] - [-1 \ -1/2 \ -3/2] \right) \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix} + [0 \ 1/2 \ 3/2] \begin{bmatrix} 40 \\ 10 \\ 10 \end{bmatrix}$$

$$= [4 \ 1/2 \ 3/2] \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix} + 5 + 15 = 4x_2 + \tfrac{1}{2}x_5 + \tfrac{3}{2}x_6 + 20$$

$$\Rightarrow Z = 4x_2 + \tfrac{1}{2}x_5 + \tfrac{3}{2}x_6 + 20$$

$$I x_B + A_B^{-1} A_N x_N = A_B^{-1} b \Rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & -1/2 & -1/2 \\ -1 & 0 & -1 \\ 2 & 1/2 & 3/2 \end{bmatrix} \begin{bmatrix} x_2 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & -3/2 \end{bmatrix} \begin{bmatrix} 40 \\ 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} x_5 - \frac{1}{2} x_6 \\ -x_2 - x_6 \\ 2x_2 + \frac{1}{2} x_5 + \frac{3}{2} x_6 \end{bmatrix} = \begin{bmatrix} 5+5 \\ 10 \\ 40-5-15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} x_1 - \tfrac{1}{2} x_5 - \tfrac{1}{2} x_6 &= 10 \\ x_3 - x_2 - x_6 &= 10 \\ x_4 + 2x_2 + \tfrac{1}{2} x_5 + \tfrac{3}{2} x_6 &= 20 \end{aligned}$$

$\Rightarrow$ constraints

$$\begin{cases} x_1 - \frac{1}{2} x_5 - \frac{1}{2} x_6 = 10 \\ -x_2 + x_3 - x_6 = 10 \\ 2x_2 + x_4 + \frac{1}{2} x_5 + \frac{3}{2} x_6 = 20 \end{cases}$$

primal LP at this stage

max

$$Z = 4x_2 + \frac{1}{2}x_5 + \frac{3}{2}x_6 + 20$$

$$Z - 4x_2 - \frac{1}{2}x_5 - \frac{3}{2}x_6 = 20$$

S/t

$$\begin{cases} x_1 - \frac{1}{2}x_5 - \frac{1}{2}x_6 = 10 \\ -x_2 + x_3 - x_6 = 10 \\ 2x_2 + x_4 + \frac{1}{2}x_5 + \frac{3}{2}x_6 = 20 \end{cases}$$

Solving using tables:

	x_1	x_2	x_3	x_4	x_5	x_6	
R_0	0	-4	0	0	$-\frac{1}{2}$	$-\frac{3}{2}$	20
R_1	1	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	10
R_2	0	-1	1	0	0	-1	10
R_3	0	2	0	1	$\frac{1}{2}$	$\frac{3}{2}$	20

choosing x_2, last row (R_3) as pivot

$$\begin{cases} R_0 \leftarrow 2R_3 + R_0 \\ R_2 \leftarrow (R_3 \div 2) + R_2 \\ R_3 \leftarrow R_3 \div 2 \end{cases}$$

x_1	x_2	x_3	x_4	x_5	x_6	
0	0	0	2	$1/2$	$3/2$	60
1	0	0	0	$-1/2$	$-1/2$	10
0	0	1	$1/2$	$1/4$	$-1/4$	20
0	1	0	$1/2$	$1/4$	$3/4$	10

Done, since all coeffciats are ≥ 0

optimal solution :

$$Z = 60$$

$$B = \{x_1, x_2, x_3\} \quad N = \{x_4, x_5, x_6\}$$

$$\begin{cases} x_1 = 10 \\ x_2 = 10 \\ x_3 = 20 \end{cases}$$