
2.

Solution:

I'll first prove the second part:

Let $X \leq_p Y$, and $Y \in \text{coNP}$.

From the definition of polynomial time reduction we know that since X is polynomial time reducible to Y , there is an algorithm for transforming an instances of X into instances of Y .

In other words, there is a polynomial reduction function f such that

$$\alpha \in X \iff f(\alpha) \in Y.$$

Now since Y is in coNP and $X \leq_p Y$, we can find a coNP algorithm for X by finding $f(\alpha)$ for any α in X and then applying the coNP algorithm of Y on it. Therefore we can find a coNP algorithm for X using the coNP algorithm we have for Y and X is in coNP .

Proof of the first part:

Let problem P be a NP-hard problem in coNP and problem M be any problem in NP .

By the definition of NP-hard, we can say that every problem in NP poly-time reduces to P . So M polynomial reduces to P .

So now we have $M \leq_p P$, and $P \in \text{coNP}$.

From the proof above, if we substitute M for X and P for Y , we can conclude that M is also in coNP .

Since M is any problem in NP and M is also in coNP , we can say that any problem in NP is also in coNP or NP is a subset of coNP .