6. In this problem we model a simple power grid. As input, we are given n houses, represented by n pairs of positive integers (x1, y1),(x2, y2), $\cdots$,(xn, yn) which are their coordinates in the plane. We are also given a list of m power plants. Each power plant is represented by
a) A pair of positive integers (ai, bi), representing their coordinates in the plane.
b) A positive integer ri, representing the maximum distance to which the power plant can send power (measured by standard Euclidean distance).
c) A positive integer ci, representing the number of houses that power plant can simultaneously power.
Design an algorithm to solve the following problem: given a list of n houses and m power plants as described above, decide if it is possible to connect each house to each power plant such that all houses receive power and no power plant is over its capacity

Solution:
I have come up with a solution inspired by the Baseball Elimination problem discussed in class.
We have to build a bipartite graph between the houses and plants. Lets put all the plants on the right and all the houses on the left. For each plant (p_i) lets compute the euclidean distance between the plant and all the houses, if for each house the distance was less than or equal to r_i we will make an edge from the house to the plant with capacity ∞, if the distance was more than r_i we will not connect the house and the plant. We will do this for all plants.
Code:
FOR ALL (plants p_i) DO
    FOR ALL (houses h_n) DO
        compute the distance between p_i and h_n
        if distance $\leq r_i$ then connect h_n to p_i (directed edge with capacity ∞)

Now that we have a bipartite graph between all possible house/plant combinations we will add a source and a sink. Will will connect the source to all the houses and set the capacity of the edges to 1. We will connect all the plants to the sink where plant p_i's edge has capacity c_i (the number of houses plant p_i can power).
Now if we run any max flow algorithm, if after the algorithm completes all the source edges are saturated (flow = 1) then we can say all houses can receive power, else there will be at least one house with no power.
Correctness:
First of all, we are respecting the distance constraint since we are only connecting houses to plants where the distance is less than or equal to the constraint r_i. So we will not have any option of getting power from a plant that is too far away.
Also, I am claiming that

$$\text{all houses will have power} \iff \text{all source edges get saturated}$$

Proof:
Lets assume that not all the source edges are saturated (there is at least one edge with capacity 0). This means that there is at least one edge from a power plant that has flow=capacity, this plant is the only available plant for the house and since that edge is full it means that it can't support any more houses.
On the other hand, if all the source edges are saturated, it means that we where able to use a combination of plants and plant edges to send flow through such that every house is connected to a plant and each plant has less that or equal to its house capacity (edge capacity).

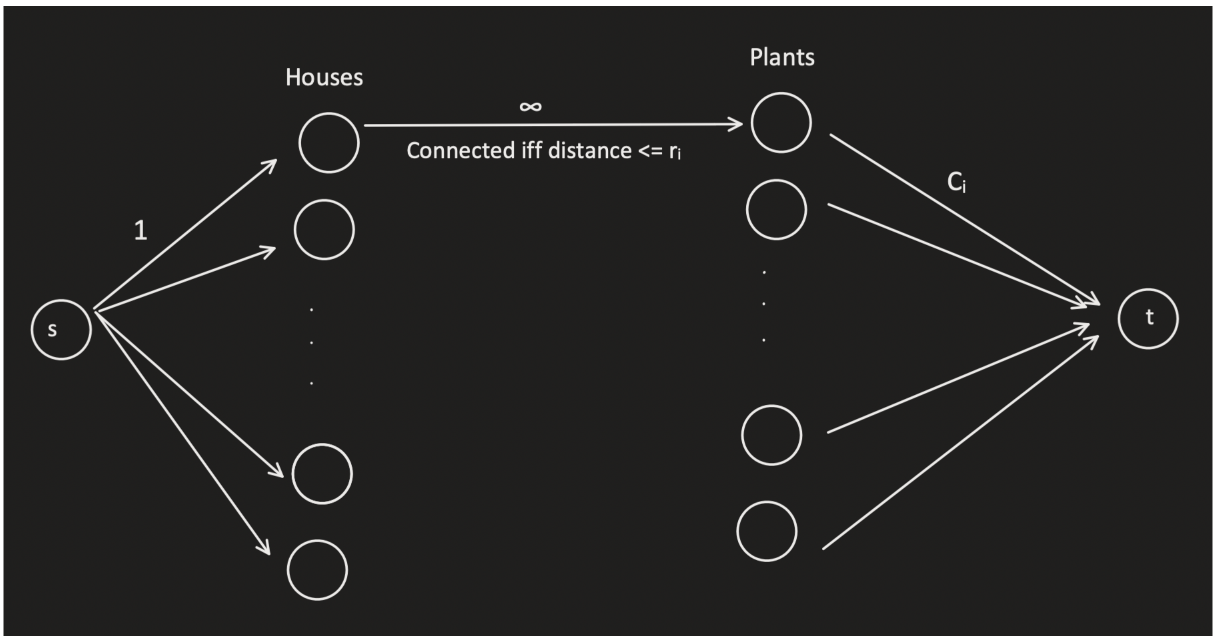

Figure 1: How the flow network would end up