6. Show that the following problem is NP-complete:
Input: An undirected graph G and an edge e.
Question: Does G have a Hamiltonian cycle that passes through the edge e.

Solution:
To prove that this problem is NP-complete:
let this problem be y.
1. I will first prove that this problem is in NP, so $(y \in NP)$
If we are given a certificate, so a Hamiltonian cycle that passes through the edge e, we can verify this certificate in polynomial time. As a verification algorithm, we can simply iterate through the cycle given and check if every node is visited once at the meanwhile we can also check if edge e is included in the cycle. This can be done in polynomial time $O(n)$, and the problem in in NP.
2. We know from the lectures that the Hamiltonian cycle problem is NP-complete (slide 34 of Intractability-II), so let the Hamiltonian cycle problem be $x \in$ NP-complete.
3. I will reduce the Hamiltonian cycle problem to this problem, so $(x \leq_p y)$.
For an input G for Hamiltonian cycle problem, we can solve the problem with the algorithm below:

For every edge e in G
    Call problem y with graph G and edge e, if the output is YES, output YES and terminate.

output NO

Given that we can solve problem x with polynomial calls to problem y then we can say that $(x \leq_p y)$.
After completing the three steps above we cay say that the problem is NP-complete.