- **Due:** Tuesday 9/7 at 10:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

First name	NAMEERA
Last name	FAISAL AKHTAR
SID	3034244256
Collaborators	SARA IMAM

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Q7.	Challenge Problem (Search)	/18

Q7. [18 pts] Challenge Problem (Search)

It is training day for Pacbabies, also known as Hungry Running Maze Games day. Each of k Pacbabies starts in its own assigned start location s_i in a large maze of size $M \times N$ and must return to its own Pacdad who is waiting patiently but proudly at g_i along the way, the Pacbabies must, between them, eat all the dots in the maze.

At each step, all k Pacbabies move one unit to any open adjacent square. The only legal actions are Up, Down, Left, or Right. It is illegal for a Pacbaby to wait in a square, attempt to move into a wall, or attempt to occupy the same square as another Pacbaby. To set a record, the Pacbabies must find an optimal collective solution.

7.1) (5 pts) Define a minimal state space representation for this problem.

- current x,y location of all k Pacbabies
- boolean indicating the following:
  - True : Food is present at that x,y location
  - False : Food is NOT present at that x,y location

7.2) (2 pts) How large is the state space?

- Each of k Pacbabies have M choices for the x coordinate of their location and N choices for the y coordinate of their location $\underbrace{(MN)\cdot(MN)\cdot\cdots\cdot(MN)}_{k \text{ times}} = (MN)^k$
- For each of the MN locations in the maze, there are 2 options: there is either food at the location or not. $\underbrace{2\cdot 2\cdot\cdots\cdot 2}_{MN \text{ times}} = 2^{MN}$

$\boxed{\therefore \text{ Total size} = (MN)^k \cdot (2)^{MN}}$

7.3) (2 pts) What is the maximum branching factor for this problem?

A) 4^k

B) 8^k

C) $4^k 2^{MN}$

D) $4^k 2^4$

There are k Pacbabies.

At each step, they can move 4 ways: Up, Down, Left, Right

$\underbrace{4\cdot 4\cdot 4\cdot\cdots\cdot 4}_{k \text{ times}} = 4^k$

7.4) (5 pts) Let $MH(p,q)$ be the Manhattan distance between positions p and q and F be the set of all positions of remaining food pellets and p_i be the current position of Pacbaby i. Which of the following are admissible heuristics?

An admissible heuristic should underestimate true cost to goal.

A) $\frac{\sum_{i=1}^{k} MH(p_i,g_i)}{k}$

B) $\max_{1\le i\le k} MH(p_i, g_i)$

C) $\max_{1\le i\le k}[\max_{f\in F} MH(p_i, f)]$

D) $\max_{1\le i\le k}[\min_{f\in F} MH(p_i, f)]$

E) $\min_{1\le i\le k}[\min_{f\in F} MH(p_i, f)]$

F) $\min_{f\in F}[\max_{1\le i\le k} MH(p_i, f)]$

A) Average distance between Pacbabies and their Pacdad.
This is an underestimate because the average is less than the maximum distance from Pacbaby to Pacdad, which is yet an underestimate to the true solution. Hence, this is $\boxed{\text{ADMISSIBLE}}$.

B) The maximum distance between any Pacbaby and its Pacdad.
This is the minimum number of steps that a Pacbaby can take to reach its goal because it assumes they don't have to make detours to eat the food first. Hence, this is $\boxed{\text{ADMISSIBLE}}$.

C) Find the maximum distance between Pacbaby i and a food pellet. Then, find the maximum of these.
It may overestimate because it assumes that the Pacbaby will eat food furthest from it, which is not true. In reality, that food pellet might be eaten by a Pacbaby who is closer to it. Hence, this is $\boxed{\text{NOT ADMISSIBLE}}$.

D) Find the minimum distance between Pacbaby i and a food pellet. Then, find the maximum of these
It may overestimate because a Pacbaby doesn't necessarily have to eat food which is far from it. In reality, a Pacbaby eats the closest foods to it then heads to the goal. So finding the max of the closest pellets to a Pacbaby may assume that a closer Pacbaby will not eat that pellet already, which is false. Hence, this is $\boxed{\text{NOT ADMISSIBLE}}$.

E) Find the minimum distance between Pacbaby i and a food pellet. Then, find the minimum of these.
This is the minimum distance that any Pacbaby will travel to eat food, so it is an underestimate because it assumes you don't have to go to the goal after eating the food. Hence, this is $\boxed{\text{ADMISSIBLE}}$.

F) Find the maximum distance between Pacbaby i and a food pellet. Then, find the minimum of these.
It may overestimate because it assumes that the Pacbaby will eat food furthest from it, which is not true. In reality, that food pellet might be eaten by a Pacbaby who is closer to it. Hence, this is $\boxed{\text{NOT ADMISSIBLE}}$.

7.5) (2 pts) Give one pair of heuristics h_i, h_j from part (7.4) such that their maximum, $h(n) = \max(h_i(n), h_j(n))$, is an admissible heuristic.

Any pair of admissible heuristics is admissible.

If h_K is an admissible heuristic, then $0 \leq h_K \leq h^*$ where h^* is true cost to goal.

So, if you take the maximum of two admissible heuristics, they will still be less than h^*, i.e:

If $0 \leq h_i \leq h^*$, $0 \leq h_j \leq h^*$ then $0 \leq \max(h_i, h_j) \leq h^*$.

$\boxed{\text{Hence, any pair of heuristics A, B, E from 7.4 can be used here.}}$

7.6) (2 pts) Is there a pair of heuristics h_i, h_j from part (7.4) such that their convex combination, defined as

$$h(n) = \alpha h_i(n) + (1 - \alpha)h_j(n)), \alpha \in [0, 1],$$

is an admissible heuristic for any value of α between 0 and 1?

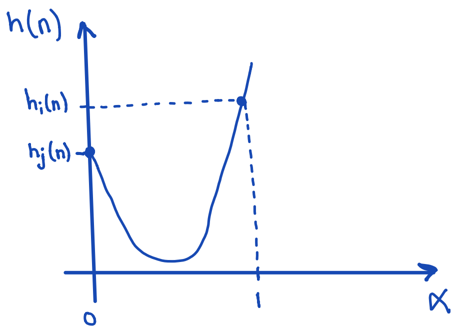
When $\alpha = 0$,

$$h(n) = h_j(n)$$

and when $\alpha = 1$,

$$h(n) = h_i(n).$$

We also know $h(n)$ is a convex combination of h_i and h_j. So, $h(n)$ can be displayed as follows:



If $h_i(n)$ and $h_j(n)$ are admissible, then $0 \leq h_i(n), h_j(n) \leq h^*$

Due to convexity, $h(n)$ also satisfies the condition $0 \leq h(n) \leq h^*$.

$\boxed{\text{Hence, any pair of heuristics A, B, E from 7.4 can be used here.}}$