CS 188 Fall 2021 | Introduction to Artificial Intelligence | Challenge Q9 HW7

- **Due:** Tuesday 10/26 at 10:59pm.
- **Policy:** Can be solved in groups (acknowledge collaborators) but must be submitted individually.
- **Make sure to show all your work and justify your answers**.
- **Note:** This is a typical exam-level question. On the exam, you would be under time pressure, and have to complete this question on your own. We strongly encourage you to first try this on your own to help you understand where you currently stand. Then feel free to have some discussion about the question with other students and/or staff, before independently writing up your solution.
- Your submission on Gradescope should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question begins on page 2.). **Do not reorder, split, combine, or add extra pages**. The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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For staff use only:

Q9.	Bayes Nets and Sampling	/25

Q9. [25 pts] Bayes Nets and Sampling

For the next three questions consider the following Bayes net.

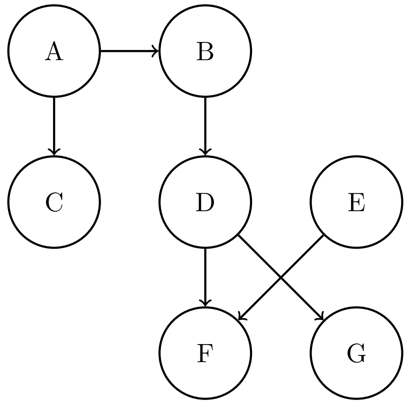


Figure 1: Bayes Net.

9.1) (1 pts) What is the joint distribution represented by this Bayes net? Write it as a product of conditional probability tables.

$$\boxed{P(A)P(B|A)P(C|A)P(D|B)P(E)P(F|D,E)P(G|D)}$$

9.2) (1 pts) If each variable can take 4 values what is the number of entries for the factors at A, D and F?

A : 4 D : 4^2 F : 4^3

A can take on 4 values.

D can take on 4 values, and B can take on 4 values

F can take on 4 values, and D can take on 4 values and E can take on 4 values

9.3) (2 pts) Which of the following independence assumptions are guaranteed to be true based on the above Bayes Net? Select all that apply.

(A) $B \perp\!\!\!\perp C$ x common cause

(B) $A \perp\!\!\!\perp F$ x causal chain

(C) $D \perp\!\!\!\perp E | F$ x common effect w/ observed

(D) $E \perp\!\!\!\perp A | D$ causal chain w/ observed

(E) $F \perp\!\!\!\perp G | D$ common cause w/ observed

(F) $B \perp\!\!\!\perp F | D$ causal chain w/ observed

(G) $C \perp\!\!\!\perp G$ x causal chain

(H) $D \perp\!\!\!\perp E$ common effect

For the next two questions, consider the following tables. You can assume that the joint probability $P(A, B, C, D)$ is equal to the product of these tables.

A	$P(A)$
+a	0.8
−a	0.2

A	B	$P(B\|A)$
+a	+b	0.9
+a	−b	0.1
−a	+b	0.6
−a	−b	0.4

B	C	$P(C\|B)$
+b	+c	0.8
+b	−c	0.2
−b	+c	0.8
−b	−c	0.2

C	D	$P(D\|C)$
+c	+d	0.25
+c	−d	0.75
−c	+d	0.5
−c	−d	0.5

Figure 2: Probability Tables.

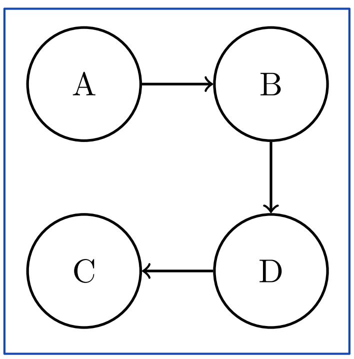
9.4) (2 pts) Mention all the independence assumptions (excluding conditional independence assumptions) that are implied by these tables.

$\boxed{B \perp\!\!\!\perp C}$ because $P(+c|+b) = P(+c|-b)$ and $P(-c|+b) = P(-c|-b)$

$\boxed{A \not\perp\!\!\!\perp B}$ because $P(+b|+a) \neq P(+b|-a)$ and $P(-b|+a) \neq P(-b|-a)$

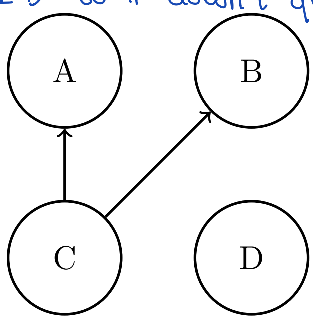
$\boxed{C \not\perp\!\!\!\perp D}$ because $P(+d|+c) \neq P(+d|-c)$ and $P(-d|+c) \neq P(-d|-c)$

9.5) (3 pts) Which of the following Bayes nets are consistent with the above probability tables (i.e.: which **can** represent a distribution consistent with the above probability tables? Select all that apply.



A

$C \perp\!\!\!\perp D$ so it doesn't qualify

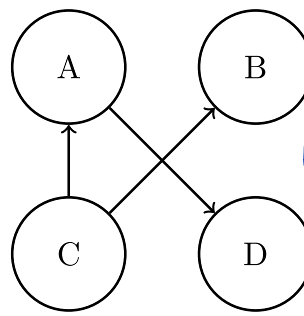


B

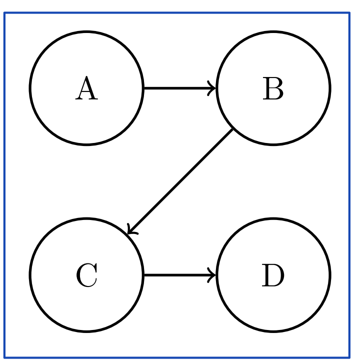
$A \not\perp\!\!\!\perp B$ not guaranteed (common cause)

$C \not\perp\!\!\!\perp D$ not guarantanteed (causal chain)

$B \perp\!\!\!\perp C$ so arrow is vacuous, so A and B not dependent ($A \perp\!\!\!\perp B$)

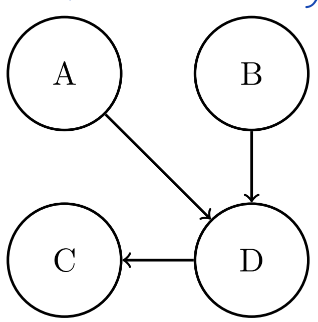


C

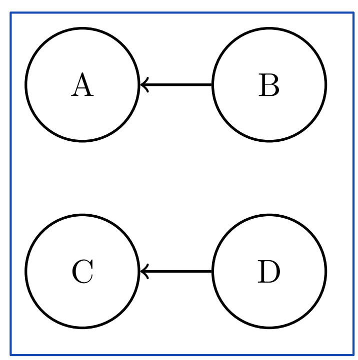


D

$A \perp\!\!\!\perp B$ (common effect)

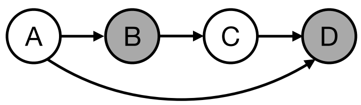


E



F

Now consider the following Bayes net and probability tables. Furthermore, assume that we have observed the following evidence $B = +b$ and $D = +d$.



$P(A)$	
$+a$	0.5
$-a$	0.5

$P(B\|A)$		
$+a$	$+b$	0.8
$+a$	$-b$	0.2
$-a$	$+b$	0.4
$-a$	$-b$	0.6

$P(C\|B)$		
$+b$	$+c$	0.1
$+b$	$-c$	0.9
$-b$	$+c$	0.7
$-b$	$-c$	0.3

$P(D\|A,C)$				
$+a$	$+c$	$+d$	0.6	sample 5
$+a$	$+c$	$-d$	0.4	
$+a$	$-c$	$+d$	0.1	sample 2, 3
$+a$	$-c$	$-d$	0.9	
$-a$	$+c$	$+d$	0.2	sample 4
$-a$	$+c$	$-d$	0.8	
$-a$	$-c$	$+d$	0.5	sample 1
$-a$	$-c$	$-d$	0.5	

Figure 3: Bayes Net and Probability Table.

9.6) (4 pts) In this question we will perform Gibbs sampling. We have initialized the variables A, B, C, and D as $+a, +b, +c, +d$ respectively. If we unassign the value for C so that we proceed to resample it, what are the following probabilities at the next step of the Gibbs sampling procedure?

$P(C = +c$ at the next step of Gibbs sampling) = $\dfrac{(0.1)(0.6)}{(0.1)(0.6) + (0.9)(0.1)} = \boxed{\dfrac{2}{5}}$

$P(C = -c$ at the next step of Gibbs sampling) = $\dfrac{(0.9)(0.1)}{(0.1)(0.6) + (0.9)(0.1)} = \boxed{\dfrac{3}{5}}$

Now we want to come up with a new sampling technique that is a hybrid between rejection sampling and likelihood-weighted sampling. In this new scheme, we first perform rejection sampling for the variables A and B. After that, we take the sampled values for A and B and add the values for variables C and D that we obtained using likelihood-weighted sampling.

9.7) (2 pts) Which of the following samples would be rejected by the rejection sampling part of our new sampling scheme?

A. $-a, -b$ not consistent w/ evidence that $B = +b$

B. $+a, +b$

C. $+a, -b$ not consistent w/ evidence that $B = +b$

D. $-a, +b$

9.8) (4 pts) Assume the we observe a new set of samples, separate from the previous part. Fill in the weights of these samples following our new sampling technique.

$-a + b - c + d$, weight= 0.5

$+a + b - c + d$, weight= 0.1

$+a + b - c + d$, weight= 0.1

$-a + b + c + d$, weight= 0.2

$+a + b + c + d$, weight= 0.6

9.9) (2 pts) Using the weights from the previous part, what is the estimate of $P(+a|+b,+d)$?

$$P(+a|+b,+d) = \frac{P(+a,+b,+d)}{P(+b,+d)} \quad \begin{array}{l} \nearrow \text{sample } 2,3,5 \\ \rightarrow \text{sample } 1,2,3,4,5 \end{array} = \frac{0.1+0.1+0.6}{0.5+0.1+0.1+0.2+0.6} = \frac{0.8}{1.5} = \boxed{\frac{8}{15}}$$

Now we design another hybrid sampling scheme that combines likelihood-weighted and rejection sampling. For the following two questions, indicate whether the sampling technique correctly approximates $P(A,C|+b,+d)$.

9.10) (2 pts) We first collect a likelihood-weighted sample for the variables A and B. After that we use rejection sampling for the variables C and D. If we reject, then we discard the values of A and B and the sample weight. We then restart by sampling from node A.

Correct

Incorrect

No problems w/ this method.

9.11) (2 pts) We first collect a likelihood-weighted sample for the variables A and B. After that we use rejection sampling for the variables C and D. If we reject, the values of A and B and the sample weight are retained. We then restart sampling from node C.

Correct

Incorrect

This does not utilize the evidence that we gain and the values of sample weights.