Mapping the International Phonetic Alphabet into the odd-limit Scale

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Contents

1 Motivation

A primary ingredient of universal composition consists of bridging different areas of human expression so they can be structured into a common composition. A precursor of such bridges is the concept of an alphabet, which establishes a correspondence between phonemes and graphic signs. Alphabets are as variable as languages, but by providing a one-to-one mapping from phonemes to graphic signs, a somewhat universal one can be devised. The standard attempt to do this is the International Phonetic Alphabet. Here we will extend this two-fold bridge by connecting it to a third area: that of pitch classes relative to a fixed pitch class, so "scale degrees" in the general sense of "members of a set of pitch classes relative to a fixed element". This way, any phoneme sequence can be translated into a pitch sequence.

There several reasons to do this: first, sonification. By identifying each letter with the corresponding phoneme, each alphabet gives rise to a corresponding scale (some care has to be taken if several letters refer to the same phonemes) and each language has its own melodic profile depending on the frequency with which it uses the various phonemes. Furthermore, this method can be used to derive motifs. Finally, and perhaps most importantly, through this method language and music can be used in counterpoint that transcends both domains, such as music that encodes text that rhymes with contemporaneous writing, which, if translated into pitch classes, would in turn be harmonious with the music.

It should also be noted that the idea of mapping linguistic units to pitch classes is neither unnatural nor unprecedented. Indeed, grapheme-pitch class synesthesia is a rare form synesthesia and mappings of the Latin alphabet to pitches have already been proposed in the middle ages, while Bach liked to use his own name as a motif. Since most people can at most develop relative pitch, the move from pitch classes to scale degrees seems justified to keep the possibility of intelligibility at least in specific alphabets.

2 Domain and Target of the Mapping

Naturally, the question arises how to best map the International Phonetic Alphabet to scale degrees. Phonemes are interrelated by their common features, while es occupy the space of an octave and are related by the intervals between them. How much of the interphonemic structure should and can be respected by such a mapping? Short consideration of the IPA chart should suffice to show that the relations between es cannot possibly contain the wealth of interphonemic relationships. However, a more conservative mapping might be achieved if es were replaced by a more general target, such as sounds in general. I have decided against such a generalization, since my goal was *not* to simply replicate phonemic structures but to codify them. Indeed, a trivial mapping of the IPA to the domain of sounds is given by mapping each letter to its corresponding phoneme. However, such a mapping does not open any new possibilities to make music from language. Instead, very few of the interphonemic structure is respected, but the target of the mapping is specific enough to be used as material for music by varying the remaining degrees of freedom, such as temporal structure, register, timbre or space. An exception are the symbols for click sounds, which are not mapped. Since click sounds are rhythmic in nature it seems fitting to express them using percussive instruments. Furthermore, the IPA contains a lot of diacritics, these should be expressed by other musical means.

Phonemes vary widely in frequency of use, and this use frequency might be the most important factor in deciding the target of a phoneme. The question arises of how to determine this use frequency. The best estimate I could find was the PHOIBLE database [1], which provides estimates for the language frequency of phonemes based on 2186 surveyed languages:

Definition 2.1 *The* language frequency *of a phoneme is the percentage of languages the phoneme appears in.*

The language frequency is not to be confused with the use frequency of a phoneme within a particular language, though the two are related, since the same factors that lead to a phoneme being used in many languages, ease of expressing and ease of distinguishing, also aid its popularity within a particular language.

The target space meanwhile carries its own questions. Clearly, the most common 12-tone equal temperament is not sufficient to encode the IPA. However, as there are infinitely many possible scale degrees and most of them are meaningless on their own, some restriction has to be implied. Furthermore, due to the natural variation of language frequency, a serialist treatment is impossible. Furthermore, due to the size of the IPA, it can be assumed that any instrument chosen to actually perform it have continuous pitch selection. Finally, since, for various reasons, such as alphabets with letters encoding the same phonemes, the IPA pitches might be insufficient, a kind of restriction is desirable that can be extended at will. For all of those reasons, equal temperament was seen as unsuitable for the task at hand. Instead, the *n*-odd limit scale was chosen.

Definition 2.2 *An n-odd interval is as an interval given by a ratio $2^m\frac{k}{l}$ for some integer m, such that neither k nor l are larger than n.*

The n-odd limit scale *on a tonic t consists of the collection of pitch classes of the higher notes of the intervals of all n-odd intervals whose lower note is t.*

The odd-limit scale *is the union of all n-odd limit scales.*

Thus, the odd-limit scale contains infinitely many scale degrees, but is graded by the n-odd limit scales in the sense that, for each pitch class p in the odd-limit scale, a number n_p exists such that the n_p-odd limit scale is the smallest n-odd limit scale that contains p[1]. The first few n-odd limit intervals can be found on the Xenharmonic wiki[2], which also contains much other information that was be used in the construction of this mapping. n_p is a fairly good measure of the degree of dissonance of the interval corresponding to p: the 1-odd intervals are the unison and the octave, the 3-odd limit adds the pure perfect fourth and fifth, the 5-odd limit the pure thirds and sixths and the 7-odd limit adds the first xenharmonic intervals. As n increases, an n-odd interval becomes more exotic, unless its pitch difference is small enough to a more consonant interval that it can be comprehended as a variation of a more consonant interval. This already hints at the necessity of a classification of intervals based on proximity. For this we will use Margo Schulter's interval classification [5]. Since no danger of e will identify a rational interval with its upper note in the odd-limit scale.

3 Mapping rules

We will now describe the rules that were used when choosing the scale degree corresponding to a phoneme. Each n-odd limit is closed under octave comple-

[1] More precisely, the odd-limit scale should be thought of as the filtered colimit of the n-odd limit scales, but the term of a filtered colimit needs some mathematical background that cannot be assumed in a general audience.

mentation, and thus its intervals can be grouped into interval pairs. In correspondence, phonemes were grouped into phoneme pairs that were mapped to their respective octave complements. The complement of a vowel was chosen like this: a front vowel is paired with a back vowel of the same height, while a central vowel is paired with a central vowel of the same height, and an unrounded vowel is paired with a rounded vowel. Thus the complement of “i” is “u”, the complement of “e” is “o” and so on. This way the complement of all vowels in regular positions is determined. The remaining vowels are paired up with each other by similarity. This pairing was chosen because it fits well with the use frequencies of the vowels. Voiced consonants were paired with their voiceless counterparts if they were available and similar consonants if they were not, and vice versa.

Perhaps the most controversial mapping rule is that the distribution of mapped phonemes with their corresponding vowel frequencies follows roughly the contours of the diatonic major scale as determined in Figure 1 of [4]. Given the variations in use frequencies of phonemes, some heuristic ordering principle seems appropriate, and while the diatonic scale is not universal, it is among the oldest scales and was already used in ancient Mesopotamia [3]. Furthermore, Schulter’s classification is essentially an expanded version of the diatonic scale. This global similarity to the diatonic scale does of course generally not translate to a similarity for any particular language, depending on the phonemes used, that scale might look very different.

Vowels are prolongable and can be changed in tone and volume, and thus are usually the nuclei of syllables. Due to this, vowels are usually among the most frequently used phonemes, even in most consonant-heavy languages, and make melisma possible, which might be important for musico-textual counterpoint. For these reasons, vowels are generally preferred to syllables, even if their language frequency is somewhat lower, and thus, they are given the most consonant intervals. So unison and octave are inhabited by the pair of most frequent vowels, “i” language frequency 92%) and “u” (88%), the pure perfect fifth and fourth are given to “e” (61%) and “o” (60%), and the pure major third and minor sixth are given to a (86%) and “ɒ” (2%). This is in line with a further, though secondary, consideration, which focuses explicitly on the English and German alphabet. While this is not a universalist factor, and thus lower in priority, the first applications of the system will probably be in these languages, and “e” being the most often used vowel is helpful in that regard. However, it also creates some problems. In particular, the higher language frequency of “a” compared to “e” is not in line with the distribution of degrees of the major scale, and while other thirds are quite frequent among lower-limit intervals, fifths are fairly rare. To avoid a large number of languages that have no fifths (or fourths) at all, the septimal super-fifth 32/21 and septimal sub-fourth 21/16 were given the pair of frequent consonants “m” and “n”, and the vicesimotertial grave fifth 34/23 and vicesimotertial acute fourth 23/17 were given “r” and “ɾ”, both pairs of intervals that can stand on their own, but also serve as replacement fifths in languages that don't use "e" or "o". Some further changes like this were applied to the result of the general procedure, which consisted of:

- Going through the n-odd limits.
- In each limit, assign to each interval pair a phoneme pair such that
  - intervals of interval classes that lie in the diatonic scale are given phonemes with higher language frequency,
  - interval pairs both of whose intervals are with phonemes that are roughly matched in language frequency,
  - more important intervals are given vowels or phonemes with high language frequency,
  - in interval pairs where either both or none of the intervals are in the diatonic scale, usually the one closer to the tonic is preferred,
  - fifths are preferred to fourths.

4 Table

Below is the table of degree-phoneme correspondences, listed along Schulter's interval classification, where scale degrees are identified with intervals. The correspondences are given in tuples (x, y, z, w) where x is the phoneme, y is its language frequency, z is the corresponding interval as a fraction and w is its approximate cent value.

Phoneme-degree correspondence Unison	
0	(i, 92, 1, 0)
Minor Seconds	
Small (60-80)	(ɦ, 4, 24/23, 74)
Middle (80-100)	(ɖ, ?, 18/17, 99), (Voiced labiodental flap, 1, 20/19, 89), (ɕ, 4, 19/18, 94), (ø, 3, 22/21, 81), (ɠ, 0, 21/20, 84)
Large (100-120)	(ɭ, 12, 16/15, 112), (ʊ, 14, 15/14, 119), (œ, 0, 17/16, 105),
Neutral Seconds	
Small (125-135)	(ɜ, 0, 14/13, 128),
Middle (135-160)	(s, 67, 12/11, 151), (ʈ, 16, 13/12, 138) Large (160-170)
(μ, 2, 11/10, 165)	
Equable heptatonic	
160-182	(ɘ, 1, 21/19, 173)
Major Seconds	
Small (180-200)	(w, 82, 10/9, 182), (ɐ, 2, 19/17, 193)
Middle (200-220)	(ɻ, 10, 9/8, 204), (Voiced alveolo-palatal lateral approximant, 7, 17/15, 217)
Large (220-240)	(t, 68, 8/7, 231)

Interseptimal	
240-260	(ɪ, 15, 15/13, 248), (ɬ, 5, 22/19, 254),
Minor thirds	
Small (260-280)	(g, 57, 7/6, 267)
Middle (280-300)	(ʡ, 0, 13/11, 289), (q, 8, 20/17, 281), (θ, 4, 19/16, 298)
Large (300-330)	(ϵ, 37, 6/5, 316)
Neutral thirds	
Small (330-342)	(ɖ, 9, 17/14, 336), (ʀ, 1, 28/23, 341)
Middle (342-360)	(v, 27, 11/9, 347), (ɨ, 16, 16/13, 359)
Large (360-372)	(ħ, 3, 26/21, 370), (ɠ, 1, 34/21, 834)
Major thirds	
Small (372-400)	(a, 86, 5/4, 386)
Middle (400-423)	(ʂ, 7, 14/11, 417), (ʎ, 5, 24/19, 404), (ʌ, 4, 19/15, 409)
Large (423-440)	(p, 86, 9/7, 435), (ɱ, 2, 23/18, 424)
Interseptimal	
440-468	(ʒ, 16, 13/10, 454), (c, 14, 17/13, 464), (β, 10, 22/17, 446), (ʜ, 0, 30/23, 460)
Perfect fourths	
Small (468-491)	(n, 78, 21/16, 477)
Middle (491-505)	(o, 60, 4/3, 498)
Large (505-523)	(ɾ, 26, 23/17, 523)
Superfourths	
528-560	(ɲ, 42, 11/8, 551), (ɳ, 13, 15/11, 537), (y, 6, 19/14, 529), (ç, 5, 26/19, 543)
Tritones	
Small (560-577)	(ɣ, 14, 18/13, 563)
Middle (577-623)	(j, 90, 7/5, 583), (ʋ, 2, 10/7, 617), (ə, 22, , 600), (χ, 7, 17/12, 603), (ʁ, 5, 24/17, 597)
Large (623-640)	(ɾ, 26, 23/17, 523)
Superfourths	
640-672	(ŋ, 63, 16/11, 649), (ɭ, 13, 22/15, 663), (ɯ, 6, 28/19, 671), (ʝ, 2, 19/13, 657)
Perfect fifths	
Small (672-695)	(r, 44, 34/23, 677)
Middle (695-709)	(e, 61, 3/2, 702)
Large (709-732)	(m, 96, 32/21, 729)
Interseptimal	
732-760	(ʃ, 37, 20/13, 746), (ɟ, 12, 26/17, 736), (ɸ, 5, 17/11, 754), (ʢ, 0, 23/15, 740)
Minor sixths	
Small (760-777)	(b, 63, 14/9, 765), (ɴ, 0, 36/23, 776)
Middle (777-800)	(ʐ, 3, 11/7, 782), (ʟ, 0, 19/12, 796), (œ, 3, 30/19, 791)
Large (800-828)	(ɒ, 2, 8/5, 814)
Neutral sixths	
Small (828-840)	(ʕ, 2, 21/13, 830)
Middle (840-858)	(f, 44, 18/11, 853), (ʉ, 2, 13/8, 841)
Large (858-870)	(ɓ, 10, 28/17, 864), (ʙ, 0, 23/14, 859)
Major sixths	

Small (870-900) Middle (900-920) Large (920-940)	(ɔ, 35, 5/3, 884) (ʔ, 37, 22/13, 911), (ɢ, 2, 17/10, 919), (ð, 5, 32/19, 902) (k, 90, 12/7, 933)
Interseptimal	
732-760	(ʏ, 1, 26/15, 952), (, 2, 19/11, 946)
Minor sevenths	
Small (960-980) Middle (980-1000) Large (1000-1025)	(d, 46, 7/4, 969) (ɹ, 2, 16/9, 996), (ɰ, 2, 30/17, 983) (ʍ, 1, 9/5, 1018), (ä, 3, 34/19, 1007)
Equable heptatonic	
732-760	(ɵ, 1, 38/21, 1027)
Neutral sevenths	
Small (1030-1043) Middle (1043-1065) Large (1065-1075)	(ᴊ, 1, 20/11, 1035) (z, 30, 11/6, 1049) (ɛ, 37, 13/7, 1072)
Major sevenths	
Small (1030-1043) Middle (1043-1065) Large (1065-1075)	(l, 68, 15/8, 1088), (æ, 7, 28/15, 1081), (ɑ, 7, 32/17, 1095) (ƫ, 10, 17/9, 1101), (ɽ, 6, 19/10, 1111), (ʑ, 2, 36/19, 1106), (ɤ, 3, 21/11, 1119), (ʄ, 2, 40/21, 1116) (h, 56, 23/12, 1126)
Octave	
1200	(u, 88, 2, 1200)

References

[1] URL: `https://phoible.org`.

[2] URL: `https://en.xen.wiki`.

[3] Leon Crickmore. New light on the babylonian tonal system, 2008. URL: `https://musicircle.net/wp-content/uploads/2018/08/Crickmore-Iconea20081.pdf`.

[4] Jon Prince and Mark Schmuckler. The tonal-metric hierarchy: A corpus analysis. *Music Perception*, 31:254–270, 02 2014. `doi:10.1525/MP.2014.31.3.254`.

[5] Margo Schulter. Regions of the interval spectrum. URL: `https://www.bestii.com/~mschulter/IntervalSpectrumRegions.txt`.