MTH 430: Numerical Analysis & Scientific Computing II
Department of Mathematics and Statistics
Indian Institute of Technology - Kanpur — January 6, 2025

LAB 1

I. Convert the problem

$$y''' + 4y'' + 5y' + 2y = -4\sin t - 2\cos t$$
$$y(0) = 1, \quad y'(0) = 0, \quad y''(0) = -1$$

to a system of first-order equations. Using Euler's method, solve this system to obtain an approximation $\hat{y}_N$ for $y(1)$ using uniform time steps $t_n = n/N, n = 0, 1, \ldots, N$ for $N = 2^j, j = 0, 1, \ldots, 10$, and report the errors $e_N = |\hat{y}_N - y(1)|$, where the exact solution is $y(t) = \cos t$.

The first line of your Matlab implementation file `lab1_exercise1.m` should read

```
function [E] = lab1_exercise1(J)
```

where the input `J` is used to define the largest $N = 2^J$ (for example, you are asked to do the computation for $J = 10$) and the output `E` is a vector of size $J + 1$ whose j-th entry $E(j)$ contains error $e_{2^{j-1}}$. What you observe about $E(j)/E(j+1)$ as j increases?

II. The populations of two species, a pray denoted by y_1 and predator denoted by y_2 can be modeled by the non-linear ODE

$$y' = \begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} y_1(\alpha_1 - \beta_1 y_2) \\ y_2(-\alpha_2 + \beta_2 y_1) \end{bmatrix} = f(y).$$

The parameters α_1 and α_2 are natural birth and death rates in isolation of prey and predators, respectively, and the parameters β_1 and β_2 determine the effect of interactions between the two populations, where the probability of interaction in proportional to product of the populations.

Implement a Matlab function that uses the Euler's method with uniform time steps $t_n = nT/N, n = 0, \ldots, N$, to solve the IVP between $t = 0$ and $t = T$. The first line of your Matlab implementation file `lab1_exercise2.m` should read

```
function [ ] = lab1_exercise2(N, T, a1, a2, b1, b2, y10, y20)
```

where the input `N` specifies the grid size N, the input `T` specifies the final time T, the inputs, `a1, a2, b1, b2` correspond to ODE parameters $\alpha_1, \alpha_2, \beta_1, \beta_2$ respectively and the inputs `y10, y20` specify the initial conditions $y_1(0), y_2(0)$ respectively. Your function should plot each of the two populations as a function of time (on the same plot) and, on a separate graph, plots the trajectory of the points $(y_1(t), y_2(t))$ in the plane as a function of time.

Use $N = 1000$, $T = 25$, the parameter values $\alpha_1 = 1$, $\beta_1 = 0.1$, $\alpha_2 = 0.5$, $\beta_2 = 0.02$, and the initial population $y_1(0) = 100$ and $y_2(0) = 10$ to display the plots and comment on your observations.