Spin waves notes

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Chapter 1

Minimization $E(\theta_{ij}, \phi_{ij})$

Spin:

$$\vec{S} = \begin{Bmatrix} \cos\theta_{ij}\cos\phi_{ij} \\ \cos\theta_{ij}\sin\phi_{ij} \\ \sin\phi_{ij} \end{Bmatrix};$$

$-J1/2\ \vec{S}_{ij} \cdot \vec{S}_{i+1j} = \sin\theta_{ij}\sin\theta_{i+1j} + \cos\theta_{ij}\cos\theta_{i+1j}\cos(\phi_{ij} - \phi_{i+1j});$

Expression for the energy has the next form, where $\sum_n^N$ sum over all elements of matrices:

$$E = \sum_n^N \left[-J1/2 \sum_{ij} \vec{S}_{i,j}\vec{S}_{i\pm1,j} + \vec{S}_{i,j}\vec{S}_{i,j\pm1} + J2/2 \sum_{ij} \vec{S}_{i,j}\vec{S}_{i\pm1,j\pm1} + J4/2 \sum_{ij} \vec{S}_{i,j}\vec{S}_{i\pm2,j} + \vec{S}_{i,j}\vec{S}_{i,j\pm2} \right] \tag{1.1}$$

First element of the Hessian:

$$Hes(\vec{S}_{ij} \cdot \vec{S}_{i\pm1j}) = J1/2 \cdot \begin{array}{c c} & \begin{array}{cccc} \boldsymbol{\theta_{ij}} & \boldsymbol{\phi_{ij}} & \boldsymbol{\theta_{i\pm1j}} & \boldsymbol{\phi_{i\pm1j}} \end{array} \\ \begin{array}{c} \boldsymbol{\theta_{ij}} \\ \boldsymbol{\phi_{ij}} \\ \boldsymbol{\theta_{i\pm1j}} \\ \boldsymbol{\phi_{i\pm1j}} \end{array} & \begin{pmatrix} \frac{\partial^2 E}{\partial\theta_{ij}^2} & \frac{\partial^2 E}{\partial\theta_{ij}\partial\phi_{ij}} & \frac{\partial^2 E}{\partial\theta_{ij}\partial\theta_{i\pm1j}} & \frac{\partial^2 E}{\partial\theta_{ij}\partial\phi_{i\pm1j}} \\ \frac{\partial^2 E}{\partial\phi_{ij}\partial\theta_{ij}} & \frac{\partial^2 E}{\partial\phi_{ij}^2} & \frac{\partial^2 E}{\partial\phi_{ij}\partial\theta_{i\pm1j}} & \frac{\partial^2 E}{\partial\phi_{ij}\partial\phi_{i\pm1j}} \\ \frac{\partial^2 E}{\partial\theta_{i\pm1j}\partial\theta_{ij}} & \frac{\partial^2 E}{\partial\theta_{i\pm1j}\partial\phi_{ij}} & \frac{\partial^2 E}{\partial\theta_{i\pm1j}^2} & \frac{\partial^2 E}{\partial\theta_{i\pm1j}\partial\phi_{i\pm1j}} \\ \frac{\partial^2 E}{\partial\phi_{i\pm1j}\partial\theta_{ij}} & \frac{\partial^2 E}{\partial\phi_{i\pm1j}\partial\phi_{ij}} & \frac{\partial^2 E}{\partial\phi_{i\pm1j}\partial\theta_{i\pm1j}} & \frac{\partial^2 E}{\partial\phi_{i\pm1j}^2} \end{pmatrix} \end{array} \tag{1.2}$$

$\frac{\partial^2 E}{\partial\theta_{ij}^2} = 0,\ \frac{\partial^2 E}{\partial\phi_{ij}^2} = 0$

$\frac{\partial^2 E}{\partial\theta_{ij}\partial\phi_{ij}} = -J1/2[\sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\phi_{ij} - \phi_{i\pm1j})],$

$\frac{\partial^2 E}{\partial\theta_{ij}\partial\theta_{i\pm1j}} = -J1/2[\cos\theta_{ij}\cos\theta_{i\pm1j} \pm \sin\theta_{ij}\sin\theta_{i\pm1j}\cos(\phi_{ij} - \phi_{i\pm1j})],$

$\frac{\partial^2 E}{\partial\phi_{ij}\partial\phi_{i\pm1j}} = -J1/2[\cos\theta_{ij}\cos\theta_{i\pm1j}\cos(\phi_{ij} - \phi_{i\pm1j})],$

$\frac{\partial^2 E}{\partial\theta_{ij}\partial\phi_{i\pm1j}} = -J1/2[-\sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\phi_{ij} - \phi_{i\pm1j})],$

$\frac{\partial^2 E}{\partial\theta_{i\pm1j}\partial\phi_{ij}} = -J1/2[\sin\theta_{i\pm1j}\cos\theta_{ij}\sin(\phi_{ij} - \phi_{i\pm1j})],$

$\frac{\partial^2 E}{\partial\theta_{i\pm1j}\partial\phi_{i\pm1j}} = -J1/2[-\cos\theta_{ij}\sin\theta_{i\pm1j}\sin(\phi_{ij} - \phi_{i\pm1j})].$

Then, matrix (1.2) has following form:

$$\begin{pmatrix} 0 & \sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\delta\phi_{ij}) & \sin\theta_{ij}\cos\theta_{i\pm1j} \pm \sin\theta_{ij}\sin\theta_{i\pm1j}\cos(\delta\phi_{ij}) & -\sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\delta\phi_{ij}) \\ \sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\delta\phi_{ij}) & 0 & \sin\theta_{i\pm1j}\cos\theta_{ij}\sin(\delta\phi_{ij}) & \cos\theta_{ij}\cos\theta_{i\pm1j}\cos(\delta\phi_{ij}) \\ \cos\theta_{ij}\cos\theta_{i\pm1j} \pm \sin\theta_{ij}\sin\theta_{i\pm1j}\cos(\delta\phi_{ij}) & \sin\theta_{i\pm1j}\cos\theta_{ij}\sin(\delta\phi_{ij}) & 0 & -\cos\theta_{ij}\sin\theta_{i\pm1j}\sin(\delta\phi_{ij}) \\ -\sin\theta_{ij}\cos\theta_{i\pm1j}\sin(\delta\phi_{ij}) & \cos\theta_{ij}\cos\theta_{i\pm1j}\cos(\delta\phi_{ij}) & -\cos\theta_{ij}\sin\theta_{i\pm1j}\sin(\delta\phi_{ij}) & 0 \end{pmatrix}$$