CS 267: Automated Verification

Lectures 5: Symbolic model checking

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Symbolic Model Checking

[McMillan et al. LICS 90]

- Basic idea: ***Represent sets of states and the transition relation as Boolean logic formulas***

- Fixpoint computation becomes formula manipulation
  - pre-condition (EX) computation: Existential variable elimination
  - conjunction (intersection), disjunction (union) and negation (set difference), and equivalence check

- Use an efficient data structure for boolean logic formulas
  - Binary Decision Diagrams (BDDs)

Example Mutual Exclusion Protocol

Two concurrently executing processes are trying to enter a critical section without violating mutual exclusion

```
Process 1:
while (true) {
   out:  a := true; turn := true;
   wait: await (b = false or turn = false);
   cs:   a := false;
}
||
Process 2:
while (true) {
   out:  b := true; turn := false;
   wait: await (a = false or turn);
   cs:   b := false;
}
```

State Space

- Encode the state space using only boolean variables

- Two program counters: `pc1, pc2` with domains `{out, wait, cs}`
  - Use two boolean variable per program counter: $pc1_0, pc1_1, pc2_0, pc2_1$
  - Encoding:

$\neg pc1_0 \wedge \neg pc1_1 \equiv$ `pc1 = out`

$\neg pc1_0 \wedge pc1_1 \equiv$ `pc1 = wait`

$pc1_0 \wedge pc1_1 \equiv$ `pc1 = cs`

- The other three variables are booleans: turn, a , b

State Space

- Each state can be written as a 7-tuple of boolean values: $(pc1_0,pc1_1,pc2_0,pc2_1,turn,a,b)$
  - For example:

    `(o,o,F,F,F)` becomes (F,F,F,F,F,F,F)

    `(o,c,F,T,F)` becomes (F,F,T,T,F,T,F)

- There are $2^7=128$ possible values for our new representation
  - Our original state space was 3*3*2*2*2=72
  - Note that the following tuples are not in our state space due to our encoding of the program counters:

    (T,F,*,*,*,*,*), (*,*,T,F,*,*,*)

    I used "*" to mean any value

Representing sets of states

- We can use boolean logic formulas on the variables $pc1_0, pc1_1, pc2_0, pc2_1$, turn, a, b to represent sets of states:

$\{(F,F,F,F,F,F,F)\} \equiv \neg pc1_0 \wedge \neg pc1_1 \wedge \neg pc2_0 \wedge \neg pc2_1 \wedge \neg turn \wedge \neg a \wedge \neg b$

$\{(F,F,T,T,F,F,T)\} \equiv \neg pc1_0 \wedge \neg pc1_1 \wedge pc2_0 \wedge pc2_1 \wedge \neg turn \wedge \neg a \wedge b$

$\{(F,F,F,F,F,F,F), (F,F,T,T,F,F,T)\} \equiv \neg pc1_0 \wedge \neg pc1_1 \wedge \neg pc2_0 \wedge \neg pc2_1 \wedge \neg turn \wedge \neg a \wedge \neg b \vee \neg pc1_0 \wedge \neg pc1_1 \wedge pc2_0 \wedge pc2_1 \wedge \neg turn \wedge \neg a \wedge b$

$\equiv \neg pc1_0 \wedge \neg pc1_1 \wedge \neg turn \wedge \neg b \wedge (pc2_0 \wedge pc2_1 \leftrightarrow b)$

Initial States

- We can write the initial states as a boolean logic formula
  - recall that, initially: `pc1=o and pc2=o`

I ≡ `{(o,o,F,F,F), (o,o,F,F,T), (o,o,F,T,F), (o,o,F,T,T), (o,o,T,F,F), (o,o,T,F,T), (o,o,T,T,F), (o,o,T,T,T)}`

$\equiv \neg pc1_0 \wedge \neg pc1_1 \wedge \neg pc2_0 \wedge \neg pc2_1$

Transition Relation

- We can use boolean logic formulas to encode the transition relation. We will use two sets of variables:
  - Current state variables: $pc1_0, pc1_1, pc2_0, pc2_1, turn, a, b$
  - Next state variables: $pc1_0', pc1_1', pc2_0', pc2_1', turn', a', b$
- For example, we can write a boolean logic formula for the statement:

```
cs:   a := false;
```

as follows:

$$pc1_0 \wedge pc1_1 \wedge \neg pc1_0' \wedge \neg pc1_1' \wedge \neg a' \wedge$$
$$(pc2_0' \leftrightarrow pc2_0) \wedge (pc2_1' \leftrightarrow pc2_1) \wedge (turn' \leftrightarrow turn) \wedge (b' \leftrightarrow b)$$

  - Call this formula R_{1c}

Transition Relation

- We can write a formula for each statement in the program
- Then the overall transition relation is

$$R \equiv R_{1o} \vee R_{1w} \vee R_{1c} \vee R_{2o} \vee R_{2w} \vee R_{2c}$$

```
        Process 1:
        while (true) {
R1o <->   out:  a := true; turn := true;
R1w <->   wait: await (b = false or turn = false);
R1c <->   cs:   a := false;
        }
        ||
        Process 2:
        while (true) {
R2o <->   out:  b := true; turn := false;
R2w <->   wait: await (a = false or turn);
R2c <->   cs:   b := false;
        }
```

Symbolic Pre-condition Computation

- Remember the function

  $EX : 2^S \rightarrow 2^S$

  which is defined as:

  $EX(p) = \{ s \mid (s,s') \in R \text{ and } s' \in p \}$
- We can symbolically compute pre as follows

  $EX(p) \equiv \exists V' \; R \wedge p[V' / V]$
  - V : current-state boolean variables
  - V' : next-state boolean variables
  - $p[V' / V]$: rename variables in p by replacing current-state variables with the corresponding next-state variables
  - $\exists V' \; f$: existentially quantify out all the variables in V' from f

Renaming

- Assume that we have two variables x, y.
- Then, $V = \{x, y\}$ and $V' = \{x', y'\}$

- Renaming example:

  Given $p \equiv x \wedge y$:

  $p[V' / V] \equiv x \wedge y\, [V' / V] \equiv x' \wedge y'$

Existential Quantifier Elimination

- Given a boolean formula f and a single variable v

  $\exists v \ f \equiv f[True/v] \vee f[False/v]$

  i.e., to existentially quantify out a variable, first set it to true then set it to false and then take the disjunction of the two results

- Example: $f \equiv \neg x \wedge y \wedge x' \wedge y'$

$\exists V' \ f \equiv \exists x' \ (\exists y' \ (\neg x \wedge y \wedge x' \wedge y'))$

$\equiv \exists x' \ ((\neg x \wedge y \wedge x' \wedge y')[T/y'] \vee (\neg x \wedge y \wedge x' \wedge y')[F/y'])$

$\equiv \exists x' \ (\neg x \wedge y \wedge x' \wedge T \vee \neg x \wedge y \wedge x' \wedge F)$

$\equiv \exists x' \ \neg x \wedge y \wedge x'$

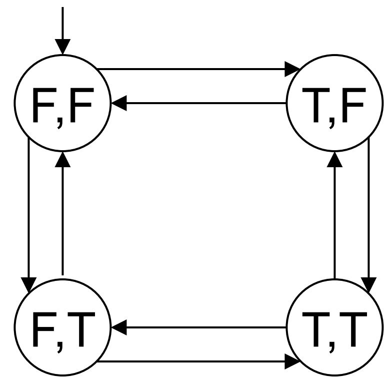
$\equiv (\neg x \wedge y \wedge x')[T/x'] \vee (\neg x \wedge y \wedge x')[F/x'])$

$\equiv \neg x \wedge y \wedge T \vee \neg x \wedge y \wedge F$

$\equiv \neg x \wedge y$

An Extremely Simple Example

Variables: x, y: boolean



Set of states:

$S = \{(F,F), (F,T), (T,F), (T,T)\}$

$S \equiv True$

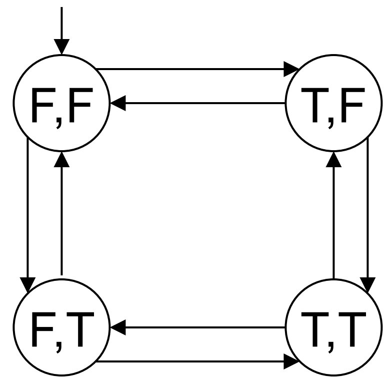
Initial condition:

$I \equiv \neg x \wedge \neg y$

Transition relation (negates one variable at a time):

$R \equiv x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y$ (= means $\leftrightarrow$)

An Extremely Simple Example



Given $p \equiv x \wedge y$, compute EX(p)

$$EX(p) \equiv \exists V' \; R \wedge p[V' / V]$$

$$\equiv \exists V' \; R \wedge x' \wedge y'$$

$$\equiv \exists V' \; (x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y) \wedge x' \wedge y'$$

$$\equiv \exists V' \; (x' = \neg x \wedge y' = y) \wedge x' \wedge y' \vee (x' = x \wedge y' = \neg y) \wedge x' \wedge y'$$

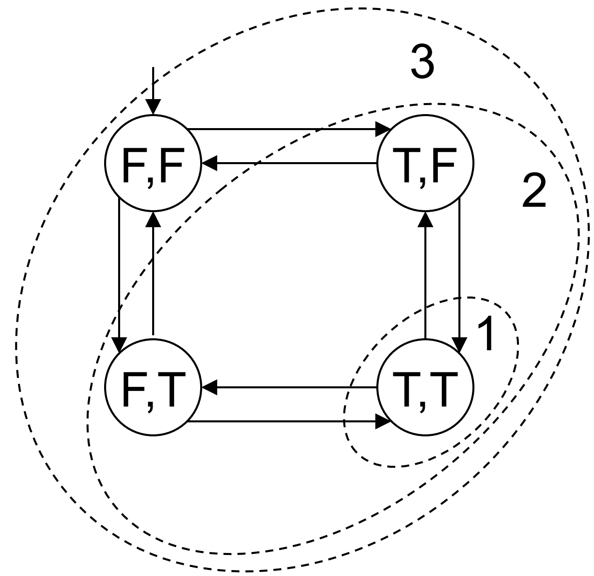
$$\equiv \exists V' \; \neg x \wedge y \wedge x' \wedge y' \vee x \wedge \neg y \wedge x' \wedge y'$$

$$\equiv \neg x \wedge y \vee x \wedge \neg y$$

$EX(x \wedge y) \equiv \neg x \wedge y \vee x \wedge \neg y$

In other words $EX(\{(T,T)\}) \equiv \{(F,T), (T,F)\}$

An Extremely Simple Example



Let's compute compute EF($x \wedge y$)

The fixpoint sequence is

False, $x \wedge y$, $x \wedge y \vee EX(x \wedge y)$, $x \wedge y \vee EX\,(x \wedge y \vee EX(x \wedge y))$, ...

If we do the EX computations, we get:

$$\underbrace{\text{False},}_{0} \quad \underbrace{x \wedge y}_{1}, \quad \underbrace{x \wedge y \vee \neg x \wedge y \vee x \wedge \neg y}_{2}, \quad \underbrace{\text{True}}_{3}$$

EF($x \wedge y$) $\equiv$ True

In other words EF({(T,T)}) $\equiv$ {(F,F),(F,T), (T,F),(T,T)}

An Extremely Simple Example

- Based on our results, for our extremely simple transition system T=(S,I,R) we have

$I \subseteq EF(x \wedge y)$ hence:

$T \models EF(x \wedge y)$

(i.e., there exists a path from each initial state where eventually x and y both become true at the same time)

$I \not\subseteq EX(x \wedge y)$ hence:

$T \not\models EX(x \wedge y)$

(i.e., there does not exist a path from each initial state where in the next state x and y both become true)

An Extremely Simple Example

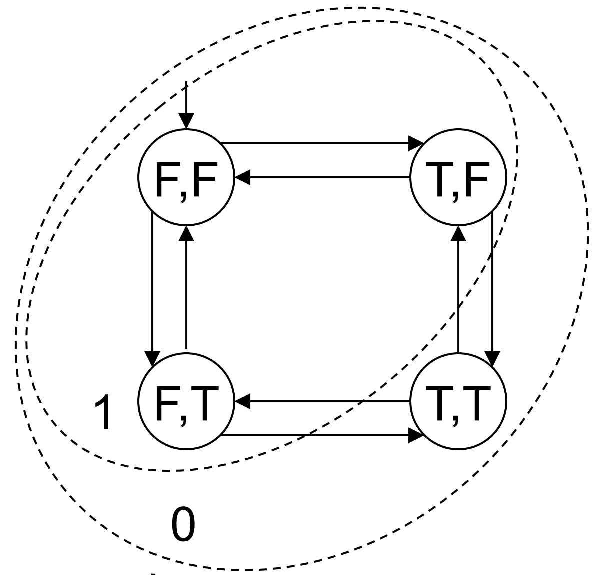
- Let's try one more property AF(x ∧ y)

- To check this property we first convert it to a formula which uses only the temporal operators in our basis:

  $AF(x \wedge y) \equiv \neg EG(\neg(x \wedge y))$

If we can find an initial state which satisfies $EG(\neg(x \wedge y))$, then we know that the transition system T, does not satisfy the property $AF(x \wedge y)$

An Extremely Simple Example



Let's compute compute EG(¬(x ∧ y))

The fixpoint sequence is

True, ¬x ∨ ¬y, (¬x ∨ ¬y) ∧ EX(¬x ∨ ¬y) , …

If we do the EX computations, we get:

$\underbrace{\text{True,}}_{0}\quad \underbrace{\neg x \lor \neg y,}_{1}\quad \underbrace{\neg x \lor \neg y,}_{2}$

EG(¬(x ∧ y)) ≡ ¬x ∨ ¬y

Since I ∩ EG(¬(x ∧ y)) ≠ ∅ we conclude that T $\not\models$ AF(x ∧ y)

Symbolic CTL Model Checking Algorithm

- Translate the formula to a formula which uses the basis
  - EX p, EG p, p EU q
- Atomic formulas can be interpreted directly on the state representation
- For EX p compute the precondition using existential variable elimination as we discussed
- For EG and EU compute the fixpoints iteratively

Symbolic Model Checking Algorithm

Check(f : CTL formula) : boolean logic formula

case: $f \in AP$	return f;
case: $f \equiv \neg p$	return $\neg$Check(p);
case: $f \equiv p \wedge q$	return Check(p) $\wedge$ Check(q);
case: $f \equiv p \vee q$	return Check(p) $\vee$ Check(q);
case: $f \equiv EX\ p$	return $\exists V'\ R \wedge$ Check(p) $[V' / V]$;

Symbolic Model Checking Algorithm

```
Check(f)
    …
    case: f ≡ EG p
        Y := True;
        P := Check(p);
        Ynew := P ∧ Check(EX(Y));
        while (Y ≠ Ynew) {
            Y := Ynew;
            Ynew := P ∧ Check(EX(Y));
        }
        return Y;
```

Symbolic Model Checking Algorithm

```
Check(f)
    …
    case: f ≡ p EU q
        Y := False;
        P := Check(p);
        Q := Check(q);
        Ynew := Q ∨ P ∧ Check(EX(Y));
        while (Y ≠ Ynew) {
                Y := Ynew;
                Ynew := Q ∨ P ∧ Check(EX(Y));
        }
        return Y;
```