CS 267: Automated Verification

Lecture 7: SMV Symbolic Model Checker, Partitioned Transition Systems, Counter-example Generation in Symbolic Model Checking

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SMV [McMillan 93]

- BDD-based symbolic model checker
- Finite state
- Temporal logic: CTL
- Focus: hardware verification
  - Later applied to software specifications, protocols, etc.
- SMV has its own input specification language
  - concurrency: synchronous, asynchronous
  - shared variables
  - boolean and enumerated variables
  - bounded integer variables (binary encoding)
    - SMV is not efficient for integers, but that can be fixed
  - fixed size arrays

SMV Language

- An SMV specification consists of a set of modules (one of them must be called `main`)
- Modules can have access to shared variables
- Modules can be composed asynchronously using the `process` keyword
- Module behaviors can be specified using the `ASSIGN` statement which assigns values to next values of variables in parallel
- Module behaviors can also be specified using the `TRANS` statements which allow specification of the transition relation as a logic formula where next state values are identified using the `next` keyword

Example Mutual Exclusion Protocol

Two concurrently executing processes are trying to enter a critical section without violating mutual exclusion

```
Process 1:
while (true) {
   out:  a := true; turn := true;
   wait: await (b = false or turn = false);
   cs:   a := false;
}
||
Process 2:
while (true) {
   out:  b := true; turn := false;
   wait: await (a = false or turn);
   cs:   b := false;
}
```

Example Mutual Exclusion Protocol in SMV

```
MODULE process1(a,b,turn)
VAR
  pc: {out, wait, cs};
ASSIGN
  init(pc) := out;
  next(pc) :=
    case
      pc=out : wait;
      pc=wait & (!b | !turn) : cs;
      pc=cs : out;
      1 : pc;
    esac;
  next(turn) :=
    case
      pc=out : 1;
      1 : turn;
    esac;
  next(a) :=
    case
      pc=out : 1;
      pc=cs : 0;
      1 : a;
    esac;
  next(b) := b;
FAIRNESS
  running
```

```
MODULE process2(a,b,turn)
VAR
  pc: {out, wait, cs};
ASSIGN
  init(pc) := out;
  next(pc) :=
    case
      pc=out : wait;
      pc=wait & (!a | turn) : cs;
      pc=cs : out;
      1 : pc;
    esac;
  next(turn) :=
    case
      pc=out : 0;
      1 : turn;
    esac;
  next(b) :=
    case
      pc=out : 1;
      pc=cs : 0;
      1 : b;
    esac;
  next(a) := a;
FAIRNESS
  running
```

Example Mutual Exclusion Protocol in SMV

```
MODULE main
VAR
  a : boolean;
  b : boolean;
  turn : boolean;
  p1 : process process1(a,b,turn);
  p2 : process process2(a,b,turn);
SPEC
  AG(!(p1.pc=cs & p2.pc=cs))
  -- AG(p1.pc=wait -> AF(p1.pc=cs)) & AG(p2.pc=wait -> AF(p2.pc=cs))
```

Here is the output when I run SMV on this example to check the mutual exclusion property

```
% smv mutex.smv
-- specification AG (!(p1.pc = cs & p2.pc = cs)) is true

resources used:
user time: 0.01 s, system time: 0 s
BDD nodes allocated: 692
Bytes allocated: 1245184
BDD nodes representing transition relation: 143 + 6
```

Example Mutual Exclusion Protocol in SMV

The output for the starvation freedom property:

```
% smv mutex.smv
-- specification AG (p1.pc = wait -> AF p1.pc = cs) & AG ... is true

resources used:
user time: 0 s, system time: 0 s
BDD nodes allocated: 1251
Bytes allocated: 1245184
BDD nodes representing transition relation: 143 + 6
```

Example Mutual Exclusion Protocol in SMV

Let’s insert an error

change `pc=wait & (!b | !turn) : cs;`

to `pc=wait & (!b | turn) : cs;`

```
% smv mutex.smv
-- specification AG (!(p1.pc = cs & p2.pc = cs)) is false
-- as demonstrated by the following execution sequence
state 1.1:
a = 0
b = 0
turn = 0
p1.pc = out
p2.pc = out
[stuttering]

state 1.2:
[executing process p2]

state 1.3:
b = 1
p2.pc = wait
[executing process p2]

state 1.4:
p2.pc = cs
[executing process p1]

state 1.5:
a = 1
turn = 1
p1.pc = wait
[executing process p1]

state 1.6:
p1.pc = cs
[stuttering]
```

```
resources used:
user time: 0.01 s, system time: 0 s
BDD nodes allocated: 1878
Bytes allocated: 1245184
BDD nodes representing transition relation: 143 + 6
```

Symbolic Model Checking with BDDs

- As we discussed earlier BDDs are used as a data structure for encoding trust sets of Boolean logic formulas in symbolic model checking

- One can use BDD-based symbolic model checking for any finite state system using a Boolean encoding of the state space and the transition relation

- Why are we using symbolic model checking?
  - We hope that the symbolic representations will be more compact than the explicit state representation on the average
  - In the worst case we may not gain anything

Symbolic Model Checking with BDDs

- Possible problems
  - The BDD for the transition relation could be huge
    - Remember that the BDD could be exponential in the number of disjuncts and conjuncts
    - Since we are using a Boolean encoding there could be a large number of conjuncts and disjuncts
  - The EX computation could result in exponential blow-up
    - Exponential in the number of existentially quantified variables

Partitioned Transition Systems

- If the BDD for the transition relation R is too big, we can try to partition it and represent it with multiple BDDs

- We need to be able to do the EX computation on this partitioned transition system

Disjunctive Partitioning

- Disjunctive partitioning:

  $R \equiv R_1 \vee R_2 \vee \ \ldots \vee R_k$

  We can distribute the EX computation since ***existential quantification distributes over disjunction***

  We compute the EX for each R_i separately and then take the disjunction of all the results

Disjunctive Partitioning

- Remember EX, let's assume that EX also takes the transition relation as input:

  EX(p, R) = { s | (s,s') $\in$ R and s' $\in$ p }

  which in symbolic model checking becomes:

  $EX(p, R) \equiv \exists V' \; R \wedge p[V' / V]$

If we can write R as $R \equiv R_1 \vee R_2 \vee \ldots \vee R_k$ then

$EX(p, R) \equiv \exists V' \; R \wedge p[V' / V]$

$\equiv \exists V' \; (R_1 \vee R_2 \vee \ldots \vee R_k) \wedge p[V' / V]$

$\equiv \exists V' \; (R_1 \wedge p[V' / V] \vee R_2 \wedge p[V' / V] \vee \ldots \vee R_k \wedge p[V' / V])$

$\equiv (\exists V' \; R_1 \wedge p[V' / V]) \vee (\exists V' \; R_2 \wedge p[V' / V]) \vee \ldots \vee (\exists V' \; R_k \wedge p[V' / V])$

$\equiv EX(p, R_1) \vee EX(p, R_2) \vee \ldots \vee EX(p, R_k)$

Disjunctive Partitioning

The purpose of disjunctive partitioning is the following:

- If we can write R as

  $R \equiv R_1 \vee R_2 \vee \ \dots \vee R_k$

  then we can use $R_1 \dots R_k$ instead of R during the EX computation and we never have to construct the BDD for R

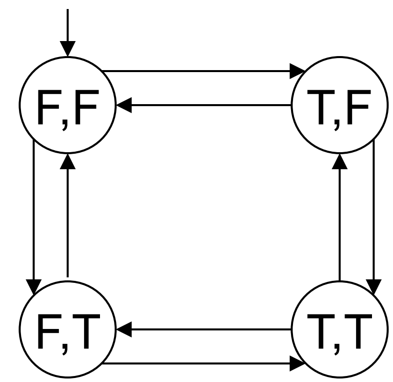
- We can use R_is to compute the EX(p, R) as

  $EX(p, R) \equiv EX(p, R_1) \vee EX(p, R_2) \vee \ \dots \vee EX(p, R_k)$

- If R is much bigger than all the R_is, then disjunctive partitioning can improve the model checking performance

Recall this Extremely Simple Example

Variables: x, y: boolean
Set of states:
$S = \{(F,F), (F,T), (T,F), (T,T)\}$
$S \equiv True$

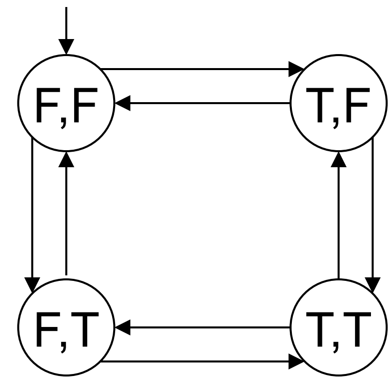


Initial condition:
$I \equiv \neg\, x \wedge \neg\, y$

Transition relation (negates one variable at a time):
$R \equiv x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y$ (= means $\leftrightarrow$)

A possible disjunctive partitioning:
$R \equiv R_1 \vee R_2$
$R_1 \equiv x' = \neg x \wedge y' = y \quad R_2 \equiv x' = x \wedge y' = \neg y$

An Extremely Simple Example



Given $p \equiv x \wedge y$, compute EX(p)

$EX(p, R) \equiv \exists V' \; R \wedge p[V' / V]$
$\equiv EX(p, R_1) \vee EX(p, R_2)$

$EX(p, R_1) \equiv (\exists V' \; R_1 \wedge x' \wedge y') \equiv (\exists V' \; x' = \neg x \wedge y' = y \wedge x' \wedge y')$
$\equiv (\exists V' \; \neg x \wedge y \wedge x' \wedge y') \equiv \neg x \wedge y$

$EX(p, R_2) \equiv (\exists V' \; R_2 \wedge x' \wedge y') \equiv (\exists V' \; x' = x \wedge y' = \neg y \wedge x' \wedge y')$
$\equiv (\exists V' \; x \wedge \neg y \wedge x' \wedge y') \equiv x \wedge \neg y$

$EX(x \wedge y) \equiv EX(p, R_1) \vee EX(p, R_2) \equiv \neg x \wedge y \vee x \wedge \neg y$
In other words $EX(\{(T,T)\}) \equiv \{(F,T), (T,F)\}$

Conjunctive Partitioning

- Conjunctive partitioning:

  $R \equiv R_1 \wedge R_2 \wedge \ \ldots \wedge R_k$

  Unfortunately EX computation does not distribute over the conjunction partitioning in general since ***existential quantification does NOT distribute over conjunction***

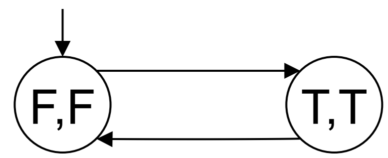
- However if each R_i is expressed on a separate set of next state variables (i.e., if a next state variable appears in R_i then it should not appear in any other conjunct)
  - Then we can distribute the existential quantification over each R_i

Conjunctive Partitioning

- If we can write R as $R \equiv R_1 \wedge R_2 \wedge \ \ldots \wedge R_k$
  where R_i is a formula only on variables V_i and V_i'
  and $i \neq j \Rightarrow V_i' \cap V_j' = \varnothing$
  which means that a primed variable does not appear in more than one R_i

- Then, we can do the existential quantification separately for each R_i as follows:
  $EX(p, R) \equiv \exists V' \ R \wedge p[V' / V]$
  $\equiv \exists V' \ p[V' / V] \wedge (R_1 \wedge R_2 \wedge \ \ldots \wedge R_k)$
  $\equiv (\exists V_k' \ \ldots (\exists V_2' \ (\exists V_1' \ p[V' / V] \wedge R_1) \ \wedge R_2) \wedge \ \ldots \wedge R_k)$

An Even Simpler Example



Variables: x, y: boolean

Set of states:

$S = \{(F,F), (F,T), (T,F), (T,T)\}$

$S \equiv True$

Initial condition:

$I \equiv \neg\, x \wedge \neg\, y$

Transition relation (negates one variable at a time):

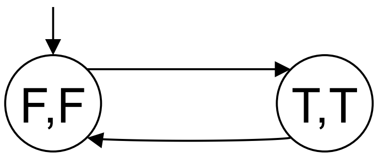
$R \equiv x' = \neg x \wedge y' = \neg y$ (= means $\leftrightarrow$)

A possible conjunctive partitioning:

$R \equiv R_1 \wedge R_2$

$R_1 \equiv x' = \neg x \quad R_2 \equiv y' = \neg y$

An Even Simpler Example



Given $p \equiv x \wedge y$, compute EX(p)

$EX(p, R) \equiv \exists V' \; R \wedge p[V' / V]$

$\equiv \exists V_2' \; (\exists V_1' \; p[V' / V] \wedge R_1) \wedge R_2$

$\equiv \exists V_2' \; (\exists V_1' \; x' \wedge y' \wedge R_1) \wedge R_2$

$\equiv \exists y' \; (\exists x' \; x' \wedge y' \wedge x' = \neg x) \wedge y' = \neg y$

$\equiv \exists y' \; (\exists x' \; x' \wedge y' \wedge \neg x) \wedge y' = \neg y$

$\equiv \exists y' \; y' \wedge \neg x \wedge y' = \neg y$

$\equiv \exists y' \; y' \wedge \neg x \wedge \neg y$

$\equiv \neg x \wedge \neg y$

$EX(x \wedge y) \equiv \neg x \wedge \neg y$

In other words EX({(T,T)}) ≡ {(F,F)}

Partitioned Transition Systems

- Using partitioned transition systems we can reduce the size of memory required for representing R and the size of the memory required to do model checking with R

- Note that, for either type of partitioning
  - disjunctive $R \equiv R_1 \vee R_2 \vee \ldots \vee R_k$
  - or conjunctive $R \equiv R_1 \wedge R_2 \wedge \ldots \wedge R_k$

  size of R can be exponential in k

- So by keeping R in partitioned form we can avoid constructing the BDD for R which can be exponentially bigger than each R_i

Other Improvements for BDDs

- Variable ordering is important
  - For example for representing linear arithmetic constraints such as x = y + z where x, y, and z are integer variables represented in binary,
    - If the variable ordering is: all the bits for x, all the bits for y and all the bits for z, then the size of the BDD is exponential in the number of bits
      - In fact this is the ordering used in SMV which makes SMV very inefficient for verification of specifications that contain arithmetic constraints
    - If the binary variables for x, y, and z are interleaved, the size of the BDD is linear in the number of bits
  - So, for specific classes of systems there may be good variable orderings

Other Improvements to BDDs

- There are also dynamic variable ordering heuristics which try to change the ordering of the BDD on the fly and reduce the size of the BDD

- There are also variants of BDDs such as multi-terminal decision diagrams, where the leaf nodes have more than two distinct values.
  - Useful for domains with more than two values
    - Can be translated to BDDs

Counter-Example Generation

- Remember: Given a transition system T= (S, I, R) and a CTL property p T |= p iff for all initial state s $\in$ I, s |= p

- Verification vs. Falsification
  - Verification:
    - Show: initial states $\subseteq$ truth set of *p*
  - Falsification:
    - Find: a state $\in$ initial states $\cap$ truth set of $\neg p$
    - Generate a counter-example starting from that state

- The ability to find counter-examples is one of the biggest strengths of the model checkers

An Example

- If we wish to check the property AG(p)

- We can use the equivalence:
  $AG(p) \equiv \neg EF(\neg p)$

If we can find an initial state which satisfies $EF(\neg p)$, then we know that the transition system T, does not satisfy the property AG(p)

Another Example

- If we wish to check the property AF(p)

- We can use the equivalence:
  AF(p) ≡ ¬ EG(¬p)

If we can find an initial state which satisfies EG(¬p), then we know that the transition system T, does not satisfy the property AF(p)

General Idea

- We can define two temporal logics using subsets of CTL operators
  - ACTL: CTL formulas which only use the temporal operators AX, AG, AF and AU and all the negations appear only in atomic properties (there are no negations outside of temporal operators)
  - ECTL: CTL formulas which only use the temporal operators EX, EG, EF and EU and all the negations appear only in atomic properties

- Given an ACTL property its negation is an ECTL property

Counter-Example Generation

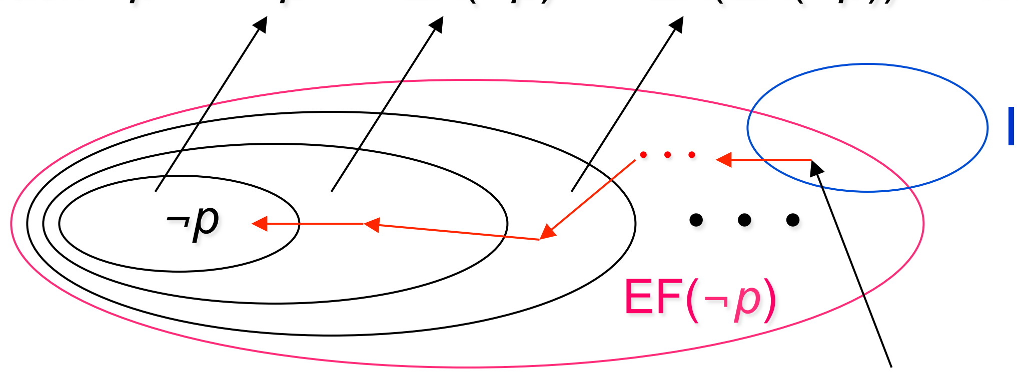
- Given an ACTL property p, we negate it and compute the set of states which satisfy it is negation ¬ p
  - ¬p is an ECTL property

- If we can find an initial state which satisfies ¬ p then we generate a counter-example path for p starting from that initial state
  - Such a path is called a *witness* for the ECTL property ¬ p

An Example

- We want to check the property AG(p)
- We compute the fixpoint for EF(¬p)
- We check if the intersection of the set of initial states I and the truth set of EF(¬p) is empty
  – If it is not empty we generate a counter-example path starting from the intersection

$EF(\neg p) \equiv$ states that can reach $\neg p \equiv \neg p \cup EX(\neg p) \cup EX(EX(\neg p)) \cup \ldots$



• In order to generate the counter-example path, save the fixpoint iterations.
• After the fixpoint computation converges, do a second pass to generate the counter-example path.

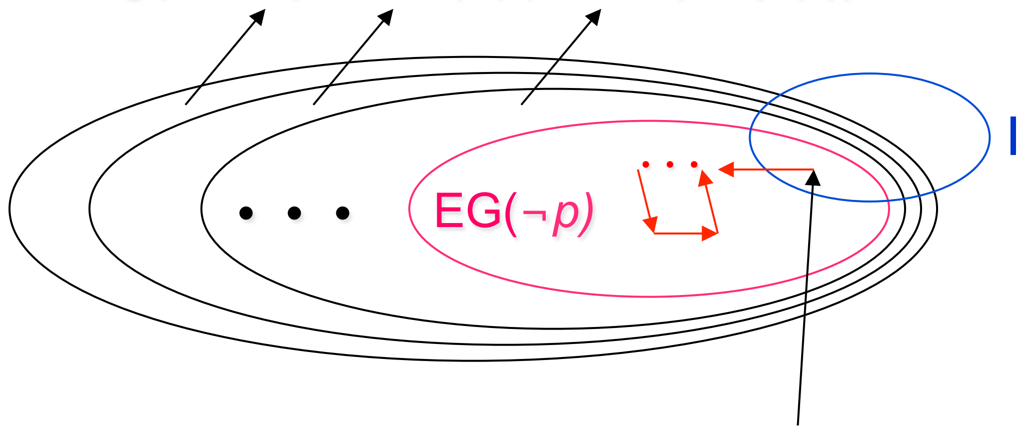
Generate a counter-example path starting from a state here

Another Example

- We want to check the property AF(p)
- We compute the fixpoint for EG(¬p)
- We check if the intersection of the set of initial states I and the truth set of EG(¬p) is empty
  – If it is not empty we generate a counter-example path starting from the intersection

EG(¬*p*) ≡ states that can avoid reaching *p* ≡ ¬*p* ∩ EX(¬*p*) ∩ EX(EX(¬*p*)) ∩ …

- In order to generate the counter-example path, look for a cycle in the resulting fixpoint



Generate a counter-example path starting from a state here

Counter-example generation

- In general the counter-example for an ACTL property (equivalently a witness to an ECTL property) is not a single path
- For example, the counter example for the property AF(AGp) would be a witness for the property EG(EF¬p)
  - It is not possible to characterize the witness for EG(EF¬p) as a single path
- However it is possible to generate tree-like transition graphs containing counter-example behaviors as a counter-example:
  - Edmund M. Clarke, Somesh Jha, Yuan Lu, Helmut Veith: "Tree-Like Counterexamples in Model Checking". LICS 2002: 19-29