Alexander Grothendieck

(1928–2014)

Mathematician who rebuilt algebraic geometry.

Alexander Grothendieck, who died on 13 November, was considered by many to be the greatest mathematician of the twentieth century. His unique skill was to burrow into an area so deeply that its inner patterns on the most abstract level revealed themselves, and solutions to old problems fell out in straightforward ways.

Grothendieck was born in Berlin in 1928 to a Russian Jewish father and a German Protestant mother. After being separated from his parents at the age of five, he was briefly reunited with them in France just before his father was interned and then transported to Auschwitz, where he died. Around 1942, Grothendieck arrived in the village of Le Chambon-sur-Lignon, a centre of resistance against the Nazis, where thousands of refugees were hidden. It was probably here, at the secondary school Collège Cévonol, that his fascination for mathematics began.

In 1945, Grothendieck enrolled at the University of Montpellier. He completed his doctoral thesis on topological vector spaces at the University of Nancy in 1953, and spent a short time teaching in Brazil. His most revolutionary work happened between 1954 and 1970, mainly at the Institute of Advanced Scientific Studies (IHÉS) in a suburb of Paris. His strength and dedication were legendary: throughout his 15 years in mainstream mathematics, he would work long hours in the unheated attic of his house seven days a week. He was awarded the Fields medal in 1966 for his work in algebraic geometry.

Algebraic geometry is the field that studies the solutions of sets of polynomial equations by looking at their geometric properties. For instance, a circle is the set of solutions of $x^2+y^2=1$, and in general such a set of points is called a variety. Traditionally, algebraic geometry was limited to polynomials with real or complex coefficients, but just before Grothendieck's work, André Weil and Oscar Zariski had realized that it could be connected to number theory if you allowed the polynomials to have coefficients in a finite field. These are a type of number that are added like the hours on a clock — 7 hours after 9 o'clock is not 16 o'clock, but 4 o'clock — and it creates a new discrete type of variety, one variant for each prime number p.

But the proper foundations of this enlarged view were unclear, and this is where, inspired by the ideas of the French mathematician Jean-Pierre Serre, but generalizing them

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enormously, Grothendieck made his first hugely significant innovation. He proposed that a geometric object called a scheme was associated to any commutative ring — that is, a set in which addition and multiplication are defined and multiplication is commutative, $a \times b = b \times a$. Before Grothendieck, mathematicians considered only the case in which the ring is the set of functions on the variety that are expressible as polynomials in the coordinates. In any geometry, local parts are glued together in some fashion to create global objects, and this worked for schemes too.

An example might help in illustrating how novel this idea was. A simple ring can be generated if we make a ring from expressions $a+b\varepsilon$, in which a and b are ordinary real numbers but ε is a variable with only 'very small' values, so small that we decide to set $\varepsilon^2=0$. The scheme corresponding to this ring consists of only one point, and that point is allowed to move the infinitesimal distance ε but no further. The possibility of manipulating infinitesimals was one great success of schemes. But Grothendieck's ideas also had important implications in number theory. The ring of all integers, for example, defines a scheme that connects finite fields to real numbers, a bridge between the discrete and classical worlds, having one point for each prime number and one for the classical world.

Probably his best-known work was his discovery of how all schemes have a topology. Topology had been thought to belong exclusively to real objects, such as spheres and other surfaces in space. But Grothendieck found not one but two ways to endow all schemes, even the discrete ones, with a topology, and especially with the fundamental invariant called cohomology. With a brilliant group of collaborators, he gained deep insight into theories of cohomology, and established them as some of the most important tools in modern mathematics. Owing to the many connections that schemes turned out to have to various mathematical disciplines, from algebraic geometry to number theory to topology, there can be no doubt that Grothendieck's work recast the foundations of large parts of twenty-first-century mathematics.

Grothendieck left the IHÉS in 1970 for reasons not entirely clear to anyone. He turned from maths to the problems of environmental protection, founding the activist group Survivre. With a breathtakingly naive spirit (that had served him well in mathematics), he believed that this movement could change the world. When he saw that it was not succeeding, in 1973, he returned to maths, teaching at the University of Montpellier. Despite writing thousand-page treatises on yet-deeper structures connecting algebra and geometry (still unpublished), his research was only meagrely funded by the CNRS, France's main basic-research agency.

Grothendieck could be very warm. Yet the nightmares of his childhood had made him a complex person. He remained on a Nansen passport his whole life — a document issued for stateless people and refugees who could not obtain travel documents from a national authority. For the last two decades of his life he broke off from the maths community, his wife, a later partner and even his children. He sought total solitude in the village of Lasserre in the foothills of the Pyrenees. Here he wrote remarkable self-analytical works on topics ranging from maths and philosophy to religion. ■

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