

# BiLSTM and CRF with Fine-Tuned BERT for Named Entity Recognition

## Explained

### Overview

Given a sequence  $X = \{x_1, \dots, x_n\}$ , and a label sequence  $y = \{\text{start}(y_0), y_1, \dots, y_n, \text{end}(y_{n+1})\}$ , where  $y_i \in \mathcal{Y}$ , and we denote the size of  $|\mathcal{Y}| = m$ .

The **score** of such label sequence is

$$\text{Score}(X, y) = \sum_{i=0}^n A[y_{i+1}][y_i] + \sum_{i=1}^n E[x_i][y_i],$$

where  $A \in \mathbb{R}^{m \times m}$  is the transition matrix, the  $i, j$  entry  $A_{i,j}$  is the unnormalized probability of transferring to label  $i$  from label  $j$ ,  $E \in \mathbb{R}^{n \times m}$  is the emission matrix, the  $i, j$  entry  $E_{i,j}$  is the unnormalized probability of  $i$ -th word being labeled with  $j$ . Both of the matrix are constructed by **trainable parameters**  $\lambda$ .

Afterwards, since we want a **probability distribution**  $p(y|X, \lambda)$ , the score should be normalized as:

$$p(y|X, \lambda) = \frac{\exp(\text{Score}(X, y))}{\sum_{\tilde{y} \in Y} \exp(\text{Score}(X, \tilde{y}))}$$

Then, denote the ground-truth label sequence of  $X$  is  $\hat{y}$ , we train the model to maximize  $p(\hat{y}|X, \lambda)$ , which is equivalent to **minimizing its negative log likelihood**:

$$\begin{aligned} \mathcal{L} &= -\log p(\hat{y}|X, \lambda) \\ &= \log \sum_{\tilde{y} \in Y} \exp(\text{Score}(X, \tilde{y})) - \text{Score}(X, \hat{y}) \end{aligned}$$

In terms of **inference**, given the unlabeled sequence  $X$ , we can simply select  $y$  with the biggest  $p(y|X, \lambda)$  to be its label sequence, formally:

$$y = \arg \max_{y \in Y} (p(y|X, \lambda))$$

## Function

- `_forward_arg`

Based on

$$\log\left(\sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} \exp(x + y)\right) = \log\left(\sum_{y \in \mathcal{Y}} \exp\left(\log\left(\sum_{x \in \mathcal{X}} \exp(x)\right) + y\right)\right),$$

we denote  $s(X, y, i) = \text{Score}(X[0 : i], y[0 : i])$ , the following equation can be derived:

$$\begin{aligned}
\log \sum_{y \in Y} \exp(\text{Score}(X, y)) &= \log \sum_{y \in Y} \exp(s(X, y, n)) \\
&= \log \left( \sum_{y_0 \in \mathcal{Y}} \sum_{y_1 \in \mathcal{Y}} \cdots \sum_{y_n \in \mathcal{Y}} \exp(s(X, y, n)) \right) \\
&= \log \left( \sum_{y_0 \in \mathcal{Y}} \sum_{y_1 \in \mathcal{Y}} \cdots \sum_{y_{n-1} \in \mathcal{Y}} \exp \left( \log \left( \sum_{y_n \in \mathcal{Y}} \exp(s(X, y, n-1)) \right) + \text{Score}(X[n], y[n]) \right) \right) \\
&= \log \left( \sum_{y[0:n-1] \in Y[0:n-1]} \exp \left( \log \left( \sum_{y_n \in \mathcal{Y}} \exp(s(X, y, n-1)) \right) + \text{Score}(X[n], y[n]) \right) \right)
\end{aligned}$$

- the equation reveals the **Optimal Substructure** of computing the score of every possible tag sequence
- therefore, we can use dynamic programming to calculate  $\sum_{\tilde{y} \in Y} \exp(\text{Score}(X, \tilde{y}))$  by iteratively computing the **log-exp-sum** of the prefix string.