Honor Code: I have completed this exam in the spirit of the Duke Community Standard and have neither given nor received aid.

Signature:

1. (20 pts.) a) Solve the equation $y' = (e^{-x} + e^{x})/(1 + y^2)$.

b) Show that if a is positive constant, and b is any real number, then every solution of the equation

$$y' + ay = be^{-at} \tag{1}$$

has the property that $y \to 0$ as $t \to \infty$.

2. (20 pts.) a) Determine whether the equation is exact. And if it is exact, solve it.

$$(ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x) + (xe^{xy}\cos 2x - 3)y' = 0.$$

b) Find the integration factor of the equation:

$$y' = e^{2x} + y - 1.$$

3. (20 pts.) a) Apply the reduction of order to determine the other homogeneous solution to the equation

$$t^2y'' - 4ty' + 6y = 0, t > 0; \quad y_1(t) = t^2.$$

b) Apply the method of undetermined coefficient to find the solution to the equation:

$$y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 1. \tag{2}$$

4. (20 pts.) a) Find the solution to the following inhomogeneous equation using the method of variation of parameters.

$$(1-t)y'' + ty' - y = 2(t-1)^2 e^{-t}, \quad 0 < t < 1, \quad y_1(t) = e^t, \quad y_2(t) = t. \tag{3}$$

b) Explain why $y' = \sqrt{y}$ does not have unique solution initiating at $y(0) = 0$. Give an example of ODE which blows up in finite time. Describe the long time behavior of $y' = y(1-y)(2-y)$.

5. (20 pts.) a) Find the series solution to the following initial value problem and determine the radius of convergence:

$$(2+x^2)y'' - xy' + 4y = 0, \quad y(0) = -1, y'(0) = 3. \tag{4}$$

b) Find the radius of convergence:

$$e^x y'' + xy = 0. \tag{5}$$