Math 353 Midterm 1 Name:____________________

Honor Code: I have completed this exam in the spirit of the Duke Community Standard and have neither given nor received aid. Only textbook, personal notes done during class, and formula sheet are allowed.

Signature:

1. (20 pts.) a) Solve the equation $y' = (x + e^x)/(1 + 4y^3), \quad y(0) = 1$.

   b) Solve the equation:

$$\frac{dy}{dx} = \frac{y^2 + 3xy + x^2}{x^2}.$$

   c) Provide an example of 1st order ODE which has two different solutions initiating from the same initial data. Explain why the uniqueness theorem does not apply here.

1.

2. (20 pts.) a) Determine whether the equation is exact. And if it is exact, solve it. Write down the final solution in the form $F(x, y)$ =Constant.

$$\left(\frac{y}{x} + 2x\right) dx + (\ln|x| - 3)\, dy = 0.$$

b) Check if the following equation is exact or not:

$$x dx + \frac{y^2}{x} dy = 0.$$

If it is not exact, find the integration factor μ.

2.

3. (15 pts.) Apply the reduction of order to determine the other homogeneous solution to the equation

$$t^2 y'' + 3ty' + y = 0, \quad t > 0; \quad y_1(t) = t^{-1}.$$

3.

4. (20 pts.) a) Find the solution to the following inhomogeneous equation using the method of variation of parameters.

$$2y'' + 6y' + 4y = e^{-t}, \quad 0 < t.$$

Explain (do not calculate) how to solve the initial value problem

$$2y'' + 6y' + 4y = e^{-t}, \quad y(0) = y_0,\, y'(0) = z_0$$

using the y_p you just got.

b) Give an example of ODE which blows up in finite time. Describe the long time behavior of $y' = (y-1)(4-y^2)$.

c) Solve the equation $y' = -y^2, \quad y(0) = \infty$.

4.

5. (25 pts.) a) Consider the following problem

$$(1-x^2)y'' - 2xy' + 6y = 0.$$

Find the recurrence relation and the first five terms (find a_2, a_3, a_4) in the general solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$. Identify the two linearly independent solutions y_1, y_2.

b) What can you say about a_{2n}, $n \geq 2$?

5.