AUTOMATIC CONTROL THEORY

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Chapter 2 Fundamental of analysis and design for control system

- 2.1 Laplace transform
- 2.2 Properties of Laplace transform
- 2.3 Application

2.3 Applications

- Solving circuit problem
- Resistor:

$$u = Ri$$

$$U(s) = RI(s)$$

$$G(s) = \frac{U(s)}{I(s)} = R$$

2.3 Applications

- Capacitor:

$$i = C\frac{du}{dt}$$

$$I(s) = C[sU(s) - u(0)]$$

$u(0) = 0$:

$$G(s) = \frac{U(s)}{I(s)} = \frac{1}{Cs}$$

2.3 Applications

- Inductor:

$$u = L\frac{di}{dt}$$

$$U(s) = L[sI(s) - i(0)]$$

$i(0) = 0$:

$$G(s) = \frac{U(s)}{I(s)} = Ls$$

2.3 Applications

- Example: A R-L-C circuit.

2.3 Applications

- Transfer function: $u(t)$ - input and y(t) - output.

$$G(s) = \frac{Y(s)}{U(s)} \qquad \text{zero initial conditions}$$

2.3 Applications

- Step response h(t) : $u(t) = 1(t)$.

$$H(s) = \frac{G(s)}{s}$$

$$h(t) = L^{-1}\{H(s)\}$$

2.3 Applications

- Impulse response g(t) : $u(t) = \delta(t)$

$$Y(s) = G(s)$$

$$\mathrm{g}(t) = L^{-1}\{G(s)\}$$

2.3 Applications

$e(t) = r(t) - y(t)$, r: reference signal, y: plant's output

Steady state error:

$$e(\infty) = \lim_{s \to 0} sE(s)$$

2.3 Applications

- Jean Baptiste Joseph Fourier (1768–1830), French Mathematician.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

- Existence condition:

$$\int_{-\infty}^{\infty} |f(t)|dt < \infty$$

- Inverse Fourier transform:

$$f(t) = F^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$$

Fourier Transform Properties

Property	$f(t)$	$F(\omega)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Scaling	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t-a)$	$e^{-j\omega a} F(\omega)$
Frequency shift	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Modulation	$\cos(\omega_0 t) f(t)$	$\frac{1}{2}[F(\omega + \omega_0) + F(\omega - \omega_0)]$
Time differentiation	$\frac{df}{dt}$	$j\omega F(\omega)$
	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(t)\, dt$	$\frac{F(\omega)}{j\omega} + \pi F(0)\,\delta(\omega)$
Frequency differentiation	$t^n f(t)$	$(j)^n \frac{d^n}{d\omega^n} F(\omega)$
Reversal	$f(-t)$	$F(-\omega)$ or $F^*(\omega)$
Duality	$F(t)$	$2\pi f(-\omega)$
Convolution in t	$f_1(t) * f_2(t)$	$F_1(\omega) F_2(\omega)$
Convolution in ω	$f_1(t) f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$

Fourier Transform Pairs

$f(t)$	$F(\omega)$
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$u(t)$	$\pi\,\delta(\omega) + \dfrac{1}{j\omega}$
$u(t+\tau) - u(t-\tau)$	$2\dfrac{\sin\omega\tau}{\omega}$
$\lvert t\rvert$	$\dfrac{-2}{\omega^2}$
$\mathrm{sgn}(t)$	$\dfrac{2}{j\omega}$
$e^{-at}u(t)$	$\dfrac{1}{a + j\omega}$
$e^{at}u(-t)$	$\dfrac{1}{a - j\omega}$
$t^n e^{-at}u(t)$	$\dfrac{n!}{(a + j\omega)^{n+1}}$
$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2 + \omega^2}$
$e^{j\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
$\sin\omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$\cos\omega_0 t$	$\pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$e^{-at}\sin\omega_0 t u(t)$	$\dfrac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
$e^{-at}\cos\omega_0 t u(t)$	$\dfrac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$