

Schwartzschild 解の Ricci テンソル

2023 年 7 月 14 日

$$\Gamma^{\rho}_{\mu\nu} = \begin{cases} \Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{2}\nu' \\ \Gamma^1_{13} = \Gamma^1_{31} = r^{-1} \\ \Gamma^2_{12} = \Gamma^2_{21} = \cot\theta \\ \Gamma^2_{23} = \Gamma^2_{32} = r^{-1} \\ \Gamma^3_{00} = -\frac{1}{2}\nu'e^{-\lambda+\nu} \\ \Gamma^3_{11} = -re^{-\lambda} \\ \Gamma^3_{22} = -re^{-\lambda}\sin^2\theta \\ \Gamma^3_{33} = \frac{1}{2}\lambda' \end{cases} \quad (1)$$

$$R_{\mu\nu} = \partial_{\rho}\Gamma^{\rho}_{\mu\nu} - \partial_{\nu}\Gamma^{\rho}_{\mu\rho} + \Gamma^{\sigma}_{\mu\nu}\Gamma^{\rho}_{\rho\sigma} - \Gamma^{\tau}_{\mu\rho}\Gamma^{\rho}_{\nu\tau} \quad (2)$$

$$R_{33} = \underbrace{\partial_\rho \Gamma_{33}^\rho}_{\rho=3} - \underbrace{\partial_3 \Gamma_{3\rho}^\rho}_{\rho=0,1,2,3} + \underbrace{\Gamma_{33}^\sigma \Gamma_{\rho\sigma}^\rho}_{\sigma=3, \rho=0,1,2,3} - \underbrace{\Gamma_{3\rho}^\tau \Gamma_{3\tau}^\rho}_{\tau=\rho=0,1,2,3} \quad (3)$$

$$\begin{aligned} &= \partial_3 \Gamma_{33}^3 - \partial_3 \Gamma_{30}^0 - \partial_3 \Gamma_{31}^1 - \partial_3 \Gamma_{32}^2 - \partial_3 \Gamma_{33}^3 \\ &\quad + \Gamma_{33}^3 \Gamma_{03}^0 + \Gamma_{33}^3 \Gamma_{13}^1 + \Gamma_{33}^3 \Gamma_{23}^2 + \Gamma_{33}^3 \Gamma_{33}^3 \\ &\quad - \Gamma_{30}^0 \Gamma_{30}^0 - \Gamma_{31}^1 \Gamma_{31}^1 - \Gamma_{32}^2 \Gamma_{32}^2 - \Gamma_{33}^3 \Gamma_{33}^3 \end{aligned} \quad (4)$$

$$\begin{aligned} &= -\partial_3 \Gamma_{30}^0 - \partial_3 \Gamma_{31}^1 - \partial_3 \Gamma_{32}^2 \\ &\quad + \Gamma_{33}^3 \Gamma_{03}^0 + \Gamma_{33}^3 \Gamma_{13}^1 + \Gamma_{33}^3 \Gamma_{23}^2 \\ &\quad - (\Gamma_{30}^0)^2 - (\Gamma_{31}^1)^2 - (\Gamma_{32}^2)^2 \end{aligned} \quad (5)$$

$$\begin{aligned} &= -\frac{1}{2} \nu'' + r^{-1} + r^{-1} \\ &\quad + \frac{1}{2} \lambda' \frac{1}{2} \nu' + \frac{1}{2} \lambda' r^{-1} + \frac{1}{2} \lambda' r^{-1} + \frac{1}{4} \lambda'^2 \\ &\quad - \frac{1}{4} \nu'^2 - r^{-2} - r^{-2} - \frac{1}{4} \lambda'^2 \end{aligned} \quad (6)$$

$$= -\frac{1}{2} \nu'' - \frac{1}{4} \nu'^2 + \frac{1}{4} \lambda' \nu' + \frac{\lambda'}{r} \quad (7)$$