Schwartzschild 解の Ricci テンソル

2023 年 7 月 7 日

$$
\Gamma^{\rho}{}_{\mu\nu} = \begin{cases}
\Gamma^{0}{}_{03} = \Gamma^{0}{}_{30} = \dfrac{1}{2}\nu' \\
\Gamma^{1}{}_{13} = \Gamma^{1}{}_{31} = r^{-1} \\
\Gamma^{2}{}_{12} = \Gamma^{1}{}_{21} = \cot\theta \\
\Gamma^{2}{}_{23} = \Gamma^{2}{}_{32} = r^{-1} \\
\Gamma^{3}{}_{00} = -\dfrac{1}{2}\nu' e^{-\lambda+\nu} \\
\Gamma^{3}{}_{11} = -re^{-\lambda} \\
\Gamma^{3}{}_{22} = -re^{-\lambda}\sin^2\theta \\
\Gamma^{3}{}_{33} = \dfrac{1}{2}\lambda'
\end{cases} \tag{1}
$$

$$
R_{\mu\nu} = \partial_\rho \Gamma^{\rho}{}_{\mu\nu} - \partial_\nu \Gamma^{\rho}{}_{\mu\rho} + \Gamma^{\sigma}{}_{\mu\nu}\,\Gamma^{\rho}{}_{\rho\sigma} - \Gamma^{\tau}{}_{\mu\rho}\,\Gamma^{\rho}{}_{\nu\tau} \tag{2}
$$

$$R_{33} = \underbrace{\partial_\rho \Gamma^\rho{}_{33}}_{\rho=3} - \underbrace{\partial_3 \Gamma^\rho{}_{3\rho}}_{\rho=0,1,2,3} + \underbrace{\Gamma^\sigma{}_{33}\,\Gamma^\rho{}_{\rho\sigma}}_{\sigma=3,\rho=0,1,2,3} - \underbrace{\Gamma^\tau{}_{3\rho}\,\Gamma^\rho{}_{3\tau}}_{\tau=\rho=0,1,2,3} \tag{3}$$

$$\begin{aligned} = {} & \partial_3 \Gamma^3{}_{33} - \partial_3 \Gamma^0{}_{30} - \partial_3 \Gamma^1{}_{31} - \partial_3 \Gamma^2{}_{32} - \partial_3 \Gamma^3{}_{33} \\ & + \Gamma^3{}_{33}\,\Gamma^0{}_{03} + \Gamma^3{}_{33}\,\Gamma^1{}_{13} + \Gamma^3{}_{33}\,\Gamma^2{}_{23} + \Gamma^3{}_{33}\,\Gamma^3{}_{33} \\ & - \Gamma^0{}_{30}\,\Gamma^0{}_{30} - \Gamma^1{}_{31}\,\Gamma^1{}_{31} - \Gamma^2{}_{32}\,\Gamma^2{}_{32} - \Gamma^3{}_{33}\,\Gamma^3{}_{33} \end{aligned} \tag{4}$$

$$\begin{aligned} = {} & - \partial_3 \Gamma^0{}_{30} - \partial_3 \Gamma^1{}_{31} - \partial_3 \Gamma^2{}_{32} \\ & + \Gamma^3{}_{33}\,\Gamma^0{}_{03} + \Gamma^3{}_{33}\,\Gamma^1{}_{13} + \Gamma^3{}_{33}\,\Gamma^2{}_{23} \\ & - (\Gamma^0{}_{30})^2 - (\Gamma^1{}_{31})^2 - (\Gamma^2{}_{32})^2 \end{aligned} \tag{5}$$

$$\begin{aligned} = {} & - \frac{1}{2}\nu'' + r^{-1} + r^{-1} \\ & + \frac{1}{2}\lambda'\frac{1}{2}\nu' + \frac{1}{2}\lambda' r^{-1} + \frac{1}{2}\lambda' r^{-1} + \frac{1}{4}\lambda'^2 \\ & - \frac{1}{4}\nu'^2 - r^{-2} - r^{-2} - \frac{1}{4}\lambda'^2 \end{aligned} \tag{6}$$

$$= - \frac{1}{2}\nu'' - \frac{1}{4}\nu'^2 + \frac{1}{4}\lambda'\nu' + \frac{\lambda'}{r} \tag{7}$$