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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

April 2024 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: CS, IS and AI &ML **Duration: 3 hrs.**
Course Code: 22MA4BSLIA **Max Marks: 100**
Course: Linear Algebra

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

		UNIT - I	CO	PO	Marks
1	a)	Determine whether the set $V=\{(x,y) \mid x,y\in\mathbb{R}\}$ is a vector space over the field of reals when the vector addition is the standard vector addition and the scalar multiplication is defined as $k\cdot(x,y)=(0,ky)$.	CO1	PO1	6
	b)	Find the basis of row space, Column space, null space for the matrix $A=\begin{bmatrix}1 & -3 & 4 & -2 & 5 & 4\\ 2 & -6 & 9 & -1 & 8 & 2\\ 2 & -6 & 9 & 1 & 9 & 7\\ -1 & 3 & -4 & 2 & -5 & 4\end{bmatrix}$.	CO1	PO1	7
	c)	Find a homogeneous system whose solution set W is spanned by $\{u_1,u_2,u_3\}=\{(1,-2,0,3),(1,-1,-1,4),(1,0,-2,5)\}$.	CO1	PO1	7
		UNIT - II			
2	a)	Verify whether the linear transformation $T:P_2(t)\to M_{2\times2}$ defined by $T(at^2+bt+c)=\begin{bmatrix}a & 2b\\ 0 & a\end{bmatrix}$ is one-one and onto.	CO1	PO1	6
	b)	Verify Rank-Nullity theorem for the linear transformation $G:R^3\to R^3$ defined by $G(x,y,z,t)=(x-y+z+t,\ 2x-2y+3z+4t\ ,3x-3y+4z+5t)$.	CO1	PO1	7
	c)	Let T be a linear operator defined on R^3 through $T(x,y,z)=(2x,\ 4x-y\ ,2x+3y-z)$. Is T invertible? If so, find a formula for T^{-1} and T^{-2}.	CO1	PO1	7
		OR			

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

	3	a)	Discuss the following maps on R^2 and represent them graphically: i) Horizontal contraction ii) Vertical contraction iii) Horizontal shear.	*CO1*	*PO1*	**6**
		b)	Let $T: P_1(t) \to P_2(t)$ be the linear transformation defined by $T[f(t)] = t\,f(t)$. Find the matrix of linear transformation with respect to the standard basis of $P_1(t)$ and $B = \{1+t,\ t-1,\ t^2\}$ for $P_2(t)$.	*CO1*	*PO1*	**7**
		c)	Consider the mapping $F: R^2 \to R^2$ defined by $F(x,y) = (3y, 2x)$. Let 'S' be the unit circle in R^2, that is the solution of $x^2 + y^2 = 1$. i) Identify $F(S)$, ii) Find $F^{-1}(S)$.	*CO1*	*PO1*	**7**
			UNIT - III			
	4	a)	Apply Cayley –Hamilton theorem to find A^4 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Obtain the Eigen space for the linear transformation $T: R^3 \to R^3$ defined by $T(x,y,z) = (2x+y,\ \ y-z,\ \ 2y+4z)$.	*CO1*	*PO1*	**7**
		c)	Find all possible Jordan canonical form of linear transformation T, whose minimal polynomial is $(t-8)^2(t+8)^2$ and algebraic multiplicity of the eigen values -8 and 8 are 5 and 4 respectively.	*CO1*	*PO1*	**7**
			OR			
	5	a)	Apply Cayley –Hamilton theorem to find A^{-1} and A^{-2} if $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Obtain the Eigen space for the linear transformation $T: P_2 \to P_2$ defined by $T(at^2+bt+c) = (2a-c)t^2 + (2a+b-2c)t + (-a+2c)$.	*CO1*	*PO1*	**7**
		c)	Find the characteristic and minimal polynomials of the matrix $A = \begin{bmatrix} 3 & 2 & 0 & 0 & 0 \\ 1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$.	*CO1*	*PO1*	**7**

<table>
<tr><td></td><td></td><td></td><td>UNIT - IV</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>Consider the following polynomial in $P_2(t)$ with the inner product $\langle f \quad g\rangle = \int_{-1}^{1} f(t)g(t)dt \quad f(t)=t+2,\ g(t)=t^2-3t+4$.
Find the matrix of $\langle f \quad g\rangle$ with respect to the basis $\{1,t,t^2\}$.</td><td>CO1</td><td>PO1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find QR decomposition of the matrix $A=\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\\1&1&1\end{bmatrix}$.</td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A sales organization obtains the following data relating the number of salespersons to annual sale. Let x and y denotes the number of salespersons and annual sales respectively. Find the least square line of the form $y=a+bx$ and estimate the annual sales when there are 14 salespersons.
<table>
<tr><td>x</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>y</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr>
</table></td><td>CO1</td><td>PO1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT-V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>Compute the Hessian matrix at the point $(1,1,1)$ of the function $f(x,y,z)=xy^2+z^3+2xy+3xz+x^2+3$.</td><td>CO1</td><td>PO1</td><td>4</td></tr>
<tr><td></td><td></td><td>b)</td><td>Determine the modal matrix that reduces the quadratic form $3x^2+3y^2+3z^2-2yz+2zx+2xy$ to its canonical form and hence find the nature of the quadratic form.</td><td>CO1</td><td>PO1</td><td>8</td></tr>
<tr><td></td><td></td><td>c)</td><td>Given the data in table, reduce the dimension from 2 to 1 using principal component analysis.
<table>
<tr><td>X</td><td>4</td><td>8</td><td>13</td><td>7</td></tr>
<tr><td>Y</td><td>11</td><td>4</td><td>5</td><td>14</td></tr>
</table></td><td>CO1</td><td>PO1</td><td>8</td></tr>
</table>

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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E. | **Semester: IV**
Branch: CS cluster except AIML | **Duration: 3 hrs.**
Course Code: 23MA4BSLAO | **Max Marks: 100**
Course: Linear Algebra and Optimization

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

			CO	*PO*	Marks
		UNIT – 1			
1	a)	Find the gradient of matrix $\boldsymbol{f} = \begin{bmatrix} x_0^2 x_1 \log(x_2) & \frac{x_1^2 x_2}{x_3} \\ x_3^2 + x_1 x_3 & x_2^3 + x_1 \end{bmatrix}$ with respect to the matrix $\boldsymbol{x} = \begin{bmatrix} x_0 & x_1 \\ x_2 & x_3 \end{bmatrix}$ and hence verify $\frac{\partial\,[\mathrm{Trace}(\boldsymbol{f}(\boldsymbol{x}))]}{\partial \boldsymbol{x}} = \mathrm{Trace}\left(\frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}}\right)$.	*1*	*1*	**7**
	b)	Given $f = x^2 + y^2 + z^2 + xy + yz + zx - 4x - 4y - 4z - 5$, i) find all the stationary points of the function, ii) find the Hessian matrix and iii) classify the stationary points and find its extreme value.	*1*	*1*	**7**
	c)	Find the equation of hyperplane normal to the vector $\boldsymbol{n} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ that separates the sets $A = \left\{ \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix} \right\}$ and $B = \left\{ \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$. Does this hyperplane strictly separate these sets?	*1*	*1*	**6**
		UNIT – 2			
2	a)	Apply Newtons method to find the minimum value of the function $f = 4x_1^2 + x_2^2 - 2x_1x_2 + x_1 + x_2$ near $(1, 1)$. Carry out two iterations.	*1*	*1*	**6**
	b)	An asteroid is entering the atmosphere of moon. The shape of the asteroid is described by the equation $4x^2 + y^2 + 4z^2 = 16$. The temperature on the surface of the asteroid after one month was observed to be represented by equation $8x^2 + 4yz - 16z + 600$. Is it possible to find the point on surface of asteroid with maximum temperature? If yes, find it?	*1*	*1*	**6**
	c)	Derive the KKT conditions to minimize the function $f(x, y) = xy$ subject to the constraints $x + y^2 \le 2$ and $x, y \ge 0$. Also find the optimum values of the function.	*1*	*1*	**8**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><td></td><td></td><td></td><td>UNIT – 3</td><td></td><td></td><td></td></tr>
<tr><td></td><td>3</td><td>a)</td><td>If $u=(1,\ 3,\ -4,\ 2)$, $v=(4,-2,2,1)$ and $w=(5,-1,-2,6)$ are vectors in $\mathbb{R}^4$ then:
i) verify the $\langle 3u-2v, w\rangle = 3\langle u,w\rangle - 2\langle v,w\rangle$ and
ii) find $\dfrac{\|3u-2v\|}{\|w\|}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Find an orthogonal basis of the subspace W spanned by the following vectors $S=\{1,\ t,\ t^2\}$ of $P_2(t)$ given $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>A sales organization obtains the following data relating the number of salespersons to annual sales.
<table>
<tr><td>x: Number of salespersons</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>y:Annual Sales (millions of dollars)</td><td>2.3</td><td>3.2</td><td>4.1</td><td>5.0</td><td>6.1</td><td>7.2</td></tr>
</table>
Find the least squares line of the form $y = a + bx$ and hence estimate the annual sales when there are 14 salespersons.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>4</td><td>a)</td><td>Find the value of α such that the matrices $A=\begin{bmatrix}\alpha & 8 & -7\\ 6 & 5\alpha & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 3\\ -4 & 5 & 6\alpha\end{bmatrix}$ are orthogonal with respect to an inner product $\langle A,B\rangle = Tr\left(B^T A\right)$. Hence find $\|A\|$ and $\|B\|$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Show that $S=\{u_1,u_2,u_3,u_4\}$ where $u_1=(1,1,1,1)$, $u_2=(1,1,-1,-1)$, $u_3=(1,-1,1,-1)$, $u_4=(1,-1,-1,1)$ is orthogonal and a basis of $\mathbb{R}^4$. Find the coordinates of the arbitrary vector $v=(a,b,c,d)$ in $\mathbb{R}^4$ relative to the basis S.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Obtain the QR factorization of the following matrix $A=\begin{bmatrix}1 & -1\\ 2 & 0\\ 3 & 1\end{bmatrix}$.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - 4</td><td></td><td></td><td></td></tr>
<tr><td></td><td>5</td><td>a)</td><td>Apply Cayley-Hamilton theorem to find A^4 given $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$.</td><td>1</td><td>1</td><td>6</td></tr>
<tr><td></td><td></td><td>b)</td><td>Given $M=\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 0\\ 0 & 0 & 2\end{bmatrix}$ and eigenvalues of M are $\lambda = 0, 2$. Find the eigenvalues and the eigenspace corresponding to each eigenvalue of $A = M^2 + \frac{1}{2}M$. Hence determine whether A is defective matrix or not.</td><td>1</td><td>1</td><td>7</td></tr>
<tr><td></td><td></td><td>c)</td><td>Determine all possible Jordan canonical forms or blocks of linear transformation T with characteristic polynomial $f(t)=(t-4)^3$ and specify the geometric multiplicity in each case.</td><td>1</td><td>1</td><td>7</td></tr>
</table>

			OR			
	6	a)	Determine the inverse of the matrix $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$ by using Cayley-Hamilton theorem.	*1*	*1*	**6**
		b)	Find an eigenspace of the linear transformation $T:P_2(t)\to P_2(t)$ given by $T\left(at^2+bt+c\right)=(a+3b+3c)t^2-(3a+5b+3c)t+(3a+3b+c)$.	*1*	*1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $A=\begin{bmatrix}2 & 5 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 4 & 2 & 0\\ 0 & 0 & 3 & 5 & 0\\ 0 & 0 & 0 & 0 & 7\end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 5			
	7	a)	Find the nature and write the canonical form of the quadratic form $3x^2+3y^2+3z^2+2xy+2xz-2yz$.	*1*	*1*	**5**
		b)	Assume that the **USN** is **XYZ2222** and then construct a matrix **A** of size 2×2 by taking last four digits of the **USN** given and hence find the singular value decomposition. Also write comment on the dimension reduction.	*1*	*1*	**8**
		c)	Apply principal component analysis to given data to reduce from two-dimension to 1-dimension: x: 4, 8, 13, 7 y: 11, 4, 5, 14	*1*	*1*	**7**

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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2024 Supplementary Examinations

Programme: B.E. | **Semester: IV**
Branch: CS / IS / AIML | **Duration: 3 hrs.**
Course Code: 22MA4BSLIA | **Max Marks: 100**
Course: LINEAR ALGEBRA

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	***CO***	***PO***	**Marks**
1	a)	Check whether $b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$ can be expressed as a linear combination of the columns of $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}$.	*1*	*1*	**6**
	b)	Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ and $v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$. Find a basis for the subspace W spanned by $\{v_1, v_2, v_3, v_4\}$.	*1*	*1*	**7**
	c)	Given $B = \{t^3 + t^2,\ t^2 + t,\ t + 1, 1\}$ is an ordered basis of vector space $P_3(t)$. Find the co-ordinate vector of $f(t)$ relative to B where $f(t) = 2t^3 + t^2 - 4t + 2$.	*1*	*1*	**7**
		UNIT - 2			
2	a)	Find the matrix of the linear transformation $T: V_2(R) \to V_3(R)$ defined by $T(x, y) = (x + y,\ x,\ 3x - y)$ with respect to $B_1 = \{(1,1),\ (3,1)\}$ and $B_2 = \{(1,1,1),\ (1,1,0), (1,0,0)\}$.	*1*	*1*	**6**
	b)	Find the basis and dimension of the Image and Kernel of the linear transformation $F: R^4 \to R^3$ defined by $F(x,y,z,t) = (x - y + z + t,\ 2x - 2y + 3z + 4t,\ 3x - 3y + 4z + 5t)$.	*1*	*1*	**7**
	c)	Verify rank-nullity theorem for the linear transformation defined by the matrix $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 4 & -1 & 2 \\ -1 & -2 & 5 & 4 \end{bmatrix}$.	*1*	*1*	**7**
		OR			
3	a)	Find the linear transformation $T: V_3(R) \to V_4(R)$ which maps $T(1,0,0) = (0,1,0,2), T(0,1,0) = (0,1,1,0)$ and $T(0,0,1) = (0,1,-1,4)$.	*1*	*1*	**6**
	b)	Show that $T: P_1(t) \to P_3(t)$ is a linear transformation if $T(at + b) = at^3 + bt^2 + at + b$ and hence find the image of $p(t) = 2 - t$.	*1*	*1*	**7**

		c)	Determine the basis and dimension of the null space of the linear transformation defined by $A = \begin{bmatrix} 1 & 4 & -1 & 3 \\ 2 & 9 & -1 & 7 \\ 2 & 8 & -2 & 6 \end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 3			
	4	a)	Find the eigenvalues and the corresponding eigenspaces of the linear operator $A = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$.	*1*	*1*	**6**
		b)	Find the characteristic and minimal polynomial of the matrix $A = \begin{bmatrix} 2 & 5 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}$.	*1*	*1*	**7**
		c)	Determine the Jordan canonical form for the matrix $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$.	*1*	*1*	**7**
			OR			
	5	a)	Find the eigenvalue and the corresponding eigenvectors of the linear transformation $T: V_2(R) \rightarrow V_2(R)$, given $A = \begin{pmatrix} 3 & 3 \\ 1 & 5 \end{pmatrix}$.	*1*	*1*	**6**
		b)	Apply Cayley-Hamilton theorem to find A^{-1} if $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$.	*1*	*1*	**7**
		c)	Compute the Jordan canonical form for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$.	*1*	*1*	**7**
			UNIT - 4			
	6	a)	Find the projection of $(1,2,3)$ on to the subspace W spanned by $S = \{u_1, u_2\}$ where $u_1 = (2,5,-1)$ and $u_2 = (-2,1,1)$.	*1*	*1*	**5**
		b)	Find the equation of line that will be best approximation of the points $(-3,70), (1,21), (-7,110)$ and $(5,-35)$ using the method of least squares.	*2*	*1*	**7**
		c)	Determine the QR-decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & -1 & 4 \end{bmatrix}$.	*2*	*1*	**8**
			UNIT- 5			
	7	a)	Orthogonally diagonalize the matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.	*2*	*1*	**10**
		b)	Find the singular value decomposition of $A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$.	*2*	*1*	**10**

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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2023 Semester End Main Examinations

Programme: B.E. **Semester: IV**
Branch: CSE/ISE **Duration: 3 hrs.**
Course Code: 19MA4BSLIA **Max Marks: 100**
Course: Linear Algebra

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

			CO	PO	Marks
		UNIT - I			
1	a)	Solve the system of equations $x + 2y + 3z = 14$, $4x + 5y + 7z = 35$ and $3x + 3y + 4z = 21$ for complete solution. Also mention the free variables and the pivot variables.	CO1	PO1	**7**
	b)	Solve the system of equations $x + y + z = 1, 3x + y - 3z = 5$ and $x - 2y - 5z = 10$ by LU-Decomposition method.	CO1	PO1	**7**
	c)	Find the basis and dimension for Column space and Row space of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.	CO1	PO1	**6**
		OR			
2	a)	Solve the system of equations $u + 3v + 3w + 2y = 1$, $2u + 6v + 9w + 7y = 5$ and $-u - 3v + 3w + 4y = 5$ for the complete solution.	CO1	PO1	**7**
	b)	Solve the system of equations $x + y + z = 9, 2x + 5y + 7z = 52$ and $2x + y - z = 0$ by Gauss elimination method.	CO1	PO1	**7**
	c)	Check linearly dependency or linearly independency of following set of vectors. i) $A = \{(1,-3,2), (2,1,-3), (-3,2,1)\}$ and ii) $B = \{(2,1,3), (1,3,2), (3,2,1)\}$.	CO1	PO1	**6**
		UNIT - II			
3	a)	Find the matrix representation of linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (2x+3y, 4x-5y)$ relative to $\{(1,2), (2,5)\}$ for both vector space in domain and codomain.	CO1	PO1	**7**
	b)	If $T: R^2 \to R^3$ is defined by $T(x, y) = (x, y, x + y)$. Show that T is linear transformation and also find its kernel.	CO1	PO1	**7**
	c)	Show that the linear transformation $T: R^2 \to R^2$ defined as $T(x, y) = (3x - 5y, -3x - 6y)$ is invertible and find T^{-1}.	CO1	PO1	**6**

			UNIT – III			
	4	a)	Find all the eigenvalues and corresponding eigenvectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.	*CO2*	*PO1*	**7**
		b)	Diagonalize the matrix $A=\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ and hence find A^4.	*CO2*	*PO1*	**7**
		c)	Find minimal polynomial and characteristic polynomial of the matrix $A=\begin{bmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$.	*CO2*	*PO1*	**6**
			OR			
	5	a)	Compute the eigenspace of the linear transformation $T:\mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by the matrix $A=\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$.	*CO2*	*PO1*	**7**
		b)	Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ also compute A^{-1} and A^4.	*CO2*	*PO1*	**7**
		c)	Write the Jordan canonical form of matrix whose characteristic and minimal polynomial are respectively $(x-1)^3(x-2)^2$ and $(x-1)^2(x-2)$.	*CO2*	*PO1*	**6**
			UNIT – IV			
	6	a)	Apply Gram-Schmidt orthogonalization to construct the orthonormal basis of the vector space spanned by the vectors $(1\ \ 0\ \ 1),(1,0,0),(2,1,0)$.	*CO3*	*PO1*	**7**
		b)	Find QR decomposition of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.	*CO3*	*PO1*	**7**
		c)	Find the least square solution of the system of equations $-x+2y=3,\ x+y=4,\ x-2y=0$ and $3x+2y=2$.	*CO3*	*PO1*	**7**
			UNIT – IV			
	7	a)	Determine the orthogonal modal matrix and hence diagonalize the matrix $\begin{bmatrix} 1 & 5 & -2 \\ 5 & 4 & 5 \\ -2 & 5 & 1 \end{bmatrix}$.	*CO3*	*PO1*	**8**
		b)	Obtain the canonical form and hence classify the nature of the quadratic form $7x^2+6y^2+5z^2-4xy-4yz$.	*CO3*	*PO1*	**4**
		c)	Find singular value decomposition of the matrix $A=\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$.	*CO3*	*PO1*	**8**

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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

December 2023 Supplementary Examinations

Programme: B.E. **Semester: IV**
Branch: CS, IS and AI&ML **Duration: 3 hrs.**
Course Code: 22MA4BSLIA **Max Marks: 100**
Course: Linear Algebra

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	CO	PO	Marks
1	a)	Show that the set $M = \left\{ \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \Big/ a, b \in \mathbb{R} \right\} \subset M_{2\times2}$ satisfies all the properties of a vector space over the field of reals under standard matrix addition and scalar multiplication.	CO1	PO1	**6**
	b)	Does there exist non-zero scalars c_1, c_2, c_3 and c_4 which proves that the vectors $v_1 = (0,1,2,3,0)$, $v_2 = (1,3,-1,2,1)$, $v_3 = (2,6,-1,-3,1)$ and $v_4 = (4,0,1,0,2)$ in $\mathbb{R}^5$ are linearly dependent? If yes, find them.	CO1	PO1	**7**
	c)	Determine a subset of $S = \{p_1, p_2, p_3, p_4\} \subset P_3(t)$, the vector space of polynomials that forms a basis of $W = span(S)$ if $p_1 = t^3 + t^2$, $p_2 = 2t^3 + 2t - 2$, $p_3 = t^3 - 6t^2 + 3t - 3$ and $3t^2 - t + 1$.	CO1	PO1	**7**
		UNIT – II			
2	a)	Consider the matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ which defines a linear operator on $\mathbb{R}^2$. Find the matrix of the linear transformation relative to the basis $S = \{u_1, u_2\} = \{(1,-2),(3,-7)\}$.	CO1	PO1	**6**
	b)	Find the basis for the range space $R(T)$, null space $N(T)$ for the linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$ given by $T(x, y, z, t) = (x - y + z + t,\ x + 2z + t,\ x + y + 3z - 3t)$ and also verify rank-nullity theorem.	CO1	PO1	**7**
	c)	Let $G: \mathbb{R}^3 \to \mathbb{R}^3$ given by $G(x, y, z) = (x + y,\ x + z,\ y + z)$. (i) Show that G is invertible. (ii) Find G^{-1}.	CO1	PO1	**7**
		OR			

	3	a)	Let $A=\begin{bmatrix}1 & -3\\ 3 & 5\\ -1 & 7\end{bmatrix}$, $u=\begin{bmatrix}2\\ -1\end{bmatrix}$, $b=\begin{bmatrix}3\\ 2\\ -5\end{bmatrix}$ and $c=\begin{bmatrix}3\\ 2\\ 5\end{bmatrix}$. Define a linear transformation $T:\mathbb{R}^2\to\mathbb{R}^3$ by $T(X)=AX$, for $X\in\mathbb{R}^2$. (i) Find the image of u under the transformation T. (ii) Find an $X\in\mathbb{R}^2$ whose image is b. Is there more than one $X\in\mathbb{R}^2$ whose image is b? (iii) Determine if c is in the range of the transformation.	*CO1*	*PO1*	**6**
		b)	Derive the matrix of the linear transformation $T:\mathbb{R}^2\to\mathbb{R}^2$, which results in horizontal shear of $(x,y)\in\mathbb{R}^2$ by 0.5 units. Determine if there exist i) a preimage of $(1,-3)$. ii) an image of $(3,-1)$.	*CO1*	*PO1*	**7**
		c)	Find the basis for the range space $R(T)$ and the Kernel $N(T)$ of the linear transformation $T:\mathbb{R}^4\to\mathbb{R}^3$ given by the matrix $A=\begin{bmatrix}1 & 2 & 3 & 1\\ 1 & 3 & 5 & -2\\ 3 & 8 & 13 & -3\end{bmatrix}$. Hence verify the rank-nullity theorem. Is T a one-one mapping? Justify.	*CO1*	*PO1*	**7**
			UNIT - III			
	4	a)	Apply Cayley-Hamilton theorem to compute A^{-1} of $A=\begin{bmatrix}1 & 1 & -2\\ -1 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Find the eigenvalues and the eigenvectors of the linear transformation $T:P_1(t)\to P_1(t)$ defined $T(at+b)=(a+2b)t+(4a+3b)$.	*CO1*	*PO1*	**7**
		c)	Find the characteristic and minimal polynomial of $\begin{bmatrix}2 & 5 & 0 & 0 & 0\\ 0 & 2 & 0 & 0 & 0\\ 0 & 0 & 4 & 2 & 0\\ 0 & 0 & 3 & 5 & 0\\ 0 & 0 & 0 & 0 & -7\end{bmatrix}$.	*CO1*	*PO1*	**7**
			OR			
	5	a)	Apply Cayley-Hamilton theorem to compute A^4 if $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 5\\ 3 & 5 & 6\end{bmatrix}$.	*CO1*	*PO1*	**6**
		b)	Determine the eigenspaces of the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x+y-z,\, 0\,,\, x+2y+3z)$.	*CO1*	*PO1*	**7**

		c)	Write all possible Jordan canonical form of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ when $\Delta(t) = (t+5)^2 (t-7)^3$ is the characteristic polynomial.	*CO1*	*PO1*	**7**
			UNIT – IV			
	6	a)	Find a basis of W of $\mathbb{R}^4$ orthogonal to $u_1 = (1,-2,3,4)$ and $u_2 = (3,-5,7,8)$.	*CO1*	*PO1*	**4**
		b)	Find an orthogonal basis and hence an orthonormal basis of the subspace W spanned by $S = \{1, 1-t, t^2\}$ of $P_2(t)$ given $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$.	*CO1*	*PO1*	**9**
		c)	In an experiment designed to determine the extent of a person's natural orientation, a subject is put in a special room and kept there for a certain length of time. He is then asked to find a way of a maze and record is made of the time it takes the subject to accomplish this task. The following data are obtained. <table><tr><td>Time in Room (hours)</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Time to find way out of maze(minutes)</td><td>0.8</td><td>2.1</td><td>2.6</td><td>2.0</td><td>3.1</td><td>3.3</td></tr></table> Let x denote the number of hours in the room and let y denote the number of minutes that it takes the subject to find his way out. i. Find the least squares line of the form $y = a + bx$. ii. Estimate the time it will take the subject to find his way out of the maze after 10 hours in the room using the equation obtained.	*CO1*	*PO1*	**7**
			UNIT – V			
	7	a)	Determine the modal matrix that reduces the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ to its canonical form and hence discuss the nature the quadratic form.	*CO1*	*PO1*	**10**
		b)	Obtain the singular value decomposition of $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$.	*CO1*	*PO1*	**10**

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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

September / October 2023 Supplementary Examinations

Programme: B.E
Branch: CS/IS/AIML
Course Code: 19MA4BSLIA
Course: Linear Algebra

Semester: IV
Duration: 3 hrs.
Max Marks: 100
Date: 14.09.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1 a) Apply LU decomposition method to solve the system of equations $2x+y+4z=12,\ 4x+11y-z=33$ and $8x-3y+2z=20$. **6**

b) Express the polynomial $v=t^2+4t-3$ in $p(t)$ as a linear combination of the polynomials $p_1=t^2-2t+5$, $p_2=2t^2-3t$, $p_3=t+3$. **7**

c) Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$ and $u_3=(3,8,-3,-5)$. **7**

i) Find the basis and dimension of W.
ii) Extend the basis of W to a basis of $\mathbb{R}^4$.

OR

2 a) Show that the vectors $u_1=(1,1,1),\ u_2=(1,2,3),\ u_3=(1,5,8)$ span $\mathbb{R}^3$. **6**

b) Show that the set $V=\left\{a+b\sqrt{2}\mid a,b\in\mathbb{Q}\right\}$ is a vector space over the field of rationals. **7**

c) Find the dimension and a basis of the solution space W of the system of equations $x+2y+2z-s+3t=0$; $x+2y+3z+s+t=0$ and $3x+6y+8z+s+5t=0$. **7**

UNIT- II

3 a) If $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,\ y,\ z)=(x-y, y+z,\ x+z)$, find $R(T)$, $N(T)$ and hence verify Rank-Nullity theorem. **6**

b) Let T be a linear operator defined on $\mathbb{R}^3$ through $T(x,\ y,\ z)=(2x+3y-z, 4y-z,\ 2z)$. Is T invertible? If so, then find a formula for T^{-1}. **7**

c) Find the matrix of linear transformation $T:V_2(\mathbb{R})\to V_3(\mathbb{R})$ defined by $T(x,\ y)=(-x+2y,\ y,\ -3x+3y)$ relative to the basis $B_1=\{(1,1),\ (-1,1)\}$ and $B_2=\{(1,1,1),\ (1,-1,1),\ (0,0,1)\}$. **7**

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

UNIT- III

4 a) Find A^4 given $A=\begin{bmatrix}1&2&3\\2&4&5\\3&5&6\end{bmatrix}$ using Cayley-Hamilton theorem. **6**

b) Find the eigen space of the linear transformation $T:R^3\to R^3$ defined by $T(x,\ y,\ z)=(3x+2y+z,\ x+4y+z,\ 2y+4z)$. **7**

c) Determine all possible Jordan canonical forms of the linear operator $T:V\to V$ whose characteristic polynomial is $\Delta(t)=(t-2)^3(t-5)^2$. **7**

OR

5 a) Find the characteristic and minimal polynomial of $A=\begin{bmatrix}3&2&0&0&0\\1&4&0&0&0\\0&0&3&1&0\\0&0&1&3&0\\0&0&0&0&4\end{bmatrix}$. **6**

b) Find eigenvalues and eigenvectors of the linear transformation $T:V_2(\mathbb{R})\to V_2(\mathbb{R})$ defined by $T(x,\ y)=(2x+5y,\ 4x+3y)$. **7**

c) Express the initial-value problem $\frac{d^2x(t)}{dt^2}-2\frac{dx(t)}{dt}-3x(t)=0$ subjected to $x(0)=4,\ \frac{dx(0)}{dt}=5$ into fundamental form and hence solve. **7**

UNIT- IV

6 a) Let W be a subspace of $\mathbb{R}^5$ spanned by $u=(1,2,3,-1,2)$ and $v=(2,4,7,2,-1)$. Find a basis of the orthogonal compliment $W^{\perp}$ of W. **6**

b) Find an orthogonal basis and hence an orthonormal basis of U, the subspace of $\mathbb{R}^4$ spanned by the vectors $v_1=(1,1,1,1)$, $v_2=(1,1,2,4)$ and $v_3=(1,2,-4,-3)$ using Gram-Schmidt orthogonalization process. **7**

c) Obtain the QR factorization of $A=\begin{bmatrix}1&1&2\\0&0&1\\1&0&0\end{bmatrix}$. **7**

UNIT- V

7 a) Orthogonally diagonalize $\begin{bmatrix}3&-2&4\\-2&6&2\\4&2&3\end{bmatrix}$. **10**

b) Find a singular value decomposition of $\begin{bmatrix}1&-1\\-2&2\\2&-2\end{bmatrix}$. **10**

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