Parameter Estimation

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(1) Given: Random Sample $(x_1 \ldots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$$

Taking natural log of likelihood function

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\right)$$

To find MLE, diff. log likelihood w.r.t. θ_1, θ_2

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \frac{(x_i-\mu)}{\sigma^2} = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\underset{\mu}{\theta_1} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

for θ_2:

$$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{(x_i-\theta_1)^2}{\theta_2}\right) - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{2} = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\theta_1)^2$$

$\hookrightarrow$ Sample Variance

(2) $$L(\theta) = \prod_{i=1}^{n} \binom{n}{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{n}{x_i} + x_i \ln(\theta) + (n-x_i)\ln(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right) = 0$$

Solving

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{n-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (n-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = n \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$\therefore$ MLE of θ is sample mean of observations.