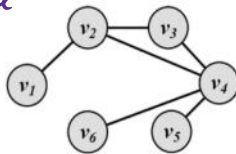


Graph representation

① Adjacency matrix — rows, columns = nodes.

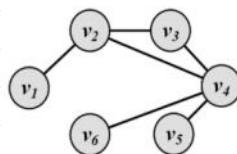
$a_{ij} = 1$  if connection b/w  $v_i, v_j$   
 $= 0$  otherwise.

Social media networks have sparse adj. matrices.



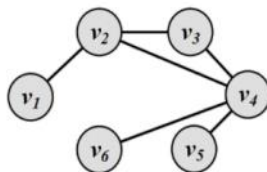
	$v_1$	$v_2$	$v_3$	$v_4$	$v_5$	$v_6$
$v_1$	0	1	0	0	0	0
$v_2$	1	0	1	1	0	0
$v_3$	0	1	0	1	0	0
$v_4$	0	1	1	0	1	1
$v_5$	0	0	0	1	0	0
$v_6$	0	0	0	1	0	0

② Adjacency list — for every node, list all connected nodes. Sort based off pref or whatever.



Node	Connected To
$v_1$	$v_2$
$v_2$	$v_1, v_3, v_4$
$v_3$	$v_2, v_4$
$v_4$	$v_2, v_3, v_5, v_6$
$v_5$	$v_4$
$v_6$	$v_4$

③ Edge list — just the set of all connected nodes.



$(v_1, v_2)$   
 $(v_2, v_3)$   
 $(v_2, v_4)$   
 $(v_3, v_4)$   
 $(v_4, v_5)$   
 $(v_4, v_6)$

Terminology

① Two nodes = adjacent if connected via edge  
 = incident if connected via endpoint.

node + edge incident if node connects to edge.

② WALK

Random walk: next step @ each node = random among neighbors.

Sequence of incident edges visited one after the other  
 repetition ✓

represent as — set of edges:  $e_1, e_2, e_3, \dots, e_n$   
 set of vertices:  $v_1, v_2, \dots, v_n$

closed

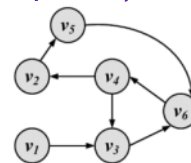
$x_i = x_f$

open

$x_i \neq x_f$

length of walk = no. of visited edges.

Length of walk = 8



③ TRAIL

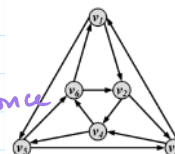
walk w/ NO REPETITION OF EDGE

unique set of edges/vertices

closed trail = tour/circuit

Eulerian Tour

— all edges traversed once  
 — Königsberg bridge.



④ PATH — walk w/ NO REPETITION OF (EDGE + VERTEX)

length = no. of edges visited.

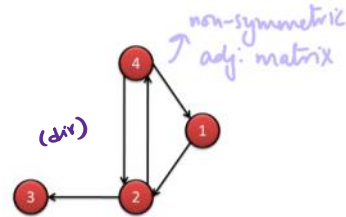
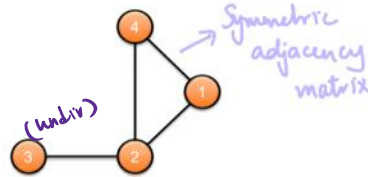
④ PATH — walk w/ NO REPETITION OF (EDGE + VERTEX)  
length = no. of edges visited.

## Types of graphs

① Null graph — empty set:  $G(V, E), V = E = \emptyset$

② Empty graph — edgeless graph. Only  $E = \emptyset$

③ Directed + undirected:



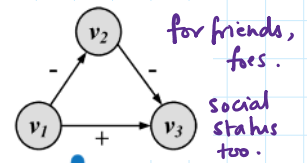
④ Simple graph — one edge b/w any node pair

⑤ Multigraph —  $\geq 1$  edge b/w node pairs. — adj. matrix can have no.  $> 1$

⑥ Weighted graph — edge associated w/ weight, weight has some meaning.

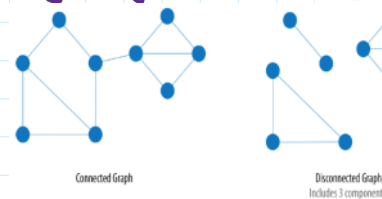
⑦ Web graph — represents internet sites — directed multigraph.

⑧ Signed graph — weighted graph w/ binary weights



⑨ Connected graph — path b/w all nodes

⑩ Disconnected graphs — if islands exist.

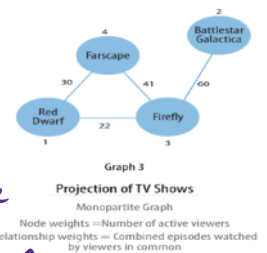
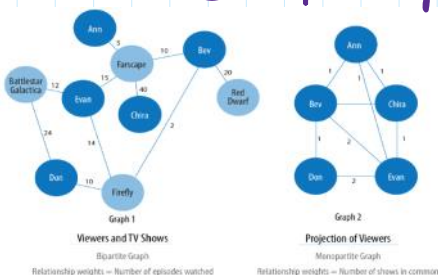


If nodes in island are connected,  
 $\Rightarrow$  components / clusters.

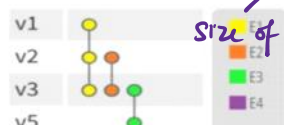
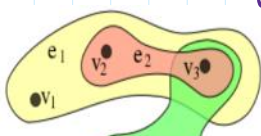
⑪ K-partite graphs — no. of node types = k.  
graph where vertices can be partitioned into k diff independent sets.

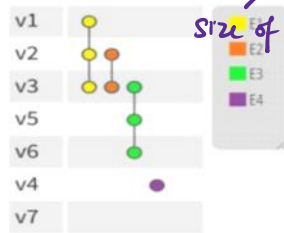
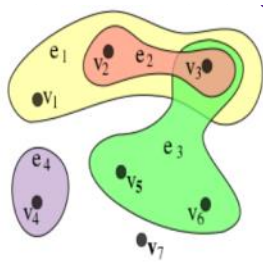
Bipartite:  $k=2$   
nodes belong to 2 sets

Monopartite:  $k=1$   
only 1 node + relation type  
project w/ inferred connections.



⑫ Hypergraph — generalization of undirected graphs — edges are subsets of  $2 \geq$  vertices.





size of vertex set = order

size of edge set = size

Undirected hypergraph  $G = (V, E)$  i.e. with

$X = \{ v_1, v_2, v_3, v_4, v_5, v_6, v_7 \}$  with  $E = \{ e_1, e_2, e_3, e_4 \} = \{ \{ v_1, v_2, v_3 \}, \{ v_2, v_3 \}, \{ v_3, v_5, v_6, v_7 \}, \{ v_4 \} \}$ . This hypergraph has order 7 and size 4 (no of hyperedges)