Row stochastic —— Σ prob for any state (row) = 1

Visible state $V_T = \{v(1) \ldots v(T)\}$

$a_{ij} = w_i \to w_j$ (transmission)

$b_{jk} = w_j$ emits v_k (emission)

① Learning problem:

I have observation sequence, Parameters unknown → forward-backward (Baum Welch) → sit on bum & observe.

② Decoding problem:

I have observat^n, I have parameters, Find states that generated it → Viterbi

③ Evaluation problem:

I have observ^n, I have sequence, I have params, Probability of getting something → forward.

Mathematics for Machine Learning   Bhaskarjyoti Das (Dept of CSE in AIML)

Q1 Consider an HMM with an explicit absorber state w_0 and null visible symbol v_0.

$$a_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0.2 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.1 \end{pmatrix} \qquad b_{jk} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.7 & 0.3 \\ 0 & 0.4 & 0.6 \end{pmatrix}$$

Assume, initial hidden state at $t=0$ is w_1

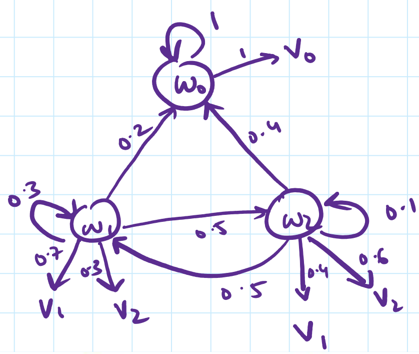
(a) What is the prob. that it will generate a sequence $\{v_2 v_1 v_0\}$

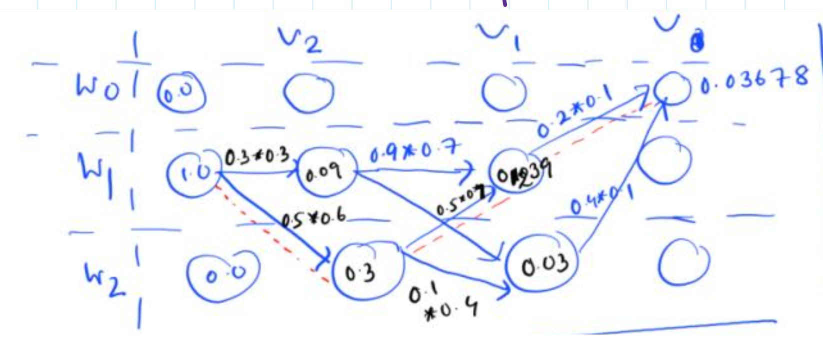
(b) What is the most likely sequence of hidden states for this visible sequence

$w_0 \; w_1 \; w_2 \to$

$$a_{ij} = \begin{array}{c|ccc} & w_0 & w_1 & w_2 \\ \hline w_0 & 1 & 0 & 0 \\ w_1 & 0.2 & 0.3 & 0.5 \\ w_2 & 0.4 & 0.5 & 0.1 \end{array}$$

$$b_{ij} = \begin{array}{c|ccc} & v_0 & v_1 & v_2 \\ \hline w_0 & 1 & 0 & 0 \\ w_1 & 0 & 0.7 & 0.3 \\ w_2 & 0 & 0.4 & 0.6 \end{array}$$

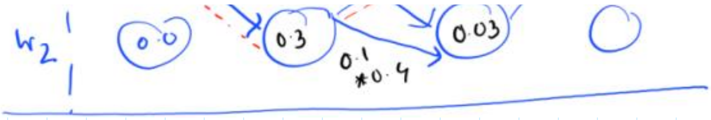




α-table or Trellis diagram

seq : $\{v_2, v_1, v_0\}$

Namita Achyuthan



seq : $\{V_2, V_1, V_0\}$

① @ $t = 0$, start @ w_1 with prob $= 1$

② from w_1 @ t_1 :
(how to get to v_2 1st)

$w_1 \rightarrow w_0$ X (absorbing state)

$w_1 \rightarrow w_1 \rightarrow v_2 = 0.3 \times 0.3 = 0.09$

$w_1 \rightarrow w_2 \rightarrow v_2 = 0.5 \times 0.6 \times 0.1 \leftarrow 0.3$

③ @ time t_2
(how to get to v_1)
from w_1

$w_1 \rightarrow w_2 \rightarrow w_1 \rightarrow v_1$ (or) + $w_1 \rightarrow w_1 \rightarrow w_1 \rightarrow v_1$

$0.3 \times 0.5 \times 0.7 + 0.3 \times 0.3 \times 0.3 \times 0.7$

$= 0.1239$

$w_1 \rightarrow w_2 \rightarrow w_2 \rightarrow v_1 + w_1 \rightarrow w_1 \rightarrow w_2 \rightarrow v_1$

$= 0.03$.

④ @ time t_3
(how to get v_0)

$0.123 \underbrace{\times 0.2}_{(w_2 \rightarrow w_0)} \times 0.1 + 0.03 \times \underbrace{0.4}_{(w_1 \rightarrow w_0)} \times 0.1$

$= 0.03678$.

⑤ Best path $= w_1 \rightarrow w_2 \rightarrow w_1 \rightarrow w_0$.