Random Graph Models → main assumpt^n : edges b/w nodes formed randomly.

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Erdos Renyi random graph model. ($G(n,p)$ model)

n pick 2 — ${}^nC_2$ — $C(n,2)$

- graph has fixed no. of nodes n. — $\binom{n}{2}$ edges can be formed independently w/ probability p.

(no. of edges not fixed)

Expected no. of edges connected to a node

$c = (n-1)p$ — all edges selected independently w/ probability p.

expected degree of node.

$G(n,p)$ $\quad p(\deg(v)=k) = {}^{n-1}C_k \cdot p^k \cdot (1-p)^{n-1-k}$ (no. of nodes fixed) contain all or none edges.

$G(n,m)$ $\quad p(|E|=m) = \binom{\binom{n}{2}}{m} \cdot p^m \cdot (1-p)^{\binom{n}{2}-m}$ (no. of edges fixed)

avg. path length $l = \dfrac{\ln|V|}{\ln c}$ expected degree

small p — no components.

bigger p — giant components, no loners.

large p — diameter shrinks → "phase transition."

when $p = \dfrac{1}{n-1}$

p = global clustering coeff

Random graph model vs scale free network

The main difference between a random and a scale-free network **comes in the *tail* of the degree distribution**, representing the **high-*k* region of p_k**. If we compare a scale free **power law graph with a Poisson function (random network),** we find that :

- **For small *k*,** the power law is above the Poisson function, indicating that a scale-free network has **a large number of small degree nodes**, most of which are absent in a random network.
- **For moderate *k*, in the vicinity of some value $\langle k \rangle$** the Poisson distribution is above the power law, indicating that in a random network **there is an excess of nodes with degree *k*≈$\langle k \rangle$ *which are in the middle range*** .
- **For large k,** the power law is again above the Poisson curve. The **difference is particularly visible if we show p_k on a log-log plot indicating that the probability of observing a high-degree node, or *hub*, is several orders of magnitude higher in a scale-free than in a random network.**

Disruption in scale free vs. random graph

- **Disruption**
  - A **random percentage** of the nodes are removed
- **How does the diameter change?**
  - Increases monotonically and linearly in **random graphs**
  - Remains almost the same in **scale-free networks**
    - Since a random sample is unlikely to pick the high-degree nodes

Attack in scale free vs. random graph

- **Attack**
  - A percentage of nodes **are removed willfully (e.g. from high degree in decreasing order of connectivity)**
- How does the diameter change?
  - For **random networks, essentially no difference from disruption as a**ll nodes are approximately same
  - **For scale-free networks, diameter doubles for every 5% node** removal!
    - This is an opportunity when you are fighting to contain spread of an epidemic