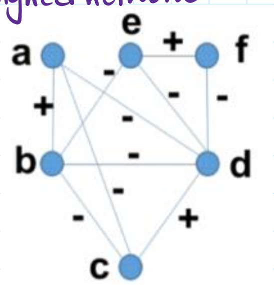
Signed network :



Dir^n of link = dir^n of info flow.
Weight = strength of info (credibility) → Normal network
↓
Does it measure perception?
No.

Captures opinion/rel^{n}ship ← A signed network can
dynamics across entities.

Two theories to measure consistency

SOCIAL BALANCE
works for directed + undirected

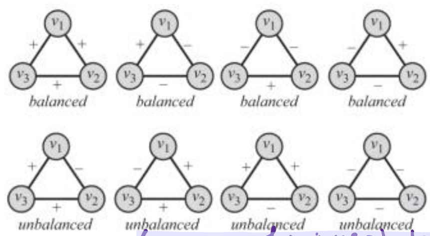
consistency in friend/foe relationship

The friend of my friend is my friend,
The friend of my enemy is my enemy,
The enemy of my enemy is my friend,
The enemy of my friend is my enemy.
→ Dwight.

W_{ij} = friendships = + 1
W_{ij} = enmity = − 1

$$\boxed{W_{ij}\, W_{jk}\, W_{ki} \geq 0 \Rightarrow \text{Balanced}}$$

all friends or 2 enemies

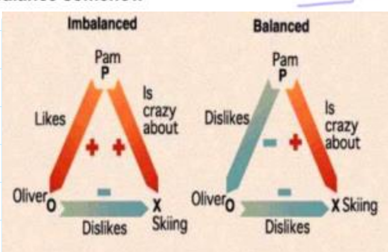


(even no. of minus = balance)

Balance theory :

If the person **perceive imbalance in his relationship, will be motivated to correct the imbalance somehow**

It's like Jim + Dwight.



+ balanced network is clusterable.

SOCIAL STATUS
works only for directed

consistency in assigning status.

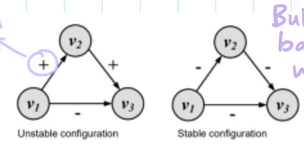
If X has a higher status than Y and Y has a higher status than Z, then X should have a higher status than Z.

$X > Y,\ Y > Z \Rightarrow X > Z$

Directed + edge from X to Y ($X \to Y$)
means $Y > X$

Directed − edge from X to Y ($X \to Y$)
means $Y < X$

$v_2 > v_1$



But social balance would say it's wrong

If X initiates a +ve link to Y,
X considers Y higher than itself

If X initiates a −ve link to Y,
X considers Y lower than itself

(g) Generation baseline = no. of +ve generated by me
(r) Receptive baseline = no. of −ve received by me

$$A_g = \frac{\text{generated}}{\text{how many sent out}}$$

$$A_r = \frac{\text{received}}{\text{how many received}}$$

Assortativity — preference of network's nodes to attach w/ another.

similar (assortative) | different (disassortative)

degree assortativity — similar degrees flock together (Rich goes w/ rich)

plot average degrees of neighbors of node w/ some degree k.

Assortative mixing

→ assortative high degree w/ high degree

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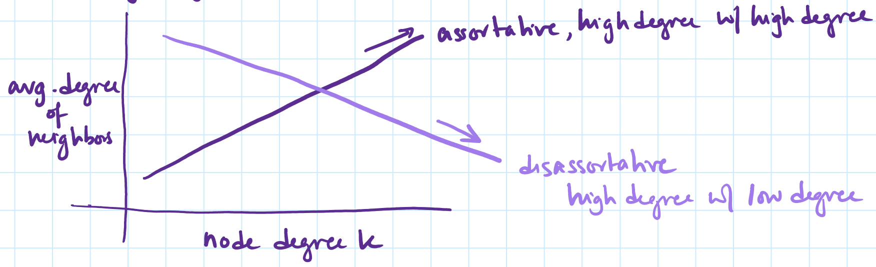
Assortative mixing

HOMOPHILY	HETEROPHILY
brown people date other brown people	women like having affairs w/ men
brown x brown	woman x man

└ plot average degrees of neighbors of node w/ some degree k.



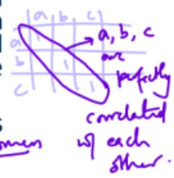
To find similarity b/w nodes: Pearson Correlation Coefficient (degree based)

$$r_{xy} = \frac{N\sum XY - \sum X \sum Y}{\sqrt{(N\sum X^2 - (\sum X)^2)(N\sum Y^2 - (\sum Y)^2)}}$$

= 1 (homo)
= -1 (hetero)
= 0 (none)

Assortative mixing

- Now that we defined what is assortativity, we can examine a network and conclude **how assortative the network is**.
- This measure is known as **assortative mixing coefficient r** → We see if like ♡ like, like ♡ unlike, or just no ♡
- In case of **perfect assortativity i.e.** only black will go with black, Asian will go with Asian etc in the previous example. **So, if we have a matrix of the association and the matrix values are normalized with total number of relations** (based on race), in the best case the diagonal will have a value of 1



- We use the concept of correlation coefficient to define r and that is the **assortative mixing coefficient** → Kevin Hart only dates black women
- If r=1 , it is **perfectly assortative** (only the diagonal elements exist)
- If all the diagonal elements of the above matrix are 0, then we have r=-1 and we have a **perfectly disassortative** network → Dwayne J only dates white women
- Degree assortativity can be used for **different attributes of the node** (in previous case, it was **race**)

Rich Club

- nodes having many connections are clustered together.

assume threshold degree k to decide who's above + below.

$$\text{Rich Club coefficient RCC} = \frac{\text{no. of edges b/w n nodes}}{\text{max no. of edges}}$$

Interpersonal Ties.

Social Computing

Strength of a tie

a tie is an information carrying connection

the closer you are, the stronger your tie

- strength of ties captures a sense of closeness among entities/people
- Simplest metric to capture the same is via Jaccard score
- Corresponding metric, called Neighborhood Overlap (NO) is defined as:

$$N(x,y) = \frac{|\Gamma(x) \cap \Gamma(y)|}{|\Gamma(x) \cup \Gamma(y)|}$$

Neighborhood overlap b/w x, y

conjunction (AND) → the stuff x,y share, the total area.

disjunction (OR) — if you don't like disjunction ∪ should go suck it

where $\Gamma(\cdot)$ denotes the neighbourhood of a node

- Higher the $NO(\cdot)$ score, higher the overlap between the nodes, and higher the chance forming a link in between

↳ high NO score →

The more this area covers up of the overall area, the stronger is the tie. Like i think ESha's friends are mine too.

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Neighborhood Overlap.

$\Gamma(\cdot)$ = neighborhood of node

$$N(x,y) = \frac{\Gamma(x) \cap \Gamma(y)}{\Gamma(x) \cup \Gamma(y)}$$

$\Gamma(x) \cap \Gamma(y)$ — only stuff that x,y share

$\Gamma(x) \cup \Gamma(y)$ — both neighborhoods combined

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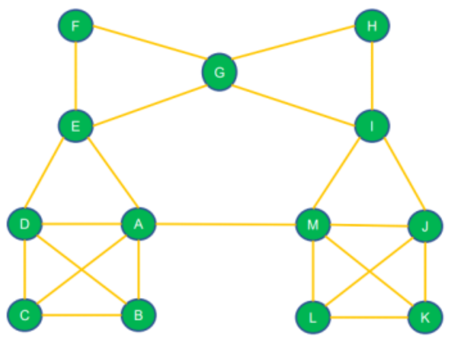


$\Gamma(A) = \{B, C, D, E, M\}$,
$\Gamma(M) = \{A, I, J, K, L\}$,
$\Gamma(E) = \{A, D, F, G\}$

$|\Gamma(A) \cap \Gamma(M)| = |\phi| = 0$
$|\Gamma(A) \cap \Gamma(E)| = |\{D\}| = 1$

$|\Gamma(A) \cup \Gamma(M)| = |\{B, C, D, E, I, J, K, L\}| = 8$
$|\Gamma(A) \cup \Gamma(E)| = |\{B, C, D, F, G, M\}| = 6$

$NO(A, M) = \frac{0}{8} = 0$
$NO(A, E) = \frac{1}{6}$

NO (D, M) :

$\Gamma(D) = \{A, B, C, E\}$
$\Gamma(M) = \{A, I, J, K, L\}$

first degree connections of D,M.

$\frac{\Gamma(D) \wedge \Gamma(M)}{\Gamma(D) \vee \Gamma(M)}$ → Both D AND M / → D OR M

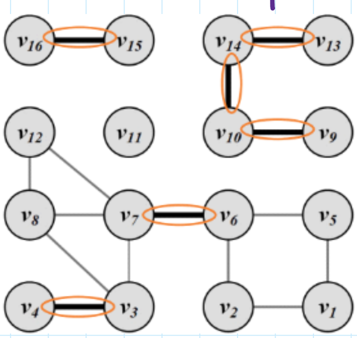
$= \frac{\{A\}}{\{A, B, C, E, I, J, K, L\}} = \frac{1}{8}$

Quantifying strength of triadic closure

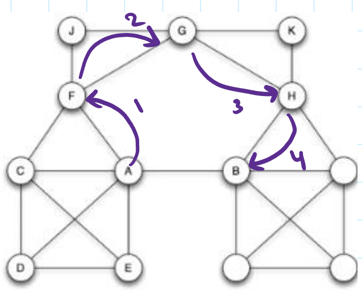
clustering coefficient : $C_A = \frac{2 \times (\text{no. of closed triplets})}{\text{no. of possible triplets}}$

Bridges:

edges whose removal will ↑ no. of connected components.



Bridges allow diffusion of info b/w disconnected communities.



AB is a local bridge — ↑ distance b/w A,B doesn't partition network.

Span of local bridge = no. of hops to connect 2 end nodes.

Edge = local bridge if neighborhood overlap $N(X,Y) = 0$

Structural Equivalence.

Social Computing

Measuring Structural Equivalence → similarity b/w two nodes based on their connection to other nodes.] CONNECTION BASED

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- This approach may not work as node v itself is excluded in N(v. For two nodes which do not share a common neighbor, will have zero structural equivalence
- Common Neighbors → v. simple – just see no. of commons
  - number of **common neighbors shared in the neighborhoods** of the nodes a and b

    $\sigma_{CN}(a, b) = |N(a) \cap N(b)|$ in a AND b's

- Jaccard Similarity

Drawback: sometimes, two nodes may not h...

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Social Computing

Measuring Structural Equivalence → Similarity b/w two nodes based on their connection to other nodes. CONNECTION BASED

- This approach may not work as node v itself is excluded in N(v. For two nodes which do not share a common neighbor, will have zero structural equivalence

Drawback: some times, two nodes may not have any mutuals so that sucks.

- Common Neighbors → v. simple – just see no. of commons
  - number of common neighbors shared in the neighborhoods of the nodes a and b

$$\sigma_{CN}(a,b) = |N(a) \cap N(b)|$$

in a AND b's neighborhood

- Jaccard Similarity
  - Normalizes the common neighbors by the combined size of the neighborhoods of the two nodes

$$\sigma_{CN}(a,b) = \frac{|N(a) \cap N(b)|}{|N(a) \cup N(b)|}$$

→ Neighborhood overlap??

- Cosine Similarity
  - **normalizes the common neighbors by the individual sizes** of the neighborhoods

$$\sigma_{CN}(a,b) = \frac{|N(a) \cap N(b)|}{\sqrt{|N(a)||N(b)|}}$$

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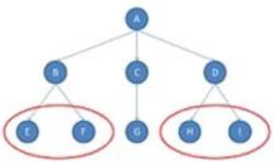
Structural Equivalence

similar relationships to everyone

like Dwight = Andy

if 1 switched w/ the other, no one would know



- Two nodes are said to be exactly structurally equivalent if they have the same relationships to all other nodes
- Two actors must be exactly substitutable in order to be structurally equivalent
- In the attached network,
  - nodes E and F are structurally equivalent, since these two nodes have same pattern ties (viz. a single tie) with the node B
  - Also, nodes H and I are structurally equivalent, since these two nodes have same pattern ties (viz. a single tie) with the node D
- Exact structural equivalence is likely to be rare (particularly in large networks)
- the degree of structural equivalence is what interests us the most

Social Computing

Structural vs. Regular Equivalence

Structural equivalence: same relationships to all other nodes

Regular equivalence: similar neighborhoods

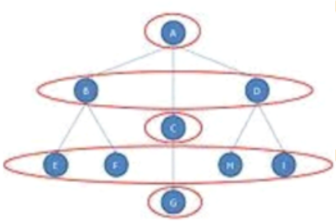
like same env. but diff font

- In **regular equivalence**, unlike structural equivalence, we do **not look at the neighborhoods shared between individuals**, but at how **neighborhoods themselves are similar.**
- Regular equivalence assesses similarity **by comparing the similarity of neighbors and not by their overlap**. One way of formalizing this is to consider nodes v_i and v_j similar when they have many similar neighbors v_k and v_l.

Similarity of neighborhoods w/o overlap.

→ Stanley is like Kevin cuz their neighborhoods overlap.

Automorphic Equivalence



- Let us interpret the network as follows: the network describe a franchise group of a restaurant chain
  - A is the general manager at central headquarters
  - B, C, and D are the managers of three different stores.
  - E and F are workers at one store; G is the lone worker at a second store; H and I are workers at the third store
- B and D are equivalent in the following sense
  - B and D report to a boss (same boss here)
  - Each has exactly two workers

school → person

- Similarly, E, F, H, and I are also equivalent in the following sense
  - They report to a store manager (different boss here)
  - Nobody report to these persons
- The above approach of equivalence is automorphic equivalence

Substructures are similar

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