Watts and Strogatz Small World Model

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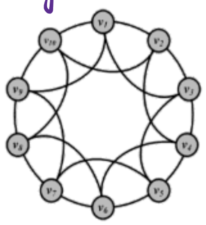
Short avg. path length ✓
High clustering coeff ✓

① Assume everyone has same no. of connections
↓
Embed individuals in regular network

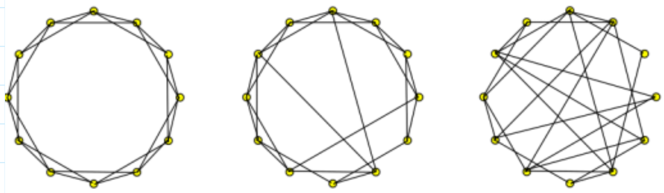
- The small-world model posits a network built on a low-dimensional regular lattice (capturing geographic or other type of social proximity), and then adding or moving random edges to create a low density of "shortcuts" that join the remote parts of the lattice to one another.
- The best studied case is a one-dimensional lattice with periodic boundary conditions, i.e., a ring.

Regular Lattice



lattice of degree c.
nodes connected to
- prev c/2
- next c/2

→ rewiring replaces existing edge w/ non-existing edge w/ probability β.



$\beta = 0$	$\beta = 0.125$	$\beta = 1$
People know their neighbors.	People know their neighbors, and a few distant people.	People know others at random.
Clustered, but not a "small world"	Clustered and "small world"	Not clustered, but "small world"

Parameters:

① N : no. of nodes.
② k : degree of each node.
③ β : probability of rewiring @ each edge.
→ β = 1 → random
→ β = 0 → regular lattice.

Summarizing the steps

1. The Initial Step in the Watts-Strogatz model is to **start with *N* nodes. Place them in a "ring."**
2. Connect **every node to its *K* neighbors (*K*/2 on each side). This is called a "ring lattice" on *N* nodes with degree *K*. This creates NK edges.**
3. Next is the **Randomization Step, where you randomly rewire each edge with probability *p* such that self-connections and duplicate edges are excluded.**
   - The rewiring procedure involves going through each edge in turn, and with probability p, moving one end of that edge to a new location chosen uniformly at random from the lattice.
   - **This finally creates nkp shortcuts**.

→

Algorithm 4.1 Small-World Generation Algorithm

Require: Number of nodes $|V|$, mean degree c, parameter β
return A small-world graph $G(V, E)$
G = A regular ring lattice with $|V|$ nodes and degree c
for node v_i (starting from v_1), and all edges $e(v_i, v_j)$, $i < j$ **do**
  v_k = Select a node from V uniformly at random.
  if rewiring $e(v_i, v_j)$ to $e(v_i, v_k)$ does not create loops in the graph or multiple edges between v_i and v_k **then**
    rewire $e(v_i, v_j)$ with probability β: $E = E - \{e(v_i, v_j)\}$, $E = E \cup \{e(v_i, v_k)\}$;
  end if
end for
Return $G(V, E)$

1. A desirable model for a real-world network should generate graphs with **high clustering coefficients** and **short average path lengths.**
2. In the simulated network, **the average path length is small and the clustering coefficient is high (as expected).**
3. However small-world model is incapable of generating a realistic degree distribution (power law) in the simulated graph

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