

Generative Models

model mechanism that drives network formation

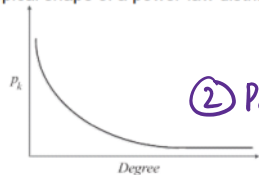
- power law
- small world property
- avg. shortest path

Properties of real world graphs:

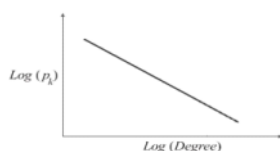
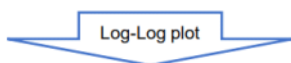
① Average Shortest Path

- Stanley Milgram : at most 6
- Lars Backstrom : at most 4

A typical shape of a power-law distribution



(a) Power-Law Degree Distribution



(b) Log-Log Plot of Power-Law Degree Distribution

② Power law distribution

- plot $\log x$ w/ $\log(\text{probability } x)$ — straight line

degree dist is a power law dist.

$$p(x) \propto \frac{1}{x^b}$$

- frequency of event changes as a negative power of attribute.
(frequency \rightarrow power law)

If $p(k)$ = fraction of individuals w/ degree k :

$$p_k = ak^{-b}$$

$$\ln p_k = -b \ln k + \ln a$$

b = power law exp.
 a = power law intercept

- Scale free network

- \rightarrow one w/ power-law degree distribution
- \rightarrow same functional form @ all scales.

- \rightarrow characterized by presence of large hubs
- \rightarrow More brittle

clustering coeff \downarrow as
node degree \uparrow

low degree nodes
belong to dense
subgraphs

- Heavy / fat tails

- \rightarrow greater than expected probabilities of extreme values.
- \rightarrow power law + tail in high k -region.
- \rightarrow outliers allowed + expected

Small world phenomena:
Avg. path = $6.35 + \log_{10} N$