Transitivity, Reciprocity, Clustering

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Density

gives us ready index of degree of dyadic connect^n in popl^n

Binary data : no. of adj / no. of pairs $= \dfrac{2\Sigma}{n(n-1)}$

But what if :

- all density from single person?
- big network ⇒ small density?

Instead find Diameter → shortest geodesic (shortest line b/w 2 points on non-linear surface)
(small diameter = cohesive network)

Transitivity (to analyze linking behavior)

"friend of a friend is my friend"

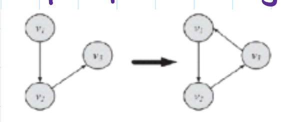


Figure : Transitive Linking.

- Perfect trans : each component triple = clique
- Partial : just ↑ in probability (triadic closure)

High transitivity ⇒ dense graph.

∴ Determine closeness by examining transitivity.

Reciprocity

Measure of likelihood of mutual linking.

"If you become my friend, I'll be yours"

$$R = \frac{1}{|E|} Tr(A^2)$$

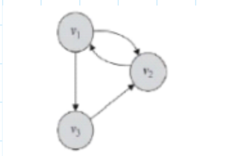


Figure : A Graph with Reciprocal Edges.

Clustering Coefficient

measure of degree to which nodes in a graph cluster together.

① Local clustering — Watts + Strogatz.
- transitivity @ node level.
- how strongly connected v's neighbors are among themselves.

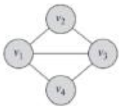
$$C(v_i) = \frac{\text{no. of pairs of neighbors of } v_i \text{ that are connected}}{\text{no. of neighbors of } v_i}$$

can be used to find structural holes.
measures how influential node i is.] more powerful = small value.

→ Take average of local clustering to get that of network.

② Global clustering — Newman, Strogatz, Watts.

$$C = \frac{|\text{closed paths of len } 2|}{|\text{paths of len } 2|} \quad \text{or} \quad = \frac{\text{no. of } \Delta^{\text{les}} \times 3}{|\text{connected triples}|}$$



$$C = \frac{|\text{Closed Paths of Length 2}|}{|\text{Paths of Length 2}|}$$

$$C = \frac{\text{Closed paths of length-2} = \{v_1v_2v_3,\ v_2v_3v_1,\ v_3v_1v_2,\ v_1v_3v_4,\ v_3v_4v_1,\ v_4v_1v_3\}}{\begin{array}{l}\text{Paths of length-2 } (v_1 \text{ centered}) = \{v_2v_1v_4,\ v_2v_1v_3,\ v_3v_1v_4\} \\ \text{Paths of length-2 } (v_3 \text{ centered}) = \{v_2v_3v_4,\ v_2v_3v_1,\ v_1v_3v_4\} \\ \text{Paths of length-2 } (v_2 \text{ centered}) = \{v_1v_2v_3\} \\ \text{Paths of length-2 } (v_4 \text{ centered}) = \{v_1v_4v_3\}\end{array}} = 6/8$$