Barabasi Albert Preferential Attachment Model

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Most known generative model for power law distribution — new nodes most likely to connect to existing nodes that has many connections

P(being joined by new node) ∝ node's degree.

$$P(v_i) = \frac{d_i}{\sum d_j}$$

- Vertices are added one by one
  - The algorithm starts with a **graph containing a small set of nodes m_0**. We **initialize** the network with **m_0 nodes with c edges** each **(c < m_0) where c is the average degree**
  - We then **add one node** at a time.
  - Each **node gets to connect to m other nodes** where m <= m_0
  - Each **edge is attached to a vertex i randomly with a probability directly proportional to i's degree k_i**

Algorithm 4.2 Preferential Attachment

Require: Graph $G(V_0, E_0)$, where $|V_0| = m_0$ and $d_v \geq 1 \; \forall \, v \in V_0$, number of expected connections $m \leq m_0$, time to run the algorithm t

1: **return** A scale-free network
2: //Initial graph with m_0 nodes with degrees at least 1
3: $G(V, E) = G(V_0, E_0)$;
4: **for** 1 to t **do**
5: $V = V \cup \{v_i\}$; // add new node v_i
6: **while** $d_i \neq m$ **do**
7: Connect v_i to a random node $v_j \in V$, $i \neq j$ (i.e., $E = E \cup \{e(v_i, v_j)\}$) with probability $P(v_j) = \frac{d_j}{\sum_k d_k}$.
8: **end while**
9: **end for**
10: Return $G(V, E)$

Key aspects:
- growth element: adding nodes as time goes by
- preferential attach: edges are added a specific way.

realistic degree distribution

small avg. path lengths

fail to exhibit high clustering coeff

Properties of BA:

1. Degree Distribution : Power law with Exponent 3 (real world graphs have this between 2 and 3)
$$P(d) = \frac{2m^2}{d^3},$$
2. Clustering coefficient is a function of time and it becomes smaller with time (not possible to exhibit high clustering coefficient in BA model)
$$C = \frac{m_0 - 1}{8} \frac{(\ln t)^2}{t},$$
3. Average Path length is reasonably short and increases logarithmically with number of nodes. This is similar to Random Graph Model
$$l \sim \frac{\ln |V|}{\ln(\ln |V|)}.$$

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