

Boolean Algebra

Set: A set is a collection of well-defined objects.

\downarrow

Property should be defined properly.

⇒ Conversion of Gray Code to Binary Code:-

$$\text{Gray code} \rightarrow g_3 \ g_2 \ g_1 \ g_0$$

$$\text{Binary code} \rightarrow b_3 \ b_2 \ b_1 \ b_0$$

$$b_3 = g_3$$

$$b_2 = b_3 \oplus g_2$$

$$b_1 = b_2 \oplus g_1$$

$$b_0 = b_1 \oplus g_0$$

~~Gray code to binary~~
Binary to Gray Code

~~110111 (Discarding carry)~~

\downarrow
101100

ex- 1001

$$b_3 = 1$$

$$b_2 = 1 \oplus 0 = 1$$

$$b_1 = 1 \oplus 0 = 1$$

$$b_0 = 1 \oplus 1 = 0$$

(a) 10111011

11000100 → corresponding binary

$$2^7 + 2^6 + 2^2 = 128 + 64 + 4 = \underline{\underline{196}}$$

⇒ Weighted Code: This obey [position weighting principle], which states the position of each number represent a specific weight.

ex- In BCD, each decimal digit is represented by group of 4 bits.

Non-weighted Code: In this, there are no specific weights assigned to the bit position. Gray code is a non-weighted code &

it is not arithmetic codes.

ex- excess 3 code.

⇒ Minimization of Boolean Expr - ① Boolean Algebra

② K-Map (can handle don't care condition)

⇒ De-Morgan's Law :- (To find complement of a complicated Expr)

$$a) \overline{A_1 + A_2 + \dots + A_n} = \bar{A}_1 \cdot \bar{A}_2 \cdot \bar{A}_3 \cdot \dots \cdot \bar{A}_n$$

$$b) \overline{A_1 \cdot A_2 \cdot A_3 \cdot \dots \cdot A_n} = \bar{A}_1 + \bar{A}_2 + \dots + \bar{A}_n$$

$$\Rightarrow i) A + BC = (A + B)(A + C)$$

$$ii) (A + B)(A + C) = A + BC$$

$$iii) A + \bar{A}B = (A + \bar{A})(A + B) = A + B$$

$$iv) \bar{A} + AB = (\bar{A} + A)(\bar{A} + B) = \bar{A} + B$$

$$v) XY + X\bar{Y}WZ = (XY + X\bar{Y})(XY + WZ) = XY + WZ$$

$$Q: V + \bar{V}W + \bar{V}\bar{W}X + \bar{V}\bar{W}\bar{X}Y + \bar{V}\bar{W}\bar{X}\bar{Y}Z$$

$$= V + \bar{V}(W + \bar{W}X + \bar{W}\bar{X}Y + \bar{W}\bar{X}\bar{Y}Z) \quad [A + \bar{A}B = A + B]$$

$$= V + W + \bar{W}X + \bar{W}\bar{X}Y + \bar{W}\bar{X}\bar{Y}Z$$

$$= V + W + \bar{W}(X + \bar{X}Y + \bar{X}\bar{Y}Z)$$

$$= V + W + X + \bar{X}Y + \bar{X}\bar{Y}Z$$

$$= V + W + X + Y + \bar{Y}Z$$

$$= V + W + X + Y + Z$$

Boolean Algebra.

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~~Gray code to binary~~
Binary to Gray code
~~g3 g2 g1 g0~~
110111 (Discarding carried)

$$\downarrow$$

$$101100$$

Ex- 1001

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(a)

$$10111011$$

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Ex- excess 3 code.

\Rightarrow Minimization of Boolean Exp:-

- ① Boolean Algebra
- ② K-Map (can handle don't care condition)

\Rightarrow De-Morgan's Law :- (To find complement of a complicated Eq)

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$$= V + W + \bar{W}(X + \bar{X}Y + \bar{X}\bar{Y}Z)$$

$$= V + W + X + \bar{X}Y + \bar{X}\bar{Y}Z$$

$$= V + W + X + Y + \bar{Y}Z$$

$$= V + W + X + Y + Z$$

⇒ K-Map with Don't Care conditions:-

Don't care conditions arises when function is not specified for certain combination of variables.

A don't care minterm is a combination of variables whose logical value is not specified.

Ex. $F(w, x, y, z) = \sum(4, 5, 6, 7, 12)$ with don't care funⁿ
 $d(w, x, y, z) = \sum(0, 8, 13)$

wz	00	01	11	10
yz	X			
00	1	1	1	1
01	1	X		
11	X			
10				

$$F = \bar{y}z + \bar{w}\bar{x}$$

* We can treat X as 0 or 1 for sake of simplicity of Boolean function.

\Rightarrow Postulates of Boolean Algebra:-

- I) $x+x' = 1$
- II) $x \cdot x' = 0$

Principle of Duality :- Every algebraic expression deducible from the postulates of boolean algebra remains true if the operators & identity elements are interchanged.

Theorem: $x \cdot x = x$

$$\begin{aligned}\Rightarrow x \cdot x &= x \cdot x + 0 \\ &= x \cdot x + x \cdot x' \\ &= x(x+x') \\ &= x \cdot 1 \\ &= x\end{aligned}$$

Theorem: $x+1 = 1$

$$\begin{aligned}\Rightarrow x+1 &= 1 \cdot (x+1) \\ &= (x+x')(x+1) \\ &= x+x' \cdot 1 \\ &= x+x' \\ &= 1\end{aligned}$$

By principle of duality

$$x \cdot 0 = 0$$

Theorem: $x + xy = x$

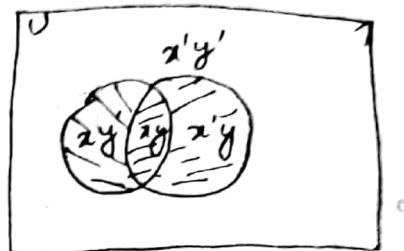
& $x \cdot (x+y) = x$

⇒ Operator Precedence :-

- 1) Parenthesis
- 2) NOT (complement -)
- 3) AND
- 4) OR

↓
Decreasing
Precedence

⇒ Venn Diagrams :-



⇒ Boolean function :-

$n^n \rightarrow$ function ; $2^{n^2} \rightarrow$ Relation

2^{2^n} functions with n variables

Relation : Any subset of $A \times B$ is relation

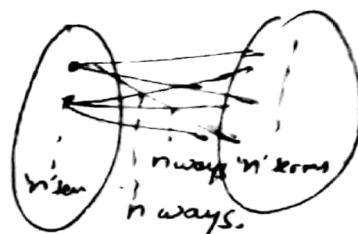
Function : A relation satisfying property of fun.

x	y	z	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$f_2 = x + y'z$$

0 0
1 1
0 0
1 1
1 1
0 1
1 0
1 1

↓

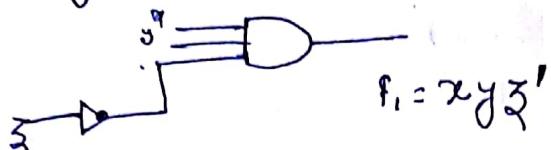


$n \times n \times n \dots n$ times.
hence n^n functions.

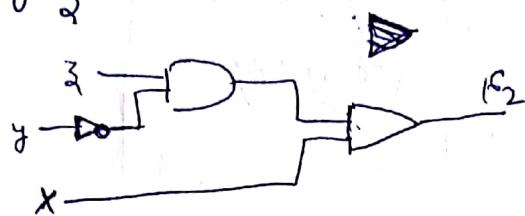
⇒ Implementation of Boolean Function :-

"Hans of
Bhartiya
Hans!"

1. with gates



$$F_2 = x + y'z$$



⇒ Literal : A literal is a primed or unprimed variable.

$$(i) x(x'+y) = xx' + xy = 0 + xy = xy$$

$$(ii) x + x'y = (x+x')(x+y) = x+y$$

$$(iii) xy + x'y + yz = y(x+x' + z) \\ = y(1+z) \\ = yz$$

$$(iv) \begin{array}{l} \text{① } xy + x'yz + yz \\ \text{② } \text{③ } \text{④ } \\ \text{⑤ } \end{array} = \begin{array}{l} xy + x'z + yz(x+x') \\ = xy + x'z + xyz + x'yz \\ = xy(1+z) + x'z(1+y) \\ = \begin{array}{l} \text{⑥ } xy + x'z \\ \text{⑦ } \text{⑧ } \text{⑨ } \end{array} \end{array}$$

$$\Rightarrow \text{Complement of a function} : (A'BC' + A'B'C)' = (A'BC')'(A'B'C)'$$

$$= (A+B'+C) \cdot (A+B+C') = A + AB + AC' + B'A + B'C' + CA + CB + CC'$$

$$= A + A(B+B'+C+C') + B'C' + BC = A + BC + B'C'$$

\Rightarrow Minterms

\downarrow (Sum of Products) & Maxterms (P.O.S)
For two variables \rightarrow

$$x'y, xy', x'y', xy.$$

$$\text{For three variables} \rightarrow x'y'z', x'y'z, x'y'z, x'yz,$$

$$xy'z', xyz', xyz, xy'z.$$

For n variables $\rightarrow 2^n$ minterms.

\rightarrow Complement of minterm is maxterm.

here, $x' \rightarrow 1$

$x \rightarrow 0$

\rightarrow Any boolean exp can be represented as standard form.

\rightarrow Standard Exp.

i) Sum of Minterms

ii) Product of maxterms

$\rightarrow F(A, B, C) = \sum(1, 4, 5, 6, 7) \rightarrow$ minterms.

1	001	$x'y'z$
4	100	$xy'z'$
5	101	$xy'z$
6	110	xyz'
7	111	xyz

$\rightarrow F = \sum(1, 3, 6, 7)$ \rightarrow conversion from one form
 $F = \prod(0, 2, 4, 5)$ to another.

~~QUESTION~~ → Subtraction with complement

two n-digit unsigned number M & N base '8'
for $M - N$

- ① Add M to 8's complement of N
8's complement of N = $8^n - N$
 $\therefore (M + 8^n - N)$

- ② If $M \geq N$, the sum will produce an end carry 8^n
which is discarded.

- ③ If $M < N$, sum does not produce an end carry.
It is equal to

$$8^n - (N - M)$$

which is 8's complement of $N - M$. To obtain the answer, take the familiar term, take the 8's complement of the sum and place -ve sign.

Q: Find $72532 - 3250$, using 10's complement

$$M = 72532$$

$$N = 03250$$

$$10\text{'s complement of } N = 10 - 03250$$

$$\begin{array}{r} 99999 \\ 03250 \\ \hline 96749 \end{array}$$

$$\begin{array}{r} \text{Now, } 72532 + 96750 \xrightarrow{\text{10's complement}} 96750 \\ = 169282 \end{array}$$

$$= 169282 - 10^5$$

$$= \underline{69282}$$

Ans.

Q2 Find $3250 - 7253_2$

10's complement

Step 1:- $M = 3250$ $\rightarrow 96750 + 1 + 1$

$$N = 7253_2 \rightarrow 27464$$

$$10^5 \text{ complement} \rightarrow \underline{\underline{27468}} + 1$$

Now,

$$3250 + 27468$$

$$= 30718$$

Step 2: Now, 10's complement of 30718

$$\begin{array}{r} 99999 \\ \underline{\underline{30718}} \\ 69281 \\ + 1 \\ \hline 69282 \end{array}$$

Step 3: Putting -ve sign $\rightarrow -69282$.

Q. $X = 1010100$

$$Y = 1000011$$

$$84 - 62 = 22$$

$$62 - 84 = -22$$

Perform the subtraction using two's complement-

(i) $X - Y$ (ii) $Y - X$.

(i) 9's complement of $Y = \cancel{010101} + 1 \quad 011100 + 1$

$$\begin{array}{r} \cancel{010101} \\ + 1 \\ \hline \cancel{010110} \end{array}$$

9's complement $\rightarrow 011101$

$$\begin{array}{r} 1010100 \\ + 0111101 \\ \hline 10010001 \end{array}$$

$$- 2^7 = (0010001)_2 \approx (17)_{10}$$

(11) 2's &

$$2^{\text{nd}} \text{ complement of } 5 = \begin{array}{r} 010101 \\ + 1 \\ \hline 010110 \\ + 1000011 \\ \hline \cancel{10} \cancel{00011} \\ - 1 \\ \hline 1010100 \\ \hline 1101111 \end{array}$$

2's complement of

1's complement $\underline{0010000}$

$$\begin{array}{r} + 1 \\ \hline - (0010001) = \end{array}$$

\Rightarrow Signed Binary No's :-

Signed magnitude Notation \rightarrow 1st bit represents
represent - magnitude.

1. Signed 1's complement :

$$+9 \rightarrow 01001$$

$$-9 \rightarrow 10\underline{1}00$$

2. Signed 2's complement :

$$+9 \rightarrow 01001$$

$$\begin{array}{r} 10110 \\ + 1 \\ \hline 10111 \end{array}$$

\Rightarrow In 8 bits, signed 1's complement representation of -9
 $+9$: 00001001

1's complement, -9 : $10001001 \boxed{111.0110}$

246

246
246

Signed two's complement of -9

$$\begin{array}{r} 11110110 \\ +1 \\ \hline 11110111 \end{array}$$

⇒ Binary Codes:

With 'n' bits of 0 & 1, how many diff. patterns are possible -

2^n patterns are possible

① → BCD: Binary coded decimal : Coding of decimal no. in binary
Total no. of bits = (no. of digits * 4)

② → Excess 3 code :- adding 3 to binary equivalent

0	0011
1	0100
2	0101
3	0110
4	0111
5	1000
6	1001
7	1010
8	1011
9	1100

③ $8421 \rightarrow$

$$\begin{array}{l} 8 \rightarrow 0000 \\ 4 \rightarrow 0100 \\ 2 \rightarrow 0010 \\ 1 \rightarrow 0001 \end{array} \rightarrow 0001-2 \times 1 = 2$$
$$0010-2 \times 1 = 1$$

④ $2421 \rightarrow$

$$\begin{array}{l} 2 \rightarrow 0000 \\ 4 \rightarrow 0001 \\ 2 \rightarrow 0010 \\ 1 \rightarrow 0001 \end{array} \rightarrow 2$$

→ When internal arithmetic in computer is done in decimal form, then BCD is used.

$$\begin{array}{r} 00010010 \\ +13 \\ \hline 00010011 \end{array}$$

→ Advantages of excess 3, 2421 & 84-2-1,
 The excess 3, 2421 and 84-2-1 are say complementary
 codes.

9's complement of a number is obtained by changing
 1's to 0's & 0's to 1's.

→ ex -

<u>1001</u>	\rightarrow 9 (Binary)	\rightarrow -1 (Signed magnitude representation)
9 (BCD)	6 (Excess 3)	+ (84-2-1)
<u>0110</u>	\rightarrow 6	9 (Q421)

$$\begin{array}{r} 0110 \\ + 1 \\ \hline 0111 \end{array} \rightarrow 7 \Rightarrow -7 \text{ (Signed 1's complement method)}$$

$$10^{\text{'}s \text{ complement}} \rightarrow 8_2 \quad \cancel{\begin{array}{r} 0101 \\ + 1 \\ \hline 1000 \end{array}}$$

$$\begin{array}{r} 23 \\ - 18 \\ \hline \end{array} \quad (M-N)$$

$$N \rightarrow 18 \quad \begin{array}{r} 8_1 \\ + 1 \\ \hline \end{array}$$

$$10^{\text{'s complement}} \rightarrow 8_2 \quad \cancel{\begin{array}{r} 0101 \\ + 1 \\ \hline 1000 \end{array}}$$

$$\begin{array}{r} 23 \\ 82 \\ \hline 105 \end{array}$$

→ 84-2-1 :

$$\begin{array}{r} 5 \\ - 4 \\ \hline \end{array} \quad \begin{array}{r} 1011 \\ 0100 \\ \hline \end{array}$$

$$\begin{array}{r} 9^{\text{'s}} \text{ complement} \\ \rightarrow \left. \begin{array}{r} 0100 \\ 1011 \\ \hline \end{array} \right\} \rightarrow 0101 \quad \left. \begin{array}{r} 0100 \\ \hline 10001 \end{array} \right\} \rightarrow 10^{\text{'s complement}} \end{array}$$

$$\rightarrow \text{discarding carry} \Rightarrow 0001 = \underline{\underline{1}}$$

→ Error Detection code :-

Parity scheme

Parity → no. of ones.

Odd parity

00 00	1
01 01	1

even parity

00 00	0
0 0 01	1

→ Gray Code:- Only one bit varies b/w successive numbers.

Gray Code

0000
0001
0010
0011
0110
0111
0101

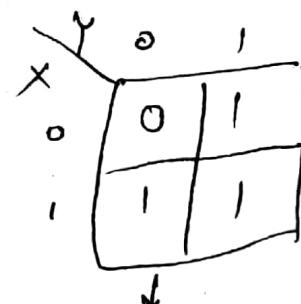
Decimal Equivalence

0
1
2
3
4
5
6

→ Two & Three variable Maps:-

$$F_1 = x + y$$

x	y	x+1
0	0	0
0	1	1
1	0	1
1	1	1

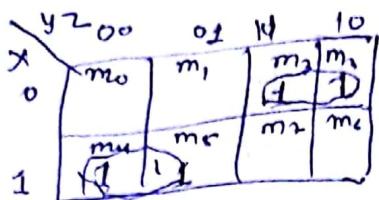


K map rep
of $x+y$

combinations are made in '2' powers $\rightarrow 2, 4, 8$ -

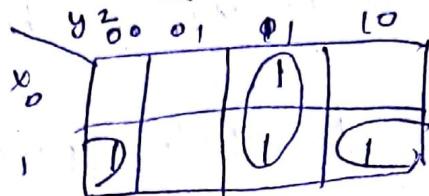
3-variable :-

Q1. $f(x, y, z) = \sum (2, 3, 4, 5)$
 $= m_0 + m_3 + m_4 + m_5$



$$x'y + y'z + xy = \overline{x}y + yz$$

Q2. $f(x, y, z) = \sum (3, 4, 5, 7)$



$$\Rightarrow yz + x\bar{z}$$

20/08/18

\Rightarrow Prime Implicant : A prime implicant is a product-term obtained by combining the maximum possible no. of adjacent squares in the map. A single 1 on a map represents a prime implicant if it is not adjacent to any 1.

If a minterm in a square is covered by only one prime implicant in that implicant is said to be an essential prime implicant.

$$f(A, B, C, D) = \{0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 14\}$$

Prime Implicants

$C'D'$, $B'C$, AD , AB'	$f = BD + B'D' + CD + AD$
$B'D$, $B'D'$	$= BD + B'D' + CD + AB'$
\downarrow Essential	$= BD + B'D' + B'C + AD$
	$= BD + B'D' + B'C + AB'$

covered
by only
one prime
implicant.

A_8	$C'D$	CD	00	01	11	10
			1		1	1
				1	1	
			.	1	1	
			1	1	1	1

⇒ Five variable Maps :-

$$A=0$$

0	1	3	2
4	5	+	6
12	13	15	14
8	9	11	10

$$A=1$$

16	17	19	18
20	21	23	22
28	29	31	30
24	25	27	26

Ques:-

$$F(A, B, C, D, E) = \{0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31\}$$

$$A=0$$

$$A'=1$$

	DE	00	01	11	10
BC					
00		1			1
01		1			1
10		1	1		
11		1			

	DE	00	01	11	10
BC					
00	
01			1	1	
11			1	1	
10		.	1	1	.

$$A'B'E' + A'B'D'E + ACE$$

there are six minterms from 0 to 15 belonging to $A=0$ part of the map. Other five terms belongs to $A=1$. The two squares in column 01 & the last two rows are common to both parts of the map.

31/08/18

CIRCUIT

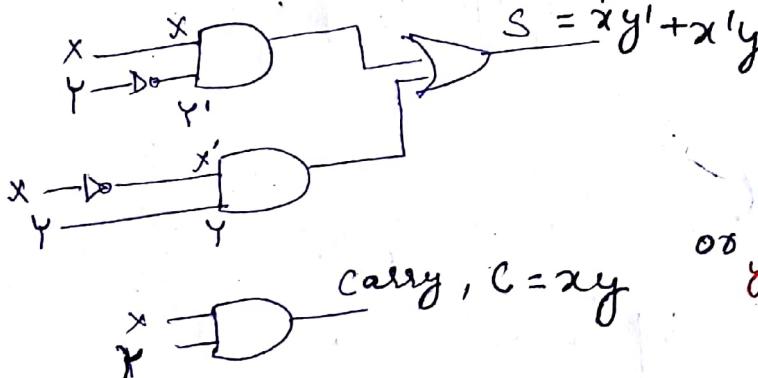
A combinational circuit performs an operation that can be specified logically by a set of Boolean functions.

In contrast, sequential storage elements in addition to logic gates.

→ Adder design:-

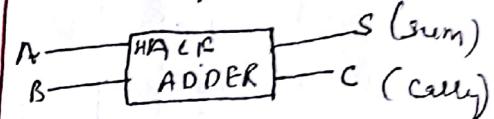
HALF ADDER	bit 1	bit 2	Sum	Carry
	0	0	0	0
	0	1	1	0
	1	0	1	0
	1	1	0	1

$$S \text{ (sum of product)} = x'y + xy' \\ C \text{ (carry)} = xy$$

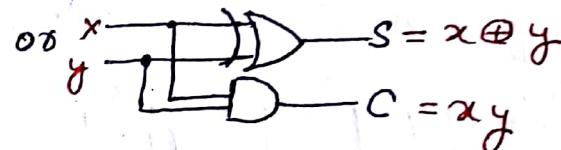


A combinational circuit that performs addition of two bits is called half adder.

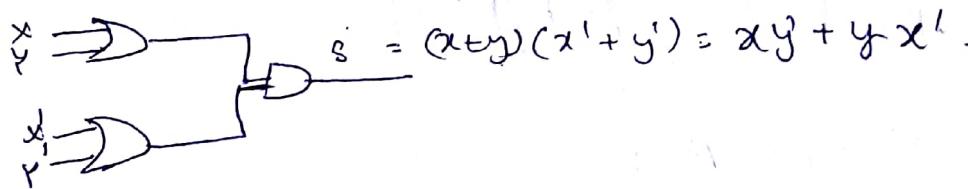
Half adder adds two single bit no.



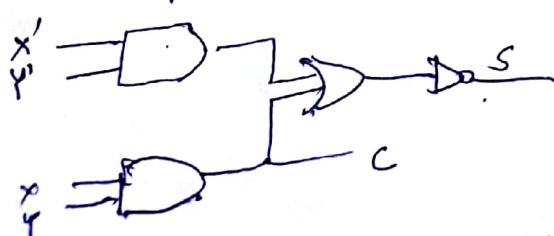
→ Implementation of half adder.



To implement using 2 OR gate & 2 AND gate

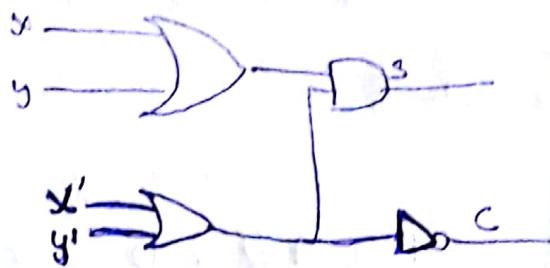


To implement using 2 AND, 1 OR & 1 inverter.



$$S = (C + x'y)' \\ = (C') \cdot (x'y)' \\ = (xy)' \cdot (x+y) \\ = (x'+y') \cdot (x+y) \\ = x'y + xy'$$

→ To implement using 2 OR gate, 1 AND gate, 1 inverter.

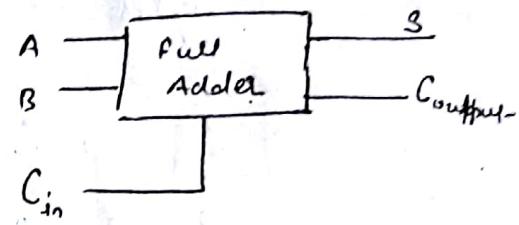


$$S = (x + y)(x'y') = xy' + y'x$$

$$C = (xy)^1 = xy$$

⇒ FULL ADDER :- It performs arithmetic sum of three bits.

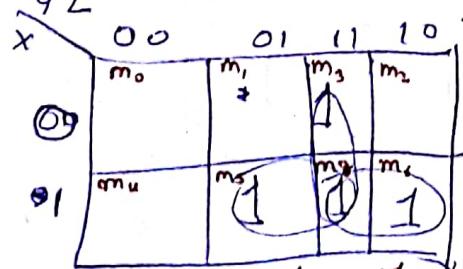
x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1



$$S = x'y'z + x'yz' + xy'z' + xyz$$

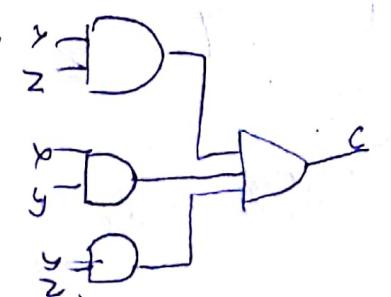
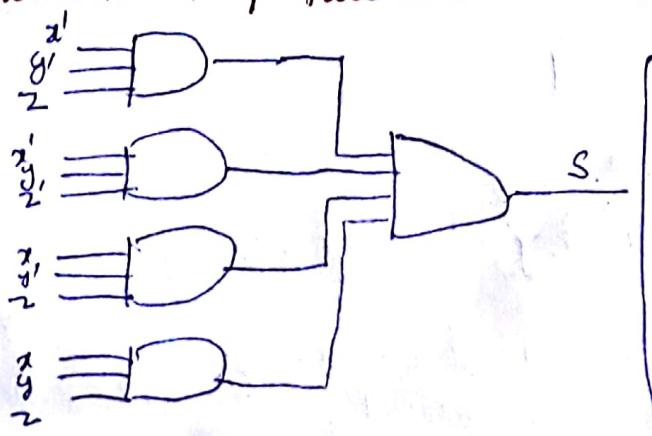
Carry

$$C = x'y'z + xy'z + xyz' + xyz$$



$$C = x_2 + xy + yz$$

⇒ Implementation of full adder in SOP form.



\Rightarrow HALF SUBTRACTOR :- (Borrow) : To subtract two binary bits.

X	Y	B	D (difference)
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

for $X - Y$

$$D = A \oplus B \quad D = X \oplus Y$$

$$B = \overline{A} \oplus B \quad B = \overline{X} Y$$

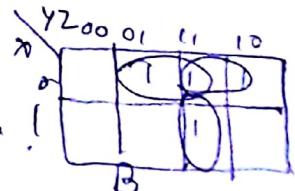
\Rightarrow FULL SUBTRACTOR :-

X	Y	Z	B	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = x'y'z + x'yz' + xy'z' + xyz$$

$$B = x'y'z + x'yz' + xy'z + xyz$$

$$B = x'z + x'y + yz \quad (\text{Simplified})$$



Code Conversion

Input - BCD				Output - excess-3 code			
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0

Sum of products (w) = $0101 \rightarrow 5$

AB	00	01	11	10
00				
01	1	1	1	
11	X	X	X	X
10	1	1	X	X

$0110 \rightarrow 6$

$0111 \rightarrow 7$

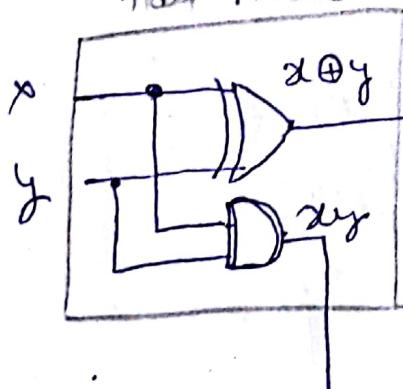
$1000 \rightarrow 8$

$1001 \rightarrow 9$

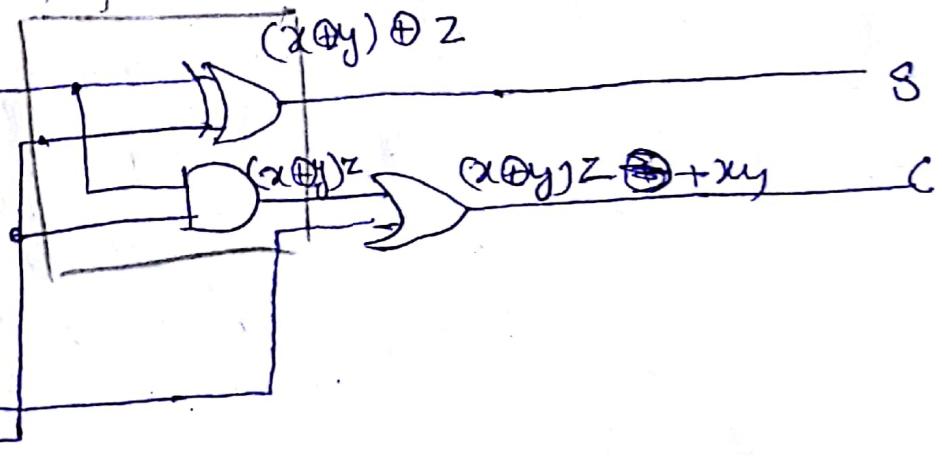
$$w = A + AB + BC$$

⇒ Implementation of full adder with two half adder & OR gate

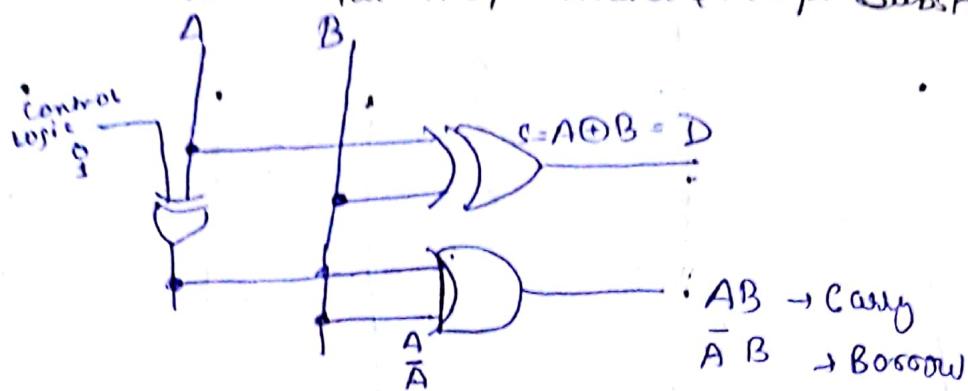
Half Adder



Half-Adder



\Rightarrow Common circuit for Half Adder & Half Subtractor.



Note: we need min \approx 5/9 no. of N AND gate to design H.A/H.S.

" " " 5/9 n N AND " " " H.A/H.S.

" " " 5/9 " NOR " " " H.A.

" " " 5/9 " NOR " " " H.S.

Exclusive OR \rightarrow commutative & associative

$$a \oplus b = b \oplus a$$

$$a \oplus (b \oplus c) = (a \oplus b) \oplus c$$

1111 \rightarrow 0111
sender receiver

single error detection code : message + parity.

message

1111
 \downarrow
1011

Parity Bit

\downarrow
even matching Parity

$$x \oplus 0 = x$$

$$x \oplus x' = 1$$

$$x \oplus 1 = x'$$

$$x \oplus y' = (x \oplus y)'$$

$$x \oplus x = 0$$

$$x' \oplus y = (x \oplus y)'$$

$$A \oplus B \oplus C$$

$$x \oplus y = x'y + xy'$$

$$\Rightarrow A \oplus (B'C + BC')$$

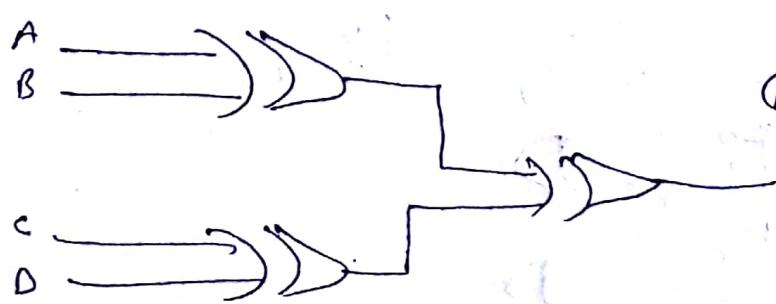
$$\Rightarrow AB'C' + A'BC' + ABC + A'B'C$$

100 010 111 001

\Rightarrow Parity generator



$$(A \oplus B) \oplus C$$



$$(A \oplus B) \oplus (C \oplus D)$$

14/Sept/18:-

Hindi: Dinesh
(18F) - 1st year

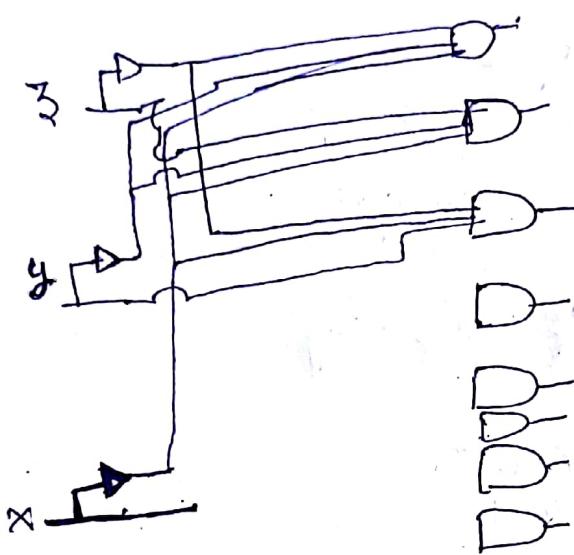
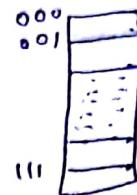
⇒ Decoder & Encoding :- (OR AND Gate)

Standard Combinational Circuits

2. DECODER:- A decoder is a combinational circuit that converts binary information from n input lines to a max^m of 2^n output lines.

$\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$
3 bits

$\Rightarrow 2^3 = 8$ can be addressed



$$\begin{aligned} D_0 &= x'y'z' \quad (000) \\ D_1 &= x'y'z \quad (001) \\ D_2 &= x'y'z' \quad (010) \\ &\vdots \\ D_7 &= \dots \end{aligned}$$

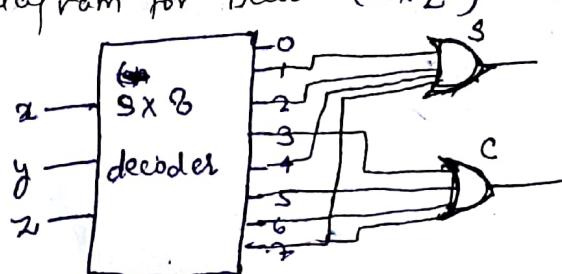
Only one output will be activated at a time
rest will be zero.

⇒ Full adder with a decoder and two OR gates.

$$S(x, y, z) = \sum 1, 2, 4, 7 \quad (\text{Sum})$$

$$C(x, y, z) = \sum 3, 5, 6, 7 \quad (\text{Carry}).$$

* Block diagram for Decoder ($n \times 2^n$)



Q. ENCODER:- (OR Gate)

$2^n \rightarrow \text{input}$

$n \rightarrow \text{output}$

A encoder is a digital circuit that performs the inverse operation of a decoder.

Truth Table for Octal to binary encoder :-

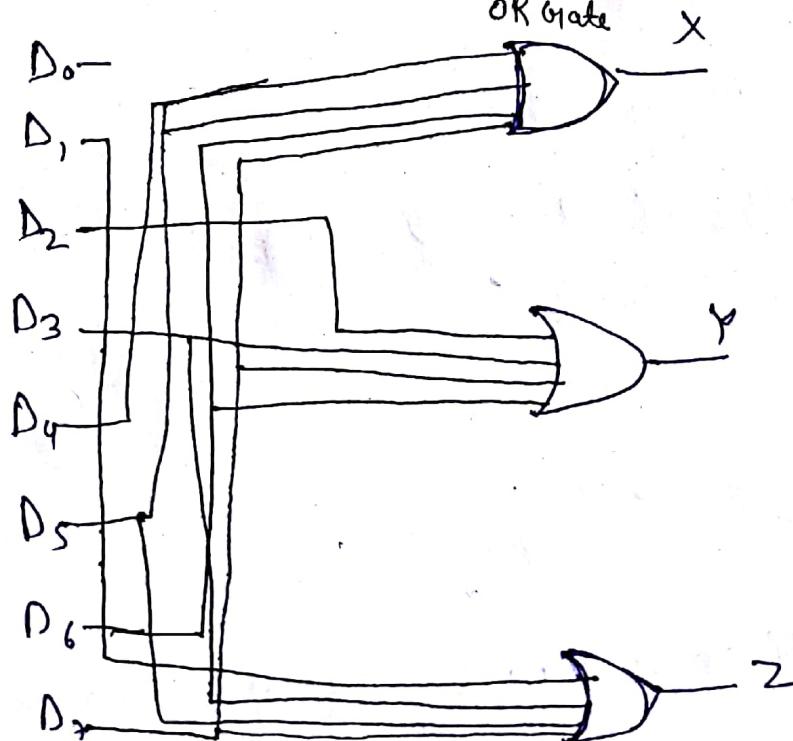
Input - s							Output -		
D_0	D_1	D_2	D_3-D_4	D_5	D_6	D_7	X	Y	Z
1	0	0	0 0	0 0	0 0	0 0	0 0 0	0 0 0	0 0 0
0	1	0	0 0	0 0	0 0	0 0	0 0 1	0 0 1	0 0 1
0	0	1	0 0	0 0	0 0	0 0	0 1 0	0 1 0	0 1 0
0	0	0	1 0	0 0	0 0	0 0	0 1 1	0 1 1	0 1 1
0	0	0	0 1	0 0	0 0	0 0	$\left\{ \begin{array}{l} D_4 \leftarrow 1 \cdot 0 \cdot 0 \\ D_5 \leftarrow 1 \cdot 0 \cdot 1 \\ D_6 \leftarrow 1 \cdot 1 \cdot 0 \\ D_7 \leftarrow 1 \cdot 1 \cdot 1 \end{array} \right.$		
0	0	0	0 0	0 1	0 0	0 0			
0	0	0	0 0	0 0	1 0	0 1			
0	0	0	0 0	0 0	0 1	1 0			
0	0	0	0 0	0 0	1 1	1 1			

$$X = D_4 + D_5 + D_6 + D_7$$

$$Y = D_2 + D_3 + D_6 + D_7$$

$$Z = D_1 + D_3 + D_5 + D_7$$

OR Gate

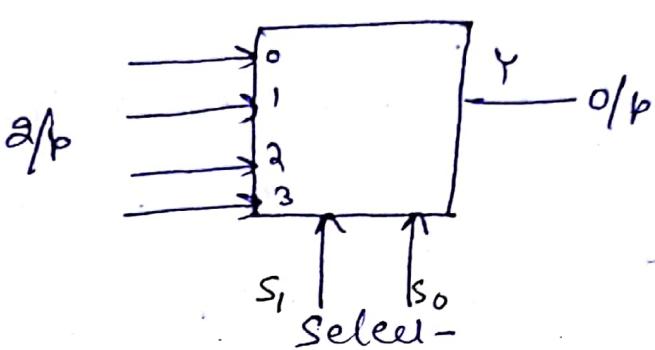


Note: D_0 never used

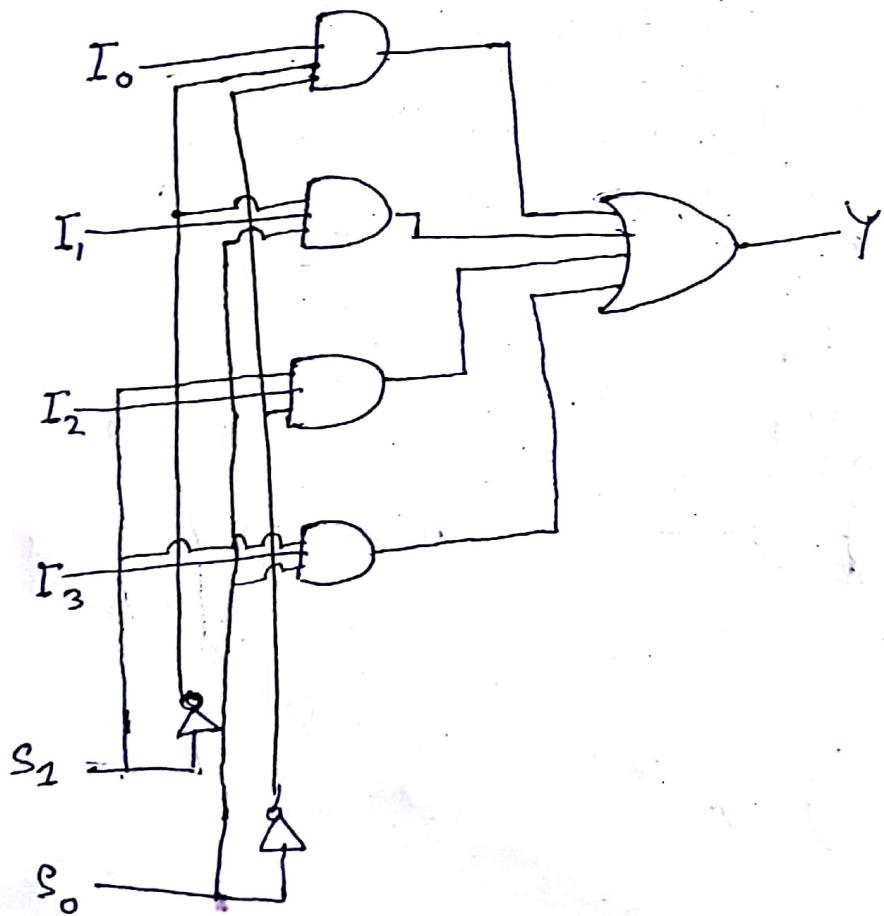
Multiplexer :-

Transmitting a large information over a small no. of lines.

Block Diagram :-



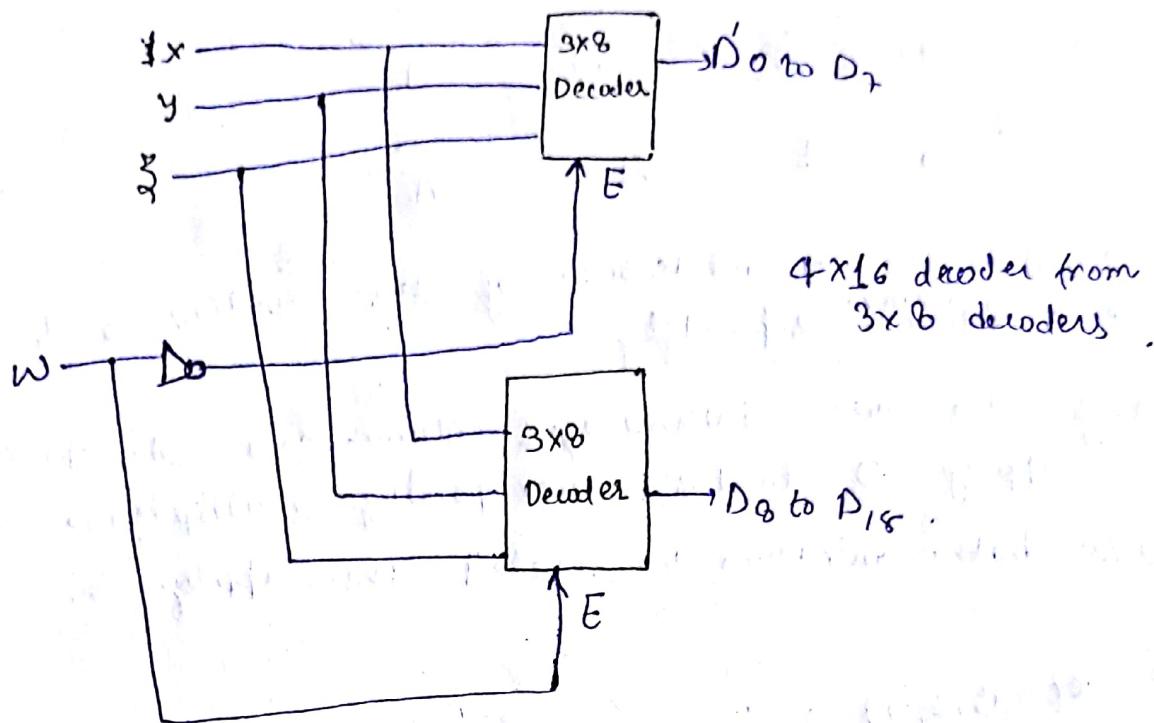
S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



⇒ At any moment, almost one of I_0, I_1, I_2 & I_3 will be activated rest will be zero.

No. of selection lines be m & no. of inputs be n , then

$$2^m = n$$



⇒ Using multiplexers for implementing any boolean function

Any boolean fun of $n+1$ variable can be implemented with 2^n to 1 multiplexer ($2^n \times 1$).

$$\text{Ex} \rightarrow F(A, B, C) = \sum 1, 3, 5, 6$$

	A	B	C	F
0.	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

We will use two variables for selection lines & one variable for input.

	I_0	I_1	I_2	I_3
A'	0	1	2	3
A	4	5	6	7
	0	1	A'	A'

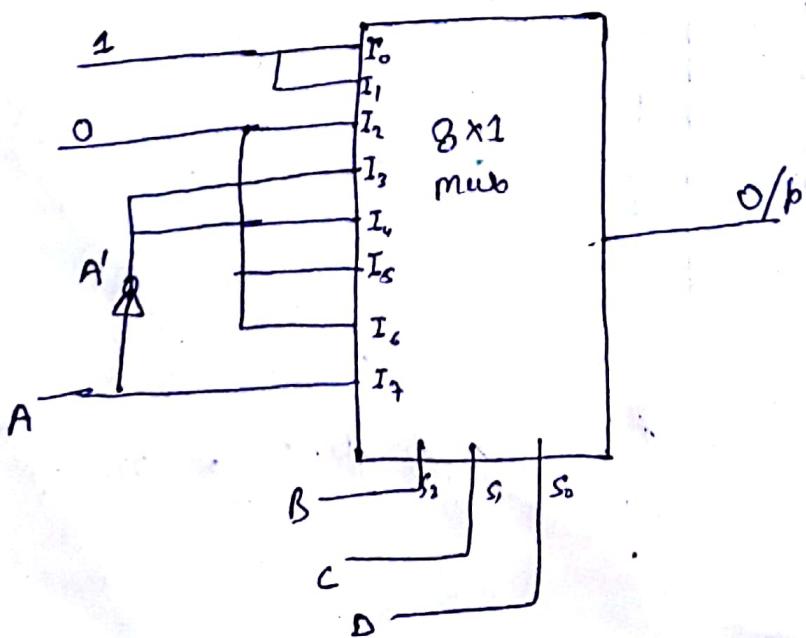
Circle all the minterms of the function and insert each element separately.

- ① If the two minterms of a column are not circled apply 0 to the corresponding multiplexer input.
- ② If both minterms are circled then apply 1.

$$Q: f(A, B, C, D) = \sum 0, 1, 3, 4, 8, 9, 15$$

SOL: No. of selection lines = 3
 No. of variables = 3
 No. of Input = 16

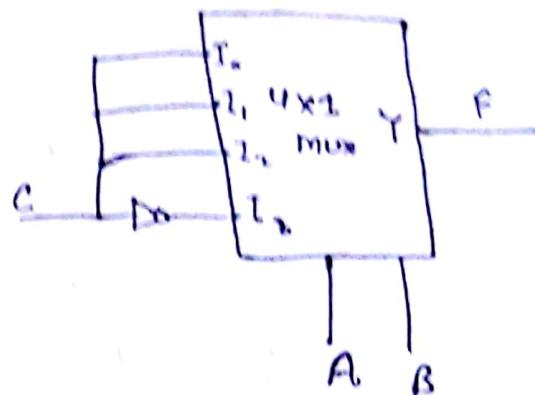
	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7
A'	0	1	2	3	4	5	6	7
A^{\neq}	8	9	10	11	12	13	14	15
	1	1	0	A'	A'	0	0	A'



$$Q \quad f(A, B, C) = (1, 3, 5, 6)$$

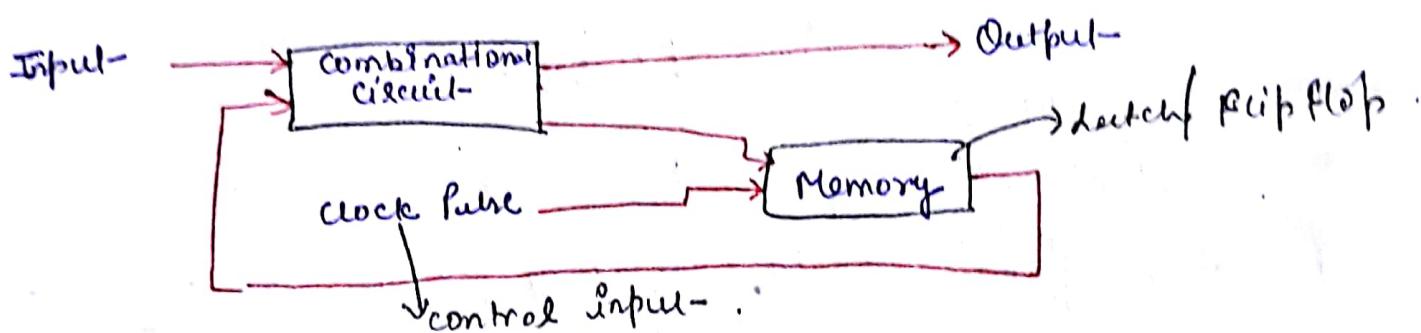
Min. no. of retention lines = 2.

	T_0	T_1	T_2	T_3
C'	0	2	4	(6)
C	1	3	5	7



SEQUENTIAL CIRCUIT

Sequential Circuit is a combinational circuit with memory.



\Rightarrow Types of latches :-

- I) S-R (Set--Reset-type)
- II) D (Delay) Type
- III) J-K type
- IV) T (Toggle) type.

5/10/18

Digital Circuits

Combinational Sequential

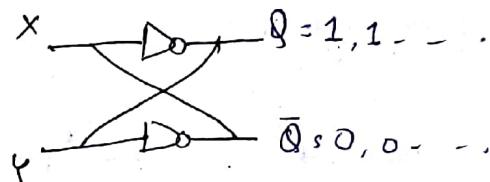
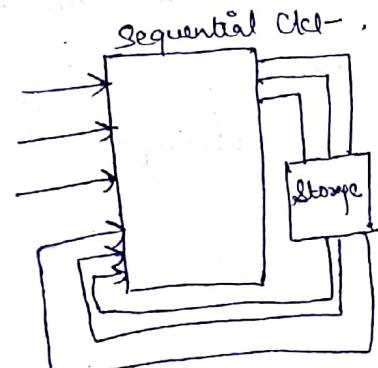
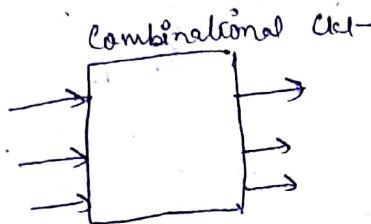
Output depends only
in the present
inputs

O/p depends on the present o/p as
well as previous o/p.
{ we have memory
to store past o/p. or storage element }

⇒ why sequential circuit are necessary.

Circuit for incrementing a number by 1.

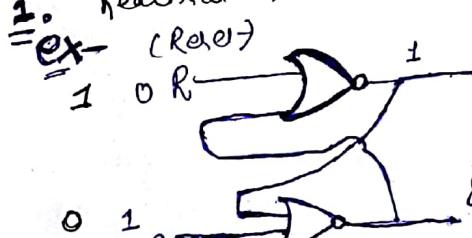
ex- Traffic light controller



{ Either 1 or 0 , value will be
held }

latch circuit
(to lock or hold something)
(data)

⇒ S-R latch → two cross coupled NOR gates. \Rightarrow State table
Realization with NOR gates :-



$S = 1$	$R = 0$	$Q = 1$	$\bar{Q} = 0$
$S = 0$	$R = 0$	$Q = 1$	$\bar{Q} = 0$ (2nd)
$S = 0$	$R = 1$	$Q = 0$	$\bar{Q} = 1$
$S = 0$	$R = 0$	$Q = 0$	$\bar{Q} = 1$

Current state

case 3 undesirable condition.

$S = 1, R = 1, Q = 0, \bar{Q} = 0$ (forbidden)

As we can see, after removing inputs we get
same o/p, hence its k/l as memory.

\rightarrow unpredictable
fast-gate will be 1 &
slower will be zero.

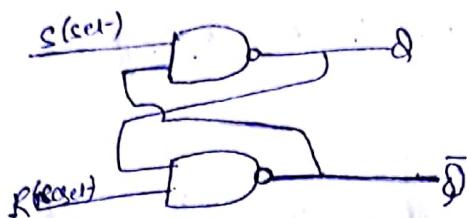
\Rightarrow Reset $\rightarrow Q = 0$

Set $\rightarrow Q = 1$

\Rightarrow when Reset = 1, o/p i.e. $Q = 0$ when Set = 1 ; $Q = 1$.

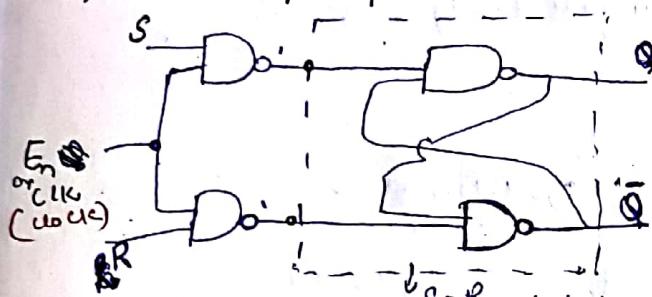
⇒ Realization with NAND Gate :-

Note the diff. in circuits
of rep. with NOR and NAND gates.



X	Y	O/p
0	0	1
0	1	1
1	0	1
1	1	0

→ control input :-



when we have En, then above circuit is SR-latch.
when we have CLK, then it is flip-flop.
⇒ Above ~~circuit~~ circuit can act as both
latch & flip-flop.

⇒ State table

S	R	Q	\bar{Q}
1	0	0	1
1	1	1	0
0	1	1	0
1	1	1	0
0	0	1	1
1	1	1	1

memory state
Quiescent state
Not used state.

I (after $S=0, R=0$)
 Q (after $S=0, R=1$)
 Q (forbidden)
unpredictable (race condition)

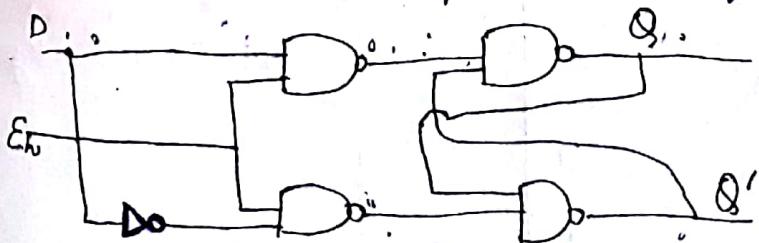
Eh	S	R	Next stage of Q
0	x	x	No Change
1	0	0	No Change
1	0	1	$Q=0$, reset stage
1	1	0	$Q=1$, set stage
1	1	1	Indeterminate

S.R. latch with control input.

6/10/18.

→ There is problem of undesirable input with both realisation hence we have D-latch.

→ hold data
→ D-latch :- (transparent-latch)



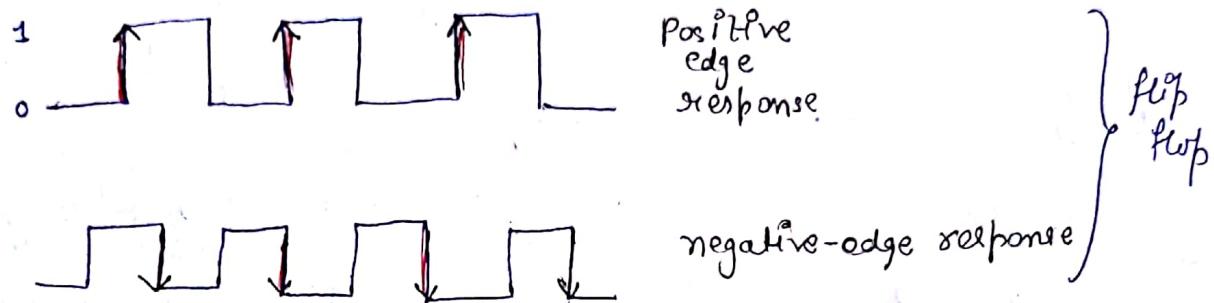
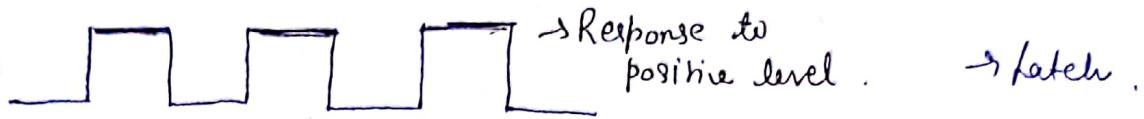
⇒ State table

Eh	D	next-state of Q
0	x	No change
1	0	$Q=0$, reset state
1	1	$Q=1$, set state

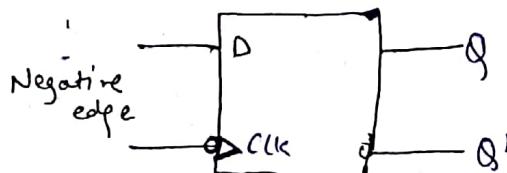
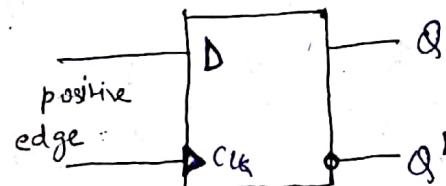
5 \Rightarrow Difference b/w latch & flip-flop :-

Both latches and flip-flops are storage elements.

~~latch~~ Latches operate with signal levels & flip flops operate with signal transitions.



\Rightarrow Flip-flop

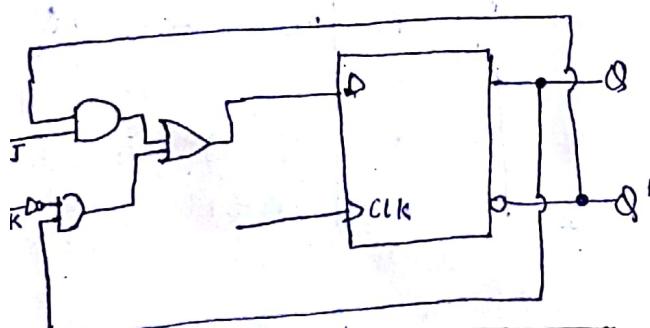


D- flip flop

D	$Q(t+1)$
0	0
1	1

holds the input -

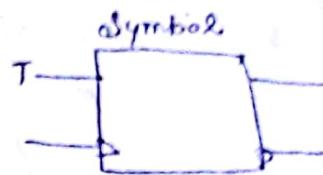
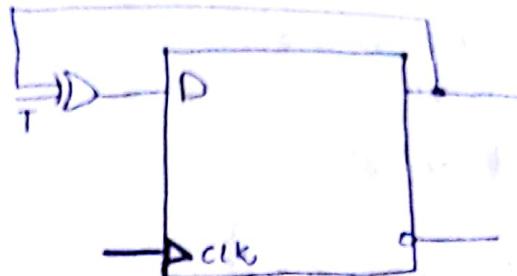
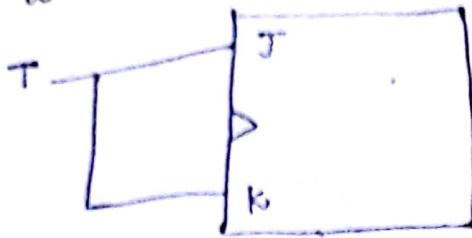
\Rightarrow JK flip flop :-



$$D = JQ' + K'Q$$

J	K	$Q(t+1)$
0	0	$Q(t)$ (memory)
0	1	0
1	0	1
1	1	$Q'(t)$

⇒ T - flip flop
Toggle (change)



T flip flop

$$T \quad Q(t+1)$$

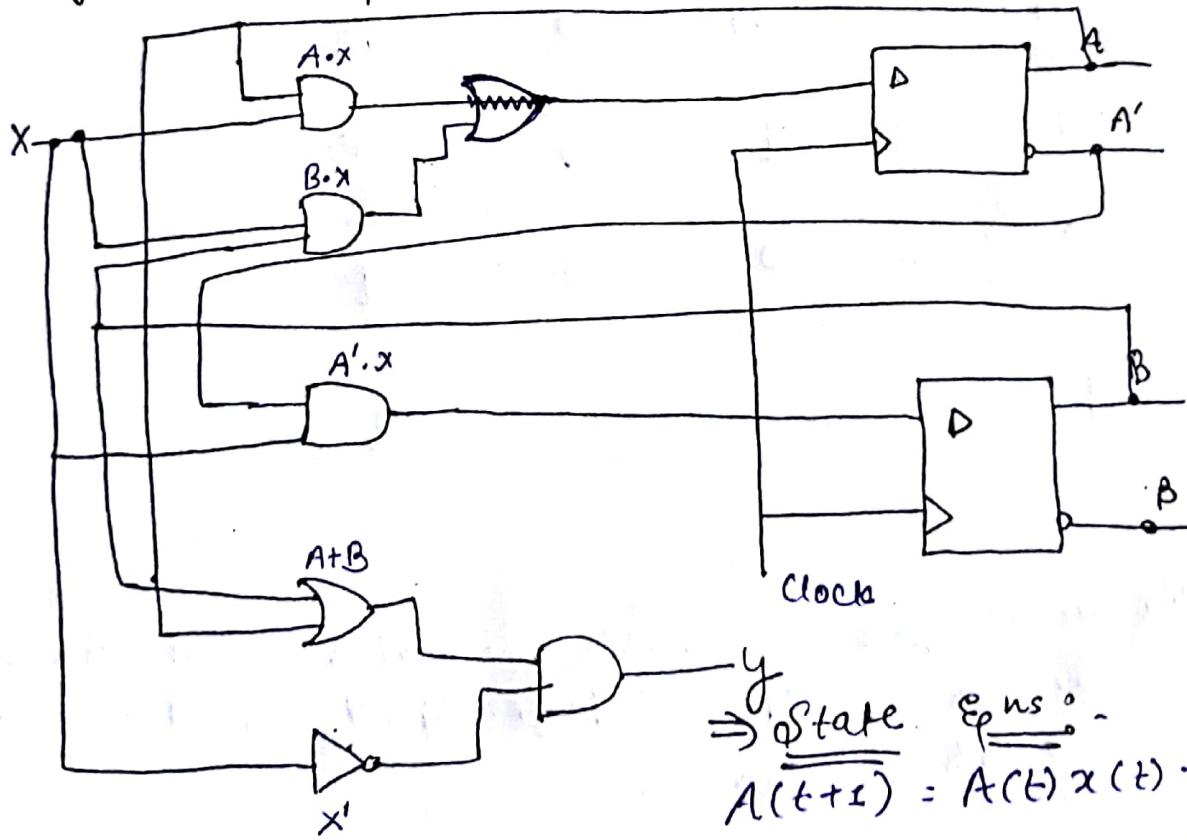
0 $Q(t)$ no change

1 $Q'(t)$ complement

T flip-flop toggle the input-, when $T=1$, Q/b output charged.

8/10/2018 : Indian Airforce Day.

⇒ Analysis of Sequential Circuit :-



⇒ State $\overset{\text{Eqns}}{=} -$
 $A(t+1) = A(t)x(t) + B(t)x(t)$

$$B(t+1) = A'(t)x(t)$$

$$y(t) = \cancel{(A+B)} [A(t)+B(t)]x(t)$$

5 or

$$A(t+1) = Ax + Bx$$

$$B(t+1) = A'x$$

$$y = (A+B)x'$$

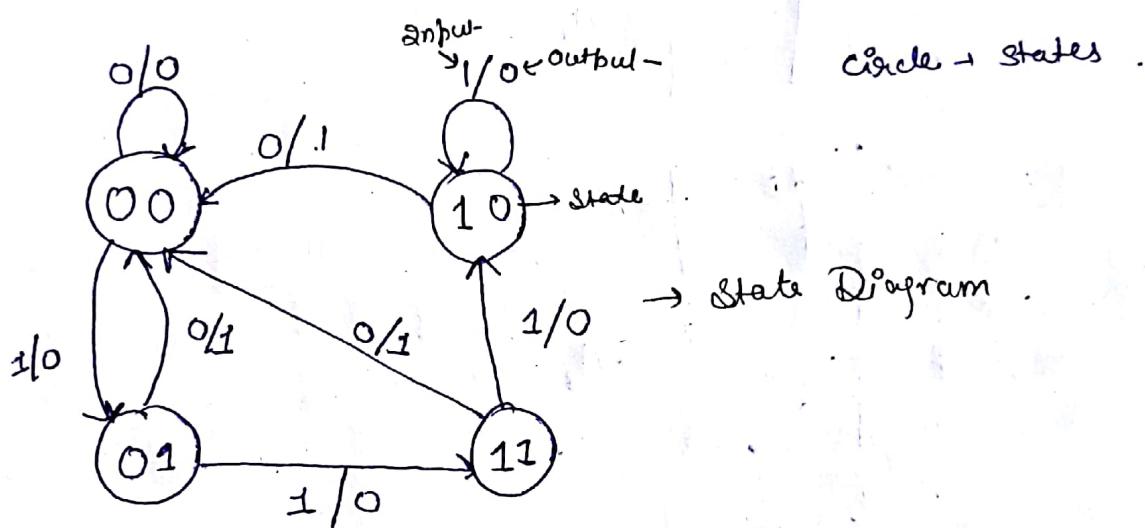
\Rightarrow Analysis of Sequential Circuit
state Table.

Present-state		input- x	Next state		Output- y
A	B		A	B	
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

\Rightarrow State Diagram:- the information available in the state table can be represented graphically in the form of State Diagram.

⇒ Second Form of State Table.

Present - state		Next - state				Output -	
		$x = 0$		$x = 1$		$x = 0$	$x = 1$
A	B	A	B	A	B	y	y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0



⇒ Steps for Analysis of Sequential Circuit:-

- ① Circuit-Diagram \rightarrow State eqns
- ② \rightarrow State table
- ③ \rightarrow State Diagram

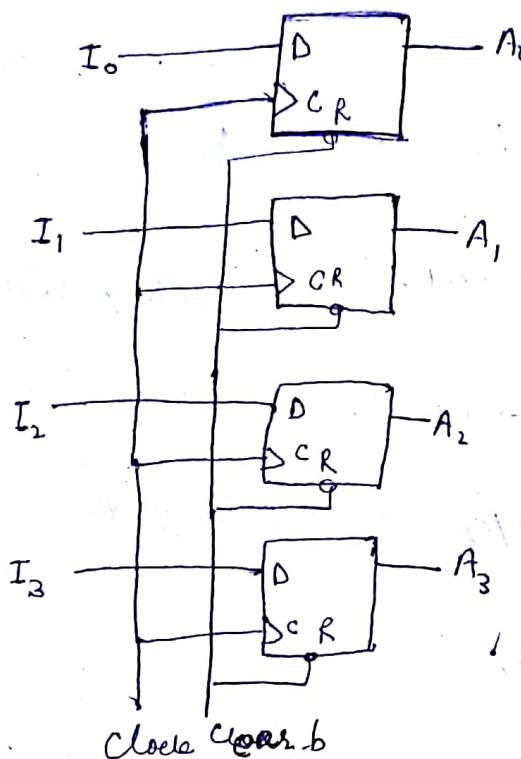
26/10/18 :-

5

Registers and Counters :-

Register :- A group of flip-flops, each one of which shares a common clock & is capable of storing 1 bit of information.
An n-bit register has n flip-flops.

Counters :- It is basically a register that goes through a predefined sequence of states.

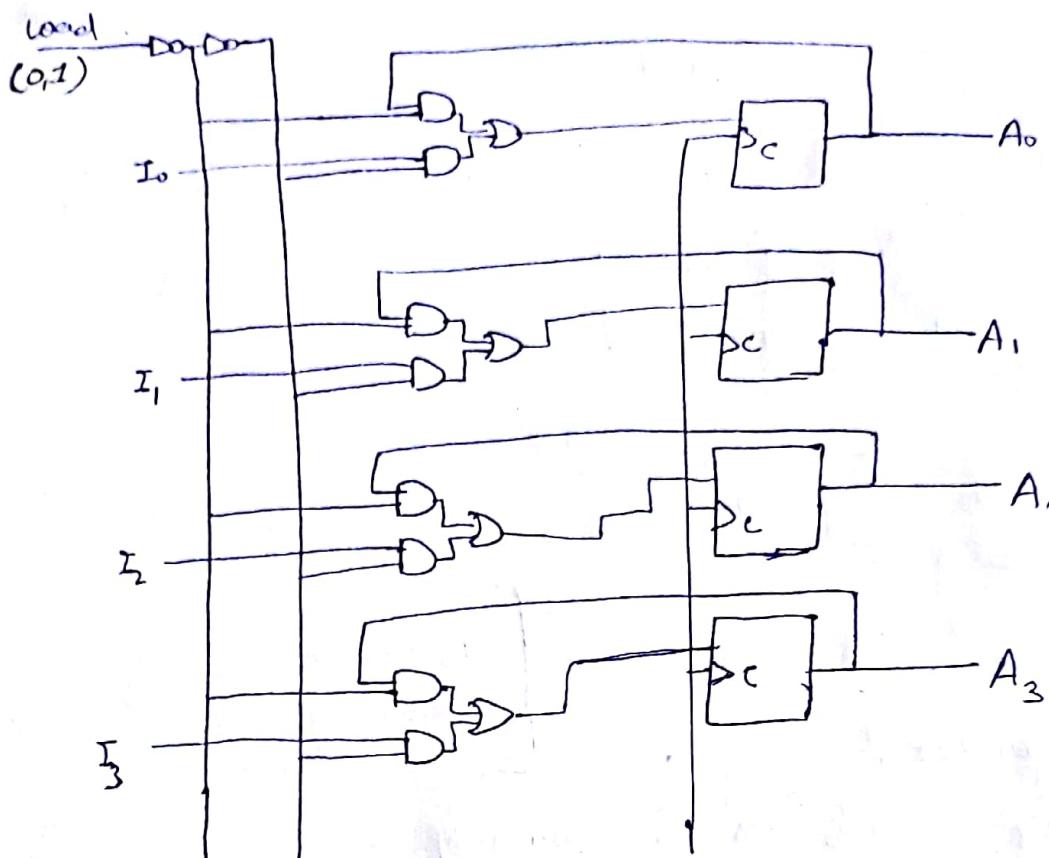


The common clock input triggers all flip-flops on the positive edge and the binary data available at the four inputs is transferred into the registers.

Clear-b input is useful for closing the register to all 0's if no free input is present.

⇒ Register with parallel load :- All the inputs should appear at q/b at same time.

Using clock input for creating the register with parallel load creates problems in synchronization.

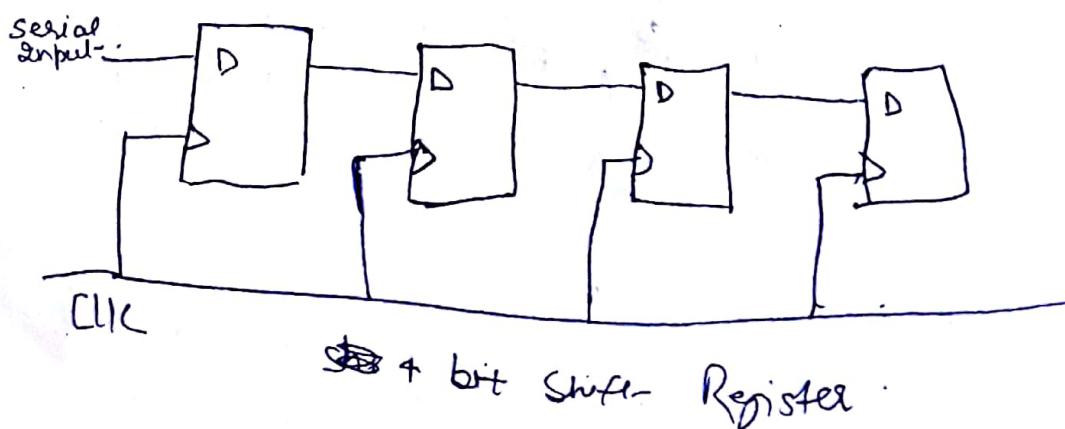


When load input is 1, the data in four inputs is transferred into the register, with next positive edge of the clock.

When load input is 0, the output of the flip-flops are connected to their respective inputs.

The feedback connection from Q/p to i/p is necessary because D flipflop do not have a no change condition.

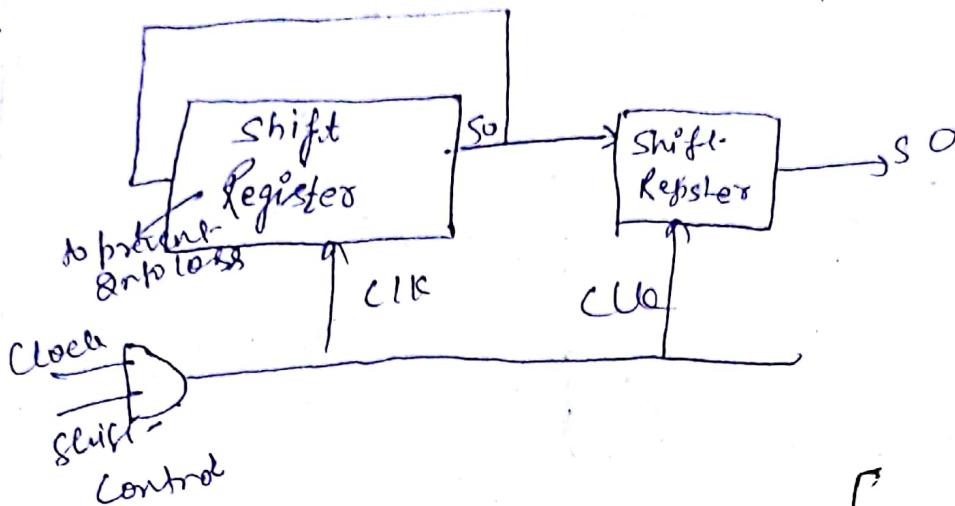
⇒ Shift- Register :- If register capable of shifting the binary information is called Shift- Register.



5

Serial Transfer:-

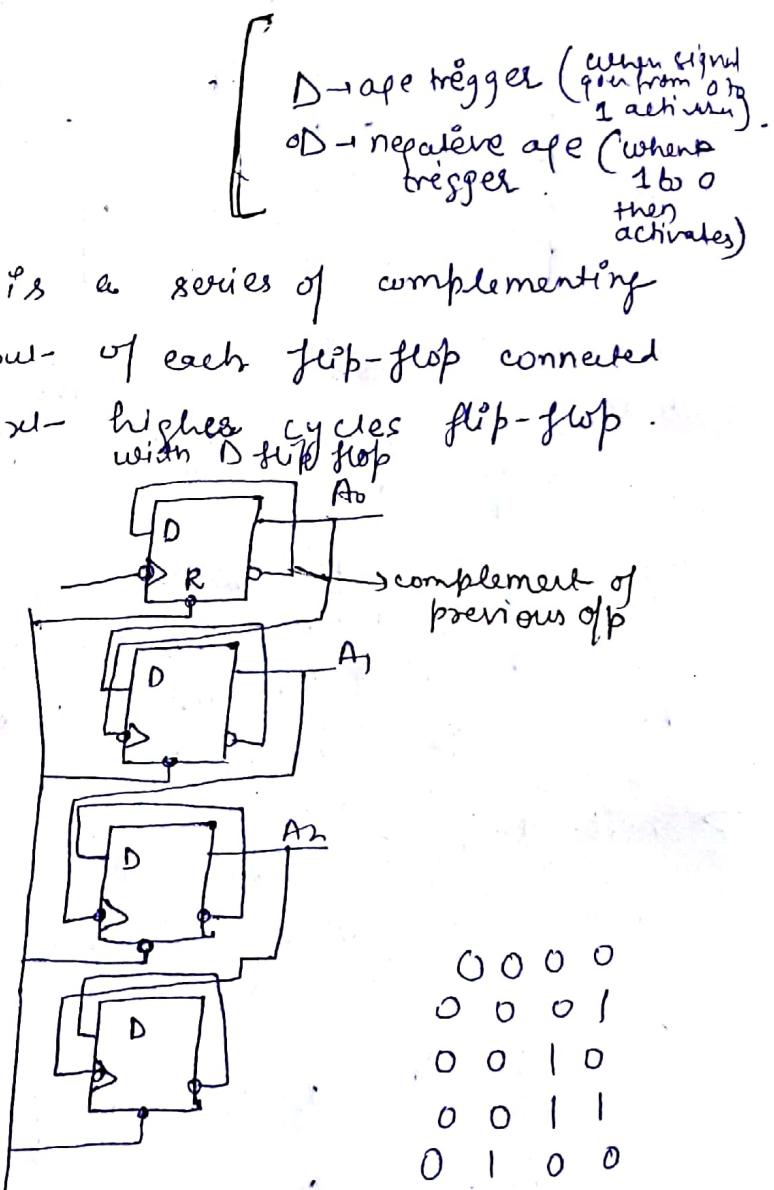
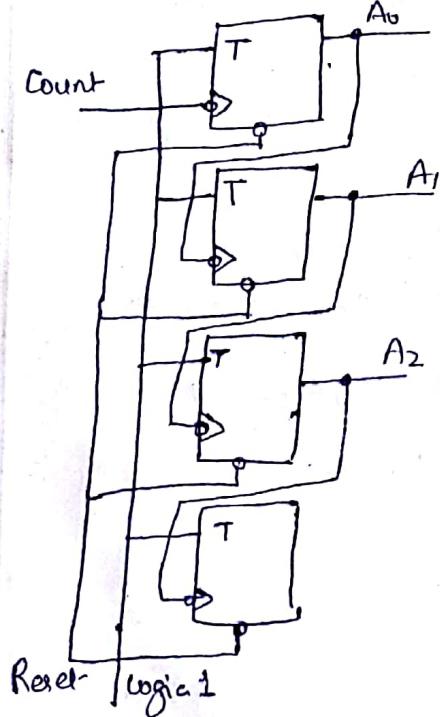
A digital system is said to operate



→ 30/10/2018 :-

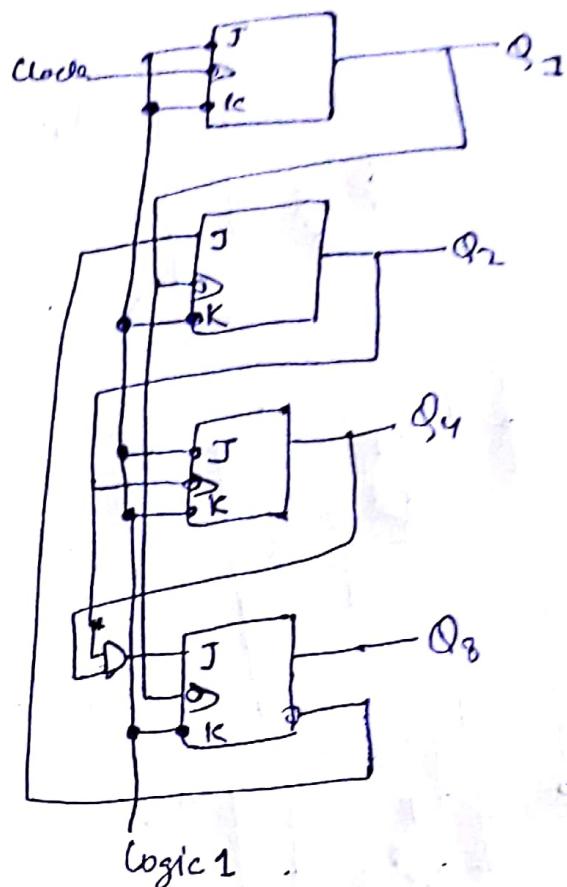
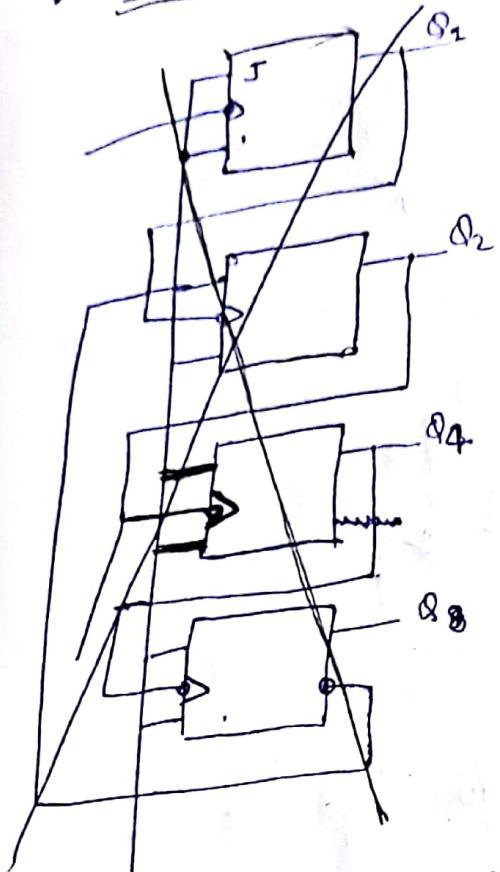
Binary Ripple Counter :-

A binary ripple counter is a series of complementing flip-flops with the output of each flip-flop connected to the C input of next flip-flop with T flip-flop.



0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
;	;	;	;
;	;	;	;
1	1	1	1
(Reset → 0000)			

→ BCD ripple counter :-

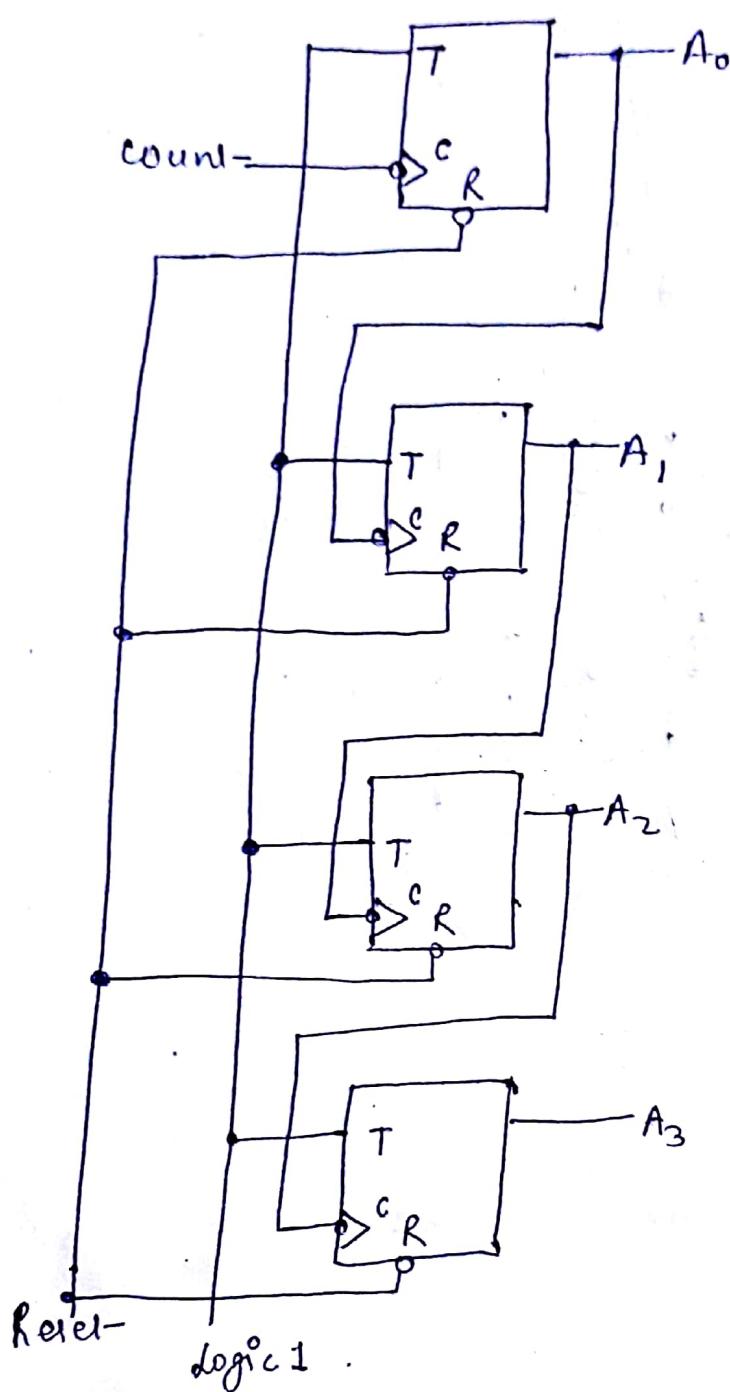


⇒ For counting 0 to 9.

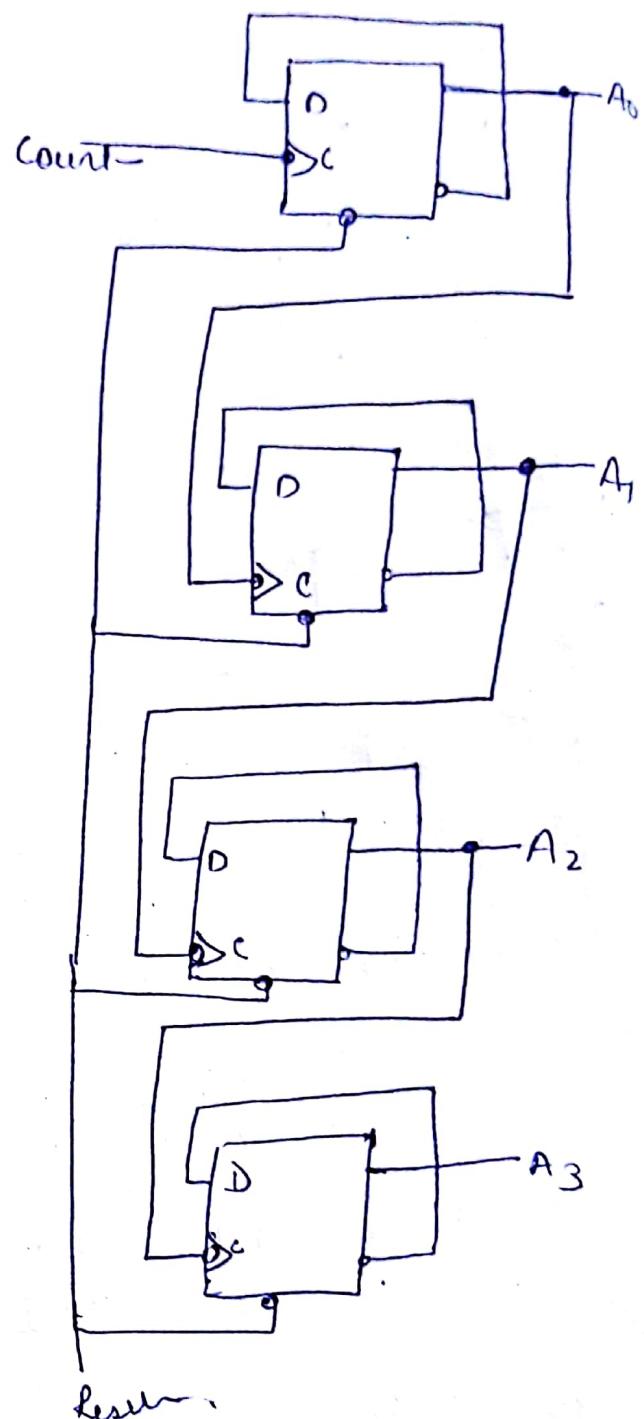
Q_8	Q_4	Q_2	Q_1
0	0	0	0

- 1) Q_1 changes state after each clockwise
--- 0 1
- 2) Q_2 complements every time Q_1 goes from 1 to 0, as long as $Q_8 = 0$.
- 3) When Q_8 becomes 1 Q_2 remains at 0.
- 4) Q_4 complements each time when Q_2 goes from 1 to 0.
- 5) Q_8 remains at 0 as long as Q_2 or Q_4 is 0.

Four bit binary ripple counter :-



with T flip-flop .



with D flip-flop .