

Baye's Theorem in Real Life

Introduction

Introduction to Probability in Real Life

- In everyday life, we often make decisions under uncertainty.
- Doctors diagnose diseases.
- Email systems detect spam.
- Courts evaluate evidence.
- These decisions involve probabilities.
- Bayes' Theorem helps us update probability when new evidence is available.

What is Bayes' Theorem?

Bayes' Theorem is a mathematical formula used to find the probability of an event based on prior knowledge of related conditions.

Formula:

$$P(A|B) = (P(B|A) * P(A)) / (P(B))$$

Where:

- $P(A|B)$ = Probability of A given B (Posterior probability)
- $P(B|A)$ = Probability of B given A (Likelihood)
- $P(A)$ = Prior probability
- $P(B)$ = Total probability of B

Chosen Real-Life Scenario: Medical Testing

We will consider a disease testing scenario.

Suppose:

- A disease affects 1% of the population.
- The medical test:
 - Is 99% accurate for infected people.
 - Has 5% false positive rate.

We want to find:

If a person tests positive, what is the probability they actually have the disease?

Step-by-Step Mathematical Calculation

1. Given Data

- $P(\text{Disease}) = 0.01$
- $P(\text{No Disease}) = 0.99$

- $P(\text{Positive} | \text{Disease}) = 0.99$
- $P(\text{Positive} | \text{No Disease}) = 0.05$

Step 1: Calculate Total Probability of Testing Positive

$$P(\text{Positive}) = (P(\text{Positive} | \text{Disease}) * P(\text{Disease})) + (P(\text{Positive} | \text{No Disease}) * P(\text{No Disease}))$$

Substitute values:

- $P(\text{Positive}) = (0.99 * 0.01) + (0.05 * 0.99)$
- $P(\text{Positive}) = 0.0099 + 0.0495$
- $P(\text{Positive}) = 0.0594$

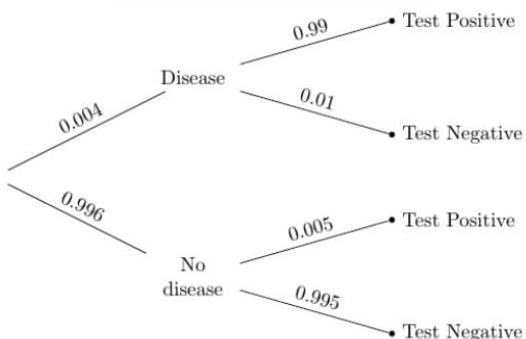
Step 2: Apply Bayes' Theorem

- $P(\text{Positive} | \text{Disease}) = (0.99 * 0.01) / (0.0594)$
- $P(\text{Positive} | \text{Disease}) = 0.0099 / 0.0594$
- $P(\text{Positive} | \text{Disease}) = 0.1666$

Final Answer

Probability that a person actually has the disease after testing positive = 16.66%

Visual Explanation of Probability Flow



- Most positives come from healthy people.
- This is because disease prevalence is very low.
- Even accurate tests can mislead when disease is rare.

Interpretation & Real-Life Meaning

Important Observation

Even though:

- Test accuracy = 99%
- False positive rate = 5%

The actual chance of having disease after positive result is only 16.66%

Why Does This Happen?

- Disease is rare (1%)
- Large number of healthy people
- Small false positive rate applied to large healthy population creates many false alarms

Real-Life Implications in Healthcare

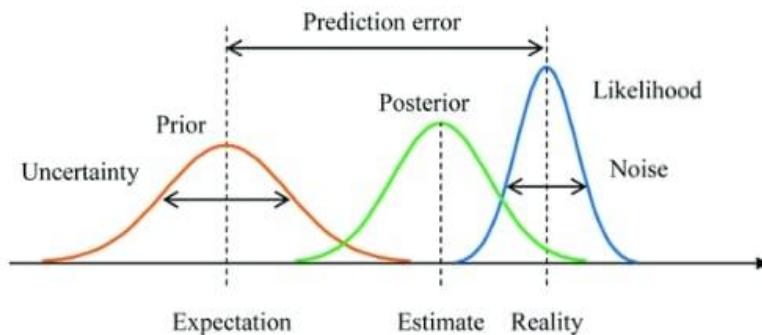
- Doctors do confirmatory tests
- Avoid panic from first test
- Consider:
 - Medical history
 - Symptoms
 - Risk factors

Decision-Making Using Bayes' Theorem

Doctors mentally apply Bayesian thinking:

- Prior belief (disease rate)
- New evidence (test result)
- Updated probability (posterior)

Prior vs Posterior Concept



- Prior probability = before evidence
- Posterior probability = after evidence
- Evidence shifts belief

Extension to Other Real-World Applications

1. Email Spam Filtering

Companies like:

- Google (Gmail)
- Microsoft (Outlook)
- Use Bayesian filtering.

Words like "FREE", "WIN", "OFFER" increase spam probability.

System updates probability as more words are analyzed.

2. Criminal Investigation

- Prior: Suspect likelihood
- Evidence: DNA match
- Posterior: Updated guilt probability

3. Machine Learning

Bayesian inference is used in:

- Predictive models
- AI systems
- Risk analysis

Advantages of Bayes' Theorem

- Handles uncertainty
- Updates beliefs logically
- Useful in small data scenarios
- Helps reduce decision errors

Conclusion

- Bayes' Theorem is not just a formula.
- It is a decision-making framework.
- In medical testing:
 - A positive result does NOT guarantee disease.

It teaches us:

- Always consider prior probability.
- Do not rely only on test accuracy.
- Understand context before making conclusions.