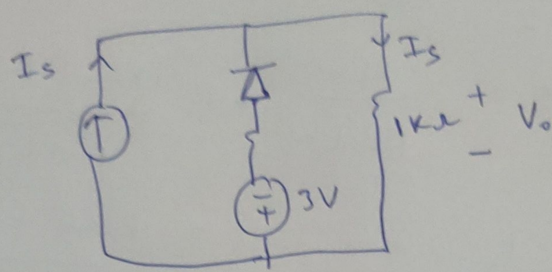
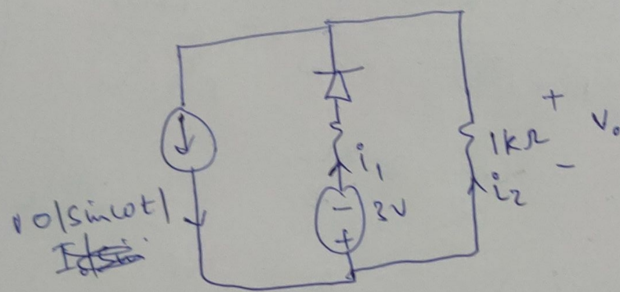
Q1

let $\frac{2n\pi}{\omega} \le t \le \frac{(2n+1)\pi}{\omega}$ where n is an integer ≥ 0



$$V_o = I_s(1K\Omega) = 10\sin\omega t \, V$$

let $\frac{(2n+1)\pi}{\omega} \le t \le \frac{2(n+1)\pi}{\omega}$



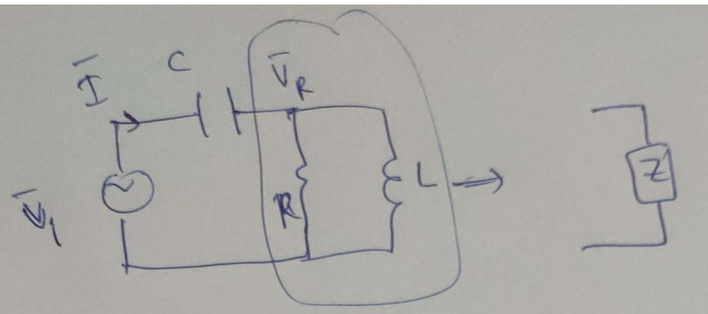
$$\begin{cases} 3 + i_1 = i_2 \\ i_1 + i_2 = 10|\sin\omega t| = -10\sin\omega t \end{cases}$$

$$2i_2 = 3 - 10\sin\omega t \Rightarrow i_2 = \frac{3 - 10\sin\omega t}{2}$$

$$V_o = -i_2 = \frac{10\sin\omega t - 3}{2}$$

$$\Rightarrow \quad V_o = \begin{cases} 10\sin\omega t \, V & \frac{2n\pi}{\omega} \le t \le \frac{(2n+1)\pi}{\omega} \\ \frac{10\sin\omega t - 3}{2} \, V & \frac{(2n+1)\pi}{\omega} \le t < \frac{2(n+1)\pi}{\omega} \end{cases}$$

Q2.



$$Z = R \parallel X_L$$

$$\bar{I} = \frac{\bar{V}_i}{Z + X_C}$$

$$\bar{V}_R = \bar{I}\, Z$$

$$\Rightarrow \frac{V_R}{V_i} = H(\omega) = \frac{Z}{Z + X_C}$$

$$= \frac{\frac{RX_L}{R+X_L}}{\frac{RX_L}{R+X_L} + X_C} = \frac{RX_L}{RX_L + RX_C + X_L X_C}$$

$$= \frac{iRL\omega}{iRL\omega - \frac{iR}{C\omega} - \frac{L}{C}} = \frac{-RL\omega}{\frac{R}{C\omega} - RL\omega - \frac{iL}{C}} = \frac{-RLC\omega^2}{R - RLC\omega^2 - iL\omega}$$

$$= \frac{-RLC\omega^2\left(R - RLC\omega^2 + iL\omega\right)}{(R - RLC\omega^2)^2 + L^2\omega^2}$$

$$= \underbrace{\frac{-RLC\omega^2\left(R - RLC\omega^2\right)}{(R - RLC\omega^2)^2 + (L\omega)^2}}_{Re(H(\omega))} + i\underbrace{\left(\frac{-RL^2C\omega^3}{(R - RLC\omega^2)^2 + L^2\omega^2}\right)}_{Im(H(\omega))}$$

$$\boxed{H(\omega) = \frac{-RLC\omega^2(R-RLC\omega^2)}{(R-RLC\omega^2)^2+(L\omega)^2} + i\,\frac{-RL^2C\omega^3}{(R-RLC\omega^2)^2+(L\omega)^2}}$$

$Re(H(\omega)) = 0$

$\Rightarrow \quad R = RLC\omega^2$

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

$Re(H(\omega)) = 0$ at $\omega = \frac{1}{\sqrt{LC}}$

$Im(H(\omega)) < 0 \quad \forall\, \omega$

$\Rightarrow \quad Im(H(\omega)) \to 0$ as $\omega \to \infty$

$Q = R\sqrt{\frac{C}{L}} = R\sqrt{\frac{1}{X_L X_C}}$

$\Rightarrow \quad R = Q\sqrt{X_L X_C}$

$$H(\omega) = \frac{X_L}{X_L + X_C + \frac{X_L X_C}{R}} = \frac{X_L}{X_L + X_C + \frac{\sqrt{X_L X_C}}{Q}}$$

~~as $Q \gg 1$~~

$$\cancel{H(\omega) \approx \frac{X_L}{X_L + X_C} = \frac{iL\omega}{iL\omega - \frac{i}{C\omega}}} \quad L^2\omega^2 \quad (i\omega)$$

$$\cancel{H(\omega) = \frac{X_L}{X_L + X_c}}$$

$$H(\omega) = \frac{\left(\frac{X_L}{X_L + X_C}\right)}{1 + \left(\frac{\sqrt{X_L X_C}}{X_L + X_C}\right)\frac{1}{Q}}$$

$<< 1$

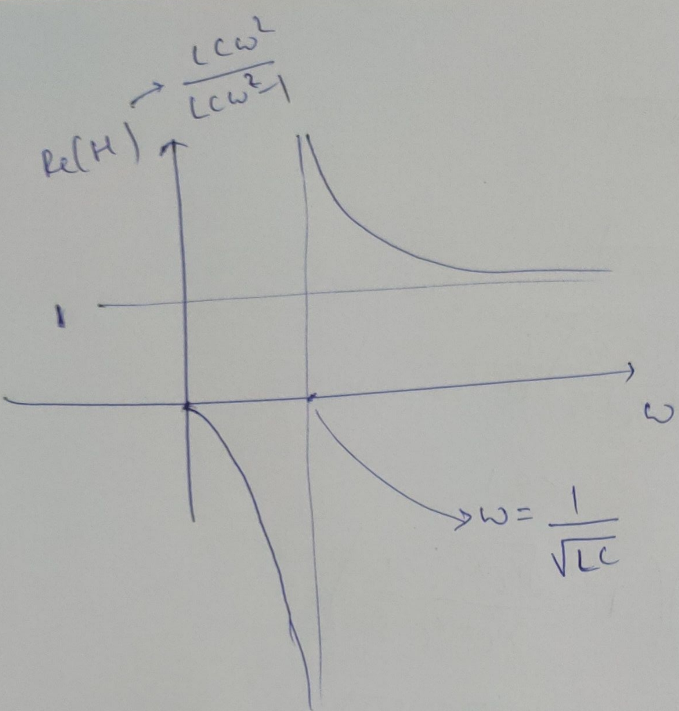
$$= \left(\frac{X_L}{X_L + X_C}\right)\left(1 - \frac{\sqrt{X_L X_C}}{X_L + X_C}\frac{1}{Q}\right)$$

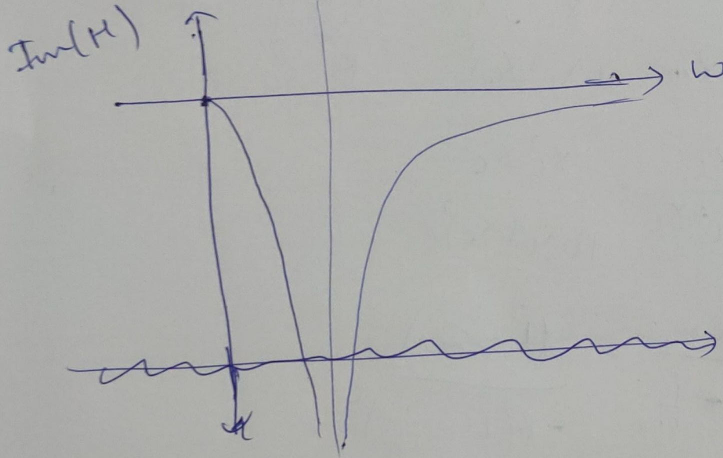
$$= \left(\frac{X_L}{X_L + X_C} - \frac{X_L \sqrt{X_L X_C}}{(X_L + X_C)^2}\frac{\sqrt{X_L X_C}}{R}\right)$$

$$= \frac{X_L}{X_L + X_C} - \frac{X_L^2 X_C}{R(X_L + X_C)^2}$$

$$= \frac{iL\omega}{iL\omega + \frac{i}{C\omega}} - \frac{i\left(L^2\omega^2/C\omega\right)}{-R\left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$= \underbrace{\frac{LC\omega^2}{LC\omega^2 - 1}}_{Re} + i\underbrace{\left(-\frac{L^2 C\omega^3}{R(LC\omega^2 - 1)^2}\right)}_{Im}$$



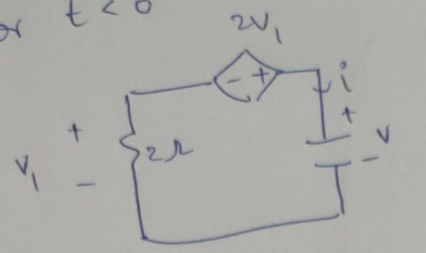


$$\cancel{H(\omega) = \frac{LC\omega^2}{(LC\omega^2-1)}\left[1 + i\,\frac{-L\omega}{R(LC\omega^2-1)}\right]}$$

$$\cancel{|H(\omega)|_{max}} = \quad H(\omega) = \frac{X_L}{X_L+X_C}\left(1 - \frac{\sqrt{X_L X_C}}{X_L+X_C}\,\frac{1}{Q}\right)$$

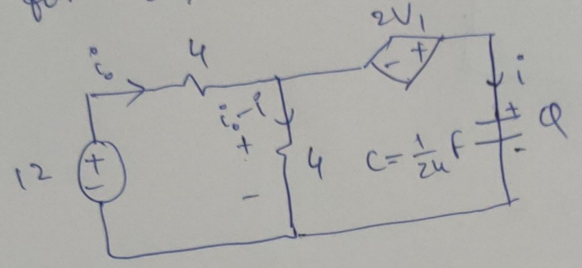
$$\boxed{|H(\omega)|_{max} \approx \frac{X_L}{X_L+X_C} = \frac{LC\omega^2}{LC\omega^2-1}}$$

Q3. for $t < 0$



$v = 0$
$i = 0$

for $t \geq 0$ (let ~~t~~ at time t)



$$12 = 4i_0 + 4i_0 - 4i = 8i_0 - 4i$$

$$v_1 = 4i_0 - 4i$$

$$12 = -8i_0 + 8i + \frac{Q}{C}$$

$$24 = 4i + \frac{Q}{C} \qquad \cancel{Q=}\ \cancel{di}\ \boxed{i = \frac{dQ}{dt}}$$

$$4\frac{dQ}{dt} = 24 - \frac{Q}{C}$$

$$\int_0^Q \frac{dQ}{24 - Q/C} = \int_0^t \frac{dt}{4}$$

$$\left(\ln(24 - Q/C)\right)_0^Q = \frac{-t}{4C}$$

$$24 - Q/C = 24e^{-t/4C}$$

$$\Rightarrow \boxed{Q = 24C\left(1 - e^{-t/4C}\right)}$$

~~$V = Q$~~

$$V(t) = \frac{Q}{C} = 24\left(1 - e^{-t/4C}\right)$$

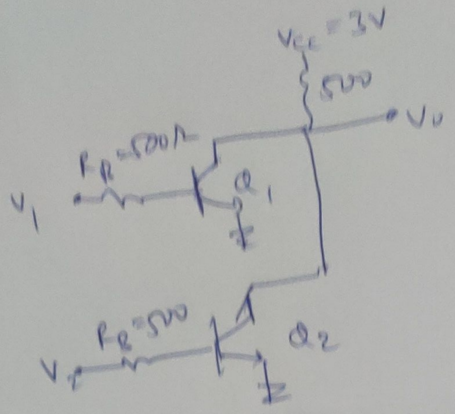
$$\boxed{V(t) = 24\left(1 - e^{-6t}\right)}$$

$$\boxed{i(t) = \frac{dQ}{dt} = 6e^{-6t}}$$

$$\Rightarrow V(t) = \begin{cases} 0 & t < 0 \\ 24\left(1 - e^{-6t}\right) & t \geq 0 \end{cases}$$

$$i(t) = \begin{cases} 0 & t < 0 \\ 6e^{-6t} & t \geq 0 \end{cases}$$

Q4.



i) $V_1 = V_2 =$ low

Assume $Q_1, Q_2 \rightarrow$ cutoff

$V_{C_1} = V_{C_2} = V_{cc} = 3V = V_o$

$V_{BE_1} = V_{BE_2} = 0.2 < 0.5$

Assumption correct.

ii) $V_1 = 3V$, $V_2 = 0.2V$

Assume $Q_2 \rightarrow$ cutoff, $Q_1 \rightarrow$ sat

$\cancel{V_{B1} = 0.8V}$ $\cancel{i_{B1} = \frac{4.2}{10k\Omega} = 0.42mA}$

$V_{B_1} = 0.8V \rightarrow i_{B_1} = \frac{2.2}{500} A$

$V_{C1} = \boxed{V_o = 0.2V}$

$i_{C_1} = \frac{(3-0.2)\times 2}{500} = \frac{2.8\times 2}{500}$

$\frac{i_{C_1}}{i_{B1}} < \beta$ & $V_{BE} = 0.2 < 0.5$

Assumption correct

(iii) $V_1 = 0.2V$ $\quad V_1 = \cancel{5}\,3V$

same as (ii)

(iv) $V_1 = 3V = V_2$

Assume $Q_1, Q_2 \to$ Sat

$V_{B1} = V_{B2} = 0.8 \;.\; i_{B1} = i_{B2} = \cancel{0.44}\ \frac{2.2}{500}$

$$\boxed{V_{C1} = V_{C2} = 0.2V = V_0}$$

$i_{C1} = i_{C2} = \cancel{500}\ \frac{2.8}{500}$

$\frac{i_{C1}}{i_{B1}} = \frac{i_{C2}}{i_{B2}} < \beta$ $\cancel{\&}$ $\cancel{V_{BE}}$,

Assumption correct

$\boxed{\text{NOR GATE}}$

~~Q~~. Q5.

$V_T(M_1, M_2) = 1V$

$V_T(M_3, M_4) = -1V$

$k = 0.25\, mA/V^2$

Assume, M_1 & M_2 in active region

$V_{GS4} = V_2 - V_{DD} = 0$

So M_4 is off (as V_{GS} should be <0 for it to be ON)

Assume M_3 to be in ohmic region.

as M_4 is off $i_{D4} = 0$ so,

$i_{D3} = i_{D2} = i_{D1}$

~~i_{T}~~

~~$i_{D1} = k[2(V_{GS1} - 1)V$~~

$i_{D1} = k(V_{GS1} - V_T)^2$

$V_{GS1} = V_1 = 4V$

$\rightarrow \boxed{i_{D1} = k(3)^2 = 9k}$

$i_{D2} = k(V_{GS2} - V_T)^2$

$V_{GS2} = V_2 - V_{DS1} = 10 - V_{DS1}$

$\rightarrow \boxed{i_{D2} = k(9 - V_{DS1})^2} = i_{D1}$

$$K(9 - V_{DS1})^2 = K(3)^2$$

$$\Rightarrow \boxed{V_{DS1} = 6\,V}$$

$$V_{GS3} = V_1 - V_{DD} = 4 - 10 = -6\,V$$

$$i_{D3} = K\left[2(V_{GS3} - V_T)V_{DS3} - V_{DS3}^2\right]$$

$$\Rightarrow 9K = K\left[-10V_{DS3} - V_{DS3}^2\right]$$

$$\Rightarrow V_{DS3}^2 + 10V_{DS3} + 9 = 0$$

$$V_{DS3} = -\frac{10 \pm 8}{2} = -1, -9$$

Let $V_{DS3} = -9$

$$V_{GS3} - V_T = -6 + 1 = -5$$

$V_{DS3} < V_{GS3} - V_T$ (contradiction)

So discard $V_{DS3} = -9$

Let $V_{DS3} = -1$

$$(V_{DS3} = -1) > (V_{GS3} - V_T = -5)$$

$$\boxed{\text{So our assumption is correct. } M_3 \text{ is indeed in ohmic mode}}$$

$V_{DS3} = V_o - V_{DD}$

$\Rightarrow \boxed{V_o = 10 - 1 = 9V}$

$(V_{DS1} = 6V) > (V_{GS1} - V_T = 3V)$

So M_1 is ~~in~~ indeed in active region

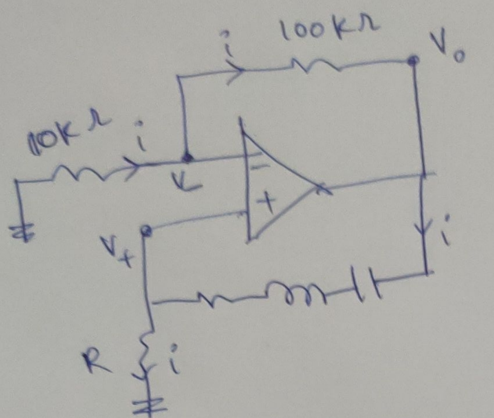
& $V_{DS2} = V_o - V_{DS1} = 9 - 6 = 3V$

$(V_{DS2} = 3V) \geq (V_{GS2} - V_T = 3V)$

So M_2 is indeed in active region.

Q6.

Assume Virtual ~~sto~~ short (ie $V_+ = V_-$)



$$V_- = V_+$$

$$i = \frac{V_-}{10} = \frac{V_o - V_-}{100} \Rightarrow \boxed{V_o = 11V_-}$$

$$\frac{V_+}{R} = \frac{V_o - V_+}{|Z_{eq}|} \Rightarrow 10R = |Z_{eq}|$$

$$(10R)^2 = 5^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \times 10^{-6}$$

$$\Rightarrow 10R \geq 5 \Rightarrow \boxed{R \geq 0.5\,k\Omega}$$

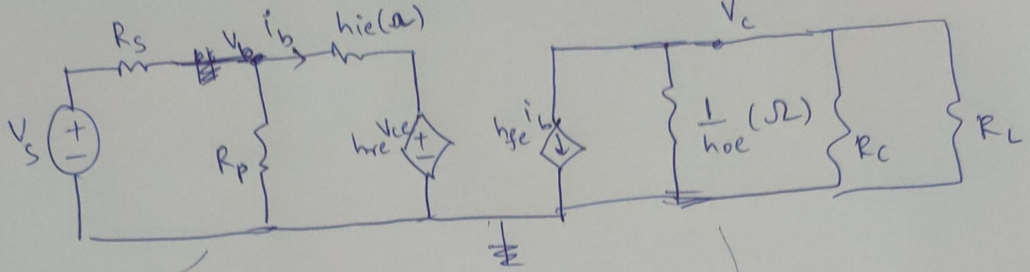
---

for $R = 500\,\Omega$, $\Rightarrow$ $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{0.2\times10^{-3}\times5\times10^{-9}}} = 10^6 \text{ rad/s}$$

Q7.

$R_p = R_1 || R_2$



$$\frac{V_s - V_b}{R_s} = \frac{V_b}{R_p} + i_b$$

$$\boxed{i_b = \frac{V_s - V_b}{R_s} - \frac{V_b}{R_p}}$$

$$i_b h_{ie} + h_{re} V_c = V_b$$

let this be R_x

$$V_c = -h_{fe} i_b \left(\frac{1}{h_{oe}} || R_c || R_L \right)$$

$$\Rightarrow V_c = -h_{fe} i_b R_x$$

$$\Rightarrow \cancel{\left(\frac{V_s - V_b}{R_s} - \frac{V_b}{R_p} \right) h_{ie} + h_{re}\left(-h_{fe} i_b R_x\right) = V_b}$$

$$\rightarrow i_b h_{ie} + h_{re}\left(-h_{fe} i_b R_x\right) = V_b$$

$$\Rightarrow \boxed{V_b = i_b \left(h_{ie} - h_{re} h_{fe} R_x \right)}$$

& $\boxed{V_c = i_b \left(-h_{fe} R_x \right)}$

$\Rightarrow$ $\cancel{\frac{V_b}{V_c}}$

ac Voltage gain $\left(\frac{V_c}{V_b}\right) = \frac{i_b(-h_{fe}R_x)}{i_b(h_{ie} - h_{re}h_{fe}R_x)}$

$$= \frac{h_{fe}R_x}{h_{re}h_{fe}R_x - h_{ie}}$$

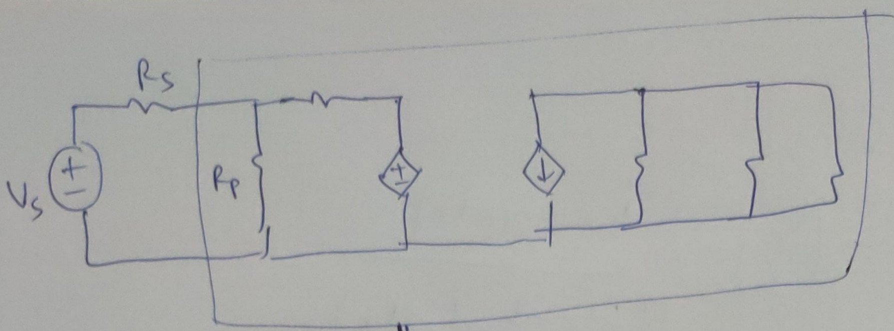
$R_x = \cancel{R_E}\ \frac{1}{h_{oe}} \| R_c \| R_L = 100 \| 2 \| 2 = 100 \| 1$

$$\boxed{R_x = \frac{100}{101}}$$

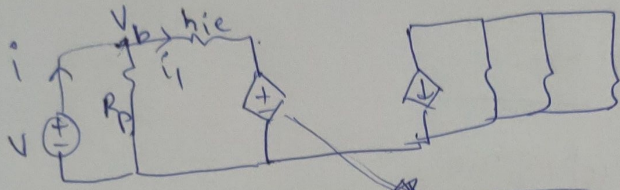
Voltage gain $= \frac{100\left(\frac{100}{101}\right)}{(10^{-2})\left(\frac{100}{101}\right) - 1.3}$ $\cancel{=}$ $\cancel{\frac{10^4}{101}}$

$$= \frac{10^4}{1 - 1.3(101)} = -\frac{10^4}{130.3} = -76.745$$

$$\boxed{\therefore \text{Voltage gain } (A_v) = -76.745}$$



$\Downarrow$



$\cancel{V_c = 0} \Rightarrow \boxed{\cancel{h_{re} V_{ce} = 0}}$

$$i = \frac{V_b}{R_p} + i_1 \qquad \boxed{V = V_b}$$

$$i_1 h_{ie} + h_{re} V_c = V$$

$$i_1 h_{ie} + h_{re} A_v V = V$$

$$\boxed{i_1 = \frac{V(1 - h_{re} A_v)}{h_{ie}}}$$

$$i = \frac{V}{R_p} + \frac{V(1 - h_{re} A_v)}{h_{ie}}$$

$$\frac{V}{i} = \frac{1}{\frac{1}{R_p} + \left(\frac{1 - h_{re} A_v}{h_{ie}}\right)} = R_{in}$$

$$R_p = R_1 \| R_2 = 10\ k\Omega$$

$$\Rightarrow \boxed{R_{in} = \frac{1}{\frac{1}{10} + \left(\frac{1 + 10^{-4}(A_v)}{h_{ie}}\right)} = 1.1426\ k\Omega}$$