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Removing Projective and Affine Distortion from Images

In the following homework we have been asked to remove the projective and the affine distortion from four different images using 3 different methods. In the following section I will explain my logic and the equations used to remove the distortions using the three methods.

1. Using Point to Point Correspondence

In this method we remove the distortion by using homography calculated using point to point correspondence between the real world and the distorted image. Mapping between the image plane and the world plane is given by,

$$X' = H\,X$$

where X is given by $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, X' is given by $\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix}$ and H is given by $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \in R^3$. We set $h_{33} = 1$, Thus we have 8 unknowns that we need to solve for. The physical coordinates are given by, $x = \frac{x_1}{x_3}$, $y = \frac{x_2}{x_3}$, $x' = \frac{x'_1}{x'_3}$ and $y' = \frac{x'_2}{x'_3}$. The physical coordinate (x, y) is represented as $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. Multiplying this vector by the homography matrix H gives us the mapped coordinates x' and y'

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1} \qquad 1$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1} \qquad 2$$

Since we have 8 unknowns to solve for, we need to consider a minimum of 4 points. Let the 4 points be $(x_1, y_1), (x_2, y_2)(x_3, y_3)(x_4, y_4)$. Following equation 1 and 2 the mapped points $(x'_1, y'_1), (x'_2, y'_2), (x'_3, y'_3), (x'_4, y'_4)$ are given by

$$\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}$$

The homography matrix is given by, $H = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}$

and

$$A = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix}, \qquad B = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4 \end{bmatrix}$$

Thus, the homography matrix H = $B^{-1}A$

Method specific to solving the problem :

1. In this problem the homography matrix is mapped from four points in the distorted image to four real world coordinates. To determine the real world coordinates we assume that each pixel corresponds to one centimeter in the real world. Once the homography is calculated we map through a scaled blank image whose dimensions are determined using the homography matrix. The inverse of the homography matrix is then applied to the points in the blank image to get the mapped points in the imaginary plane and the corresponding pixel values are obtained from the distorted image.

2. Two Step Method

Step 1 : Finding the projective homography matrix using the vanishing line

Another method for determining the homography matrix to remove the projective distortion in an image is by finding the vanishing line. To determine the parameters of the vanishing line we will need a two vanishing points. The vanishing point is the intersection of a pair of parallel lines. Thus, to determine the vanishing line, we will need two pairs of parallel lines. The equation below demonstrates how to calculate the parameter of the vanishing line from the points in the distorted image.

A line passing through two points p and q is given by, $l = p \times q$

A vanishing point is the intersection of two parallel lines, l and m is given by, $p_v = l \times m$

The vanishing line passing through two vanishing points is p_v and q_v is given by, $vl = p_v \times q_v$

Once the vanishing line is calculated, the homography to get rid of the projective distortion is given by,

$$Hp = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ vl_1 & vl_2 & vl_3 \end{bmatrix}$$

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Step 2: Finding the affine homography matrix

To determine the homography matrix that will remove the affine distortion from the image we need to determine two pairs of orthogonal lines. Given two sets of perpendicular lines $l = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ and $m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$ the angle between the two lines is given by,

$$\cos\theta = \frac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* m)(m^T C_\infty^* l)}} = 0$$

Since $l^T = l'^T H$ and $m = H^T m'$ substituting we get,

$$l'^T H C_\infty^* H^T m' = 0$$

$$\text{Where, } Ha = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Thus}[l_1' \quad l_2' \quad l_3'] \begin{bmatrix} AA^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} m_1' \\ m_2' \\ m_3' \end{bmatrix} = 0$$

$$\text{Where, } AA^T = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}, s_{22} = 1 \text{ and } s_{21} = s_{12}$$

$$\text{Thus, } [l_1'm_1' \quad l_1'm_2' + l_2'm_1'] \begin{bmatrix} s_{11} \\ s_{12} \end{bmatrix} = -l_2'm_2'$$

Since there are two unknowns we would need 2 pairs of perpendicular lines.

Since $A = VDV^T$, $S = VD^2V^T$, we need to use SVD decomposition to solve for V and D and in turn find A.

Method specific to solving the problem :

1. The projective homography matrix is mapped from plane with projective distortion to the plane without projective distortion. Thus, like the method followed in the point to point correspondence mentioned above, I map through a scaled blank image using the inverse of the homography matrix to get the mapped points in the imaginary plane. The corresponding pixel values are obtained from the distorted image. This will create the intermediate image without projective distortion.
2. The affine homography matrix Ha, is mapped from the plane without projective or affine distortion to the plane with affine distortion, hence the effective homography that will be used will be equal to inv(Ha). Thus, I first find the bounding image by multiplying inv(Ha) to the corner points of the image without the projective distortion. I then scale the image to ensure it not too big or too small. Finally map through a scaled blank image using the homography matrix to get the mapped points in the imaginary plane and the corresponding pixel values are obtained from the image without the projective distortion.

3. Single Step Method

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In case of the single step method we use the dual conic to derive the parameters of the combined Hpa matrix. The dual conic is given by,

$$C_\infty^{*\prime} = HC_\infty^* H = \begin{bmatrix} AA^T & Av \\ v^T A^T & v^T v \end{bmatrix} = \begin{bmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{bmatrix}$$

Given two sets of perpendicular lines $l = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$ and $m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$ in the plane without the affine and projective distortion. Since the cos of the angle between the two lines is 0.

$$l'^T HC_\infty^* H^T m' = l'^T \begin{bmatrix} a & \frac{b}{2} & \frac{d}{2} \\ \frac{b}{2} & c & \frac{e}{2} \\ \frac{d}{2} & \frac{e}{2} & f \end{bmatrix} m', \text{ where f = 1}$$

$$\begin{bmatrix} l_1' m_1' & \frac{l_1' m_2' + l_2' m_1'}{2} & l_2' m_2' & \frac{l_1' m_3' + l_3' m_1'}{2} & \frac{l_2' m_3' + l_3' m_2'}{2} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = -l_3' m_3'$$

Since there are five unknown we need five pairs of perpendicular lines.

Once the conic is derived AA^T, can be derived using SVD. Then A and v can be derived from $v^T A^T$, and once A and v are calculated we can derive Hpa which is given by,

$$Hpa = \begin{bmatrix} A & 0 \\ v^T & 1 \end{bmatrix}$$

Method specific to solving the problem :

1. The homography matrix is mapped from plane with distortion to the plane without distortion. I first find the bounding image by multiplying Hpa to the corner points of the image without the distortion. Then, like the method followed in the point to point correspondence mentioned above, I map through a scaled blank image using the inverse of the homography matrix to get the mapped points in the imaginary plane and the corresponding pixel values are obtained from the distorted image. This will create an image without projective or affine distortion.

Observations between the two-step method and the one-step method

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The two-step method is more efficient that the one step method since the one step method is very sensitive to the points that are selected. Hence it takes a longer time to use the one step method even though it has fewer no of steps compared to the two-step method.

Result

1. **Image 1:**

Figure 1: Input image 1 with the parallel lines and perpendicular lines to calculate the homography matrix

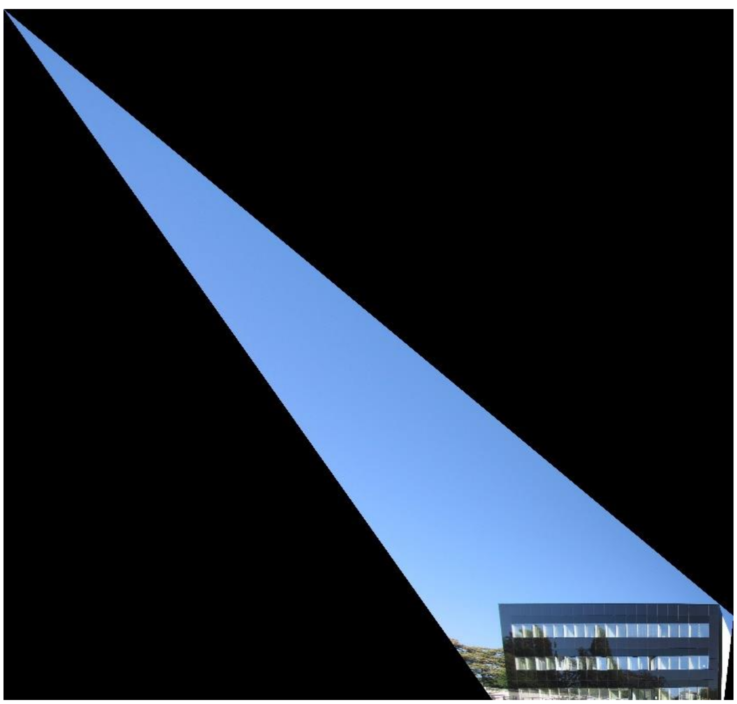

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Figure 2: Output image 1 without affine and projective distortion using point to point correspondence method

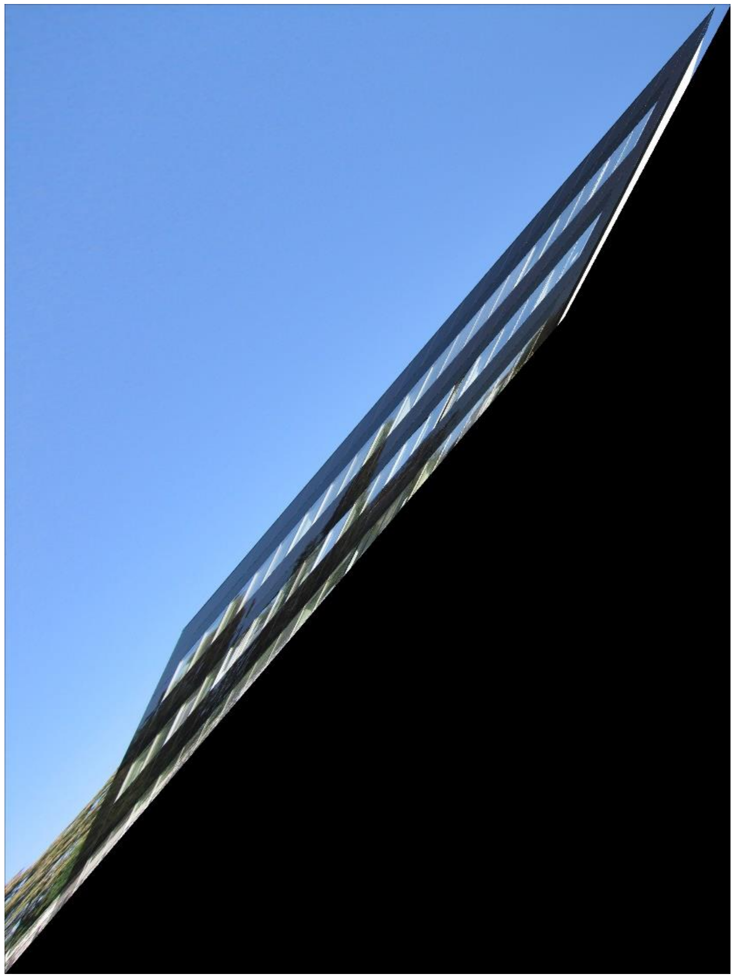

Figure 3: Output image 1 without projective distortion using 2 step method

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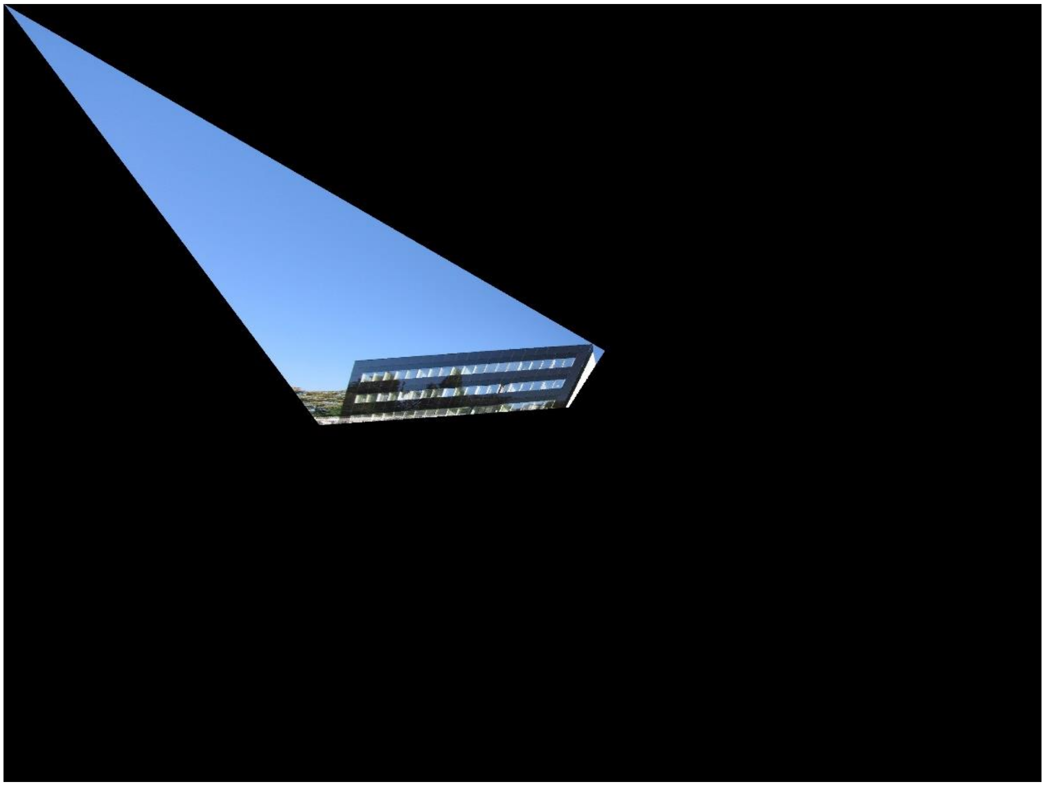

Figure 4: Output image 1 without affine and projective distortion using 2 step method

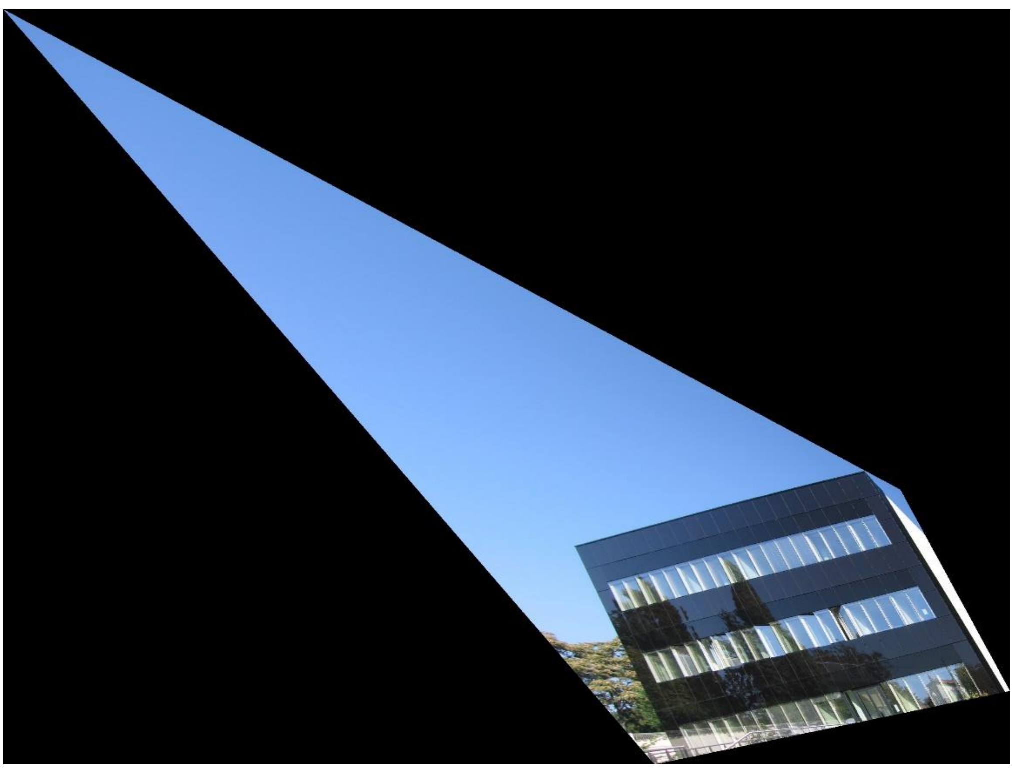

Figure 5: Output image 1 without affine and projective distortion using 1 step method

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2. Input Image 2

Figure 6: Input image 2 with the parallel lines and perpendicular lines to calculate the homography matrix

Figure 7: Output image 2 without affine and projective distortion using point to point correspondence method

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Figure 8: Output image 2 without projective distortion using 2 step method

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Figure 9: Output image 2 without affine and projective distortion using 2 step method

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Figure 10: Output image 2 without affine and projective distortion using 1 step method

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3. Input Image 3

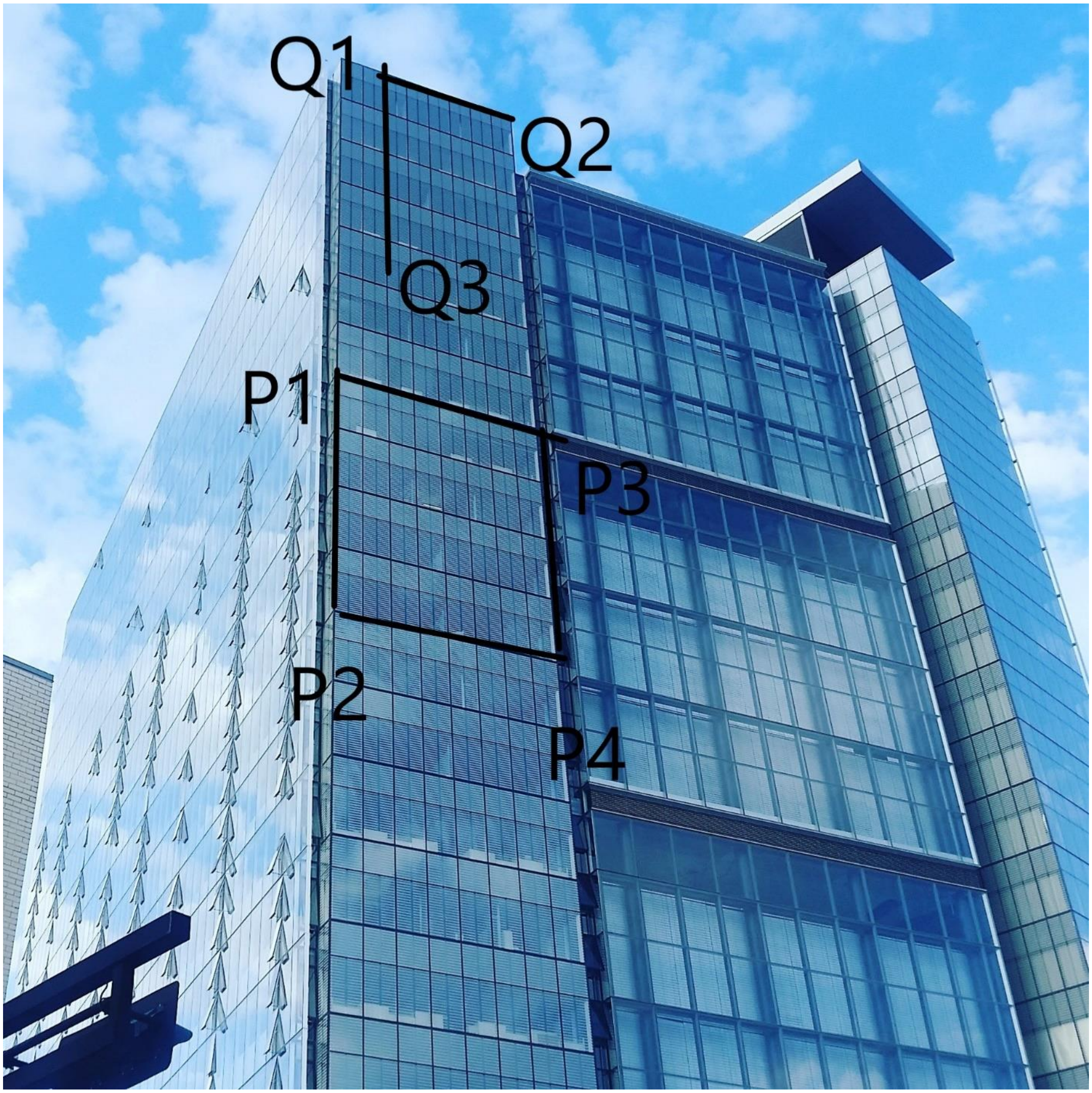

Figure 11: Input image 3 with the parallel lines and perpendicular lines to calculate the homography matrix

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Figure 12: Output image 3 without affine and projective distortion using point to point correspondence method

Figure 13: Output image 3 without projective distortion using 2 step method

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Figure 14: Output image 3 without affine and projective distortion using 2 step method

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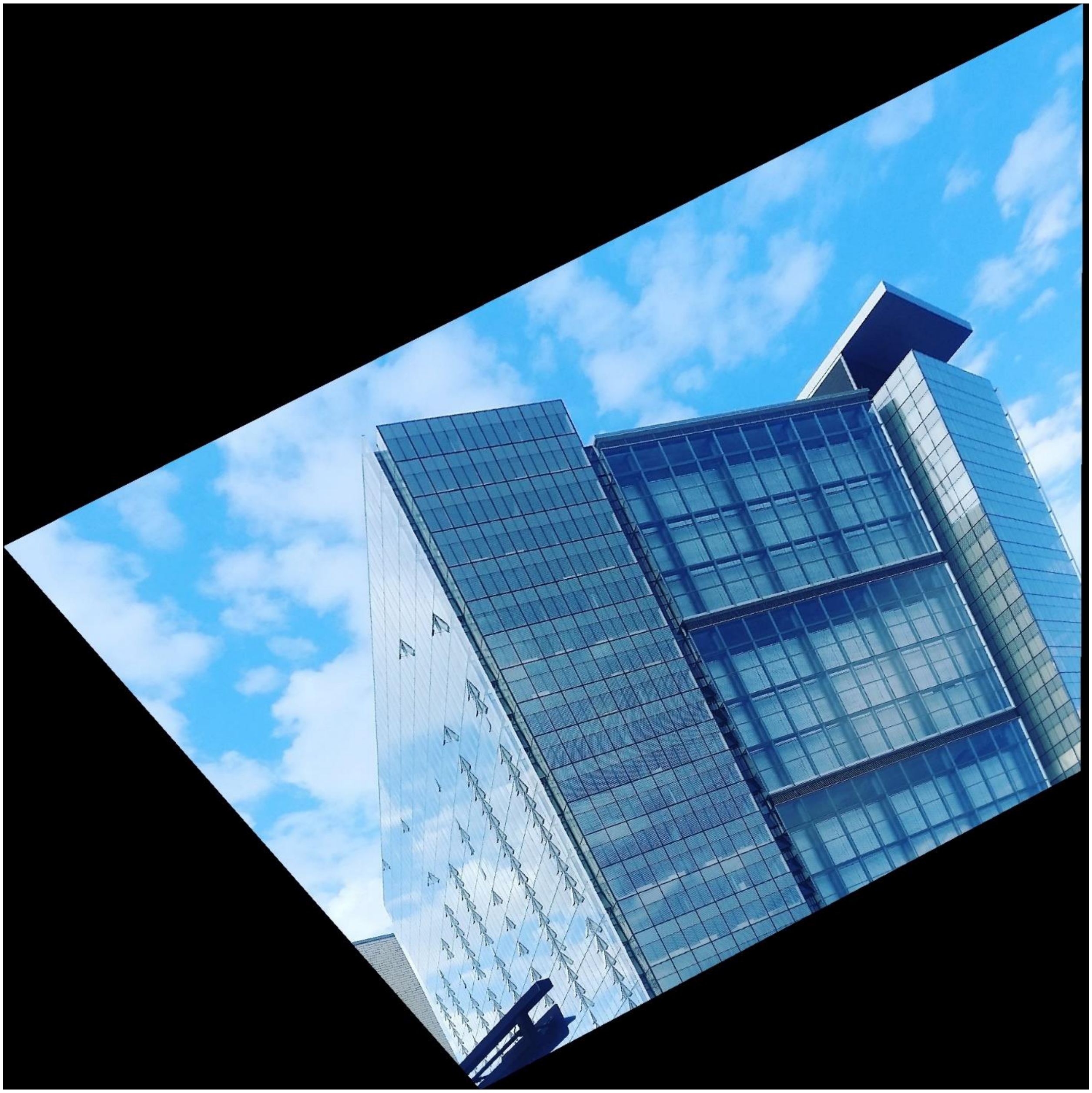

Figure 15: Output image 2 without affine and projective distortion using 1 step method

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4. Input Image 4

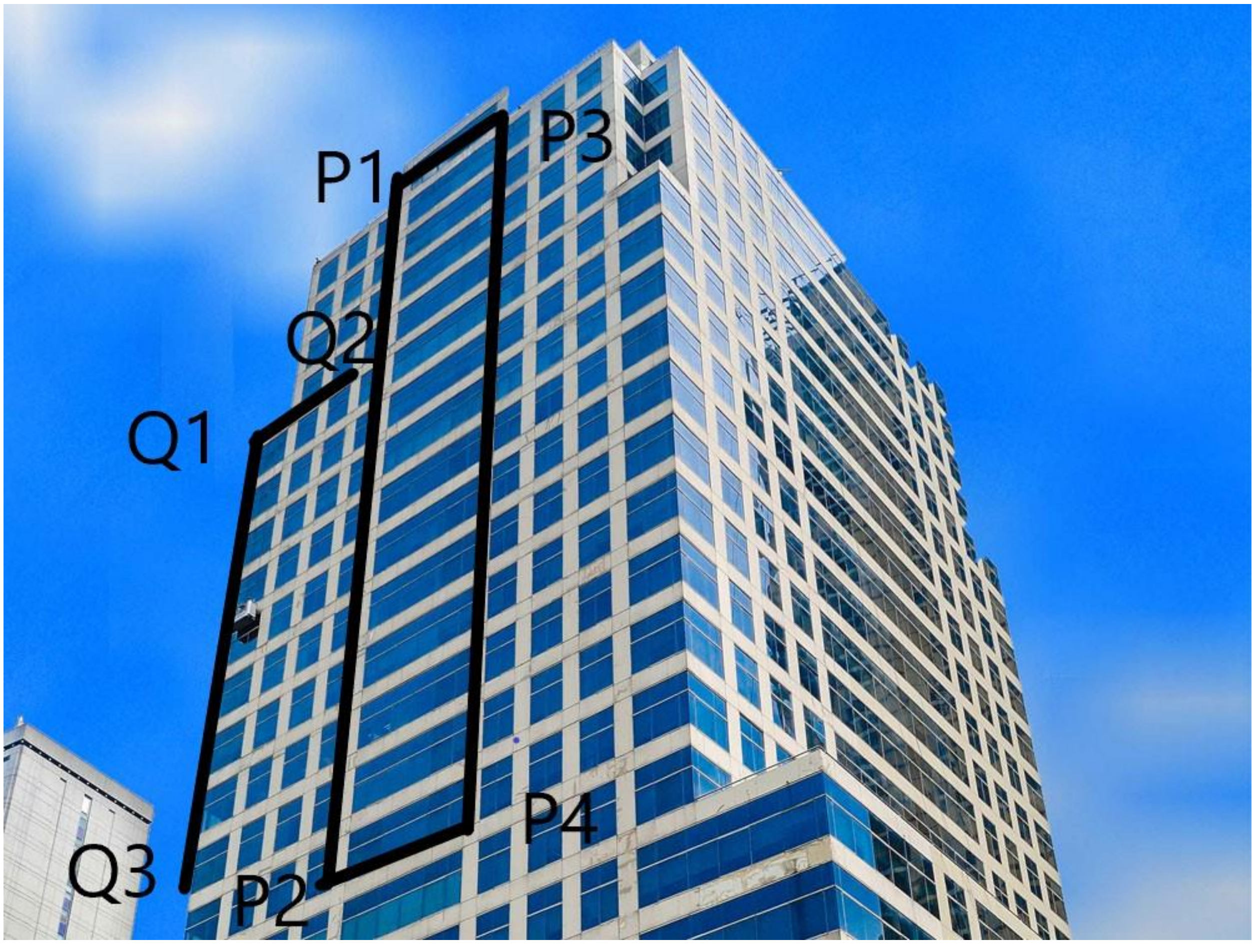

Figure 16: Input image 4 with the parallel lines and perpendicular lines to calculate the homography matrix

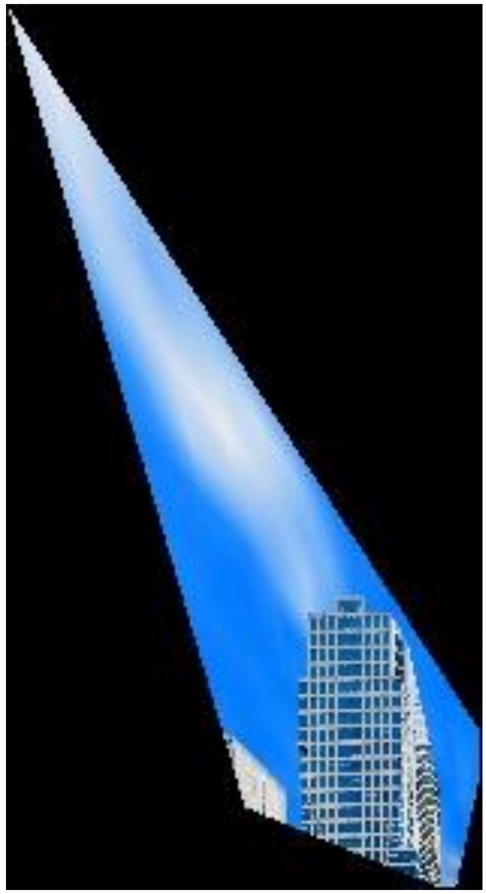

Figure 17: Output image 4 without affine and projective distortion using point to point correspondence method

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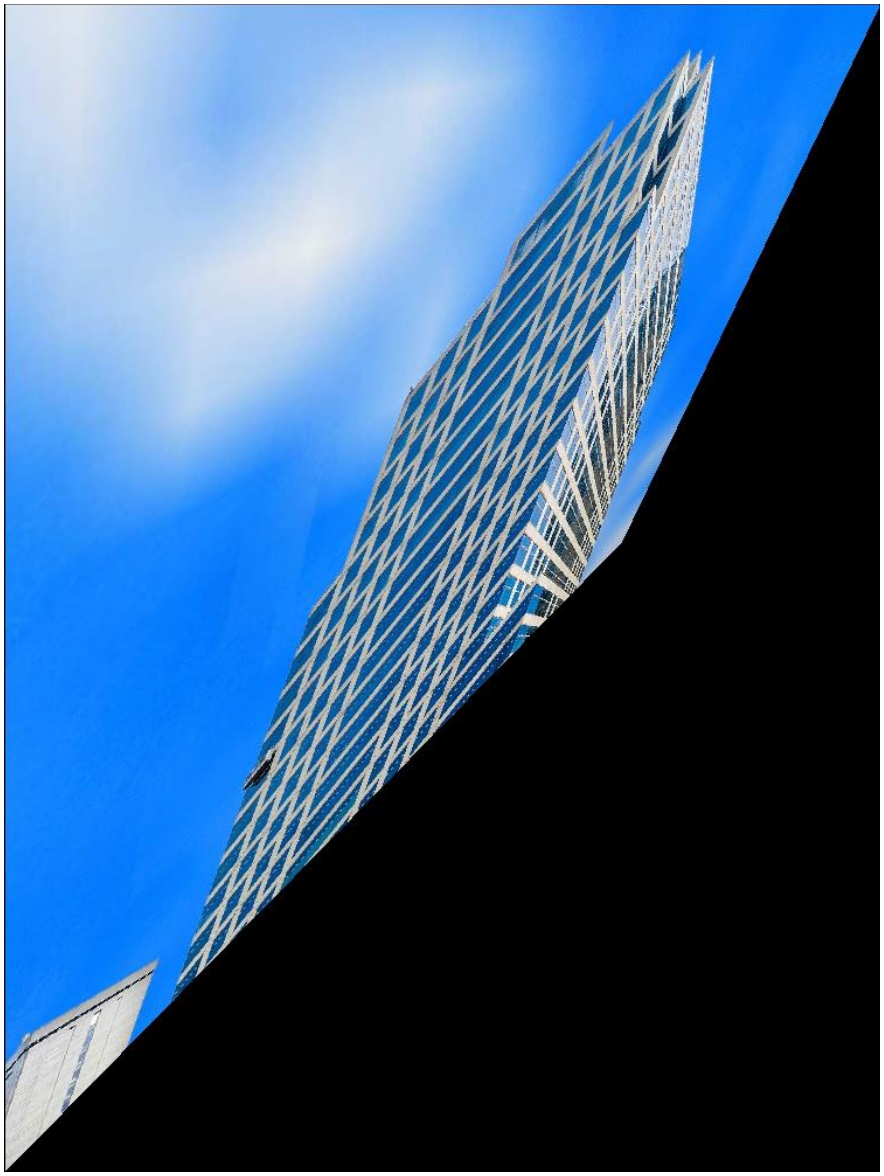

Figure 18: Output image 4 without projective distortion using 2 step method

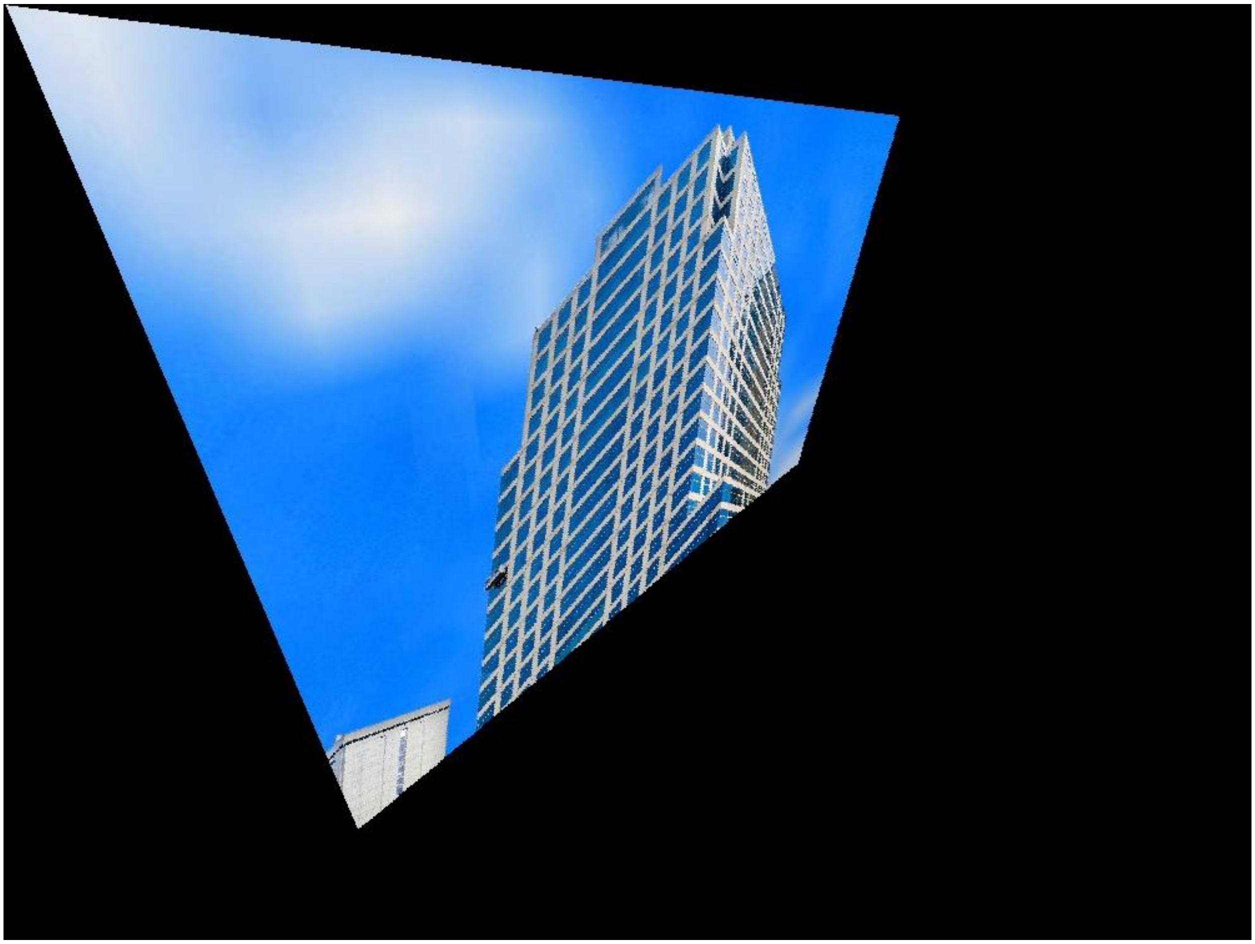

Figure 19: Output image 4 without affine and projective distortion using 2 step method

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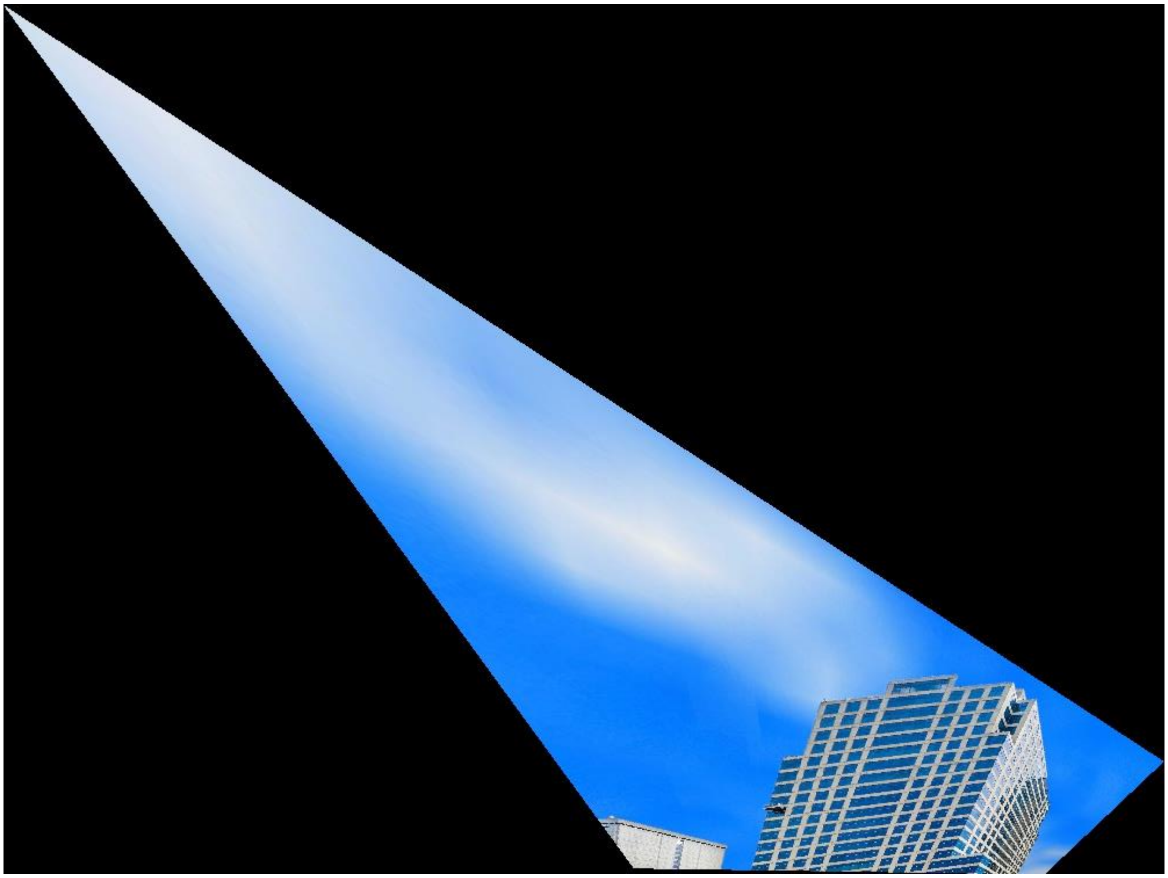

Figure 20: Output image 2 without affine and projective distortion using 1 step method