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Computer Vision Basics
Understanding Homography

Method for calculating the homography matrix

To map one, two-dimensional plane onto another we need to use a planar projective transformation known as Homography. Thus, to project a point X from one to plane to another plane as X', we require a 3 x 3 nonsingular matrix H.

$$X' = H X$$

where X is given by $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, X' is given by $\begin{bmatrix} x'_1 \\ x'_2 \\ x'_3 \end{bmatrix}$ and H is given by $\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \in R^3$. We set $h_{33} = 1$, Thus we have 8 unknowns that we need to solve for. The physical coordinates are given by, $x = \frac{x_1}{x_3}$, $y = \frac{x_2}{x_3}$, $x' = \frac{x'_1}{x'_3}$ and $y' = \frac{x'_2}{x'_3}$. The physical coordinate (x, y) is represented as $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$. Multiplying this vector by the homography matrix H gives us the mapped coordinates x' and y'

$$x' = \frac{h_{11}x + h_{12}y + h_{13}}{h_{31}x + h_{32}y + 1} \qquad 1$$

$$y' = \frac{h_{21}x + h_{22}y + h_{23}}{h_{31}x + h_{32}y + 1} \qquad 2$$

Since we have 8 unknowns to solve for, we need to consider a minimum of 4 points. Let the 4 points be $(x_1, y_1), (x_2, y_2)(x_3, y_3)(x_4, y_4)$. Following equation 1 and 2 the mapped points $(x'_1, y'_1), (x'_2, y'_2), (x'_3, y'_3), (x'_4, y'_4)$ are given by

$$\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix} = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}$$

The homography matrix is given by, $H = \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \end{bmatrix}$

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and

$$A = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ x'_3 \\ y'_3 \\ x'_4 \\ y'_4 \end{bmatrix}, \qquad B = \begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x'_1 & -y_1x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y'_1 & -y_1y'_1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x'_2 & -y_2x'_2 \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y'_2 & -y_2y'_2 \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x'_3 & -y_3x'_3 \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y'_3 & -y_3y'_3 \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x'_4 & -y_4x'_4 \\ 0 & 0 & 0 & x_4 & y_4 & 1 & -x_4y'_4 & -y_4y'_4 \end{bmatrix}$$

Thus, the homography matrix H = $B^{-1}A$.

Method for obtaining pixel values

The pixel values are obtained using approximation, ie we round the pixel value to value of the nearest pixel. If the x and y coordinates are decimal values, then we round them to the nearest integer and obtain the pixel value at that point. This method is accurate enough to get and efficient mapping of the face in one image to the frame in the other image.

Task (a)

Goal: To project the face of Jackie Chan in image 1d to the frames in image 1a, 1b, 1c

Method:

1. Find the homography matrix H using the formula X' = H X, where X' are the four corners of the Jackie Chan image while X are the four points P, Q, R, S in each image 1a, 1b, 1c. The method for calculating the entries of the homography matrix is explain in the section above*, Method for calculating the homography matrix*
2. Once the homography matrix between the destination (image with the frame) and source image (image with Jackie Chan's face) is found, create a new blank image with a bounded area having the same dimension as the frame in the source image.
3. Loop over all the pixels in the source image (one with the frame) and only change those that are in bounded area PQRS. The point in the bounded region is mapped to the source image using the homography matrix H and then color of the pixel is retrieved from the source image. The method for obtaining the pixel RGB value is explained in the section*, Method for obtaining pixel values.*

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Input images:

Figure 1: Destination image 1a with the frame PQRS

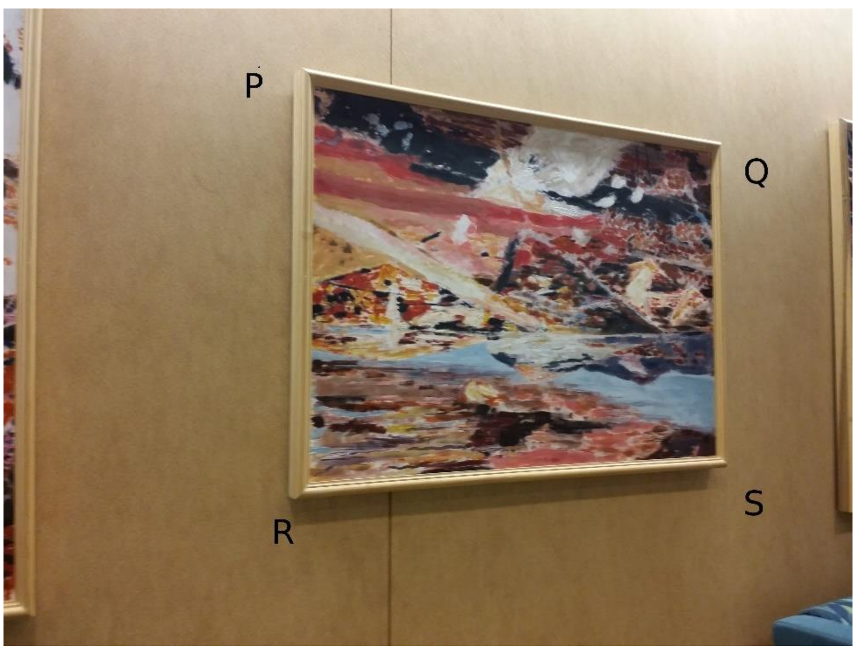

Figure 2: Destination image 1b with the frame PQRS

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Figure 3: Destination image 1c with the frame PQRS

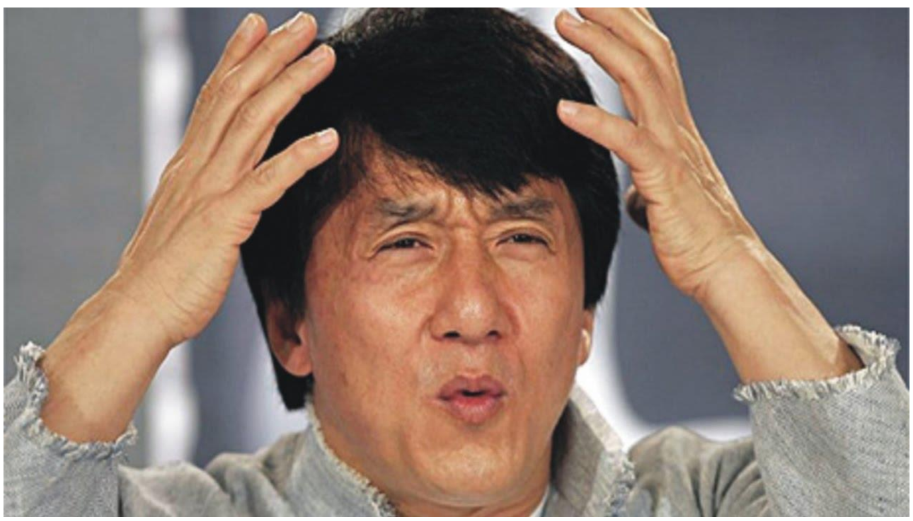

Figure 4: Input image 1d of Jackie Chan's face

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Output Images:

Figure 5: Final image with Jackie Chan's face from image 1d mapped onto the frame in image 1a

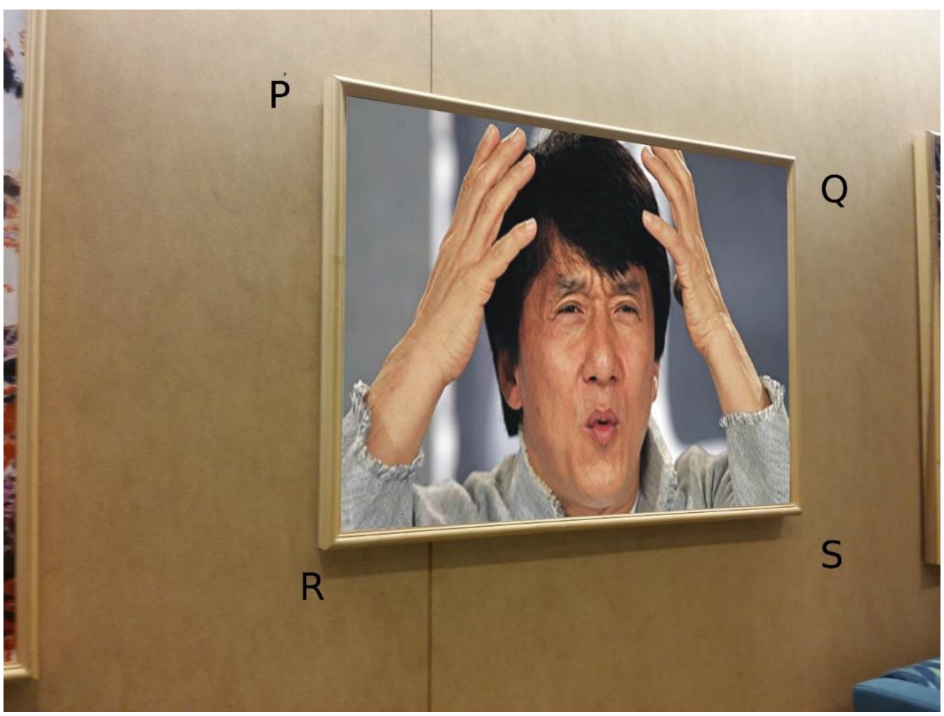

Figure 6: Final image with Jackie Chan's face from image 1d mapped onto the frame in image 1b

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Figure 5: Final image with Jackie Chan's face from image 1d mapped onto the frame in image 1c

Task (b)

Goal: To find the product of the homography matrices between images 1a and 1b and images 1b and 1c and to use the resulting matrix to map image 1a

Method:

1. Find the homography matrix H using the formula X' = H X, where X' and X are the four points P, Q, R, S in images 1b and 1a. Repeat the process with images 1b and 1c. The method for calculating the entries of the homography matrix is explain in the section above*, Method for calculating the homography matrix.*
2. Calculate the product of the two homography matrices obtained in step 1 to calculate the resulting homography matrix.
3. Once the homography matrix is found, create a new blank image with the same dimension as the image 1a.
4. Loop over all the pixels in the blank image and change the points using the homography matrix H. Next obtain color of the pixel from image1a. The method for obtaining the pixel RGB value is explained in the section*, Method for obtaining pixel values.*
5. The resulting image should have the same orientation of the image 1c.

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Input images:

Figure 6: Destination image 1a with the pixel points P, Q, R, S

Figure 7: Destination image 1b with the pixel points P, Q, R, S

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Figure 8: Destination image 1c with the pixel points P, Q, R, S

Output Image:

Figure 9: Final image like the image 1c

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Task (c)

Goal: To project the face of Nicole Kidman in image 1d to the frames in image 1a, 1b, 1c

Method:

1. Find the homography matrix H using the formula X' = H X, where X' is the four corners of the Jackie Chan image while X are the four points P, Q, R, S in each image 1a, 1b, 1c. The method for calculating the entries of the homography matrix is explain in the section above*, Method for calculating the homography matrix*
2. Once the homography matrix between the destination (image with the frame) and source image (image with Nicole Kidman) is found, create a new blank image with a bounded area having the same dimension as the frame in the source image.
3. Loop over all the pixels in the source image (one with the frame) and only change those that are in bounded area PQRS. The point in the bounded region is mapped to the source image using the homography matrix H and then color of the pixel is retrieved from the source image. The method for obtaining the pixel RGB value is explained in the section*, Method for obtaining pixel values.*

Input images:

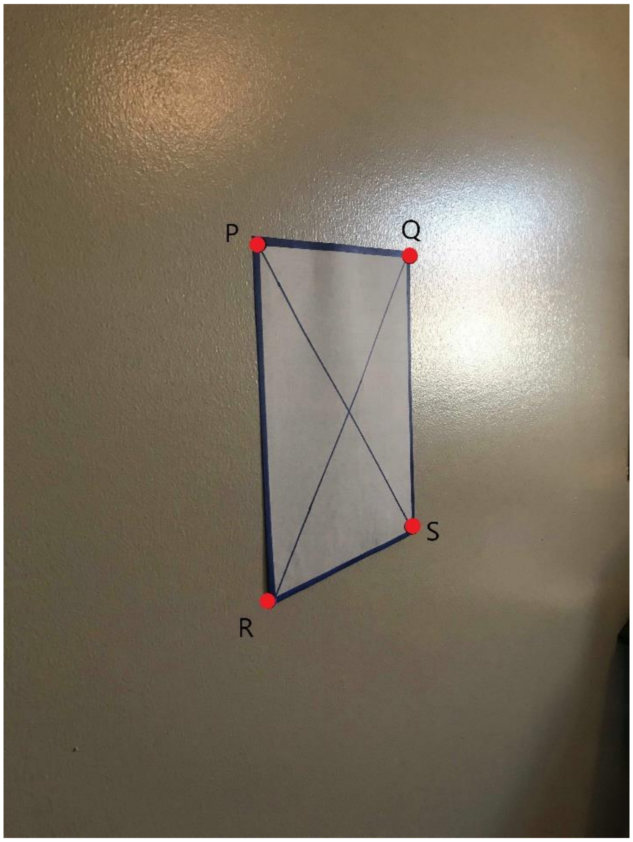

Figure 10: Destination image 1a with the frame PQRS

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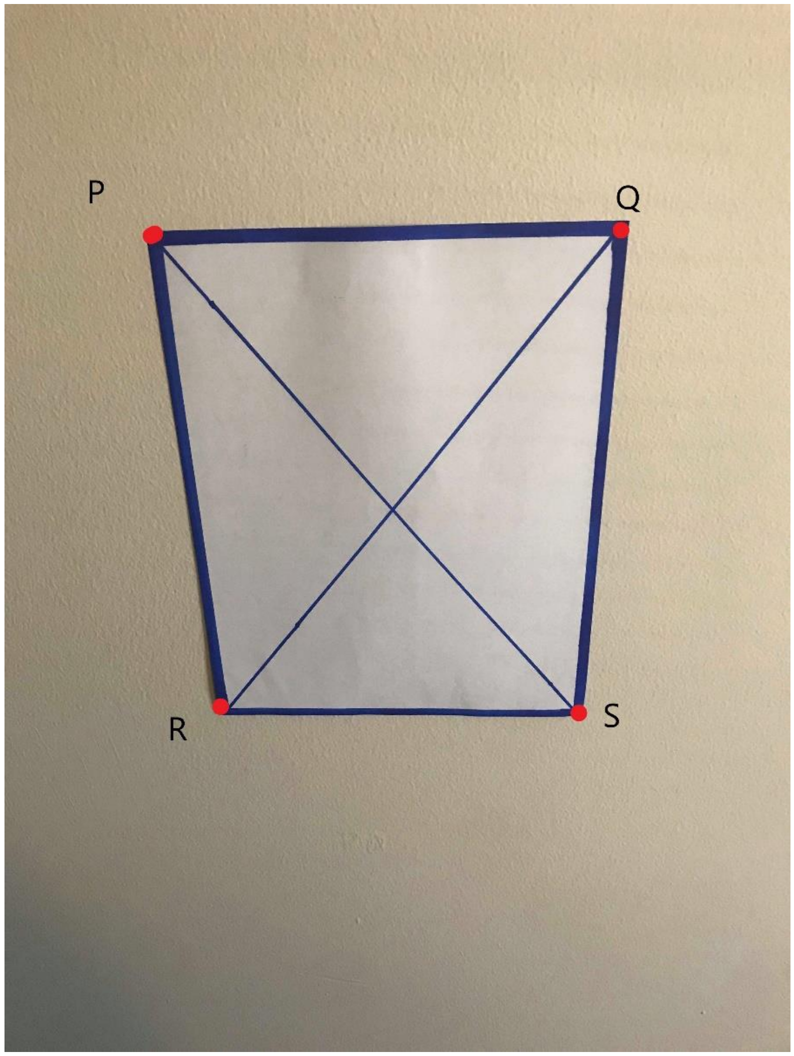

Figure 11: Destination image 1b with the frame PQRS

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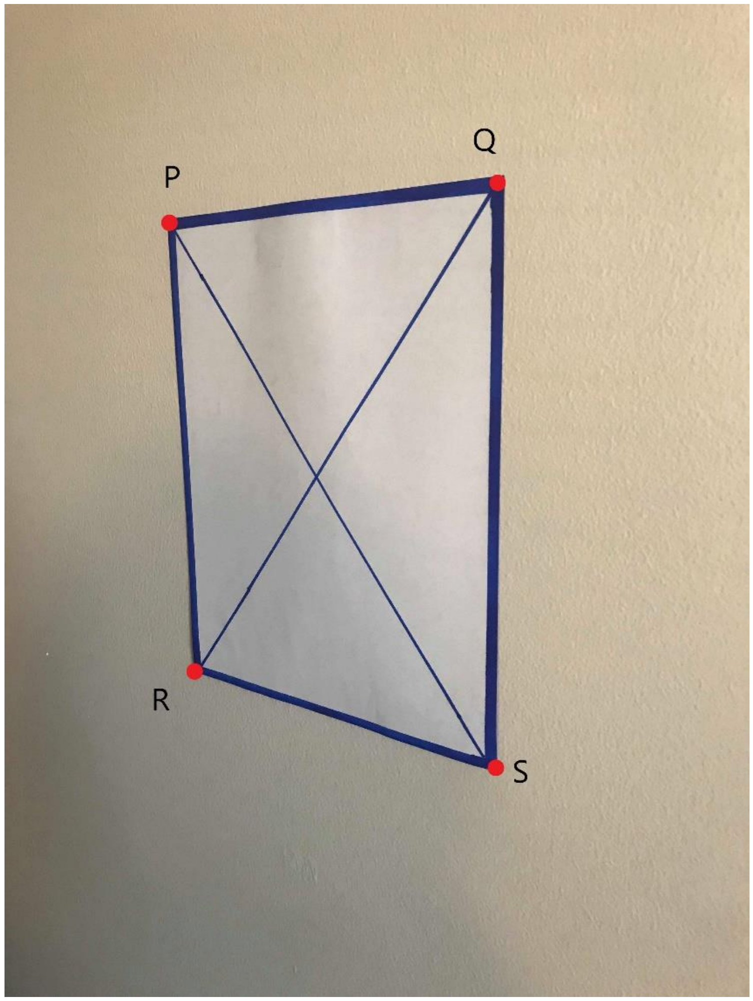

Figure 12: Destination image 1c with the frame PQRS

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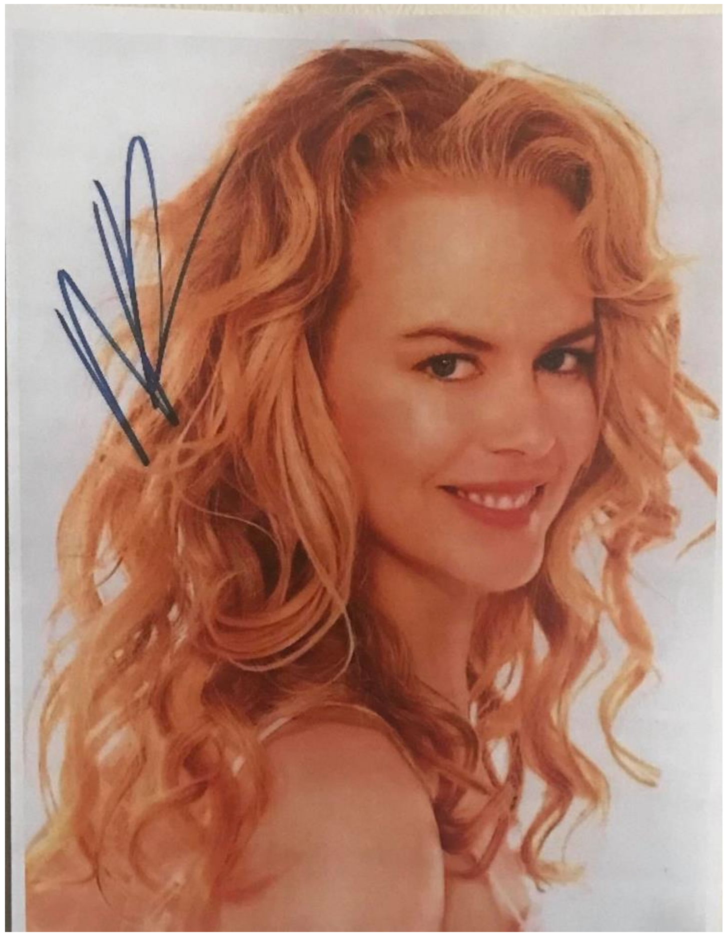

Figure 13: Input image 1d of Nicole Kidman’s face

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Output Images:

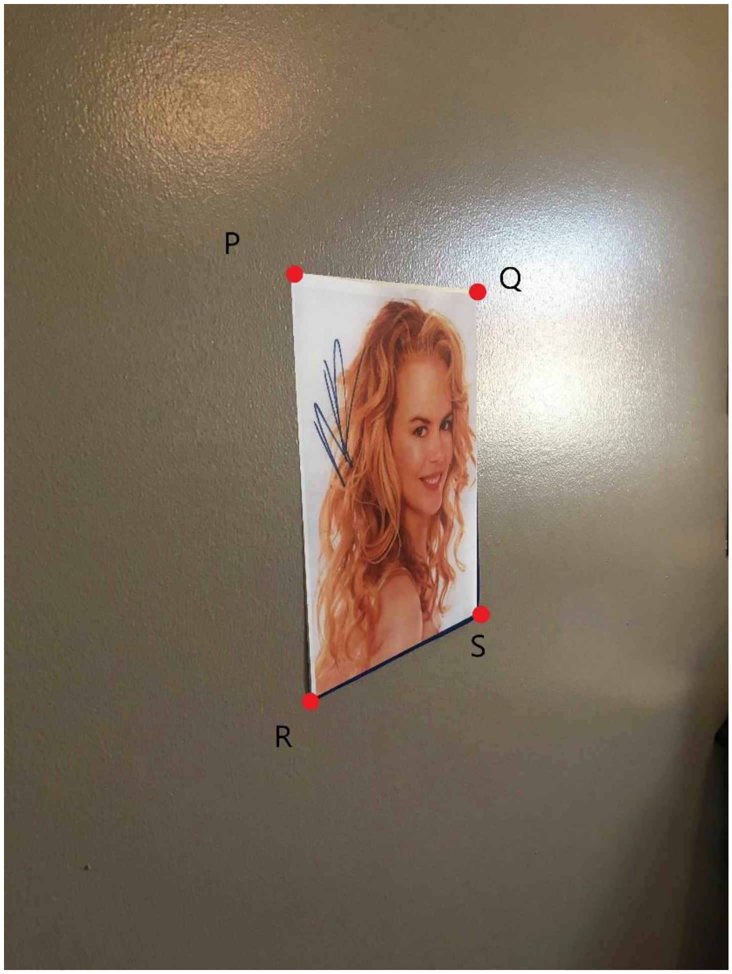

Figure 14: Final image with Nicole Kidman's face from image 1d mapped onto the frame in image 1a

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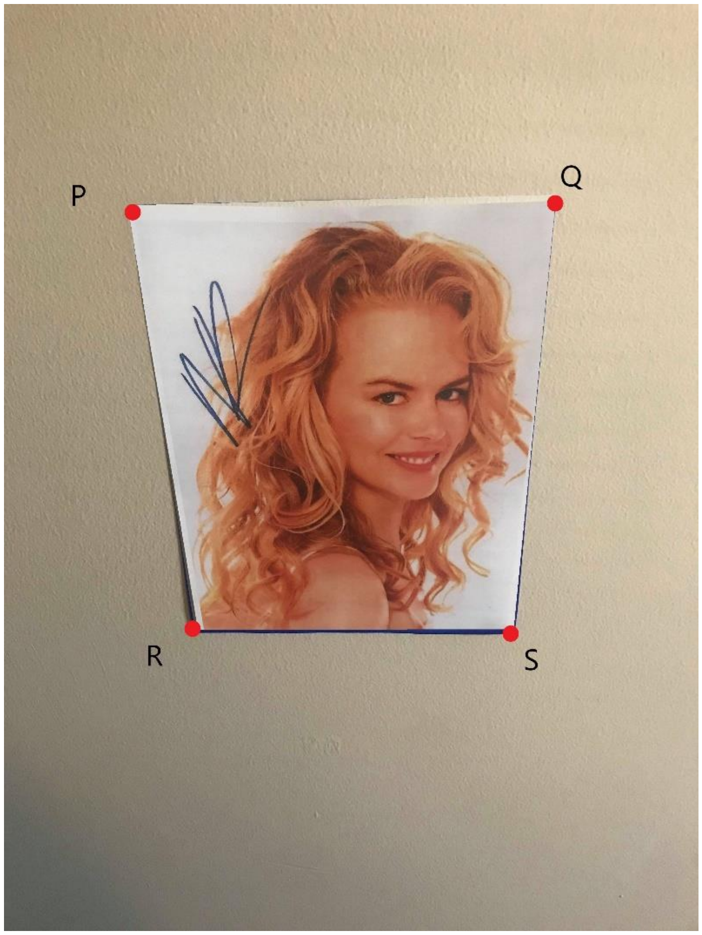

Figure 15: Final image with Nicole Kidman's face from image 1d mapped onto the frame in image 1b

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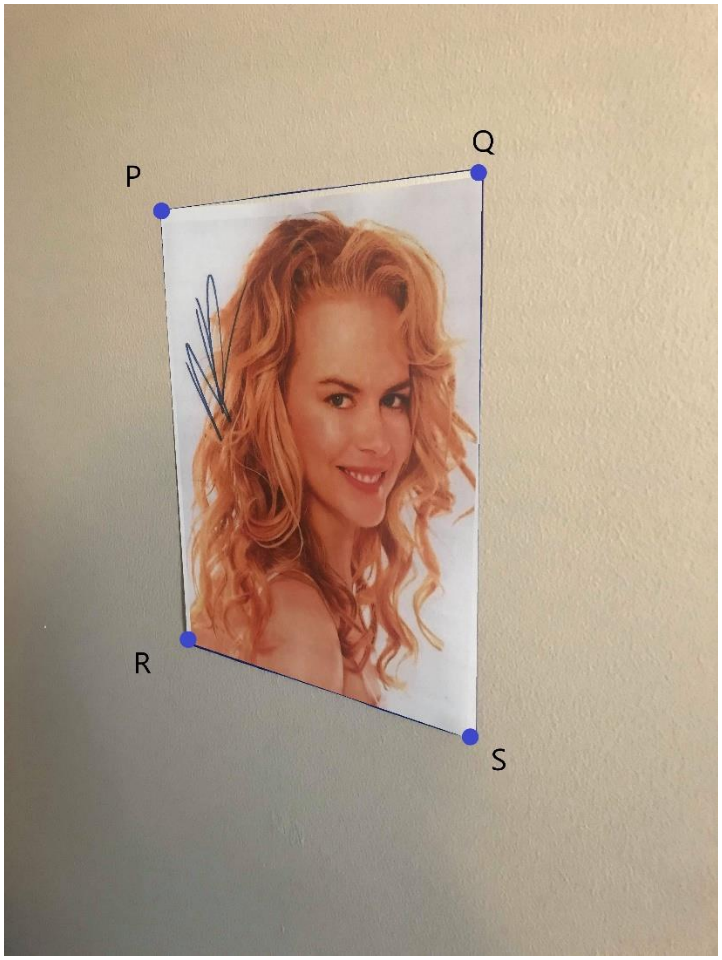

Figure 16: Final image with Nicole Kidman's face from image 1d mapped onto the frame in image 1c