ETH
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Urban Computation

Linear and linear integer optimization

Conclusion
Main points

- Many problems in urban management can be modelled as optimisation problems

- Modelling the problems increases the probability that optimal solutions will be found.

- Expectation of you (after this class, reading the script, and doing the assignments):
  - To be able to formulate linear integer optimization problems, including the use of binary variables to model logical relationships

  - To be able to explain the basic principals behind the simplex algorithm

  - To be able to conduct a sensitivity analysis.

Integrality conditions

- An integrality condition indicates that some (or all) of the variables in the formulation must attain only integer values.

Number of houses type A to execute intervention

Number of house type B to execute intervention

$$
\begin{array}{lllllll}
\text{Max}: & 2 & \cdot x_1 & +3 & \cdot x_2 & & \\
\text{Subject to}: & & x_1 & +3 & \cdot x_2 & \leq & 8.25 \\
 & 2.5 & \cdot x_1 & + & \cdot x_2 & \leq & 8.75 \\
 & & x_1 & & , x_2 & \geq & 0 \\
 & & \boxed{x_1} & & \boxed{, x_2} & & \boxed{\text{int.}}
\end{array}
$$

Budget constraint

Time constraint

Non-negativity constraints

Integrality constraints

Relaxation approach

- Ignore the integrality constraint and round the answer

- The objective function value for the optimal solution to its ILP relaxation problem can never be better than the objective function value for the optimal solution to its LP relaxation

- Solving the LP relaxation provides information that helps determine the quality of integer solutions

Branch and bound algorithm

- relax all integrality constraints
- determine optimal solution to the linear optimization problem, if this solution consists of only integer values stop, otherwise continue to step 3
- modify the problem so that the corner points of the feasible region for the relaxed problem are integer feasible solutions.
- determine optimal solution to the new problems. If the solutions to these problems are both integer solutions stop and select the optimal solution to the original problem. If not continue to step 3 with the problem that does not yet have an integer solution.

Branch and bound algorithm – Example

- Any integer variable in an ILP that assumes a fractional value in the optimal solution to the relaxed problem can be designated as a “branching variable”

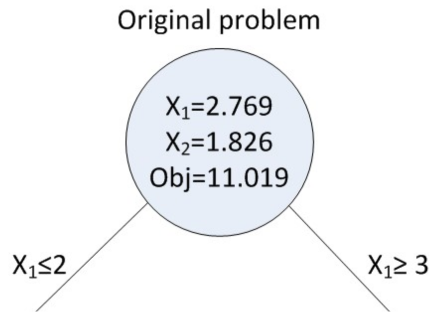
$$x_1 = 2.769,\ x_2 = 1.826$$

$$\begin{array}{lrlrlcl}
\text{Max:} & 2 & \cdot x_1 & +3 & \cdot x_2 & & \\
\text{Subject to:} & & x_1 & +3 & \cdot x_2 & \leq & 8.25 \\
 & 2.5 & \cdot x_1 & + & x_2 & \leq & 8.75 \\
 & & x_1 & & & \leq & 2 \\
\hline
 & & x_1 & & , x_2 & \geq & 0 \\
 & & x_1 & & , x_2 & & \text{int.}
\end{array}$$

$$\begin{array}{lrlrlcl}
\text{Max:} & 2 & \cdot x_1 & +3 & \cdot x_2 & & \\
\text{Subject to:} & & x_1 & +3 & \cdot x_2 & \leq & 8.25 \\
 & 2.5 & \cdot x_1 & + & \cdot x_2 & \leq & 8.75 \\
 & & x_1 & & & \geq & 3 \\
\hline
 & & x_1 & & , x_2 & \geq & 0 \\
 & & x_1 & & , x_2 & & \text{int.}
\end{array}$$

Linear integer optimisation

Branch and bound algorithm – Example



$$Z = 10 \qquad x_1 = 2.00, x_2 = 2.00$$

Binary variables

- Logical conditions
  - No more than 1 of multiple options
  - One of multiple options
  - Either both options should be selected or neither
  - If one option is chosen so is another

- Linking constraints
  - a relationship between the x_i and the y_i

$$x_i \leq M_i \cdot y_i$$

  - Value of M_i indicates the upper bound on the values of x_i

Example: Bridge repair with crew mobilization costs

- You, as a manager in a road authority, are constructing a work program for its bridges in the upcoming five year period. You have three types of bridges, metal, concrete and masonry bridges that are candidates for intervention. Each of which require engineer design time, contractor time and project manager time.
- You know that engineers need roughly 200 hours per steel bridge, 300 hours per concrete bridge, and 600 hours per masonry bridges, that contractors need 600 hours per steel bridge, 300 hours per concrete bridge and 400 hours per masonry bridge and project managers need roughly 500 hours per steel bridge, 600 hours per concrete bridge and 200 hours per masonry bridge. Their availability to execute the interventions is, of course, not unlimited. The engineer time, the contractor time and the project manager time that is available is a maximum of 60’000 hours, 30’000 hours and 40’000 hours, respectively.
- Since you cannot execute interventions on all bridges that require them you would like to ensure that you are providing the users of the road network with the maximum level of service possible. It has been has determined that the difference between the variable costs of executing an intervention on each type of bridge and the benefits obtained by the users of each type of bridge of type 1, 2 and 3 are 4’800, 5’500 and 5’000 mus, respectively.
- Through past experience you also know that the fixed costs of executing interventions, regardless of the number, on bridges of type 1, 2 and 3 are 10’000 mu, 80’000 mu, and 90’000 mu, respectively.

Examples

Sand – salt ordering problem

- You are the manager that orders salt and sand for roads in the winter, in a group of three newly amalgamated communities. Due to the agreement made at amalgamation you need to order the salt and sand from three different suppliers in each of the communities (A, B and C) for the next two years, for
- Sand: 200 CHF/t, 250 CHF/t, 300 CHF/t from suppliers 1, 2 and 3, respectively
- Salt : 160 CHF/t, 140 CHF/t, 170 CHF/t from suppliers 1, 2 and 3, respectively.
- Afterwards you are free to buy from, whoever gives you the best price.
- According to the agreement, you need to buy at least 50 tons of salt and 50 tons of sand from each supplier in each year. You also have to give exactly the same amount of money to each of the suppliers in each of the two years, although they do not have to be the same in year 1 and year 2.
- You have determined that you need:
- 500 tons of salt in year 1 and 700 tons of salt in year 2, and
- 350 tons of sand in year 1, and 600 tons of sand in year 2.

- How much salt and sand do you need to order from each supplier in year 1 and year 2 in order to have the minimal overall costs?

Examples

Road surfacing problem

- An infrastructure management company is awarded a contract to do resurfacing of 20 km of road in a high-density residential area. According to the environmental regulation, all activities must be in compliance with environmental regulation. This regulation was enforced in order to protect the stakeholders against environmental issues such as pollution, energy consumption, etc. These regulations mean that the total CO_2 emission from the intervention cannot exceed 13 tons.
- In addition to the environmental regulations you are know that the intervention cannot cost more than 2.86 million mu, or otherwise you will lose money, and that you have to have the project completed within 25 days.
- You are considering using two different techniques to resurface the road 1) Hot mixed asphalt concrete (HMAC); and 2) Warm mixed asphalt concrete (WMAC). The first one is faster but produces more CO_2 than the second one. It is known that producing and paving 1 m^3 of HMAC discharges approximately 2000 kg of CO_2, and that producing and paving 1 m^3 of WMAC discharges 1600 kg of CO_2.
- The maximum amount of HMAC that can be produced and paved is 550 tons/days and the maximum amount of WMAC is 450 tons/day. Pavement with HMAC is 5 cm thick. Pavement with WMAC is 5 cm thick. The road is 6.5 m wide.
- The cost to produce and pave HMAC and WMAC are 20 and 25 mu/m^2. The profit from you expect from each is 3 and 4 mu/m^3, respectively.
- The density of asphalt is 2'400 kg/m3

- How much of the 20 km road section should you pave with HMAC and WMAC to maximize your profit? How many days will you produce and pave with HMAC and WMAC?

Examples

Snow removal problem (1/3)

- Snow removal and disposal are important and expensive activities in your city. Even though snow can be cleared from streets and sidewalks by plowing and shoveling, in prolonged sub-freezing temperatures, the resulting banks of accumulated snow can impede pedestrian and vehicular traffic and must be removed.
- To allow timely removal and disposal of snow, you have divided your city into several sectors and snow removal operations are carried out concurrently in each sector.
- Accumulated snow is loaded onto trucks and hauled away to disposal sites, e.g., rivers, quarries, sewers, and surface areas. The different types of disposal sites can accommodate different amounts of snow, either because of the physical size of the disposal facility, or because of the environmental restrictions on the amount of snow (often contaminated by salt and de-icing chemicals). The annual capacities for the five different snow disposal sites available to you are
- Storage capacity of disposal sites

Disposal sites[1]				
1	2	3	4	5
350	250	500	400	200

[1]in 1’000 m^3

Snow removal problem (2/3)

- The cost of removing and disposing of snow depends mainly on the distance it must be trucked. You have decided to use the straight line distance between the centers of each sector to each of the various disposal sites as an approximation of the cost involved in transporting snow between these locations.
- Distance from each sector to each disposal site

[1]in km

sector	Disposal site				
	1	2	3	4	5
1	3.4	1.4	4.9	7.4	9.3
2	2.4	2.1	8.3	9.1	8.8
3	1.4	2.9	3.7	9.4	8.6
4	2.6	3.6	4.5	8.2	8.9
5	1.5	3.1	2.1	7.9	8.8
6	4.2	4.9	6.5	7.7	6.1
7	4.8	6.2	9.9	6.2	5.7
8	5.4	6.0	5.2	7.6	4.9
9	3.1	4.1	6.6	7.5	7.2
10	3.2	6.5	7.1	6.0	8.3

- You have estimated, using past snow fall records, that the annual volume of snow requiring removal in each sector as four times the length of the roads in the sectors in meters (i.e. it is assumed that each meter of road generates four cubic meters of snow to remove over the entire year). It costs 0.10 mu to transport 1 m^3 of snow one kilometer.
- Quantity of snow to be removed per year

Sector	1	2	3	4	5	6	7	8	9	10
Snow to be removed	153	152	154	138	127	129	111	110	130	135

Snow removal problem (3/3)

- Question A
  - What is the most efficient way to remove snow? and how much will it cost?

- Question B
  - If there were no constraints on the amount of snow that could be dumped at each site what would the most efficient way be to remove snow? and how much would it cost?

- Question C
  - If you could increase the capacity of a single disposal site by 100 m3, which would it be? How much should you be willing to pay to do this?

Power generation problem

- As the owner of a power company you are considering how to increase your generating capacity to meet expected demand in your growing service area. Currently, you have 750 MW of generating capacity but predict you will need the minimum generating capacities in each of the next five years

- Minimum required generating capacity

Year	1	2	3	4	5
Capacity (MW)	780	860	950	1‘060	1‘180

- You can increase your generating capacity by purchasing generators of four different types: 10 MW, 25MW, 50MW and/or 100MW. The cost of acquiring and installing a generator of each type in the next five years

- Cost of generators

Generator type	Costs of generator (in 1’000 mu) in year				
	1	2	3	4	5
10 MW	300	250	200	170	145
25MW	460	375	350	280	235
50MW	670	558	465	380	320
100MW	950	790	670	550	460

- How many generators of each type should you buy and install each year to meet demand but to minimize purchasing cost?

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