Urban Computation

Network optimisation

Script

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(ECTS-3KP)

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Disclaimer:

This document has been compiled to provide students taking the above mentioned course in understanding the examples given in class and assignments. All questions presented have been prepared by the authors based to varying degrees on different sources. This document is being issued so that students taking the course may better prepare for the exam. As it has not yet undergone the complete rigorous control processes of IBI this document is to be considered a work in progress and is hence labeled as a DRAFT. This chapter is based on the following sources:

Hillier F.S., and Liebermann, G.J. (2010), Introduction to operations research, 9th edition, McGraw-Hill, 1047 pages.

Lethanh, N, Adey, B.T., Sigrist, M. (2014), A Mix-integer Linear Model for Optimizing Work Zone Interventions on a Transportation Network, International Conference on Engineering and Applied Sciences Optimisation, Kos Island, Greece, June 4-6.

Ragsdale, C.T., (2008), Spreadsheet Modelling and decision analysis: A practical introduction to management science, 5th edition, 820 pages.

Revelle C.S., Whitlach, E.E., and Wright J.R., (2004), Civil and Environmental Systems Engineering, Pearson International, 552 pages.

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1 Network optimisation

1.1 General

Network optimisation has a particularly important role in the management of infrastructure as a considerable portion of the infrastructures are network infrastructures, e.g. road networks, rail networks, and water distribution networks. Indeed, many problems cannot be properly tackled unless they are modelled as networks. In addition to this, it is advantageous to think of many other problems as networks, in order to solve them.

A general network optimisation problem consists of start nodes, i.e. the points of entry into the network, intermediary nodes, i.e. the nodes that represent intersections within the network, and end nodes, i.e. the nodes of exit from the network. These are also sometimes referred to as supply nodes, link nodes and demand nodes, respectively. They are also sometimes referred to as origin nodes, middle nodes and destination nodes, respectively. In network models the nodes are connected through links, and each node and link have capacities.

In minimum cost network optimisation problems there must be at least one constraint per node in the network, and it is necessary to apply, so called, balance of flow rules. Balance of flow rules ensure that items transported across the network move as intended. They are:

1) if the total supply to the network is greater than the demand from the network, than inflow – outflow >= supply or demand.
2) if the total supply to the network is less than the demand from the network, the inflow – outflow <= supply or demand
3) if the total supply to the network equals the demand from the network, the inflow – outflow = supply or demand

Four typical types of network optimisation problems are:

1) shipment problems – where the goal is to transport items across a network using all paths for the lowest cost,
2) shortest path problems – where the goal is to transport items across a network using one path for the lowest cost,
3) maximal flow problems – where the goal is to transport as many items as possible across a network,
4) assignment problems – where the goal is to determine where to produce items and how to transport them across a network using all paths for the lowest cost.

Examples of the first three are given in Example 1. An example of the fourth is given in example 2. An additional, somewhat more complex example of a shortest path problem is given in example 3.

There are numerous things to keep in mind to ensure that the models are built correctly when developing network models. They are:

1) to put constraints (minimum or maximum) on the number of items entering or leaving a node, dummy nodes are required to ensure that the balance of flow rules are not violated. Otherwise it is necessary to impose side constraints on the model.

2) multiple flows between nodes cannot be modelled directly. It is necessary to implement dummy nodes so that only one flow goes from node *i* to node *j*.
3) in cases where it is uncertain which balance of flow rules should be applied (often due to restrictions on the links) it is advantageous to use dummy nodes to siphon off excess supply at very high costs.

1.2 Example 1 – Cement production and shipment problem

1.2.1 Problem

Your company wants to transport truckloads of cement from a production plant to a warehouse. From the plant to the warehouse, there are many routes that your trucks can take. There are costs associated with travelling on each route that are route dependent. These are indicated as the number not in the brackets beside each link in Table 1. There are also restrictions on the number of trucks that you can send over each link. These are also given in Table 1.

Table 1. Amount to transport over each link

Link		Unit cost of transport on link (mu)	Link capacity (units)
From node	To node		
1	2	5	250
1	3	2	300
1	4	1	80
1	5	4	150
2	6	2	250
2	7	1	320
3	2	2	150
3	7	3	130
4	3	2	90
4	5	2	200
4	7	6	180
5	9	12	200
6	8	1	255
7	8	4	350
7	9	9	240
7	10	7	250
8	10	6	300
9	10	4	250

The graphical representation of the problem is shown in Figure 1. In the figure, numbers inside nodes denote the name of nodes. Values of units of transport on links and link capacities are shown in the brackets.

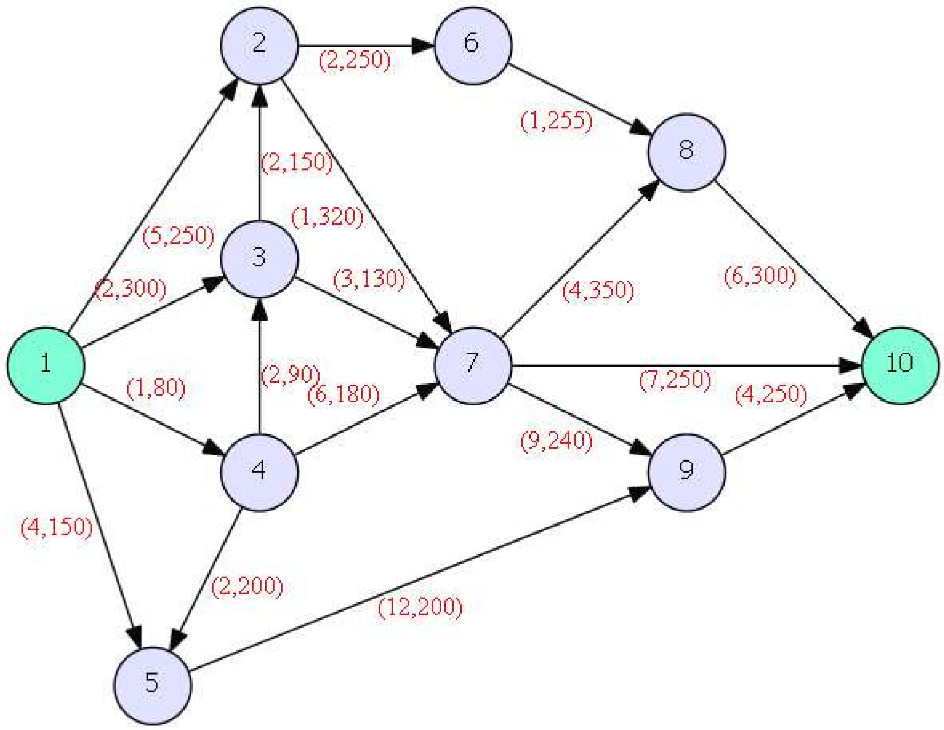


Figure 1. Physical road network

1.2.2 Question A – Lowest cost shipment problem

What is the least expensive way for you to transport 700 truckloads of cement from the plant (node 1) to the warehouse (node 10)?

1.2.3 Answer A

1.2.3.1 Conceptual model

The answer to this problem can be found by modelling the network and setting the plant site as the start node and the warehouse as the end node. The roads connecting the plant to the warehouse are the links of the network. All other nodes are the intermediary nodes.

1.2.3.2 Specific mathematical model

The objective function is

Maximize

$$\begin{aligned} Z = {} & 5\cdot x_{12} + 2\cdot x_{13} + 1\cdot x_{14} + 4\cdot x_{15} + 2\cdot x_{26} + 1\cdot x_{27} + 2\cdot x_{32} + 3\cdot x_{37} + 2\cdot x_{43} + 2\cdot x_{45} \\ & + 6\cdot x_{47} + 12\cdot x_{59} + 1\cdot x_{68} + 4\cdot x_{78} + 9\cdot x_{79} + 7\cdot x_{7-10} + 6\cdot x_{8-10} + 4\cdot x_{9-10} \end{aligned} \tag{1}$$

subject to the following functional constraints

$$
\begin{aligned}
&700+0 = x_{12}+x_{13}+x_{14}+x_{15}+0,\\
&0+x_{12}+x_{32} = x_{26}+x_{27}+0,\\
&0+x_{13}+x_{43} = x_{32}+x_{37}+0,\\
&0+x_{14} = x_{43}+x_{47}+x_{45}+0,\\
&0+x_{15}+x_{45} = x_{59}+0,\\
&0+x_{26} = x_{68}+0,\\
&0+x_{27}+x_{37}+x_{47} = x_{78}+x_{79}+x_{7-10}+0,\\
&0+x_{68}+x_{78} = x_{8-10}+0,\\
&0+x_{59}+x_{79} = x_{9-10}+0,\\
&0+x_{7-10}+x_{8-10}+x_{9-10} = 700,
\end{aligned}
\tag{2}
$$

and functional and non-negativity constraints

$$
\begin{aligned}
&0 \le x_{12} \le 250,\\
&0 \le x_{13} \le 300,\\
&0 \le x_{14} \le 80,\\
&0 \le x_{15} \le 150,\\
&0 \le x_{26} \le 250,\\
&0 \le x_{27} \le 320,\\
&0 \le x_{32} \le 150,\\
&0 \le x_{37} \le 130,\\
&0 \le x_{43} \le 90,\\
&0 \le x_{45} \le 200,\\
&0 \le x_{47} \le 180,\\
&0 \le x_{59} \le 200,\\
&0 \le x_{68} \le 255,\\
&0 \le x_{78} \le 350,\\
&0 \le x_{79} \le 240,\\
&0 \le x_{7-10} \le 250,\\
&0 \le x_{8-10} \le 300,\\
&0 \le x_{9-10} \le 250
\end{aligned}
\tag{3}
$$

1.2.3.3 Solution

The least expensive way to transport 700 truckloads of cement to the warehouse is shown in Table 2 and will cost 10’230 mus. The AMPL Code is given in the appendix (section 2.1.1). The graphical representation of the solution is shown in Figure 2, where the link constraint is replaced with the amount transported on the link.

Table 2. Amount to transport over each link

Link		Unit cost of transport on link (mu)	Capacity (units)	Amount to transport over each link (units)	Costs incurred on each link (mu)
From node	To node				
1	2	5	250	250	1’250
1	3	2	300	280	560
1	4	1	80	80	80
1	5	4	150	90	360
2	6	2	250	250	500
2	7	1	320	150	150
3	2	2	150	150	300
3	7	3	130	130	390
4	3	2	90	0	0
4	5	2	200	60	120
4	7	6	180	20	120
5	9	12	200	150	1’800
6	8	1	255	250	250
7	8	4	350	50	200
7	9	9	240	0	0
7	10	7	250	250	1’750
8	10	6	300	300	1’800
9	10	4	250	150	600
Total					10‘230

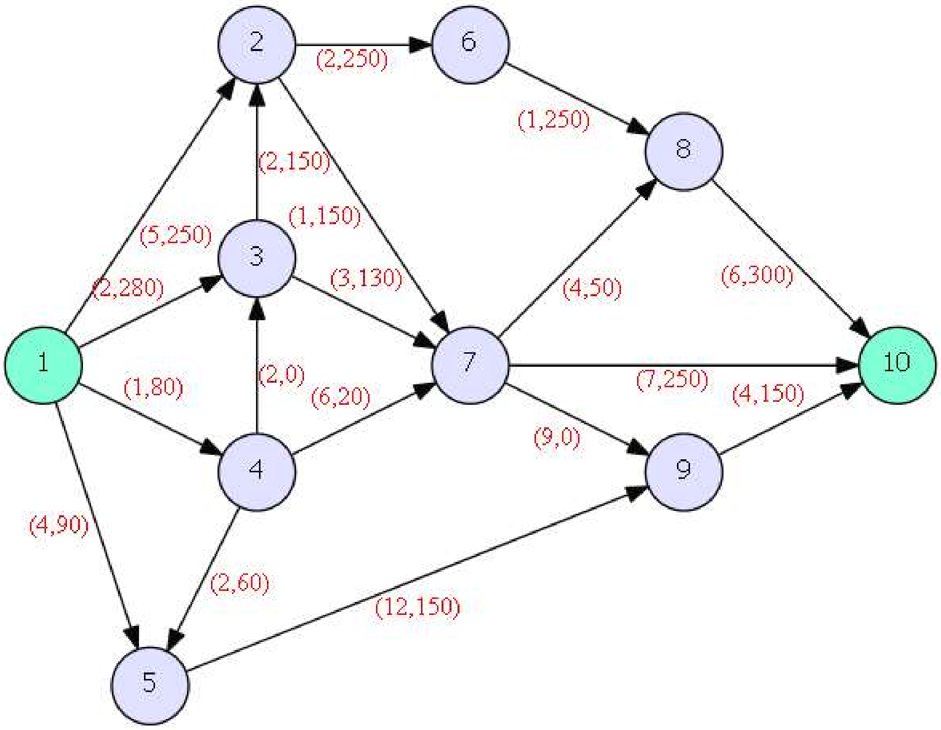


Figure 2. Graphical representation of the optimal solution-question A

1.2.4 Question B

What is the least expensive way for you to send one truckload of cement to the warehouse?

1.2.5 Answer B

1.2.5.1 Specific mathematical model

The objective function is

$$\begin{aligned}&\text{Maximize}\\ &Z = 5\cdot\delta_{12} + 2\cdot\delta_{13} + 1\cdot\delta_{14} + 4\cdot\delta_{15} + 2\cdot\delta_{26} + 1\cdot\delta_{27} + 2\cdot\delta_{32} + 3\cdot\delta_{37} + 2\cdot\delta_{43} + 2\cdot\delta_{45}\\ &\quad + 6\cdot\delta_{47} + 12\cdot\delta_{59} + 1\cdot\delta_{68} + 4\cdot\delta_{78} + 9\cdot\delta_{79} + 7\cdot\delta_{7-10} + 6\cdot\delta_{8-10} + 4\cdot\delta_{9-10}\end{aligned} \tag{4}$$

subject to the following functional constraints

$$\begin{aligned}&1+0 = \delta_{12} + \delta_{13} + \delta_{14} + \delta_{15} + 0,\\ &0+\delta_{12} + \delta_{32} = \delta_{26} + \delta_{27} + 0,\\ &0+\delta_{13} + \delta_{43} = \delta_{32} + \delta_{37} + 0,\\ &0+\delta_{14} = \delta_{43} + \delta_{47} + \delta_{45} + 0,\\ &0+\delta_{15} + \delta_{45} = \delta_{59} + 0,\\ &0+\delta_{26} = \delta_{68} + 0,\\ &0+\delta_{27} + \delta_{37} + \delta_{47} = \delta_{78} + \delta_{79} + \delta_{7-10} + 0,\\ &0+\delta_{68} + \delta_{78} = \delta_{8-10} + 0,\\ &0+\delta_{59} + \delta_{79} = \delta_{9-10} + 0,\\ &0+x_{7-10} + x_{8-10} + x_{9-10} = 1,\end{aligned} \tag{5}$$

and non-negativity constraints

$$\begin{aligned}&0\le x_{12},\, 0\le x_{13},\, 0\le x_{14},\, 0\le x_{15},\, 0\le x_{26},\, 0\le x_{27},\, 0\le x_{32}\\ &0\le x_{37},\, 0\le x_{43},\, 0\le x_{45},\, 0\le x_{47},\, 0\le x_{59},\, 0\le x_{68},\, 0\le x_{78}\\ &0\le x_{79},\, 0\le x_{7-10},\, 0\le x_{8-10},\, 0\le x_{9-10}\end{aligned} \tag{6}$$

1.2.5.2 Solution

The least expensive way to send one truckload of cement to the warehouse is shown in Table 3 and costs 12 mus. The AMPL Code is given in the appendix (section 2.1.2). The graphical representation of the optimal solution is shown in Figure 3, where the link constraint is replaced with the amount transported on the link.

Table 3. Amount to transport over each link

Link		Unit cost of transport on link (mu)	Capacity (units)	Amount to transport over each link (units)	Costs incurred on each link (mu)
From node	To node				
1	2	5	unlimited	0	0
1	3	2	unlimited	1	2
1	4	1	unlimited	0	0
1	5	4	unlimited	0	0
2	6	2	unlimited	0	0
2	7	1	unlimited	1	1
3	2	2	unlimited	1	2
3	7	3	unlimited	0	0
4	3	2	unlimited	0	0
4	5	2	unlimited	0	0
4	7	6	unlimited	0	0
5	9	12	unlimited	0	0
6	8	1	unlimited	0	0
7	8	4	unlimited	0	0
7	9	9	unlimited	0	0
7	10	7	unlimited	1	7
8	10	6	unlimited	0	0
9	10	4	unlimited	0	0
Total					12

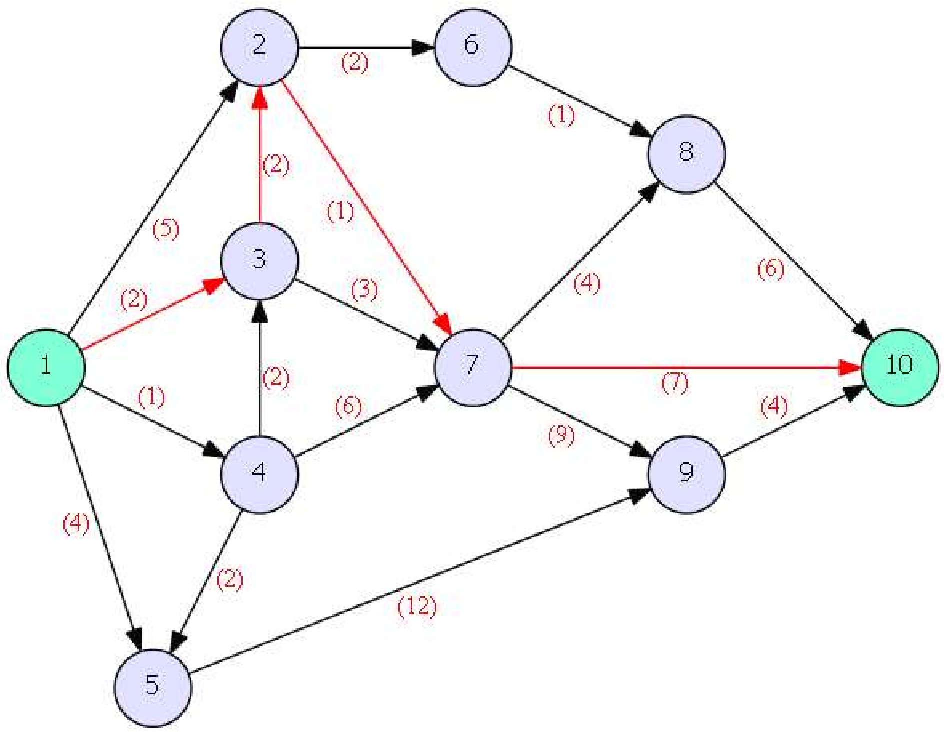


Figure 3. Graphical representation of the optimal solution-question B

1.2.6 Question C – Maximum shipment problem

What is the maximum number of truckloads of cement that you can send to your warehouse?

1.2.7 Answer C

1.2.7.1 Specific mathematical model

The objective function is

$$\text{Maximize} \quad \begin{aligned} &Z = x_{12} + x_{13} + x_{14} + x_{15}, \; or \\ &Z = x_{10-1} \end{aligned} \tag{7}$$

subject to the following functional constraints

$$\begin{aligned}
&x_{10-1} + 0 = x_{12} + x_{13} + x_{14} + x_{15} + 0, \\
&0 + x_{12} + x_{32} = x_{26} + x_{27} + 0, \\
&0 + x_{13} + x_{43} = x_{32} + x_{37} + 0, \\
&0 + x_{14} = x_{43} + x_{47} + x_{45} + 0, \\
&0 + x_{15} + x_{45} = x_{59} + 0, \\
&0 + x_{26} = x_{68} + 0, \\
&0 + x_{27} + x_{37} + x_{47} = x_{78} + x_{79} + x_{7-10} + 0, \\
&0 + x_{68} + x_{78} = x_{8-10} + 0, \\
&0 + x_{59} + x_{79} = x_{9-10} + 0, \\
&0 + x_{7-10} + x_{8-10} + x_{9-10} = x_{10-1},
\end{aligned} \tag{8}$$

and functional and non-negativity constraints

$$\begin{aligned}
&0 \le x_{12} \le 250, \\
&0 \le x_{13} \le 300, \\
&0 \le x_{14} \le 80, \\
&0 \le x_{15} \le 150, \\
&0 \le x_{26} \le 250, \\
&0 \le x_{27} \le 320, \\
&0 \le x_{32} \le 150, \\
&0 \le x_{37} \le 130, \\
&0 \le x_{43} \le 90, \\
&0 \le x_{45} \le 200, \\
&0 \le x_{47} \le 180, \\
&0 \le x_{59} \le 200, \\
&0 \le x_{68} \le 255, \\
&0 \le x_{78} \le 350, \\
&0 \le x_{79} \le 240, \\
&0 \le x_{7-10} \le 250, \\
&0 \le x_{8-10} \le 300, \\
&0 \le x_{9-10} \le 250
\end{aligned} \tag{9}$$

1.2.7.2 Solution

The maximum number of truckloads of cement that can be sent to the warehouse is 760. How they are shipped is shown in Table 4. The least expensive way to do this is 11’440 mus, which is given by substituting the results in Table 4 into:

$$Z = 5 \cdot x_{12} + 2 \cdot x_{13} + 1 \cdot x_{14} + 4 \cdot x_{15} + 2 \cdot x_{26} + 1 \cdot x_{27} + 2 \cdot x_{32} + 3 \cdot x_{37} + 2 \cdot x_{43} + 2 \cdot x_{45}$$
$$+ 6 \cdot x_{47} + 12 \cdot x_{59} + 1 \cdot x_{68} + 4 \cdot x_{78} + 9 \cdot x_{79} + 7 \cdot x_{7-10} + 6 \cdot x_{8-10} + 4 \cdot x_{9-10}.$$

The AMPL Code is given in the appendix (section 2.1.3). The graphical representation of the optimal solution is shown in Figure 4.

Table 4. Amount to transport over each link

Link		Unit cost of transport on link (mu)	Capacity (units)	Amount to transport over each link (units)	Costs incurred on each link (mu)
From node	To node				
1	2	5	250	250	1’250
1	3	2	300	280	560
1	4	1	80	80	80
1	5	4	150	150	600
2	6	2	250	250	500
2	7	1	320	150	150
3	2	2	150	150	300
3	7	3	130	130	390
4	3	2	90	0	0
4	5	2	200	50	100
4	7	6	180	30	180
5	9	12	200	200	2’400
6	8	1	255	250	250
7	8	4	350	50	200
7	9	9	240	10	90
7	10	7	250	250	1’750
8	10	6	300	300	1’800
9	10	4	250	210	840
Total					11’440

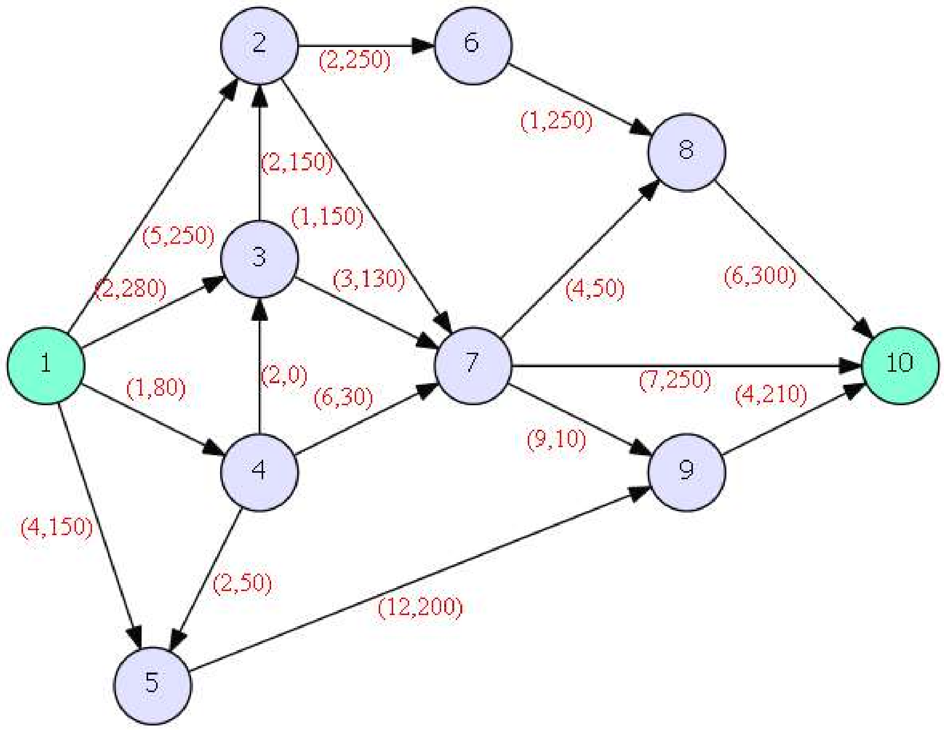


Figure 4. Graphical representation of the optimal solution-question C

1.3 Example 2 – Cement and steel production and delivery problem

1.3.1 Problem

Your company has now expanded. You produce and deliver both cement and reinforcement. In order to maximize your profits (or minimize the total cost) you want to ensure that your plants and the transport of the materials work in a way that you will at the same time ensure that the needs of your customers are satisfied but that for the least cost.

You are currently producing both cement and steel at three different locations (A, B, and C) and have contracts to deliver both materials in different quantities to six different destinations (1, 2, 3, 4, 5 and 6). Plant A can be operated 20 hours a day and can produce cement and reinforcement at 140 and 130 units per day. Plant B can be operated 15 hours a day and can produce cement and reinforcement at 130 and 160 units per day. Plant C can operate for 20 hours a day and can produce 160 and 170 units per day. The demand at the 6 construction sites is given in Table 6. The cost per truckload on each route between your production plants and the construction sites are given in Table 7. The costs of production are given in Table 5.

Table 5. Costs of production at each plant

Plant	Units of cement	Units of reinforcing steel
A	170	180
B	170	185
C	180	185

Table 6. Demand at the construction sties

Construction site	Required truckloads of cement per day	Required truckloads of reinforcing steel per day
1	500	100
2	750	100
3	400	0
4	250	50
5	950	200
6	850	100

Table 7. Cost of transport per path between plants and construction sites

Plant	Construction Site	Cost per truckload cement (mu)	Cost per truckload reinforcement (mu)
A	1	35	45
	2	12	22
	3	9	20
	4	30	37
	5	13	26
	6	42	54
B	1	27	29
	2	9	9
	3	12	13
	4	9	9
	5	26	28
	6	95	99
C	1	26	41
	2	7	14
	3	9	17
	4	12	13
	5	11	31
	6	23	104

1.3.2 Question

What is the least expensive way that you should produce and deliver the required truckloads of cement and reinforcing steel?

1.3.3 Answer

1.3.3.1 Specific mathematical model

The objective function is:

$$\text{Minimize}\ \begin{aligned}
&170\cdot x_{11}+170\cdot x_{21}+180\cdot x_{31}+180\cdot x_{12}+185\cdot x_{22}+185\cdot x_{32}\\
&+35\cdot y_{111}+27\cdot y_{211}+26\cdot y_{311}+45\cdot y_{112}+29\cdot y_{212}+41\cdot y_{312}\\
&+12\cdot y_{121}+9\cdot y_{221}+7\cdot y_{321}+22\cdot y_{122}+9\cdot y_{222}+14\cdot y_{322}\\
&+9\cdot y_{131}+12\cdot y_{231}+9\cdot y_{331}+20\cdot y_{132}+13\cdot y_{232}+17\cdot y_{332}\\
&+30\cdot y_{141}+9\cdot y_{241}+12\cdot y_{341}+37\cdot y_{142}+9\cdot y_{242}+13\cdot y_{342}\\
&+13\cdot y_{151}+26\cdot y_{251}+11\cdot y_{351}+26\cdot y_{152}+28\cdot y_{252}+31\cdot y_{352}\\
&+42\cdot y_{161}+95\cdot y_{261}+23\cdot y_{361}+54\cdot y_{162}+99\cdot y_{262}+104\cdot y_{362}
\end{aligned}$$

Subject to the functional constraints

Production constraints

$$\frac{1}{140}\cdot x_{11}+\frac{1}{130}\cdot x_{12}\le 20$$
$$\frac{1}{130}\cdot x_{21}+\frac{1}{160}\cdot x_{22}\le 15$$
$$\frac{1}{160}\cdot x_{31}+\frac{1}{170}\cdot x_{32}\le 20$$

Supply constraints

$$\sum_{j=1}^{J} y_{ijk}=x_{ik}$$
$$y_{111}+y_{121}+y_{131}+y_{141}+y_{151}+y_{161}=x_{11}$$
$$y_{112}+y_{122}+y_{132}+y_{142}+y_{152}+y_{162}=x_{12}$$
$$y_{211}+y_{221}+y_{231}+y_{241}+y_{251}+y_{261}=x_{21}$$
$$y_{212}+y_{222}+y_{232}+y_{242}+y_{252}+y_{262}=x_{22}$$
$$y_{311}+y_{321}+y_{331}+y_{341}+y_{351}+y_{361}=x_{31}$$
$$y_{312}+y_{322}+y_{332}+y_{342}+y_{352}+y_{362}=x_{32}$$

Demand constraints

$$y_{111}+y_{211}+y_{311}=500$$
$$y_{112}+y_{212}+y_{312}=100$$
$$y_{121}+y_{221}+y_{321}=750$$
$$y_{122}+y_{222}+y_{322}=100$$
$$y_{131}+y_{231}+y_{331}=400$$
$$y_{132}+y_{232}+y_{332}=0$$
$$y_{141}+y_{241}+y_{341}=250$$
$$y_{142}+y_{242}+y_{342}=50$$
$$y_{151}+y_{251}+y_{351}=950$$
$$y_{152}+y_{252}+y_{352}=200$$
$$y_{161}+y_{261}+y_{361}=850$$
$$y_{162}+y_{262}+y_{362}=100$$

1.3.3.2 Solution

The truckloads of cement and reinforcing steel to be produced at each plant and delivered across each path in the network to minimize costs are shown in Table 8. This results in 810’600 mu of costs, which is comprised of 737’750 mus for production and 72’850 mus for transport. The AMPL Code is given in the appendix (section 2.2).

Table 8. Amount of each material to be produced at each plant and delivered to each construction site

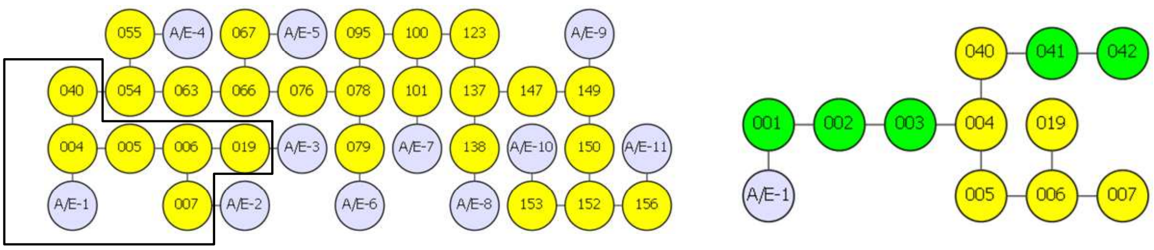
Variables				Plants (i) 1	2	3
Truckloads of material (k) to be produced		Cement	x_1	1350	1500	850
		Steel	x_2	300	250	0
Truckloads of material (k) to be transported to each site j	site 1	Cement	y_{i11}	0	500	0
		Steel	y_{i12}	0	100	0
	site 2	Cement	y_{i21}	0	750	0
		Steel	y_{i22}	0	100	0
	site 3	Cement	y_{i31}	400	0	0
		Steel	y_{i32}	0	0	0
	site 4	Cement	y_{i41}	0	250	0
		Steel	y_{i42}	0	50	0
	site 5	Cement	y_{i51}	950	0	0
		Steel	y_{i52}	200	0	0
	site 6	Cement	y_{i61}	0	0	850
		Steel	y_{i62}	100	0	0

1.4 Example 3 – Work program construction problem

1.4.1 Problem

1.4.1.1 Network

As a road manager you are responsible for the development of a work program for your road network consisting of 160 road segments, where each road segment consists of one object (Figure 5-a, Table 9-Table 10). The yellow nodes in Figure 5-a represent the road segments that are either the first or the last road segments in a group of road segments connected in series. The road segments between these are suppressed for illustration purposes. The grey nodes indicate entry and exist points to the network. A partial, but complete, model for the network within the area in Figure 5-a indicated with the black line is given in Figure 5-b, where the green nodes are the nodes that have been suppressed in Figure 5-a.



a-Condensed model of the complete b-Partial complete model

Figure 5. Physical road network

Table 9. Objects (1-40)

No.	Road type	Object type	Length (km)	DTV	CS	No.	Road type	Object type	Length (km)	DTV	CS
1	HW	tunnel	1.2	38’790	2	41	HW	road	6.6	35’350	3
2	HW	road	5.5	38’790	2	42	HW	bridge	0.3	35’350	2
3	HW	road	6.4	38’790	1	43	HW	road	6.6	37’760	4
4	HW	road	1.2	38’790	2	44	HW	road	4.5	37’760	2
5	CR	road	3.2	13’630	3	45	HW	bridge	1.8	37’760	1
6	CR	tunnel	2.2	13’630	3	46	HW	tunnel	0.7	24’230	4
7	RR	road	1.4	3’820	2	47	HW	bridge	5	24’230	4
8	RR	road	1.7	3’820	1	48	HW	road	7.2	24’230	3
9	MR	road	1.8	3’050	3	49	HW	tunnel	2.5	11’680	4
10	MR	road	1.3	3’050	4	50	HW	bridge	1.3	11’680	3
11	MR	road	2.5	3’050	4	51	CR	road	1.5	10’030	3
12	MR	road	2.6	3’050	5	52	CR	road	1.2	10’030	5
13	MR	road	1.6	3’050	3	53	CR	road	2.1	10’030	4
14	MR	road	2.2	3’050	4	54	CR	road	1.8	10’030	3
15	RR	road	2.5	3’050	4	55	RR	road	1.3	4’030	2
16	RR	road	1	3’050	2	56	RR	road	2.1	4’030	2
17	RR	tunnel	1.2	3’050	1	57	RR	road	2.3	4’030	2
18	RR	road	1.7	3’050	1	58	MR	road	1.8	3’270	3
19	RR	road	1.2	14’390	2	59	MR	road	2.3	3’270	5
20	RR	road	2.2	14’390	2	60	RR	road	1.7	3’270	1
21	RR	road	1.9	12210	2	61	RR	road	1.9	3’270	4
22	RR	bridge	2	12210	4	62	RR	road	1.6	3’270	3
23	RR	tunnel	1.6	12210	3	63	CR	road	2.8	17’010	5
24	RR	road	1.7	12210	4	64	CR	road	2.3	17’010	4
25	RR	road	1.2	6’000	2	65	CR	road	2.6	17’010	3
26	RR	tunnel	0.8	6’000	3	66	CR	road	1.3	17’010	2
27	RR	tunnel	0.6	6’000	1	67	RR	road	1	8’610	5
28	RR	road	1.8	6’000	3	68	RR	road	0.6	8’610	2
29	RR	road	1.1	3’050	4	69	MR	road	1.8	4’800	3
30	RR	road	1.8	3’050	4	70	MR	road	1.4	4’800	4
31	RR	road	1.3	3’050	1	71	RR	tunnel	1.5	4’800	4
32	RR	road	2	3’050	5	72	RR	road	1.2	4’800	4
33	RR	road	1.9	3’050	3	73	RR	tunnel	1	4’800	5
34	RR	road	1.1	3’050	2	74	RR	tunnel	1.3	4’800	5
35	RR	road	2.5	1’740	4	75	RR	road	0.8	4’800	4
36	RR	tunnel	0.6	1’740	4	76	CR	road	3.4	15’920	5
37	RR	road	2.1	1’740	2	77	CR	road	2.7	15’920	2
38	RR	tunnel	5.2	1’740	2	78	CR	road	3	17’770	3
39	RR	tunnel	4.6	1’740	4	79	RR	tunnel	3.2	3’380	1
40	HW	road	6.1	35’350	2	80	RR	bridge	1.2	9’590	4

Note: HW – highway, CR-Cantonal road, RR-regular road, MR-mountain road; CS-condition state

Table 10. Objects (81-160)

No.	Road type	Object type	Length (km)	DTV	CS	No.	Road type	Object type	Length (km)	DTV	CS
81	RR	road	1.9	9’590	5	121	RR	tunnel	3.1	2’500	3
82	RR	road	1.2	9’590	5	122	CR	tunnel	2.6	13410	2
83	RR	bridge	1.7	9’590	4	123	CR	bridge	2.2	7’960	4
84	RR	bridge	1.4	7’410	4	124	RR	road	2.1	7’740	4
85	RR	tunnel	2.3	5’230	2	125	RR	road	2.5	7’630	4
86	RR	road	2.4	5’230	4	126	RR	road	2.1	7’630	4
87	RR	road	1.9	5’230	2	127	RR	bridge	1.8	7’630	4
88	RR	road	2.5	4’360	1	128	MR	bridge	1.8	7’630	4
89	RR	road	2.1	4’360	5	129	RR	road	2.4	7’630	4
90	RR	road	2.5	4’360	2	130	RR	bridge	1.7	4’910	5
91	RR	road	2.1	4’360	3	131	MR	road	2.4	3’600	2
92	RR	bridge	1	3’930	5	132	RR	road	2.7	3’600	5
93	RR	road	2	3’930	3	133	RR	tunnel	2.7	3’600	4
94	RR	road	1.5	3’930	2	134	RR	tunnel	2.8	2’940	5
95	CR	road	1.4	11’670	2	135	RR	road	2.4	2’940	2
96	CR	road	1.9	16’680	2	136	RR	road	2.8	2’940	4
97	CR	road	2.2	21’700	2	137	RR	road	3	2’940	3
98	HW	road	1.4	14’310	1	138	RR	road	1	1’420	1
99	HW	tunnel	1	14’310	2	139	MR	road	1.7	1’420	2
100	HW	road	0.7	14’310	1	140	MR	road	1.4	1’420	5
101	CR	tunnel	1.8	2’500	1	141	MR	road	1.3	1’420	1
102	CR	tunnel	3.2	2’500	5	142	MR	road	1.2	1’420	2
103	RR	road	2.2	2’500	5	143	MR	road	2	1’420	3
104	RR	tunnel	1.3	2’500	2	144	MR	road	1.8	1’420	2
105	RR	tunnel	1.8	2’500	1	145	MR	road	1.8	1’420	2
106	RR	road	2.5	2’500	3	146	MR	road	1.8	1’420	3
107	RR	bridge	0.7	2’500	3	147	RR	road	2.2	2’510	4
108	RR	bridge	2.5	2’500	4	148	RR	tunnel	1.2	2’510	3
109	RR	bridge	2	2’500	2	149	RR	tunnel	1.6	2’510	1
110	RR	tunnel	0.7	2’500	3	150	MR	road	2.6	2’510	4
111	RR	tunnel	1.4	2’500	3	151	MR	road	1.2	2’510	4
112	RR	tunnel	1.7	2’500	4	152	MR	road	2.1	2’510	5
113	RR	road	4.6	2’500	5	153	MR	road	1.8	1’740	4
114	RR	tunnel	1.6	2’500	3	154	MR	road	2.1	1’740	4
115	RR	road	3.4	2’500	5	155	MR	road	2.1	1’740	1
116	RR	tunnel	1.3	2’500	5	156	MR	road	1.9	2’400	2
117	RR	tunnel	1.1	2’500	4	157	MR	road	2.8	2’400	3
118	RR	tunnel	1.6	2’500	4	158	MR	road	1.9	2’400	2
119	RR	tunnel	1.4	2’500	1	159	MR	road	1.9	2’400	2
120	RR	tunnel	2.2	2’500	2	160	MR	road	1.8	2’400	3

1.4.1.2 Condition states

All objects are considered to be in one of five discrete condition states (CS): CS 1 - 5 indicate increasingly poor CSs, where CS 1 is the best condition and CS 2 is the worst condition.

1.4.1.3 Interventions and traffic configurations

The interventions to be executed on each type of object for each type of road segment when it is in each CS are given in Table 11-Table 14, along with their costs (intervention costs, i.e. the costs of the manual labour, materials and equipment required to execute the interventions; travel time costs, i.e. the costs due to users not being able to travel as normal; accident costs, i.e. the costs due to increases in the accident rate. No interventions are to be executed on objects that are in CS 1 or 2. Traffic configurations considered possible are given in Table 15 and Table 16. Intervention costs are approximated based on historical data and norms. They are broken down into fixed costs and variable costs, where fixed costs are those that are independent of the size of object and variable costs are those dependent on the size of the object.

Table 11. Highway interventions

Object type	CS	IT	TC	Intervention		Loss of travel time	Accidents
				Fixed	Variable		
				CHF	CHF/m^2	CHF/km/day/	CHF/km/day/car
Road section	1, 2,3,4,5	0	0	0	0		
			1	0	0	1.03	0.33
			2	0	0	0.69	0.34
	3	1	1	3’500	8	1.03	0.33
			2	4’200	10	0.69	0.34
	4	2	1	4’100	52	1.03	0.33
			2	4’900	62	0.69	0.34
	5	3	1	9’600	108	1.03	0.33
			2	11’500	130	0.69	0.34
Bridge	1, 2	0	0	0	0	0	0
			1	0	0	0.82	0.34
			2	0	0	2.06	0.36
	3	1	1	20’000	2’100	0.82	0.34
			2	24’000	25’200	2.06	0.36
	4	2	1	30’000	2’800	0.82	0.34
			2	36’000	4’400	2.06	0.36
	5	3	1	40’000	3’500	0.82	0.34
			2	48’000	4’200	2.06	0.36
Tunnel	1, 2	0	0	0	0	0	
			1	0	0	0.82	0.34
			2	0	0	4.11	0.29
	3	1	1	100’000	20’000	0.82	0.34
			2	120’000	24’000	4.11	0.29
	4	2	1	150’000	35’000	0.82	0.34
			2	180’000	42’000	4.11	0.29
	5	3	1	200’000	50’000	0.82	0.34
			2	240’000	60’000	4.11	0.29

Note: CS- condition state; IT-intervention type; TC-traffic configuration

Table 12. Cantonal road interventions

Object type	CS	IT	TC	Intervention		Loss of travel time	Accidents
				Fixed	Variable		
				CHF	CHF/m^2	CHF/km/day/	CHF/km/day/car
Road section	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	1.03	0.33
			2	0	0	0.44	0.34
	3	1	1	3’500	8	1.03	0.33
			2	4’200	10	0.44	0.34
	4	2	1	4’100	52	1.03	0.33
			2	4’900	62	0.44	0.34
	5	3	1	9’600	108	1.03	0.33
			2	11’500	130	0.44	0.34
Bridge	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	0.82	0.34
			2	0	0	4.11	1.76
	3	1	1	20’000	2’100	0.82	0.34
			2	24’000	25’200	4.11	1.76
	4	2	1	30’000	2’800	0.82	0.34
			2	36’000	4’400	4.11	1.76
	5	3	1	40’000	3’500	0.82	0.34
			2	48’000	4’200	4.11	1.76
Tunnel	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	0.82	0.34
			2	0	0	4.11	4.05
	3	1	1	100’000	20’000	0.82	0.34
			2	120’000	24’000	4.11	4.05
	4	2	1	150’000	35’000	0.82	0.34
			2	180’000	42’000	4.11	4.05
	5	3	1	200’000	50’000	0.82	0.34
			2	240’000	60’000	4.11	4.05

Table 13. Rural road interventions

Object type	CS	IT	TC	Intervention		Loss of travel time	Accidents
				Fixed	Variable		
				CHF	CHF/m^2	CHF/km/day/	CHF/km/day/car
Road section	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	3.29	0.31
			2	0	0	4.94	2.77
	3	1	1	3’500	8	3.29	0.31
			2	4’200	10	4.94	2.77
	4	2	1	4’100	52	3.29	0.31
			2	4’900	62	4.94	2.77
	5	3	1	9’600	108	3.29	0.31
			2	11’500	130	4.94	2.77
Bridge	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	1.23	0.33
			2	0	0	4.94	1.76
	3	1	1	20’000	2’100	1.23	0.33
			2	24’000	25’200	4.94	1.76
	4	2	1	30’000	2’800	1.23	0.33
			2	36’000	4’400	4.94	1.76
	5	3	1	40’000	3’500	1.23	0.33
			2	48’000	4’200	4.94	1.76
Tunnel	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	1.23	0.33
			2	0	0	4.94	1.76
	3	1	1	100’000	20’000	1.23	0.33
			2	120’000	24’000	4.94	1.76
	4	2	1	150’000	35’000	1.23	0.33
			2	180’000	42’000	4.94	1.76
	5	3	1	200’000	50’000	1.23	0.33
			2	240’000	60’000	4.94	1.76

Table 14. Mountain road interventions

Object type	CS	IT	TC	Intervention		Loss of travel time	Accidents
				Fixed	Variable		
				CHF	CHF/m^2	CHF/km/day/	CHF/km/day/car
Road section	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	1.23	0.33
			2	0	0	4.94	4.05
	3	1	1	3’500	8	1.23	0.33
			2	4’200	10	4.94	4.05
	4	2	1	4’100	52	1.23	0.33
			2	4’900	62	4.94	4.05
	5	3	1	9’600	108	1.23	0.33
			2	11’500	130	4.94	4.05
Bridge	1, 2,3,4,5	0	0	0	0	0	0
			1	0	0	1.23	0.33
			2	0	0	4.94	2.26
	3	1	1	20’000	2’100	1.23	0.33
			2	24’000	25’200	4.94	2.26
	4	2	1	30’000	2’800	1.23	0.33
			2	36’000	4’400	4.94	2.26
	5	3	1	40’000	3’500	1.23	0.33
			2	48’000	4’200	4.94	2.26

Table 15. Differences of traffic configuration

Road type			Object	Traffic configuration	
Description	Number of normal lanes	Number of emergency lanes		1	2
Highway	4	2	Road section	4-0	H2-N2
			Bridge	4-0	H2-N2
			Tunnel	4-0	H2-N2
		0	Road section	2-0	H2-N1
			Bridge	NA	NA
			Tunnel	2-0	H2-N1
Cantonal road	4	0	Road section	2-0	H2-N1
			Bridge	NA	NA
			Tunnel	NA	NA
	3	0	Road section	1-1	N1-N1
			Bridge	1-1	N1-N1
			Tunnel	1-1	N1-N1
	2	1	Road section	1-1	N1-N1
			Bridge	NA	NA
			Tunnel	NA	NA
	2	2	Road section	NA	NA
			Bridge	NA	NA
			Tunnel	NA	NA
Rural road	2	0	Road section	N2	1-0
	2	0	Bridge	NW	1-0
	2	0	Tunnel	NW	1-0
Mountain road	2	0	Road section	NW	1-0
	2	0	Bridge	NW	1-0
	2	0	Tunnel	NA	NA

Table 16. Types of traffic configurations

TC	Description
4-0	All three lanes of one direction are completely closed. One side is free of traffic which allows performing all interventions nearly undisturbed. The open side with its originally one emergency and two regular lanes is regrouped to four narrower lanes, two in each direction. Due to the missing emergency lanes and the smaller lane width the speed limit is reduced from 120 km/h to 80 km/h.
H2-N2	One lane at the time is closed on one side of the highway while the other side remains completely open. Within three steps the intervention for one travel direction can be completed. One third of the road is closed for work zones, two narrower lanes with lower speed limits (80 km/h) are used for traffic. Only one travel direction of the highway is affected but six instead of two steps are necessary to complete an intervention on the whole object.
2-0	One side of the road is closed for intervention while on the other side two lanes – one in each travel direction – are being operated.
H2-N1	This TC requires four steps to complete an intervention on the whole road. Two lanes in one travel direction remain untouched, in the other travel direction one lane is closed and the other remains open, but is slightly narrower than in regular operation.
1-1	Two lanes – one in both travel directions – remain open; the third lane provides the required space for the intervention.
N1-N1	Half of the road is closed. On one half the intervention takes place, on the other half two narrow lanes are installed. The speed limit is significantly lower than in the "1-1" setup due to the narrower lanes.
1-0	One lane remains open for the traffic in both travel directions; a traffic light system regulates the traffic flow.
N2	A third of the road is closed for intervention, two narrow lanes are installed on the remaining two third of the road.
NW	The whole intervention only takes place during the nighttime, during the day the road is completely open to the traffic. The works have to be divided in a way, that during daytime cars and trucks are still able to cross the intervention zone.

1.4.2 Question

What are the interventions that should be included in the work program to minimize total costs when 1) there is no budget constraint but the maximum length of work site is 15 km, 2) there is no budget constraint but the maximum length of the work site is 10 km, 3) there is a budget constraint of 55.9 x 10^6 mus and a maximum length of a work site of 15 km (Table 17).

Table 17: Descriptions of scenarios

Scenarios	Budget [x10^6 CHF]	L^{max} (km)
1	unlimited	15
2	unlimited	10
3	55.9	15

1.4.3 Answer

1.4.3.1 Conceptual model

In order to answer this question it is necessary to conceptually build the model. In this model, a road network is considered to consist of road segments. Each segment consists of one infrastructure object (e.g. a road section, a culvert, a bridge, a tunnel). Due to deterioration, the physical condition of objects deteriorate over time and interventions must be executed in order to ensure that the network is able to provide an adequate level of service (LOS). When an intervention is executed on a road segment, it is considered that interventions are executed on all objects within the road segment. There are different types of interventions that can be executed on each object in the network. For example, a road section can be renewed or can be upgraded with partial depth repair. For each type of intervention different types of traffic configurations (TCs) are possible. For example, if crack filling of pavement on a two lane road is to be done there may be either the reduction of the width of both lanes but two lanes remain open, or the closure of one lane and no reduciton in the width of the remaining lane. This concept is summarized in Table 18.

Table 18. Hierarchy of road segments, objects, interventions and traffic configurations

Road segment	Objects	Interventions	Traffic configurations
$l = 1,\ldots,L$	$n_l = 1,\ldots,N_l$	$k_{l,n} = 1,\ldots,K_{l,n}$	$t_{l,n} = 1,\ldots,T_{l,n}$
L: Total number of segments in the network	Nl: Total number of infrastructure objects n segment l	Kl,n: Total number of interventions on object n in segment l	Tl,n: Total number of TCs possible for object n in segment l
		$k_{l,n} = 1$ refers to "no intervention"	$t_{l,n} = 1$ refers to the default TC

In the model, each node represents a specific road segment composed of multiple objects. The node also represents an intevention to be executed and a TC to be put in place during the execution of the intervention. The link represents the change in TC that occurs between the two objects. A further illustration of the hierarchy is given in Figure 6. This hierarchy makes it easy to use to relate the attributes of one model element to another. Example attributes are given in in Table 19.

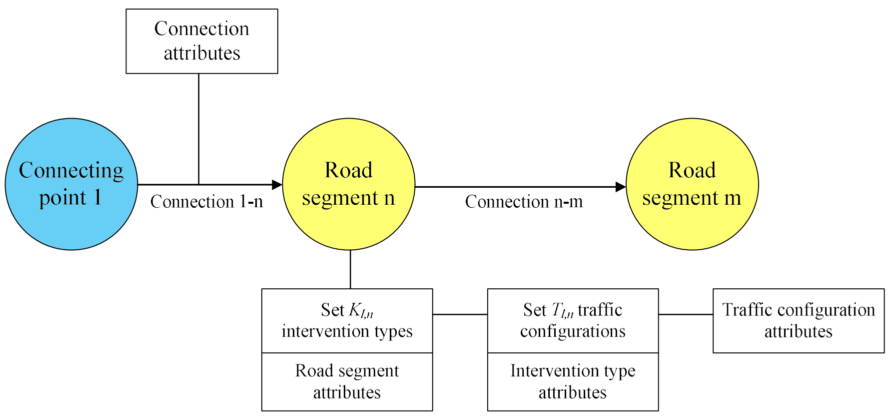


Figure 6: Illustration of the hierarchy

Table 19. Attribute types

	Type	Examples
nodes	Road segment	Length, number of lanes, DTV, type of infrastructure (e.g. tunnel, bridge, etc.), CS
	Intervention type	Costs (e.g. CHF/m^2), improvement in CS (e.g. from CS 4 to CS 1), production capacity (e.g. m^2/day)
	Traffic configuration	Travel speed, change in accident rate, work space per meter of road (as a result of closing one or more lanes)
links	Connection	Specific attributes that result from a change in TC, e.g. costs for traffic jam at the beginning of a work zone, higher accident rates resulting from the change of the TC

An example of the network model is shown in Figure 7. It is built for a network with two road segments *n* and *m* and each segment consists of exactly one object. Two types of interventions can be executed on each object, i.e. do nothing and execute an intervention. There is one possible traffic configuration TC1 when no intervention is executed, and there are two possible traffic configurations (TC2 and TC3) when an intervention is executed, i.e. 2 lane restricted traffic flow and 1 lane unrestriced traffic flow. For these two road segments, there are 9 possible work zones that can be formed. A work zone encompasses the entire area from between the normal traffic configurationand the normal trafic configuration.

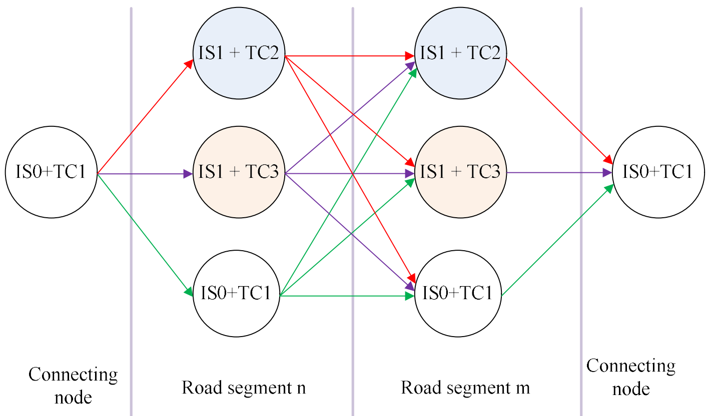


Figure 7. Example network model

The model of the intervention and traffic configurations for the partial network, where a node represents a possible intervention - traffic configuration combination for a road segment, is shown in Figure 8. The node number are comprised of three parts (XXX.Y.Z). XXX indicates the physical road segment, Y indicates the intervention type, and ZZ indicates the traffic configuration.

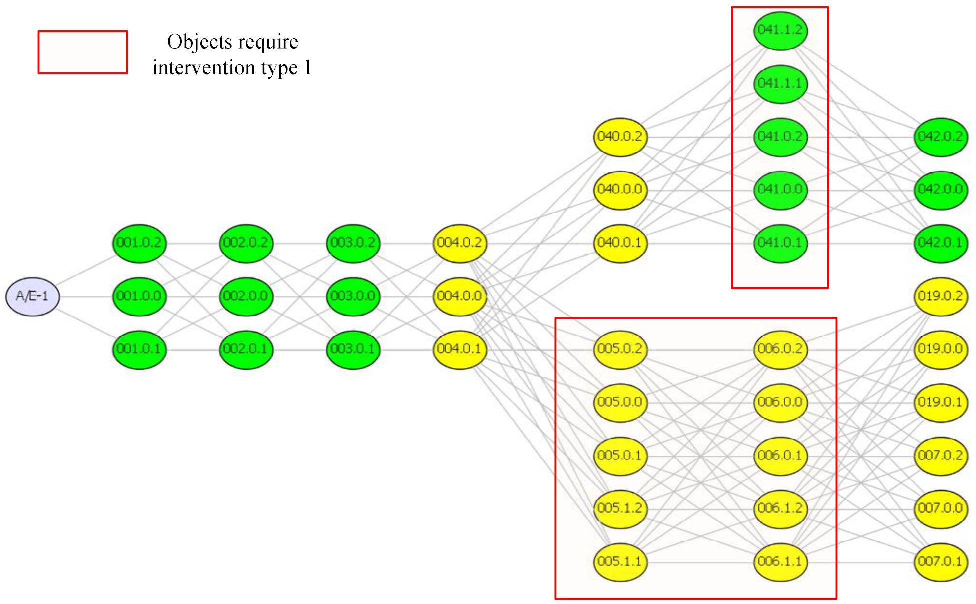


Figure 8: Nodes and links representing the possible interventions and traffic configurations

For example, the Y = 0 in the node reference indicates an intervention of type 0 (which is equivalent to "do-nothing"), whilst, Y = 1 in the node reference (e.g. nodes with XXX = 005, 006 and 041) indicates an intervention of type 1 (refer to Table 11-Table 14).

1.4.3.2 General mathematical model

The objective function is:

$$Maximize\ \ Z = \sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T}\delta_{l,n,k,t}\cdot M_{l,n,k,t} - \sum_{e=1}^{E}\gamma_{l,n,k,g-e}\cdot \overrightarrow{N}_{l,n,k,g-e} \tag{10}$$

where

$\delta_{l,n,k,t}$ is a binary variable, which has a value of 1 if an intervention of type k is executed on road segment n in link l, and traffic configuration t is used, and 0 otherwise.

$\gamma_{l,n,k,g-e}$ is a binary variable, which has a value of 1 if edge e leaving node $[l,n,k,t]$ is selected, and 0 otherwise. E equals the total number of links leaving node $[l,n,k,t]$

$M_{l,n,k,t}$ is the long term benefit of executing an intervention of type k with traffic configuration t on object n in section l.

$\overrightarrow{N}_{l,n,k,g-e}$ is the cost, associated with changes between, nodes in the network, for example the cost related to the change in traffic configuration between a road segment with no intervention and a road segment with an intervention

Constraint (11) ensures that only one intervention of type k and one traffic configuration of type t is selected for road segment n of link l at one time.

$$\sum_{k=1}^{K} \delta_{l,n,k,t} = 1 \quad \forall\ l,t,n \tag{11}$$

Constraint (12) ensures that the sets of road segments upon which interventions are executed are within a WZ, i.e. that if a node is selected than so is an edge leaving the node.

$$\gamma_{l,n,g-e} = \delta_{l,n,k,t} \cdot \delta_e \tag{12}$$

Constraint (13) ensured that the costs of each type, e.g. owners costs, stay within given limits. The number of cost constraints in a model depend on the number of stakeholders and cost types that are to be modelled.

$$\sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T}\left\{\delta_{l,n,k,t}\cdot\left|P^{s}_{l,n,k,t}\right| + \sum_{e=1}^{E}\varphi_{l,n,k,t-e}\cdot\left|\vec{P}^{s}_{l,n,k,t-e}\right|\right\} \le B^{s} \tag{13}$$

where

$P^{s}_{l,n,k,t}$	is the total cost incurred by stakeholder *s* attributed to the execution of an intervention of type *k* with a traffic configuration of type *t* on object n in link *l*.
$\vec{P}^{s}_{l,n,k,t}$	is the total cost incurred by stakeholder *s* attributed to changes between, nodes in the network.
B^{s}	is the maximum allowable costs incurred by stakeholder *s*.

The values of $P^{s}_{l,n,k,t}$ and $\vec{P}^{s}_{l,n,k,t}$ are in fact a part of $M_{l,n,k,t}$ and $\overrightarrow{N}_{l,n,k,t-e}$ in Eq. (10), respectively.

Constraint (14) ensures that the WZs do not exceed a specified maximum length. WZs may be limited in maximum length, for example, due to regulations imposed by governing organisations, or due to the capacity of construction companies to run such long construction projects.

$$\sum_{l=a_l^w}^{e_l^w}\sum_{n=a_n^w}^{e_n^w} \lambda_{l,n} \le \Lambda^{MAX} \quad \forall w \tag{14}$$

where

$\lambda_{l,n}$	is the length of the road segment [*l,n*];
$a^{w}\ \left(l = a_l^w, n = a_n^w\right)$	is the first road segment of the WZ $w = (1,...,W)$, and road segment
$e^{w}\ \left(l = e_l^w, n = e_n^w\right)$	is the last road segment in the WZ w.
Λ^{MAX}	is maximum allowable length of the WZ.

1.4.3.3 Solution

The optimal work program for each scenario are shown in Figure 9. This figure simplifies the graph in Figure 8 by representing only the possible traffic configurations for each object, i.e. it is possible to tell that an intervention is to be executed but not which type of intervention. If the nodes belong to one object go vertically; the middle node represents “IS0- TC0”, the upper and lower node infer YY-TC2 and IS.YY.TC1, respectively. If the nodes belong to one object go horizontally; the middle node represent “IS0- TC0”, the right and left node infer YY-TC2 and YY-TC1, respectively. YY represent intervention type on actual condition state of the object.

In the figure, the red line indicates the traffic configuration over the length of the entire network. The differences between the graphs are emphasized by coloring the nodes in different colors (pink for scenario 2 and purple in scenario 3) that are different in comparison with those for scenario 1. A detail of how a work zone is different for each scenario is further illustrated in Figure 10, Figure 11, and Figure 12. No R Code is given in the appendix as the problem was solved used Excel and the Add-in “What’s Best”[1]. The network is constructed using Python [2].

Under scenario 1, where the budget is equal or greater than 74.4 million mus and the maximum length of work zone is 15 km, the expected total benefit is 370.46 million mus. When the maximum length constraint is reduced from 15 km to 10 km (scenario 2), while having the same budget constraint as in scenario 1, the optimal work program is no longer the same as those in scenario 1. For example, object 041 is no longer included (Figure 11). It can also be seen that the reduction in the length constraint results in a reduction in expected total benefit, in this case, 6%.

The reduction of budget from 25% of that in scenario 1 (scenario 3) results an expected total benefit of 340.41 million CHF, which is 8% lower than that of scenario 1. By comparing the optimal work programs for both scenarios (Figure 12), the changes in the objects to have interventions can be seen. For example, object 006 is no longer included in a work zone due to the tighter budget restrictions.

[1] http://www.lindo.com/
[2] https://www.python.org/

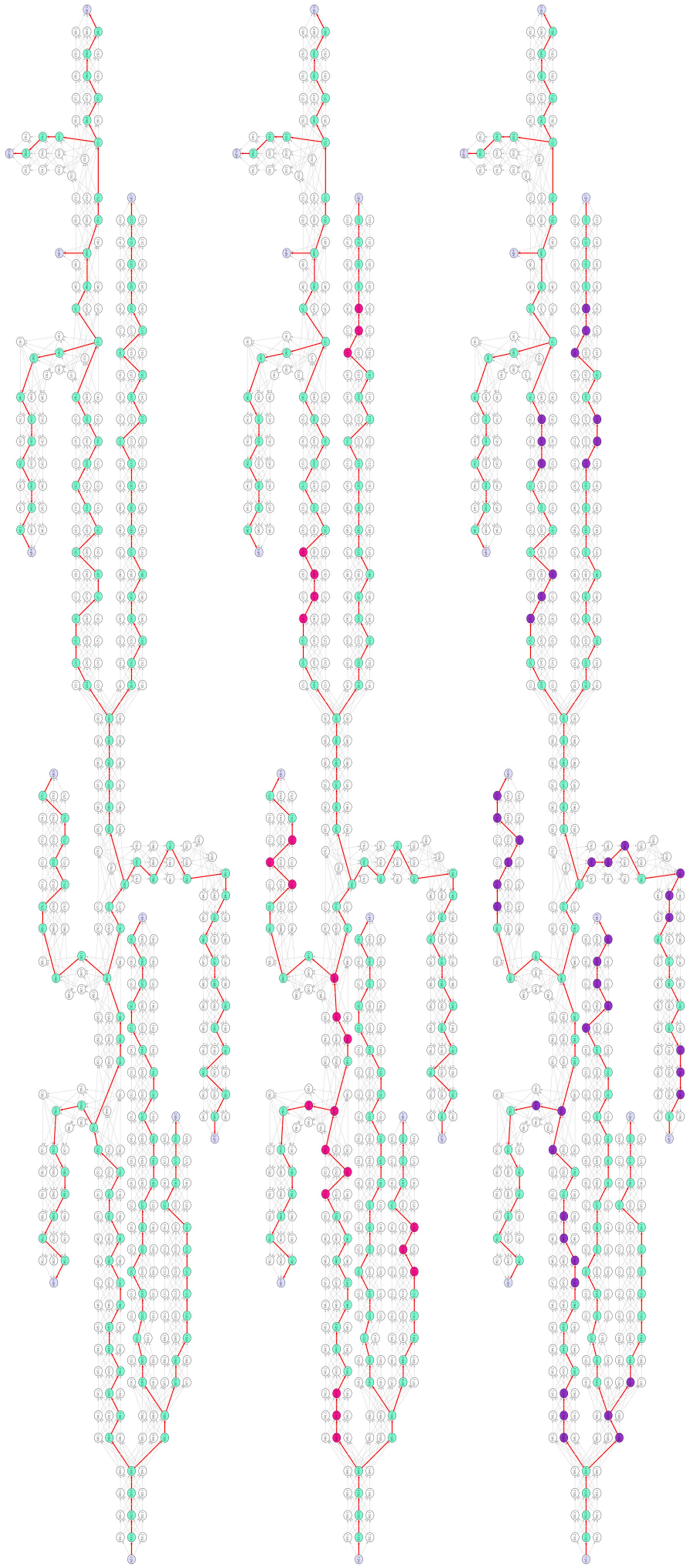


Figure 9: Graphical representation of optimal work programs under SC1, SC2, and SC3

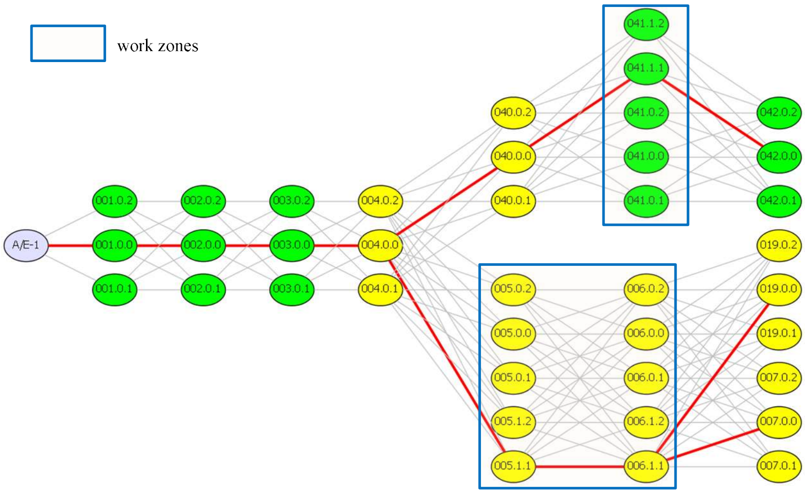


Figure 10. Optimal work zone - scenario 1

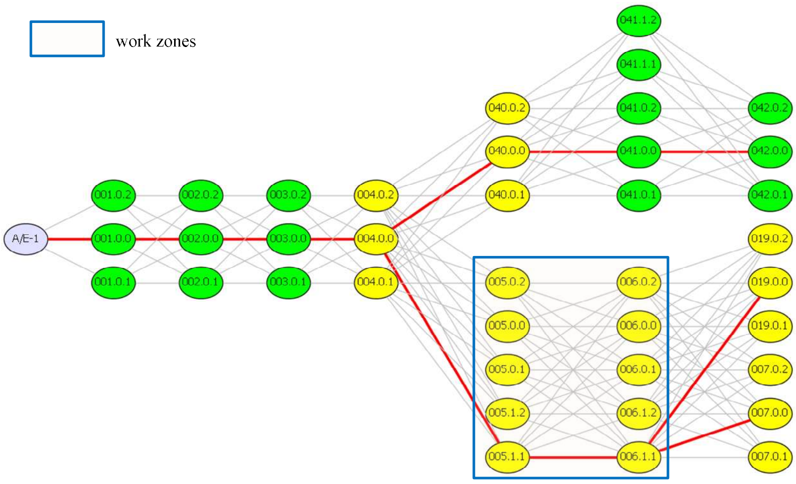


Figure 11. Optimal work zone - scenario 2

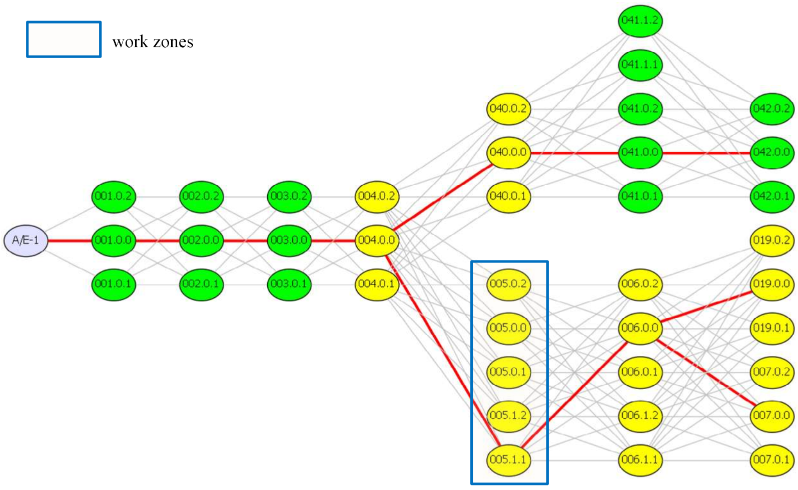


Figure 12. Optimal work zone - scenario 3

2 Appendix

2.1 Cement production and shipment problem

2.1.1 Question A

2.1.1.1 Model

```
set nodes;
set links within (nodes cross nodes);
param s {nodes} >= 0;   # amounts available at nodes (supply)
param d {nodes} >= 0;   # amounts required at nodes (demand)
check: sum {i in nodes} s[i] = sum {j in nodes} d[j];
param c {links} >= 0;      #
param Q {links} >= 0;  # capacity of the link
var x {(i,j) in links} >= 0, <= Q[i,j];
minimize totalcost:    sum {(i,j) in links} c[i,j] * x[i,j];
subject to balance {k in nodes}:    s[k] + sum {(i,k) in links} x[i,k]     = d[k] + sum {(k,j) in links} x[k,j];
```

2.1.1.2 Data

```
data;
set nodes := N1  N2 N3  N4 N5 N6 N7 N8 N9 N10 ;
set links := (N1,N2) (N1,N3) (N1,N4) (N1,N5)  (N2,N6) (N2,N7)   (N3,N2) (N3,N7)   (N4,N3) (N4,N5)
(N4,N7)    (N5,N9)  (N6,N8)  (N7,N8)  (N7,N9) (N7,N10)  (N8,N10)(N9,N10);
param s  default 0 := N1 700 ; # ton
param d  default 0 := N10 700;
param:                  c          Q  :=
N1          N2          5          250
N1          N3          2          300
N1          N4          1          80
N1          N5          4          150
N2          N6          2          250
N2          N7          1          320
N3          N2          2          150
N3          N7          3          130
N4          N3          2          90
N4          N7          6          180
N4          N5          2          200
N5          N9          12         200
N6          N8          1          255
N7          N8          4          350
N7          N9          8          240
N7          N10         7          250
N8          N10         6          300
N9          N10         4          250;
```

2.1.1.3 Results

```
MINOS 5.5: optimal solution found.
10 iterations, objective 10230
ampl: display x;
x :=
N1 N2    250
N1 N3    280
N1 N4     80
N1 N5     90
N2 N6    250
```

```
N2 N7    150
N3 N2    150
N3 N7    130
N4 N3      0
N4 N5     60
N4 N7     20
N5 N9    150
N6 N8    250
N7 N10   250
N7 N8     50
N7 N9      0
N8 N10   300
N9 N10   150
;
```

2.1.2 Question B

2.1.2.1 Model

```
set nodes;  # intersections
param entr symbolic in nodes;          # entrance to road network
param exit symbolic in nodes, <> entr;  # exit from road network
set links within (nodes diff {exit}) cross (nodes diff {entr});
param l {links} >= 0;        # length
var Use {(i,j) in links} >= 0;   # 1 iff (i,j) in shortest path
minimize Total_l: sum {(i,j) in links} l[i,j] * Use[i,j];
subject to Start:  sum {(entr,j) in links} Use[entr,j] = 1;
subject to Balance {k in nodes diff {entr,exit}}:
sum {(i,k) in links} Use[i,k] = sum {(k,j) in links} Use[k,j];
```

2.1.2.2 Data

```
data;
set nodes := N1  N2 N3  N4 N5 N6 N7 N9 N8 N10 ;
param entr := N1 ;
param exit := N10 ;
param:    links:     l   :=
  N1          N2          5
  N1          N3          2
  N1          N4          1
  N1          N5          4
  N2          N6          2
  N2          N7          1
  N3          N2          2
  N3          N7          4
  N4          N3          2
  N4          N7          6
  N4          N5          2
  N5          N9          12
  N6          N8          1
  N7          N8          4
  N7          N9          8
  N7          N10         7
  N9          N10         4
  N8          N10         6;
```

2.1.2.3 Results

```
Load Avg: ( 0.0 , 0.16 , 0.36 )
File exists
You are using the solver gurobi_ampl.
Executing AMPL.
processing data.
processing commands.
```

```
18 variables, all linear
9 constraints, all linear; 33 nonzeros
  9 equality constraints
1 linear objective; 18 nonzeros.

Gurobi 5.5.0: outlev=1
threads=4
Optimize a model with 9 rows, 18 columns and 33 nonzeros
Presolve removed 9 rows and 18 columns
Presolve time: 0.00s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    1.2000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.00 seconds
Optimal objective  1.200000000e+01
Gurobi 5.5.0: optimal solution; objective 12
Use :=
N1 N2    0
N1 N3    1
N1 N4    0
N1 N5    0
N2 N6    0
N2 N7    1
N3 N2    1
N3 N7    0
N4 N3    0
N4 N5    0
N4 N7    0
N5 N9    0
N6 N8    0
N7 N10   1
N7 N8    0
N7 N9    0
N8 N10   0
N9 N10   0
;
```

2.1.3 Question C

2.1.3.1 Model

```
set INTER; # intersections
param entr symbolic in INTER; # entrance to road network
param exit symbolic in INTER, <> entr; # exit from road network
set ROADS within (INTER diff {exit}) cross (INTER diff {entr});
param cap {ROADS} >= 0; # capacities
var Traff {(i,j) in ROADS} >= 0, <= cap[i,j]; # traffic loads
maximize Entering_Traff: sum {(entr,j) in ROADS} Traff[entr,j];
subject to Balance {k in INTER diff {entr,exit}}:
sum {(i,k) in ROADS} Traff[i,k] = sum {(k,j) in ROADS} Traff[k,j];
```

2.1.3.2 Data

```
data;
set INTER := N1  N2 N3  N4 N5 N6 N7 N8 N9 N10 ;
param entr := N1 ;
param exit := N10 ;
param: ROADS: cap :=
N1 N2 250, N1 N3 300, N1 N4 80, N1 N5 150
N2 N6 250, N2 N7 320
N3 N2 150, N3 N7 130
```

```
N4 N3 90, N4 N5 200, N4 N7 180
N5 N9 200
N6 N8 255
N7 N8 350, N7 N9 240, N7 N10 250
N8 N10 300
N9 N10 250;
```

2.1.3.3 Results

```
MINOS 5.51: optimal solution found.
9 iterations, objective 760
ampl: display Traff
ampl? ;
Traff :=
N1 N2    250
N1 N3    280
N1 N4     80
N1 N5    150
N2 N6    250
N2 N7    150
N3 N2    150
N3 N7    130
N4 N3      0
N4 N5     50
N4 N7     30
N5 N9    200
N6 N8    250
N7 N10   250
N7 N8     50
N7 N9     10
N8 N10   300
N9 N10   210;
```

2.2 Cement and steel production and delivery problem

2.2.1 Model

```
set plant;   #
set customer;   #
set material;   #
param r {plant,material} > 0;     # tons per unit time (hour, day)
param Q {plant} >= 0;        # available time of plant i
param D {customer,material} >= 0;  # tons required for customer
param c {plant,material} >= 0;        # manufacturing cost/ton
param v {plant,customer,material} >= 0;  # transportation cost/ton
var x {plant,material} >= 0;       # tons material at plant i
var y {plant,customer,material} >= 0; # tons transported
minimize Total_Cost:   sum {i in plant, p in material} c[i,p] * x[i,p] +
                       sum {i in plant, j in customer, p in material}v[i,j,p] * y[i,j,p];
subject to Time {i in plant}:   sum {p in material} (1/r[i,p]) * x[i,p] <= Q[i];
subject to Supply {i in plant, p in material}:   sum {j in customer} y[i,j,p] = x[i,p];
subject to Demand {j in customer, p in material}:   sum {i in plant} y[i,j,p] = D[j,p];
```

2.2.2 Data

```
set plant := A B C ;
set customer := c1 c2 c3 c4 c5 c6 ;
set material := cement steel ;
param Q :=  A 20  B 15  C 20 ; #hours
param D (tr):
                 c1     c2     c3     c4     c5     c6 :=
  cement         500    750    400    250    950    850
  steel          100    100    0      50     200    100;
```

```
param r (tr):           A       B       C :=
        cement          140     130     160
        steel           160     160     170;

param c (tr):
                        A       B       C :=
        cement          170     170     180
        steel           180     185     185;

param v :=

[*,*,cement]:   c1  c2  c3  c4  c5  c6 :=
    A           35  12  9   30  13  42
    B           27  9   12  9   26  95
    C           26  7   9   12  11  23

[*,*,steel]:    c1  c2  c3  c4  c5  c6 :=
    A           45  22  20  37  26  54
    B           29  9   13  9   28  99
    C           41  14  17  13  31  104;
```

2.2.3 Results

```
MINOS 5.51: optimal solution found.
11 iterations, objective 810600
ampl: display x;
x :=
A concrete      1350
A steel         300
B concrete      1500
B steel         250
C concrete      850
C steel         0
;
ampl: display y;
y [*,*,concrete] (tr)
:   A     B     C    :=
c1  0     500   0
c2  0     750   0
c3  400   0     0
c4  0     250   0
c5  950   0     0
c6  0     0     850

 [*,*,steel] (tr)
:   A     B     C    :=
c1  0     100   0
c2  0     100   0
c3  0     0     0
c4  0     50    0
c5  200   0     0
c6  100   0     0
;
```

2.3 Conference facility construction problem

2.3.1 Model

```
set ROOM; #type of rooms
set OUTCOME; #type of outcome
set DEVIATION; #numbers of deviations

#parameters
param w1 {ROOM, DEVIATION}; # weighting factors assigned to room
param w2 {OUTCOME, DEVIATION}; # weighting factors assigned to outcome
param N {ROOM}; #numbers of room
param M {OUTCOME}; #values of outcome
param c {ROOM,OUTCOME}; #values of outcome

# Decision variables
var d1{ROOM, DEVIATION} integer >=0;  #number of conference
var d2{OUTCOME, DEVIATION} >=0;  #number of outcome
var x{ROOM} integer >=0;  #integer value

# Objective function
minimize obj: sum {i in ROOM} (1/N[i]*(sum{k in DEVIATION} (w1[i,k]*d1[i,k]))) + sum {j in
OUTCOME}(1/M[j]*(sum{k in DEVIATION} (w2[j,k]*d2[j,k])));

subject to room {i in ROOM}:        x[i] +sum {k in DEVIATION} d1[i,k] = N[i];
subject to outcome {j in OUTCOME}:        (sum {i in ROOM} (x[i]*c[i,j])) + sum {k in DEVIATION}
d2[j,k] = M[j];
```

2.3.2 Data

```
data;
set ROOM := small medium large;
set OUTCOME := area expenditure;
set DEVIATION := over under;
param N :=  small 5          medium 10        large 15;
param M :=  area 9000        expenditure 3000000;
param w1 :=
  [*,*]:                    over                 under:=
              small         0                    1
              medium        0                    1
              large         0                    1;

param w2 :=
  [*,*]:                         over                 under    :=
              area               1                    1
              expenditure        100                  1;

param c :=
  [*,*]:                    area                 expenditure:=
              small         125                  60000
              medium        250                  100000
              large         350                  152000;
```

2.3.3 Results

```
MINOS 5.51: optimal solution found.
5 iterations, objective 0.490000
ampl: display x;
x :=
small      5
medium    10
large     11
;
```

```
ampl: display d1, d2;

d1 :=     under   over
small     0       0
medium    0       0
large     4       0
;

d2 :=           under   over
area            2025    0
expenditure     29100   0
;
```

2.4 Communication tower problem

2.4.1 Question A

2.4.1.1 Model

<u>+ When all areas are not necessary to be covered</u>

```
param T;
#parameters
param c; # unit construction cost for one tower
param R {1..T,1..T};

# Decision variables
var x{1..T,1..T} binary;  #area to be covered
var y{1..T,1..T} binary;  #location of tower in which area
var z{1..T,1..T} integer >=0;  #times cover by areas

# Objective function
maximize profit: sum {i in 1..T,j in 1..T} (x[i,j]*R[i,j]-y[i,j]*c);

subject to revenue {i in 1..T,j in 1..T}:           x[i,j]*R[i,j] <= R[i,j];
subject to equality {i in 1..T,j in 1..T}: z[i,j] >= x[i,j];

subject to times1 {i in 1..1,j in 1..1}: z[i,j] = y[i,j]+y[i,j+1]+y[i+1,j];
subject to times2 {i in 1..1,j in T..T}: z[i,j] = y[i,j-1]+y[i,j]+y[i+1,j];
subject to times3 {i in T..T,j in 1..1}: z[i,j] = y[i-1,j]+y[i,j]+y[i,j+1];
subject to times4 {i in T..T,j in T..T}: z[i,j] = y[i-1,j]+y[i,j-1]+y[i,j];

subject to times5 {i in 1..1,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i+1,j];
subject to times6 {i in T..T,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i-1,j];
subject to times7 {i in 2..(T-1),j in 1..1}: z[i,j] = (y[i-1,j])+(y[i,j]+y[i,j+1])+(y[i+1,j]);
subject to times8 {i in 2..(T-1),j in T..T}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j])+(y[i+1,j]);

subject to times9 {i in 2..(T-1),j in 2..(T-1)}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j]+y[i,j+1])+(y[i+1,j]);
```

<u>+ When all areas have to be covered</u>

```
param T;
#parameters
param c; # unit construction cost for one tower
param R {1..T,1..T};

# Decision variables
var x{1..T,1..T} binary;  #area to be covered
var y{1..T,1..T} binary;  #location of tower in which area
var z{1..T,1..T} integer >=0;  #times cover by areas
```

```
# Objective function
maximize profit: sum {i in 1..T,j in 1..T} (x[i,j]*R[i,j]-y[i,j]*c);

subject to coverage {i in 1..T,j in 1..T}:        x[i,j= 1;
subject to revenue {i in 1..T,j in 1..T}:         x[i,j]*R[i,j] <= R[i,j];
subject to equality {i in 1..T,j in 1..T}: z[i,j] >= x[i,j];

subject to times1 {i in 1..1,j in 1..1}: z[i,j] = y[i,j]+y[i,j+1]+y[i+1,j];
subject to times2 {i in 1..1,j in T..T}: z[i,j] = y[i,j-1]+y[i,j]+y[i+1,j];
subject to times3 {i in T..T,j in 1..1}: z[i,j] = y[i-1,j]+y[i,j]+y[i,j+1];
subject to times4 {i in T..T,j in T..T}: z[i,j] = y[i-1,j]+y[i,j-1]+y[i,j];

subject to times5 {i in 1..1,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i+1,j];
subject to times6 {i in T..T,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i-1,j];
subject to times7 {i in 2..(T-1),j in 1..1}: z[i,j] = (y[i-1,j])+(y[i,j]+y[i,j+1])+(y[i+1,j]);
subject to times8 {i in 2..(T-1),j in T..T}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j])+(y[i+1,j]);

subject to times9 {i in 2..(T-1),j in 2..(T-1)}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j]+y[i,j+1])+(y[i+1,j]);
```

2.4.1.2 Data

```
data;
param T:= 5;
param c:= 150;
param R :=
[*,*]:   1     2     3     4     5:=
         1     34    43    62    42    34
         2     64    43    71    48    65
         3     57    57    51    61    30
         4     32    38    70    56    40
         5     68    73    30    56    44;
```

2.4.1.3 Results

<u>+ When all areas are not necessary to be covered</u>

```
Optimal solution found (tolerance 1.00e-04)
Best objective 3.770000000000e+02, best bound 3.770000000000e+02, gap 0.0%
Optimize a model with 50 rows, 75 columns and 180 nonzeros
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    3.7700000e+02   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.00 seconds
Optimal objective  3.770000000e+02
Gurobi 5.5.0: optimal solution; objective 377
31 simplex iterations
ampl: display x;
x :=     1     2     3     4     5

1        0     0     1     0     0
2        1     1     1     1     0
3        1     1     1     1     0
4        1     1     1     1     1
5        1     1     1     1     0 ;

ampl: display y;
y :=     1     2     3     4     5
```

1	0	0	0	0	0
2	0	0	1	0	0
3	1	0	0	0	0
4	0	0	0	1	0
5	0	1	0	0	0 ;

1	0	0	0	0	0
2	0	0	1	0	0
3	1	0	0	0	0
4	0	0	0	1	0
5	0	1	0	0	0;

ampl: display z;

y :=	1	2	3	4	5
1	0	0	1	0	0
2	1	1	1	1	0
3	1	1	1	1	0
4	1	1	1	1	1
5	1	1	1	1	0;

<u>+ When all areas have to be covered</u>

Optimal solution found (tolerance 1.00e-04)
Best objective 2.190000000000e+02, best bound 2.190000000000e+02, gap 0.0%
Optimize a model with 25 rows, 50 columns and 130 nonzeros

Iteration	Objective	Primal Inf.	Dual Inf.	Time
0	2.1900000e+02	0.000000e+00	0.000000e+00	0s

Solved in 0 iterations and 0.00 seconds
Optimal objective 2.190000000e+02
Gurobi 5.5.0: optimal solution; objective 219
38 simplex iterations
ampl: display x;

x :=	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1 ;

ampl: display y;

y :=	1	2	3	4	5
1	0	1	0	0	1
2	0	0	1	0	0
3	1	0	1	1	0
4	0	0	0	0	0
5	0	1	0	0	1 ;

ampl: display z;

z :=	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	2	2	2	1
4	1	1	1	1	1
5	1	1	1	1	1;

2.4.2 Answer-Question B

2.4.2.1 Model

```
param T;
#parameters
param c; # unit construction cost for one tower
param R {1..T,1..T};
param Maxcoverage;
param Maxprofit;
param w1; #weight for coverage
param w2; #weight for profit

# Decision variables
var x{1..T,1..T} binary;  #area to be covered
var y{1..T,1..T} binary;  #location of tower in which area
var z{1..T,1..T} integer >=0;  #times cover by areas

# Objective function
minimize weight: (w1/Maxcoverage)*(Maxcoverage-sum{i in 1..1,j in 1..1}x[i,j])+
(w2/Maxprofit)*(Maxprofit-sum{i in 1..T,j in 1..T}(x[i,j]*R[i,j]-y[i,j]*c));
subject to revenue {i in 1..T,j in 1..T}:        x[i,j]*R[i,j] <= R[i,j];
subject to equality {i in 1..T,j in 1..T}: z[i,j] >= x[i,j];

subject to times1 {i in 1..1,j in 1..1}: z[i,j] = y[i,j]+y[i,j+1]+y[i+1,j];
subject to times2 {i in 1..1,j in T..T}: z[i,j] = y[i,j-1]+y[i,j]+y[i+1,j];
subject to times3 {i in T..T,j in 1..1}: z[i,j] = y[i-1,j]+y[i,j]+y[i,j+1];
subject to times4 {i in T..T,j in T..T}: z[i,j] = y[i-1,j]+y[i,j-1]+y[i,j];
subject to times5 {i in 1..1,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i+1,j];
subject to times6 {i in T..T,j in 2..(T-1)}: z[i,j] = y[i,j-1]+y[i,j]+y[i,j+1]+y[i-1,j];
subject to times7 {i in 2..(T-1),j in 1..1}: z[i,j] = (y[i-1,j])+(y[i,j]+y[i,j+1])+(y[i+1,j]);
subject to times8 {i in 2..(T-1),j in T..T}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j])+(y[i+1,j]);
subject to times9 {i in 2..(T-1),j in 2..(T-1)}: z[i,j] = (y[i-1,j])+(y[i,j-1]+y[i,j]+y[i,j+1])+(y[i+1,j]);
```

2.4.2.2 Data

```
data;
param T:= 5;
param c:= 150;
param Maxcoverage = 25;
param Maxprofit =377;
param w1 =1;
param w2 =2;
param R :=
[*,*]:  1    2    3    4    5:=
        1    34   43   62   42   34
        2    64   43   71   48   65
        3    57   57   51   61   30
        4    32   38   70   56   40
        5    68   73   30   56   44;
```

2.4.2.3 Results

```
Optimal solution found (tolerance 1.00e-04)
Best objective 1.000000000000e+00, best bound 1.000000000000e+00, gap 0.0%
Optimize a model with 50 rows, 75 columns and 180 nonzeros
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    1.0000000e+00   0.000000e+00   0.000000e+00      0s
```

```
Solved in 0 iterations and 0.00 seconds
Optimal objective  1.000000000e+00
Gurobi 5.5.0: optimal solution; objective 0.207777
31 simplex iterations
absmipgap = 8.88e-16, relmipgap = 8.88e-16ampl: display x;
x :=    1    2    3    4    5

1       0    0    1    1    1
2       1    1    1    1    1
3       1    1    1    1    0
4       1    1    1    1    1
5       1    1    1    1    0 ;

ampl: display y;
y :=    1    2    3    4    5
1       0    0    0    0    1
2       0    0    1    0    0
3       1    0    0    0    0
4       0    0    0    1    0
5       0    1    0    0    0 ;

ampl: display z;
z :=    1    2    3    4    5

1       0    0    1    1    1
2       1    1    1    1    1
3       1    1    1    1    0
4       1    1    1    1    1
5       1    1    1    1    0 ;
```