

Performance indicators

Reliability





- When using reliability as a performance indicator
 - prioritization of interventions with respect to net benefit is rarely possible
 - for an item, there are multiple ways to estimate the reliability
 - for a sub-item – don't forget that the LOS it provides depends on how it is integrated into the item as a whole and the reliability of the other sub-items in the item.

- Expectation of you (after this class, reading the script, the handouts, and doing the assignments):
 - to be able to estimate the reliability of interconnected sub-items

 - to be able to determine which objects within an interconnected sub-item should be repaired to maximize reliability

Introduction

Items and sub-items / Systems and elements / Networks and objects 

Super-Items $\xrightarrow{\text{consist of}}$ **Items** $\xrightarrow{\text{consist of}}$ **Sub-items**

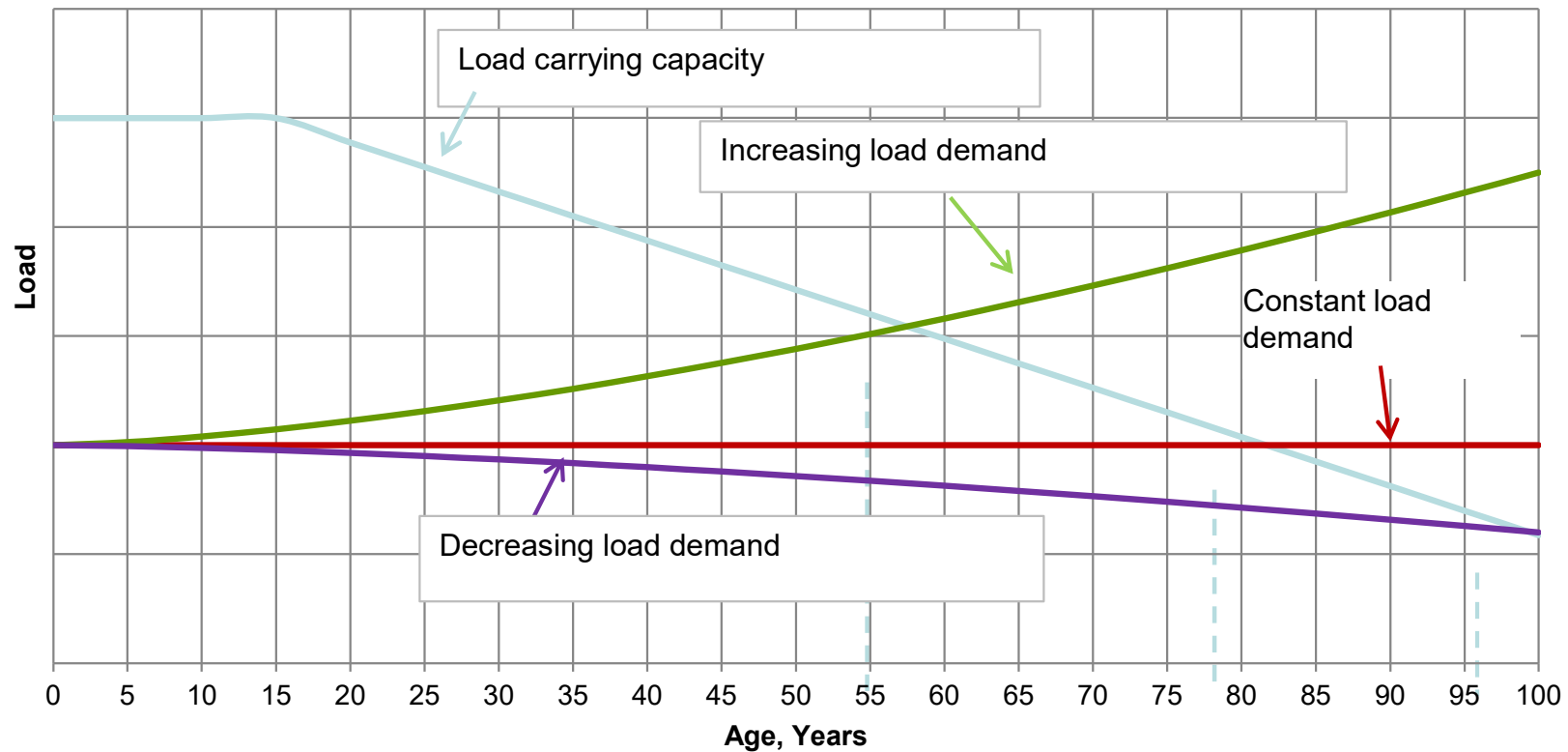




- **Total impacts**
 - the total impacts related to the performance of the item
- **Reliability**
 - the ability of an item to provide an adequate level of service
- **Availability**
 - the proportion of time an item provides an adequate level of service
- **Maintainability**
 - the ease with which an item can be maintained

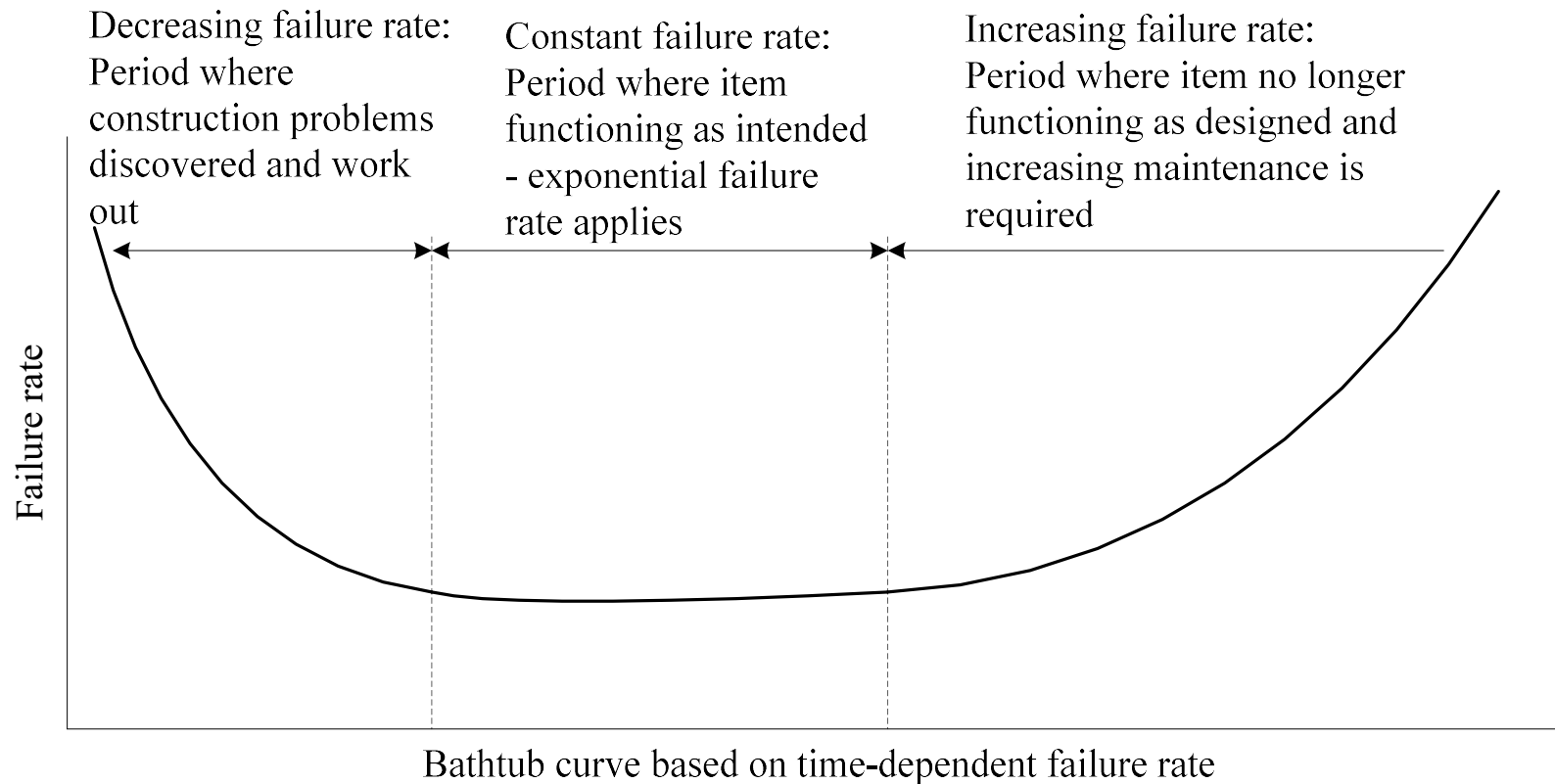


- How do you determine the reliability of an item?
- How do you determine the probability of failure of an item?
- How do you determine the amount of time until the first time an item reaches a state in which it provides an inadequate LOS?
- How do you determine the amount of time until the first failure of an item?
- How do you determine the service life of an item?



Variation of service life with varying demand

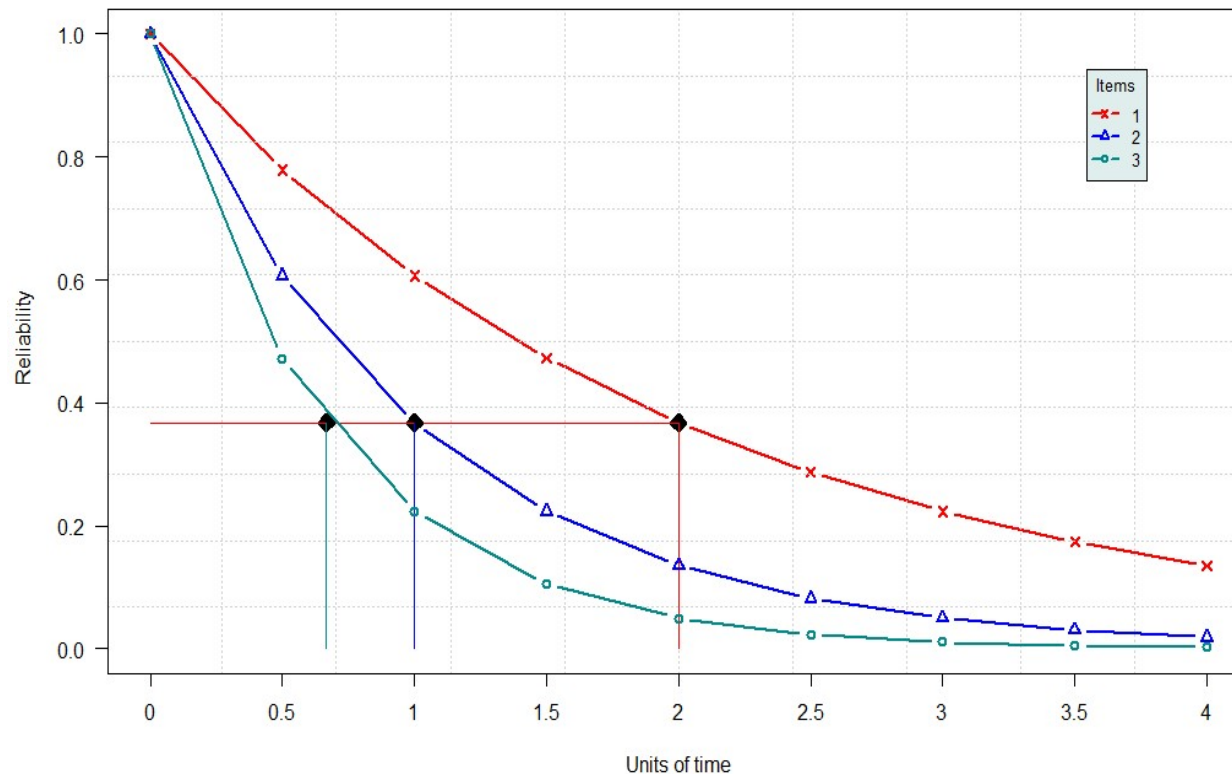
Failure rates are not always constant





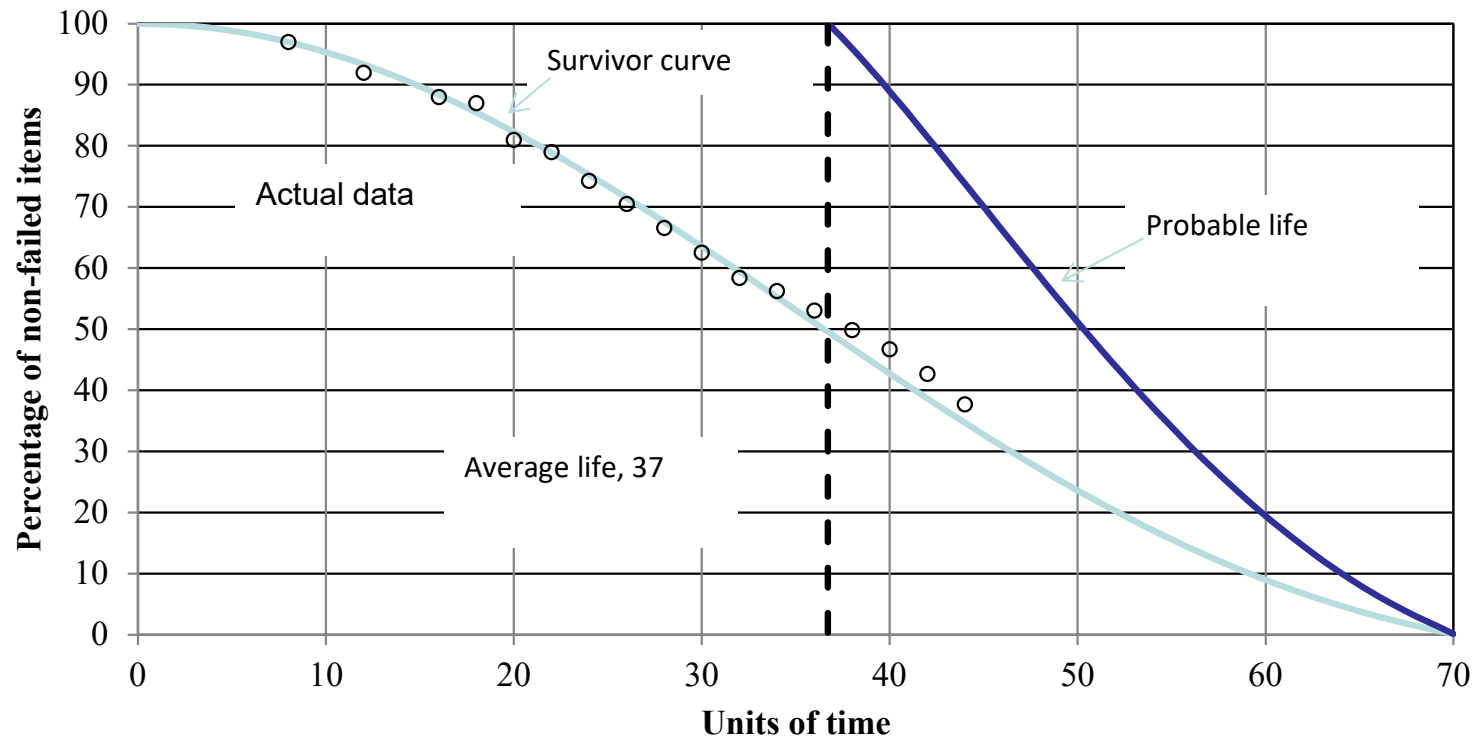
- The reliability of item i , which has an exponentially distributed lifetime, is:

$$R(t) = R\left(\frac{1}{\theta}\right) = \exp\left(-\theta \cdot \frac{1}{\theta}\right) = \exp(-1) = 0.3678794 \approx 0.37$$



Failure rates
 Item 1 – 0.5
 Item 2 – 1
 Item 3 – 1.5

Mean life times
 Item 1 – 2
 Item 2 – 1
 Item 3 – 0.667



Life expectancy of non-failed items at t = remaining area at t / % of non-failed items at t

Probable life of non-failed items at t = life expectancy of non-failed items at $t + t$



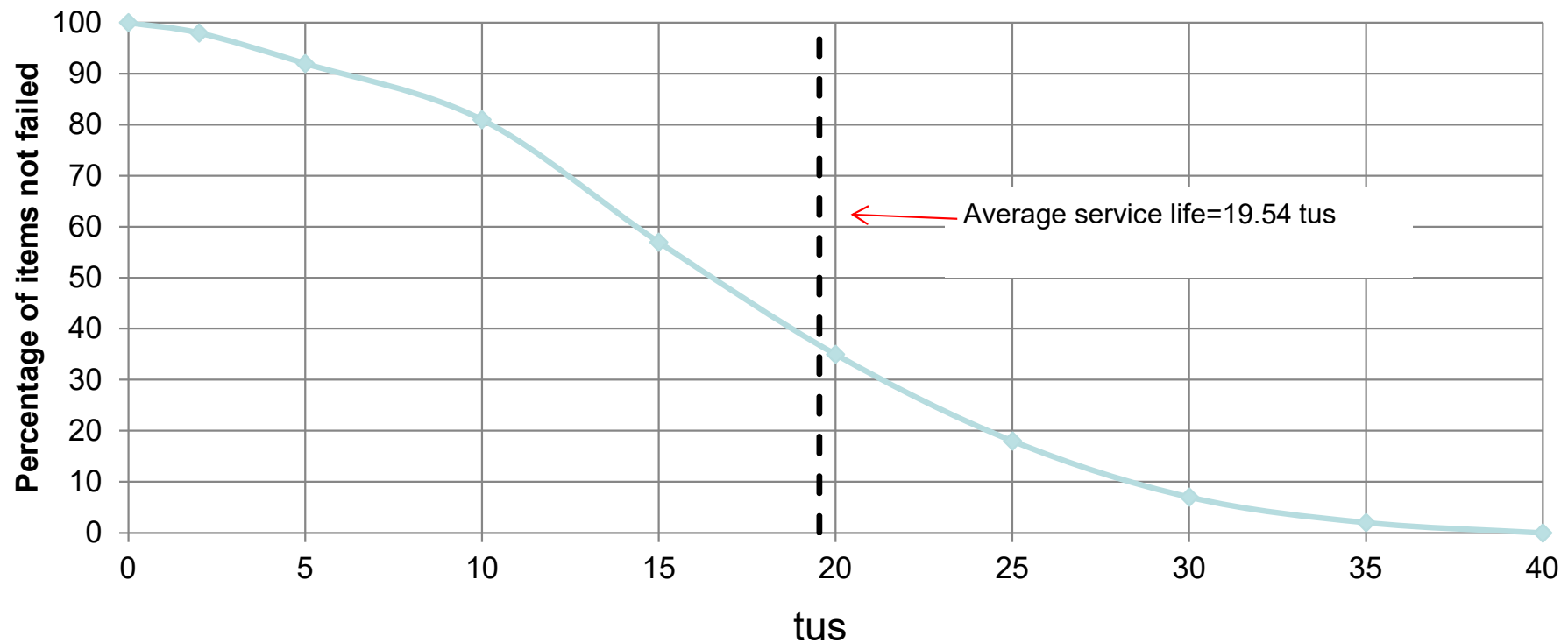
Units of time (A)	Number of failed items (B)	Cum. number of failed items $B(t)+C(t-1)$ (C)	Number of non-failed items $\text{Sum}(B)-C$ (D)	% of non- failed items $D/\text{sum}(B)^*100$ (E)	$A*B$ (F)	Total remaining amount of service <i>Integration</i> (G)	Life expectancy of non-failed items G/E (H)	Probable life of non-failed items $A+H$ (J)
0	0							
2	2							
5	6							
10	11							
15	24							
20	22							
25	17							
30	11							
35	5							
40	2							
Sum(B)	100			Avg. life				



Units of time (A)	Number of failed items (B)	Cum. number of failed items $B(t)+C(t-1)$ (C)	Number of non-failed items $\text{Sum}(B)-C$ (D)	% of non- failed items $D/\text{sum}(B)^* 100$ (E)	$A*B$ (F)	Total remaining amount of service <i>Integration</i> (G)	Life expectancy of non-failed items G/E (H)	Probable life of non-failed items $A+H$ (J)
0	0	0	100	100	0			
2	2	2	98	98	4			
5	6	8	92	92	30			
10	11	19	81	81	110			
15	24	43	57	57	360			
20	22	65	35	35	440			
25	17	82	18	18	425			
30	11	93	7	7	330			
35	5	98	2	2	175			
40	2	100	0	0	80			
Sum(B)	100			Avg. life	$1954/100 = 19.54$			

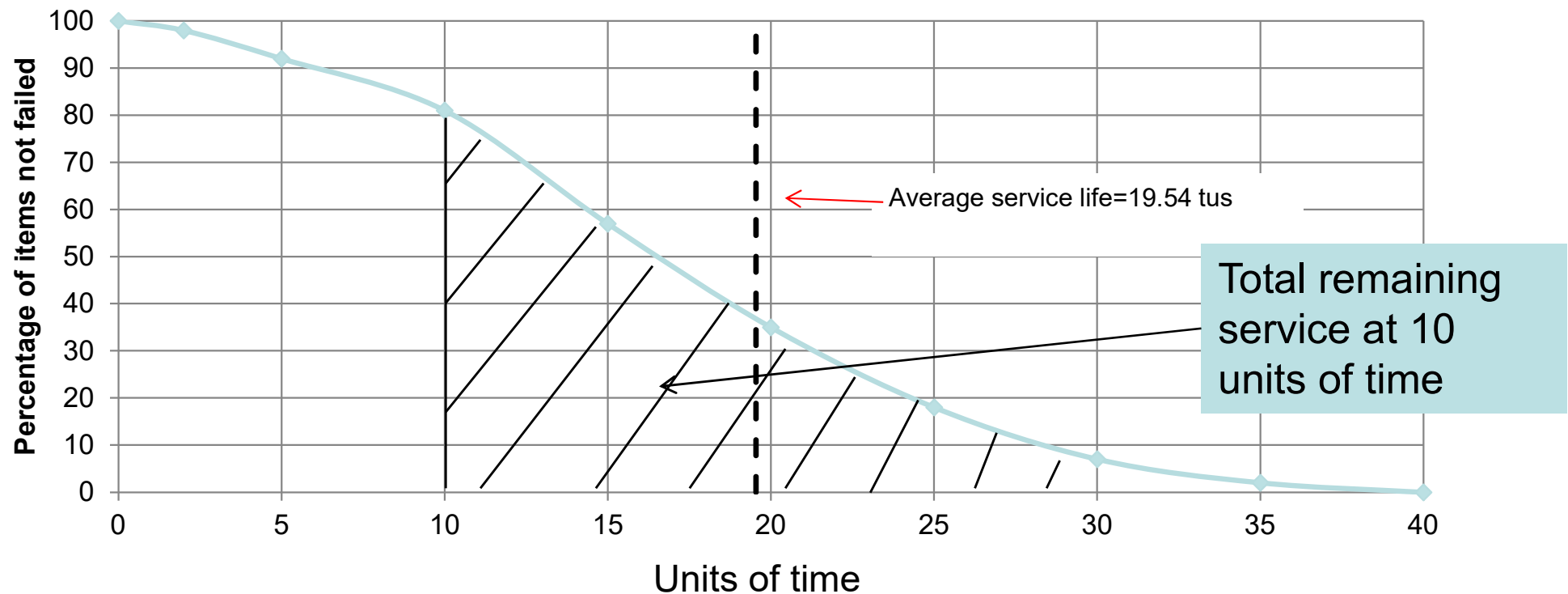


- Average service life = $\frac{\sum N \times f}{\sum f} = \frac{1954}{100} = 19.54 \text{ tus}$



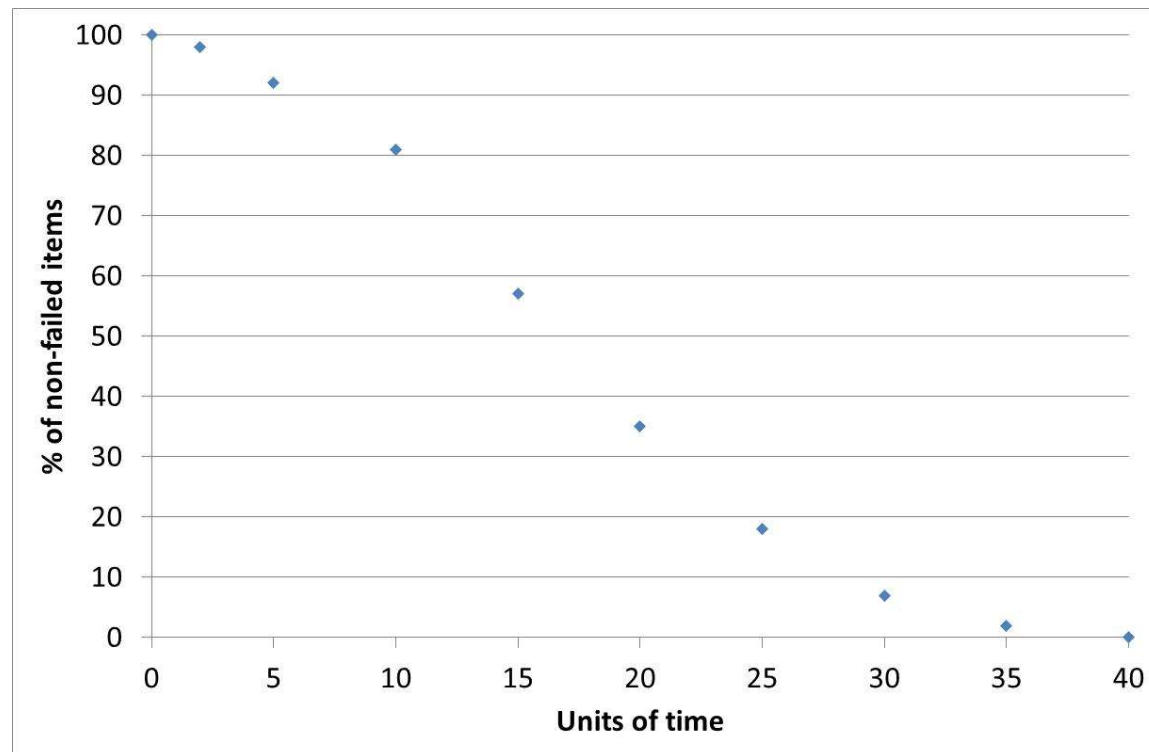


- Total remaining amount of time an item will provide an adequate LOS at $t =$
 - the area under the curve (*assumed as a triangle*) to the right of the time ordinate
 - $0.5 * (T - t) * (\% \text{ items not failed at time } t)$
 - $0.5 * (40 - 0) * 100 = 2'000$



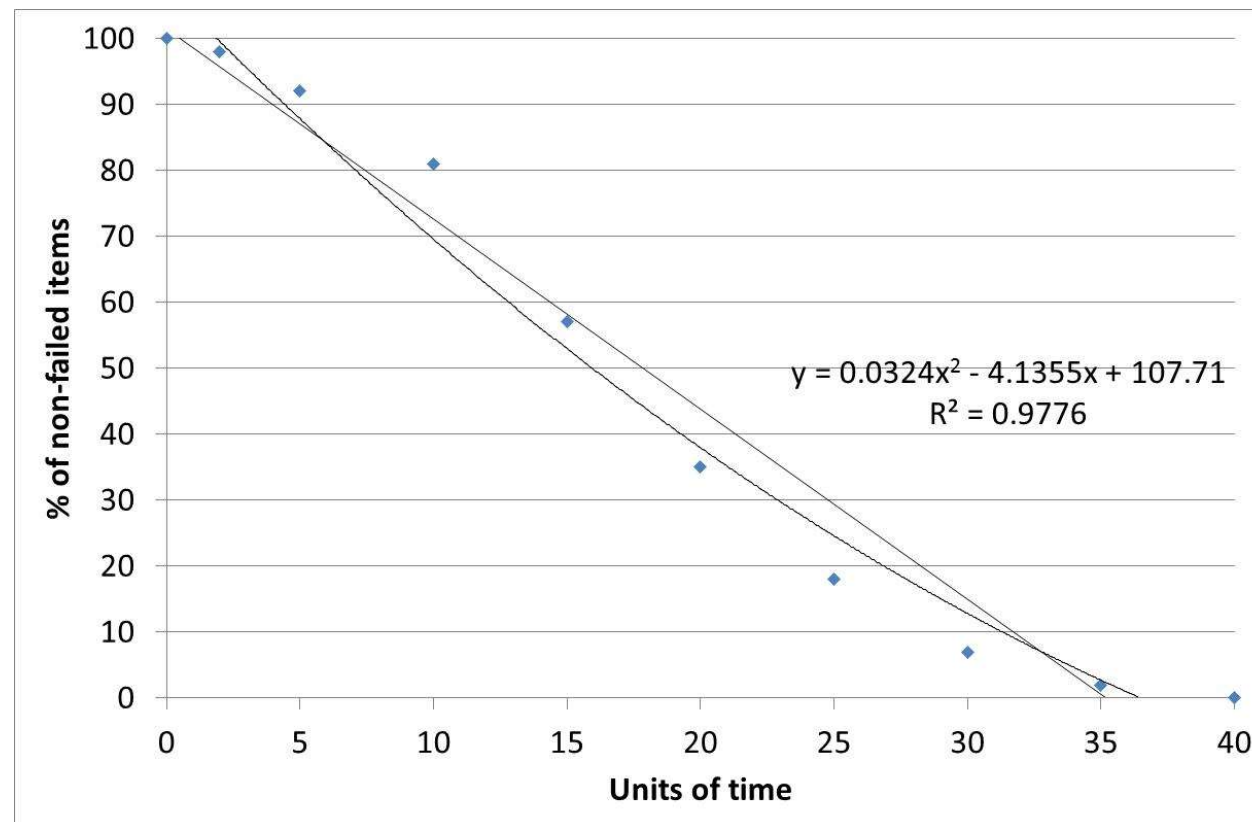


- Area calculation by integration
 - Enter values in Excel
 - Draw a scatter plot for % of non-failed items (E) against time (A) using the following Excel option





- Area calculation by integration
 - Enter values in Excel
 - Draw a scatter plot for % of non-failed items (E) against time (A) using the following Excel option
 - Determine best fit regression curve (polynomial of order 2)





- Area at any t can be calculated by integrating the regression equation between two age limits

$$t_1 = \int_{t=t_1}^{t=T} y \cdot dt \quad y = 0.0324 \cdot t^2 - 4.1355 \cdot t + 107.71$$

$$\left[0.0324 \cdot \frac{t^3}{3} - 4.1355 \cdot \frac{t^2}{2} + 107.71 \cdot t \right]_t^T \Rightarrow \left[0.0324 \cdot \frac{2^3}{3} - 4.1355 \cdot \frac{2^2}{2} + 107.71 \cdot 2 \right] \frac{40}{2}$$

1484 % - units of time



Units of time (A)	Number of failed items (B)	Cum. number of failed items $B(t)+C(t-1)$ (C)	Number of non-failed items $\text{Sum}(B)-C$ (D)	% of non- failed items $D/\text{sum}(B)^*$ 100 (E)	$A*B$ (F)	Total remaining amount of service <i>Integration</i> (G)	Life expectancy of non-failed items G/E (H)	Probable life of non-failed items $A+H$ (J)
0	0	0	100	100	0	1691		
2	2	2	98	98	4	1484		
5	6	8	92	92	30	1203		
10	11	19	81	81	110	810		
15	24	43	57	57	360	504		
20	22	65	35	35	440	278		
25	17	82	18	18	425	122		
30	11	93	7	7	330	29		
35	5	98	2	2	175	-9		
40	2	100	0	0	80	0		
Sum(B)	100			Avg. life	$1954/100 = 19.54$			



Units of time (A)	Number of failed items (B)	Cum. number of failed items $B(t)+C(t-1)$ (C)	Number of non-failed items $\text{Sum}(B)-C$ (D)	% of non- failed items $D/\text{sum}(B)^* 100$ (E)	$A*B$ (F)	Total remaining amount of service <i>Integration</i> (G)	Life expectancy of non-failed items G/E (H)	Probable life of non-failed items $A+H$ (J)
0	0	0	100	100	0	1691	16.91	
2	2	2	98	98	4	1484	15.14	
5	6	8	92	92	30	1203	13.08	
10	11	19	81	81	110	810	10.00	
15	24	43	57	57	360	504	8.85	
20	22	65	35	35	440	278	7.93	
25	17	82	18	18	425	122	6.78	
30	11	93	7	7	330	29	4.18	
35	5	98	2	2	175	-9	-4.35	
40	2	100	0	0	80	0	0.00	
Sum(B)	100			Avg. life	$1954/100 = 19.54$			



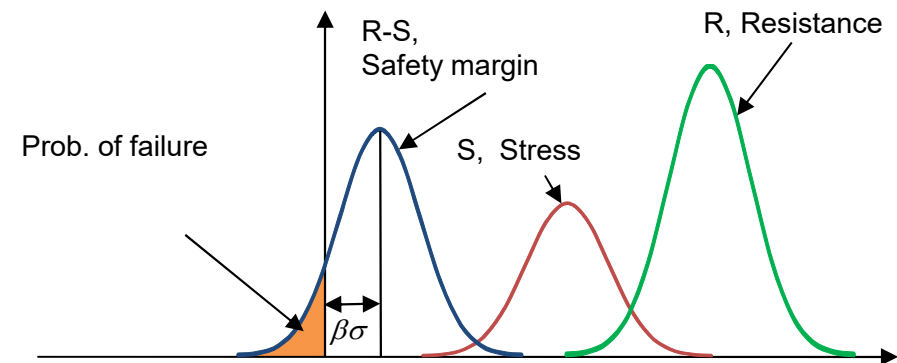
Units of time (A)	Number of failed items (B)	Cum. number of failed items $B(t)+C(t-1)$ (C)	Number of non-failed items $\text{Sum}(B)-C$ (D)	% of non- failed items $D/\text{sum}(B)^* 100$ (E)	$A*B$ (F)	Total remaining amount of service <i>Integration</i> (G)	Life expectancy of non-failed items G/E (H)	Probable life of non-failed items $A+H$ (J)
0	0	0	100	100	0	1691	16.91	16.91
2	2	2	98	98	4	1484	15.14	17.14
5	6	8	92	92	30	1203	13.08	18.08
10	11	19	81	81	110	810	10.00	20.00
15	24	43	57	57	360	504	8.85	23.85
20	22	65	35	35	440	278	7.93	27.93
25	17	82	18	18	425	122	6.78	31.78
30	11	93	7	7	330	29	4.18	34.18
35	5	98	2	2	175	-9	-4.35	30.65
40	2	100	0	0	80	0	0.00	40.00
Sum(B)	100			Avg. life	$1954/100 = 19.54$			



- Based on data



- Based on models and uncertainty of parameters



- Based on reliability sub-items



Reliability: Item comprised of sub-items

Assumptions



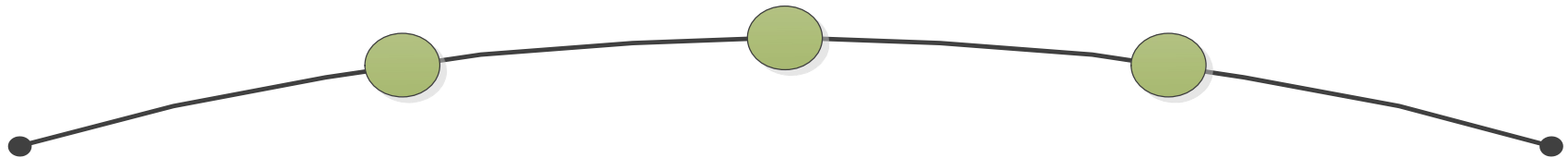
- Assume:
 - sub-items are either operational or non-operational, i.e. two states
- The state of sub-item i will be described by a binary variable x_i :
 - $x_i = 1$ if sub-item operational,
 - $x_i = 0$ if sub-item non-operational
- Assume:
 - item is either operational or non-operational, i.e. two states
- The dependence of an item's state on the states of its sub-items is determined by means of a structure function $\phi(\mathbf{x})$,
 - where $\mathbf{x} = (x_1, x_2, \dots, x_n)$
 - $\phi(\mathbf{x}) = 1$ if item operational, and
 - $\phi(\mathbf{x}) = 0$ if item non-operational.

Reliability: Item comprised of sub-items

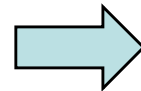
Sub-items in series



- An item with sub-items connected in series is operational if and only if all of its sub-items are operational



$$\phi(\vec{x}) = \prod_{i=1}^n x_i = \min_{1 \leq i \leq n} x_i$$



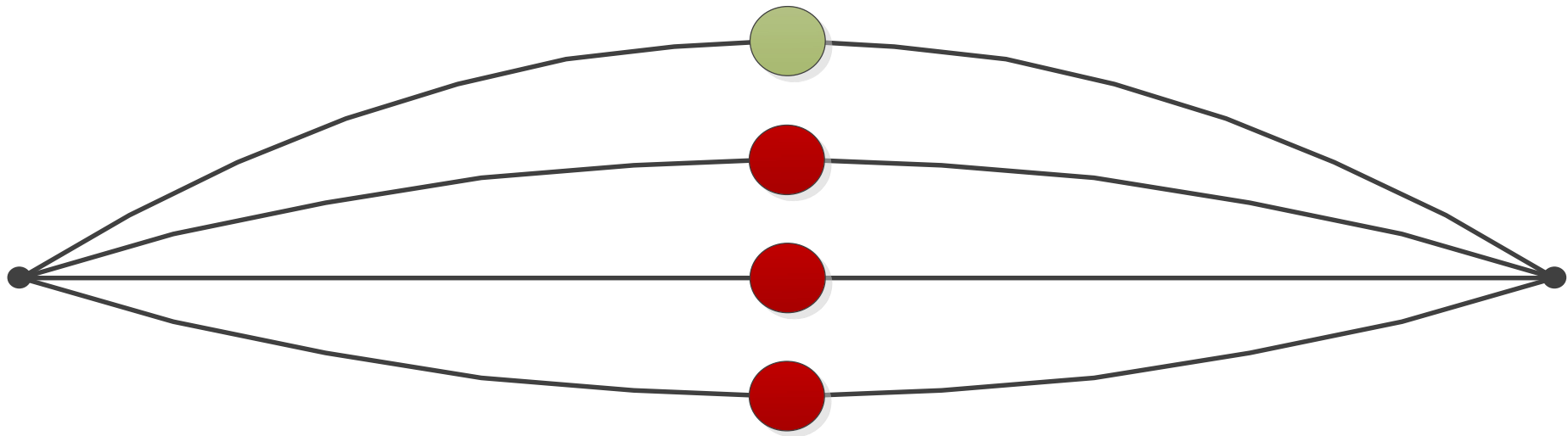
$$\vec{R}(t) = \prod_{i=1}^n R_i(t)$$

Reliability: Item comprised of sub-items

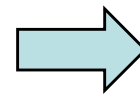
Sub-items in parallel



- An item with sub-items connected in parallel is operational if and only if at least one sub-item is operational



$$\phi(\vec{x}) = 1 - \prod_{i=1}^n (1 - x_i) = \max_{1 \leq i \leq n} x_i$$



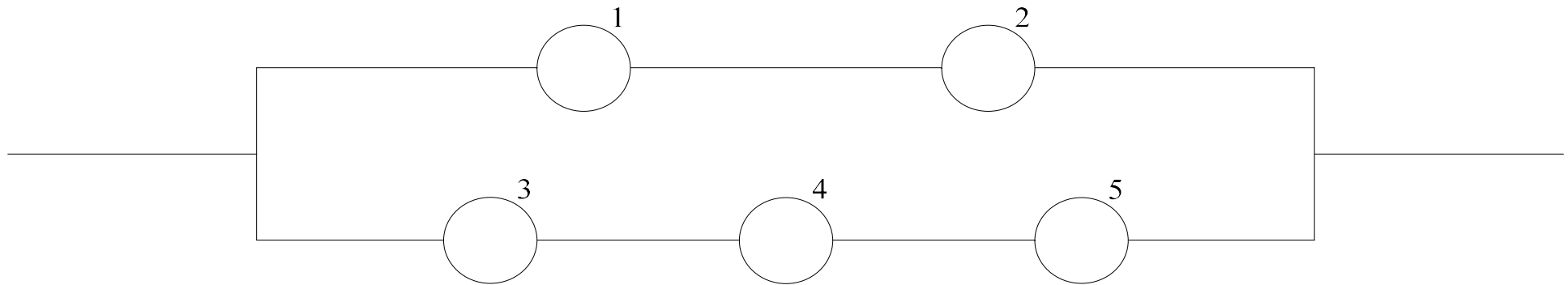
$$\vec{R}(t) = 1 - \prod_{i=1}^n (1 - R_i(t))$$



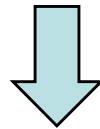
$$\phi(\bar{x}) = [1 - (1 - x_1) \cdot (1 - x_2)] \cdot [1 - (1 - x_3) \cdot (1 - x_4)]$$



$$R = [1 - (1 - R_1) \cdot (1 - R_2)] \cdot [1 - (1 - R_3) \cdot (1 - R_4)]$$



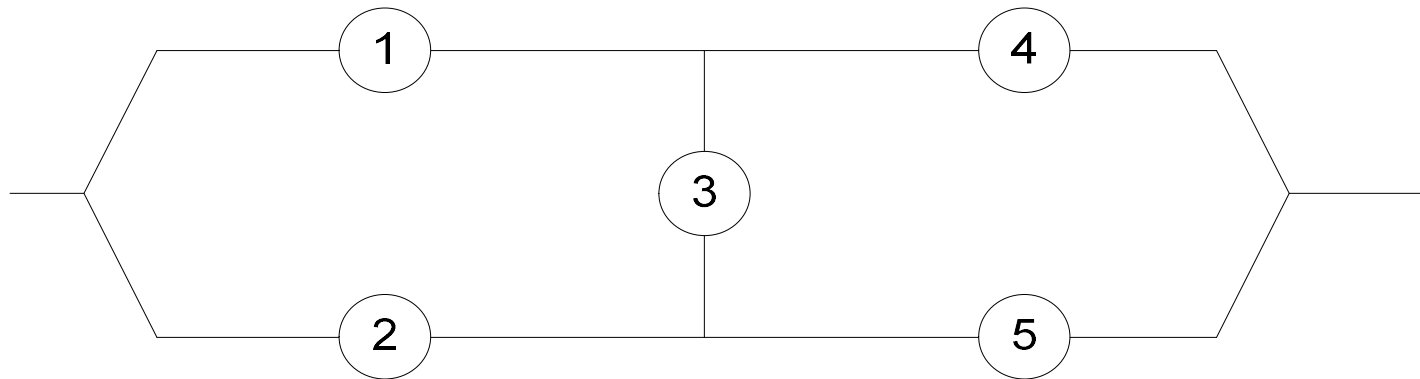
$$\phi(\vec{x}) = 1 - (1 - x_1 \cdot x_2) \cdot (1 - x_3 \cdot x_4 \cdot x_5)$$

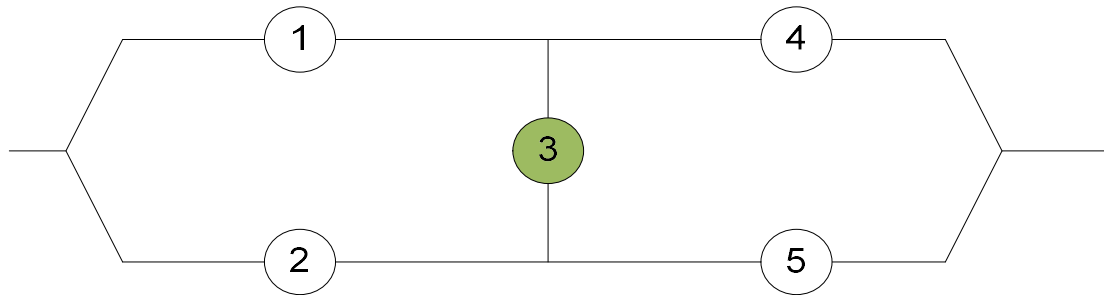


$$R = 1 - (1 - R_1 \cdot R_2) \cdot (1 - R_3 \cdot R_4 \cdot R_5)$$

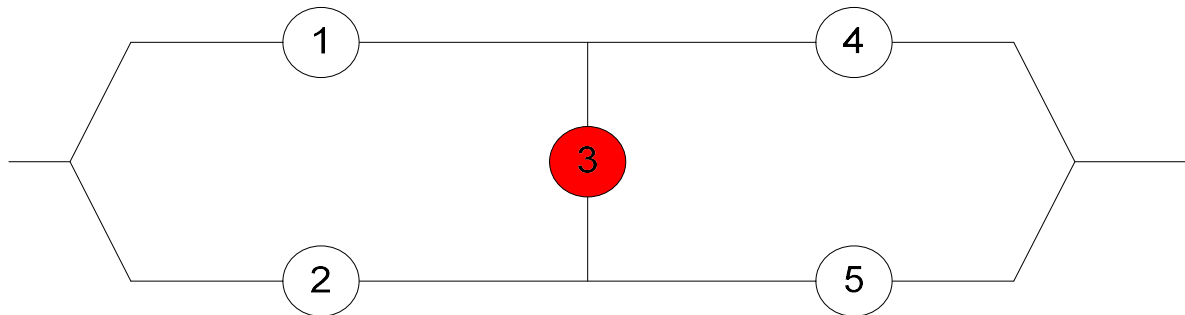
Reliability: Item comprised of sub-items

Complex configurations – pivotal decomposition

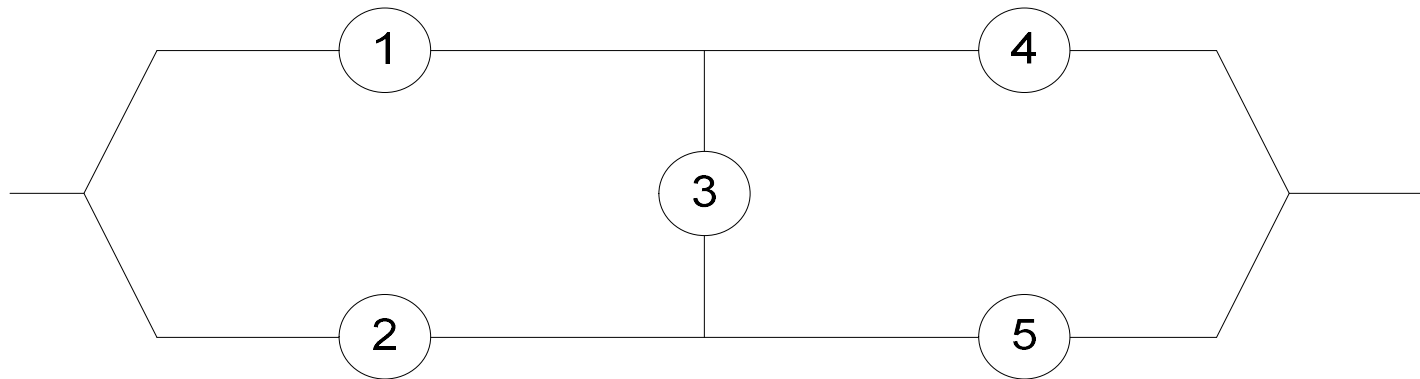




$$R(1_3, \vec{x}) = [1 - (1 - R_1)(1 - R_2)] \cdot [1 - (1 - R_4)(1 - R_5)]$$



$$R(0_3, \vec{x}) = [1 - (1 - R_1 \cdot R_4)] \cdot [1 - (1 - R_2 \cdot R_5)]$$



$$\begin{aligned} R &= R_3 \cdot R(1_3, \vec{x}) + (1 - R_3) \cdot R(0_3; \vec{x}) \\ &= R_1 \cdot R_3 \cdot R_4 + R_1 \cdot R_3 \cdot R_5 + R_2 \cdot R_3 \cdot R_4 + R_2 \cdot R_3 \cdot R_5 + R_1 \cdot R_2 \cdot R_4 \cdot R_5 \\ &\quad - R_1 \cdot R_2 \cdot R_3 \cdot R_4 - R_1 \cdot R_3 \cdot R_4 \cdot R_5 - R_1 \cdot R_2 \cdot R_3 \cdot R_5 - R_2 \cdot R_3 \cdot R_4 \cdot R_5 \end{aligned}$$

Reliability: Item comprised of sub-items

Minimal paths and minimal cuts

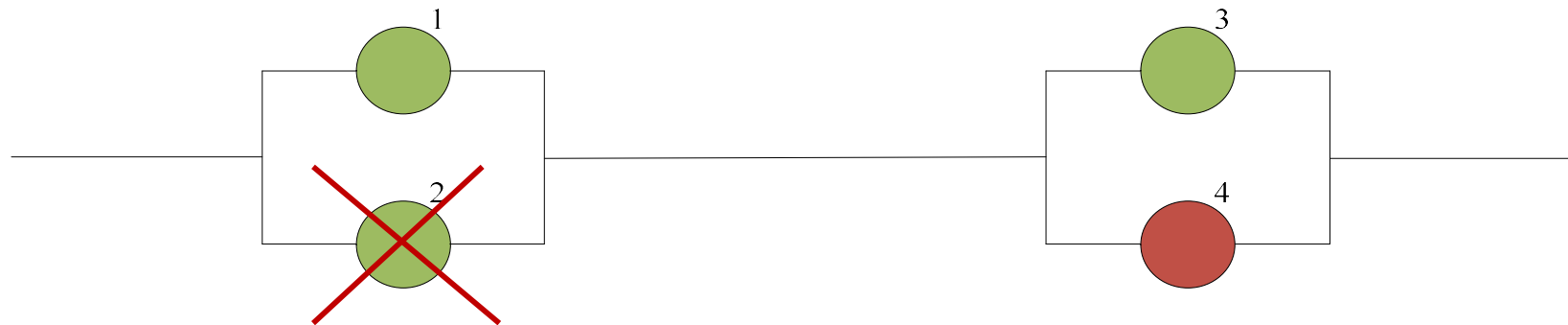


- Any monotone item can be represented in two equivalent ways:
 - as parallel sub-items connected in series each being a minimal cut set, or
 - as series sub-items connected in parallel each being a minimal path set
- Let P_1, P_2, \dots, P_s = the minimal path sets of the item. Then

$$R(\vec{x}) = 1 - \prod_{j=1}^s \left(1 - \prod_{i \in P_j} R_i \right)$$

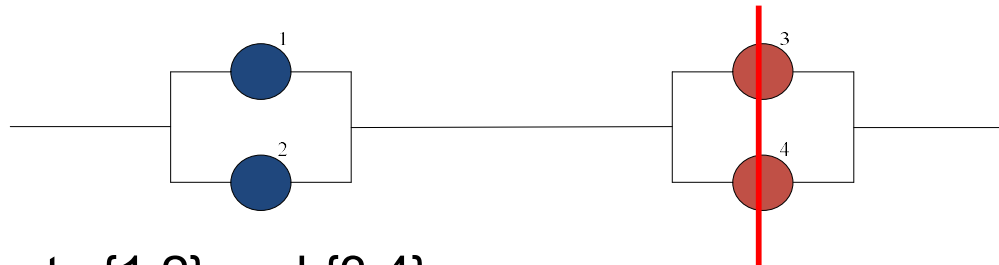
- Let C_1, C_2, \dots, C_k = the minimal cut sets of the item. Then

$$R(\vec{x}) = \prod_{j=1}^k \left(1 - \prod_{i \in C_j} (1 - R_i) \right)$$





- The structure function for these parallel sub-items connected in series can be based



- on minimal cuts $\{1,2\}$ and $\{3,4\}$,

$$R(\vec{x}) = \left[1 - (1 - R_1) \cdot (1 - R_2) \right] \cdot \left[1 - (1 - R_3) \cdot (1 - R_4) \right]$$



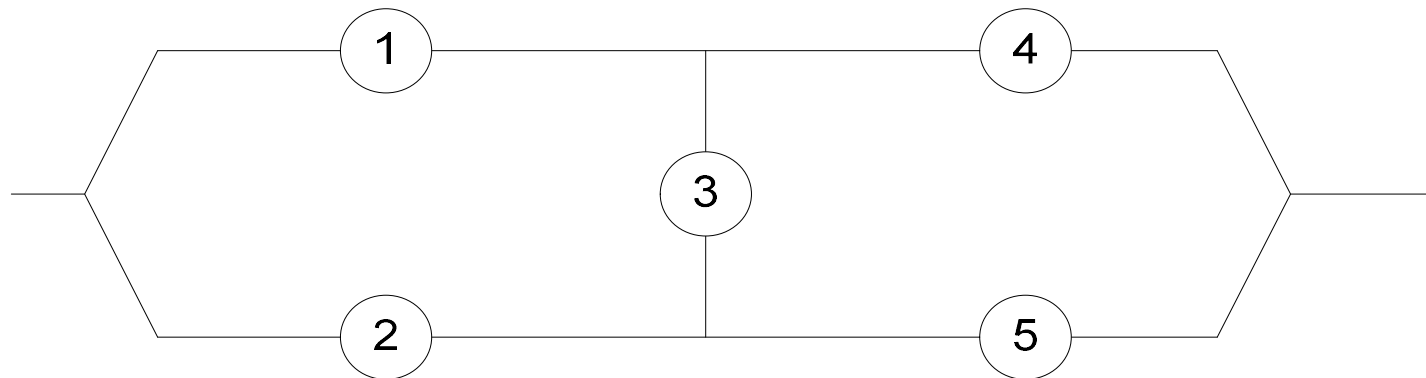
- on minimal paths: $\{1,3\}$, $\{1,4\}$, $\{2,3\}$, $\{2,4\}$.

$$R(\vec{x}) = \left[1 - (1 - R_1 \cdot R_3) \cdot (1 - R_1 \cdot R_4) \cdot (1 - R_2 \cdot R_3) \cdot (1 - R_2 \cdot R_4) \right]$$



Challenge:

Show that the minimal path method will give the same answer as pivotal decomposition



$$R(\overline{X}) = 1 - (1 - R_1 \cdot R_3 \cdot R_5) \cdot (1 - R_2 \cdot R_3 \cdot R_4) \cdot (1 - R_2 \cdot R_5) \cdot (1 - R_1 \cdot R_4)$$

Reliability: Item comprised of sub-items Time!



- What is the reliability of a road link with 4 objects that have mean life times of 60, 45, 105 and 32 *tus*?



Reliability

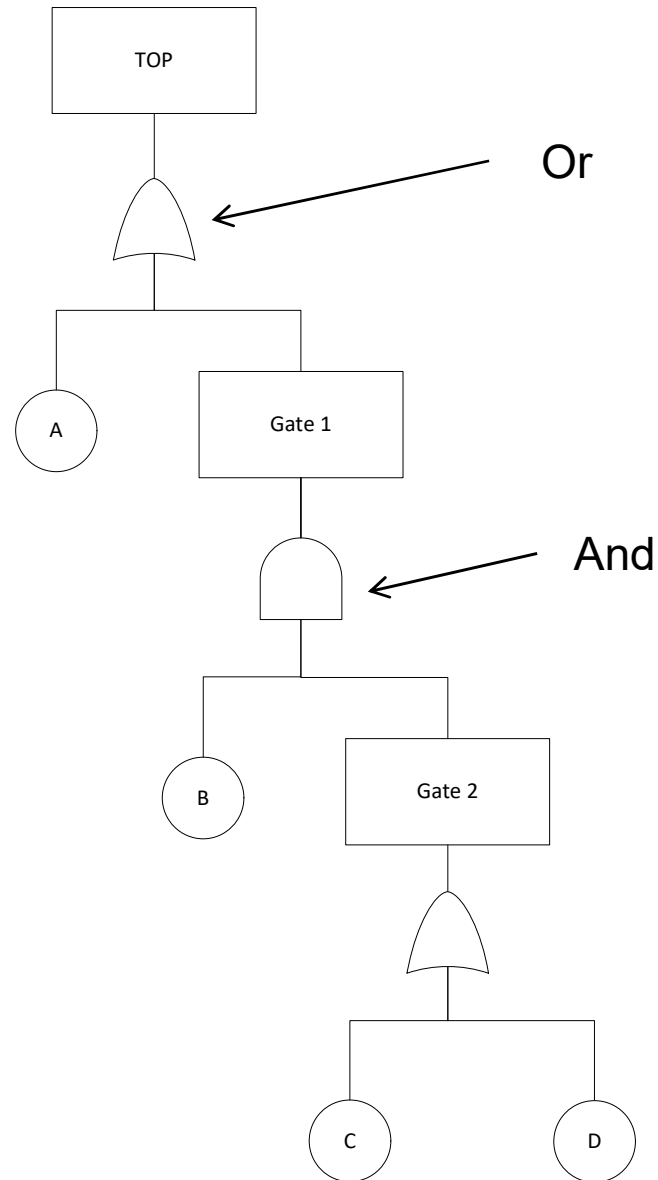
Sub-item importance



Top event is failure

$$q_a, q_c = 0.1$$

$$q_b, q_d = 0.2$$



Reliability: Sub-item importance

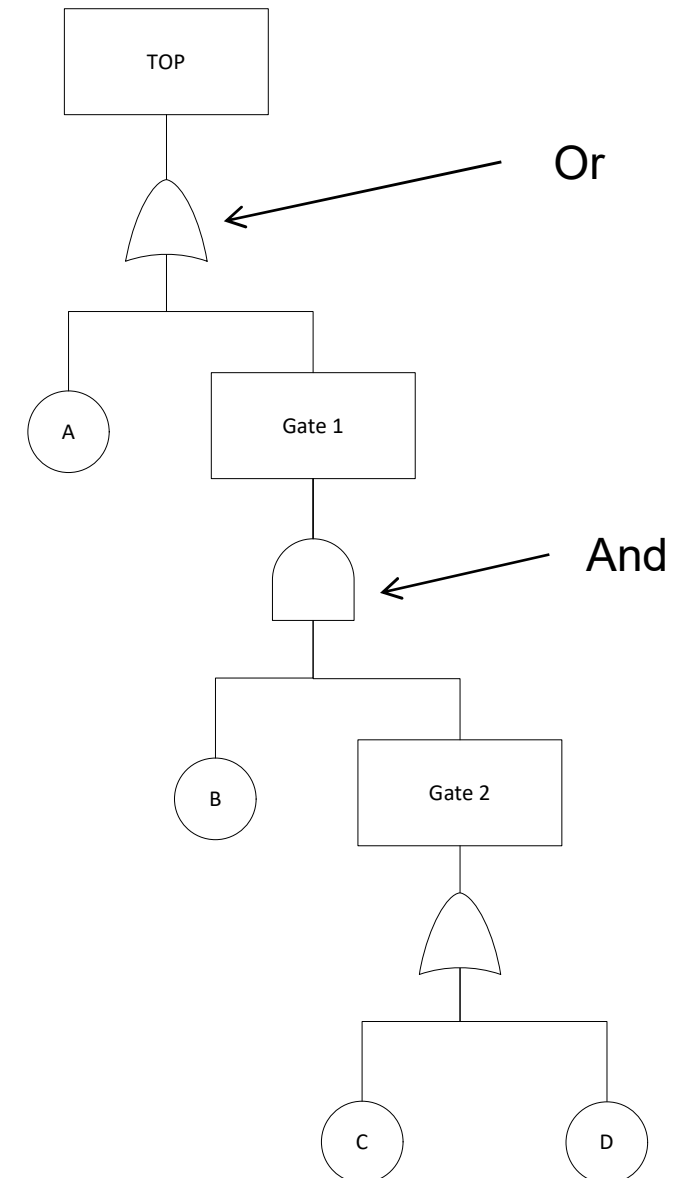
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item A

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	B	C	D	
1				
2				
3				
4				
5				
6				
7				
8				



Reliability: Sub-item importance

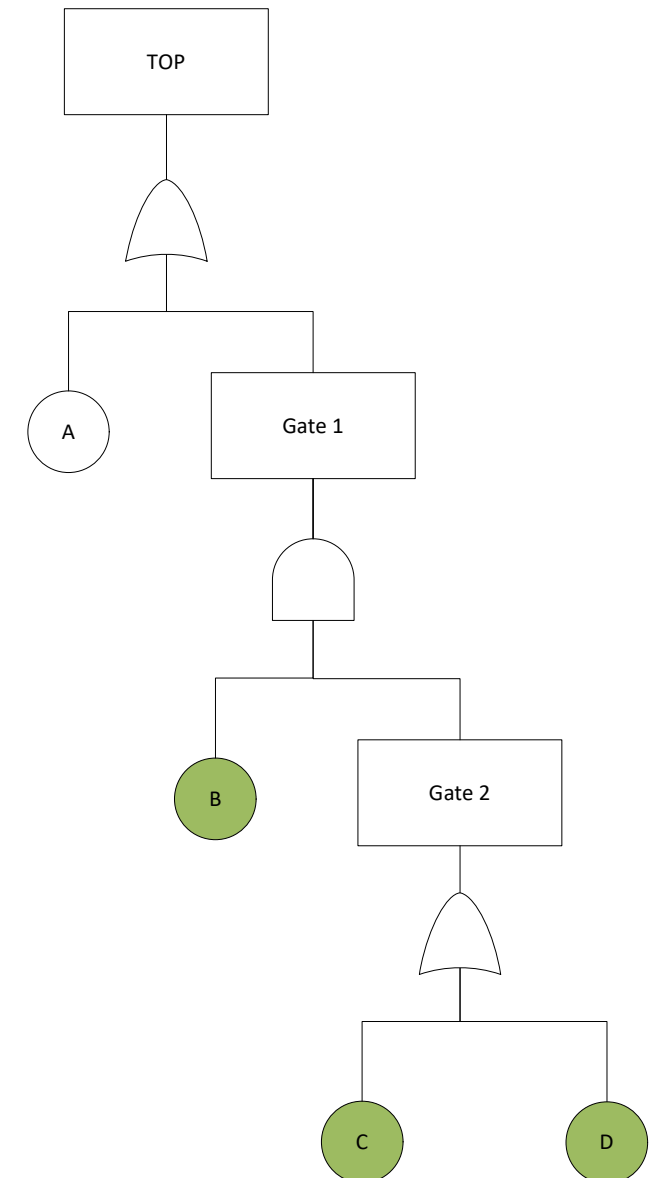
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item A

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	B	C	D	
1	W	W	W	Y
2				
3				
4				
5				
6				
7				
8				



Reliability: Sub-item importance

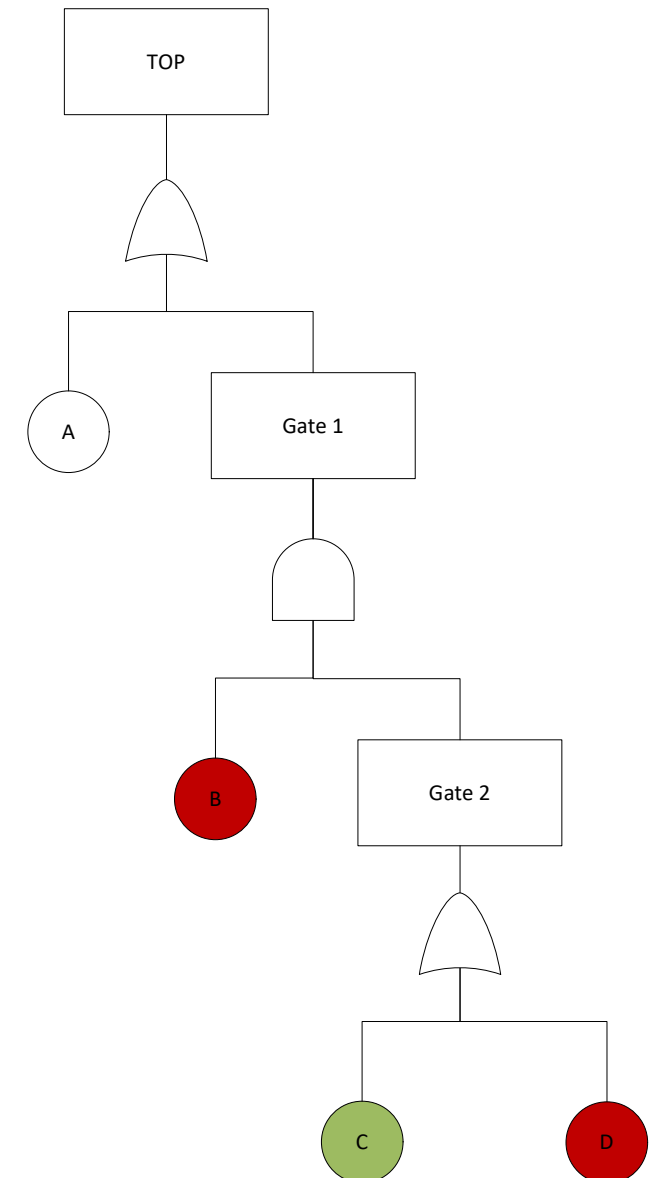
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item A

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	B	C	D	
1	W	W	W	Y
2	W	W	F	Y
3	W	F	W	Y
4	W	F	F	Y
5	F	W	W	Y
6	F	W	F	N
7				
8				



Reliability: Sub-item importance

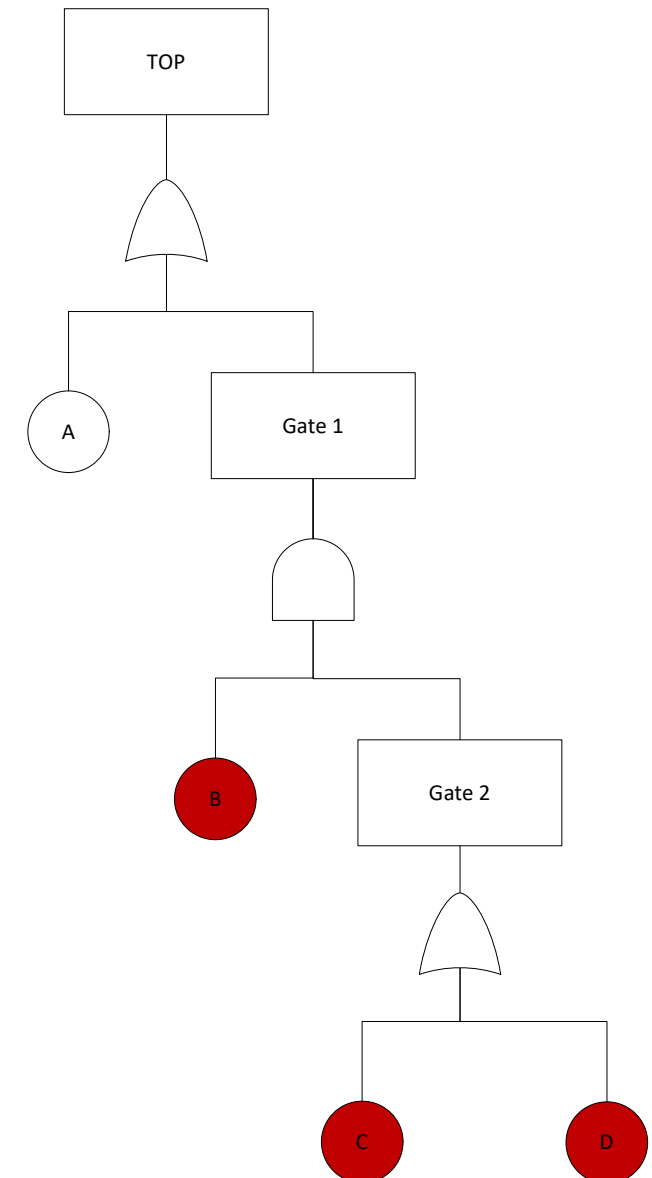
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item A

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	B	C	D	
1	W	W	W	Y
2	W	W	F	Y
3	W	F	W	Y
4	W	F	F	Y
5	F	W	W	Y
6	F	W	F	N
7	F	F	W	N
8	F	F	F	N



Reliability: Sub-item importance

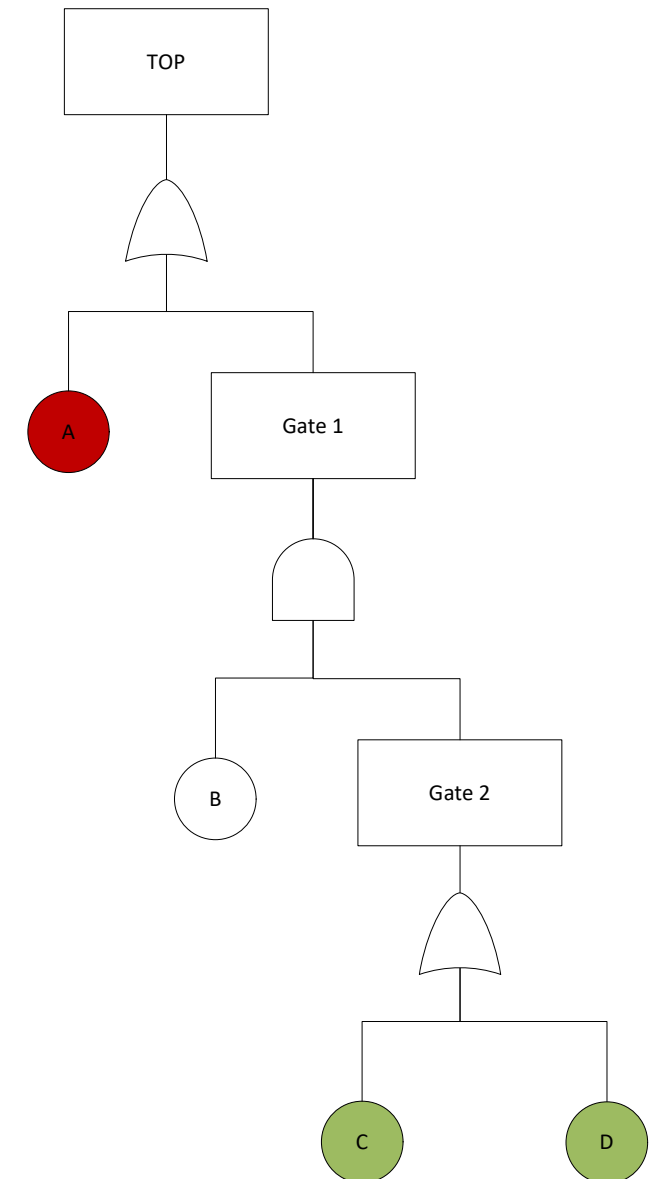
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item B

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	A	C	D	
1	W	W	W	N
2	W	W	F	Y
3	W	F	W	Y
4	W	F	F	Y
5	F	W	W	N



Reliability: Sub-item importance

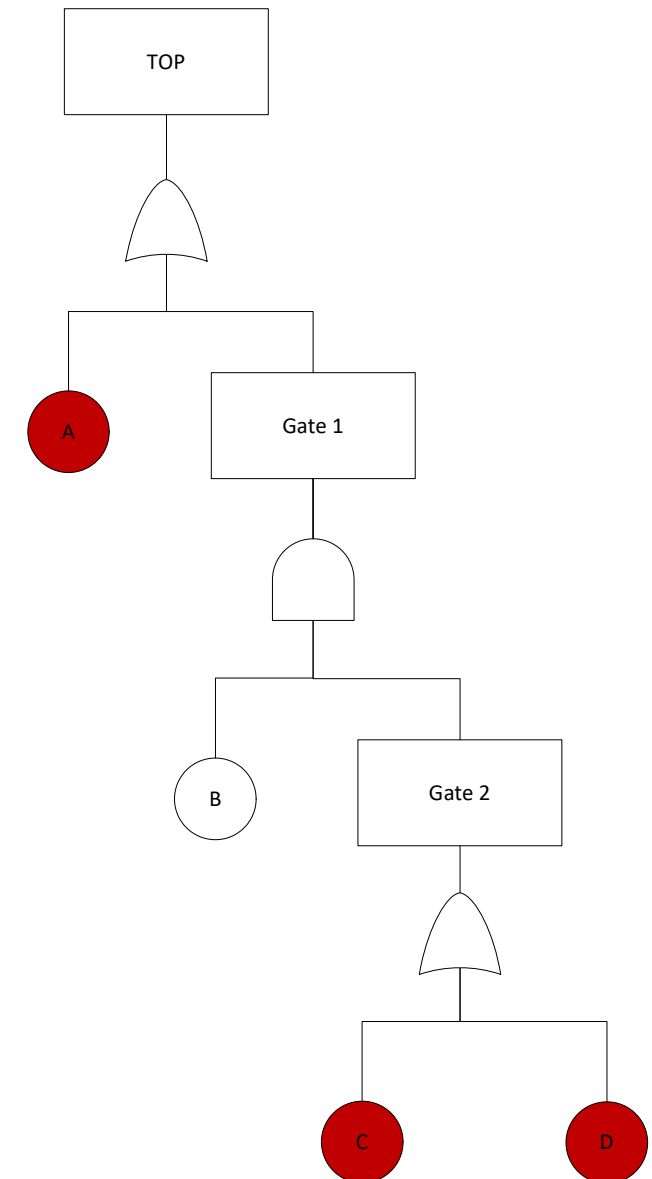
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item B

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	A	C	D	
1	W	W	W	N
2	W	W	F	Y
3	W	F	W	Y
4	W	F	F	Y
5	F	W	W	N
6	F	W	F	N
7	F	F	W	N
8	F	F	F	N



Reliability: Sub-item importance

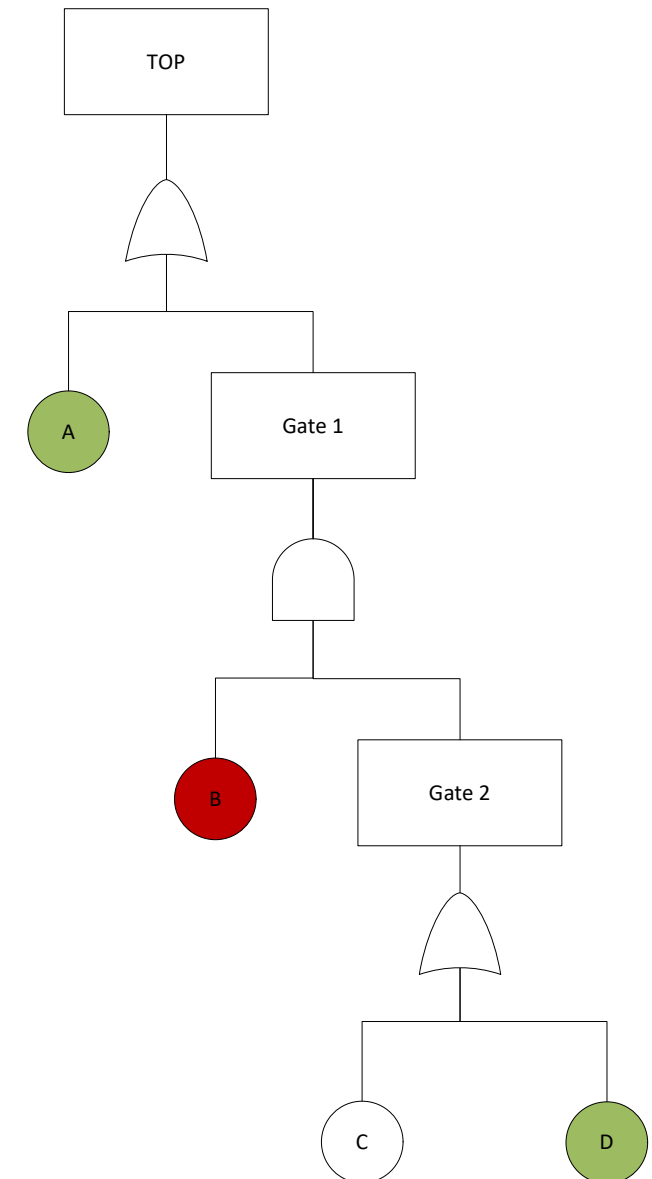
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item C

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	A	B	D	
1	W	W	W	N
2	W	W	F	N
3	W	F	W	Y



Reliability: Sub-item importance

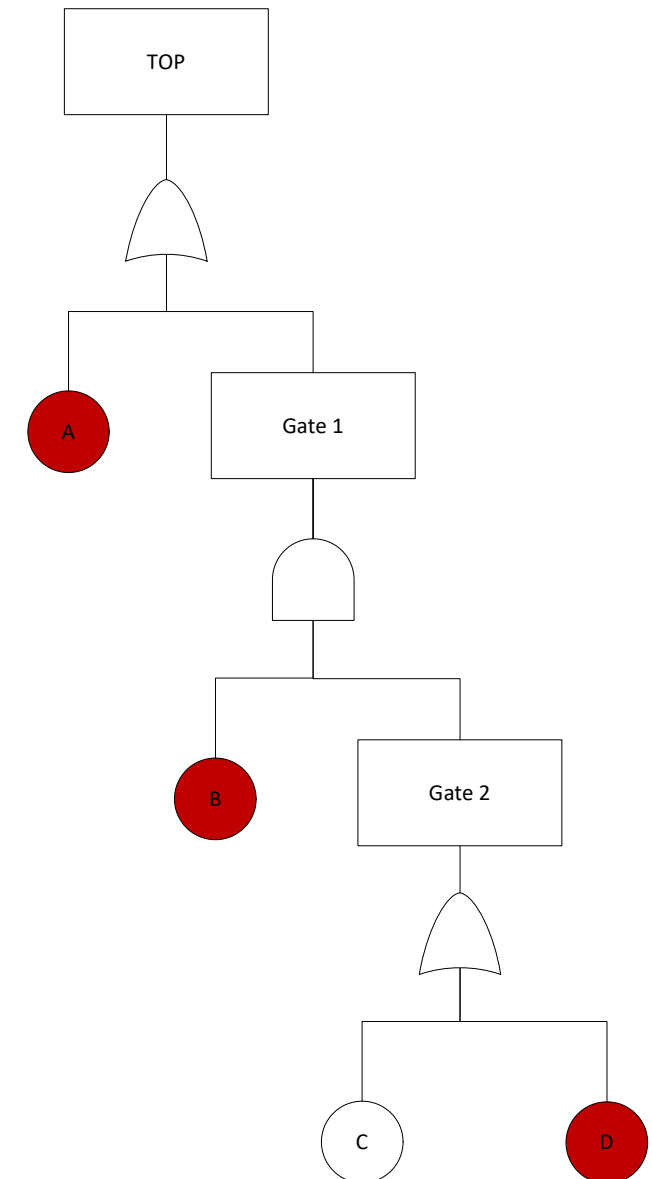
Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Importance of sub-item C

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	A	B	D	
1	W	W	W	N
2	W	W	F	N
3	W	F	W	Y
4	W	F	F	N
5	F	W	W	N
6	F	W	F	N
7	F	F	W	N
8	F	F	F	N



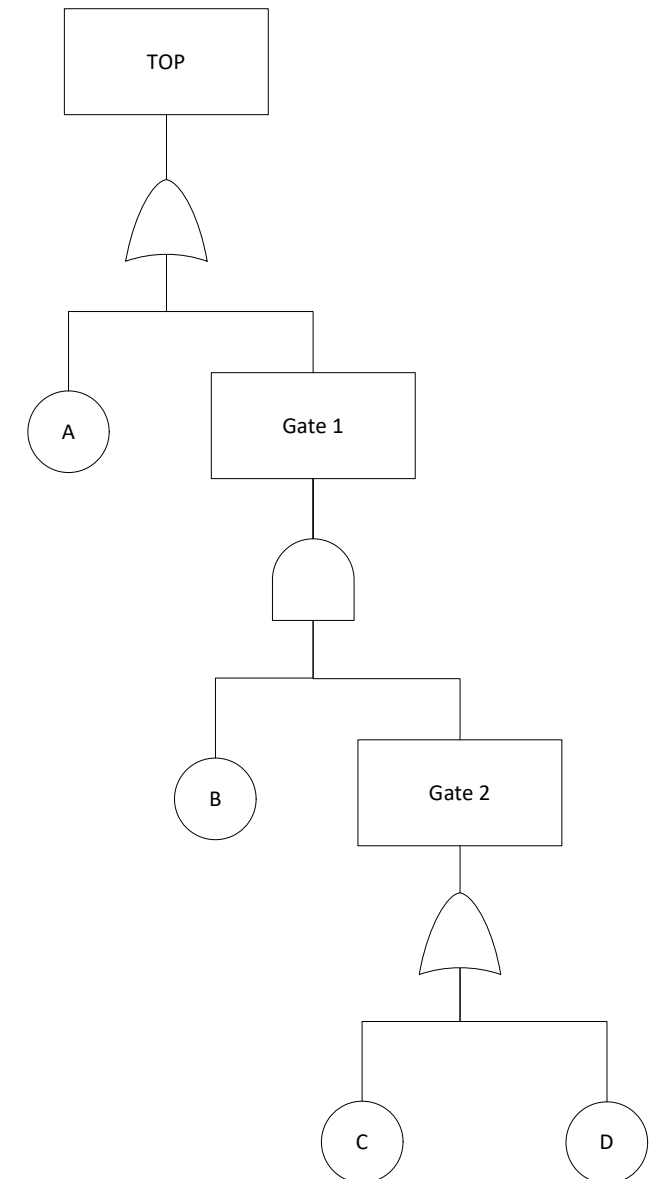
Reliability: Sub-item importance

Structural measure of importance



$$I_i = \frac{\text{number of critical states for sub-item } i}{\text{total number of states for the } (n - 1) \text{ remaining sub-items}}$$

Measure	Sub-items			
	A	B	C	D
Structural importance	0.625	0.375	0.125	0.125



Reliability: Sub-item importance

Birnbaum measure of importance



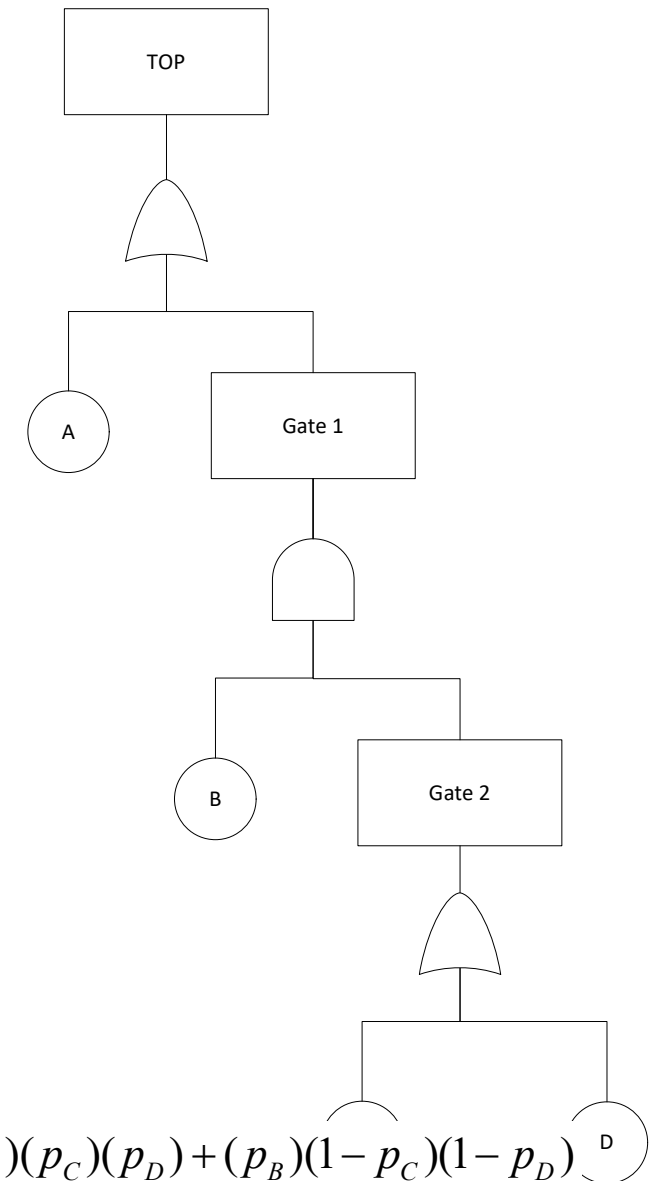
- the probability that the item is in a state in which the failure of sub-item i will cause failure of the item

$$I_i^B = \sum_j^J p_j^{cs_i} \quad j = \text{item state}$$

Item state	Sub-item states			Does failure of sub-item A cause failure of item?
	B	C	D	
1	W	W	W	Y
2	W	W	F	Y
3	W	F	W	Y
4	W	F	F	Y
5	F	W	W	Y
6	F	W	F	N
7	F	F	W	N
8	F	F	F	N

$$I_A^B = (1 - p_B)(1 - p_C)(1 - p_D) + (1 - p_B)(1 - p_C)(p_D) + (1 - p_B)(p_C)(1 - p_D) + (1 - p_B)(p_C)(p_D) + (p_B)(1 - p_C)(1 - p_D) + (p_B)(1 - p_C)(p_D) + (p_B)(p_C)(1 - p_D) + (p_B)(p_C)(p_D)$$

$$= 0.944$$

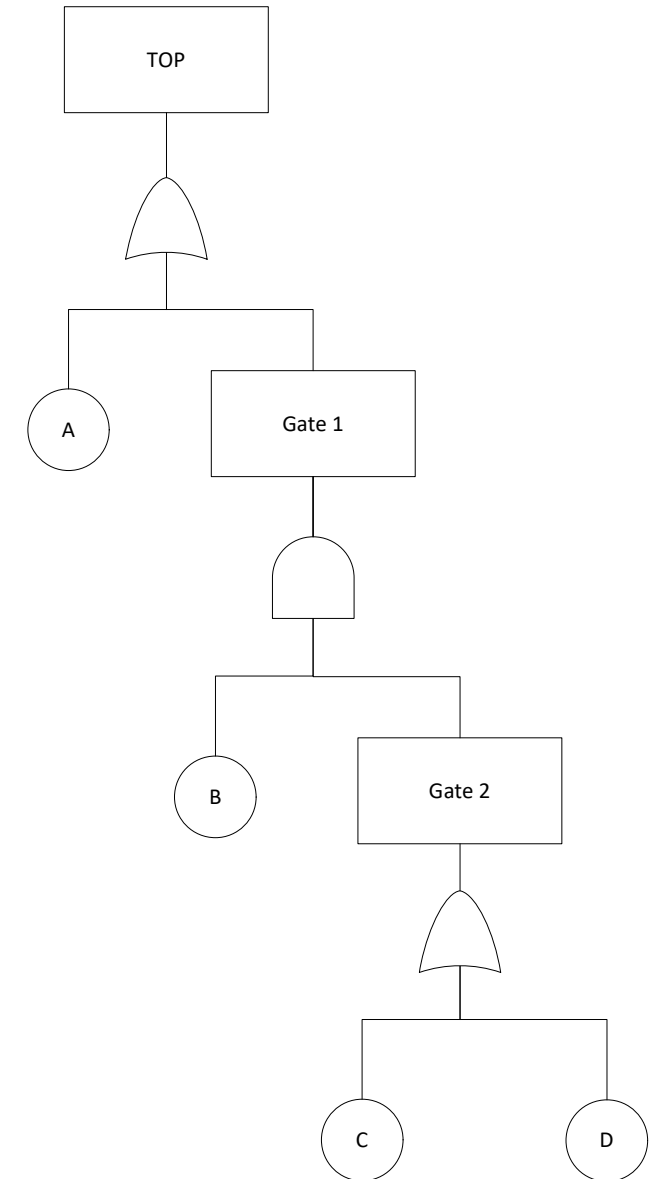


Reliability: Sub-item importance

Birnbaum measure of importance



Measure	Sub-items			
	A	B	C	D
Structural importance	0.625	0.375	0.125	0.125
Birnbaum importance	0.944	0.252	0.144	0.162



Reliability: Sub-item importance

Criticality measure of importance



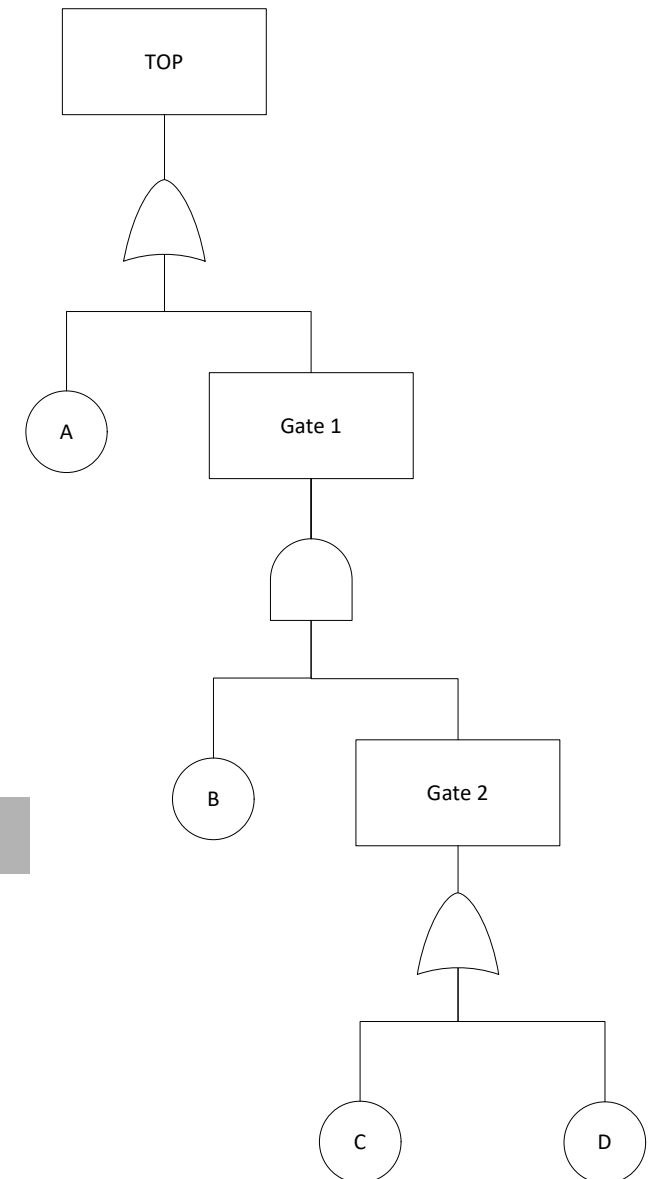
- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

Prob. of item being in a state where failure of sub-item would cause failure

Prob. of failure of sub-item

Prob. of failure of item



Reliability: Sub-item importance

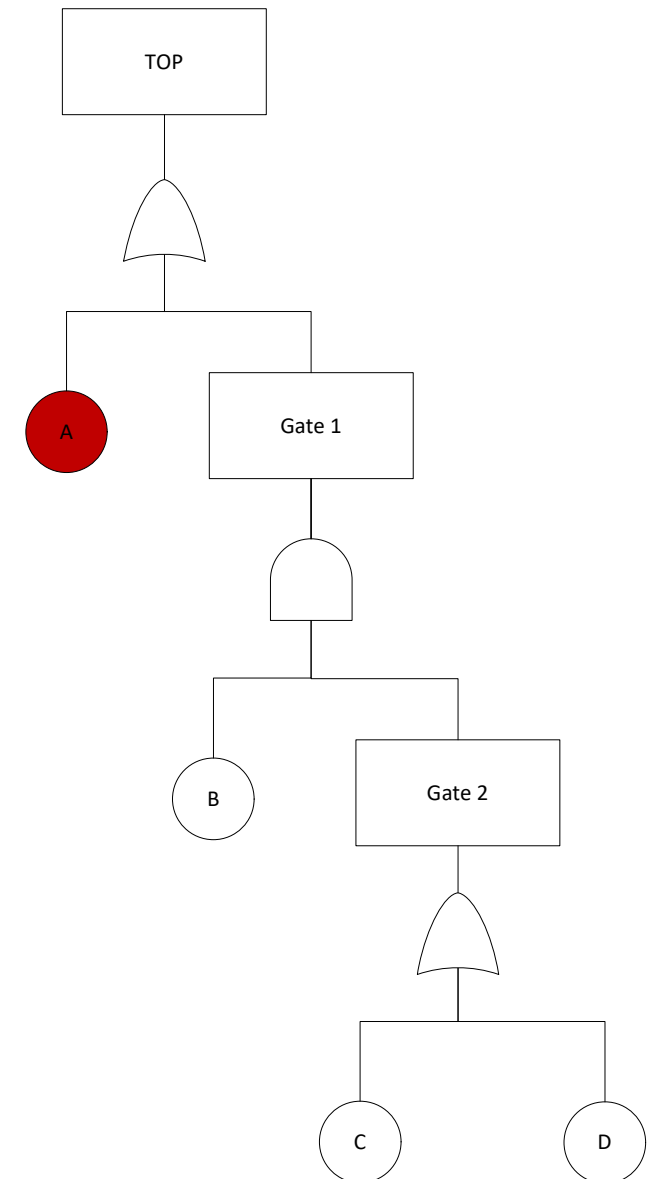
Criticality measure of importance



- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

- The minimal cut sets are



Reliability: Sub-item importance

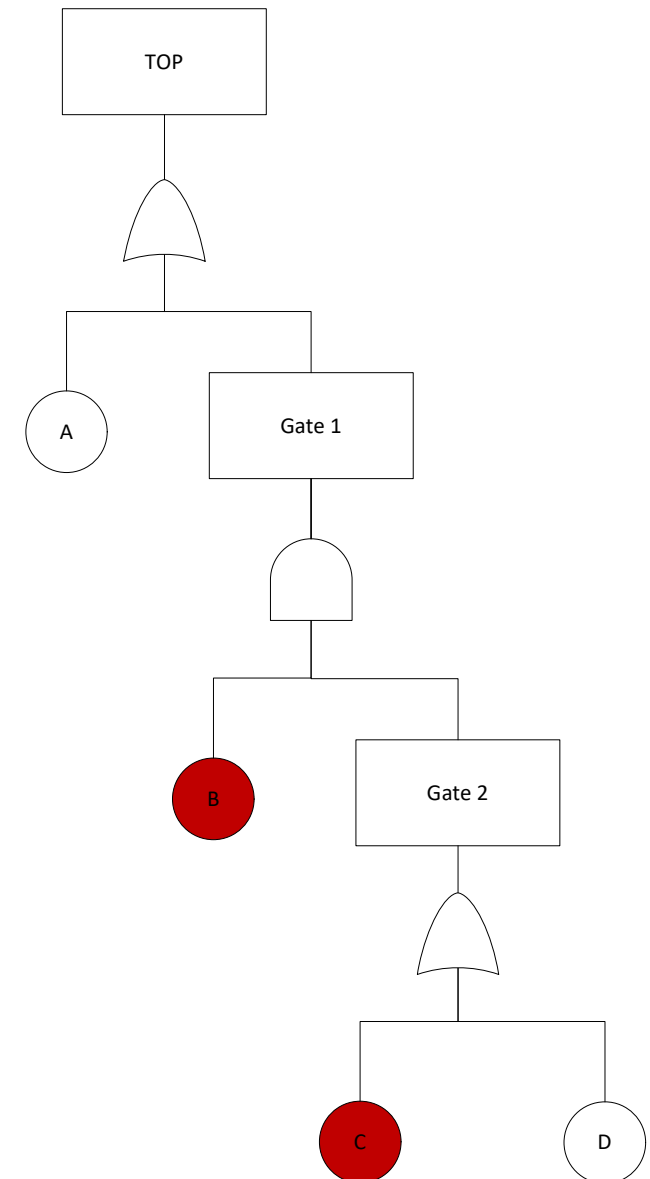
Criticality measure of importance



- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

- The minimal cut sets are



Reliability: Sub-item importance

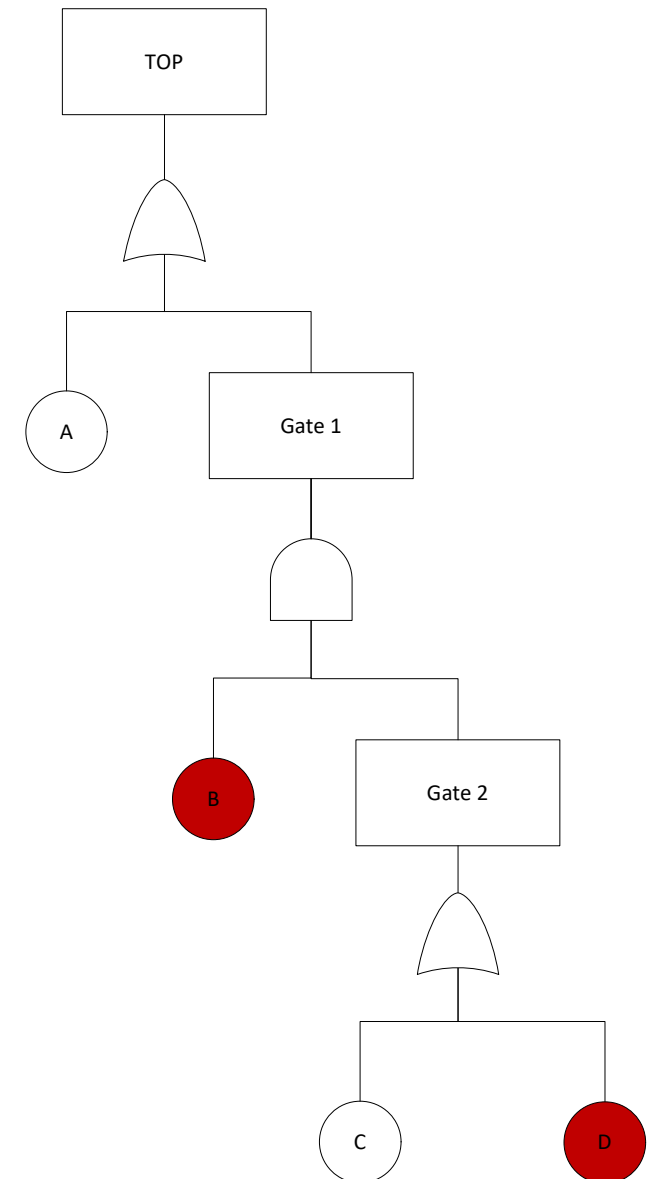
Criticality measure of importance



- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

- The minimal cut sets are



Reliability: Sub-item importance

Criticality measure of importance



- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

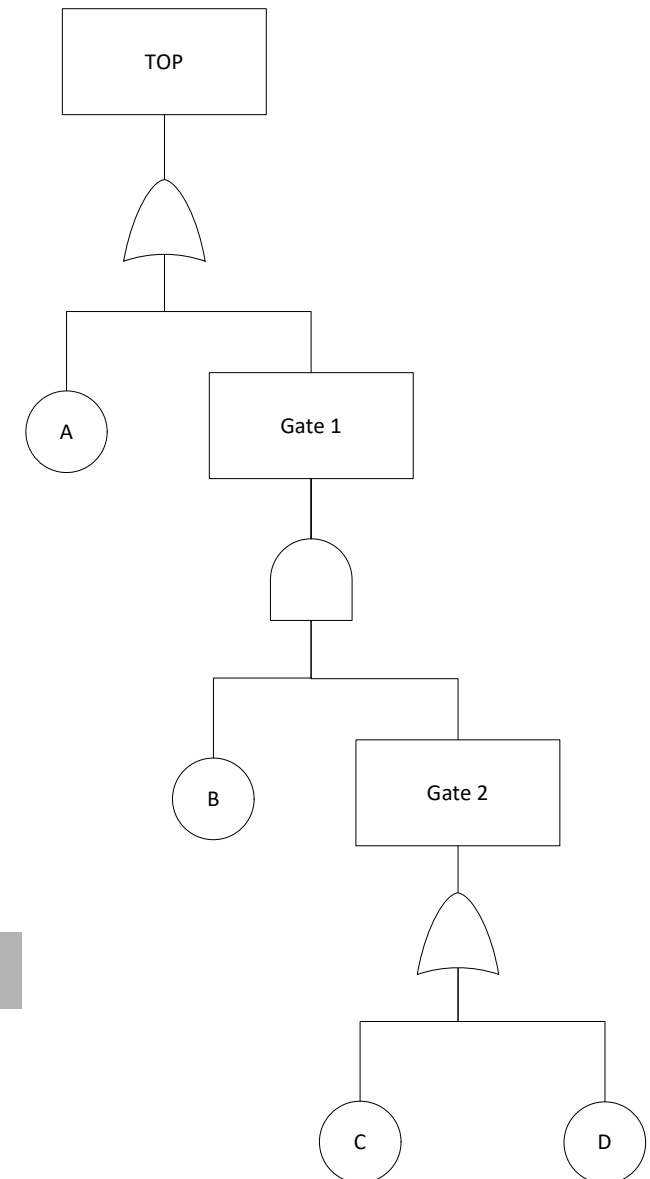
$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

- The minimal cut sets are

$$P_i = p_A + p_B q_C + p_B p_D - p_A p_B p_C - p_A p_B p_D - p_B p_C p_D + p_A p_B p_C p_D$$

$$P_i = 0.1 + 0.02 + 0.04 - 0.002 - 0.004 - 0.004 + 0.0004$$

$$P_i = 0.1504$$



Reliability: Sub-item importance

Criticality measure of importance



- the contribution of the sub-item to the probability of failure of the item due to the item being in a state in which the failure of the sub-item would cause failure of the item and the sub-item failing

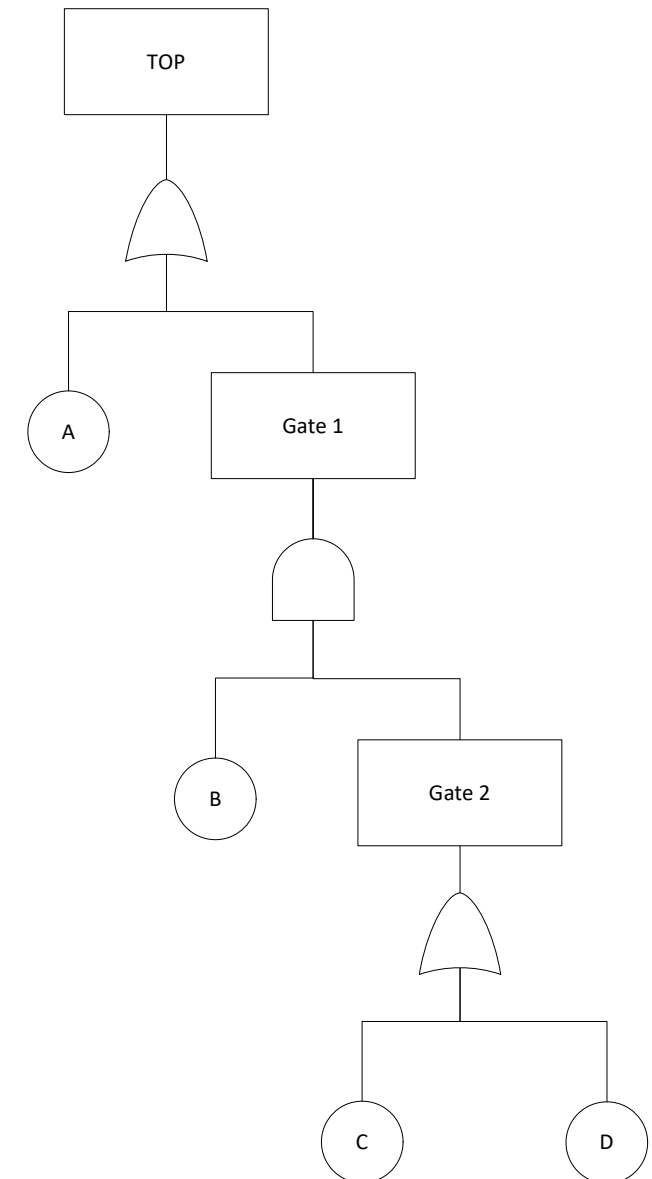
$$I_i^{CR}(t) = \frac{\sum_j^J p_j^{cs_i} \cdot (1 - R_i(t))}{1 - R(t)}$$

$$I_A^{CR}(t) = \frac{0.944 \cdot 0.1}{0.1504} = 0.6277$$

$$I_B^{CR}(t) = \frac{0.252 \cdot 0.2}{0.1504} = 0.3351$$

$$I_C^{CR}(t) = \frac{0.144 \cdot 0.1}{0.1504} = 0.0957$$

$$I_D^{CR}(t) = \frac{0.162 \cdot 0.2}{0.1504} = 0.2154$$





- the sum of the probabilities of each of the minimal cut sets with the sub-item occurring divided by probability of failure of the item

$$I_i^{FV}(t) = \frac{\sum_{j=1}^J F_i^j(t)}{F(t)}$$

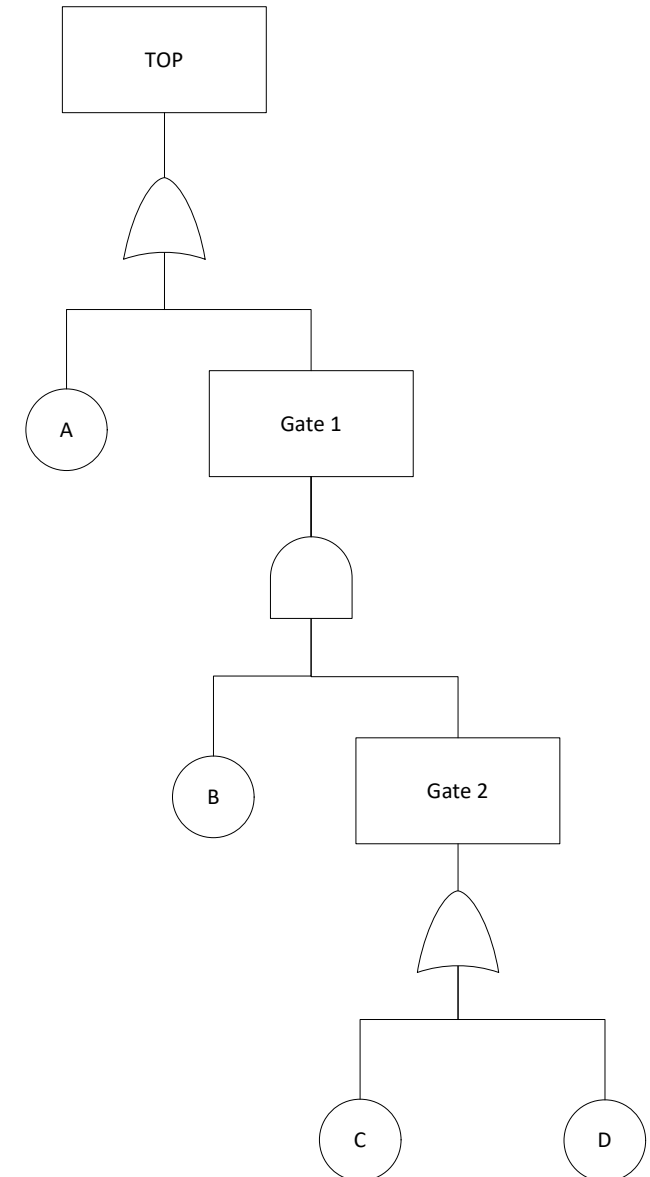
j = the minimal cut sets with the sub-item

$$I_A^{FV}(t) = \frac{p_A}{P} = \frac{0.1}{0.1504} = 0.6649$$

$$I_B^{FV}(t) = \frac{p_B \cdot (p_C + p_D - p_C \cdot p_D)}{P} = \frac{0.2 \cdot (0.1 + 0.2 - 0.002)}{0.1504} = 0.3723$$

$$I_C^{FV}(t) = \frac{p_C \cdot p_B}{P} = \frac{0.1 \cdot 0.2}{0.1504} = 0.1330$$

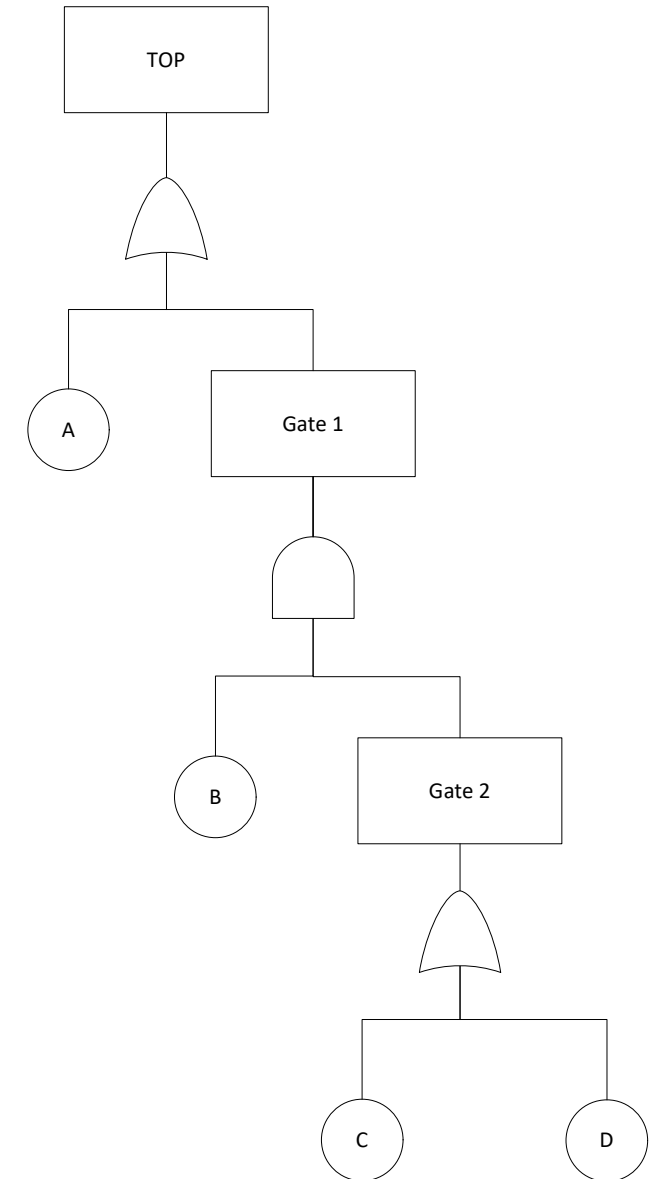
$$I_D^{FV}(t) = \frac{p_D \cdot p_B}{P} = \frac{0.02 \cdot 0.02}{0.1504} = 0.2660$$



Reliability: Sub-item importance Comparison



Measure	Sub-items			
	A	B	C	D
Structural importance	0.625	0.375	0.125	0.125
Birnbaum importance	0.944	0.252	0.144	0.162
Criticality importance	0.6277	0.3351	0.0957	0.2154
Fussel-Vessely importance	0.6649	0.3723	0.1330	0.2660





- When using reliability as a performance indicator
 - prioritization of interventions with respect to net benefit is rarely possible
 - for an item, there are multiple ways to estimate the reliability.
 - or a sub-item – don't forget that the LOS it provides depends on how it is integrated into the item as a whole and the reliability of the other sub-items in the item.

- Expectation of you (after this class, reading the script, the handouts, and doing the assignments):
 - to be able to estimate the reliability of interconnected sub-items

 - to be able to determine which objects within an interconnected sub-item should be repaired to maximize reliability