Optimal Long-Term Scheduling of Bridge Deck Replacement and Rehabilitation

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(Reviewed by the Highway Division)

Abstract: A mixed-integer mathematical model is presented to optimally schedule long-term bridge deck rehabilitation and replacement activities. This model is applicable to multiple bridge decks and is driven by overall cost considerations. Activities are scheduled based on the current condition and perceived future deterioration of each bridge deck. The model uses logical constraints to incorporate an "if-then-else" scenario into the rehabilitation model. These constraints directly relate the maintenance decisions in any year to the perceived condition of each individual bridge deck. Model structure and solution method were discussed, and an example using three highway bridges in North Carolina is presented. Model solutions indicate bridge deck replacement and rehabilitation activities throughout the planning horizon for each deck considered. Model results include the development of a trade-off relationship between long-term costs and the level of deterioration at which a bridge deck is replaced.

Introduction and Scope

The deterioration of the nation's infrastructure has become an enormous problem with repair-or-replace estimates reaching as high as $2 trillion (*Research Needs* 1984). Presently, the problem is so immense that even with significant increases in financial and technological support, the problem will exist for many years (Steinthal 1984).

Traditionally, the term *infrastructure* refers to constructed facilities such as buildings, airports, sewage systems, reservoirs, streets, roadways, and bridges, including all appurtenant facilities vital to the successful operation of these systems. One component of the infrastructure system that has received much publicity in recent years are this country's bridges. The maintenance and repair of these bridges are vital to the successful operation of the country's transportation systems.

Many states have bridges that are in poor condition, or that are so heavily used that they are difficult to maintain. In 1983, North Carolina reported having approximately 16,856 bridges of all types of which 5,316 were found to be structurally deficient and an additional 5,581 were found to be functionally obsolete (*North Carolina's* 1983). This means that approximately 65% of the state's bridges were in need of rehabilitation or replacement by federal standards. More recent figures provided by the North Carolina Department of Transportation indicate that as of August 1990, North Carolina had 16,832 state regulated bridges of which 8,202 were considered substandard. Scheduling the maintenance and rehabilitation activities for even a small subset of these bridges is a complex problem including many objectives and budget and material resource constraints.

A key bridge component that is subject to substantital cyclic loadings and harsh environmental conditions is the deck. The deterioration of bridge

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decks has been the focus of several papers within recent years (Cady and Weyers 1983, 1984; Bazant 1979). Cady and Weyers (1984) studied the deterioration rates of bridge decks and subject to de-icing chemicals. In this work, the times at which bridge deck rehabilitation and replacement programs should be implemented are determined as a function of average deterioration rates and the percent of deck deterioration at replacement. The time between the attainment of the chloride corrosion threshold concentration and the development of cracking in the concrete has been investigated by Bazant (1979) and is a vital component of Cady and Weyer's work. Although chemical reactions and reinforcement corrosion play an important role in the deterioration of decks, loading cycles and magnitudes also contribute to their deterioration.

A number of recent papers have focused on bridge, bridge deck, and pavement rehabilitation decision making (Cady 1985; Gunaratne et al. 1985; Carnahan 1987). A cost-effective decision model for the maintenance, rehabilitation, and replacement of bridges based on minimum life-cycle costs has been proposed by Weyers et al. (1984). This work uses standard engineering economic analysis to compare rehabilitation and replacement alternatives for individual bridges. A similar method for evaluating rehabilitation and replacement alternatives for individual bridge decks has also been proposed by Cady (1985). Another approach proposed by Manning and Ryell (1980) uses field studies and tests of the deck to develop a decision criteria for deck rehabilitation. Although each of the methods discussed here provide valuable information concerning rehabilitation and replacement of individual bridges and decks, no information is provided concerning the long-term scheduling of rehabilitation and replacement activities for many bridges under the jurisdiction of a single governing body.

The problem of scheduling and prioritizing rehabilitation and replacement activities has been considered and investigated by many. Fuzzy-set mathematics and dynamic programming have been applied to determine the priority of pavement sections for replacement (Gunaratne et al. 1985; Feighan et al. 1988). Tee et al. (1988) have applied fuzzy logic to assess the condition of concrete slab bridges. Mohan and Bushnak (1985) have applied multiattribute utility theory based on benefit-cost analysis and state sufficiency rating to prioritize pavement sections for rehabilitation. A mixed-integer linear programming model has been developed to schedule rehabilitation alternatives within a specified year for a system of dams based on perceived reliabilities and future deteriorations of each dam (Jacobs 1988).

Although literature covering bridge deck rehabilitation and replacement is abundant, no formalized approach has been developed to schedule deck rehabilitation and replacement activities over a long-term planning horizon. This paper presents a mathematical model to determine optimal long-term strategies for deck replacement and rehabilitation activities. The model is applicable to multiple decks in various states of deterioration.

Model Development

Consider the task for scheduling the rehabilitation or replacement of decks for many individual bridges over a period of several years. Replacement or rehabilitation decisions for each bridge in each year may be characterized by three actions: replace the deck, rehabilitate the deck, or do nothing. Formally, the decision to replace or rehabilitate a deck in any year can be represented by three binary (0–1) decision variables X_{jt}, Y_{jt}, and Z_{jt} where

$$X_{jt} = 1 \quad \text{if bridge deck } j \text{ is replaced in year } t \tag{1a}$$

$$X_{jt} = 0 \quad \text{otherwise} \tag{1b}$$

$$Y_{jt} = 1 \quad \text{if bridge deck } j \text{ is rehabilitated in year } t \tag{2a}$$

$$Y_{jt} = 0 \quad \text{otherwise} \tag{2b}$$

$$Z_{jt} = 1 \quad \text{if nothing is done to bridge deck } j \text{ in year } t \tag{3a}$$

$$Z_{jt} = 0 \quad \text{otherwise} \tag{3b}$$

Although the deterioration of bridge decks with time is a nonlinear function, it can be approximated by piecewise linear segments. Assuming an average rate of deterioration, the condition of the deck in any year can be described as a piecewise linear function. The deteriorated condition of each deck in any year can be defined as

$$D_{j,t+1} = D_{j,t} + \Delta D_{j,t} - R_j X_{jt} - B_j Y_{jt} + A_j Z_{jt} \qquad \forall\, j,t \tag{4a}$$

where D_{jt} = the deteriorated condition; and ΔD_{jt} represents the average deterioration rate of bridge deck j in year t. The terms R_j, B_j, and A_j represent the improvement from deck replacement, the improvement from deck rehabilitation, and the accelerated deterioration from "doing nothing" for any deck j, respectively. The values for R_j, B_j and A_j can be either estimated by the decision maker or derived from maintenance records of bridges of similar construction and subject to similar environmental conditions.

In some cases, it can be advantageous to rehabilitate or replace a bridge deck before its condition warrants such measures. This option can be incorporated into the model by defining the deteriorated condition of the deck as a lower bound. Using this idea, (4*a*) may be restated as the inequality:

$$D_{jt+1} \geq D_{jt} + \Delta D_{jt} - R_j X_{jt} - B Y_{jt} + A_j Z_{jt} \qquad \forall\, j,t \tag{4b}$$

Eq. (4*b*) allows deck replacement and rehabilitation to be scheduled early. This means that rehabilitation or replacement alternatives may be implemented before the deteriorated condition of the deck reaches the threshold levels for rehabilitation or replacement. In addition, by defining the deteriorated condition as a lower bound, the model guarantees that rehabilitation or replacement activities will not be scheduled late.

To guarantee that only one activity (replace, rehabilitate, do nothing) is scheduled for each bridge in any year and the maintenance decisions are based on the condition of the deck, five logical constraints are required. They may be stated as:

$$D_{jt} - MX_{jt} \leq L_R \qquad \forall\, j,t \tag{5}$$

$$D_{jt} + MY_{jt} + MZ_{jt} \geq L_R \qquad \forall\, j,t \tag{6}$$

$$D_{jt} - MX_{jt} - MY_{jt} \leq L_m \qquad \forall\, j,t \tag{7}$$

$$D_{jt} + MZ_{jt} \geq L_m \qquad \forall\, j,t \tag{8}$$

$$X_{jt} + Y_{jt} + Z_{jt} = 1 \qquad \forall\, j,t \tag{9}$$

where M = an arbitrarily large number; and L_R and L_m = the deterioration thresholds at which the deck is replaced or rehabilitated, respectively.

These five logical constraints construct and "if-then-else" scenario into the rehabilitation model. This provides the decision maker with the ability to simultaneously consider multiple rehabilitation alternatives. The deterioration thresholds for bridge deck replacement and rehabilitation represent the condition of the deck at which these maintenance activities become viable alternatives (Fig. 1).

The final model constraint limits the total amount of monetary resources used in any year to less than or equal to the available resources. Defining F_t as the total amount of funds available for deck replacement or rehabilitation in year t, this constraint can be mathematically stated as:

$$\sum_{j=1}^{n} \frac{1}{(1+i)^t} (C_{rjt}X_{jt} + C_{mjt}Y_{jt}) \leq F_t \qquad \forall\, t \quad \text{(10)}$$

In general terms, one possible objective is to make the best use of limited financial resources while adequately maintaining all the bridges within the systems. One possible way to address this objective is to minimize the total long-term cost of maintaining all the bridge decks being considered. This may be stated as:

$$\text{minimize cost} = \sum_{t=1}^{T} \sum_{j=1}^{n} \frac{1}{(1+i)^t} (C_{Rjt}X_{jt} + C_{mjt}Y_{jt}) \quad \text{(11)}$$

where C_{Rjt} and C_{mjt} represent the projected costs of deck replacement and maintenance, respectively. Defining T as the planning horizon and i as the discount rate, the objective function represents the total present worth of maintaining all n bridges throughout T. The complete model may now be presented in standard form, as in (11). Subject to (4b)–(10) and

$$X_{jt}, Y_{jt}Z_{jt}\varepsilon(0,1) \qquad \forall\, j,t \quad \text{(12)}$$

$$D_{jt} \geq 0 \qquad \forall\, j,t \quad \text{(13)}$$

The model developed here will optimally schedule deck replacement and rehabilitation activities throughout a system of several bridges at a minimum present-worth cost. To illustrate the model's performance, a case study is presented using three bridges of similar type and subject to similar conditions.

Case Study and Analysis

To illustrate the aforementioned replacement or rehabilitation scheduling model, an example was formulated and solved using three highway bridges

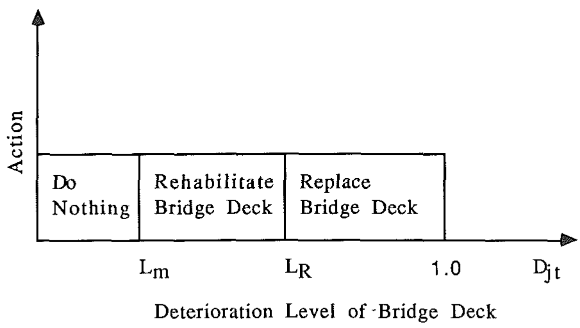


FIG. 1. Decision Criteria for Bridge Deck Replacement and Rehabilitation

in North Carolina. Each of the three bridges considered were of similar construction and subject to similar environmental and loading conditions. A planning horizon of 10 years and a discount rate of 5.25% per year were used throughout the study.

For this study, bridge deck deterioration was measured in terms of the fraction of deck surface area that has deteriorated. The average deterioration rate of each deck surface was assumed to be 0.021 per year (Cady and Weyers 1984). The threshold parameter for deck replacement L_R and rehabilitation L_m were assumed to be 0.35 and 0.08, respectively. Only bituminous patching was considered as a rehabilitation alternative for this example. In addition, this case study considers only the replacement or rehabilitation of each deck without regard to parameters such as serviceability requirements, user costs, and load capacity, which impact overall maintenance strategies. Table 1 presents the pertinent data for each deck considered in this model.

The replacement and rehabilitation costs were estimated based on the total surface area of each bridge deck and a nominal unit cost for completing the work. Bituminous patching was found to cost $1.23/sq. ft. (13.24/sq m) and deck replacement with epoxy-coated reinforcing bars was found to cost approximately $27.85/sq. ft. ($299.77/sq. m). The average patching cost for each deck was used throughout the model. A budget of $445,000 was assumed for the first year and appropriately discounted for each subsequent year at a rate of 5.25% per year.

For this example, R_j, B_j, and A_j were assumed to be 1.0, 0.014, and 0.009, respectively. These parameters represent the reduction or acceleration of the yearly deterioration of the bridge deck surface. Although these parameters were assumed for this example, in application these parameters would be estimated from past rehabilitation records. For example, B_j would represent the average reduction in the deck deterioration rate due to patching. This parameter could be determined by comparing the year-end deteriorated condition of two similar decks, where one has been maintained by patching and the other has not.

The (0–1) binary model formulation was solved using the XMP/ZOOM (Marsten 1987) mathematical programming package. The model was solved interactively on a UNIX-based Silicon Graphics Personal Iris workstation.

TABLE 1. Case-Study Bridge Deck Data

Bridge specification (1)	Bridge #62 (2)	Bridge #203 (3)	Bridge #284 (4)
Location	U.S. 64, Raleigh, (N.C.) Beltline	Melbourne Street U.S. 1, Raleigh (N.C.)	U.S. 1
Construction	Reinforced concrete slab/plate girders	Reinforced concrete slab/prestressed girders	Reinforced concrete slab/prestressed girders
Length	520 ft (158.5 m)	224 ft 6 in. (68.4 m)	307 ft 6 in. (93.7 m)
Bridge deck width	32 ft (9.8 m)	32 ft (9.8 m)	32 ft (9.8 m)
Age	28 years	30 years	28 years
Assumed initial condition of bridge deck	0.09	0.19	0.26

Fig. 2 presents the optimal deck replacement or rehabilitation schedule for the three bridges considered in this example. The model results completely specify deck replacement or rehabilitation activities for each bridge and each year throughout the planning horizon. These results represent the long-term (10-year) deck replacement or rehabilitation strategies for each of the bridges that result in a minimum total cost of $858,350.

A trade-off can be developed to illustrate the relationship between long-term costs and the level of deterioration at which a bridge deck is replaced. To develop this relationship, the replacement or rehabilitation model was solved for numerous replacement threshold values between 0.22 and 0.99. Fig. 3 presents the trade-off relationship between long-term cost and the deck replacement threshold.

The trade-off between long-term cost and the bridge deck replacement threshold clearly shows that the total long-term cost decreases as the replacement threshold increases. For threshold values greater than approximately 0.55, the long-term cost becomes constant. This illustrates that for deck replacement thresholds above 0.55 the total long-term cost remains constant and is independent of the replacement threshold. Figs. 4 and 5 present the long-term deck replacement or rehabilitation schedules for replacement thresholds of 0.45 and 0.22, respectively. These results clearly illustrate the variety of possible replacement or rehabilitation schedules depending on the deck replacement threshold. Similar trade-off relationships can be developed for long-term cost versus the deck rehabilitation threshold.

The model presented herein can be expanded to include more than three rehabilitation alternatives. Consider the example in which the decision maker may choose to replace, overlay, patch, or "do nothing" to each bridge deck. To include these four rehabilitation alternatives, one additional decision variable is defined as:

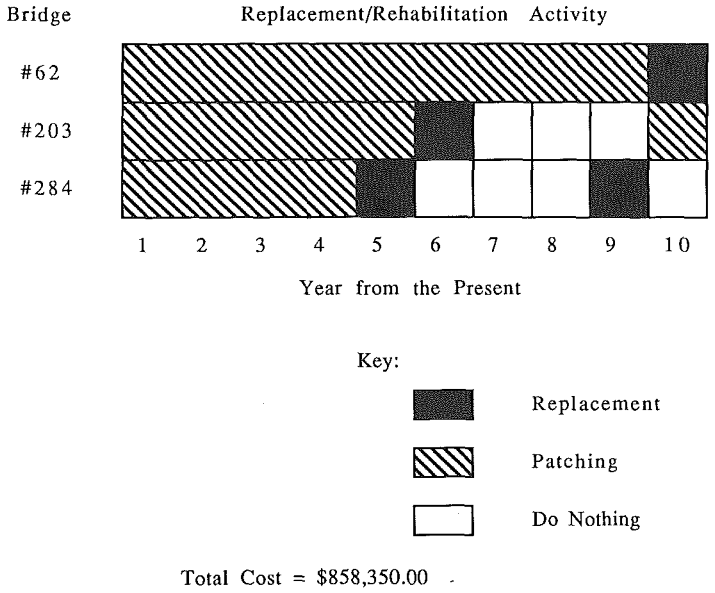


FIG. 2. Bridge Deck Replacement or Rehabilitation Schedule

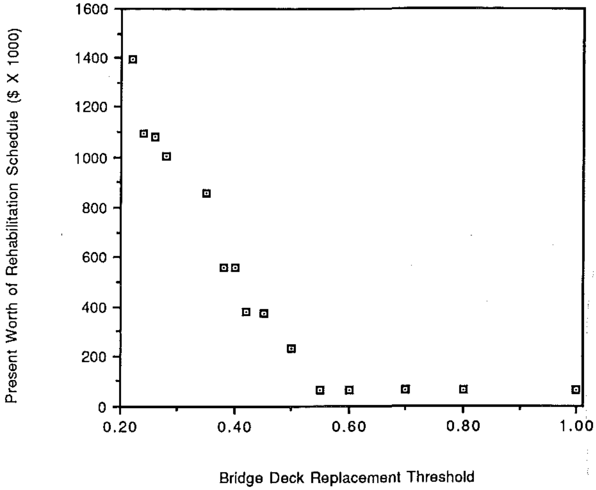


FIG. 3. Maintenance Cost versus Bridge Deck Replacement Threshold

$$V_{jt} = 1 \quad \text{if bridge deck } j \text{ is overlayed in year } t \tag{13a}$$

$$V_{jt} = 0 \quad \text{otherwise} \tag{13b}$$

Constraint equations (5)–(9) are then replaced by:

$$D_{jt} - MV_{jt} - MX_{jt} \leq L_l \quad \forall\, j,t \tag{14}$$

$$D_{jt} + MZ_{jt} + MY_{it} \geq L_l \quad \forall\, j,t \tag{15}$$

$$D_{jt} - MY_{jt} \leq L_R \quad \forall\, j,t \tag{16}$$

$$D_{jt} + MZ_{jt} \geq L_m \quad \forall\, j,t \tag{17}$$

$$D_{jt} + MV_{jt} + MZ_{jt} + MY_{jt} \geq L_R \quad \forall\, j,t \tag{18}$$

$$D_{jt} - MV_{jt} - MX_{jt} - MY_{jt} \leq L_m \quad \forall\, j,t \tag{19}$$

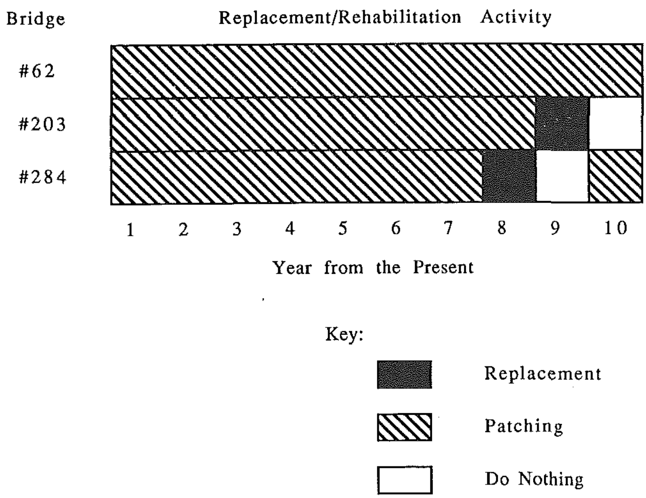
$$X_{jt} + V_{jt} + Y_{jt} + Z_{jt} = 1.0 \quad \forall\, j,t \tag{20}$$

where L_l = the deterioration threshold that delineates bituminous patching and overlaying the deck. All other variable definitions remain unchanged.

Due to the (0–1) binary nature of the decision variables used in this model formulation, the number of possible solutions grows exponentially with each additional decision variable, and optimal solutions are not obvious for relatively small problems (Garey and Johnson 1979). Such combinatorics can become computationally cumbersome and infeasible to apply in practice.

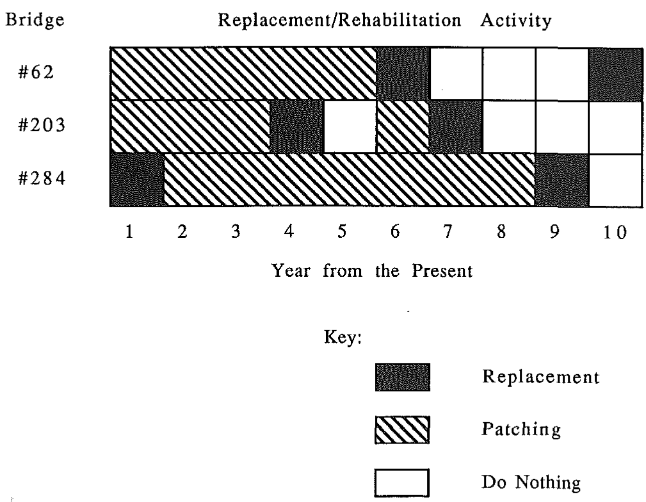
The maximum number of bridges that may be considered with this model is dependent on the number of rehabilitation alternatives considered for deck, the number of time increments within the planning horizon, and the available computer hardware and software. With this in mind, it is difficult to precisely determine the absolute maximum number of bridges that can be considered. However, by limiting the number of rehabilitation alternatives for each deck, the decision maker can restrain the problem size and implement the model for numerous bridges on a real-time basis.

Although the model formulation presented here assumed a piecewise linear deterioration function and described deterioration in terms of the



Total Cost = $368,978.00

FIG. 4. Bridge Deck Replacement or Rehabilitation Schedule for Replacement Thresholds of 0.45



Total Cost = $1,393,616.00

FIG. 5. Bridge Deck Replacement or Rehabilitation Schedule for Replacement Thresholds of 0.22

fraction of bridge deck surface area that is deteriorated, other functions and methods of describing the condition of the deck may be used. One alternative may be to represent the condition of the deck in terms of reliability or risk. However, modeling the deterioration of the decks in terms of the fraction of deteriorated deck surface provides the engineer with a method of tentatively scheduling long-term maintenance activities based solely on the results of field investigations.

Other extensions to this model include numerous alternative objective functions and added constraints to limit the frequency of bridge deck rehabilitation or replacement. The decision maker may wish to maximize the performance of a system of bridges or minimize the probability of unexpected bridge closings. In many cases the decision maker may with to consider more than one objective simultaneously. In this instance, the decison maker would determine numerous efficient solutions, the locus of which would define a set of noninferior solutions.

Another extension would be to incorporate rehabilitation costs that were proportional to the degree of rehabilitation. This extension would result in a nonlinear optimization problem and could be solved using established nonlinear programming techniques.

By adding constraints, the decision maker may limit the frequency with which rehabilitation or replacement activities can be scheduled. In some cases the decision maker may allow for patching to be scheduled every two years. An alternative would be to formulate the model based on two-year time increments. Many other extensions exist for implementation within the model.

The bridge deck replacement or rehabilitation model presented here can be used as an aid in the long-term scheduling of deck maintenance activities. By using this modeling technique, engineers involved in the scheduling of bridge maintenance activities can estimate long-term resource needs and develop tentative schedules. It is appropriate to emphasize that this proposed model can be used to quickly estimate long-term rehabilitation resource needs and that the model is easily adapted to information available to the decision maker. The tentative schedules are based on the current conditions of the bridge and the estimated future deterioration. Further study is needed to develop a better understanding of how bridge decks deteriorate with time.

Conclusion

A (0–1) binary decision model has been formulated to determine optimal long-term deck replacement or rehabilitation schedules for a system of independent bridges. The model is applicable to multiple decks and is driven by over-all long-term cost considerations. Maintenance activities are scheduled based on the current condition and future deterioration of each deck. To illustrate the model, a simple example is presented using three bridges in North Carolina and a piecewise linear deterioration function. Possible extensions using nonlinear deterioration functions and additional rehabilitation alternatives have been discussed. Results of this model illustrate that it can be used by engineers as an aid in the long-term scheduling of bridge maintenance activities.

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Appendix II. Notation

The following symbols are used in this paper:

A_j = acceleration in bridge deck deterioration of bridge deck j due to not implementing rehabilitation strategies;

B_j = reduction in bridge deck deterioration to bridge deck j due to rehabilitation;

C_{mjt} = estimated cost of rehabilitating bridge deck j in year t;
C_{Rjt} = estimated cost of replacing bridge deck j in year t;
D_{jt} = percent of bridge deck j that is deteriorated in year t;
F_t = total amount of funds available for bridge deck rehabilitation or replacement
i = discount rate throughout planning horizon;
j = bridge deck j;
L_l = deterioration threshold that delineates bituminous patching and overlaying bridge deck;
L_m = bridge deck rehabilitation threshold;
L_R = bridge deck replacement threshold;
M = arbitrarily large number used to maintain decision logic throughout planning horizon (equal to 10 for this formulation);
n = number of bridges considered in formulation;
R_j = reduction in bridge deck deterioration to bridge deck j due to bridge deck replacement (usually considered to be 1.0);
T = planning horizon;
t = year t;
V_{jt} = binary decision variable indicating overlaying of bridge deck j in year t;
X_{jt} = binary decision variable indicating replacement bridge deck in year t;
Y_{jt} = binary decision variable indicating rehabilitation of bridge deck j in year t;
Z_{jt} = binary decision variable indicating that no repairs will be implemented on bridge deck j in year t; and
ΔD_{jt} = average yearly deterioration of bridge deck j in year t.