Optimal allocation of resources in reduction of the seismic risk of highway networks

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The optimization of preventive upgrading interventions on the bridges of a highway network in an earthquake prone area is tackled. The bridges are assumed to be the vulnerable elements of the network, whose main purpose is to connect two sites. If the probability of collapse of each bridge under an earthquake of given intensity is known, the topology of the network allows us to calculate its reliability, i.e., the probability of maintaining the connection between the source node and the destination node. To increase the reliability, preventive upgrading interventions must be designed: the main purpose of this study is to present a procedure, based on dynamic programming, which is able to distribute the interventions among the bridges so that, for a given total amount of employed resources, the increase in reliability is maximized. The applicability of the procedure is demonstrated by some complete numerical examples. Alternative choices of the objective function and other further developments are also discussed.

Keywords: structural reliability, network reliability, lifelines, optimal design, bridges, dynamic programming

It is generally recognized that among the most damaging effects of earthquakes are the disruptions of communication networks and other lifeline systems: indeed, this type of damage can have direct and indirect consequences of utmost importance to the economy, as well as to the lives and the well-being, of the whole area affected by an earthquake. These consequences may not only be dramatic in emergencies, but may also last for months and years. Moreover, the great vulnerability of such systems in the case of moderate and strong events has been observed.

Therefore, it is essential that the reliability of lifeline networks under the expected seismic action be conveniently high: to this aim, the structures that are essential parts of such networks and often represent their most vulnerable elements (say, bridges for highways, pylons for electric lines) must also be highly reliable. This applies to both the design of new structures, and the planning of maintenance and repairs. Sometimes, objective exigencies or an increased public sensitivity can require appropriate interventions in order to meet a more satisfactory reliability level.

Over the last decade, a large theoretical and analytical effort has been devoted to the seismic risk assessment and design of lifeline systems, in order to ensure their continuous and safe operation during and after earthquakes. The main aspects of the problem that have been considered are: the use of network reliability methods for assessing lifelines' performance; the earthquake hazard mitigation; the emergency response planning; the experimental performance of the critical components and equipment of lifelines.

The main results of these studies have been discussed in

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a number of papers, presented in seminars and symposia specially dedicated to the subject: as a brief reference, attention should be paid to the proceedings of the three sessions of the 10th WCEE devoted to the seismic reliability assessment of lifelines[1] and to the proceedings of the 3rd US–Japan Workshop on Earthquake Disaster Prevention for Lifeline Systems[2]. Also the repair and retrofit procedures and the postevent service restoration, as well as the ordering and the efficiency of component strengthening has been dealt with[3], in order to upgrade the serviceability of an existing lifeline after an earthquake.

The rational planning of the interventions on the network (i.e., on its vulnerable elements), whether aimed at simple maintenance or at improving its reliability, requires a comparison of the expected consequences of the seismic event with or without interventions, i.e., an estimate of the increase in reliability due to the interventions, and the evaluation of the costs of the latter.

On the other hand, the occurrence time and intensity of earthquakes and other accidental actions, and the response of structures are all highly random: therefore, a rational strategy for planning the interventions and distributing the available resources in an optimal way, requires a number of choices under uncertainties[4].

In principle, the problem of optimal allocation of resources among the elements of a network is not much different from the problem of optimizing the allocation among existing buildings, whose solution has been illustrated and exemplified in several previous papers[5–7]. The basic conceptual difference is that in the case of buildings, the initial vulnerability, the consequences of failures, and the benefit derived from an intervention on any element of the ensemble can be assumed to be independent from each other. However, the study of the malfunction conditions (failures) of networks and of the preventive measures appropriate to reduce the relevant risks, requires a system approach: the consequences of the failure of a network element depend on the relevance of that element in the logical diagram of the network. Thus, the vulnerability of each structure and of the network as a whole must be evaluated and taken into account at the same time.

Environmental risk mitigation for lifeline systems therefore requires a multidisciplinary approach: in fact, it is necessary to identify the affected network with its topology and the connections between its elements, then recognize its functionality conditions, and finally identify the critical structures and their vulnerability (perhaps by means of fragility curves). Only at this point would it be possible to evaluate the reliability of the network as a whole, and to think of a strategy for its improvement: but for this it is necessary not only to estimate the costs and the benefits of possible preventive measures (in terms of their effects on the vulnerability of critical elements and of the whole system), but also to choose one or more objectives for the improvement.

The purpose of the present paper is to give a methodological contribution in this respect, and propose a general procedure for the optimal allocation of the resources available for improving the reliability of an existing network. For the sake of clarity and simplicity, the procedure will be formulated with reference to a specific case: namely, a number of bridges are assumed to be the critical elements of a highway network, whose configuration cannot be changed, and the problem is the planning of preventive upgrading interventions on the bridges in such a way that, under an earthquake of given (*design*) intensity, an appropriately defined reliability of the network is maximized for any given total amount of resources employed (a preliminary version of the procedure has been presented elsewhere[8,9]).

As usual in tackling such problems, most quantities and relationships will be assumed to be discontinuous: for instance, the earthquake intensity is measured according to a discrete scale (the Mercalli or an analogous scale), the probability of occurrence of each intensity being in turn known at most in the form of discrete (mass) functions, rather than continuous densities. Moreover, in order to formulate intervention strategies of sufficient generality, it is necessary to estimate how much each possible intervention costs, and how much it reduces the vulnerability of each structure, and, consequently, the expected damage: thus, it is convenient to model the interventions into a few types. It is evidently impossible to use differential optimization procedures: rather, the problem can be formulated as a multistage decisional process, and so dynamic programming becomes the best optimization tool[10].

It should be noted that the planning of maintenance inspections and repairs, which would follow a similar logic to the improvement measures, will not be explicitly dealt with in this paper.

General description of the problem

The relevant road system (*Figure 1(a), (b)*) is represented as a network, in general multiconnected (*Figure 1(c)*), and it is assumed that, when an earthquake occurs, some of the network nodes may fail, while the branches always remain able to connect to their terminal nodes. In other words, the critical elements of the network (indicated by squares in *Figure 1(c)*) are by definition all located in nodes: the possible vulnerability of a link is taken into account by introducing a critical element, so that the link is divided in two branches connected by a vulnerable node (in the example, *Figures 1(b)* and *(c)*, all nodes play this role). The network thus individuated may or may not coincide with a diagrammatic map of the highways in the relevant area.

It is also assumed that the main purpose of the network is to establish a connection between a *source* and a *destination* node, so that the network reliability is defined as the probability of maintaining such a connection under the prescribed conditions, and in particular when an earthquake hits the area.

Throughout the paper, the failures of the elements and of the connection established by the network are assumed to be yes-or-no events: in other words, a binary logic is followed and gradual or partial failures (like the reduction of load capacity on a bridge, or of traffic capacity in a road network) are not considered. Thus, the presented approach is particularly suitable for studying the connections that can be exploited in an emergency, e.g., taking into account links that are little used and may be neglected in normal conditions (in the concluding section of the paper, some possible alternatives and refinements will be discussed).

Then, the procedure for planning preventive interventions can be divided into three main stages

(i) Individuation of the relevant network and its logical diagram, and determination of its present reliability taking into account the probability of failure (fragility) of each critical element under the relevant

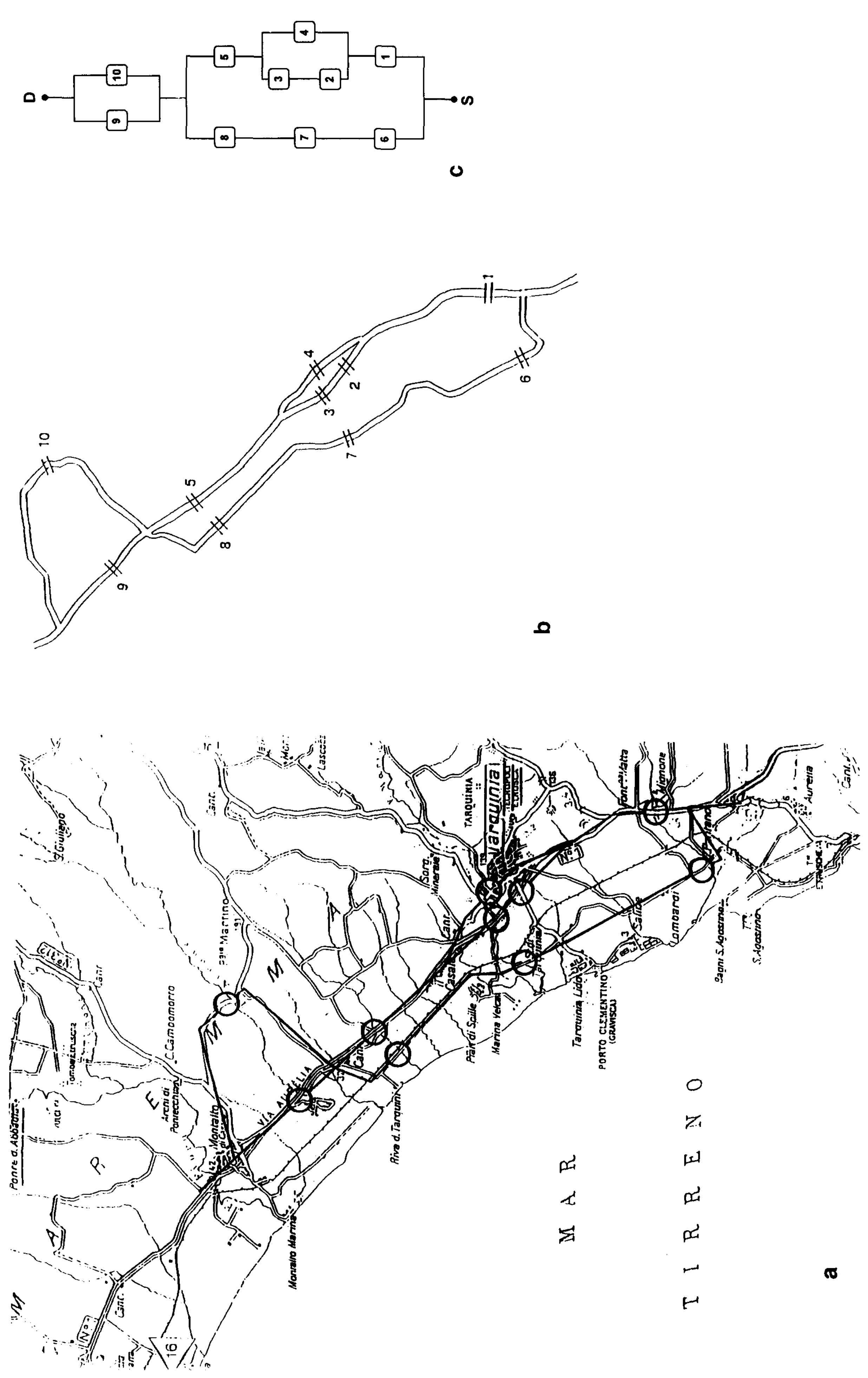


Figure 1 (a) Example map; (b) simplified road network; (c) corresponding logical network, with critical nodes indicated by squares

actions: if this reliability is unsatisfactory, the procedure is continued.

(ii) The design of structural interventions efficient in terms of cost/benefits, including the evaluation of their costs.

(iii) The allocation of the available resources between the several possible interventions on each element: the allocation is optimized when the largest possible increase of the network reliability is achieved.

Evaluation of the reliability of the network

Preliminary steps

In the first stage of the procedure, the road system to be considered must be delimited, and the links forming the logical network as defined above drawn. The possible part of the highway system outside the considered network is substituted by fictitious nodes[11].

The vulnerable (or critical) elements are next identified and located in the network. In a road network, these elements may be bridges, tunnels, trenches in unstable terrain, embankments, etc. (in the application developed later in this paper, the critical elements will only be identified with bridges).

The functional logic of the network can now be established. Since it has been assumed that the only purpose of the network is to maintain a connection between a source node, say S, and a destination node, say D, this logic is constituted by an ensemble of the minimal paths to collapse, obtained by inspection or an appropriate algorithm[12].

Calculation of the present reliability of the network

As already stated, in some nodes of the network there are concentrated critical elements that may fail when subjected to an earthquake (or other accidental load). According to previous definitions, the reliability of the network is equal to the probability that at least one of the paths connecting source and destination is not severed by the failure of one of the critical elements encountered on it.

Then, if it is also assumed that the failures of each element under the considered action are stochastically independent events, the logical structure of the network allows us to write the formula relating the conditional probability of failure $P_{sys}(a)$ of the network and the conditional probabilities of failure $P_i(a)$ of each ith element, subject to an earthquake of intensity a

$$P_{sys}(a) = F[P_i(a)] \tag{1}$$

where $F[\cdot]$ is a combination law depending on the topology of the network, of which some examples will be presented in the section on example networks.

If the fragility curve $P_i(a)$ of the ith element (i.e., its conditional probability of failure as a function of the earthquake intensity) is convoluted with the probability density (or mass) function $f(a)$ of the earthquake intensities in the area, the expected probability of failure $\boldsymbol{P}_i$ of that element is obtained

$$\boldsymbol{P}_i = \Sigma_a P_i(a) f(a) \tag{2}$$

where Σ_a can be either an integral or a sum over the range of intensities a's[4].

However, to obtain the expected probability of failure $\boldsymbol{P}_{sys}$ (or the seismic reliability: $1 - \boldsymbol{P}_{sys}$) of the network, one cannot introduce the values of $\boldsymbol{P}_i$ directly into the RHS of equation (1), because the relevant combination law $F[\cdot]$ is derived under the assumption that the failures are stochastically independent events. Instead, the curve of the conditional probability $P_{sys}(a)$ must be calculated by repeated applications of equation (1), and then convoluted with $f(a)$

$$\boldsymbol{P}_{sys} = \Sigma_a P_{sys}(a) f(a) \tag{3}$$

with the already defined meaning of Σ_a.

Design and optimization of structural interventions

If an element of the network is modified (e.g., by deterioration or retrofitting), its conditional probability of failure $P_i(a)$ changes: if this is the case even for only one element, the whole calculation of the system probability of failure $\boldsymbol{P}_{sys}$, equation (3), should be repeated. This makes it impossible to optimize the upgrading interventions with respect to $\boldsymbol{P}_{sys}$. Therefore, in the following, the procedure is formulated for optimization with respect to the conditional probabilities of failure under a given seismic intensity.

Although this approach may appear to severely limit the significance of the achieved optimization, it is in practice consistent with the usual 'engineering' procedure, which introduces, for each site, a well defined value of the intensity, usually prescribed by a Building Code. The prescribed given intensity might vary between the different elements of the network, if it extends over a large area: this does not introduce any conceptual difficulty, but only makes the formulae and calculations more cumbersome. Therefore, in the following description and in the illustrative examples, the intensity parameter a will be assumed to be equal for all elements; otherwise, different parameters should be introduced for different elements.

Interventions and their consequences

There is no general rule to be followed to design the upgrading interventions on a structure: the variety of possible means and devices is so large that good design can be obtained only in an empirical, heuristic way. Of course, in the most efficient design the smallest expenditures yield the largest increase of reliability under the relevant actions.

To formulate a rational strategy for the use of resources, a number of possible types k of interventions must be chosen, and their cost and gain (i.e., increase of reliability) calculated for each element of the network: in other words, the fragility of each element must be calculated, in the present and possibly upgraded conditions, and for each relevant value of the earthquake intensity.

Cost/benefit analysis

Let H_{ik} indicate the cost of intervention k on element i, and $P_{ik}(a)$ the conditional probability of failure (under an earthquake of intensity a) of the ith element after intervention k: for each element of the network and each value of a, it is thus possible to draw a gain plot, which will appear as a step function of the type shown in *Figure 2* (because to perform an intervention, a defined finite amount must be spent). Inspection of such a plot indicates immediately which interventions are possible if a sum C_i is available to be spent on the element (intervention 2 would be the most convenient in *Figure 2*), and also makes clear that a more expensive intervention is significant only if it yields a larger decrease in P_{ik}.

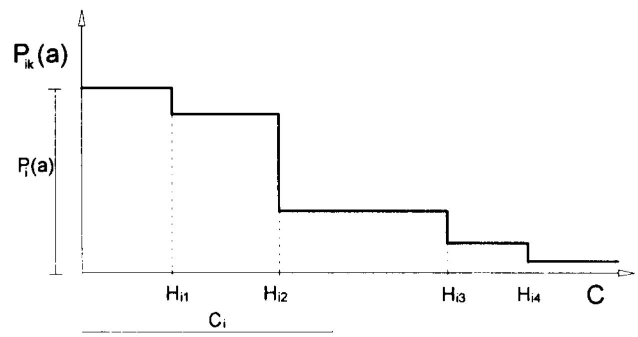


Figure 2 Conditional probability of failure of element *i* under earthquake of given intensity *a*, versus amount *C* spent in an upgrading intervention

The total cost of a set $\boldsymbol{K}$ of interventions is $\Sigma_i H_{ik}$; let $P_{sys}(a\boldsymbol{K}) = F[P_{ik}(a)]$ be the conditional probability of failure of the network, when the set $\boldsymbol{K}$ of interventions has been performed.

If C_{max} is the total amount of the resources available for upgrading the network elements, their optimal allocation (under an earthquake of intensity a) is achieved by the set $\boldsymbol{K}_{opt|a}$ of interventions which maximizes the decrease of the corresponding conditional failure probability of the network (i.e., the increase of reliability)

$$P_{sys}(a) - P_{sys}(a,\boldsymbol{K}) = F[P_i(a)] - F[P_{i\boldsymbol{K}}(a)] \qquad (4a)$$

subject to the constraint

$$\Sigma_i H_{ik} \leqslant C_{max} \qquad (4b)$$

Dynamic programming optimization procedure

The objective function of the optimization problem, equations (4), is a nonlinear function of discontinuous variables (possible interventions, or their effects). Therefore, the usual differential techniques cannot be employed to find the optimal solution, which would instead require an exhaustive search and a comparison between all possible sets of interventions.

Fortunately, an optimization problem in which all relevant quantities are discrete-value variables (and for this purpose also, the resources that can be allocated must be discretized into finite amounts) leads often to a multistage decisional process, in which each stage of decision-making is independent of the previous ones: such a process is optimal if, whatever the decisions taken at stage x, further decisions correspond to an optimal solution compatible with the state of the system after the decisions taken up to stage x (*optimality principle*[10]). The optimization algorithm is then furnished by *dynamic programming*, which involves comparatively few operations.

Augusti *et al.*[5–7] have applied this technique to examples in which the objective function was the sum of independent quantities, as many as the involved buildings (or building classes), and each was only a function of the resource assigned to the *i*th building: thus, it was immediately obvious that the optimization was a multistage decisional process.

The procedure is here extended to network problems, but in this case a systemic approach must be followed. In particular, the position and importance of each element in the logic of the network must be taken into account to define its influence on the objective function: for instance, in the case of a group of parallel elements (bundle), each element can be bypassed and the reliability of the bundle varied without acting on that element; on the contrary, the reliability of a number of elements in series (chain) depends on the reliability of each element of the chain.

To apply dynamic programming to the defined highway network, this must be divided into elementary meshes, in which the vulnerable elements appear in parallel and/or series, as illustrated in detail in Reference 9: simple examples will be presented later in the paper.

Conditional and unconditional optimization

Bearing in mind what was stated at the end of the section on the calculation of the present reliability of the network, the optimization is performed for a given seismic intensity: therefore, the procedure leads to a conditional optimization under the chosen intensity a. The choice of the relevant seismic intensity may derive from an analysis of the seismicity of the concerned area or from the prescriptions of a Seismic Building Code.

The optimal solutions $\boldsymbol{K}_{opt|a}$ for different intensities can be compared with each other, taking into account the seismicity of the relevant area (defined as the probability of occurrence of earthquakes of each intensity in a given time interval, e.g. one year).

To this end, take a set of interventions $\boldsymbol{K}_{opt|a_j}$ (conditionally optimized with reference to intensity a_j) and calculate the consequent increase in network reliability for several values of a, by repeated applications of equation (4a); convolution of these results with $f(a)$ (analogous to equation (3)) will yield the reduction of the expected probability of failure (i.e., the expected increase in reliability)

$$R(\boldsymbol{K}_{opt|a_j}) = \Sigma_a \{P_{sys}(a) - P_{sys}(a,\boldsymbol{K}_{opt|a_j})\} f(a) \qquad (5)$$

The values of $R(\boldsymbol{K}_{opt|a_j})$ obtained for each optimized set of interventions $\boldsymbol{K}_{opt|a_j}$ can be compared with each other in order to recognize the best among the sets $\boldsymbol{K}_{opt|a_j}$ (which however, in full rigour, does not necessarily coincide with the true optimal solution): thus, the set optimized with reference to the most frequent earthquakes may easily be the most convenient.

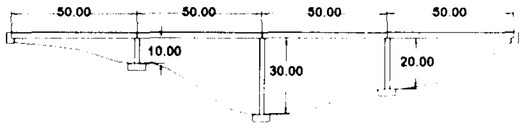
Examples of application

Examples will now be presented for the optimal allocation of resources among the bridges located on four simple road networks. Five types of bridges have been distributed on the networks; three values of the peak ground acceleration have been considered in particular for the design earthquake (namely $a = 0.15$, 0.25 and 0.35g, corresponding to low, medium and high seismicity zones, respectively).

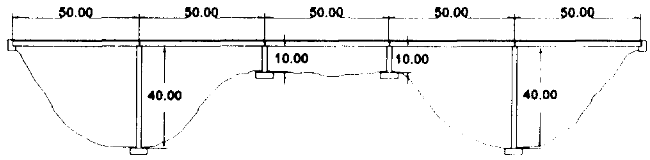
Bridges: fragility curves and interventions

The five bridge schemes considered in the examples consist of prestressed concrete box decks, simply supported on reinforced concrete box piers of constant section (*Figures 3* and *4*). This is by far the most common structural type of highway bridges and viaducts built in Italy in the 1970s and 1980s; it is worth noting that many of these bridges now require upgrading either because of deterioration or because the seismicity of their sites has been reevaluated. The five example bridges have been designed, in accord with the current Italian Regulations, for a horizontal ground acceleration of 0.10g.

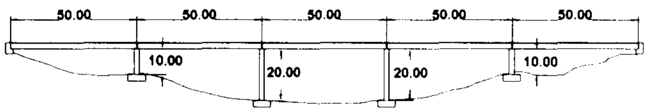
In the calculations, the bridges have been considered as originally designed, or upgraded in one of three ways, which follow two different techniques, namely[13]



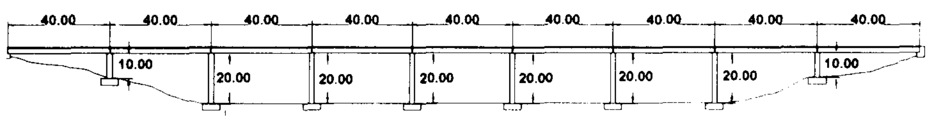
Bridge a
(piers type A)



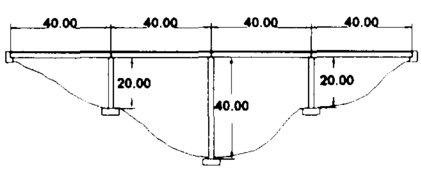
Bridge b
(piers type B)



Bridge c
(piers type A)



Bridge d
(piers type A)



Bridge e
(piers type B)

Figure 3 Structural diagrams of example bridges

- Jacketing of the piers with shotcrete cover and addition of longitudinal reinforcement to improve the pier flexural capacity and shear strength (the reinforcement is increased by 50% in intervention I; by 100% in intervention II)
- The introduction of isolation/dissipation devices on piers to replace the existing supports and elimination of expansion joints between the decks (intervention III)

The seismic fragility of the existing bridge and of the same bridge upgraded using either technique, is evaluated adopting two different models, described in detail in Reference 13: one involves piers carrying the mass of the deck span at their tops, the other is the continuous deck bridge.

To simulate the behaviour of existing bridges with simply-supported deck, single piers have been modelled as a one degree-of-freedom (DOF) oscillator characterized by a mass, a damping factor and the force–displacement relationship referred to the pier top. Plastic hinging of the piers is assumed to occur at the base only. The force–displacement relationship at the pier top is then obtained by combining the elastic response of the upper portion of the pier with an elastoplastic stiffness degrading moment-curvature (Takeda-type) relationship associated with a constant length of the plasticized zone. The mass corresponds to the weight of the deck (2000 or 1600 kN), and to a fraction of the weight actually distributed along the pier, equal approximately to 24.7% of the total weight. Viscous damping has been conventionally assumed to be 5% of critical, this is also the value underlying the spectrum used to generate artificial accelerograms. To account for cracking, the elastic portion of the pier has been attributed a stiffness equal to the uncracked value divided by a factor 2.5.

In the latter case, to simulate the behaviour of continuous deck bridges, the superstructure is introduced into the model through the corresponding stiffness matrix, whose terms govern the displacements of the pier tops in the longitudinal or transverse directions. Isolating and dissipating

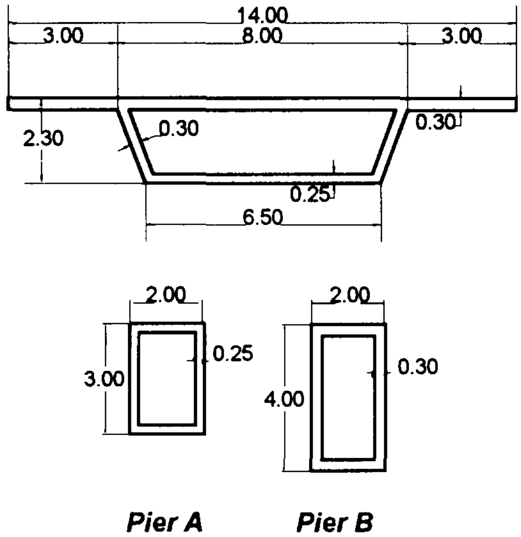


Figure 4 Cross-sections of bridge deck and piers

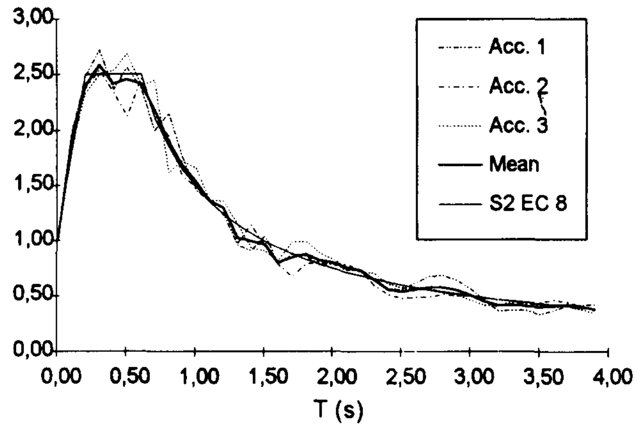


Figure 5 Comparison of reference and calculated spectra[13]

devices are inserted between each pier or abutment and the deck; the hysteretic behaviour of these devices is described by the well known Menegotto-Pinto (1973) model.

For simplification, it is assumed that the seismic response of the bridge can be analysed separately in the longitudinal and transverse directions, and that the fragility of the piers depends essentially on their response to the transverse actions.

The damage in the critical sections (the base sections of the piers) is defined by an indicator Δ, obtained as a weighted sum of the ratios between: the maximum attained curvature ϑ_{max} and the ultimate curvature ϑ_u of the plastic hinge; the energy dissipated and the limit plastic energy

$$\Delta = \frac{\vartheta_{max}}{\vartheta_u} + \lambda \frac{\int M d\vartheta}{M_y \vartheta_u} \qquad (6)$$

λ is an empirical parameter, here assumed to be a random variable, depending on the degree of confinement of the concrete core. Failure of the bridge is assumed to occur when the damage indicator Δ reaches the threshold value $\Delta = 0.4$ in the base section of only one pier (series system). The value $\Delta = 0.4$ has been considered as representative of a heavy level of damage, which compromises bridge operation and requires extensive repair work.

The analyses have been performed using a seismic input characterized by a definite value of the peak ground acceleration in the range 0.10–0.45 g, and by a frequency content corresponding to the amplification spectrum given in Eurocode 8 (1988) for intermediate soils. Three artificial accelerograms have been considered, scaled to the selected peak value, with a total duration of 27 s and a central stationary portion of 20 s. The reference spectrum is compared in *Figure 5* with the spectra of the artificial accelerograms and their average.

Then, bridge seismic fragility has been assessed by applying the Monte Carlo method, improved by means of directional simulation and importance sampling. For each evaluation of the probability of failure $P_i(a)$ at least 200, and at most 1000 simulated experiments for each accelerogram have been run. In any case the assessment was stopped if the coefficient of variation of $P_i(a)$ was less than 0.15.

The random variables assumed in the structural model are: the translational K_T and rotational K_φ stiffness of the soil springs; the elastic flexural rigidity EJ of the piers; the length L_p of the plastic hinge; the yielding moment M_y and the ultimate curvature ϑ_u of the plastic hinges; the empirical parameter λ; the elastic stiffness K_{is} and the yield displacement δ_{is} of the isolating devices. These last variables make it possible to take into account the uncertainty in the maximum force transmitted by the superstructure to the pier. After a series of trial calculations, the mean level of force assumed for the design of the isolation/dissipation devices has been set at 50% of the present pier yield strength.

All random variables have been assumed to be log-normally distributed: the mean values, the coefficients of variation and the correlation coefficients ρ_{xx} are shown in *Tables 1* and *2*.

The calculated fragility curves of the five bridges, in the original and upgraded conditions, are shown in *Figure 6* as functions of the peak ground acceleration a; *Table 3* reports in particular the conditional probabilities of failure for $a = 0.15$, 0.25 and $0.35g$. Since the bridges were designed for $a = 0.10g$, some of the calculated probabilities are very large, and even equal to 1 for all practical purposes.

The comparative construction costs of the five bridges have been evaluated: they are reported in *Table 4*, putting conventionally the cost of the most expensive bridge (bridge *d*) equal to 100 resource units (r.u.). The costs of the interventions have also been evaluated in the same units, and are also reported in *Table 4*.

Table 1 Mean values, coefficients of variation and correlation coefficients of relevant random variables (correlation coefficients ρ_{xx} are defined between the same r.v. in two different piers, or between two different r.v.'s – K_T, K_φ – in the same * or in two different piers**)

	Mean	COV	ρ_{xx}
K_T	12 000 MN/m	0.30	0.80
K_φ	520 000 Mn/m	0.30	0.80
E	3000 MPa	0.20	0.75
L_p	0.10 H	0.25	0.70
M_y	see *Table 2*	0.15	0.90
ϑ_u	see *Table 2*	0.20	0.75
λ	0.12	0.60	0.00
K_T–K_φ			0.80*–0.60**

Table 2 Mean values of M_y (MN·m) and ϑ_u (m^{-1}) for the present (0) and upgraded piers

	Pier A		
	$H = 10$ m	$H = 20$ m	$H = 30$ m
0	22.3/0.0267	22.9/0.0259	23.6/0.0251
I	30.3/0.0231	31.3/0.0223	32.3/0.0217
II	41.5/0.0205	42.2/0.0201	42.9/0.0197
	Pier B		
	$H = 10$ m	$H = 20$ m	$H = 40$ m
0	32.7/0.0230	34.0/0.0220	36.5/0.0207
I	44.8/0.0195	46.4/0.0188	49.9/0.0175
II	60.9/0.0176	63.0/0.0169	67.5/0.0157

Networks

Four simple networks, shown in *Figure 7*, have been studied. The combination laws, yielding the (conditional) probability of failure of each network, are presented below: recall that bridge failures under a given earthquake are assumed stochastically independent of each other. For simplicity's sake, the dependence of the conditional probabilities on the peak ground acceleration a is not explicitly indicated in the following formulae.

The first network, denoted network *SE*, is an elementary chain of elements in series, and may correspond to bridges located along an isolated highway stretch. It fails if any one of the bridges fail: therefore, its (conditional) probability of survival $1 - P_{SE}$ is equal to the product of the probabilities of survival of all elements, whence

$$P_{SE} = 1 - \Pi_i\{1 - P_i\} \tag{7a}$$

where P_i is the probability of failure of element i subjected to an earthquake of given peak ground acceleration, and Π_i indicates the product from $i = 1$ to $i = 5$.

The second network, denoted *PA*, is an elementary bundle of elements in parallel, and may represent the situation of a city cut by a river. The connection between the two banks fails if all bridges fail, whence

$$P_{PA} = \Pi_i P_i \tag{7b}$$

The third network, *SP*1, is a simple series-parallel mesh, which includes five bridges, as the previous networks: two paths connect source *S* and destination *D*, namely 1-2-3-5 and 1-4-5. The failure probability of the network can be obtained by considering a series chain 1-*m*-5, whose intermediate element *m* is the mesh formed by element 4, in parallel with 2 and 3, in turn in series with each other. Indicating the failure probability of mesh *m* by

$$P_m = P_4[1 - (1 - P_2)(1 - P_3)] \tag{7c}$$

the failure probability of network *SP*1, in analogy with equation (7*a*), is given by

$$P_{SP1} = 1 - [(1 - P_1)(1 - P_m)(1 - P_5)] \tag{7d}$$

The fourth network, which corresponds to the real road network of *Figure 1*, contains ten bridges, of five different types: six different paths connect *S* and *D*: namely [6-7-8-

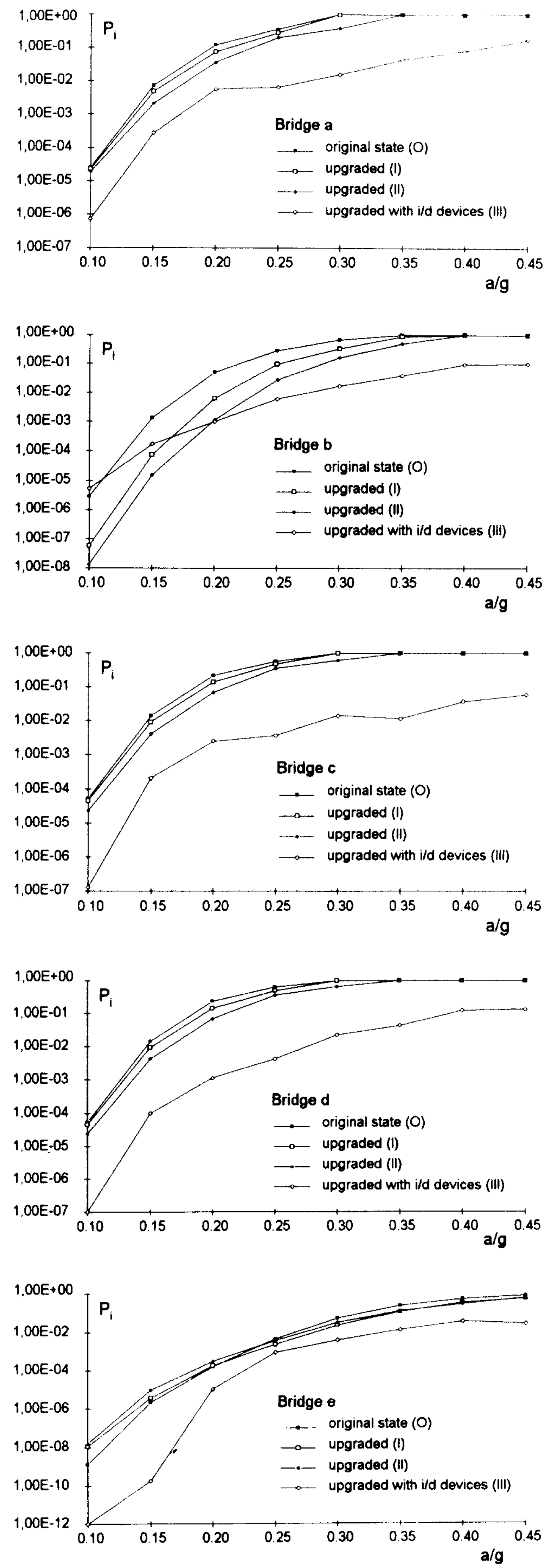


Figure 6 Fragility curves (conditional probabilities of failure) of example bridges in original state (O) and after upgrading intervention (I, II and III, respectively)[13]

Table 3 Conditional probabilities of failure ($\Delta = 0.4$) of original (0) and upgraded bridges (I, II and III, respectively)

Bridge scheme		$a/g = 0.15$	$a/g = 0.25$	$a/g = 0.35$
a				
	0	$7.05 \cdot 10^{-3}$	$3.15 \cdot 10^{-1}$	1.00
	I	$4.60 \cdot 10^{-3}$	$2.77 \cdot 10^{-1}$	1.00
	II	$2.09 \cdot 10^{-3}$	$1.94 \cdot 10^{-1}$	1.00
	III	$2.04 \cdot 10^{-4}$	$7.29 \cdot 10^{-3}$	$3.02 \cdot 10^{-2}$
b				
	0	$1.38 \cdot 10^{-3}$	$2.82 \cdot 10^{-1}$	1.00
	I	$7.68 \cdot 10^{-5}$	$9.62 \cdot 10^{-2}$	$8.72 \cdot 10^{-1}$
	II	$1.60 \cdot 10^{-5}$	$2.71 \cdot 10^{-2}$	$4.94 \cdot 10^{-1}$
	III	$8.86 \cdot 10^{-5}$	$2.33 \cdot 10^{-3}$	$1.54 \cdot 10^{-2}$
c				
	0	$1.38 \cdot 10^{-2}$	$5.60 \cdot 10^{-1}$	1.00
	I	$9.17 \cdot 10^{-3}$	$4.71 \cdot 10^{-1}$	1.00
	II	$4.18 \cdot 10^{-3}$	$3.49 \cdot 10^{-1}$	1.00
	III	$1.58 \cdot 10^{-4}$	$2.66 \cdot 10^{-3}$	$1.14 \cdot 10^{-2}$
d				
	0	$1.43 \cdot 10^{-2}$	$6.29 \cdot 10^{-1}$	1.00
	I	$9.32 \cdot 10^{-3}$	$4.96 \cdot 10^{-1}$	1.00
	II	$4.22 \cdot 10^{-3}$	$3.59 \cdot 10^{-1}$	1.00
	III	$1.39 \cdot 10^{-5}$	$3.10 \cdot 10^{-3}$	$2.50 \cdot 10^{-2}$
e				
	0	$2.09 \cdot 10^{-6}$	$4.43 \cdot 10^{-3}$	$2.42 \cdot 10^{-1}$
	I	$3.56 \cdot 10^{-6}$	$2.30 \cdot 10^{-3}$	$1.15 \cdot 10^{-1}$
	II	$9.69 \cdot 10^{-6}$	$3.69 \cdot 10^{-3}$	$1.22 \cdot 10^{-1}$
	III	$4.93 \cdot 10^{-10}$	$3.40 \cdot 10^{-4}$	$7.57 \cdot 10^{-3}$

Table 4 Construction and upgrading costs of example bridges (cost of bridge $d = 100$ r.u.)

Bridge scheme	Construction cost	Upgrading cost I	II	III
a	56	3	4	7
b	72	6	8	9
c	66	3	4	9
d	100	7	9	14
e	48	5	6	7

9], [6-7-8-10], [1-4-5-9], [1-4-5-10], [1-2-3-5-9] and [1-2-3-5-10].

Applying the same arguments that have led to equation (*7d*), it can be shown the reliability $(1 - P_{SP2})$ can be evaluated by means of the inclusion-exclusion principle[12].

Table 5 shows the failure probabilities of the four networks, in the original design conditions (*O*) and after interventions of the same type on all bridges.

Optimal design of interventions and discussion of results

The question is now which of the described interventions should be applied to which bridges, in order to maximize the increase in reliability of a network.

It has been described earlier in this paper how this question can be answered by dynamic programming, assuming an earthquake and any given amount of available resources. This has been done for the four example networks, introducing the element failure probabilities given by *Table 3*, the costs given by *Table 4*, and the combination laws of equation (7), or applying the inclusion-exclusion principle: the conditional failure probabilities of the networks are plotted in *Figures 8–11* versus the amount of resources. It is also interesting to see how dynamic programming automatically suggests how and where to intervene, and some-

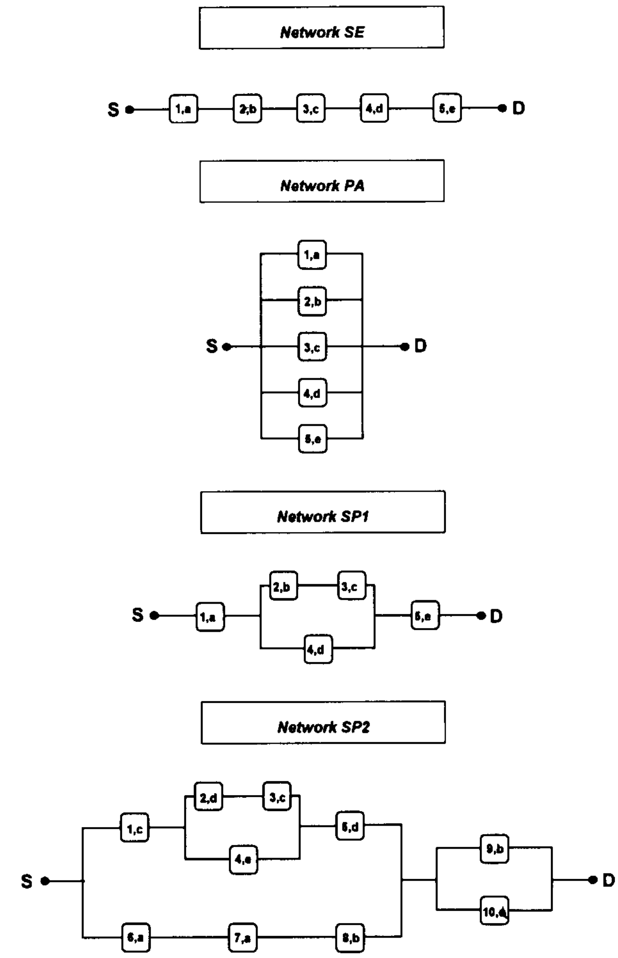


Figure 7 Example networks (nodes with critical elements are labelled by a number, while the bridge scheme is indicated by a, b, c, d or e)

Table 5 Conditional probabilities of network failure in the original state (0) and after the same type of interventions (I-II-III) on all bridges

Network		$a/g = 0.15$	$a/g = 0.25$	$a/g = 0.35$
SE				
	0	$3.63 \cdot 10^{-2}$	$9.20 \cdot 10^{-1}$	1.00
	I	$2.33 \cdot 10^{-2}$	$8.26 \cdot 10^{-1}$	1.00
	II	$1.14 \cdot 10^{-2}$	$6.74 \cdot 10^{-1}$	1.00
	III	$3.90 \cdot 10^{-4}$	$1.56 \cdot 10^{-2}$	$8.66 \cdot 10^{-2}$
PA				
	0	$\cong 0$	$1.39 \cdot 10^{-4}$	$2.42 \cdot 10^{-1}$
	I	$\cong 0$	$1.43 \cdot 10^{-5}$	$1.00 \cdot 10^{-1}$
	II	$\cong 0$	$2.43 \cdot 10^{-6}$	$6.03 \cdot 10^{-1}$
	III	$\cong 0$	$\cong 0$	$1.00 \cdot 10^{-9}$
SP1				
	0	$7.47 \cdot 10^{-3}$	$6.32 \cdot 10^{-1}$	1.00
	I	$5.04 \cdot 10^{-3}$	$4.65 \cdot 10^{-1}$	1.00
	II	$3.07 \cdot 10^{-3}$	$3.03 \cdot 10^{-1}$	1.00
	III	$2.09 \cdot 10^{-4}$	$7.64 \cdot 10^{-3}$	$3.82 \cdot 10^{-2}$
SP2				
	0	$4.28 \cdot 10^{-4}$	$5.85 \cdot 10^{-1}$	1.00
	I	$1.71 \cdot 10^{-4}$	$3.87 \cdot 10^{-1}$	1.00
	II	$3.45 \cdot 10^{-5}$	$2.15 \cdot 10^{-1}$	1.00
	III	$1.10 \cdot 10^{-6}$	$1.10 \cdot 10^{-3}$	$2.81 \cdot 10^{-3}$

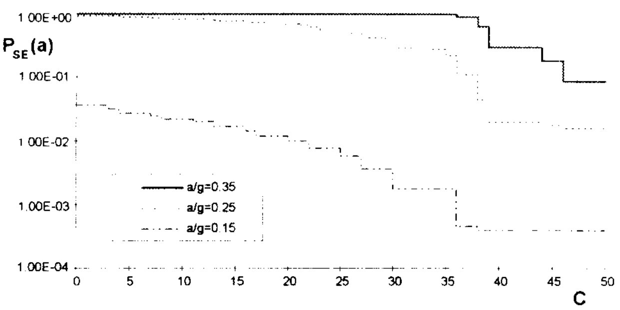


Figure 8 Probability of failure of network *SE* versus employed resources: peak ground acceleration $a = 0.15$, 0.25 and $0.35g$

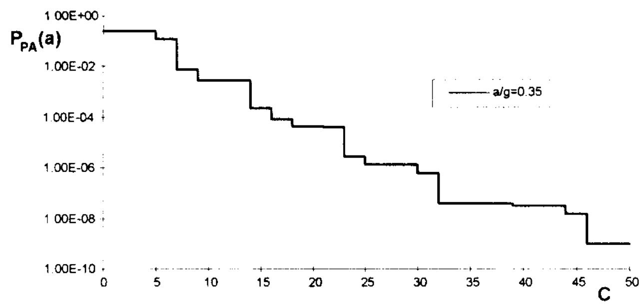


Figure 9 Probability of failure of network *PA* versus employed resources: peak ground acceleration $a = 0.35g$

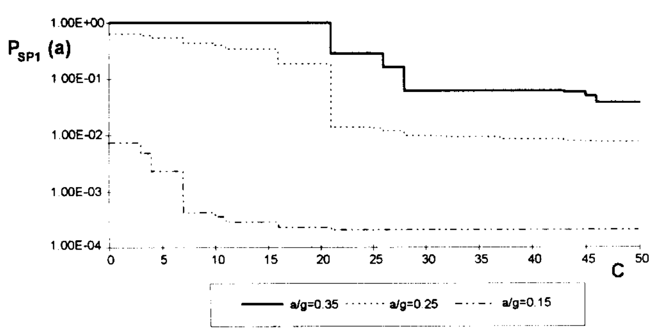


Figure 10 Probability of failure of network *SP1* versus employed resources: peak ground acceleration $a = 0.15$, 0.25 and $0.35g$

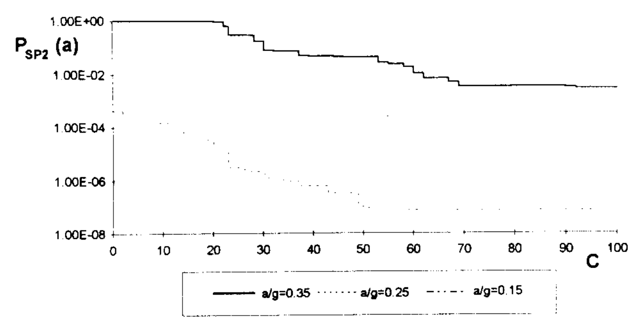


Figure 11 Probability of failure of network *SP2* versus employed resources: peak ground acceleration $a = 0.15$, 0.25 and $0.35g$

times drastically changes the distribution of the interventions when the amount of the resources varies, as shown by *Tables 6–9*.

Some of the results obtained are discussed below. To fully obtain their meaning, it must be recalled that the bridges had been originally designed for a peak ground acceleration $a = 0.10g$.

Figure 8 shows the variation of the failure probability P_{SE} of network *SE* subject to earthquakes of three intensities. It can be remarked that for $a = 0.15g$ and $a = 0.25g$, P_{SE} decreases in a rather regular way with increase of available resources, up to the 46 resource units needed to apply intervention III to all five bridges. The most critical element is bridge 3 (*Table 6*), to which interventions of type I or II are applied as soon as the resources are sufficient; with further increase of resources, bridges 1 and 4 are upgraded, while interventions on bridges 2 and 5 are performed only when intervention III has been applied to the other three bridges. But, if $a = 0.35g$, *Table 3* indicates that, in the original design condition, bridges 1, 2, 3 and 4 fail with probability 1: the network failure probability can be reduced only if the available resources are sufficient to intervene on them all (i.e., from $C = 36$ onwards).

Network *PA* has been studied only for $a = 0.35g$, because its reliability under weaker earthquakes is comparatively very large (*Figure 9, Table 7*). A gradual decrease of P_{PA} is shown, and it can be noted that at $C = 23$ P_{PA} is already reduced to a small fraction of the original value: it might be inferred that perhaps it is not worth devoting more than 23 resource units to the preventive upgrading of this network (and the argument would be even more valid for 32 units, which corresponds to another big decrease).

Figure 10 and *Table 8* refer to network *SP*1. If $a = 0.15g$, seven resource units appear to be enough to decrease the failure probability P_{SP1} of this network to a sufficiently small value (with intervention III on bridge 1), while further resources do not increase the reliability significantly. An analogous situation occurs at $C = 21$ when $a = 0.25g$ (intervention III on bridges 1 and 4). When $a = 0.35g$, P_{SP1} remains equal to 1 unless the resources are sufficient for intervention III on bridges 1 and 4, i.e., up to $C = 21$; P_{SP1} decreases further at $C = 28$, then remains practically constant for larger amounts of resources. The three plots agree in suggesting that it is not worth devoting more than 28 resource units to reduce the seismic risk of this network. It can also be noted that, under weak earthquakes it is convenient to operate on the reliability of the path [1-2-3-5], and on the path [1-4-5] for stronger ones.

Finally, the optimal allocation of resources in network *SP*2 is described by *Figure 11* and *Table 9*. Analogous indications on the efficiency of the resources employed can be derived from the plots of the failure probability P_{SP2}: in particular, the three plots indicate again that it is not worth allocating to this network an amount of resources above a certain limit ($C = 68$ r.u. in the specific case; note that interventions III on all ten bridges would correspond to $C = 92$ r.u.).

The advantage of the optimal allocation is very clear if one compares the increases in network reliability derived from *Table 5* with those of *Figure 11*. For instance, for $a = 0.15g$, if interventions I are applied on all bridges (thus employing 48 r.u.), the probability of failure is reduced from 4.3×10^{-4} to 1.7×10^{-4}; if the same resources are employed with optimal distribution, the failure probability becomes 3.2×10^{-7}; if interventions II are applied on all bridges (employing 62 r.u.), the failure probability becomes 3.5×10^{-5}, while 62 r.u., optimized, bring the failure probability down to 1.1×10^{-6}. The analogous figures for $a = 0.25g$, are, respectively: 5.9×10^{-1} (original probability);

3.9×10^{-1} (interventions I); 1.4×10^{-3} (48 r.u. optimized); 2.1×10^{-1} (interventions II); 1.1×10^{-4} (62 r.u. optimized). When $a = 0.35g$, and the original probability of failure is 1, interventions I and II do not decrease the network failure probability, while 48 and 62 r.u. distributed optimally decrease it, respectively, to 4.4×10^{-2} and 6.9×10^{-3}.

Inspection of *Table 9* is also of interest. For either $a = 0.25g$ and $a = 0.35g$, it can be noted that when the available amount is rather small, the interventions aim at increasing the reliability of the branch [6-7-8]; for the next higher amounts, interventions focus on branch [1-4-5]; and for even greater amounts, both branches are taken care of. With regard to the final mesh (in which bridges 9 and 10 are in parallel), bridge 10 is upgraded for the smallest amounts, bridge 9 for the next higher amounts, then both bridges.

Conclusions

In this paper, the failure of a network has been identified with loss of connection between source and destination; and optimization has been aimed only at reducing the probability of such failure.

In order to formulate the problem in this binary logic, the condition of failure of each critical element of the network must also be sharply and unequivocally defined. In the examples (in which the critical elements were bridges) it was assumed that they failed when a well-defined damage indicator became 0.40.

It has been thus possible to develop fully realistic examples, including the structural analysis and the design of the upgrading interventions, the evaluation of costs and benefits, etc. The results obtained can be looked at from at least four alternative points of view: firstly, assuming the use of a well-defined amount of resources, the corresponding column in *Tables 6–9* gives the distribution of interventions which maximizes the system reliability. Secondly, the same Tables (and *Figures 8–11*) also yield the minimum amount of resources necessary to bring the system reliability up to an assigned level. Thirdly, for given intervention type(s), an analogous treatment can suggest their optimal distribution with respect to minimization of

Table 6 Optimized interventions on each bridge (1-5) of network *SE* vs. employed resources (3-46 r.u.) for $a = 0.15g$, $a = 0.25g$ and $a = 0.35g$

	$a = 0.15g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	–	–	II	III	–	II	III	II	II	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	I	III	III	II	III
3	I	II	II	II	II	II	II	II	III	III	III	III	III	III	III	III
4	–	–	–	–	II	II	II	III	III	III	III	III	III	III	III	III
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	III	III
	$a = 0.25g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	–	–	–	I	–	–	I	–	II	III	II	III	III	III	III	III
2	–	–	–	–	I	–	–	–	–	–	I	I	III	III	III	III
3	I	II	III	III	III	II	II	III	III	III	III	III	III	III	III	III
4	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	I	III
	$a = 0.35g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	–	–	–	–	–	–	–	–	–	–	–	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	I	III	III	III	III
3	–	–	–	–	–	–	–	–	–	–	–	III	III	III	III	III
4	–	–	–	–	–	–	–	–	–	–	–	III	III	III	III	III
5	–	–	–	–	–	–	–	–	–	–	–	–	–	–	I	III

Table 7 Optimized interventions on each bridge (1-5) of network PA vs. employed resources (3-46 r.u.) for $a = 0.35g$

	$a = 0.35g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	–	–	–	I	III	–	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	III	–	–	III	III	III	III	III	III	III	III
3	–	–	III	III	–	III	III	III	III	III	III	III	III	III	III	III
4	–	–	–	–	–	–	–	–	–	–	–	–	III	III	III	III
5	–	I	–	–	III	–	I	III	–	I	III	III	–	–	I	III

Table 8 Optimized interventions on each bridge (1-5) of network *SP*1 vs. employed resources (3-46 r.u.) for $a=0.15g$, $a=0.25g$ and $a=0.35g$

	$a=0.15g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	I	II	III	III	III	III	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	II	III
3	–	–	–	I	–	I	–	III	III	–	I	I	III	III	III	III
4	–	–	–	–	–	–	I	–	–	III	III	III	III	III	III	III
5	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III
	$a=0.25g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	I	II	III	III	II	III	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	II	III
3	–	–	–	II	III	III	–	I	–	–	II	II	III	III	III	III
4	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III
5	–	–	–	–	–	–	–	–	I	III	III	III	III	III	III	III
	$a=0.35g$															
	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	46
1	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	II	III
3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	III	III
4	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III
5	–	–	–	–	–	–	–	–	I	III	III	III	III	III	III	III

Table 9 Optimized interventions on each bridge (1-10) of network *SP*2 vs. employed resources (5-92 r.u.) for $a=0.25g$ and $a=0.35g$

	$a=0.25g$																		
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	92
1	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	III	III	III
3	–	–	–	–	–	–	–	–	–	–	–	–	II	–	I	III	III	III	III
4	–	–	–	–	–	III	III	–	–	–	–	II	III	III	III	III	–	II	III
5	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III	III	III	III
6	–	II	III	III	–	–	–	I	III	III	III	III	III	III	III	III	III	III	III
7	I	III	III	III	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III
8	–	–	–	II	–	–	–	II	I	I	III	III	III	III	III	III	III	III	III
9	–	–	–	–	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III
10	–	–	–	–	–	–	II	–	–	II	–	–	–	III	III	III	III	III	III
	$a=0.35g$																		
	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	92
1	–	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	III	III
3	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	III	III
4	–	–	–	–	–	–	–	III	III	III	–	III	III	III	III	III	III	II	III
5	–	–	–	–	–	–	–	III	III	III	III	III	III	III	III	III	III	III	III
6	–	–	–	III	III	III	III	–	–	–	III	III	III	III	III	III	III	III	III
7	–	–	–	III	III	III	III	–	–	–	III	III	III	III	III	III	III	III	III
8	–	–	–	II	III	III	III	–	–	–	III	III	III	III	III	III	III	III	III
9	–	–	–	–	–	–	III	III	III	III	III	–	III	III	III	III	III	III	III
10	–	–	–	–	–	III	–	–	II	III	–	III	–	III	III	III	III	III	III

employed resources or maximization of system reliability. And finally, inspection of the plot of the optimized reduction of failure probability versus the total amount of resources employed, allows us to evaluate the efficiency of the expenditure and to suggest suitable amounts to be allocated.

It is therefore fair to say that, notwithstanding the many simplifying assumptions that have been pointed out in the text, the procedure could already be applied to real problems of resource allocation, when the reliability of the connection is the main concern.

The procedure could, however, be extended with comparative ease to cover other cases, introducing alternative objective function(s) of the optimization process.

For instance, the possibility of several degrees of damage (of the elements and/or the network) could be considered: as already hinted, these degrees could be related to the reduction of load capacity of bridges and of traffic capacity of the road network, and take into account also the velocity of the connection (which may be of importance, while in the simple form in which the procedure has been presented, there is no difference if source and destination are connected by a motorway or a country road). The optimization of the interventions might then aim at, say, the largest expected traffic capacity between source and destination after the earthquake has occurred: thus, the objective function includes a final convolution integral.

The network may also aim to establish connections between several points: in this case, the definition of the objective function should take into account all relevant connections. Another quantity that should be taken into consideration is the expected *out-of-service* time, i.e., the length of repair time necessary to restore the connection when it is severed as a consequence of the earthquake. And, of course, there are the costs of reconstruction and repair, and the other economical consequences of the failures, which must include the direct costs related to the possible disruption of the economy of the concerned area.

Note, however, that, in general, the costs of the interventions and the gain they produce are not commensurable quantities: the former are usually monetary, but the latter take account of variations of reliability, which may involve the safety and well being of people (the intangible quantities defined elsewhere[14,15]). Therefore, a direct comparison is seldom possible, and the efficiency of the measures taken can only be evaluated indirectly: in this paper we chose to tackle the problem by optimizing the gain, given the expenditures.

It also appears evident that in many problems more objective functions should be considered at the same time. If, as often happens, the quantities to optimize are not commensurable with each other and therefore cannot be combined by appropriate weights, multi-objective optimization must be pursued. Further research is planned to tackle all these aspects.

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