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A Simultaneous Network Optimization Approach for Pavement Management Systems

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ABSTRACT

In the context of sequential decision-making under uncertainty, Markov decision process (MDP) is a widely used mathematical framework. The MDP-based approaches in the infrastructure management literature can be broadly categorized as either top-down or bottom-up. The former, while efficient in incorporating system-level budget constraints, provide randomized policies, which must be mapped to individual facilities using additional sub-routines. On the other hand, although state-of-the-art bottom-up approaches provide facility-specific decisions, the disjointed nature of their problem formulation does not account for budget constraints in the future years. In this paper, a simultaneous network-level optimization framework is proposed, which seeks to bridge the gap between the top-down and bottom-up MDP-based approaches in infrastructure management. The salient feature of the approach is that it provides facility-specific policies for the current year of decision-making, while utilizing the randomized policies to calculate the expected future costs. Finally, the proposed methodology is compared with a state-of-the-art bottom-up methodology using a parametric study involving varying network sizes.

Keywords: Pavements, Maintenance, Markov decision process, Optimization.

INTRODUCTION

Transportation infrastructure management refers to the process of allocating a limited set of resources to a system of deteriorating facilities for maintenance, rehabilitation and replacement (MR&R) activities. The concept of infrastructure management was first demonstrated in the context of highway preservation in the state of Arizona. Golabi et al. (1982) developed a linear programming (LP) formulation for the Arizona pavement management system (PMS), which resulted in a $14 million savings in its first year of operations (Way 1983). The framework, represented as a discrete-state discrete-time Markov decision process (MDP),

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provides aggregate MR&R policies, which are also referred to as *randomized policies*. In a randomized policy setting, the optimal strategy for a given state is provided as a probability distribution over two or more actions. Golabi et al. (1982) interpreted these probabilities as fractions of the network in a given state to which the corresponding actions are to be applied. However, the translation of these fractional policies into facility-specific decisions necessitates the presence of additional sub-routines, which imparts a "top-down" characteristic to the methodology. The LP approach to MDP has since been extended to other scenarios in infrastructure management, as shown in Pontis (Golabi and Shepard 1997), Smilowitz and Madanat (2000), Kuhn and Madanat (2005) and Madanat et al. (2006), among others.

Kallenberg (1994) shows that linear programming formulations for solving finite horizon MDP problems with constraints provide optimal solutions in the form of randomized policies. In the context of incorporating budget constraints, randomization of optimal policies can be interpreted as enabling a complete utilization of the available budget, since a purely deterministic policy can either underutilize or exceed the budget (Dimitrov and Morton 2009). Mbwana and Turnquist (1996) formulated an alternate linear programming problem to develop facility-specific MR&R policies for an infinite horizon problem. However, the MDP literature indicates that the introduction of additional constraints leads to the randomization of steady-state policies as well (Ross 1989).

An alternate MDP-based methodology was developed in the case of the Indiana Bridge Management System (Sinha et al. 1988), which provides facility-specific policies to the decision-makers. Using a two stage optimization approach, the framework decouples the resource allocation problem into facility-level and system-level problems. However, while the approach assumes a Markovian process of deterioration, it transforms the Markovian transition probabilities into deterministic pavement performance models developed through regression techniques. Yeo et al. (2012) also developed a two stage MDP-based optimization framework for pavement management systems, which uses evolutionary algorithms to deal with the computational complexity of the system-level budget allocation problem. Since

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these approaches build upon single facility optimization procedures to develop policies at the system-level, they are also referred to as "bottom-up" approaches. Other researchers have also formulated bottom-up methodologies, especially in the context of developing deterministic optimal control policies at the system-level (Ouyang and Madanat 2004, Sathaye and Madanat 2011, 2012), and reliability-based MR&R decision-making of bridge systems (Robelin and Madanat 2008).

A simultaneous optimization approach to system-level decision-making has also been attempted using reinforcement learning algorithms (Gao and Zhang 2009, Kuhn 2010), wherein a system-level dynamic programming formulation is approximated using simulation techniques and lower dimensional approximations of value functions. These approaches have the advantage of incorporating complex network constraints, but it is hard to provide optimality guarantees upon implementation of these methods.

More recently, efforts have been made to account for network effects in MR&R decision-making processes. **?**) suggests that interactions between individual facilities in an infrastructure network can be classified into three different types: economic dependence (benefits/costs associated with joint maintenance), structural dependence (set of facilities collectively determining system performance such as connectivity or capacity) and stochastic dependence (presence of correlated deterioration factors like environment, loading). In this regard, **?**) addressed the role of economic and structural interdependence in coordinating maintenance activities on a road network, using a quadratic programming formulation with a deterministic deterioration model. Ouyang (2007) used a deterministic infinite horizon dynamic program with multidimensional continuous state and control variables to integrate travelers' route choices and the agency's resource allocation decisions. Ng et al. (2009) incorporated traffic dynamics within infrastructure maintenance planning by using a cell transmission model in a mixed-integer programming formulation.

Given this emerging interest in looking beyond pure resource allocation problems, traditional top-down approaches have become less used due to their inability to provide facility-

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specific policies. In fact, one of the major motivating factors for bottom-up approaches, such as Yeo et al. (2012), has been the need to develop MDP frameworks that allocate MR&R activities to individual facilities. At the same time, given their common underlying modeling assumptions, it is also important to verify that the various Markovian approaches, both top-down and bottom-up, provide similar policies. It is in this context that a simultaneous network-level optimization (SNO) framework is proposed in this paper, which seeks to bridge the gap between the top-down and bottom-up MDP-based approaches in infrastructure management.

METHODOLOGY

In order to arrive at the problem formulation for the SNO framework, it would be instructive to compare and contrast the formulations of the top-down and bottom-up approaches. The frameworks presented below are the state-of-the-art approaches in their respective categories, and provide the necessary insights to develop the SNO framework. The discussion focuses on finite planning horizon problems, since the infinite planning horizon problem provides long-term policies, which are not always readily implementable.

Top-Down Approach

As shown previously, the LP-based approach, proposed by Golabi et al. (1982), has been widely used in the area of infrastructure management. In order to solve the decision-making problem in any year t of a finite planning horizon of T years, the optimization problem can be formulated as follows:

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$$\min_{w} \quad N\left(\sum_{s\in\mathbf{S}}\sum_{a\in\mathbf{A}}\left(\sum_{\tau=t}^{T}\alpha^{\tau-t}\left(c(s,a)+u(s)\right)w_{sat}+\alpha^{T+1-t}\,\tilde{V}(s)w_{sa\,T+1}\right)\right) \tag{1}$$

$$\text{subject to} \quad \sum_{a\in\mathbf{A}} w_{sat} = f_s^t \tag{2}$$

$$\sum_{s\in\mathbf{S}}\sum_{a\in A} w_{sa\tau} = 1 \qquad \forall\tau = t+1,\ldots,T, \tag{3}$$

$$N\left(\sum_{s\in\mathbf{S}}\sum_{a\in\mathbf{A}} c(s,a)w_{sa\tau}\right) \leq B_\tau \qquad \forall\tau = t,\ldots,T, \tag{4}$$

$$\sum_{r\in\mathbf{S}}\sum_{a\in\mathbf{A}} p_a\left(r,s\right)w_{ra\tau} = \sum_{a\in\mathbf{A}} w_{sa\tau+1} \qquad \forall s\in\mathbf{S}, \forall\tau = t,\ldots,T, \tag{5}$$

$$w_{sa\tau} \in [0,1] \qquad \forall s\in\mathbf{S}, \forall a\in\mathbf{A}, \forall\tau = t,\ldots,T+1,$$

where,

$w_{sa\tau}$: fraction of facilities in state s to which action a is applied in year τ (*randomized policies*),

f_s^t: fraction of facilities in state s in year t (the first year of optimization),

$c(s,a)$: cost incurred by the agency to implement action a, when a facility is in state s,

$u(s)$: cost incurred by users due to vehicle wear-and-tear, when a facility is in state s,

B_τ: agency's annual budget in year τ,

$p_a(r,s)$: probability of a facility transitioning from state r to s, when action a is selected,

α: discount amount factor,

$\tilde{V}(s)$: salvage value associated with state s at the end of the planning horizon,

N: number of facilities in the network,

$\mathbf{A}$: action space associated with a facility (including do-nothing),

$\mathbf{S}$: state space associated with a facility.

Herein, equation 1 refers to the objective, which is to minimize the expected system-level user-plus-agency costs, incurred from year t to the end of the planning horizon; equation 2 represents the state of the system at the start of the optimization; equation 3 ensures that the randomized policies sum up to one for each year; equation 4 forces the agency expenditure to be within the annual budget constraint; and equation 5 represents the Chapman-Kolmogorov

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equations, which relate the policies of a given year with the policies of the subsequent year.

In order to implement the recommendations from the top-down approach, the randomized policies need to be associated with individual pavement sections, either using engineering judgement, or with the help of additional sub-programs within the PMS. If the size of the network is sufficiently large, the policies obtained for the future time periods should also be consistent with the distribution of condition states observed in the subsequent years, due to the law of large numbers. However, since the Chapman-Kolmogorov relationship (equation 5) models the evolution of the system in an expected sense, it is possible that the proportion of the road network in a given state s, as realized in the future, may differ from the shares predicted by the randomized policies. Under such a scenario, it is recommended that the optimization be repeated in that year using the information available about the current state of the network.

The LP formulation provides an optimal as well as a computationally attractive framework for solving the constrained MDP problem. The aggregation of policies allows for budget constraints to be imposed on all future actions, while maintaining the Markovian evolution of the state of the system. As a result, it provides agencies with a defensible procedure for preparing multi-year budget plans for MR&R decision-making. However, a limitation of the top-down approach is that the use of randomized policies precludes the identification of facility-specific actions from the optimization results.

Two Stage Bottom-Up Approach

In order to determine facility-specific policies for a pavement network, Yeo et al. (2012) formulated a two stage bottom-up (TSBU) approach, which consists of a facility-level and a system-level problem. In the first stage, the facility-level problem is solved to obtain optimal and near-optimal policies for each facility, which act as inputs for the second stage. The system-level problem is then represented as a multi-choice knapsack problem, which incorporates the budget constraint for the current year. The decoupled nature of the formulation was motivated by the curse of dimensionality associated with solving a system-level dynamic

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programming problem involving a multidimensional state space.

Facility-level Problem

The objective of the facility-level optimization problem is to identify the optimal and sub-optimal policies for each facility, along with their associated to-go costs for each time period of a finite planning horizon. Herein, the optimal policy is defined by the action which minimizes the expected cost-to-go from the current year to the end of the planning horizon, for a given state s and year t. The motivation behind identifying the alternate policies is to provide greater flexibility with budget allocation at the system-level, since the sum of all the optimal policies might exceed the available budget. The optimization problem, represented as a discrete-state discrete-time MDP, can be solved using a backward-recursive dynamic programming approach with the following formulation:

$$a_{k\tau}(s) = \underset{a \in \mathbf{A} - \{a_{j\tau}, j \le k-1\}}{\arg\min} \left[c(s,a) + u(s) + \alpha \sum_{r \in S} \left(p_a(s,r) V_{1\tau+1}(r) \right) \right], \quad \forall k = 1, \ldots, |\mathbf{A}|, \quad \forall s \in \mathbf{S}, \forall \tau = t, \ldots, T, \tag{6}$$

$$V_{k\tau}(s) = \min_{a \in \mathbf{A} - \{a_{j\tau}, j \le k-1\}} \left[c(s,a) + u(s) + \alpha \sum_{r \in S} \left(p_a(s,r) V_{1\tau+1}(r) \right) \right], \quad \forall k = 1, \ldots, |\mathbf{A}|, \quad \forall s \in \mathbf{S}, \forall \tau = t, \ldots, T, \tag{7}$$

$$V_{1T+1}(s) = \tilde{V}(s), \quad \forall s \in \mathbf{S}, \tag{8}$$

where,

$a_{k\tau}(s)$: k^{th} optimal action when a facility is in state s in year τ ($k = 1$ is optimal),

$V_{k\tau}(s)$: expected cost-to-go associated with the k^{th} optimal action, from year τ to the end of the planning horizon, when a facility is in state s,

$V_{1T+1}(s)$: salvage value associated with state s at the end of the planning horizon, $\tilde{V}(s)$.

In the absence of system-level constraints in the facility-level formulation, an assumption being made is that the future costs correspond to an optimal policy implementation, as

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denoted by $V_{1\tau+1}(r)$ (1=optimal) in equations 6 and 7. In effect, the formulation implies that optimality/sub-optimality is only restricted to the current year, and in the subsequent years, the budget would be sufficient for selecting the optimal actions for each facility. Finally, equation 8 specifies a state-dependent salvage value to the cost to-go function associated with the end of the planning horizon.

System-level Problem

The objective of the system-level problem is to allocate the annual budget for MR&R activities, so as to minimize the expected cost-to-go for the entire network. Using the ranked set of actions from the facility-level problem as an input, the problem is formulated as a multiple-choice knapsack problem:

$$\min_{x} \quad \sum_{k=1}^{|A|}\sum_{i=1}^{N} V_{kt}^{(i)}\left(\mathbf{s}_t(i)\right) x_{a_{kt}^{(i)}} \tag{9}$$

$$\text{subject to} \quad \sum_{i=1}^{N}\sum_{k=1}^{|A|} c(\mathbf{s}_t(i), a) x_{ia} \leq B_t, \tag{10}$$

$$\sum_{a\in\mathbf{A}} x_{ia} = 1 \qquad \forall i = 1,\ldots,N, \tag{11}$$

$$x_{ia} \in \{0,1\} \quad \forall i = 1,\ldots,N, \forall a \in \mathbf{A},$$

where,

$a_{kt}^{(i)}, V_{kt}^{(i)}$: k^{th} optimal action and the corresponding expected cost-to-go for facility i, obtained from the facility-level problem for the year of decision-making, t,

$x_{a_{kt}^{(i)}}$: 1 if the action corresponding to $a_{kt}^{(i)}$ is selected for facility i; 0 otherwise,

$\mathbf{s}_t(i)$: condition state associated with facility i in year t.

Here, equation 9 represents the objective function, defined as the expected cost-to-go for the network, based on the actions selected for each facility; equation 10 indicates that the total amount spent on MR&R activities should be within the annual budget, and equation 11 ensures that exactly one action (including do-nothing) is selected for each facility.

The system-level problem assumes that information about the condition state of each

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facility is obtained at the beginning of each year through annual inspections. Hence, in order to implement the two stage bottom-up approach, the system-level optimization needs to be re-solved in each year of the planning horizon. On the other hand, since the facility-level problem is solved for the entire planning horizon, the optimal and alternative policies for each facility need not be calculated again.

The use of an integer programming formulation has the benefit of selecting policies for individual facilities. However, the disjointed nature of this approach suffers from the limitation that the facility-level policies are developed without acknowledging the interdependencies introduced by limited resources at the network level, such as a finite budget. In particular, it is difficult to justify the optimistic assumption in the facility-level formulation of implementing optimal policies in the future years.

Obtaining Facility-Specific Policies using Top-Down Approach: A Simultaneous Network Optimization Approach

Based on the discussion above, it can be inferred that facility-specific policies need to be developed in accordance with the financial constraints imposed on the current as well as future years. Herein, the LP-based top-down approach satisfies all requirements, except for providing facility-specific policies. At the same time, determining facility-specific policies alludes to an integer programming formulation. Keeping this in mind, an approach can be developed by modifying the LP formulation into a mixed-integer linear programming formulation, as shown below:

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$$
\begin{aligned}
\min_{x,w} \quad & \sum_{i=1}^{N} \sum_{a\in\mathbf{A}} \left(c\left(\mathbf{s}_t(i),a\right)+u\left(\mathbf{s}_t(i)\right)\right) x_{iat} \\
& + N\left(\sum_{s\in\mathbf{S}}\sum_{a\in\mathbf{A}}\left(\sum_{\tau=t+1}^{T} \alpha^{\tau-t}\left(c(s,a)+u(s)\right)w_{sat}+\alpha^{T+1-t}\,\tilde{V}(s)w_{sa\,T+1}\right)\right) && (12)\\
\text{subject to} \quad & \sum_{a\in\mathbf{A}} x_{iat} = 1 \qquad \forall i=1,\ldots,N, && (13)\\
& \frac{1}{N}\left(\sum_{i=1,\ldots,N|\mathbf{s}_t(i)=r} x_{iat}\right) = w_{rat} \qquad \forall r\in\mathbf{S},\forall a\in\mathbf{A}, && (14)\\
& \sum_{s\in\mathbf{S}}\sum_{a\in\mathbf{A}} w_{sa\tau} = 1 \qquad \forall \tau = t+1,\ldots,T, && (15)\\
& N\left(\sum_{s\in\mathbf{S}}\sum_{a\in\mathbf{A}} c(s,a)w_{sa\tau}\right) \le B_\tau \qquad \forall\tau=t,\ldots,T, && (16)\\
& \sum_{r\in\mathbf{S}}\sum_{a\in\mathbf{A}} p_a(r,s)\,w_{ra\tau} = \sum_{a\in\mathbf{A}} w_{sa\tau+1} \qquad \forall s\in\mathbf{S},\forall\tau=t,\ldots,T, && (17)\\
& w_{sa\tau} \in [0,1] \qquad \forall s\in\mathbf{S},\forall a\in\mathbf{A},\forall\tau=t,\ldots,T+1,
\end{aligned}
$$

where,

x_{iat}: 1 if action a is selected for facility i; 0 otherwise (t refers to the current year),

$w_{sa\tau}$: fraction of the network in state s to which action a is applied in year τ, where τ is representative of all the future years.

In terms of the objective function and the resulting optimal solution, SNO is identical to the approach provided by Golabi et al. (1982). The only modification in the problem formulation is with regards to the use of binary integer variables for the current year, t, as is evident from the objective function (equation 12). The constraint of interest is equation 14, which defines the randomized policies for the current year in terms of the integer variables. Once the relationship between the two sets of variables is established, it is then possible to determine the expected future costs in terms of the randomized policies.

The salient feature of SNO is that it provides facility-specific policies for the current year using a single optimization routine, while utilizing the randomized policies to calculate the expected future costs. This allows for budget constraints to be imposed on the future years,

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as well as retaining the optimal nature of the LP formulation. In comparison, TSBU is internally inconsistent, as it does not account for the system-level inderdependencies at the facility-level problem.

The SNO framework needs to be implemented for every year of the planning horizon, since the condition state associated with each facility, $\mathbf{s}_t(i)$, needs to be identified at the beginning of each year.

PARAMETRIC STUDY

For evaluating the proposed methodology and comparing its performance with TSBU, a parametric study was conducted. The condition state of the facilities was defined using an eight point ordinal index, where 1 is the best state and 8 is deemed to be an unacceptable state by the agency. For the purpose of illustration, four types of activities were considered: do-nothing, routine maintenance, rehabilitation and reconstruction. The agency and user cost structure, shown in table 1, was taken from Madanat (1993). Herein, maintenance and rehabilitation activities become prohibitively more expensive as the state worsens, whereas reconstruction incurs a constant cost. The user cost also increases as the facility deteriorates, and a high penalty cost is imposed when the facility is in the non-permissible condition state ($s = 8$). The transition probability matrices for the different MR&R alternatives, as shown in tables 2-5, were also adapted from Madanat (1993), but were suitably modified to reflect the increasing levels of maintenance effectiveness. The planning horizon consisted of 15 years and the discount rate was 5%. The salvage value at the end of the planning horizon was set equal to the user costs, which can be interpreted as a proxy for the quality of the terminating state of the facility.

In order to generate the different scenarios, the annual budget was fixed at $B = 250$ units, while the number of facilities, n, was varied to be 10, 50, 100, 500 and 1000 (which is comparable to the Arizona PMS network of 7400 sections (Yeo et al. 2012)). Hence, for a network of 10 facilities, an annual budget of 250 units would be sufficiently high, whereas, for n=1000, the same budget would be severely constraining. The initial condition of the

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TABLE 1. Cost structure for numerical example

Maintenance Activity	Pavement State 1	2	3	4	5	6	7 Acceptable	8 Unacceptable
Do-Nothing	0	0	0	0	0	0	0	0
Maintenance	0.04	0.15	0 .31	0.65	0.83	1.4	2	6.9
Rehabilitation	3.81	3.91	4.11	6.64	9.06	10.69	12.31	21.81
Replacement	25.97	25.97	25.97	25.97	25.97	25.97	25.97	25.97
User Costs	0	2	4	8	14	22	25	100

facilities was uniformly distributed between states 1 and 7 (the non-permissible condition state 8 was excluded), so as to represent a wide range of condition states in the system.

Given the stochastic nature of deterioration, the results were generated using a Monte Carlo simulation method. Monte Carlo simulation is a popular sampling technique, wherein random information is generated using an artificial process (typically, a uniform distribution), so as to pick a random observation from a population(**?**). In the context of the parametric study, based on the facility-specific actions recommended by SNO and TSBU for a given

TABLE 2. Do nothing transition matrix

s_τ	$s_{\tau+1}$: 1	2	3	4	5	6	7	8
1	0.6	0.4	0	0	0	0	0	0
2	0	0.5	0.5	0	0	0	0	0
3	0	0	0.4	0.6	0	0	0	0
4	0	0	0	0.35	0.65	0	0	0
5	0	0	0	0	0.3	0.7	0	0
6	0	0	0	0	0	0.2	0.8	0
7	0	0	0	0	0	0	0.1	0.9
8	0	0	0	0	0	0	0	1

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TABLE 3. Routine maintenance transition matrix

s_τ	$s_{\tau+1}$							
	1	2	3	4	5	6	7	8
1	0.85	0.15	0	0	0	0	0	0
2	0	0.73	0.37	0	0	0	0	0
3	0	0	0.62	0.38	0	0	0	0
4	0	0	0	0.52	0.48	0	0	0
5	0	0	0	0	0.43	0.57	0	0
6	0	0	0	0	0	0.35	0.65	0
7	0	0	0	0	0	0	0.29	0.71
8	0	0	0	0	0	0	0	1

TABLE 4. Rehabilitation transition matrix

s_τ	$s_{\tau+1}$							
	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	0.85	0.15	0	0	0	0	0	0
3	0	0.85	0.15	0	0	0	0	0
4	0	0	0.85	0.15	0	0	0	0
5	0	0	0	0.85	0.15	0	0	0
6	0	0	0	0	0.85	0.15	0	0
7	0	0	0	0	0	0.85	0.15	0
8	0	0	0	0	0	0	0.85	0.15

year of decision-making, the condition states for the next year are simulated using a uniform random number generator. The process is then repeated the following year, and similarly, for every year till the end of the planning horizon. For each scenario, 1000 simulations were carried out to determine the average system-level user-plus-agency costs incurred by the agency (in net present value).

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TABLE 5. Reconstruction transition matrix

s_τ	$s_{\tau+1}$							
	1	2	3	4	5	6	7	8
1	1	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0
6	1	0	0	0	0	0	0	0
7	1	0	0	0	0	0	0	0
8	1	0	0	0	0	0	0	0

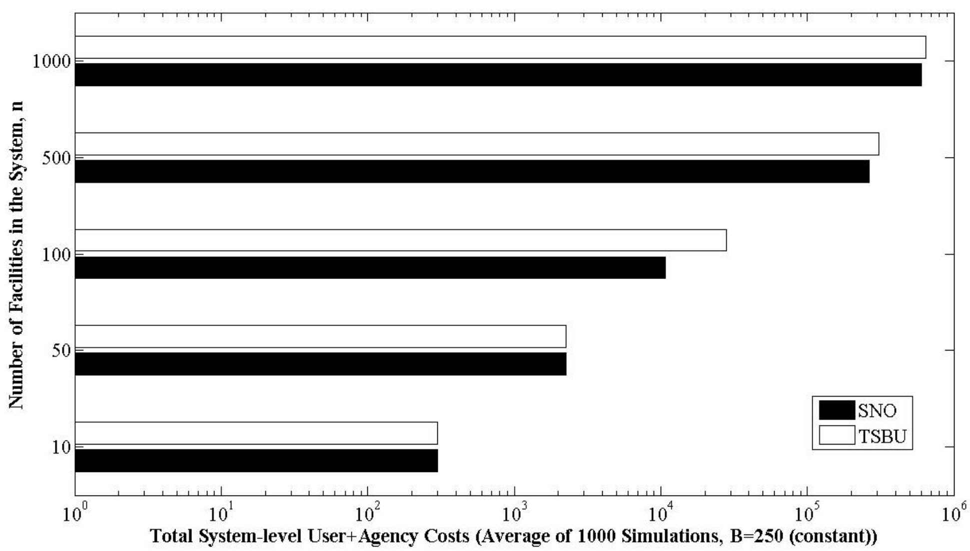

FIG. 1. Comparison of the average system-level costs incurred by implementing SNO and TSBU

Results

Figure 1 represents the average system-level costs incurred by the agency using SNO and TSBU, respectively. As the budget is kept constant, the costs for both approaches, represented on a log-scale, increase with an increase in the number of facilities in the system. For n=10 and 50, both SNO and TSBU, perform equally well. However, as the budget

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constraint becomes more severe, SNO starts providing lower costs than TSBU.

A more informative assessment of the two approaches can be made by comparing the distributions of the simulation results, as illustrated in figure 2. Herein, a box plot representation shows the median (the horizontal line inside the box), the lower and upper quartiles (the edges of the box), and the overall spread of the simulation results (the whiskers extending above and below the box). In addition, a dot, signifying the expected system-level costs, predicted by each optimization routine at $t = 1$, has also been marked on the plot. As the number of simulations tends to infinity, it would be anticipated that the average of the costs realized through simulation and the expected cost predicted by the optimization would be identical. Hence, using the expected costs as a metric, the two simulations can also be evaluated on how closely the realized costs match with the a-priori expected minimum costs.

Figure 2(a) shows the box plot corresponding to $n = 10$. In this case, the distribution of the simulated costs is identical for both SNO and TSBU. Also, the medians of the box plots coincide with the costs expected at $t = 1$, indicating that both approaches predict the future costs accurately. For $n = 50$ (figure 2(b)), while the simulation results are identical for both approaches, differences between the predicted and the realized costs begin to emerge for the TSBU approach. In fact, it can be observed that TSBU becomes increasingly inconsistent with its predictions as the budget constraint becomes tighter (i.e., as the number of facilities increase). Consequently, its policy selection becomes increasingly sub-optimal as well. In comparison, SNO always provides lower costs, and is also consistent with the costs predicted by its optimization routine. These differences are best illustrated in figures 2(d) and 2(e), where the costs obtained using SNO are significantly lower than TSBU. Also, while SNO's estimates of expected cost at $t = 1$ are mostly within the range of its simulated outcomes, TSBU's forecasts are lower by an order of magnitude.

From a methodological perspective, these results indicate that benchmarking and internal consistency checks are useful tools while evaluating a stochastic optimization approach. In the absence of any theoretical guarantee on optimality, comparing the performance of an

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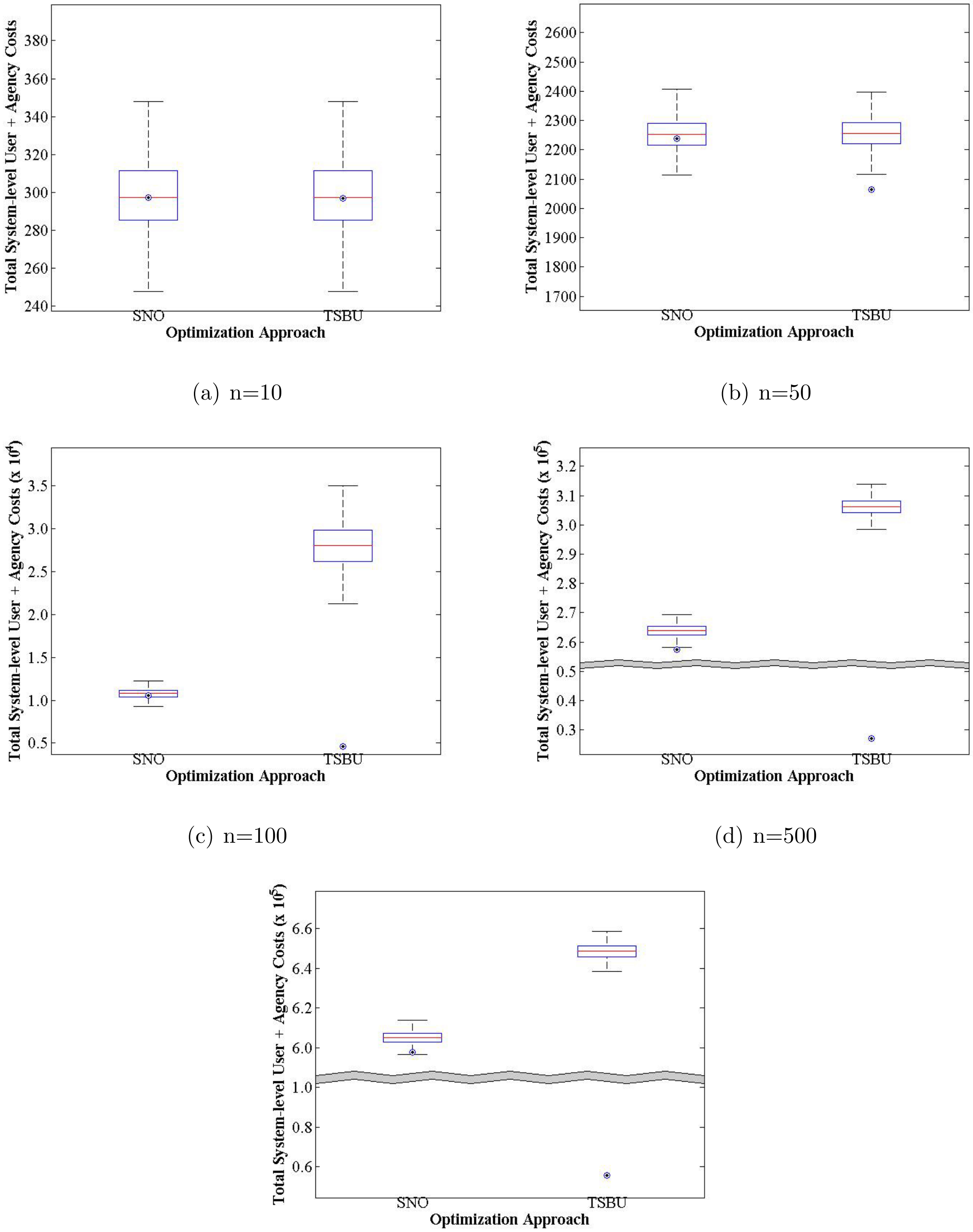

(a) n=10

(b) n=50

(c) n=100

(d) n=500

(e) n=1000

FIG. 2. Comparison between the simulated costs and the expected system-level costs predicted at t=1 for SNO and TSBU

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approach with other state-of-the-art methods provides a good measure of its efficacy. In contrast, internal consistency checks ensure that the observed costs are always consistent with the a-priori expectations, even if it is known to be sub-optimal. In the case of TSBU, its assumptions on future costs proved to be inaccurate, leading to sub-optimal policies. In comparison, SNO's problem formulation used aggregate system-level policies to effectively capture the evolution of the system, and hence was able to provide optimal solutions.

A comparison between the facility-specific policies obtained from SNO and TSBU is illustrated in figure 3. Herein, the two plots show the evolution of the condition state of a facility starting in state 7, and the corresponding actions recommended by SNO and TSBU, respectively. The results correspond to the $n = 100$ scenario. The optimal policy recommended by TSBU for state 7 in year $t = 1$, is reconstruction (4), followed by rehabilitation (3), routine maintenance (2), and do-nothing (1). In comparison, the system-level analysis of SNO recommends the optimal actions to be distributed between do-nothing (1) and reconstruction (4). Owing to the budget constraints, TSBU implements the rehabilitation action (second optimal), while SNO implements do-nothing for the facility under consideration. As the simulation evolves over time, SNO implements a reconstruction action in year 3, whereas the TSBU policies result in the facility staying in state 8 for prolonged periods of time. The resulting user+agency cost accrued through SNO and TSBU are 178.57 and 609.42 units respectively.

While the above discussion corresponds to a single simulation run for a facility representing a worst-case scenario, some insights can still be obtained on the underlying decision-making process of both approaches. The policy matrix generated by TSBU assumes that an optimal policy will be implemented in the future, which in the case of a significantly deteriorated condition state is to reconstruct. However, the available budget prevents the selection of a reconstruction action, leading to multiple implementations of the rehabilitation action. In comparison, SNO recognizes the budget constraints in the current, as well as the future years, and concludes that neither routine maintenance nor rehabilitation activities will lead

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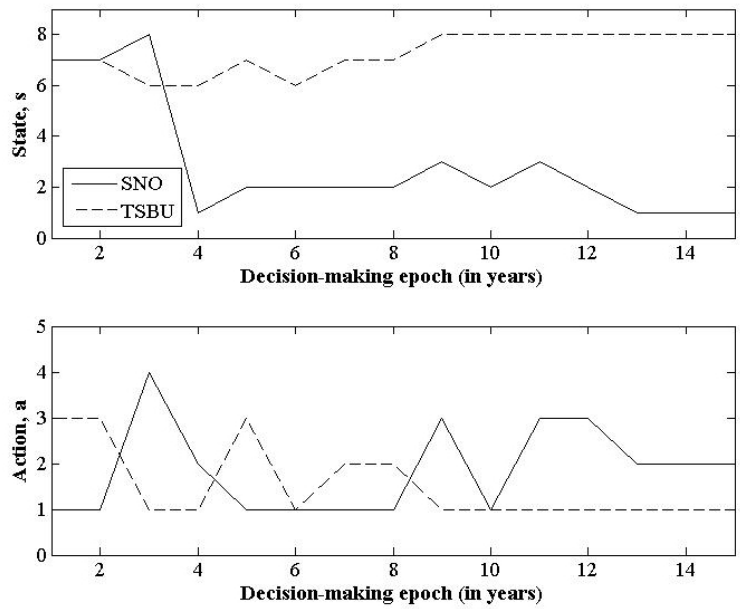

FIG. 3. Comparison between facility-specific policies of SNO and TSBU for a facility from $n = 100$ scenario; For the action set, 1: do-nothing, 2: routine maintenance, 3: rehabilitation, and 4: reconstruction

to a significant improvement in the condition state of the facility in the long run. Consequently, SNO recommends either doing nothing, or reconstructing, for facilities in state 7, which ensures that over a period of time, most facilities will be reconstructed.

Implementation Issues

In the parametric study undertaken above, the optimization was carried out over a finite planning horizon. However, from an agency's perspective, infrastructure assets, like pavements and bridges, may not have predefined useful lives. In such cases, a more realistic accounting practice would be to use a rolling planning horizon, wherein at every decision epoch, a new T-year planning horizon is solved for. A long enough planning horizon ensures that issues pertaining to salvage value selection become insignificant due to the discounting of future costs. In addition, steady-state policies and costs can also be incorporated into the SNO framework as a proxy for salvage values, as demonstrated in Golabi et al. (1982).

CONCLUSIONS AND FUTURE WORK

Through this research, the top-down approach proposed by Golabi et al. (1982) has been

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extended to accommodate facility-specific decision-making. Using numerical examples, the SNO approach was shown to provide consistently optimal results for varying network sizes. In addition, a state-of-the-art MDP-based approach was shown to be sub-optimal as well as inconsistent for scenarios with constrained financial resources.

A major contribution of this work lies in resolving the dichotomy between top-down and bottom-up methodologies in MDP-based MR&R decision-making frameworks. While the Golabi et al. (1982) approach has been successfully implemented previously, the absence of facility-specific policies is often cited as a limitation of the approach. By addressing the identification issues associated with randomized policies, SNO provides a suitable framework for looking beyond resource allocation problems in a MDP setting. In particular, incorporating economies of scale and accounting for the impact of traffic disruptions are important practical considerations, and are being investigated as part of on-going research.

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