A Nonlinear Model for Predicting Pavement Serviceability

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Abstract
A recursive nonlinear model was developed to predict pavement deterioration. Pavement deterioration was assessed in terms of loss of serviceability – expressed as a function of traffic characteristics, pavement structural properties and environmental conditions. The model highlights some of the advantages of relaxing the linear restriction usually placed on the model specification. First, a functional form that better represents the physical deterioration process can be used. Second, the estimated parameters are unbiased because of proper specification and the use of sound estimation techniques. Finally, the standard error of the regression was reduced by half that of the equivalent existing linear model.
The model developed enables the determination of an unbiased exponent of the so-called power law, and equivalent loads for different axle configurations. The estimated exponent confirms the value of 4.2 traditionally used. However, equivalent loads estimated for different axle configurations differed from traditionally used values. The estimated equivalent load for a single axle with single wheels is 44 kN, while the equivalent load for a tandem axle with dual wheels is 148 kN.

Introduction
The objective of this research was to develop accurate pavement deterioration models to be used primarily for the effective management of the road infrastructure. Pavement deterioration is assessed in terms of loss of serviceability (Carey and Irick, 1960; HRB, 1962).
At the network level, deterioration prediction is key for adequate activity planning, project prioritization and budget allocation. At the project level, it is important for establishing the specific corrective actions that need to be taken. Vehicle operating costs and the costs of transporting goods increase as road deteriorates. These costs are often one order of magnitude greater than the cost of maintaining the road to an acceptable quality level (Paterson, 1987). Both, the rate of deterioration over time and the contribution of the various factors to such deterioration, are also essential inputs to road pricing and regulation studies.

Background
Commonly used data sources. A number of data sources had been used for the development of pavement deterioration models: (i) randomly selected in-service

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pavement sections, (ii) in-service pavement sections selected according to an experimental design, (iii) purposely built pavement test sections subjected to the action of actual traffic and the environment, and (iv) test sections subjected to the accelerated action of traffic and environmental conditions.

Data from in-service pavement sections subjected to the combined actions of highway traffic and environmental conditions represent most closely the actual deterioration process of pavements in the field. All other data sources would produce models that would suffer from some kind of bias or restrictions, unless special considerations are taken into account during the parameter estimation process. Some of the most common problems encountered in deterioration models developed from randomly selected in-service pavement sections are caused by (i) multicollinearity between explanatory variables, (ii) unobserved events typical of such data sets (censoring), and (iii) endogeneity bias generated by the use of endogenous variables as explanatory variables (Paterson, 1987; Ramaswamy and Ben-Akiva, 1990; Prozzi and Madanat, 2000).

For these reasons, it was decided to estimate the parameters of the model based on data from a statistically design experiment. A number of alternative data sources were evaluated and the data from the AASHO Road Test was finally selected. The AASHO Road Tests provided the most comprehensive a reliable pavement performance data source available to date.

The AASHO Road Test. The AASHO Road Test was sponsored by the American Association of State Highway Officials (AASHO) and took place in the late 1950s and early 1960s near Ottawa, Illinois (HRB, 1962). The site was chosen because the soil and the climate in the area were representative of large areas of the United States. Hence, the use of the results outside these conditions should be subjected to detailed assessment of their applicability.

The test tracks consisted of six loops. Each loop was a segment of a four-lane divided highway whose north tangents were surfaced with asphalt concrete (AC) and the south tangents with Portland cement concrete (PCC). Only five loops were subjected to traffic and all vehicles assigned to any one traffic lane had the same axle arrangement-axle load configuration. Most of the sections on the flexible pavement tangents were part of a complete experimental design. The design factors were surface asphalt thickness (T_a), base thickness (T_b) and subbase thickness (T_s). The materials used for the construction of the asphalt surface, base and subbase layers were the same for all the sections.

Original serviceability model. The first pavement deterioration model was developed based on the data from the AASHO Road Test. The model predicts deterioration in terms of a dimensionless variable (***g***) referred to as damage. The damage variable was defined as the loss in serviceability at any given time according to the following equation:

$$g_t = \frac{p_0 - p_t}{p_0 - p_f} = \left(\frac{N_t}{\rho}\right)^{\omega} \qquad (1)$$

g_t : dimensionless damage,

p_t : serviceability at time t,
p_0, p_f : initial and terminal serviceability,
N_t : traffic applied to the section until time t, and
ρ, ω : regression parameters.

The serviceability (p) is measured by the present serviceability index (PSI) and it is a function of the observed slope variance, surface rutting, cracking and patching of the pavement section (Carey and Irick, 1960). The parameters ρ and ω were estimated as a function of pavement structural properties and traffic load configuration (HRB, 1962).

Proposed Nonlinear Model Specification

The following recursive specification form was developed for the deterioration model proposed in this research:

$$p_t = p_{t-1} + \alpha\, N_{t-1}^{\delta}\, \Delta N_t \qquad (2)$$

ΔN_t : traffic increment from time *t-1* to time *t*, and
α, δ : parameters or functions to be estimated.

The specific specification forms used to account for the various parts of Equation (2) (initial serviceability, pavement strength, environmental conditions, and traffic) are described in the following sections.

Initial serviceability. The initial serviceability (present serviceability index after construction) of a pavement section greatly depends on the thickness of the asphalt surface. Therefore, the following expression is proposed to capture this aspect:

$$p_0 = \beta_1 + \beta_2 \exp\{\beta_3\, T_a\} \qquad (3)$$

p_0 : initial serviceability,
T_a : thickness of the asphalt surface,
β_1-β_3 : regression parameters.

Specification for pavement strength. Previous research demonstrated that the rate of pavement deterioration is a decreasing function of the strength of the pavement. That is, the serviceability of weaker pavements decreases more rapidly than that of stronger pavements. Hence, the deterioration rate was specified as a decreasing function of the equivalent thickness (ET) according to the following expression:

$$\alpha = ET^{\beta_7} = \left(1 + \beta_4\, T_a + \beta_5\, T_b + \beta_6\, T_s\right)^{\beta_7} \qquad (4)$$

ET : equivalent thickness,
T_a, T_b, T_s : thickness of the surface, base and subbase layers (in mm), and
β_4-β_7 : parameters to be estimated.

The concept of equivalent thickness (ET) is introduced to differentiate the current specification from the well-known structural number (AASHTO, 1981). Although the concepts are equivalent, the layer strength coefficients cannot be directly compared.

Environmental considerations. From the observed data, three phases were distinguished in the serviceability trend of the pavement sections: (i) a *normal* phase, characteristic of the summer and fall periods, when pavement serviceability of the sections decreased slowly, (ii) a *stable* phase, characteristic of the winter period, when serviceability remained stable, and (iii) a *critical* phase, during the spring months when the rate of deterioration increased significantly.

It was also observed that the above three phases were almost identical to the periods of (i) zero frost penetration, (ii) increasing frost penetration, and (iii) decreasing frost penetration, respectively. Consequently, the frost gradient was chosen to capture environment effect on serviceability by means of an environmental factor (F_e):

$$F_e = \exp(\beta_8\, G) \tag{5}$$

G : frost gradient (in mm per day), and

β_8 : parameter to be estimated.

During the AASHO Test, the depth of frost penetration was measured every two weeks. The frost gradient (G) is then determined as the change of the depth of frost penetration within each two-week period (in mm per day).

Specification for aggregate traffic. In the present research, different standard loads (denominator of the power law) are used to convert different axle configurations into their equivalent number of single axle loads (ESALs). Three axle configurations were used during the AASHO Road Test: single axles with single wheel, single axles with dual wheels and tandem axles with dual wheels. To date, these three configurations represent the majority of traffic axle configurations in the United States.

Furthermore, it was decided to estimate the exponent of the power-law from the data instead of using a pre-estimated value because a number of studies have shown the close dependence of the exponent on the type of distress being considered and on the pavement structure (AASHTO, 1981; Christison, 1986; Prozzi and de Beer, 1997; Archilla, 2000).

Based on these considerations, the concept of the equivalent damage factor (EDF) was introduced. The EDF is a dimensionless factor, which depends on the characteristics of the truck and, which, when multiplied by the number of trucks, yields the equivalent number of standard axles. The following equation applies:

$$EDF = \left(\frac{FA}{\beta_{10}\, 80}\right)^{\beta_{12}} + n_1 \left(\frac{SA}{80}\right)^{\beta_{12}} + n_2 \left(\frac{TA}{\beta_{11}\, 80}\right)^{\beta_{12}} \tag{6}$$

EDF : equivalent damage factor,

FA : load in kN of the front axle (single axle with single wheels),

SA : load in kN of the rear single axle with dual wheels,

TA : load in kN of the rear tandem axles with dual wheels,

β_{10}-β_{12} : parameters to be estimated, and

n_1, n_2 : number of single axles and tandem axles per truck, respectively

Final model specification. In this section the final model specification is given considering that the data used consists of a panel data set:

$$p_{it} = \beta_1 + \beta_2\, e^{\beta_3 T_{ai}} + \left(1 + \beta_4 T_{ai} + \beta_5 T_{bi} + \beta_6 T_{si}\right)^{\beta_7} \sum_{r=0}^{r=t-1} \exp(\beta_8 G_r)\, N_{ir}^{\beta_9}\, \Delta N_{i,r+1} \tag{7}$$

Where the first subscript, i, indicates the pavement test section and the second subscript, t, indicates the time period. G_r is the frost gradient in period r. $N_{it} = \sum_{r=0}^{t-1} \Delta N_{i,r}$, represents the cumulative traffic in ESALs for section i in period t, obtained by multiplying the EDF of each truck configuration by the actual number of truck passes over the pavement test section as follows:

$$\Delta N_{it} = n_{it} \left(\left(\frac{FA_i}{\beta_{10}\, 80} \right)^{\beta_{12}} + D_i \left(\frac{SA_i}{80} \right)^{\beta_{12}} + D_i \left(\frac{TA_i}{\beta_{11}\, 80} \right)^{\beta_{12}} \right) \tag{8}$$

β_{10}-β_{12} : parameters to be estimated,
n_{it} : actual number of truck passes for section i at time period t, and
D_i : dummy variable (D = 1 for one rear axle, D = 2 for two rear axles).

Estimation

Ordinary Least-squares (OLS). The most popular technique used to estimate the parameters is by combining all time series data and cross sectional data and carrying out ordinary least-squares estimation (OLS), therefore, assuming the intercept to be the same for all sections. This assumption is not entirely unreasonable, as it considers that the serviceability of all pavements is the result of the same process and that it only depends on the variables that are observed. However, unobserved heterogeneity is often present as a result of unobserved section-specific variables. Due to the large number of sections, it is expected significant heterogeneity across pavement sections.
Equations (7) and (8) represent the conditional expectation of the serviceability at a given time t for a given section i. By applying the conditional expectation to all observations, the following equation can be used to represent the problem:

$$p_{it} = E\left(p_{it} \mid X_{it}, \underline{\beta}\right) + \varepsilon_{it} \tag{9}$$

X_{it} : observed explanatory variables,
$\underline{\beta}$: set of parameters, and
ε_{it} : random error term.

This equation is nonlinear in the parameters so the estimation does not have a closed-form solution. A nonlinear minimization routine was applied to estimate the parameters. Two assumptions were necessary to proceed with the OLS estimation: (i) the random error term (ε_{it}) is assumed to have mean zero and constant variance, and (ii) the covariance of the error terms is zero across sections and along time.
Under the assumption of normality, the values of the parameters that minimize the sum of square deviations will be the maximum likelihood estimators as well as the nonlinear least squares estimators (Greene, 2000). The sum of squared deviations is given by:

$$F\left(\underline{\beta}\right) = \sum_{i=1}^{S} \sum_{t=1}^{T_i} \left\{p_{it} - E\left(p_{it} \mid X_{it}, \underline{\beta}\right)\right\}^2 \tag{10}$$

Where S in the number of sections, and T_i: number of observation periods for section i. Since the panel data set is unbalanced, in general, $T_i \neq T_j$ for $i \neq j$.

Random Effects (RE). Two methods are commonly used to assess the effect of the unobserved heterogeneity: fixed effects (FE) and random effects (RE). Both methods assume that the differences across sections can be captured by differences in the intercept term. The fixed effects method is based on the estimation of section specific intercepts and, therefore, it does not provide much information about the population. On the other hand, the random effects method assumes that the intercept term is randomly distributed in the population. In this research, it was assumed that it is normally distributed so it can be characterized by a mean and a standard deviation.
The RE approach was used in this research for two reasons: (i) in the fixed effects approach the estimated intercepts are specific to the given sample and are not necessarily applicable to the population, and (ii) there is a high cost (reduction of degrees of freedom) associated with FE estimation, especially when the number of sections is large. Using the random effects approach (RE), the specification becomes:

$$p_{it} = \beta_1 + u_i + f\left(X_{it}, \beta_2, \ldots, \beta_{12}\right) + \varepsilon_{it} \tag{11}$$

u_i : section specific error, and

ε_{it} : overall regression error.

Results

The parameter estimates and their asymptotic t-values are given in Table 1. It is important to emphasize that the standard error of the original serviceability model (HRB, 1962) was approximately 1.0 PSI, which is almost double the value obtained in this research (the standard errors for the OLS and the RE are 0.50 and 0.52, respectively, as can be determined from Table 1). It is also important to note that the original serviceability model and the model developed in this research used the same data set and the same number of explanatory variables.

Table 1. Estimated parameters and corresponding statistics

Parameter	OLS estimates	t-value	RE estimates	t-value
β_1	4.45	57.1	4.24	165.4
β_2	-1.47	-16.5	-1.43	-8.9
β_3	-2.18	-6.2	-3.37	-8.4
β_4	.0898	14.1	.0547	17.6
β_5	.0305	10.8	.0129	14.4
β_6	.0215	11.3	.0107	15.2
β_7	-2.67	-29.5	-3.03	-35.2
β_8	-.00732	-49.0	-0.00680	-47.7
β_9	-.473	-39.8	-.512	-49.5
β_{10}	.790	22.3	.552	29.6
β_{11}	1.72	101.2	1.85	109.4
β_{12}	3.57	46.0	4.15	54.6
σ_ε^2	.248		.142	
σ_u^2	N/A		.126	

The parameters β_1, β_2, and β_3 represent the intercept term and the effect of initial asphalt thickness on the initial serviceability. As expected, the expected initial serviceability increases as the asphalt thickness increases, however, it never reaches the maximum theoretical value of 5.0 PSI.

The parameter of the surface asphalt layer (layer coefficient, β_4) estimated using the RE approach is more than four times the parameter of the untreated granular base layer (β_5). This indicates that the contribution of one millimeter of asphalt is equivalent to that of approximately four millimeters of granular base.

The signs of the parameters β_7, β_8, and β_9 are as expected. In addition, the three parameters are very significant.

The parameter corresponding to the exponent of the power law (β_{12}) is 3.6 for the OLS approach and 4.2 for the RE approach. This is significant, not only from the statistical point of view, but also for its impact on the allocation of the cost responsibilities of deterioration produced by the various vehicle classes. This effect is twofold. On one hand the damaging effect of heavy axles (>80 kN) is underestimated by the OLS approach and therefore the damage caused by heavy vehicles is greater than that OLS estimates. On the other hand, the damaging effect of lighter vehicles (axle loads less than 80 kN) is overestimated with the OLS approach, thus over-predicting the contribution of lighter traffic in the estimation of the design traffic.

The formulation of the equivalent load in terms of the EDF (though parameters β_{10}, β_{11}, and β_{12}) enabled equivalent loads for various axle and wheel configurations to be determined. The equivalent load of a single axle with single wheels is estimated to be 44 kN (RE approach). This value indicates that a single axle with single wheels with a 44 kN load has the same effect on serviceability as an 80 kN single axle with dual wheels. The equivalent load for a tandem axle with dual wheels is estimated to be 148 kN, which is close to the equivalent load used during the original analysis of the AASHO Road Test data (142 kN). It should be noted that this estimates are based on the experimental data used. Its application to actual in-service pavements is subjected to the validation and updating of the proposed models.

Conclusions

The results obtained with this model highlight the three most important aspects in the development of pavement deterioration models: (i) a *physically realistic model* specification, (ii) an *adequate data source*, and (iii) *statistically sound estimation techniques*. The model specification should be supported by engineering knowledge of the materials behavior under load and environmental conditions. The data should be obtained from a well-conceived experimentally designed test aimed at addressing all the important variables that have been identified during the development of the theory.

This research proved the importance of considering these three aspects when developing a model. The prediction error of the developed model was reduced by half although the same data set and variables were used. By halving the prediction error of pavement deterioration models, agencies can realize significant budget savings through timely intervention and accurate planning.

The model presented in this research, like any other deterioration model, is only an approximation of the actual physical phenomenon of deterioration. There is an error associated with the model, however, unlike deterministic models, this error can be estimated from actual data. The estimation of the error depends, however, on the assumptions made during the estimation process (e.g. OLS vs. RE). Different assumptions may lead to significantly different results, thus, especial attention should be given to the estimation procedure used.

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