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Probabilistic models for tunnel construction risk assessment

Olga Špačková [a,b,*], Eva Novotná [a], Michal Šejnoha [a], Jiří Šejnoha [a]

[a] Faculty of Civil Engineering, Czech Technical University in Prague, Czech Republic
[b] Engineering Risk Analysis Group, Faculty of Civil, Geo and Environmental Engineering, Technische Universität München, Germany

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ABSTRACT

The paper introduces different probabilistic models for prediction of tunnel construction risk. First, a simple probabilistic model for the estimation of the damage due to tunnel construction failures (e.g. cave-in collapses) is proposed. It can be used in conjunction with a deterministic estimate of the construction time/costs as a support for decision-making in tunnel construction projects. The occurrence of failures is modelled as an inhomogeneous Poisson process. The model takes into account the heterogeneity of the environment along the tunnel (changing geological conditions, changing damage potential) and it includes the influence of common factors such as human and organisational aspects. The damages caused by the failures are modelled as uncertain and they are thus represented by full probability distributions in the model.

Second, the decision-making under uncertainty in construction projects is discussed. The use of the concept of utility for considering the attitude of the stakeholder to risk is demonstrated. The simple probabilistic model and the decision-making concept are applied to a case study of construction of a 480-m-long tunnel.

Third, stochastic models for specific problems of tunnel construction, such as impacts of excavation on surface structures or probabilistic prediction of thickness of rock overburden, are introduced. The use of the models is illustrated on an example from Blanka tunnel in Prague.

1. Introduction

In compliance with ISO [8] the risk is defined as "the effect of uncertainty on objectives". To be more specific, the risk in this paper is perceived as an expected damage due to the construction process failures and damage stands for financial losses related to a delay in construction time and/or exceeding the construction budget. Alternatively, the risk is expressed as an expected utility. The following fields are mutually interconnected while estimating risk – mechanics and economics. The former provides us with phenomena giving rise to damage, such as cave – in collapse and a vast subsidence trough yielding extensive deformations of the surface structures, and the latter arranges for the estimates of financial losses.

In current practice, the tunnel project risks are commonly analysed on a qualitative basis using different rating systems [4,1,17,6]. Such qualitative analysis is an irreplaceable basis for prioritizing the risks, for the development of risk treatment strategies and for allocating the responsibilities [24]. However, the major decisions made during planning and construction of the infrastructure should ideally be based on a consistent quantitative basis, i.e. on quantification of the risk [26].

There are several complex models for a probabilistic assessment of construction time and costs taking into account both the common variability of the construction performance and the occurrence of failures, for example Isaksson and Stille [7], Moret and Einstein [15] or Špačková and Straub [23]. These models are able to account for the high complexity of the construction process and provide a detailed quantitative analysis of construction uncertainties, but they require gathering of a significant amount of input information and, as such, their application is not always justifiable in practice.

In some cases it can be sufficient to disregard the common variability of the construction process and to analyse only the effect of construction failures. For example Sousa and Einstein [18] introduce a dynamic Bayesian networks (DBN) model for the quantification of the risk of construction failure. The model includes the uncertainty in the geological conditions but it does not consider the uncertainty in the damage caused by a failure. Eskesen et al. [4] assess the expected value of construction risk using, in principle, the same procedure as in the qualitative risk assessment. Arguably, this approach is likely to lead to an incorrect estimation of the total risk. The reason is that the identified hazards are often overlapping, they are not identified on the same level of detail and the

* Corresponding author at: Engineering Risk Analysis Group, Faculty of Civil, Geo and Environmental Engineering, Technische Universität München, Germany.
E-mail address: olga.spackova@tum.de (O. Špačková).

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relations amongst them are not described. Therefore, a pure summation of the individual risks in the database is not possible, as they do not fulfil the condition of mutual exclusivity.

Some models analyse specific failure mechanisms. For example, Jurado et al. [12] estimate the probability of ground water related hazards using the Fault Tree Analysis (FTA). Sturk et al. [26] present assessment of the probability of environmental damages, namely of damages to trees during the tunnel construction. Šejnoha et al. [16] presented a FTA for estimating the probability of the occurrence of cave-in collapses and their consequences, especially of excessive deformations of surface structures. These models primarily focus on the probability of failure, they do not quantify the overall risk.

This paper suggests a simple probabilistic model suitable for assessing total damage related to the tunnel construction (Section 2). Application of the simple model is useful for example in early phases of the project when many different alternatives of the tunnel project are considered and one has only a little information about these alternatives. The model may also be sufficient for the assessment of risk in smaller tunnel construction projects where application of the complex models mentioned above is not justifiable. The model takes into account inhomogeneity of the geological conditions along the tunnel, the uncertainty in the prediction of damage, and allows for human and other external factors, all introducing dependencies into the construction process. Especially the latter two aspects have not been addressed by most of the available models. A simplified version of the model was previously published in Špačková [19].

Even if the damage caused by tunnel construction is correctly quantified, a common framework how to use this information for making decisions (e.g. for the selection of construction technology, for the selection of a contractor, as well as for the allocation of resources) is missing. The problem of decision-making under uncertainty in the construction projects has not been appropriately studied so far. A complex study of this topic exceeds the scope of this paper. We thus only briefly introduce the concept of utility and its use in decision-making (Section 3).

A tunnel constructed in the Czech Republic, denoted here as "TUN 3", was selected to demonstrate the applicability of the model. A complex case study was deliberately split into application examples 1–5 to concisely illustrate theoretical approaches suggested in the specific sections.

Detailed stochastic models for the prediction of undesirable impacts of the tunnel construction on surface structures are discussed in Section 4. The following three problems are analysed: the impact of cave – in on a surface structure, the inference of jointed rock in a compact layer above the tunnel, and the effect of randomly varying depth of the rock overburden which tends to decrease along the tunnel axis. These problems had to be solved within the construction of the Blanka tunnel in Prague as is shown in application example 6.

2. Simple model of damage due to tunnel construction failures

The occurrence of failures in the course of a construction process is modelled as inhomogeneous Poisson process (Section 2.1). The estimate of the number of failures is further updated after the construction starts and the actual performance is observed.

Section 2.2 presents a procedure for the assessment of damages caused by the tunnel construction failures. It is applicable for the case, where the surroundings of the tunnel are homogeneous from the point of view of the damage potential, and the damage can therefore be considered as independent on the position, where the failure occurs. Section 2.3 applies to the tunnels which pass under different regions (e.g. partly under an agricultural land and partly under an urban area). In such a tunnel the expected damage is highly dependent on the location where the failure occurs.

2.1. Number of failures

The probability of occurrence of k failures during the construction of a tunnel can be estimated as

$$\Pr[N_F = k|\lambda, L] = \frac{(\lambda L)^k}{k!}\exp(-\lambda L), \tag{1}$$

where N_F is the number of failures, L is the length of the tunnel, and λ is the failure rate, i.e. the number of failures per a unit length of the tunnel.

The probability of occurrence of one or more failures in the tunnel then equals

$$\Pr[N_F \geqslant 1|\lambda, L] = 1 - \Pr[N_F = 0|\lambda, L] = 1 - \exp(-\lambda L). \tag{2}$$

Eqs. (1) and (2) hold under following assumptions: (1) the probability of occurrence of two or more failures in one time/space unit is small, (2) the failure rate does not change in time/space, i.e. the process is homogeneous, and that (3) the number of failures in any interval of time/space is independent of the number of failures in any other non-overlapping interval of time/space, i.e. the process is memory-less. The assumption (1) is easily fulfilled when rare events are modelled, which is also the case of modelling tunnel construction failures. However, the assumptions (2) and (3) are likely to be violated in reality. Adjustments to the homogeneous memory-less Poisson process corresponding to the Eqs. (1) and (2) are therefore proposed.

Conditions affecting the failure occurrence vary along the tunnel axis due to the changes in geological conditions. The failure rate varies accordingly, i.e. the Poisson process is inhomogeneous. For modelling purposes, it is convenient to divide the tunnel into so-called quasi-homogeneous geological zones, i.e. sections for which the failure rate is considered to be constant. The probability of occurrence of k failures then equals

$$\Pr[N_F = k|\lambda, L] = \frac{\left(\sum_{i=1}^{i=n_Z}\lambda_i L_i\right)^k}{k!}\exp\left(-\sum_{i=1}^{i=n_Z}\lambda_i L_i\right), \tag{3}$$

where L_i is the length of the ith quasi-homogenous zone, λ_i is the failure rate within this zone and n_Z is the number of quasi-homogenous geological zones in the tunnel; $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{n_Z}\}$ and $L = \{L_1, L_2, \ldots, L_{n_Z}\}$. The average failure rate for the whole tunnel is:

$$\bar{\lambda} = \frac{\sum_{i=1}^{i=n_Z}\lambda_i L_i}{L}. \tag{4}$$

The construction performance and the occurrence of failures are influenced by human, organisational and other external factors. These factors affect the failure rate and introduce dependencies into the construction process (violation of the assumption 3). To give an example, the selection of a less experienced construction company or a suboptimal construction technology is likely to lead to higher failure rate. The general performance of the construction company and the appropriateness of the technology are uncertain in the planning phase, therefore, the parameters λ_i of the Poisson process are uncertain as well. After the construction starts, the performance observed in the first section of the tunnel influences the expectation about the performance (and related failure rate) also in the remaining part of the tunnel and we can thus update our predictions with these observations.

To include these dependencies into the model, we introduce a discrete random variable called "human factor" H. The human factor can be classified into three categories (states), "1: unfavourable", "2: neutral" and "3: favourable", and it is supposed to be in the same state throughout the entire tunnel construction. This simple model reflects the fact that the influence of this common factor cannot be directly measured and can only be deduced from the average performance over long sections of the tunnel excavation. The conditional probability of occurrence of k failures for a given $H = j$ equals:

$$\Pr[N_F = k|\lambda, L, H = j] = \frac{\left(\sum_{i=1}^{i=n_Z} \lambda_{ij} L_i\right)^k}{k!} \exp\left(-\sum_{i=1}^{i=n_Z} \lambda_{ij} L_i\right). \tag{5}$$

And, after un-conditioning, we arrive at:

$$\Pr[N_F = k|\lambda, L] = \sum_{j=1}^{j=n_H} \Pr[H = j] \frac{\left(\sum_{i=1}^{i=n_Z} \lambda_{ij} L_i\right)^k}{k!} \exp\left(-\sum_{i=1}^{i=n_Z} \lambda_{ij} L_i\right), \tag{6}$$

where λ_{ij} is the failure rate for the ith geological zone and the human factor H being in a state j, n_H is the number of states of variable H (here $n_H = 3$), L_i is the length of the ith geological zone, n_Z is the number of zones and L and λ encapsulate the parameters of the model such that $L = \{L_1, L_2, \ldots, L_{n_Z}\}$ and

$$\lambda = \begin{bmatrix} \lambda_{11} & \ldots & \lambda_{1n_H} \\ \vdots & \vdots & \vdots \\ \lambda_{n_Z 1} & \ldots & \lambda_{n_Z n_H} \end{bmatrix}. \tag{7}$$

Note that human factor H and the number of failures N_F are discrete random variables with probability mass functions (PMFs) $p_H(j)$, and $p_{N_F}(k)$, respectively. For example, the PMF of N_F is defined as $p_{N_F}(k) = \Pr[N_F = k]$. The PMF is arranged in a probability vector p_{N_F}.

At least three different approaches are available for estimating the failure rates in the matrix of Eq. (7): expert judgment, reliability analysis and statistical approach using the data from constructed tunnels. Each of the approaches has its strengths and weaknesses. Ideally, multiple approaches should be employed and results should be compared and critically examined. The expert judgement was discussed for example in [20,21] and it turns out to be unreliable if it is not supported by data. The reliability of the construction process of a tunnel has not been, to the authors' knowledge, published in the freely accessible sources. Such analysis would require modelling of the different phases of the tunnel construction process in time, it would thus be computationally demanding and require detailed information about the structure and technology. Additionally, the reliability analysis might fail in modelling the influence of the quality of the works and organisation, because there is a lack of accurate information about the effect of these factors. In the example 1 we therefore use the statistical approach, which learns the failure rates from historic data.

2.1.1. Updating prediction with observed performance

Once the construction starts, we can deduce more information about the influence of common factors (human, organisational etc.) on the construction performance. This can be done probabilistically by updating the human factor H. For example, if we observe a failure after construction of a short section of the tunnel, the probability of an "unfavourable" human factor increases, indicating a systematic problem in the construction process. With this new information, a prediction of the number of failures for the remaining part of the tunnel construction is updated.

Let l_{OBS} be the length of the constructed/observed section of the tunnel and $n_{F,OBS}$ be the number of failures that occurred during the construction of this section. The updated PMF of human factor H is obtained as

$$\Pr[H = j|N_F = n_{F,OBS}] \propto \mathrm{L}(H = j|N_F = n_{F,OBS}) \Pr[H = j] = \Pr[N_F = n_{F,OBS}|\lambda_I, L_I, H = j] \Pr[H = j] \tag{8}$$

where $\mathrm{L}(H = j|n_{F,OBS})$ is the likelihood of $H = j$ conditionally on the observations, $\Pr[H = j]$ is the prior probability of H being in state j and $\propto$ is a sign for proportionality indicating that the updated PMF of H must be normalised to ensure that the sum of the PMF over all states of H is equal to one. The likelihood can be computed as the probability that $n_{F,OBS}$ failures occur in the observed section of the tunnel conditionally on $H = j$: $\Pr[N_F = n_{F,OBS}|\lambda_I, L_I, H = j]$, see Eq. (5). $L_I = \{L_1, L_2, \ldots, L_{n_{Z,I}}\}$ is the vector of lengths of geological zones in the constructed section of the tunnel, $n_{Z,I}$ is the number of these geological zones and λ_I is the matrix of failure rates corresponding to these zones, i.e. it is part of the matrix from Eq. (7). Note that the sum of lengths in L_I equals l_{OBS}.

With the updated PMF of H we can now update the prediction of the number of failures for the remaining section of the tunnel (compare with Eq. (6))

$$\Pr[N_F = k|\lambda_{II}, L_{II}, n_{F,OBS}] = \sum_{j=1}^{j=n_H} \Pr[\mathrm{H} = j|n_{F,OBS}] \Pr \frac{\left(\sum_{i=1}^{i=n_{Z,II}} \lambda_{ij} L_i\right)^k}{k!} \exp\left(-\sum_{i=1}^{i=n_{Z,II}} \lambda_{ij} L_i\right), \tag{9}$$

where $L_{II} = \{L_1, L_2, \ldots, L_{n_{Z,II}}\}$ is the vector of lengths of geological zones in the remaining section of the tunnel, $n_{Z,II}$ is the number of the geological zones, λ_{II} is the matrix of failure rates corresponding to these zones, $\Pr[\mathrm{H} = j|n_{F,OBS}]$ is the updated probability of H and n_H is the number of states of variable H.

2.1.2. Application example 1 – The number of failures

The analysed tunnel denoted as "TUN3" is 480 m long and it is built as part of an underground extension project. The cross-section are of the tunnel varies around 40 m^2. First section of the tunnel serves as an access tunnel and it will not be utilised after the completion of the project. Remaining section of the tunnel will be used as a ventilation plant and as a dead-end rail track.

The tunnel is constructed in homogeneous conditions of sandstones and clay stones. Based on the geotechnical survey, the tunnel is divided into seven quasi-homogeneous zones. The predicted borders of the zones are depicted in Fig. 1. The lengths L_i of the quasi-homogeneous zones are summarised in Table 1.

All states of the human factor H are assigned the same probability, i.e. $p_H = [1/3, 1/3, 1/3]$. The failure rates λ_{ij} for individual zones and human factors are summarised in Table 2. They are assessed based on analysis of data presented in Špačková et al. [22]. The data analysis exercises a significant uncertainty in the failure rate estimates: the estimated lower bound is of the order of 10^{-2} failures per km, more pessimistic estimates based on more detailed data from the Czech Republic are approximately 30 times higher. The analysed tunnel is constructed in favourable geotechnical conditions. Its cross-section area is rather smaller compared with common road and railway tunnels, indicating smaller instability problem of the tunnel. Additionally, the design of the supporting system is on the conservative side in this case. All this considered, the estimated failure is closer to a lower bound of the possible interval.

The probability distribution of the number of failures is assessed using Eq. (6). The result is shown in Fig. 2. The probability that one failure occurs during excavation is $\Pr[N_F = 1] = 0.018$; the probability that more than one failure occur is negligible.

2.2. Position independent damage

The failure of the tunnel construction can cause severe damages such as a delay of the project and financial loss. This section describes a general approach for estimating damage. We now assume

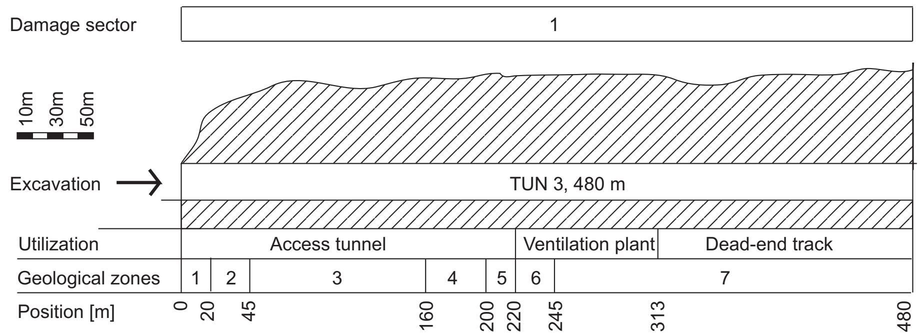

Fig. 1. Scheme of the modeled tunnel.

Table 1
Lengths L_i of geological zones in (m).

Zone	1	2	3	4	5	6	7
Length (m)	20	25	115	40	20	25	235

Table 2
Failure rates in (km^{-1}) for different zones and human factors.

Zone	1	2	3	4	5	6	7
H = "*unfav.*"	0.104	0.069	0.046	0.104	0.069	0.046	0.069
H = "*neutr.*"	0.052	0.035	0.023	0.052	0.035	0.023	0.035
H = "*fav.*"	0.026	0.017	0.012	0.026	0.017	0.012	0.017

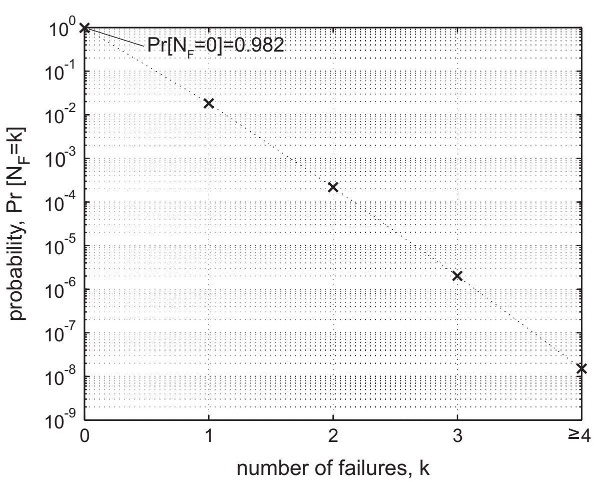

Fig. 2. Estimate of number of failures for tunnel TUN3.

that damages caused by individual failures are independent and that they do not depend on the position where the failure occurs.

The total damage due to tunnel construction failures equals

$$D_{TOT} = \sum_{i=1}^{N_F} D_i, \tag{10}$$

where D_i is the damage caused by the ith failure and N_F is the number of failures. Remember that N_F is a discrete random variable. D_i and D_{TOT} are continuous random variables with probability density functions (PDFs) $f_{D|N_{F_i}}(d|N_F = 1)$ and $f_{D_{TOT}}(d)$, respectively.

The PDF of the total damage D_{TOT} is calculated as

$$f_{D_{TOT}}(d) = \sum_{k=1}^{\infty} \Pr[N_F = k] f_{D|N_F}(d|N_F = k), \tag{11}$$

where $\Pr[N_F = k]$ is the probability that k failures occur throughout the whole tunnel, which is obtained from Eq. (6). $f_{D|N_F}(d|N_F = k)$ is the PDF of damage given that k failures occur, which is calculated as the convolution of the PDFs of individual damages [5]; [11]:

$$f_{D|N_F}(d|N_F = k) = f_{D_1|N_F} * f_{D_2|N_F} * \ldots * f_{D_k|N_F}(d|N_F = 1), \tag{12}$$

where $*$ denotes the convolution operator

Because damages are described by identical PDFs $f_{D|N_F}(d|N_F = 1)$, Eq. (12) can be calculated successively as

$$f_{D_{TOT}|N_F}(d|N_F = k) = f_{D|N_F}(d|N_F = k-1) * f_{D|N_F}(d|N_F = 1). \tag{13}$$

The convolution of Eq. (13) is computed as:

$$f_{D_{TOT}|N_F}(d|N_F = k) = \int_{-\infty}^{\infty} f_{D|N_F}(\delta|N_F = k-1) f_{D|N_F}(d-\delta|N_F = 1) d\delta. \tag{14}$$

2.2.1. Damage caused by a single failure

The PDF of damage caused by one failure $f_{D|N_{F_i}}(d|N_F = 1)$ can be obtained from historic data or assessed by experts. Statistical analysis of a delay due to tunnel construction failures is presented in Špačková et al. [22]. In application example 2 we apply the expert approach using the Event Tree Analysis (ETA), which allows us to analyse different scenarios caused by failure. The PDF of damage is here obtained as

$$f_{D|N_F}(d|N_F = 1) = \sum_{i=1}^{m_{Sc}} \Pr[Sc = i|N_F = 1] f_{D|Sc}(d|Sc_F = i), \tag{15}$$

where $\Pr[Sc = i|N_F = 1]$ is the conditional probability of the ith scenario given that one failure occurs, $f_{D|Sc}(d|Sc_F = i)$ is the PDF of the damage caused by the ith scenario (here modelled with two-parametric lognormal distribution), and m_{Sc} is the total number of scenarios obtained from ETA.

2.2.2. Application example 2 – A delay caused by failures

This example is a continuation of example 1. Here, we first estimate the delay initiated by a single failure, and then we proceed with an estimate of the total delay threatening the tunnel construction as a whole. The analysed tunnel "TUN3" is built in a city area. Sport centre, several residential buildings and the local infrastructure are located in the area being potentially influenced by the tunnel construction. Wherever a failure occurs in the tunnel, it is likely to cause a severe delay. The delay is thus considered to be independent on the position of the failure occurrence.

2.2.2.1. Delay due to one failure. The most likely types of the tunnel construction failures in the analysed tunnel TUN3 are cave-in collapse and excessive deformations of the tunnel tube. Because the tunnel is built in a developed area, the deformations of the overburden brought about by such a failure are likely to do a lot

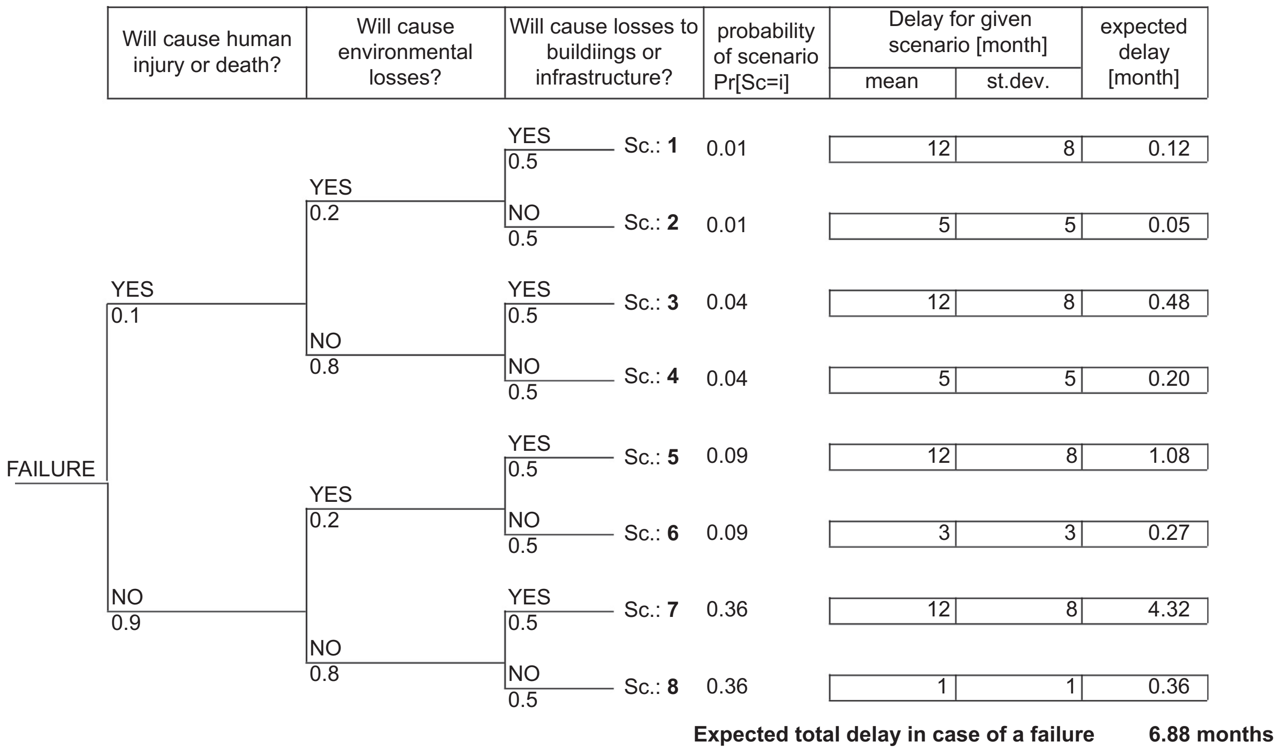

Fig. 3. ETA for the analysis of damage caused by a failure in tunnel TUN3. PDFs of delay for individual scenarios are depicted in Fig. 4.

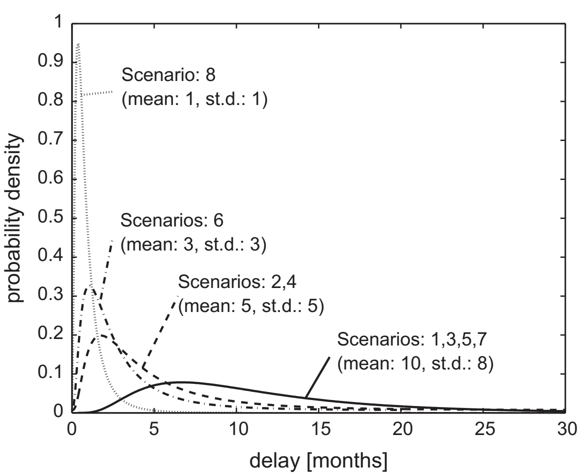

Fig. 4. Lognormal PDFs of delays of the construction process for different scenario caused by a failure, $f_{D|Sc}(d|sc)$.

of damage on the surface structures and infrastructure. Additionally, the failure can threaten the health and life of the workers and inhabitants and negatively influence the underground water and the environment. An Event Tree examining possible scenarios following a failure is shown in Fig. 3.

The measures taken after a failure occurs include for example reconstruction of the tunnel tube itself, reconstruction of the buildings and infrastructure, investigations into the failure event by authorities and a change of the design. The delay of the construction process is uncertain. For each scenario, the delay is described by a lognormal distribution with PDF $f_{D|Sc}(d|sc)$. The means and standard deviations of the delay resulting from different scenarios are summarised in Fig. 3; the respective PDFs are shown in Fig. 4.

ETA allows us to directly assess the expected delay caused by one failure, $E[D_{TOT}|N_F = 1] = 6.88$ months. The conditional PDF of delay given one failure occurs, $f_{D|N_F}(d|N_F = 1)$ is obtained using Eq. (15). It is depicted with the solid line in Fig. 5.

2.2.2.2. Total delay. Because the probability of two and more failures is small, the expected total delay can be approximated directly from the results of ETA (see Fig. 3) as $E[D_{TOT}] \approx E[D_{TOT}|N_F = 1]. \Pr[N_F = 1] = 6.88 * 0.018 = 0.124$ month.

To include also the possibility of occurrence of more than one failure and to assess the full probability distribution of the delay, $f_{D_{TOT}}(d)$, the procedure described in Section 2.2 is applied. The conditional PDFs of the total delay, $f_{D|N_F}(d|N_F = k)$, for $k > 1$, i.e. for more than 1 failure, are calculated using Eq. (13); the PDFs are depicted in Fig. 5.

Finally, the unconditional PDF of the total delay $f_{D_{TOT}}(d)$ is obtained using Eq. (11); it is shown in Fig. 6.

The expected value of the total delay shown in Fig. 6 is slightly higher than the one predicted using ETA. This difference is caused by the possible occurrence of two or more failures, which was not included in the ETA analysis. Note that the probability of zero delay equals the probability that no failure occurs (compare with Fig. 2).

2.3. Position dependent consequences

In the previous section we assumed that damage is independent of the position where the failure occurs. However, this assumption is not always valid. Many, mostly longer, tunnels pass under very different regions, as is illustrated in Fig. 7. In each region the expected damage differs.

To model the dependence of damage on the position of failure, we divide the tunnel into quasi-homogeneous damage sectors. The damage sector is a section of the tunnel where the damages caused by individual failures do not depend on the position where the failure occurs within this sector.[1]

[1] If just one damage sector along the whole tunnel is considered, i.e. $n_s = 1$, we arrive to the model presented in Section 2.2, which is a special case of the model described here.

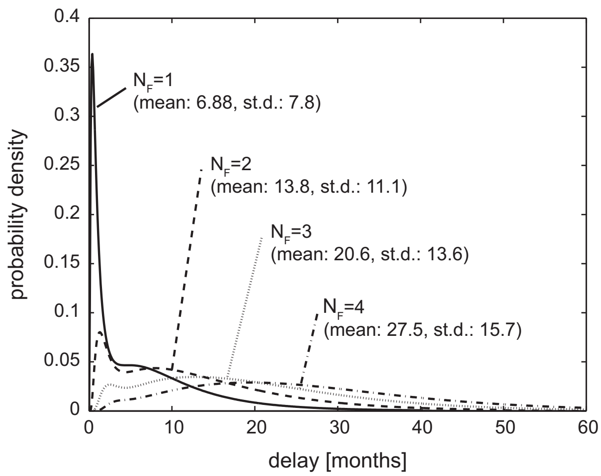

Fig. 5. PDF of delay caused by k failures, $f_{D|N_F}(d|N_F = k)$ for $k = 1, 2, 3, 4$.

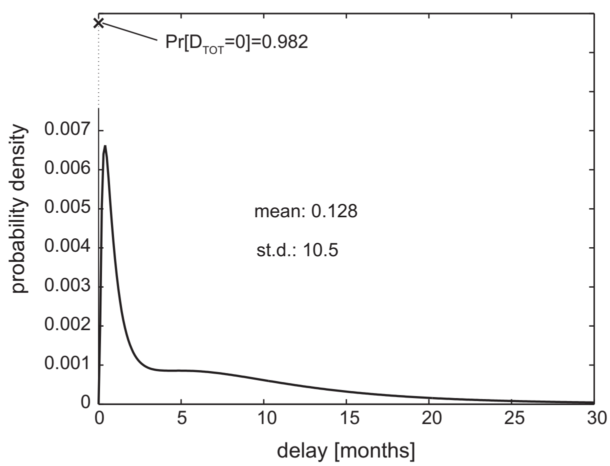

Fig. 6. PDF of total delay, $f_{D_{TOT}}(d)$.

Now the total damage can be expressed as (compare with Eq. (10)):

$$D_{TOT} = \sum_{l=1}^{n_S} D_{TOT}^{(l)} = \sum_{l=1}^{n_S} \sum_{i=1}^{N_F^{(l)}} D_i^{(l)}, \tag{16}$$

where $D_{TOT}^{(l)}$ is the total damage in sector l, $D_i^{(l)}$ is the damage caused by the ith failure in sector l, $N_F^{(l)}$ is the number of failures in the sector l, and n_S is the number of damage sectors which the tunnel passes. Analogously with Eq. (10), $N_F^{(l)}$ is a discrete random variable and $D_i^{(l)}$ and D_{TOT} are continuous random variables with PDFs $f_{D^{(l)}|N_F}(d|N_F = 1)$, and $f_{D_{TOT}}(d)$, respectively. The PDF of the damage $D_i^{(l)}$ caused by one failure for each of the damage sectors $l = 1, 2, \ldots, n_S$ can be assessed using ETA as specified in the application example 2. It can also be directly predicted by experts or determined from statistical data.

Because we assume fully dependent human factors during the construction process, the PDF of total damage D_{TOT} equals:

$$f_{D_{TOT}}(d) = \sum_{j=1}^{j=n_H} \Pr[H = j] f_{D_{TOT|H}}(d|H = j), \tag{17}$$

where n_H is the number of states of variable H and $f_{D_{TOT|H}}(d|H = j)$ is the PDF of the total damage for a given human factor $H = j$. This is calculated as convolution of the PDFs of the total damage in individual damage sectors conditionally on H

$$f_{D_{TOT|H}}(d|H = j) = f_{D_{TOT}^{(1)}|H} * f_{D_{TOT}^{(2)}|H} * \ldots * f_{D_{TOT}^{(n_S)}|H}(d|H = j). \tag{18}$$

The PDF of the total damage in sector l for a given H is obtained as

$$f_{D_{TOT}^{(l)}|H}(d|H = j) = \sum_{k=1}^{\infty} \Pr[N_F^{(l)} = k|S = l, H = j] f_{D|N_F,S}(d|N_F = k, S = l), \tag{19}$$

where $\Pr[N_F = k|S = l, H = j]$ is the probability of the occurrence of k failures in sector l given $H = j$ and $f_{D|N_F}(d|N_F = k, S = l)$ is a PDF of delay, which is caused by k failures in sector l. This PDF is obtained by the convolution of PDFs of individual damages for a given sector, $f_{D^{(l)}|N_F}(d|N_F = 1)$, analogously with Eqs. (12)–(14).

The probability of occurrence of k failures in sector l conditionally on $H = j$ reads:

$$\Pr[N_F^{(l)} = k|\lambda, L, S = l, H = j] = \frac{\left(\sum_{i=1}^{i=n_Z} \lambda_{ij} L_{il}\right)^k}{k!} \exp\left(-\sum_{i=1}^{i=n_Z} \lambda_{ij} L_{il}\right), \tag{20}$$

where n_Z is the number of geological zones, λ_{ij} is the failure rate within the ith geological zone for H being in a state j, λ is the matrix of failure rates shown in Eq. (7), and L_{il} is now the length of the ith geological zone within the damage sector l. To illustrate it on the example, in Fig. 7 only geological zones 2–4 coincide with the damage sector 2, therefore $L_{i2} > 0$ for $i = 2, 3, 4$ and $L_{i2} = 0$ otherwise. The lengths are organised in a matrix

$$L = \begin{bmatrix} L_{11} & \ldots & L_{1n_S} \\ \vdots & \ddots & \vdots \\ L_{n_Z 1} & \ldots & L_{n_Z n_S} \end{bmatrix}. \tag{21}$$

3. Utility theory in decision analysis

The methods introduced in the previous sections serve for modelling the failure probability and for quantifying damages. However, how can this knowledge be used for making decisions? How can the preferences of the decision makers be expressed in a consistent and quantitative way?

For this purpose, the concept of utility can be successfully applied [13]. Utility is an abstract measure of satisfaction. It is a formalised quantitative characteristic of the decision maker(s). Utility allows for modelling the decision maker's attitude to uncertain parameters, for example his perception of the potential high losses or uncertain gains. Additionally, utility allows incorporating different criteria (attributes) into the decision analysis: Different quantities influencing the decision (e.g. costs, time, environmental impacts) can all be expressed as the dimensionless utility; the transformation is made in such a way that it reflects the relative importance of each criterion. If the utility is properly modelled, the option with highest expected utility is the most optimal one from the analysed set of options [2].

The concept of utility will be illustrated on transformation of gains/losses from a point of view of two entities: a construction company and an insurer. The example is adapted from Straub [25]. The utility functions are depicted in Fig. 8 for a range of loss/gain of -10^6 to 10^6 Euro. The construction company is significantly smaller than the insurer, a loss in order of 10^6 Euro is liquidating for the company, and the utility of high losses therefore decreases very quickly. The benefit from an additional income generally decreases with the wealth of the company. Hence, the utility function of the construction company is concave. For the insurance company, loss/gain in the displayed interval is an everyday reality.

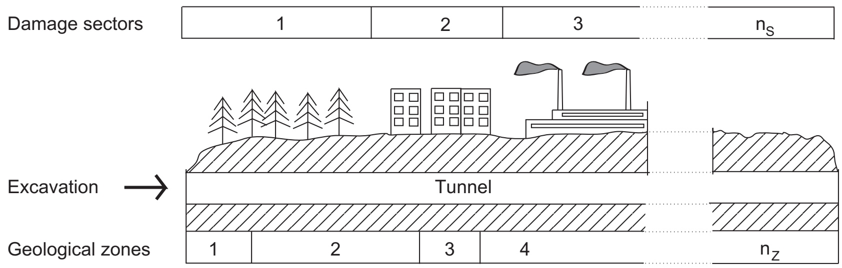

Fig. 7. Inhomogeneity of tunnelling conditions: illustration of different geological zones and damage sectors along the tunnel construction.

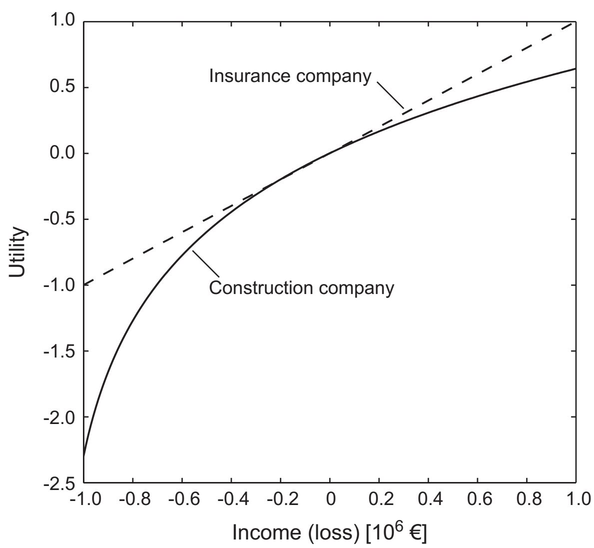

Fig. 8. Utility function of a construction company and a large insurer; from [25].

Indeed, the size of the insurer and the ability to cover losses of the clients is the core of its activity and the utility function of the insurer is thus linear in a given range.

Determining the utility function is a challenging task, which has not been studied sufficiently in the civil engineering field as yet. Generally, the utility can be determined from previous decisions of the company and/or it can be based on questioning the decision-makers using a set of so-called lottery questions.[2] Determining the utility for society or a public body is even more difficult. Moreover, the utility function is likely to change in time. In spite of the difficulties, introducing the utility concept into decision-making in civil engineering projects would allow making consistent and rational decisions.

In Fig. 8 the utility concept is only illustrated for monetary values. However, other non-monetary values such as humańs health and lives or environmental effects can also be expressed in form of the utility. Transforming and comparing different criteria is the topic of Multi-attribute utility theory (MAUT). MAUT is relevant for the infrastructure projects planning, but its proper discussion exceeds the scope of this paper. For more details the reader is referred to [13] or [11]. Applications of the utility concept in construction projects can be found for example in Dozzi et al. [3] or in Lambropoulos [14].

3.1. Application example 3 – Risk quantification

The construction risk for the tunnel "TUN3" introduced in application examples 1 and 2 will be investigated from the viewpoints of different participants involved in the construction process: (1) investor; (2) contractor; (3) the contractor evincing aversion to higher losses.

The risk is modelled as a function of delay. The uncertainty in the normal performance is not considered, the delay due to failures is therefore assumed to be the addition to the deterministic estimate of the construction time, which equals 200 days. This deterministic estimate was accepted by the contractor and agreed in the contract. The estimates of financial losses and selected monetary values used in the following examples only serve for illustration.

3.1.1. Case 1: Risk to the investor, R_1

The risk is analysed from the investor's prospective. A majority of the financial losses due to failures (i.e. reconstruction of the tunnel and the overburden, damage to the third party property, compensations to people) is transferred to the contractor. In spite of the transfer of the risks, the delay of the commencement of tunnel operation leads to additional monthly costs $C_1 = 100,000€$ to the investor. Here, C_1 includes the costs of traffic disruption, compensations for disturbing effects in the vicinity of the construction site and the costs of the investor's resources used for managing and controlling the construction project. The risk to the investor is expressed as the expected financial loss:

$$R_1 = E[B_1] = E[D_{TOT}] \times C_1 = 12,800€, \tag{a}$$

where $E[D_{TOT}] = 0.128$ months is the expected total delay due to failures (see Fig. 6) and $E[B_1]$ is the expected value of the investor's financial loss B_1.

3.1.2. Case 2: Risk of the contractor, R_2

The risk is analysed from the perspective of the contractor. The contractor is insured against the direct financial loss caused by the construction failures. The insurance covers the costs for reconstruction of the tunnel and overburden, damage to the third party property and compensations to the injured people. The deductible of the contractor is $\epsilon = 10,000€$ for every failure.

The prolongation of the construction brings, however, additional monthly costs $C_2 = 150,000€$ that are not covered by the insurance. These costs consist of the costs of labour and machinery bound to the project and of the penalty the contractor must pay to the investor in case of a delay. The financial loss of the contractor

[2] To illustrate this let's set up the following problem: The company can either get 5000 with certainty or participate in a lottery, where it can win 10,000 with a probability p or loose (get 0) with the probability $(1 - p)$. The decision-maker is asked to give the value of p for which both options have the same utility (i.e. the decision-maker is indifferent between the options). If the company has a linear utility in this range, the manager should put $p = \frac{1}{2}$, because $\frac{1}{2}u(10000) + \frac{1}{2}u(0) = u(5000)$. A risk averse manager (with a concave utility function) would require $p > 1/2$, because for him the utility of gaining 10,000 is less than twice as big as the utility of 5000.

B_2 thus contains both the additional costs and contractor's deductible. Hence

$$R_2 = E[B_2] = E[D_{TOT}] \times C_2 + \sum_{k=1}^{\infty} \Pr[N_F = k] \times k \times \epsilon = 19,440€, \qquad \text{(b)}$$

where $E[B_2]$ is the expected value of the contractor's financial loss B_2 and $\Pr[N_F = k]$ is the probability of the occurrence of k failures.

3.1.3. Case 3: Risk to the contractor including risk aversion, R_3

The risk to the contractor is evaluated alternatively in the form of expected utility. Using the utility U, it is possible to take into account the fact that high losses are dangerous for the contractor because they threaten the company's operability, goodwill and liquidity. The danger resulting from the loss does not increase linearly with the height of the loss.[3] The contractor's utility function is:

$$U = -B_3^w, \qquad \text{(c)}$$

where B_3 is the contractor's financial loss, i.e. the overrun of the budget allocated for a given project, and $w = 1.1$. The utility function can be determined by analysing the previous decisions and by questioning the company managers. The shape of the utility function reflects the possible consequences of an unsuccessful project, for example the increase of the price of insurance, if the company becomes more risky with regard to the insurer, the need of using operational loan, if the company's liquidity is insufficient, or the costs of delayed payments to the subcontractors and suppliers.

Because the deductible of the contractor ϵ is significantly smaller than C_2, we can neglect the dependence of the financial loss on the number of failures and express it as a function of delay (compare with Eq. (b)):

$$B_3 = C_2 \times D_{TOT}. \qquad \text{(d)}$$

The utility is also a function of delay:

$$U = g(D_{TOT}) = -(C_2 \times D_{TOT})^w. \qquad \text{(e)}$$

The dependence of loss B_3 and utility U on the delay is displayed in Fig. 9. Note that the delay D_{TOT} and utility U are random variables and C_2 and w are constants. The expected value of the utility then equals [2]:

$$E[U] = E[g(D_{TOT})] = \int_{-\infty}^{\infty} g(\xi) f_{D_{TOT}}(\xi) d\xi, \qquad \text{(f)}$$

where $f_{D_{TOT}}(\xi)$ is the PDF of D_{TOT} shown in Fig. 6.

The risk is obtained as a negative value of the expected utility:

$$R_3 = -E[U] = 81,590 \qquad \text{(g)}$$

3.2. Application example 4 – Decision-making

Let us assume that there is an alternative technology of construction of the tunnel denoted as technology B. This technology is cheaper but it has double failure rates than the original technology A (i.e. double the rates shown in Table 2). The costs saving resulting from the selection of technology B is 100,000 €. If the alternative technology is accepted, the contract price would be decreased by 40,000 €. Therefore, the costs saving for the investor is S_V = 40,000 € and the costs saving for the contractor is S_C = 60,000 €.

The delay due to failures and the associated risk for technology B are evaluated following the procedure described in the preceding application examples 1–3. The results for both technologies are summarised in Table 3.

[3] Note that in the cases 1 and 2 we implicitly assumed that the utility corresponds to the negative of financial loss, i.e. that the company/investor has a linear utility: $U = -B$.

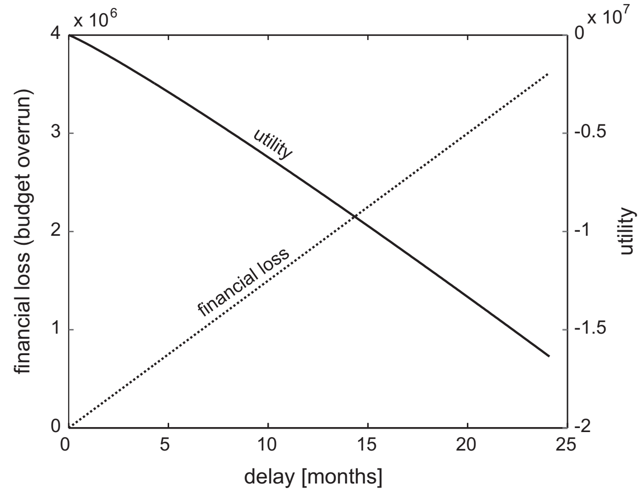

Fig. 9. Dependence of losses B_3 and utility U on the delay D_{TOT}.

The options are compared based on the difference of risk and cost savings as shown in the last row of Table 3. If for option B the cost saving is higher than the increase of risk with respect to option A, the option B is more advantageous. On the contrary, if for option B the cost saving is lower than the increase of risk, it is better to select option A.

As evident from the results, the decision is disputable. If the utility is considered to be linear to the potential losses (as in the cases R_1 and R_2), the option B appears to be more advantageous for both the investor and contractor. However, taking into account the contractor's aversion to higher losses, which is modelled by the power utility function (and included in R_3), the option B turns to be too risky for the contractor. In this case, the interests of the investor and contractor are contradicting and the contractor is likely not to accept the alternative technology (option B).

3.3. Application example 5 – Updating prediction with observed performance

Let us assume that the technology A was selected and the construction has started. After excavation of l_{OBS} = 100 m of the tunnel a failure occurs, i.e. $n_{F,OBS} = 1$. We now wish to update the prediction of number of failures and of the delay for the remaining part of the tunnel.

First we update the PMF of the variable human factor according to Eq. (8). The prior PMF of the human factor is $p_H = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$ – see application example 1. As shown in Fig. 1, the constructed section of the tunnel with length of 100 m ends approximately in the middle of the 3rd geological zone. Therefore the number of geological zones is $n_{Z,I} = 3$ and the length of the zones are $L_I = \{20, 25, 55\}$ [m] – compare with Table 1. The matrix of failure rates λ_I corresponds to the columns 1–3 in Table 2.

The updated PMF of the human factor $p_{H|N_{F,obs}}(j|n_{F,OBS})$ is shown in Fig. 10. Due to the observed failure, the probability of an "unfavourable" human factor increases indicating a worse than originally expected construction performance.

The updated PMF of the human factor is now used for predicting the number of failures in the remaining section of the tunnel, following Eq. (9). In the remaining section of the tunnel the lengths of the geological zones are $L_{II} = \{60, 40, 20, 25, 235\}$ [m] – compare with Table 1. The matrix of failure rates λ_{II} corresponds to the columns 3–7 in Table 2. The updated PMF of the number of failures is depicted in Fig. 11. Compared with the prior estimate for the whole tunnel shown in Fig. 2, the updated

Table 3
Comparison of alternative tunnelling technologies.

Technology	A			B		
Expected delay (months)	0.125			0.25		
Risk of	Investor R_1 [t. €]	Contractor R_2 [t. €]	Risk-averse contractor R_3 (utility) [–]	Investor R_1 [t. €]	Contractor R_2 [t. €]	Risk-averse contractor R_3 (utility) [–]
Risk	12.8	19.4	81.6	25.6	38.8	163.4
Cost saving against op. A	0	0	0	40	60	60*
Increase of risk against op. A	0	0	0	12.8	19.4	81.8
Cost saving – increase of risk	0	0	0	27.2	40.6	−21.8

* The utility of cost saving is considered to be linear, i.e. $U = S_C$.

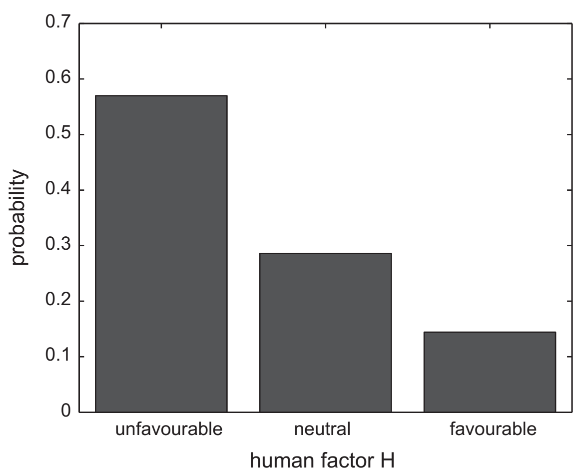

Fig. 10. Updated PMF of the human factor $p_{H|N_{F,obs}}(j|n_{F,OBS})$ with observations from construction of 100 m long section of the tunnel, where one failure was observed.

probabilities of failure change very little. However, remember that now the estimate applies only to the remaining part of the tunnel, which implies that the average failure rate in the remaining section of the tunnel is higher (due to the higher probability of "unfavourable" human factor).

The updated estimate of the number of failures can now be used as the input for assessing the delay in the remaining section of the tunnel following the procedure shown in application example 2. The updated PDF of a delay is shown in Fig. 12.

We can further update the estimate of risk for the remaining part of the tunnel according to the procedure described in application example 3; the results are shown in Table 4.

4. Impacts of tunnel collapse on surface structure

In foregoing sections we focused on simplified models for the quantification of risks to the tunnel itself and its environs without considering interactions with the surface structures. This section aims at modelling specific problems emerging in conjunction with the tunnel construction. Section 4.1 describes the effect of extraordinary subsidence trough evolved in the vicinity of cave-ins on an upper structure undergone by the tunnel. The subsequent paragraphs propose simple stochastic models to capture insufficient thickness of the sound overburden: Section 4.2 deals with jointed rock randomly interfering in the compact rock layer overlaying the tunnel and Section 4.3 addresses a related problem – the effect of a random interface between the sound rock layer and a soft material above it.

4.1. Probability of failure in surface structure due to cave-in collapse

Cave-in is a less frequent variant of possible accidents developing during tunnel excavation. However, a series of three collapses recorded within the construction of the Blanka tunnel in Prague implies that this mode of failure becomes a serious menace which needs to be properly analysed. To this end, an efficient methodology will now be developed by extending the approach proposed in the previous sections.

When estimating the costs induced by a cave-in collapse, two factors are to be taken into account. The most important one is a

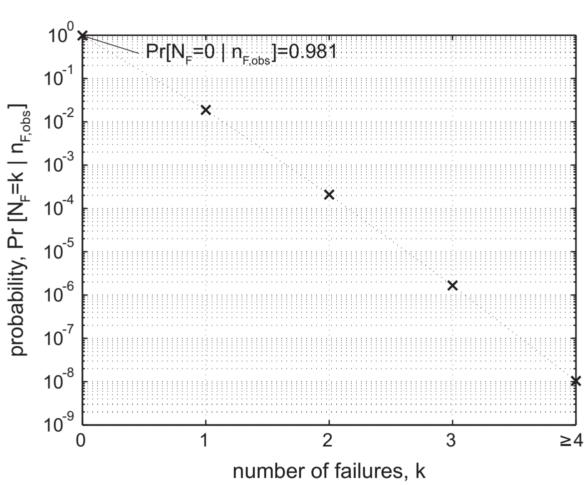

Fig. 11. Updated estimate of the number of failures in the remaining section of the tunnel TUN3 (380 m long).

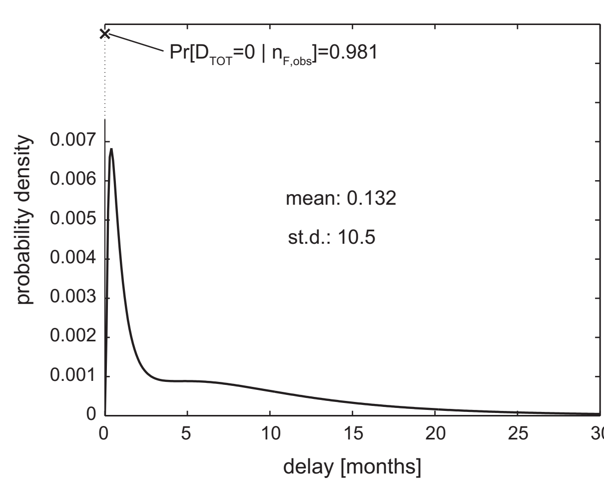

Fig. 12. Updated PDF of total delay D_{TOT} in the remaining section of the tunnel TUN3.

Table 4
Updated risk for the remaining section of the tunnel TUN3.

Technology	A		
Expected delay (months)	0.132		
Risk of	Investor R_1 [t. €]	Contractor R_2 [t. €]	Risk-averse contractor R_3 (utility) [–]
Risk	13.2	20.0	94.0

certain predisposition of the tunnel surrounding rock to fail. This property, along with other factors, gives rise to the cave-in collapse.

The sequence of collapses is further described by means of the Poisson model, i.e. a random variable distance between two collapses, U, is exponentially distributed with the intensity of occurrence λ as the only parameter. Then

$$F_U(u) = P(U \leqslant u) = 1 - e^{-\lambda u}. \tag{22}$$

The probability that a random position of impending cave-in falls in a small interval $\langle u, u + \mathrm{d}u\rangle$ is

$$f_U(u)du = P(u \leqslant U \leqslant u + du) = \lambda e^{-\lambda u}du. \tag{23}$$

If more detailed information on susceptibility of the surrounding soil to collapse is available, Eq. (23) deserves to be modified. Let P_1 be the probability that a cave-in occurs within an interval $\langle u_1 \leqslant U \leqslant u_2\rangle$, see Fig. 14, and

$$f_U(u)du = P_1 g(u)du, \quad u_1 \leqslant U \leqslant u_2 \tag{24}$$

is an expert surrogate for Eq. (23). Because

$$\int_{u_1}^{u_2} f_U(u)du = P_1 \int_{u_1}^{u_2} g(u)du = P_1 \tag{25}$$

the localisation function g fulfills this normalisation condition

$$\int_{u_1}^{u_2} g(u)du = 1. \tag{26}$$

A tentative shape of the localisation function is shown in Fig. 14.

The second factor relates to the failure of individual segments of a surface structure. The respective probability depends not only on the properties of the structure itself (geometrical arrangement, material properties etc.) but also on the position of the incoming cave-in with respect to the selected segment. Hereafter, this position is denoted by x, see Fig. 13.

The following two scenarios should be distinguished with regard to the cave-in location.

– The cave-in position is sufficiently far away from the selected segment. Then a discretized prediction of the subsidence trough may be obtained using the 2D/3D interaction approach proposed in Janda et al. [9] to arrive at the conditional probability of failure in the surface structure, $p_f(x|u) \leqslant 1$ given the position of cave-in is u.

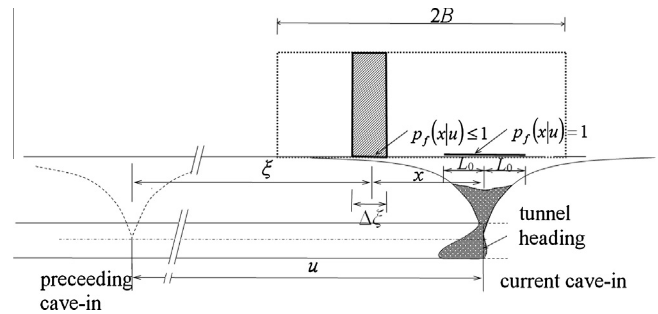

Fig. 13. Cave-in collapse under a long building.

– Particular attention must be drawn to the most dangerous scenario when a cave-in occurs under the segment itself and/or in its close vicinity ($|x| < L_0$). Then the simplest upper-bound estimate for the conditional probability reads $p_f(x|u) \approx 1$.

Making use of Eq. (23) and/or (24), the unconditional probability of failure in a selected segment $\xi = u - x$ becomes

$$p_f(\xi) = \int_{u_1}^{u_2} p_f(\xi|u) f_U(u)du + 2L_0 f_U(\xi). \tag{27}$$

The interval of integration has been extended by $2B$ in order to include an edge effect into the overall behaviour of the surface structure.

The overall risk depends on the number of failures recorded in individual segments of the structure. Here, a failure is understood as exceeding the structure's resistance by the loading effect. Despite the failures being mutually interconnected, we again adopt the Poisson model to estimate the measure of structure's damage, given the location u of cave-in is fixed. To this end, the smeared model of a row building well known from the 1970th [10] happens to be a suitable tool to interconnect both probabilistic and mechanics-based models. The intensity of failure in a segment of surface structure λ^* is a function of two variables, ξ and u. It relates to the conditional failure probability as

$$p_f(\xi|u) = \lambda^*(\xi|u)\xi, \tag{28}$$

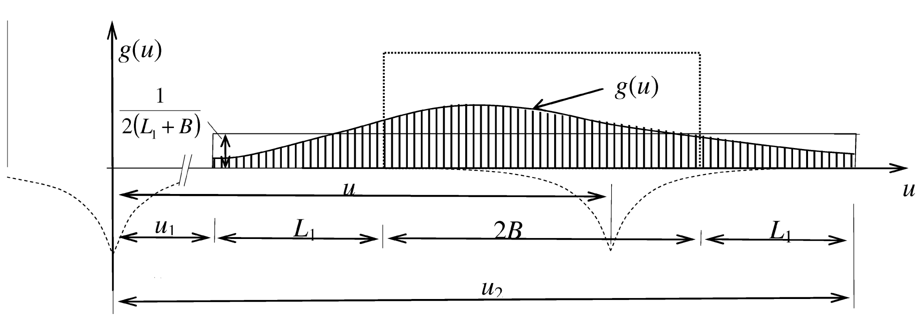

Fig. 14. Expert-based identification of cave-in collapse.

where $\Delta\xi$ is a very small width of the selected segment. Denote as

$$\Lambda^*(u) = \int_0^{2B} \lambda^*(\xi|u)d\xi, \tag{29}$$

the average number of failures provided the cave-in is located at u. Then the conditional probability that the number of failures (within a smeared model) is n can be expressed as

$$P(N = n|u) = \frac{[\Lambda^*(u)]^n e^{-\Lambda*(u)}}{n!}. \tag{30}$$

Finally, the estimate for the overall probability of the number of failures assumes this form:

$$P(N = n) = \int_{u_1}^{u_2} \frac{[\Lambda^*(u)]^n e^{-\Lambda*(u)}}{n!} f_U(u)du. \tag{31}$$

4.2. Stochastic modelling of the compact layer above tunnel disrupted by jointed rock

Let jointed rock interfere in the compact rock mass of the depth $h - a$. The basic question arises, what is the probability that a joint will disrupt compactness of the rock layer forming the intermediate overburden of the tunnel. We can observe the location of the bottom of a particular joint V (Fig. 15). Let N_f be the number of joints breaking the barrier to the compact layer (specified by depth a) with probability

$$p_f = P[V > a]. \tag{32}$$

If joints occur with a constant intensity λ, the probability that the number of barrier up-crossings is j on a length L reads

$$P[N_f = j] = \frac{(\lambda L p_f)^j e^{-\lambda L p_f}}{j!} \tag{33}$$

for p_f being a constant.

If the mean of location of the bottom joint end V is not constant, one proceeds as follows. First, introduce a new random function

$$\widetilde{V} = V - \mu_V(x), \tag{34}$$

such that $\mu_{\widetilde{V}} = 0, \sigma_{\widetilde{V}} = \sigma_V = \text{const}$. Then

$$p_f(x) = P[V(x) > a] = p[\widetilde{V}(x) > a - \mu_V(x)] = 1 - F_{\widetilde{V}}[a - \mu_V(x)] \tag{35}$$

Providing $p_f = p_f(x)$, Eq. (33) must be replaced by

$$P(N_f = j) = \frac{(\Lambda_f)^j e^{-\Lambda_f}}{j!}, \tag{36}$$

where

$$\Lambda_f = \int_0^L \lambda(x)p_f(x)dx \tag{37}$$

is the cumulative intensity and $\lambda = \lambda(x)$ is the intensity of a non-homogenous Poisson process.

Finally, the probability that within the tunnel length L at least one joint breaks the compactness of the rock layer is

$$P[N_f > 1] = \sum_{j=1}^{\infty} \frac{(\Lambda_f)^j e^{-\Lambda_f}}{j!} = 1 - e^{-\Lambda_f}. \tag{38}$$

4.3. Stochastic modelling of the compact layer above tunnel disrupted by soft material

Let us consider a randomly varying depth of the rock overburden which tends to decrease (see Fig. 16). To guarantee reliable excavation of the tunnel, the depth of the rock layer should not be smaller than a given limit $h - a$ (compare with Section 4.2).

There is a load of theoretical models suitable for the prediction of the first barrier up-crossing probability. Because of their mathematically tractable properties, the most common continuous process is the differentiable normal (Gaussian) process $V = V(x)$. For a constant level a, the barrier up-crossing rate, v_a^+, is given by Rice's formula

$$v_a^+ = \int_0^{\infty} \dot{v} f_{V\dot{V}}(a, \dot{v})d\dot{v} \tag{39}$$

where $f_{V\dot{V}}(v, \dot{v})$ is the joint probability density function of $V, \dot{V}$ and the dots denote differentiation with respect to x.

In the case of a non-stationary process (i.e. thickness of the bearable rock boundary along the tunnel axis) with a mean $\mu_V(x)$ and a constant standard deviation σ_V, the intensity of up-crossing the level a can be calculated as

$$v_a^+(x) = \frac{1}{2\pi}\frac{\sigma_{\dot{V}}}{\sigma_V} \exp\left[-\frac{(a - \mu_V(x))^2}{2\sigma_V^2}\right]. \tag{40}$$

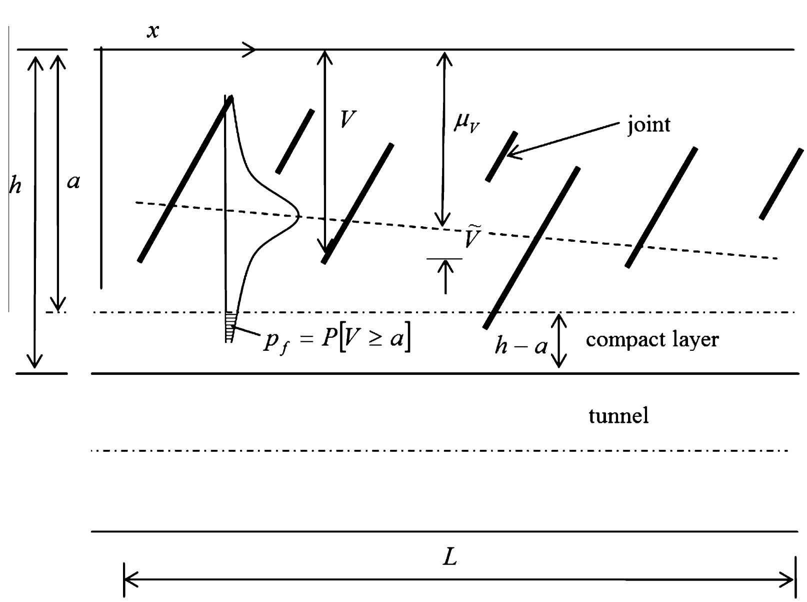

Fig. 15. Up-crossing of a barrier by joints.

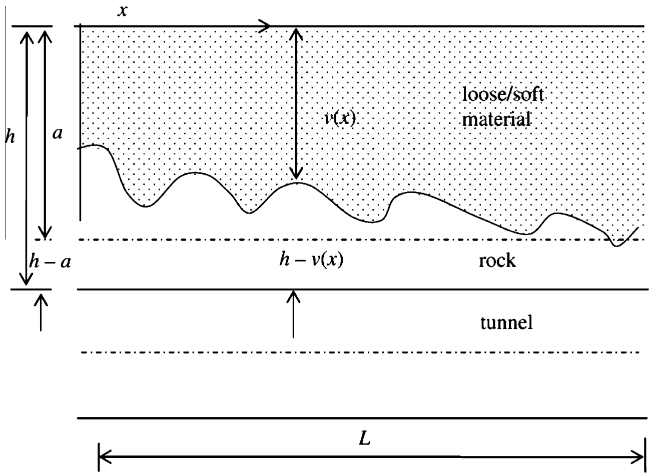

Fig. 16. Decreasing level of tunnel overburden.

Further, introduce the spectral density function $S_V(\omega)$ and corresponding relations

$$\sigma_V^2 = \int_{-\infty}^{\infty} S_V(\omega)d\omega, \quad \sigma_{\dot{V}}^2 = \int_{-\infty}^{\infty} \omega^2 S_V(\omega)d\omega. \tag{41}$$

After some manipulation and considering a narrow band process characterised by frequency ω_0 we arrive at $\sigma_{\dot{V}}/\sigma_V \cong \omega_0$.

Apparently, v_a^+ could also be considered as the intensity of a Poisson process. It is then possible to calculate the first-passage probability as

$$P[N_f > 1] = 1 - \exp[-\widetilde{v}_a^+ L] \tag{42}$$

where $\widetilde{v}_a^+ = (1/L)\int_0^L v_a^+(x)dx$.

The number of failures on a given length which is then

$$N_f(v_a^+(x), L) = \int_0^L v_a^+(x)dx. \tag{43}$$

4.3.1. Application example 6 – Disrupting of compact layer

Stochastic model discussed in Section 4.3 was applied to the excavation process on a segment of the Blanka tunnel in Prague.

The interface between the rock overburden and soft material detected by in situ measurements is displayed in Fig. 17. A solid line obtained by the linear regression demonstrates a variable mean function of Gauss' process.

The results of a case study depicted in Fig. 18 show sensibility of the model to the wave length L_0. The standard deviation of Gauss' process σ = 0.841 m was evaluated numerically and the depth of the rock layer was drawn as $h - a$ = 6 m.

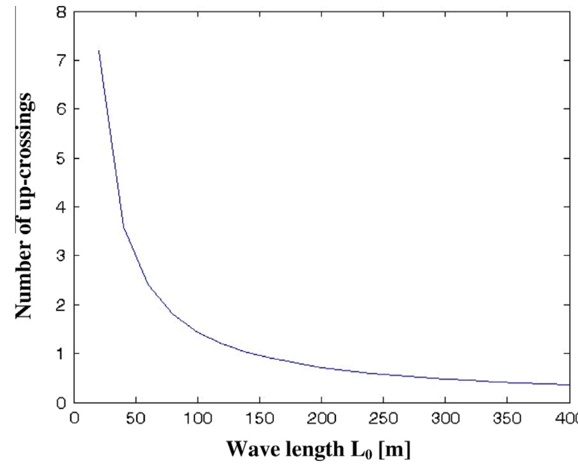

Fig. 18. The number of up-crossings of constant level as a function of wave length L_0.

5. Discussion and conclusions

The paper introduced different stochastic models for prediction of tunnel construction risk related to construction failures including for example cave-in collapses. The failures of construction process are a severe problem in the tunnelling and they should thus be considered when decisions are made in the course of the project. A decision-making framework based on utility theory was further discussed.

The simple model introduced in Sections 2 is intended for fast probabilistic estimates of damages (delay, financial loss) caused by the tunnel construction failures. The model takes into account the variability of the failure rate in different sections of the tunnel, depending on the changes of geotechnical conditions. The model further includes the epistemic uncertainty in the estimation of the failure rate. The epistemic uncertainty comes from our limited knowledge about the overall construction performance and about the influence of common factors such as human and organisational

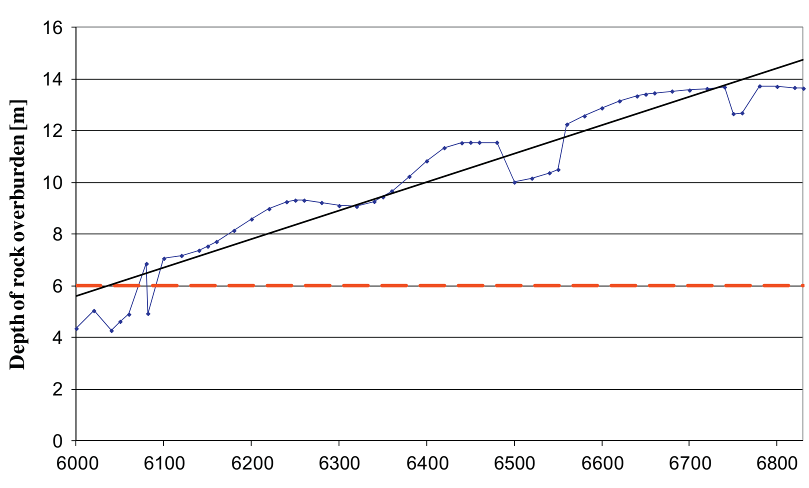

Fig. 17. Random boundary between rock overburden (bottom) and soft material (upper).

factors or other external effects. Impact of these factors is uncertain in the design phase giving rise to the uncertainty in the selection of the failure rates. The epistemic uncertainty is represented by a discrete random variable called "human factor". This uncertainty decreases once the construction starts and the variable can thus be updated with observed performance as illustrated in Fig. 10.

The proposed model further takes into account the uncertainty in the estimated damage ensuing from a failure (Fig. 3). So far, the consequences of the construction failures were commonly represented by their mean values or evaluated qualitatively. Estimating the mean value of damage is, however, not sufficient when the attitude of the decision maker to risk is to be included. The model also allows for considering the dependence of damage on the position, where the failure occurs (Section 2.3). This should be considered if tunnels pass under different regions (built up regions, agricultural region etc.). In application examples the dependence of damage on the position of failure is not included, because the modelled tunnel TUN3 is built in a relatively homogeneous environment (Fig. 1).

The concept of utility and decision making-under uncertainty is introduced in Section 3. A quantitative approach to decision-making is not common in the construction projects. This paper thus aims at demonstrating how the probabilistic estimates of the construction time/cost obtained from available models (e.g. the one introduced in Section 2) should be utilised in the tunnel project management. In the application example 4, the optimal tunnelling technology is selected in regards to the expected risk and cost (Table 3). The risk is quantified for three cases. In cases 1 and 2, the risk is defined as the expected damage. In case 3, the risk is defined as a negative value of expected utility. By introducing the utility, the aversion of the contractor to high financial losses can be taken into account. To include the risk aversion, the full probabilistic estimate of damage, which can be obtained from the model proposed in Section 2, is needed. However, to realistically capture the whole complexity of the decision problems in construction projects, additional research is necessary. The optimal quantitative decision concept, possibly based on the utility theory, should be proposed. The decision concept should allow for taking into account the measurable criteria, such as construction time and cost or maintenance costs, as well as soft criteria covering environmental or social impacts.

Probabilistic problems related to the failures of surface structures and ensuing from the impacts of the tunnel excavation and mutual interactions are discussed in Section 4 along with other specific tasks. Throughout the paper the occurrence is modelled as an inhomogeneous Poisson process, (alternatively as Gauss process), thus allowing for heterogeneity of the environment along the tunnel (changing geological conditions, changing damage potential), incl. variable geometry of geological layers.

Estimation of model parameters is a crucial part in the probabilistic modelling. The failure rate in tunnel construction and the damage caused by the failures have not been studied systematically in the past and cannot be assessed reliably from available data as discussed in Špačková et al. [22]. To obtain a realistic prediction from the probabilistic models, including the ones presented in this paper, a systematic analysis of data from construction projects is necessary. To achieve this, the understanding of benefits of probabilistic modelling among stakeholders should be raised to motivate them to more systematically manage and statistically analyse data from available projects.

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