Stochastic Duration Modeling of Pavement Overlay Crack Initiation

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Abstract: In this paper we present the development of an empirical pavement overlay crack initiation model using condition survey data for the highway system in the state of Washington. We use a semiparametric Cox model and show that its hazard function represents the sample hazard function, which follows a bathtub shape. This is a shape that commonly used parametric models cannot predict. Empirical data complications, such as the endogeneity of overlay design variables and data censoring, are corrected for using appropriate econometric techniques. Our research shows that a specification that captures the main factors responsible for the overlay crack initiation process, combined with careful analysis of the data, can produce models of sufficient realism for maintenance and rehabilitation action selection purposes.

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Introduction

Infrastructure management systems (IMS) are decision-support tools that aid public agencies in planning maintenance activities of their facilities. A complete IMS facilitates the following tasks: facility inspection and data collection, deterioration prediction through performance models, and maintenance, repair and reconstruction (MR & R) policy selection over the planning horizon.

Several IMSs have been developed and applied to actual infrastructure networks. The Arizona pavement management system (PMS) was implemented in the 1980s with estimated savings of about $200 million in maintenance and rehabilitation costs in 5 years (OECD 1987). Pontis, a system for maintenance optimization and improvement of bridge network, has been used effectively for bridge improvement and maintenance planning in 40 states in the United States (Golabi and Shepard 1997). In California, $188 million of pavement rehabilitation contracts were awarded in the 2002–2003 fiscal year (California Department of Transportation 2003). There is a potential for these expenses to be reduced if an IMS is developed and implemented.

Performance models are a core component of PMS. There are two types of empirical performance models used in PMS: models based on field data, and those based on experimental data. Experimental data allow researchers to define and select most variables relevant to pavement performance. In addition, they tend to have fewer measurement errors. Experimental data however are likely to suffer from biases, as they do not represent the true deterioration mechanisms of pavements. Data from actual in-service pavement sections subjected to the combined actions of highway traffic and environmental conditions are more representative of the actual deterioration process.

However, models based on field data also have some limitations. The most common problems encountered in models developed from in-service pavement sections are caused by unobserved events, such as data censoring (Small and Winston 1988), the presence of serial correlation among contiguous sections, and biases caused by the use of endogenous variables as explanatory variables (Prozzi and Madanat 2004). These problems can be addressed using proper statistical techniques such as those that will be discussed in this paper.

Several crack initiation models have been developed in the literature. As part of the AASHO road test, an accelerated loading test experiment, a crack initiation model was developed (HRB 1962). The crack initiation model uses traffic repetitions as the dependent variable and pavement thickness and load type as explanatory variables. The AASHO model suffered from several significant problems: the analysis did not account for right censoring that occurs when pavement sections have not cracked by the end of the experiment. If censoring is not accounted for correctly in the statistical estimation of model parameters, the estimates can be expected to be biased (Greene 1997). Censoring is discussed in more detail in this paper. In addition, the form of the AASHO crack initiation model was arbitrary, and one of the explanatory variables consisted of the sum of two quantities with different units. A number of authors have addressed the problem of censoring in pavement performance modeling, including Small and Winston (1988), and Prozzi and Madanat (2000).

The GEIPOT (1982) and Queiroz (1981) models have separate regression equations that predict crack initiation and the rate of crack progression. The crack initiation model used the number of equivalent single axles loads to initiation as the dependent variable, and the structural number (SN), defined as a function of the thickness of pavement layers, as the explanatory variable. Several important explanatory variables, such as climate variables, were not included in these models.

Some models of pavement cracking have combined the initia-

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tion and progression phases in a single function. In the Kenya road transport cost study, a combined model for cracking and patching initiation and progression was developed (Hodges et al. 1975; Parsley and Robinson 1982). Crack initiation and crack progression, however, are generated by different physical mechanisms, and a separate model for each is more appropriate.

Paterson's empirical work, based on data from the HDM project, was one of the most comprehensive attempts to develop performance models for different types of pavement distresses (Paterson 1987). Paterson developed separate models for roughness progression, rutting progression, and crack initiation and progression. Paterson used stochastic duration modeling instead of ordinary least squares (OLS) regression to develop the crack initiation model. Shin and Madanat (2003) developed a stochastic crack initiation model based on using experimental data from the AASHO road test.

The objective of this paper is to develop an empirical model for initiation of longitudinal and alligator cracking in asphalt concrete pavement overlays. Overlay cracking occurs due to a combination of different types of cracking such as: fatigue cracking, which is caused by a combination of traffic and environmental cyclic stresses; and reflection cracking, which occurs due to the propagation of cracking from underlying asphalt layers. There has been little research in the literature on the subject of reflection cracking, and thus its physical mechanisms are not well understood. This calls for the use of empirical methods, rather than mechanistic analysis, to develop models of pavement overlay crack initiation. For the purpose of this paper, pavement maintenance data from the Washington State Pavement Management System are used.

Methodology

The initiation of pavement distress is a highly variable event; that is, distress occurs at different times at various locations along a homogeneous piece of road. Hence, the time of distress initiation should be represented by a probability density function rather than by a point estimate. Stochastic duration models are thus a sound approach for modeling distress initiation.

Most stochastic duration models that have been developed to predict pavement crack initiation have assumed a Weibull distribution for the hazard function. Its advantage is that it allows the modeling of lifetimes having constant, strictly increasing, and strictly decreasing hazard functions. Parametric models like the Weibull model allow, in many cases, closed form estimation of the survival and hazard functions. Moreover, they can be interpreted in a direct and simple manner, and are easy to use for prediction purposes. Parametric models, however, impose restrictions and structure on the survival and the hazard rate functions. Such restrictions might not be appropriate in certain cases when the data are more complex, as is often the case with field data.

In these cases, more general models that impose less structure on the survival and the hazard functions are more appropriate. Semiparametric models allow this flexibility; the Cox model, also known as the proportional hazard model, is one of the most flexible and the most used models in the semiparametric family of models. In the Cox model, no assumptions regarding the form of the hazard function are necessary for the analysis, which is the main reason behind the flexibility of this model. In this paper, the Cox model is used, because as will be later seen, it is a more appropriate model to predict the empirical hazard function distribution of the data.

Hazard Rate Models

Define T as the cumulative equivalent single axle loads (ESALs) to cracking of a pavement or overlay, where T=random variable that takes values in the interval $(0, \infty)$. It has a cumulative distribution $F(t)$ and a density function $f(t)$. $F(t)$ is given by (Meeker and Escobar 1998)

$$F(t) = \int_0^t f(s)ds = \text{Prob}(T \leq t) \tag{1}$$

The probability that cracking occurs after time t is given by the survival function

$$S(t) = 1 - F(t) = \text{Prob}(T \geq t) \tag{2}$$

We define $g(t)$ as the probability that a pavement cracks in the next small interval, Δt, given it lasts at least until time t

$$g(t) = \text{Prob}(t \leq T < t + \Delta t | T \geq t) \tag{3}$$

The instantaneous rate of change of $g(t)$ defined as the hazard rate function $h(t)$ is given by

$$h(t) = \lim_{\Delta t \to 0} \frac{g(t)}{\Delta t} = \frac{f(t)}{S(t)} \tag{4}$$

The hazard rate quantifies the instantaneous risk that the pavement sections crack at time t.

Censoring

Censoring occurs frequently in condition survey data because it is sometimes impossible or impractical to observe the lifetimes of all the pavement overlays. A censored observation occurs when the length of time from construction to crack initiation, i.e., the lifetime of the overlay, is only known to be greater than, or less than, a known bound. A *complete data set* is when all overlays' lifetimes are known. A data set is called *censored* if there are one or more censored observations. There are several types of censoring. The most frequent type, in studies of pavement performance, is *right censoring.* Right censoring occurs when there are one or more pavement sections for which the lifetime is known only to be longer than the observation time (Leemis 1995).

Our data set consists of observations of time to crack initiation for the last overlay built for each pavement section in the sample. Thus, we know the year when the overlay was built for all sections. On the other hand, not all of these overlays had cracked by the time of the last condition survey. Overlays that have not cracked by the end of the last condition survey are right censored. The other observations were treated as complete, though in fact they are interval censored. That is because inspections are conducted annually, and thus for a section i that had not cracked at time $t-1$ and cracked at time t, all we know is that it cracked in the interval $[t-1, t]$. By treating interval-censored observations as complete observations we, in effect, assumed continuous condition surveys. Though this leads to some loss of precision, it is justified in that the inspection interval, 1 year, is significantly shorter than the life of a typical overlay in our sample. Therefore, the effect of this approximation on the accuracy of our results should be small.

We used the method of maximum likelihood to estimate the parameters of the hazard rate model. The likelihood function for such a case with n observations is obtained by multiplying the respective contributions of values of density function f for uncensored observations and values of survival function S for right-censored observations

$$L = \prod_{\delta_i=1} f(t_i) \prod_{\delta_i=0} S(t_i) = \prod_{i=1}^{n} [f(t_i)]^{\delta_i} [S(t_i)]^{1-\delta_i} \tag{5}$$

where δ_i=dummy variable that takes the values 1 if observation i is uncensored; and 0 if it is right censored (Kalbfleisch and Prentice 2002).

The Log likelihood function is

$$l = \text{Log}(\text{L}) = \sum_{i=1}^{n} \{\delta_i \,\text{Log}[f(t_i)] + (1-\delta_i)\text{Log}[S(t_i)]\} \tag{6}$$

Cox Model

The Cox model consists of two parts: a parametric component and a nonparametric component. The researchers specify the parametric component as a combination of explanatory variables that is log linear in the parameters. The nonparametric component is not restricted to take any distribution; rather, it is the empirical curve that best fits the variation in the data with respect to time. The flexibility of the nonparametric component is behind the many advantages of the Cox model over purely parametric models (such as the Weibull, the gamma of log-logistic models). An additional advantage is that the Cox model does not assume homoscedasticity, because the nonparametric component is distribution free; thus the model is estimated while relaxing the homoscedasticity assumption required for estimating purely parametric models. The Cox hazard function is given by (Crowder et al. 1991)

$$h(t) = h_0(t)\Psi(\mathbf{x}) \tag{7}$$

where the nonparametric component $h_0(t)$=arbitrary unspecified baseline hazard function which will be estimated. The parametric component is given by

$$\Psi(\mathbf{x}) = e^{\mathbf{x}'\boldsymbol{\beta}} \tag{8}$$

where $\mathbf{x}$=vector of explanatory variables; and $\boldsymbol{\beta}$=vector of parameters that will be estimated by maximum likelihood.

The cumulative distribution function, the density function, and the survival function are, respectively, given by

$$F(t) = 1 - [S_0(t)]^{\Psi(\mathbf{x})} \tag{9}$$

$$f(t) = f_0(t)\Psi(\mathbf{x})[S_0(t)]^{\Psi(\mathbf{x})-1} \tag{10}$$

$$S(t) = [S_0(t)]^{\Psi(\mathbf{x})} \tag{11}$$

where S_0, and f_0=baseline survival and density functions, respectively, and are equal to $S(t)$, and $f(t)$, respectively, when $\Psi(\mathbf{x})=1$, $(\mathbf{x}=\mathbf{0})$

The baseline hazard function is related to the baseline survival and density functions

$$h_0(t) = \frac{f_0(t)}{S_0(t)} \tag{12}$$

Endogeneity of Explanatory Variables

An explanatory variable X is said to be endogenous in a model to predict Y, if a vector of covariates $\mathbf{Z}$ used in the model of Y, was first used for the model of X.

Mathematically, the endogeneity of an explanatory variable can be formulated as

$$X = f(\boldsymbol{\beta},\mathbf{Z},\mathbf{U}) + \varepsilon_1 \tag{13}$$

$$Y = f(\boldsymbol{\alpha},X,\mathbf{Z},\mathbf{V}) + \varepsilon_2 \tag{14}$$

where X=endogenous variable; $\boldsymbol{Z}$=vector of explanatory variables used for the prediction of X and Y; $\boldsymbol{U}$=vector of explanatory variables used only for the prediction of X; $\boldsymbol{V}$=vector of explanatory variables used only for the prediction of Y; $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$=vectors of parameters to be estimated; and ε_1 and ε_2=random error terms accounting for unobserved effects.

Econometrically, the endogeneity problem arises because dependency is likely to exist between the disturbance terms ε_1 and ε_2, which violates the main assumption of the random distribution of error terms. This leads to biased and inconsistent estimates of the vector of parameters $\boldsymbol{\alpha}$.

In order to correct for the endogeneity of a continuous explanatory variable, the method of instrumental variables (IVs) is used (Madanat et al. 1995). Basically, the IV method consists of regressing the endogenous variable X on $\boldsymbol{Z}$ and $\boldsymbol{U}$, and then using the predicted value of X as a substitute for the measured value for the estimation of the parameters of model [Eq. (14)].

When the endogenous variable X is a discrete variable generated by a choice process, the endogeneity correction can be done by computing the probability of X being selected (Madanat and Mishalani 1998). This approach is feasible assuming that the discrete selection of X can be represented by a multinomial logit (MNL) model. In order to obtain nonbiased estimates of $\boldsymbol{\alpha}$, the probability of choosing X, obtained from the MNL model, will be used as a substitute for the measured value for the estimation of the parameters of model [Eq. (14)].

Washington PMS Data Description

The Washington DOT has performed pavement condition surveys on a yearly basis since 1983. The condition surveys cover most of the state highways. Highways were segmented into 0.1-mi sections and the survey measured variables such as pavement cracking and maintenance activities. This allowed the identification of different pavement types (concrete, asphalt . . .), and the types of cracking (alligator, longitudinal, and transverse). The Washington PMS also includes traffic information. Climate variables were created based on climate contour maps for the state of Washington.

Following is a description of the relevant variables found in the Washington PMS database, as well as the climate variables that we created:

1. E-Long and E-Alli: existing longitudinal and alligator crack before rehabilitation, respectively. These variables represent the last measured cracking before the last rehabilitation activity was performed. They represent the distress level of the pavement before the overlay. They are important variables in modeling overlay cracking because part of overlay cracking is due to reflection cracking, which requires the existence of cracking in the previous pavement surface layer and its propagation through the overlay;

2. Long, and Alli: overlay longitudinal, and alligator crack, respectively. Each of these overlay cracks is reported on a yearly basis until the end of the experiment, which is defined by either the occurrence of another maintenance activity or the absence of more surveys (Northwest Pavencent Management Systems and Kay 1992);
3. Y-ESAL: traffic in ESALs in year 1999. This variable reflects a yearly traffic load in ESALs. It is important since it affects the choice of the overlay thickness and the material type of the highway sections;
4. CUM-ESAL: cumulative ESALs to initiation. CUM-ESAL is the sum of the ESAL from the year of the last overlay to the year when crack initiation occurs. If cracking does not occur by the end of the experiment then CUM-ESAL is the sum of the ESAL from the last overlay to the end of the experiment;
5. SURFTHK: layer thickness of the last overlay (ft);
6. ULT: sum of the thickness of the underlying asphalt concrete pavement layers (ft);
7. Untrthick: the thickness of the nontreated base (ft);
8. Actbthick: the thickness of asphalt concrete treated base (ft);
9. Pctbthick: the thickness of portland cement treated base (ft);
10. BA, AA: dummy variables that take the value of one if the material type of the overlay is "BA" or "AA" respectively, and 0 otherwise. The material types "BA" and "AA" are defined in the Washington PMS as asphalt concrete cement (ACP) which have the same binder type (AR4000W), but with different mix classes: "BA" is a class B mix and has a maximum aggregate size of 5/8 in. and is described as a standard mix, while "AA" is a type A mix which also has a maximum aggregate size of 5/8 in., but that is a higher grade mix with more fractured rocks;
11 Tmin: average monthly minimum temperature of the coldest month (December) in °C. Tmin is an important variable. Pavement cracking increases with low temperatures since the tensile stress caused by cold or rapidly declining temperatures sometimes exceeds the strength of the asphalt pavement;
12 Prep: annual precipitation (mm). Precipitation and moisture in asphalt pavements can cause significant loss of strength of the underlying granular layers and the subgrade, thus weakening support for the asphalt concrete layers. This will result in higher cracking;
13 FTCycle: annual number of freeze-thaw cycles (number of cycles per year). Water that accumulates in the voids and cracks of the pavement freezes and increases in volume creating more stresses and cracking;
14 FTprep: product of FTCycle and Prep: FTCycles are particularly bad for the pavement in the presence of water; the interaction of precipitation with freeze-thaw cycles is suspected to increase cracking. To reflect this in the model, the variable FTprep was created as the product of FTCycle and Prep;
15 Prob-ba, Prob-aa, Prob-other: the probability of choosing material types BA, AA, or some other type respectively. These variables will be further explained in the following section; and
16 Newoverlay1: instrumented overlay thickness (ft). This variable will be further explained in the following section.

Model

Endogeneity Bias Correction

When dealing with overlay crack initiation of in-service pavement sections, the main two explanatory variables that are likely to be endogenous are the overlay thickness and the pavement material type, since they are the main design variables. To apply the method of instrumental variables, one would like to have, ideally, the exact formula and the explanatory variables that were used to select these endogenous variables. Unfortunately this is not always available, and proxy variables that are likely to influence the overlay design can be used instead.

In order to correct for the endogeneity in the observed thickness of the overlay, we predicted the overlay thickness using Eq. (15), and named this predicted value Newoverlay1. This variable will be used as a substitute for the measured value of the overlay thickness in the crack initiation model

$$\begin{aligned}\text{Log}(\text{SURFTHK}) = \alpha_0 + \alpha_1 \text{Log}(\text{E}-\text{Alli}) + \alpha_2 \text{Log}(\text{actbthick}) \\ + \alpha_3 \text{Log}(\text{pctbthick}) + \alpha_4 \text{Log}(\text{untrthick}) \\ + \alpha_5 \text{Log}(\text{Y}-\text{ESAL}) + \alpha_6 \text{Log}(\text{ULT}) \\ + \alpha_7 \text{Log}(\text{tmin}) + \alpha_8 \text{Log}(\text{Ftprep}) + \varepsilon \qquad (15)\end{aligned}$$

where $\text{Log}(s)$=natural logarithm of s; $\alpha_0-\alpha_8$=parameters to be estimated; and ε=error term. SURFTHK, E-Alli, actbthick, pctbthick, untrthick, Y-ESAL, ULT, tmin and Ftprep were all defined in the previous section.

In order to correct for the endogeneity in the asphalt overlay material type, the probability of the agency choosing a certain material type given certain structural conditions, climate variables, and yearly traffic was computed using a MNL model. In the Washington PMS data, there were two dominant material types, BA and AA, which were described in the previous section. The other material types seem to have been used less often, and thus formed smaller fractions of the data. Thus, they were all grouped together in the "others" group, which was used as the reference or the base group, for the MNL model. Under this specification, the parameters of the base group "others" were set to zero. The MNL model is given by

$$\text{Prob}(i) = \frac{\exp(\nu_i)}{\sum_{j=1}^{J} \exp(\nu_j)} \qquad (16)$$

where $\text{Prob}(i)$=probability of selecting material type i; $\exp(s)$=exponential of s; J=set of all material types; i and j=indexes for the material types, where the material types are BA, AA, and others, as defined in the previous section. ν is defined by

$$\nu_i = \theta_0 + \theta_1 \text{ Newoverlay1} + \theta_2 \text{ Y}-\text{ESAL} + \theta_3 \text{ FTprep} \quad \text{for } i = BA \qquad (17)$$

$$\nu_i = \phi_0 + \phi_1 \text{ Newoverlay1} + \phi_2 \text{ Y}-\text{ESAL} + \phi_3 \text{ FTprep} \quad \text{for } i = AA \qquad (18)$$

$$\nu_i = 0 \quad \text{for } i = \text{other} \qquad (19)$$

where $\phi_0-\phi_3$ and $\theta_0-\theta_3$=parameters to be estimated, and Newoverlay1, Y-ESAL, and Ftprep were all defined previously.

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Table 1. Parameters Estimate of Overlay Thickness Regression

Variable	Coefficient	t statistics
Constant	4.24E−02	3.19E+00
Log(E-Alli)	3.28E−03	6.71E+00
Log(actbthick)	−2.86E−02	−3.24E+00
Log(pctbthick)	−7.64E−03	−1.74E+00
Log(untrthick)	−1.16E−02	−5.00E+00
Log(ULT)	−1.66E−02	−4.55E+00
Log(tmin)	−1.46E−02	−6.63E+00
Log(Ftprep)	5.66E−03	9.63E+00

Results and Interpretations of Endogeneity Corrections

Our models were developed using around 7,000 observations (1/3 of the data) from the Washington data source described earlier. Observations were chosen based on systematic sampling. (The first observation from every set of three observations was picked.) This sampling technique was used in order to reduce the risk of serial correlation in the data since the WSDOT database records are contiguous sections.

Results of Overlay Thickness Endogeneity Correction

sTypically, overlay design methods are based on three types of factors: predicted traffic, current condition of the existing pavement, and environmental factors. A higher yearly traffic load, higher existing cracking in the surface layer, higher precipitation, and freeze-thaw cycles will require a thicker overlay. Therefore, we expect that the signs of these variables' coefficients will be positive. As the minimum temperature increases, the occurrence of low-temperature cracking in the pavement decreases, which can be resisted by a thinner overlay. Likewise, a thicker base, and thicker previous asphalt concrete layers, strengthen the pavement structure and a thinner overlay is thus required to resist the same applied loads. Thus, we would expect these coefficients to be negative.

Table 1 shows the results of the estimation of the parameters of Eq. (15). The results of Table 1 are consistent with our intuition, in terms of the correctness of the signs. Furthermore, the t statistics show that each variable is a significant explanatory variable for the selection of overlay thickness at the 10% significance level. While the goodness-of-fit measures presented in Table 2 suggest that the model prediction power is relatively poor (low R-squared values), it should be emphasized that the model of Eq. (15) is an auxiliary model used to correct for the endogeneity of explanatory variables and will not be used for prediction purposes.

Results of Material Type Endogeneity Corrections

Material type AA is stronger than material type BA because of its higher-grade mix. Thus, for pavements carrying higher loads, overlays are more likely to be designed using type AA mixes. On the other hand, overlay thickness can be a substitute for stronger materials; thus, thicker overlays are more likely to be built with material type BA. Yearly traffic as well as the interaction between freeze-thaw cycles and precipitation imply a higher probability of choosing type AA.

Table 2. Goodness-of-Fit Measures of Overlay Thickness Regression

Number of observations	R-squared	Adjusted R-squared	Root MSE
7,162	0.0659	0.0648	0.04365

Table 3 shows the results of the estimation of the parameters from the MNL models. The results of Table 3 confirm our expectations in term of the correctness of the signs. Moreover, the t statistics show that each variable is a significant explanatory variable of material type selection at the 5% significance level.

Crack Initiation Model and Results

In our duration model, the dependent variable is the number of Cumulative ESALs to failure, where failure is the occurrence of 5% alligator or longitudinal cracking, whichever occurs first. The 5% threshold was used instead of 0% because, in our data, we found instances where low levels of cracking ($<5\%$) were recorded in 1 year's survey and then appeared to disappear in subsequent years. We suspect that these instances are due to measurement errors. To avoid including such cases in our data set, we used a 5% threshold value.

The model specification we selected for the Cox hazard function is

$$\begin{aligned}\Psi(\mathbf{x}) = \mathrm{Exp}(&\beta_1\,\mathrm{E-Alli} + \beta_2\,\mathrm{E-Long} + \beta_3\,\mathrm{actbthick}\\ &+ \beta_4\,\mathrm{pctbthick} + \beta_5\,\mathrm{untrthick} + \beta_6\,\mathrm{ULT} + \beta_7\,\mathrm{Tmax}\\ &+ \beta_8\,\mathrm{Tmin} + \beta_9\,\mathrm{FTprep} + \beta_{10}\,\mathrm{Prob\text{-}ba} + \beta_{11}\,\mathrm{Prob\text{-}aa}\\ &+ \beta_{12}\,\mathrm{newoverlay1})\end{aligned} \tag{20}$$

The variables E-Alli, E-Long, actbthick, pctbthick, untrthick, ULT, Tmax, Tmin, and FTprep were defined in the "Washington PMS" Data Description section. The variables newoverlay1, Prob-ba, and Prob-aa are the instrumented variables for SURFTHK, BA, and AA, respectively, and were discussed above.

Our expectations of the effects of explanatory variables on the overlay life (measured in cumulative ESALs to failure) are described in this paragraph. We expect that a stronger structure will increase the overlay life. Accordingly an increase in the thickness of the overlay, an increase in the thickness of the untreated or treated base, and an increase in the thickness of underlying asphalt concrete layers, will increase the overlay life by increasing the strength of the pavement. An increase in the existing cracking before rehabilitation is expected to decrease the life of the overlay. We would also expect that as the minimum temperature increases, the occurrence of low-temperature cracking in the pavement decreases, which increases the life of the overlay. Precipitation is expected to decrease the life of the overlay and to accelerate cracking since water infiltrates to the granular layers and the subgrade and softens them, thus weakening support for the asphalt layers and rendering them more susceptible to cracking. Freeze-thaw cycles, in the presence of water from precipitation, should also decrease the life of the overlay since they lead to an increase in volume, widening existing cracks (see Table 4).

Table 5 shows the results of the estimation of the parameters of Eq. (20). The results of Table 5 confirm our expectations in term of the correctness of the signs. Furthermore, the t statistics show that each variable is a significant explanatory variable of crack initiation at the 5% significance level. The results indicate that the AA material type is better than the BA material type, but that they are both worse than the average material type in the sample. In addition, a treated base appears to be significantly more effective in extending the life of the overlay than a non-

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Table 3. Results of Multinomial Logit Model

Variable	Material type BA		Material type AA	
	Coefficient	*t* statistics	Coefficients	*t* statistics
Constant	3.35E+00	7.62E+00	4.04E+00	7.93E+00
Newoverlay1	−1.03E+01	−3.14E+00	−2.77E+01	−7.34E+00
Y-ESAL	−1.09E−06	−6.25E+00	1.11E−06	6.98E+00
Ftprep	−3.03E−05	−1.70E+01	−1.21E−05	−6.16E+00

treated base, and asphalt concrete treated bases appear to be slightly more effective than portland cement treated bases.

In order to evaluate the quality of the Cox model, the estimated hazard function (Fig. 2) was plotted and compared to the nonparametric estimate of the hazard function, i.e., the sample (empirical) hazard function (Fig. 1). It is clear that the model has good predictive power, because it predicts well the shape of the empirical hazard function. This comparison confirms the use of the Cox model as more appropriate than parametric hazard models. Parametric hazard models with a monotonic hazard function, such as the Weibull model, cannot predict this shape of the hazard function. The same is true of parametric models with a unimodal hazard function, such as the log-logistic model (see Table 6).

The survival function estimates indicate that around 15% of the sections survive past 500,000 cumulative ESALs, and less than 5% past 1,000,000 cumulative ESALs. Fig. 2 shows a hazard rate that decreases initially for cumulative ESALs less than 500,000 and then starts an increasing trend past this value. The hazard rate shoots high past 1,000,000 cumulative ESALs. This trend of the hazard rate suggests that weak overlays and overlays with low construction quality fail early, and those overlays that live long enough have a lower probability of failure until a certain point after which they deteriorate rapidly leading to a sharp increase in their probability of failure. This hazard rate shape is often observed in natural phenomena (example: the hazard function depicting human life) and is referred to as a bathtub hazard function. An explanation for this behavior of the hazard function is that sections that live long enough will be subjected to routine maintenance that will further extend their life, reducing their hazard rate. Past a certain life (500,000 cumulative ESALs) these sections start deteriorating too rapidly and routine maintenance becomes ineffective.

Table 4. Goodness-of-Fit Measures of Multinomial Logit Model

Number of observations	Pseudo *R*-squared	Likelihood ratio
7,162	0.065	945

Table 5. Cox Model Coefficients Estimate

Variable	Coefficient	*t* statistics
E-Alli	2.79E−02	1.38E+01
E-long	6.40E−03	1.13E+02
actbthick	−2.44E+00	−1.17E+01
pctbthick	−7.91E−01	−8.57E+00
untrthick	−5.37E−01	−1.33E+01
ULT	−5.50E−01	−7.99E+00
tmax	−5.63E−02	−1.21E+01
tmin	−1.88E−01	−2.43E+01
ftprep	6.62E−05	2.01E+01
newoverlay1	−3.31E+01	−1.08E+01
Prob-aa	8.73E+00	1.33E+01
Prob-ba	1.11E+01	1.64E+01

Model Predictions

In this section we illustrate the use of our performance model for infrastructure maintenance policy selection.

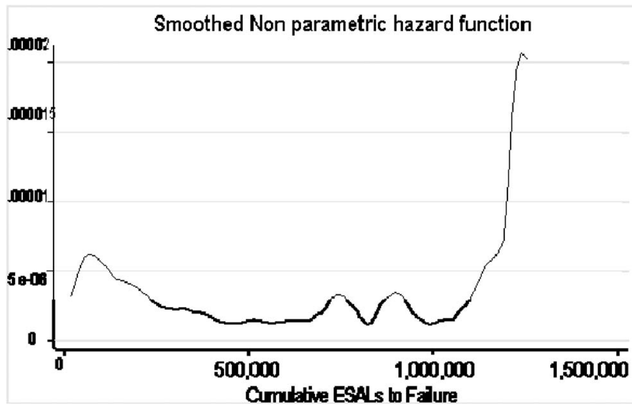

Fig. 1. Nonparametric plot of sample's hazard function (value on *X* axis is number of ESALs)

Table 6. Goodness-of-Fit Measures of Cox Model

Number of observations	Number of right censored observations	Likelihood ratio
7,162	1,563	1,706

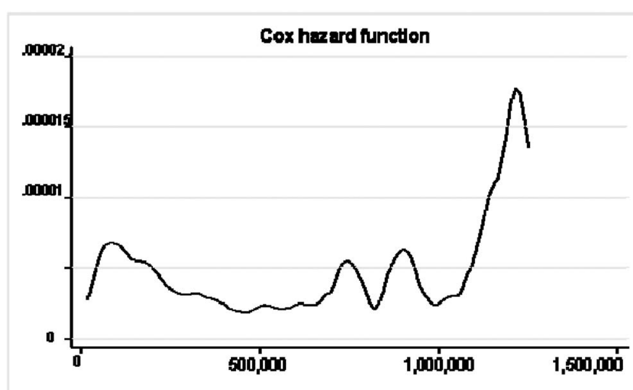

Fig. 2. Model prediction of hazard function (value on *X* axis is number of ESALs)

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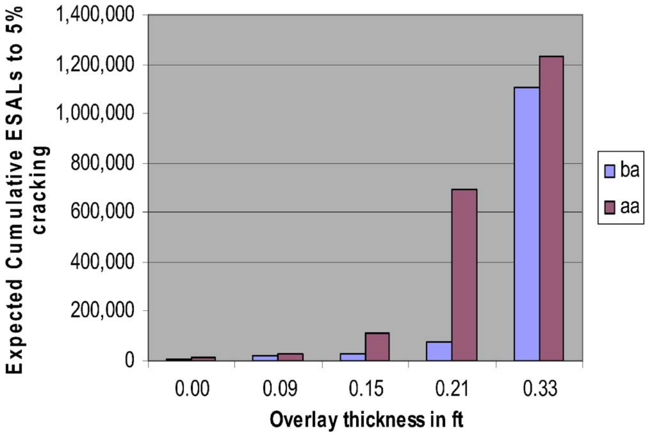

Fig. 3. Effect of AC overlay thickness on overlay life

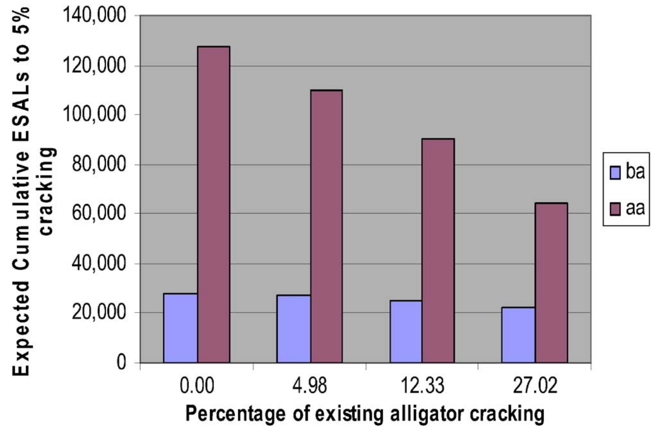

Fig. 5. Effect of percentage of existing alligator cracking on overlay life

We compute the expected cumulative ESALs to 5% cracking for each variable at its mean value in the sample, mean ± 1 SD and mean ± 3 SD, while keeping all other explanatory variables fixed at their mean values. For all graphs produced (Figs. 3–5), we have also varied the material type of the overlay (AA and BA). It should be noted that for some variables, the mean SD or mean 3 SD fall outside a meaningful range (such as a negative value for the overlay thickness) and are thus omitted from the graphs, and replaced by the minimum meaningful value within the range.

The results presented in Fig. 3 show that the overlay thickness has a large effect on the life of the overlay. The overlay material type is another very important variable: choosing a different material type can more than triple the life of the overlay. The thickness of the underlying AC layers also has a significant effect (Fig. 4). These findings are particularly important since surface thickness, material type, and the thickness of the underlying layers are among the main design variables. The decrease of overlay life with an increasing percentage of existing alligator cracking (Fig. 5) confirms the hypothesis that overlay cracking is at least partly due to reflection cracking.

Conclusions

This paper presents the development of a stochastic duration model for overlay crack initiation using data from the *Washington Sate Pavement Management System* (Washington State Department of Transportation Materials Lab 1999). Correction for the endogeneity bias problem, which is commonly present in data from in-service facilities, was also performed. A Cox model was developed, and the results were compared to nonparametric estimates of the hazard function. The Cox model appeared to be a good fit for the data.

The ability of the Cox model to capture the shape of the hazard function has engineering implications. Weibull models, which have been typically used in hazard rate modeling of facility deterioration, impose the assumption of monotonicity. Monotonically increasing hazard functions have been explained by the fact that materials degrade over time. Monotonically decreasing hazard functions are sometimes attributed to poor construction quality. In infrastructure facilities, both phenomena are important and present, but Weibull models or other models that impose the monotonicity assumption can capture only one of them. This often leads to misinterpretation of important observed phenomena. In this paper, we were able to capture both phenomena by using a semiparametric model, whose nonparametric hazard function captured the "bathtub" shape of the sample hazard function.

Result show the importance of existing cracking as an explanatory variable, confirming the hypothesis that reflection cracking is a component of overlay cracking. Structural variables such as overlay thickness, thickness of the underlying AC layers, and overlay material type, all have a large influence on the life of the overlay. These variables are the main design variables, and the proper choice of a combination of these variables can reduce the costs of maintenance and improve the life of overlays. While it does not include some variables such as routine maintenance and construction quality, this model provides a basis for selecting pavement rehabilitation policies.

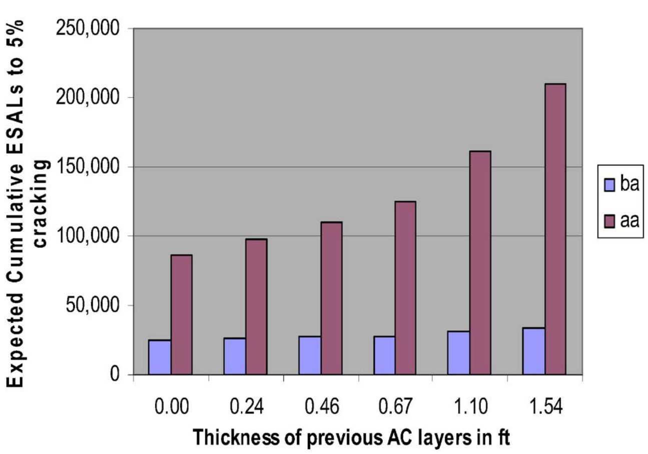

Fig. 4. Effect of thickness of previous AC layers on overlay life

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