Connectivity-Based Optimal Scheduling for Maintenance of Bridge Networks

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Abstract: This paper addresses the issue of connectivity- and cost-based optimal scheduling for maintenance of bridges at the transportation network level. Previous studies in the same field have considered the connectivity just between two points or other network performance indicators, such as the total travel time. In this paper, the maximization of the total network connectivity is chosen as the objective of the optimization, together with the minimization of the total maintenance cost. From a computational point of view, several numerical tools are combined to achieve efficiency and applicability to real cases. Random field theory and numerical models for the time-dependent structural reliability are used to handle the uncertainties involved in the problem. Latin hypercube sampling is used to keep the computational effort feasible for practical applications. Genetic algorithms are used to solve the optimization problem. Numerical applications to bridge networks illustrate the characteristics of the procedure and its applicability to realistic scenarios. **DOI: 10.1061/(ASCE)EM.1943-7889.0000271.** © *2013 American Society of Civil Engineers.*

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Introduction

All over the world, the economic resources allocated to the maintenance of the civil infrastructure are limited and usually very far from being sufficient for an adequate preservation of reliability and service levels required during the entire life cycle. For this reason, the optimal allocation of the available funding has become a goal of utmost importance. When dealing with transportation networks, resources must be distributed among the various components (in particular bridges), and the maintenance interventions should be scheduled in an optimal way (Frangopol and Bocchini 2012). A first study on the topic was performed by Augusti et al. (1998) that compared several maintenance strategies on bridges of a simplified transportation network. Liu and Frangopol (2005, 2006) proposed to use optimization techniques to improve the maintenance planning of a bridge network, considering deterioration, failure cost, maintenance cost, user cost, and connectivity within a simplified network model. Gao et al. (2010) formulated a network-level optimization for maintenance and rehabilitation of transportation systems with emphasis on pavements. Peeta et al. (2010) applied a two-stage stochastic optimization to the strengthening of highway networks with some simplifying assumptions, such as total independence of the failures. Orcesi and Cremona (2010) presented a method for bridge network maintenance optimization based on visual inspection and Markov chain theory. Bocchini and Frangopol (2011a) developed a general framework for the optimal scheduling of preventive maintenance (PM) actions on bridges of a transportation network, considering also the correlation among the bridge states, structural deterioration, complete traffic flow assignment and distribution, and the uncertainty involved in the problem. This latter paper focused on a metric for the network performance based on the ability to effectively redistribute flows and minimize the time spent and the distance covered by the users, whereas the present paper emphasizes the concept of connectivity.

When all the components of a transportation network are in service, it is possible to find a route to reach every node of the network (i.e., cities and intersections) from every other node. However, extreme events and aging (e.g., corrosion, material deterioration, fatigue) can set some bridges out of service, and this, in turn, can determine disconnections in the network. When natural (e.g., earthquakes, hurricanes, floods) or technological (e.g., large-scale terrorist attacks) extreme events damage a region, the possibility to reach all the points of the area using the highway system is of paramount importance for emergency response and the socioeconomic recovery of the region. For this reason, bridge maintenance plans should always consider the maximization of network connectivity as one of their main goals, even in the case of extreme events. In this paper, a general framework and a comprehensive computational procedure for the optimal scheduling of the bridge maintenance actions is presented at the transportation network level. The design variables are the years of application of the PM actions. The two conflicting objectives of the optimization are the minimization of the expected maintenance cost and the maximization of the network connectivity index.

Besides introducing the connectivity index, the originality of the proposed approach resides in its ability to merge and combine all the significant aspects of the problem that were previously considered only separately. In fact, the presented procedure includes the effects of deterioration on the structural reliability of the bridges, considers

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the different importance of the highway routes depending on their traffic evolution in time, takes into account the correlation of the in/out of service state of bridges belonging to the same transportation network, and handles the uncertainties involved in the problem with a complete probabilistic analysis.

The first numerical application is used to present and validate the procedure, whereas the second one proves its applicability to more complex networks.

Life-Cycle Bridge Reliability

The natural aging processes and external stressors cause the deterioration of construction materials and structural capacity. Some of the most common sources of deterioration for bridge structures and their components are steel corrosion, fatigue, and chloride penetration in concrete. Hence, it is well known that if no maintenance is applied, the time-dependent reliability index profiles of each bridge and of the entire network are monotonically decreasing (Frangopol and Liu 2007; Frangopol 2011; Bocchini and Frangopol 2011c). In a series of papers, Akgül and Frangopol (2004a, b, 2005a, b) presented a comprehensive study to assess the value of the reliability index profile $\beta(t)$ along the residual life of bridges of any kind. If such a study is not possible, for instance because of limited information, probabilistic models can be used. These models describe the time-dependent profile $\beta(t)$ as a function of random parameters whose probabilistic characterization can be assessed depending on structural characteristics and investigated limit state (Frangopol et al. 2001). The most popular models are the following (Bocchini and Frangopol 2011d):

$$\text{Bilinear:}\quad \beta(t)=\beta^0-\mathcal{H}\left(t-T^I\right)\times R^L\times\left(t-T^I\right) \tag{1}$$

$$\text{Quadratic:}\quad \beta(t)=\beta^0-\mathcal{H}\left(t-T^I\right)\times R^Q\times\left(t-T^I\right)^2 \tag{2}$$

$$\text{Square root:}\quad \beta(t)=\beta^0-\mathcal{H}\left(t-T^I\right)\times R^S\times\sqrt{t-T^I} \tag{3}$$

Exponential:

$$\beta(t)=\beta^0-\mathcal{H}\left(t-T^I\right)\times\left\{R^E\times\left(t-T^I\right)+Q-Q\times\exp\left[-\left(\frac{t-T^I}{S}\right)^2\right]\right\} \tag{4}$$

where β^0 = initial value of the reliability index; $\mathcal{H}(\cdot)$ = Heaviside step function; T^I = initiation time when the deterioration starts to affect the structural reliability; R^L, R^Q, R^S, and R^E = reliability degradation rates; Q = parameter that tunes the shape of the exponential model during the first years after T^I; and S = shape parameter associated with the position of the inflection point in the exponential model.

To counteract the deterioration process, PM actions are applied to every bridge. PM actions can have several effects on the reliability of the bridge (Kong and Frangopol 2003). The first effect is a constant improvement in the value of $\beta(t)$. This improvement is usually very limited and associated with the replacement or repair of a few damaged connections or components. The second effect of PM is a reduction of the deterioration rate for a limited time. This effect can be modeled by the superposition of a linear or quadratic function to the profile of $\beta(t)$. For instance, a silane treatment produces this type of effect. Finally the deterioration process can be arrested for a certain duration, and therefore the decrease in $\beta(t)$ is delayed. A maintenance action that produces this effect is the repainting of a steel bridge. In general, the effect of a PM intervention can be modeled as superposition of all the mentioned components

$$\text{Constant:}\quad \widehat{\beta}(t)=\beta(t)+\mathcal{H}(t-TPM)\times M_0 \tag{5}$$

$$\text{Linear:}\quad \widehat{\beta}(t)=\beta(t)+\mathcal{H}(t-TPM)\times\min[M_1\times(t-TPM),M_1\times D_1] \tag{6}$$

$$\text{Quadratic:}\quad \widehat{\beta}(t)=\beta(t)+\mathcal{H}(t-TPM)\times\min\left[M_2\times(t-TPM)^{0.5},M_2\times D_2^{0.5}\right] \tag{7}$$

$$\begin{aligned}\text{Delay:}\quad \widehat{\beta}(t)=&\ \mathcal{H}(TPM-t)\times\beta(t)+\mathcal{H}(t-TPM)\\&\times\mathcal{H}(TPM+D_0-t)\times\beta(TPM)\\&+\mathcal{H}(t-TPM-D_0)\times\beta(t-D_0)\end{aligned} \tag{8}$$

where $\widehat{\beta}(t)$ = modified reliability index that includes the effect of the PM actions; $\mathcal{H}(\cdot)$ = Heaviside step function; TPM = time of application of PM; M_0 = parameter that determines the magnitude of the constant improvement; M_1 and M_2 = rates of improvement for the linear and quadratic components, respectively; D_1 and D_2 = time intervals after which the linear and quadratic components become constant, respectively; and D_0 = time interval during which the deterioration is suppressed. Parameters M_0, M_1, M_2, D_0, D_1, and D_2 in Eqs. (5)–(8) are affected by uncertainty and are modeled as random variables. In fact, it is very difficult to accurately foresee the impact of a maintenance action on the reliability index, such as the exact duration of the deterioration delay as a result of repainting. Depending on the components that are included in the model, each PM intervention involves up to six random parameters.

Along its life cycle, each bridge can reach a serviceability limit state, and it might be necessary to apply rehabilitation and corrective maintenance (CM). This type of interventions is usually more expensive than PM and is aimed at restoring a bridge performance level as close as possible to the original one. In this paper, CM actions are modeled as a restoration of the original $\widehat{\beta}(t)$ profile, reduced by a random quantity

$$\tilde{\beta}(t>TCM)=\widehat{\beta}(t-TCM)-\Delta\times\left[\widehat{\beta}(0)-\widehat{\beta}(TCM)\right] \tag{9}$$

where $\tilde{\beta}$ = profile of the reliability index that includes the effect of CM; TCM = time at which the limit state is reached and rehabilitation is applied; and Δ = random variable.

The uncertainty introduced in the problem by all the random variables in Eqs. (1)–(9) is managed through simulation. To better represent the stochastic space with a limited number of samples, Latin hypercube sampling (LHS) is implemented. Bocchini et al. (2011) showed that in this kind of problem, LHS improves by an order of magnitude the convergence of the statistics of the output variables with respect to the number of samples. Using the samples generated for the random parameters and Eqs. (1)–(9), it is possible to compute samples $\tilde{\beta}_{b,i}$ of the reliability index for each bridge b.

Network Connectivity

Because of the aging effect and the occurrence of extreme events, the bridges of a transportation network can be out of service and require

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CM before the end of the prescribed service life. The in/out of service state of each bridge b of each sample i used for simulation is described by the binary variable

$$s_{b,i}(t) = \begin{cases} 0, & \text{if bridge } b \text{ of sample } i \text{ is out of service} \\ 1, & \text{if bridge } b \text{ of sample } i \text{ is in service} \end{cases} \tag{10}$$

The value of $s_{b,i}(t)$ depends on two factors: the reliability of the bridge and the history of external stressors to which it has been exposed (e.g., aggressive environment, traffic loads, and extreme events). Bridges belonging to the same transportation network experience the same extreme events, are subject to correlated traffic loads, are exposed to similar environmental conditions, and are likely to be designed and built with similar techniques. For these and other reasons, their in/out of service state is correlated, and it has been proved that such correlation can significantly impact the network performance indicators (Bocchini and Frangopol 2010, 2011b). This correlation is included in the model by the simulation of a random field $d(\mathbf{x},t)$, where the vector of coordinates $\mathbf{x}$ spans over the region covered by the transportation network, and in the time domain t only selected discrete values at distance t_{ana} are considered. Therefore, for each investigated time instant t, a set of two-dimensional random samples $d_i(\mathbf{x},t)$ is generated (Bocchini and Deodatis 2008; Bocchini 2008). The marginal distribution of field $d(\mathbf{x},t)$ is uniform in the interval (0,1), and the spatial autocorrelation can be assessed as suggested by Bocchini and Frangopol (2011b). The values of $d(\mathbf{x},t)$ can be compared with the reliability of the individual bridges to assess the in/out of service state (Bocchini et al. 2011)

$$s_{b,i}(t) = \mathcal{H}\left\{\Phi\left[\tilde{\beta}_{b,i}(t)\right] - d_i(\mathbf{x}_b,t)\right\} \tag{11}$$

where $\mathcal{H}\{\cdot\}$ = Heaviside step function; $\Phi[\cdot]$ = standard Gaussian cumulative distribution function; $\tilde{\beta}_{b,i}(t)$ = time-dependent reliability index associated with bridge b of sample i; and $d_i(\mathbf{x}_b,t)$ = value of the sample i for time t of the random field, calculated at the location $\mathbf{x}_b$ of bridge b.

When a highway bridge is out of service (i.e., $s_{b,i}=0$), the associated highway segment is out of service too. Thus, when several bridges are out of service some nodes of the network might be disconnected from others

$$L_{j,k,i}(t) = \begin{cases} 1, & \text{if } \exists \text{ at least one route in service from } j \text{ to } k \text{ for sample } i \\ 0, & \text{if } \nexists \text{ any route in service from } j \text{ to } k \text{ for sample } i \end{cases} \tag{12}$$

Given the in/out of service state of the bridges $s_{b,i}(t)$, the values of $L_{j,k,i}(t)$ can be easily assessed using algorithms developed for transportation engineering and graph theory, such as a simplified version of the Warshall-Floyd procedure (Warshall 1962; Floyd 1962; Dreyfus 1969).

In any transportation network, some highway connections are more important than others, because they carry higher traffic flows and/or connect important urban centers that absolutely need to be connected even after extreme events (e.g., health care facilities and emergency rooms with high capacity are located in large cities). To account for this aspect, an importance factor has been included in the model. The importance of each connection is assumed to be proportional to the amount of travels that it generates (in ordinary conditions, with all the bridges in service). To compute the origin-destination matrix **OD** that collects the number of car-equivalent vehicles that go from each node j of the network to each other node k in a fixed time interval (e.g., 1 h), a gravitational model is used (Levinson and Kumar 1994). The numbers of travels originated $O(t)$ and attracted $D(t)$ by each node in 1 h are given by

$$O_j(t) = O_j(0) \times \left(1+g_j\right)^t \quad \forall \text{ node } j \tag{13}$$

$$D_k(t) = D_k(0) \times (1+g_k)^t \quad \forall \text{ node } k \tag{14}$$

where g = annual growth rate of the city that accounts for the differential development and recession of the cities of the region, and t = time in years. The values of $O(0)$, $D(0)$, and g can be assessed depending on the urban characteristics and the population of the areas (McNally 2000) or by means of direct surveys. All the departing travelers have to reach a destination. To be compliant with this constraint, the value of $D_k(t)$ is normalized as follows:

$$\overline{D}_k(t) = D_k(t) \times \frac{\sum_j O_j(t)}{\sum_k D_k(t)} \tag{15}$$

Eq. (15) guarantees that the constraint $\sum_j O_j(t) = \sum_k \overline{D}_k(t)$ holds for every time t. Next, according to the gravitational model, the elements of the **OD** matrix can be assessed as

$$OD_{j,k}(t) = X_j^O(t)X_k^D(t)O_j(t)\overline{D}_k(t)\exp\left(\alpha^{OD} - \beta^{OD}\left|\mathbf{x}_j - \mathbf{x}_k\right|\right) \tag{16}$$

where $\mathbf{x}_j$ and $\mathbf{x}_k$ = locations of nodes j and k, respectively; α^{OD} and β^{OD} = parameters of the model, herein assumed equal to 0 and 0.15, respectively; $X_j^O(t)$ and $X_k^D(t)$ = balancing factors that can be computed by solving iteratively the following set of equations:

$$X_j^O(t) = \left[\sum_k X_k^D(t)\,\overline{D}_k(t)\exp\left(\alpha^{OD} - \beta^{OD}\left|\mathbf{x}_j - \mathbf{x}_k\right|\right)\right]^{-1} \tag{17}$$

$$X_k^D(t) = \left[\sum_j X_j^O(t)\,O_j(t)\exp\left(\alpha^{OD} - \beta^{OD}\left|\mathbf{x}_j - \mathbf{x}_k\right|\right)\right]^{-1} \tag{18}$$

Combining the information on the possibility to go from any node j to any other node k with the information on the importance of this connection, it is possible to assess the overall connectivity level $C_i(t)$ for each sample i

$$C_i(t) = \sum_j \sum_k L_{j,k,i}(t) \times OD_{j,k}(t) \tag{19}$$

where subscripts j and k run over all the nodes of the network. The value of the connectivity level is then normalized as

$$\overline{C_i}(t) = \frac{C_i(t) - C^0(t)}{C^{100}(t) - C^0(t)} \tag{20}$$

where C^{100} and C^0 = values of $C_i(t)$ when all the bridges are in service and out of service, respectively.

The time-dependent and sample-dependent connectivity values $\overline{C_i}(t)$ must be condensed in a synthetic (i.e., scalar) index to drive the optimization procedure. Among the various indexes that could be considered, the following are the most interesting. The first one is the minimum value, over time, of the expected connectivity index

$$netconn_1 = \min_t \left[\sum_{i=1}^{n_S} \frac{\overline{C_i}(t)}{n_S} \right] \tag{21}$$

where n_S = total number of samples. The maximization of index $netconn_1$ guarantees high connectivity levels over the entire life cycle. However, this index is not very sensitive to the distribution of $\overline{C_i}(t)$, therefore it does not fully exploit the available information. To overcome this issue, a second index is introduced. It is based on the definition of a limit state for the connectivity

$$g_{conn}(t) = \overline{C_i}(t) - \overline{C}_{\text{threshold}} = 0 \tag{22}$$

where $\overline{C}_{\text{threshold}}$ = minimum value that is considered acceptable for the connectivity level $\overline{C_i}(t)$. Based on $g_{conn}(t)$, it is possible to compute the annual probability of failure in terms of connectivity as

$$P_f^{\text{annual}}(t) = \frac{\text{number of samples with } g_{conn}(t) < 0}{n_S} \tag{23}$$

Then, the annual reliability index for the network is assessed as follows

$$\beta^{\text{annual}}(t) = \Phi^{-1}\left[1 - P_f^{\text{annual}}(t)\right] \tag{24}$$

where Φ^{-1} = inverse standard Gaussian cumulative distribution function. The second network connectivity performance indicator is the minimum annual reliability index over the life cycle

$$netconn_2 = \min_t \left[\beta^{\text{annual}}(t)\right] \tag{25}$$

The annual probability of failure in terms of connectivity presented in Eq. (23) is also closely related to the hazard function $h(t)$ (Ang and Tang 1984). The product $h(t) \times dt$, by definition, is the conditional probability of having a failure in the interval $(t, t + dt)$, given that the failure occurs after t

$$h(t) \times \mathrm{d}t = \mathcal{P}[t < T\langle t + \mathrm{d}t | T \rangle t] \tag{26}$$

where T = time to failure. With this definition, $P_f^{\text{annual}}(t)$ can be seen as the discrete version of $h(t) \times dt$, where the time interval is 1 year. From reliability theory

$$\mathrm{CDF}_T(t) = 1 - \exp\left[-\int_0^t h(\tau)\mathrm{d}\tau\right] \tag{27}$$

where CDF_T = cumulative distribution function of the time to failure. The value of CDF_T at the end of the life cycle (or of the considered time horizon) is the cumulative probability of having a failure with respect to the limit state in Eq. (22)

$$P_f^{\text{life cycle}} = \mathrm{CDF}_T(t_h) = 1 - \exp\left[-\sum_{t_z=0}^{t_h} P_f^{\text{annual}}(t_z) \times t_{ana}\right] \tag{28}$$

where t_h = investigated time horizon and t_{ana} = distance of the time instants at which P_f^{annual} is evaluated (e.g., 5 years). To increase the accuracy, the first and last terms of the summation (i.e., for $t_z = 0$ and $t_z = t_h$) are actually multiplied by $t_{ana}/2$. The cumulative probability of failure over the entire life cycle can be used to compute the life-cycle reliability $R^{\text{life cycle}}$. This is the third connectivity index

$$netconn_3 = R^{\text{life cycle}} = 1 - P_f^{\text{life cycle}} \tag{29}$$

Alternatively, the reliability index associated with Eq. (29) can be used

$$netconn_4 = \beta^{\text{life cycle}} = \Phi^{-1}\left(1 - P_f^{\text{life cycle}}\right) \tag{30}$$

The maximization of any of the four proposed connectivity indexes can be used as objective of the optimization procedure. Different choices for the index might yield slightly different optimum solutions. However, a complete comparison among the various indexes does not fit the purposes of this paper. The last index takes the maximum advantage of the available information and is considered the overall most representative. Therefore, in the reminder of the paper $netconn_4$ is adopted as the network connectivity index and simply called *netconn*.

Maintenance Cost

Although the effect of each PM action is modeled as uncertain, its cost for a given bridge and type of action is assumed to be prescribed. In general, different PM actions can be scheduled along the life cycle of an individual bridge. The cost of the pth PM action on bridge b is assumed to be given as $cost\ PM_{b,p}$.

The cost of CM actions, instead, depends on the amount of deterioration that must be fixed. Therefore, the cost of each CM intervention depends on the random variable Δ and is itself uncertain

$$cost\ CM_{i,b,q} = \overline{cost\ CM} \times \left(1 - \Delta_{i,b,q}\right)\left[\beta_{b,i}(0) - \beta_{b,i}\left(TCM_{i,b,q}\right)\right] \tag{31}$$

where subscript i runs over the samples used for simulation; subscript b runs over the bridges of the network; index q runs over the various CM interventions performed on one bridge of one sample; and $\overline{cost\ CM}$ = reference cost for CM that depends on the total value of the bridge.

Because the time horizon of the analysis is the entire life cycle of the bridge network, the effect of the discount rate of money cannot be disregarded. Hence, the expected value of the total cost of maintenance can be assessed as

$$maintcost = \underbrace{\frac{1}{n_S}\sum_{i=1}^{n_S}\sum_{b=1}^{n_B}\sum_{p}\frac{cost\ PM_{b,p}}{(1+r)^{TPM_{i,b,p}}}}_{\text{expected cost PM}} + \underbrace{\frac{1}{n_S}\sum_{i=1}^{n_S}\sum_{b=1}^{n_B}\sum_{q}\frac{cost\ CM_{i,b,q}}{(1+r)^{TCM_{i,b,q}}}}_{\text{expected cost CM}} \tag{32}$$

where n_S = number of samples considered in the simulation; n_B = number of bridges in the network; index p runs on the various PM interventions applied to one bridge of one sample; $TPM_{i,b,p}$ = time of the pth application of PM on bridge b of sample i; $TCM_{i,b,q}$ = time of the qth application of CM on bridge b of sample i; r = annual discount rate of money; $cost\ PM_{b,p}$ is the unit cost of the fixed pth PM intervention for bridge b; and $cost\ CM_{i,b,q}$ is given by Eq. (31). The minimization of the expected total maintenance cost *maintcost* is the second objective of the optimization.

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Problem Formulation and Solution Procedure

The multiobjective optimization problem is solved by means of genetic algorithms (GAs). They are numerical procedures that mimic the biological concepts of natural selection and survival of the fittest (Goldberg 1989). The search procedure is based on the repeated evaluation of the objectives at trial points of the feasible search domain. GAs are particularly suitable in problems similar to the one at hand, where it is not possible to compute the objectives in closed form. Multiobjective GAs can handle conflicting optimization criteria and provide the Pareto front of optimal solutions as the final result. Each Pareto-optional solution represents a PM schedule (for each bridge of the network) for which it is not possible to find another schedule that yields a better result for one of the two objectives without worsening the other. Decision makers eventually choose the solution that provides the overall best trade-off between cost and performance for the given scenario.

The design variables of the problem are the time of application of PM on each bridge of the network. Because it is not realistic to assign with extreme accuracy the starting time of a maintenance application (e.g., exactly after 68 years, 4 months, and 13 days), the time domain for the possible design variables is discretized with a prescribed resolution (e.g., 6 months, 1 year, 10 years). For the numerical applications, the resolution $t_{des} = 5$ years has been chosen. Moreover, the proposed procedure optimizes also the number of PM actions that should be applied. In fact, the analyst has to provide a maximum number of PM interventions for each bridge n_{MPM} and the procedure automatically pushes all the unnecessary applications beyond the investigated time horizon t_h.

A large amount of samples of the bridge reliability indexes $\beta_{b,i}$ are generated, as described previously. Then, the objective function applies PM to each bridge with a trial schedule $TPM_{b,p}$ and computes $\widehat{\beta}_{b,i}$. Next, at discrete time steps defined by the analysis resolution t_{ana} (for the numerical applications $t_{ana} = 5$ years, but in general t_{ana} does not need to be equal to t_{des}), the network connectivity index is evaluated using Eqs. (11)–(30). If a bridge is found to be out of service at time $\bar{t}$ [i.e., $s_{b,i}(\bar{t}) = 0$], CM is applied. Therefore, the reliability index $\tilde{\beta}_{b,i}(t)$ is updated using Eq. (9); the maintenance cost associated with sample i is increased by the result of Eq. (31); and the schedule of all the subsequent PM actions is postponed by $\bar{t}$ (Bocchini and Frangopol 2011a). Eventually, the two objectives *netconn* and *maintcost* are computed, and GAs determine whether the trial schedule belongs to the tentative Pareto front or not. After a series of GAs' generations, the final Pareto front is provided.

The analytical formulation of the optimization problem is the following:

Given:
- Network layout;
- Number of nodes n_N, bridges n_B, and samples n_S;
- Location of nodes $\mathbf{x}_j$ and bridges $\mathbf{x}_b$;
- Number of travels originated $O(0)$ and attracted $D(0)$ by each node and associated growth rate g;
- Time-dependent reliability index model $\beta_b(t)$ and probability distributions of the random parameters for each bridge;
- Bridge in/out of service state correlation;
- Time horizon t_h;
- Resolution of the analysis time domain t_{ana};
- Resolution of the design variables time domain t_{des};
- Lower threshold for the network connectivity $\overline{C}_{\text{threshold}}$;
- Maximum number of PM actions per bridge n_{MPM};
- PM effect components and probability distributions of their random parameters;
- Probability density function of the CM random parameter Δ;
- Cost of PM actions $cost\,PM_{b,p}$;
- Reference cost of CM $\overline{cost\,CM}$;
- Annual discount rate of money r;
- Available budget over life cycle $cost_{\max}$; and
- Parameters and settings for GAs (e.g., population size, number of generations).

Find:
- $TPM_{b,p}$ with $b = 1, 2, \ldots, n_B$

In order for:
- $netconn$ = maximum; and
- $maintcost$ = minimum.

Subject to:
- $TPM_{b,p} \le t_h \; \forall b, p$;
- $TPM_{b,p1} < TPM_{b,p2} \; \forall b, p1 < p2$;
- $(TPM_{b,p}/t_{des}) \in \mathbb{N}$;
- $maintcost \le cost_{\max}$; and
- $p \le n_{MPM}$.

For the numerical applications, the GAs library distributed with *MATLAB*, which is a modified version of the NSGA-II algorithm (Deb 2001; Deb et al. 2002), has been used. In addition to the objective function, special crossover and mutation functions have been developed to obtain schedules with the desired resolution in the discrete time domain. The bookkeeping technique (Bocchini and Frangopol 2011a) is applied to achieve higher computational efficiency.

Numerical Applications

The problem at hand is very complex, and it is difficult to validate its results on a large network. Therefore, the first numerical application deals with a five-bridge network, for which the results can be investigated in depth. On the other hand, problems that involve network analysis, uncertainty, and optimization are often affected by numerical issues that make the approach impractical as the complexity increases. Thus, a second application on a realistic network is presented to prove that the framework remains feasible.

Validation on a Five-Bridge Network

The layout of the network considered to validate the approach is shown in Fig. 1. It consists of five highway segments (modeled as 10 unidirectional edges) that connect six cities, represented by Numbers 1–6 in Fig. 1. Cities 3 and 4 are assumed to be large cities that attract travelers from the suburbs. Moreover, northern cities (1 and 5) and large cities (3 and 4) are expected to grow in the future, whereas the southern cities (2 and 6) are expected to reduce their capacity to attract and originate travels. These data are collected in Table 1. The length of each highway segment is 15 km, and each of them is carried by a bridge that is located exactly in the middle of the segment

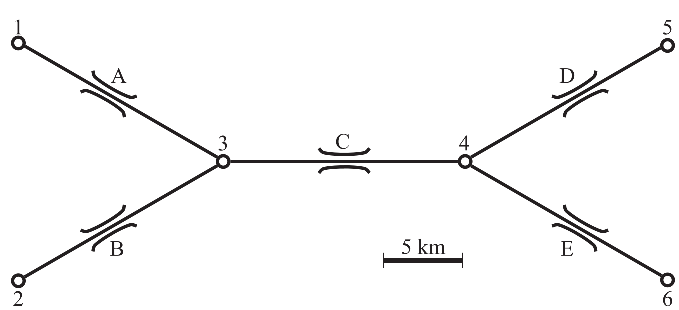

Fig. 1. Layout of the five-bridge network used for validation

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(i.e., each bridge carries two unidirectional edges). Bridges A and D have a bilinear reliability index model; Bridges B and E have a quadratic reliability index model; and Bridge C is characterized by an exponential model. The parameters of all the models are collected in Table 2. The correlation length of the random field is $\lambda = 15$ km, and the autocorrelation function R of the field is assumed to be

$$R(\xi_x, \xi_y) = \frac{1}{12}\exp\left(-\frac{\xi_x^2 + \xi_y^2}{\lambda^2}\right) \tag{33}$$

where ξ_x and ξ_y = distances between two locations in the x- and y-directions, respectively. For each bridge of the network, a maximum of $n_{MPM} = 2$ PM interventions has been considered. Hence, the problem has $n_B \times n_{MPM} = 5 \times 2 = 10$ design variables. The PM actions are supposed to have an effect modeled as the superposition of a small improvement in β and a temporary interruption of the deterioration, as described in Eqs. (5) and (8). The PM parameters and their probabilistic characterizations are collected in Table 2. CM parameter Δ in Eq. (9) has a triangular distribution with minimum 0, maximum 0.5, and mode 0.2. For all the random variables, $n_S = 4{,}000$ samples has been generated by LHS. The cost of each PM action is assumed to be always the same and equal to $cost\,PM_{b,p} = \overline{cost\,PM} = \$50{,}000$, whereas the reference cost for CM actions is $\overline{cost\,CM} = \$500{,}000$. The annual discount rate of money is $r = 2\%$. No upper limit has been imposed on the available funding. The time horizon of 70 years has been discretized using the same resolution both for the analysis and the PM actions domain ($t_{ana} = t_{des} = 5$ years). The lower threshold for the network connectivity is $\overline{C}_{\text{threshold}} = 98\%$.

After 100 generations composed by 300 individuals each, GAs provide the optimal Pareto front shown in Fig. 2. As expected, to achieve higher values for the performance indicator *netconn*, a larger amount of money is required. Table 3 and Fig. 3 present the PM schedules associated with three representative solutions indicated by S1, S2, and S3 in Fig. 2. Solution S1 requires the lowest amount of economic resources among all the optimal solutions. The GAs correctly identify Bridges A and D as the most important ones, because the cities that they connect are expected to grow along the investigated time period. Therefore, Solution S1 schedules two PM interventions on Bridges A and D, whereas there is planning for only one intervention on the other bridges. Solution S2 differs from the first one, because the PM interventions are anticipated. The discount rate of money over such a long time period can have a significant effect on the expected cost. In fact, because of the PM anticipation, S2 is expected to cost \$36,000 more than S1. However, the timely application of PM yields higher values of the bridge reliabilities and, in turn, a higher value of the network performance indicator *netconn*. Optimal Solution S3 is the overall most expensive, but also the one that guarantees the highest connectivity index. This better performance is achieved by means of an additional PM intervention on the central Bridge C. However, although this additional intervention increases the expected cost remarkably, it does not have a significant impact on the reliability index. Thus, decision makers should prefer a different PM schedule.

For the solutions of the Pareto front, Fig. 4 shows that higher connectivity corresponds to higher PM costs, but lower CM costs. In fact, Fig. 5 shows that if solutions on the right of the Pareto front are selected, the rate of cases in which the bridges of the network are out of service decreases.

Table 1. Characteristics of Nodes of Network in Fig. 1

Node (city)	$O(0)$	$D(0)$	g (%)
1	10,000	5,000	4
2	10,000	5,000	−2
3	5,000	15,000	3
4	5,000	15,000	3
5	10,000	5,000	4
6	10,000	5,000	−2

Table 2. Random Parameters Used for Network in Fig. 1

Parameter	Distribution	Mean	SD
β^0	Lognormal	6.5	1.04
T^I	Lognormal	15	5
R^L	Uniform[a]	0.0525	0.027
R^Q	Uniform[b]	0.00033	0.000113
R^E	Normal	0.05	0.005
S	Normal	16.5	1.65
Q	Normal	12.5	1.25
M_0	Uniform[c]	0.1	0.0577
D_0	Triangular[d]	7.67	1.0274
Δ	Triangular[e]	0.23	0.1027

[a]Minimum = 0.005, maximum = 0.1.
[b]Minimum = 0.000135; maximum = 0.000527.
[c]Minimum = 0, maximum = 0.2.
[d]Minimum = 5, mode = 8; maximum = 10.
[e]Minimum = 0, mode = 0.2; maximum = 0.5.

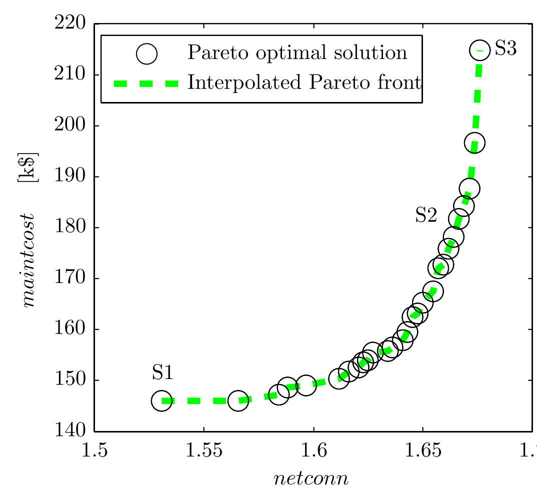

Fig. 2. Pareto front associated with the network in Fig. 1

Table 3. Representative Optimal Solutions S1–S3: Years of Preventive Maintenance Application and Associated Objectives

	Years of preventive maintenance application					
Bridge	S1		S2		S3	
A	35	55	20	35	15	25
B	65	—	45	—	45	—
C	65	—	65	—	55	60
D	40	50	20	35	15	30
E	60	—	60	—	60	—
netconn	1.53		1.67		1.68	
maintcost (\$)	146,000		182,000		215,000	

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Finally, the solutions of the Pareto front can be compared with other schedules selected only by engineering judgment. The first case that is investigated consists in a PM plan according to which all the bridges receive PM after 25 and 45 years. This solution appears reasonable, because the first intervention is scheduled some years after the expected initiation of the deterioration (Year 15), and the second PM improves the reliability in the last period of the service life. Using this schedule as a trial, with the same set of 4,000 samples for the random parameters, the resulting expected maintenance cost is *maintcost* = \$270,500, and the network connectivity index is *netconn* = 1.64. An optimal solution from the Pareto front allows to achieve the same *netconn* savings of more than \$100,000. A second trial is performed assuming only one PM intervention on all the bridges after 40 years, approximately at the middle of their deterioration phase. The result associated with this schedule is *maintcost* = \$161,500, and network connectivity index is *netconn* = 1.16. The Pareto optimal schedules that have similar expected cost guarantee a connectivity index of *netconn* = $\beta^{\text{life cycle}} \cong 1.65$, which indicates a much lower probability of downcrossing the connectivity threshold. In conclusion, with respect to PM schedules based only on common sense and engineering judgment, the use of the proposed framework provides optimal solutions that guarantee better network performance and lower maintenance cost.

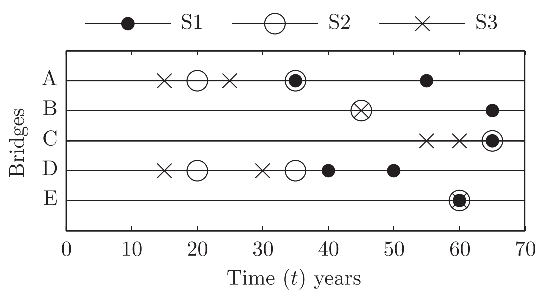

Fig. 3. Preventive maintenance schedules associated with the representative Solutions S1, S2, and S3

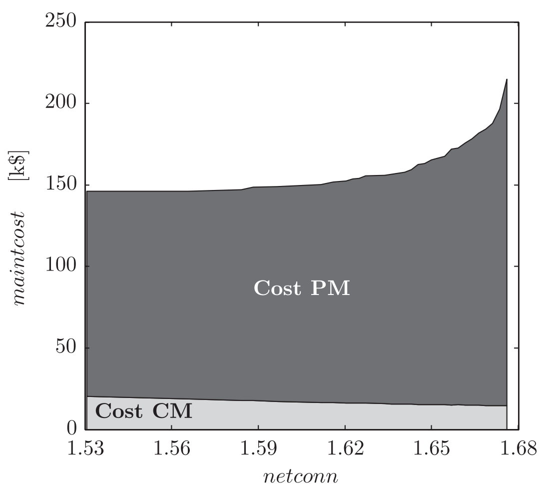

Fig. 4. Total cost divided between preventive maintenance and corrective maintenance cost, associated with the network in Fig. 1

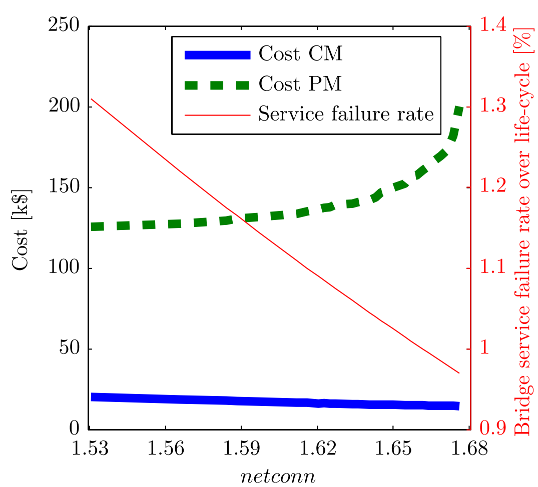

Fig. 5. Cost breakdown (preventive maintenance and corrective maintenance) and bridge service failure rate over the investigated 75 years, associated with the network in Fig. 1

Feasibility on a Complex Network

A transportation network located in Lehigh Valley, Pennsylvania, and involving 13 bridges, six nodes, and eight highway segments is considered in this second example (Fig. 6). Nodes 1 and 4 represent the cities of Allentown and Easton, respectively. Nodes 2 and 6 collect the traffic generated and attracted by the northern and southern parts of the city of Bethlehem. The remaining two nodes (i.e., 3 and 5) are highway junctions. Data associated with the traffic originated and attracted by the nodes are collected in Table 4. The location of the bridges is shown in Fig. 6, and the characteristics of the associated reliability models are presented in Table 5. The correlation length of the random field is 15.8 km (Bocchini and Frangopol 2011b), and the autocorrelation function is given by Eq. (33). The maximum number of PM interventions on each bridge has been set to $n_{MPM} = 2$, thus the problem has a total of $n_B \times n_{MPM} = 13 \times 2 = 26$ design variables. In this case, PM actions are assumed to determine a reduction of the deterioration rate modeled as a linear improvement, as in Eq. (6). The probabilistic descriptions of all the random parameters are collected in Table 5;

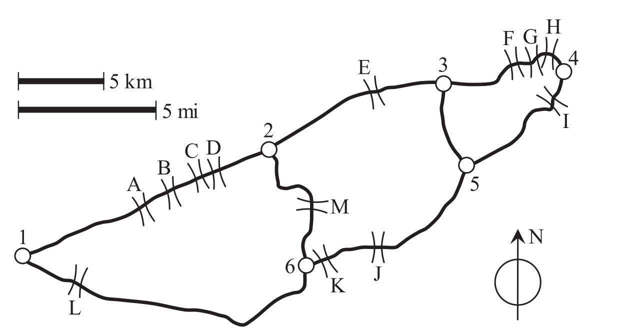

Fig. 6. Layout of the network located in Lehigh Valley, Pennsylvania

Table 4. Characteristics of Nodes of Network in Fig. 6

Node	$O(0)$	$D(0)$	g (%)
1	10,500	10,500	−2
2	12,000	12,000	3
3	5,000	5,000	−1
4	8,500	8,500	−2
5	9,500	9,500	−1
6	10,000	10,000	3

Table 5. Random Parameters Used for Network in Fig. 6

Parameter	Distribution	Mean	SD
Bilinear model, Bridges A, C, and H			
β^0	Lognormal	3.5	0.56
T^I	Lognormal	15	5
R^L	Uniform[a]	0.0525	0.027
Quadratic model, Bridges B, F, and K			
Parameter	Distribution	Mean	SD
β^0	Lognormal	3.5	0.56
T^I	Lognormal	15	5
R^Q	Uniform[b]	0.00033	0.000113
Square root model, Bridges D, E, G, I, J, L, and M			
Parameter	Distribution	Mean	SD
β^0	Deterministic	3.5	0
T^I	Deterministic	15	0
R^S	Deterministic	0.3	0
Maintenance			
Parameter	Distribution	Mean	SD
M_1	Uniform[c]	0.5	0.116
D_1	Uniform[d]	10	2.887
Δ	Triangular[e]	0.23	0.1027

[a]Minimum = 0.005, maximum = 0.1.
[b]Minimum = 0.000135; maximum = 0.000527.
[c]Minimum = 0.3, maximum = 0.7.
[d]Minimum = 5, maximum = 15.
[e]Minimum = 0, mode = 0.2; maximum = 0.5.

for each random variable $n_S = 5{,}000$ samples has been generated by LHS. The cost of all PM actions is assumed to be $costPM_{b,p} = \overline{cost\ PM} = \$50{,}000$, whereas the reference cost for CM actions is $\overline{cost\ CM} = \$500{,}000$. The annual discount rate of money is $r = 2\%$. The upper limit of the available funding has been set to \$1.150 million. The lower threshold for the network connectivity is $\overline{C}_{\text{threshold}} = 98\%$. The time horizon of 75 years has been discretized using the same resolution both for the analysis and the PM actions domain ($t_{ana} = t_{des} = 5$ years).

In 749 min, a personal computer produced in 2006 with a single-core 3.4-GHz CPU and 2 GB of random-access memory evaluated 50 generations composed by 300 individuals each. The resulting Pareto front is shown in Fig. 7. Multiplying the number of samples by the number of individuals and the number of generations, the code evaluated a total of 75 million life-cycle scenarios, each one consisting of 16 analyses at discrete time steps, for a grand total of 1.2 billion network connectivity evaluations. Three representative optimal schedules have been selected, and the details are provided in Fig. 8 and Table 6. Solution S6 is not admissible, because the expected total cost is beyond the limit. In this case, the shape of the Pareto front indicates that the solutions providing the best compromise between objectives are those around S5. In fact, this schedule determines a significant increase in the connectivity, with respect to the cheapest Solution S4, raising the expected total maintenance cost only by \$14,000. Solutions on the extreme right of the Pareto front, instead, are associated with much higher costs and limited improvements in the connectivity index. In particular, the most expensive solution, S6, prescribes several PM actions in the first years of the investigated time horizon to mitigate the fast deterioration because of the square root reliability index model

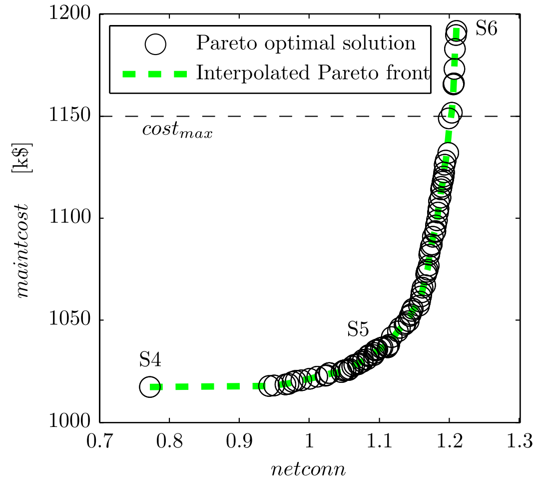

Fig. 7. Pareto front associated with the network in Fig. 6

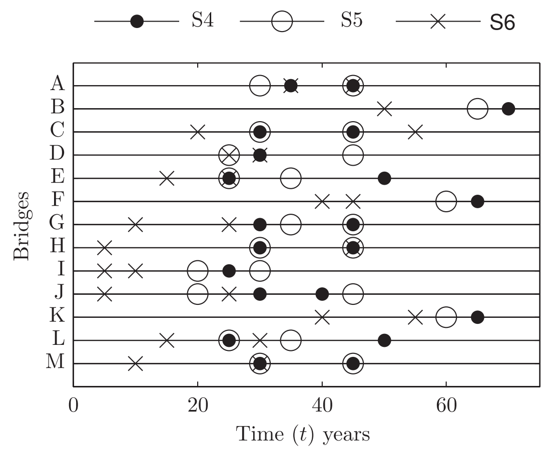

Fig. 8. Preventive maintenance schedules associated with the representative Solutions S4, S5, and S6

Table 6. Representative Optimal Solutions S4–S6: Years of Preventive Maintenance Application and Associated Objectives

	Years of preventive maintenance application					
Bridge	S4		S5		S6	
A	35	45	30	45	35	45
B	70	—	65	—	50	—
C	30	45	30	45	20	55
D	30	—	25	45	25	30
E	25	50	25	35	15	25
F	65	—	60	—	40	45
G	30	45	35	45	10	25
H	30	45	30	45	5	45
I	25	—	20	30	5	10
J	30	40	20	45	5	25
K	65	—	60	—	40	55
L	25	50	25	35	15	30
M	30	45	30	45	10	30
netconn	0.77		1.08		1.21	
maintcost (\$)	1.017 million		1.031 million		1.192 million	

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(Bridges G, I, and J). These interventions are very expensive, because they benefit only minimally by the discount rate. In addition to the high cost, PM actions likely to be performed before the deterioration initiation time have a low impact on the network performance statistically. Therefore, Solution S6 and all the most expensive solutions determine an increase in the costs that is not justified by a significant increase in the performance. The Pareto front makes all this information readily available to decision makers.

To summarize the results, the proposed technique provides a set of optimal solutions (Fig. 7) among which decision makers identify the best for the specific scenario. For instance, assuming that S5 is selected, Table 6 provides a schedule for the application of the prescribed PM action on each bridge of the network. Adopting the chosen plan, the expected optimum cost of maintenance is \$1.031 million. This includes the discounted cost of PM actions (\$553,000) and the discounted cost of CM actions that are assumed to be performed any time a bridge is out of service (\$478,000). With PM Schedule S5, the proposed technique assesses that, on average, the probability for each bridge of reaching the service limit state during 75 years is 16%. Finally, $netconn = 1.08$, which means that the probability of having a network connectivity level $\overline{C}(t)$ that is less than 98% at least once over the entire life cycle is only 14%.

Conclusions

A comprehensive probabilistic computational framework for the connectivity- and cost-based optimal scheduling for PM of deteriorating bridge networks has been presented. To the best of the authors' knowledge, this is the first time when all the following aspects are combined in a unified approach.

- The connectivity among all the pairs of network nodes is accounted for simultaneously, through a complete network model.
- The most significant sources of epistemic and aleatory uncertainty associated with the quantities involved in the problem have been considered throughout the proposed methodology and handled by means of efficient simulation techniques.
- The correlation among the in/out of service states of the bridges has been included in the model by means of random field theory.
- The network connectivity has been estimated by a novel indicator that synthetically represents all the data obtained by the full probabilistic analysis. Furthermore, the information associated with the socioeconomic, time-dependent importance of the different highway segments is embedded in the connectivity index as well.
- Network connectivity and maintenance cost have been optimized simultaneously as two conflicting criteria. The result of the biobjective simulation is a Pareto front of solutions, all equally optimal, among which decision makers can choose the one that fits the needs of the real scenario better, considering also the minor aspects that cannot be realistically included in the model.
- The deterioration of the individual bridge reliability along the life cycle is taken into account by means of fixed and/or probabilistic time-dependent reliability index models.
- The effect of PM actions (whose schedule is the design variable) is combined with the effect associated with CM. Therefore, it has been possible to remove the assumption that bridges never reach any limit state and never undergo CM along their life cycle. This assumption is very common and convenient in studies addressing bridge maintenance and maintenance optimization, but it is also not realistic. Therefore, the removal of this assumption is a paramount step toward the practical application of the proposed method.

The presented numerical applications prove that the proposed framework can truly improve funding allocation. An optimal schedule of the PM actions (including both time and number of applications for each bridge) yields higher reliability for the network and lower maintenance costs. Given the layout of any complex network, the proposed methodology automatically finds the most critical bridges and distributes the maintenance interventions along the life cycle. This, in turn, increases the chances to always be able to reach all the nodes of the highway network, even after the occurrence of extreme events, and to facilitate the prompt recovery of the region.

Further studies in this field should aim at combining the presented results with other network performance indicators, such as those presented in Bocchini and Frangopol (2011a), and bridge performance indicators. Moreover, additional constraints that focus on the reliability of individual bridges and aim at avoiding the failure of the most critical bridges should be considered. Finally, the application of the proposed framework to other real or realistic scenarios will be very useful for validation purposes.

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