PERGAMON

Transportation Research Part A 36 (2002) 525–535

TRANSPORTATION
RESEARCH
PART A

www.elsevier.com/locate/tra

A steady-state solution for the optimal pavement resurfacing problem

Yuwei Li, Samer Madanat *

Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720, USA

Received 7 July 1999; received in revised form 9 April 2001; accepted 11 April 2001

Abstract

This paper presents a solution approach for the problem of optimising the frequency and intensity of pavement resurfacing, under steady-state conditions. If the pavement deterioration and improvement models are deterministic and follow the Markov property, it is possible to develop a simple but exact solution method. This method removes the need to solve the problem as an optimal control problem, which had been the focus of previous research in this area. The key to our approach is the realisation that, at optimality, the system enters the steady state at the time of the first resurfacing. The optimal resurfacing strategy is to define a minimum serviceability level (or maximum roughness level), and whenever the pavement deteriorates to that level, to resurface with a fixed intensity. The optimal strategy does not depend on the initial condition of the pavement, as long as the initial condition is better than the condition that triggers resurfacing. This observation allows us to use a simple solution method. We apply this solution procedure to a case study, using data obtained from the literature. The results indicate that the discounted lifetime cost is not very sensitive to cycle time. What matters most is the best achievable roughness level. The minimum serviceability level strategy is robust in that when there is uncertainty in the deterioration process, the optimal condition that triggers resurfacing is not significantly changed.

Keywords: Steady state; Markov; Pavement; Resurfacing

1. Introduction

This paper presents a steady-state solution approach for the problem of optimising the frequency and intensity of pavement resurfacing, for the case of continuous time and continuous

* Corresponding author. Tel.: +1-510-643-1084; fax: +1-510-642-1246.
E-mail address: madanat@ce.berkeley.edu (S. Madanat).

0965-8564/02/$ - see front matter
PII: S0965-8564(01)00020-9

state space and deterministic pavement deterioration. If the deterioration and improvement models are deterministic and follow the Markov property, it is possible to develop a simple but exact solution method. This method eliminates the need to solve the problem as an optimal control problem, which had been the focus of previous research in this area (Markow and Balta, 1985; Tsunokawa and Schofer, 1994). We applied our solution procedure to the case study described in the work of Tsunokawa and Schofer. We also examined the impact of uncertainty in the deterioration process. Our results provide a theoretical foundation for the pavement management practice that is based on resurfacing pavements whenever a minimum allowable serviceability level is reached.

This paper is focused exclusively on the continuous time, continuous state problem, and for the case of deterministic Markovian deterioration and rehabilitation-only policy, for which we show that a simple trial and error solution procedure exists. The stochastic deterioration optimal maintenance and rehabilitation problem, which is more general, has been addressed and solved by various researchers, including (Golabi et al., 1982; Carnahan, 1988; Madanat and Ben-Akiva, 1994).

This paper is organised as follows. In Section 2, we present a review of the relevant literature, in order to clarify the relation of this research to previous work in the field. Section 3 describes the model formulation. Section 4 describes the solution procedure, and Section 5 applies it to a case study. In Section 6, we analyse the sensitivity of our solution to the uncertainty in the deterioration process. Finally, Section 7 concludes the paper.

2. Literature review

The focus of this paper is on rehabilitation activities, and specifically pavement resurfacing (or overlays). When we refer to the intensity of rehabilitation, we mean the thickness of the resurfacing. The problem of determining the optimal temporal frequency of pavement rehabilitation has received substantial attention from researchers in the field of infrastructure management. The problem is usually defined as follows: given a known deterioration curve and rehabilitation effectiveness, what is the frequency and intensity of pavement rehabilitation activities that minimises the discounted social (agency plus user) cost over a long planning horizon? The problem can also be stated as a standard determination problem: what is the optimal pavement condition at which rehabilitation should be performed, and what should be the intensity of this rehabilitation? The two statements are equivalent if the deterioration of the facility can be predicted deterministically and the system is in a steady state, because for this case, a constant frequency yields a unique standard. The advantage of posing the problem as one of optimal standard determination is that, as shown in this paper, it leads to clearer insights into the nature of the steady-state solution. Furthermore, the concept of maintenance and rehabilitation standards is well known in pavement engineering practice (Paterson, 1987).

The problem of optimal frequency of pavement resurfacing for the continuous time and continuous state case has been addressed by a number of researchers. Friesz and Fernandez (1979) formulated the problem of finding an optimal maintenance profile over time as an optimal control problem. They later extended their work to solve for the optimal timings of highway stage construction (Fernandez and Friesz, 1981). Markow and Balta (1985) used optimal control to solve for the optimal timing for a single pavement rehabilitation event in a finite horizon. Markow et al. (1993) used an optimal control formulation to solve for the optimal timing of bridge deck re-

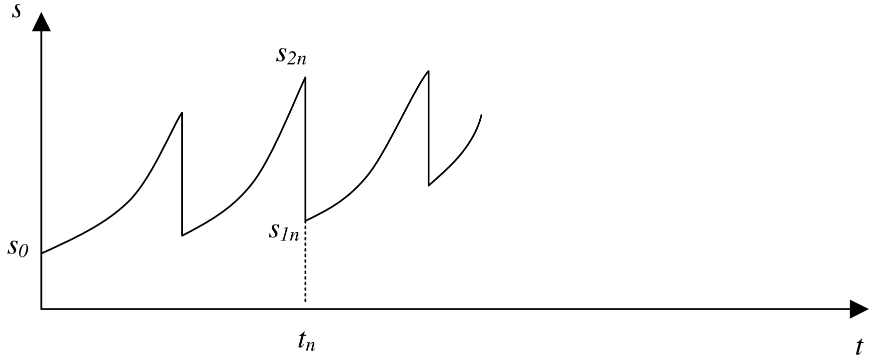

Fig. 1. Saw-tooth trajectory of pavement roughness.

habilitation, but solved the problem using numerical methods rather than analytically. More recently, Tsunokawa and Schofer (1994) used a continuous approximation to solve the problem of optimal timings and intensity of pavement resurfacing. Their approach was to approximate the saw-tooth curve of pavement condition, shown in Fig. 1, by a continuous function passing through the midpoints of the spikes. The problem of finding the timings and intensities of resurfacing activities is replaced by one of solving for an optimal rehabilitation rate. With this approximation, the problem becomes a standard optimal control problem. Based upon the solution obtained for the continuous approximation, the value of condition improvement caused by resurfacing (and thus its thickness) and the optimal frequency of resurfacing can be calculated.

In this paper, we show that the same problem can be solved exactly, without using any approximation and without having to use optimal control. It is clear that, under time homogeneous pavement deterioration and improvement functions, the saw-tooth curve representing pavement condition over time reaches a steady-state situation at some point in time. For a steady-state situation, solving for the optimal frequency and intensity of overlays is relatively easy, because these are constant and do not depend on time. The key to our approach is the realisation that, unless the facility condition at the beginning of the planning horizon is below the optimal standard, the steady-state is reached as soon as the first resurfacing is performed. This is the case because our solution is based on an optimal standard (i.e., level of serviceability that triggers resurfacing), rather than optimal frequency.

In the following section, we present the mathematical formulation of the problem. Our formulation is the same as Tsunokawa and Schofer's (1994). This is intentional, and will allow us to compare our results with theirs. We used the deterioration and rehabilitation effectiveness (i.e. improvement) models used by Tsunokawa and Schofer, despite the fact that some aspects of these models may have been unrealistic.

3. Model formulation

As described by Tsunokawa and Schofer (1994), the roughness (or unserviceability), s, of a given highway pavement follows a saw-tooth trajectory over time as the pavement deteriorates and receives resurfacings. An example of this trajectory is given in Fig. 1.

At time t_n, a resurfacing of intensity w_n is applied to the pavement so the roughness changes from s_{2n} to s_{1n}.

The goal is to minimise the net present value of life-cycle costs for the agency and the user attributable to the pavement, as described below:

$$\min \quad J = \sum_{n=1}^{\infty} \left\{ \int_{t_{n-1}}^{t_n} C(s(t))\mathrm{e}^{-rt}\,\mathrm{d}t + M(w_n)\mathrm{e}^{-rt_n} \right\} \tag{1}$$

$$\text{s.t.} \quad \frac{\mathrm{d}s(t)}{\mathrm{d}t} = F(s(t)), \tag{2}$$

$$s_{2n} - s_{1n} = G(w_n, s_{2n}), \tag{3}$$

$$s(0) = s_0, \tag{4}$$

where J is the present value of agency and user cost over an infinite horizon, t_n the time of the nth resurfacing, $s(t)$ the pavement roughness as a function of time, $C(s(t))$ the user cost rate as a function of pavement roughness, w_n the intensity of nth resurfacing, $M(w_n)$ the agency cost as a function of resurfacing intensity, r the discount rate, $F(s(t))$ the deterioration rate as a function of pavement roughness, $G(w_n, s_{2n})$ is the improvement in pavement condition as a function of resurfacing intensity and pavement roughness immediately prior to resurfacing.

The decision variables are t_n and w_n. Eq. (1) defines the object function. Eq. (2) says that the deterioration rate depends only on the current condition of the pavement, which means that our deterioration model is memory-less. Eq. (3) states that the reduction in roughness depends only on the resurfacing intensity and the condition of the pavement when resurfacing starts. Eq. (4) specifies the initial condition.

4. Solution procedure

Solving the problem defined by Eqs. (1)–(4) as an optimal control model with discrete jumps in the state variable is quite cumbersome (Bryson and Ho, 1975). As discussed in Tsunokawa and Schofer (1994) efficient solution procedures have not yet been developed for this problem. However, it is possible to solve this problem without using optimal control if certain observations are made.

This model possesses two important properties:

1. The planning horizon is infinite.
2. It is a deterministic Markov decision process (MDP).

The major advantage of using an infinite planning horizon is that we can avoid the problem of having to specify a salvage value for the pavement at the end of a planning horizon, which allows us to solve for steady-state policies.

A MDP is defined by a set of states, S, a set of actions, A, and for each particular action, a set of discrete probability distributions, P, over the set S. Associated with each action while in a given state is a cost $c(S, A)$. Given a MDP, the goal is to determine a policy, $a(s)$, to minimise total expected discounted cost. In our model, the set of states is the set of roughness levels that the pavement may be at. The set of actions includes doing nothing and resurfacing at certain intensity levels. The post-action probability distribution of the states is deterministic.

The property that history does not influence the system evolution is known as the Markovian property. It plays a key role in this model. For a steady-state MDP, at optimality, the action at time t only depends on the current state $s(t)$. In other words, whenever the system is in the same state, the same action must be taken.

This property has an important role in our problem. Suppose s_0 equals s_{new}, the roughness of the pavement when it is new. Assume that the pavement deteriorates until roughness reaches s_2 at time t_1, and that it is optimal to resurface it with intensity w so that the roughness level is reduced to s_1, where $s_1 > s_{\text{new}}$. Because the evolution of pavement roughness is Markovian, if this standard is optimal for the first resurfacing, it must be optimal for all resurfacings. In other words, the system enters a steady state at t_1, as shown in Fig. 2. The optimal steady-state resurfacing strategy is therefore to define a minimum serviceability standard (or maximum roughness level s_2), and whenever the pavement deteriorates to that standard, to resurface it with the same intensity.

The problem is thus reduced to solving for the maximum roughness level, as well as the optimal resurfacing intensity. The usual solution method for such a problem is:

1. For each s_2 (and therefore t_1), solve for the optimal cycle time τ (and therefore resurfacing intensity, or roughness after resurfacing) to minimise the discounted cost incurred since the first resurfacing (including the first resurfacing), evaluated at t_1.
2. Find the optimal s_2 (or t_1) to minimise the total discounted cost in current dollars at t_0.

We developed another solution method that is mathematically not more difficult than the one above but offers greater theoretical clarity in answering the following question: are strategies different for different initial pavement conditions? Previous research seems to suggest that the answer is yes (Darter et al., 1985). However, we will show that the initial state s_0 does not affect the optimal resurfacing strategy, at least when the initial condition $s_0 < s_{2^*}$, where s_{2^*} is the optimal value of s_2.

The proposed solution method consists of two simultaneous equations with s_1 and s_2 as variables. One equation derives from the optimality condition when we minimise the cost incurred

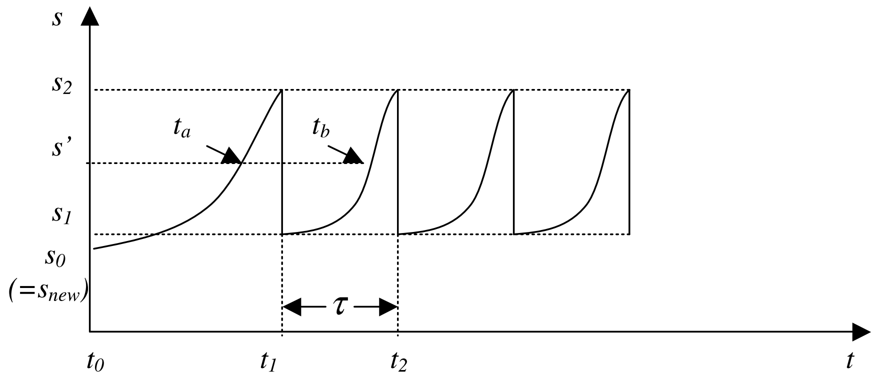

Fig. 2. System enters the steady state at the time of the first resurfacing.

since the first resurfacing (including the cost of first resurfacing) in current dollars at t_1. The other equation derives from the optimality condition when we minimise the cost incurred after the first resurfacing (excluding the cost of the first resurfacing) in current dollars at t_1.

Although the equations are easy to write, it is hard to express the solution in a simple analytical form. However, it is possible to use a spreadsheet to solve the two equations simultaneously by trial and error, or one can use software packages to come up with numerical solutions. As we will see, for the model specified in our case study, the problem possesses a mathematical property that simplifies the solution.

This solution method makes it clear that the minimum serviceability level and the optimal resurfacing intensity depend on the pavement deterioration process and the resurfacing effectiveness, but not on the initial condition of the pavement. Referring to Fig. 2, let s_{2^*} be the optimal value of s_2, when the initial condition is s_{new}. If we do not resurface at t_b, we should not resurface at t_a, since pavement condition is s' in both cases. If the initial state is s' instead of s_{new}, and $s' < s_{2^*}$, it is still optimal to do nothing till s_{2^*} is reached, irrespective of the value of s'. Clearly, if this result holds for s', it must hold for any value of s_0, as long as $s_0 < s_{2^*}$. Although the initial condition does not affect the minimum serviceability level or the optimal resurfacing intensity, it affects the total cost. In the following case study, we will assume $s_0 = s_1$ for simplicity, since we are only interested in the minimum serviceability level and the optimal resurfacing intensity, not in the value of the objective function at optimality.

5. Case study

For purposes of comparison with previous research, we used the cost functions, deterioration function and improvement function used by Tsunokawa and Schofer (1994):

Deterioration function: $$F(s) = f_1 s, \tag{5}$$

Improvement function: $$G(w,s) = g_1\sqrt{w} + g_2 s + g_3, \tag{6}$$

User cost rate function: $$C(s) = c_1 s + c_2, \tag{7}$$

Agency cost function: $$M(w) = m_1 w + m_2. \tag{8}$$

When roughness s is measured in QI units, time t in years, overlay thickness in millimetres, and cost in dollars for 1 km of pavement, the parameter values are:

$$r = 0.07, \quad f_1 = 0.05, \quad g_1 = 5.0, \quad g_2 = 0.78, \quad g_3 = -66.0,$$
$$c_1 = 1000, \quad m_1 = 3000, \quad m_2 = 150{,}000.$$

Because c_2 is a constant, it does not affect the result of the optimisation. It is therefore set to zero.

The deterioration model seems realistic. For example, a good pavement with a roughness of 25 QI will deteriorate to a poor condition of 100 QI in 28 years, which is reasonable. The improvement model, on the other hand, is not as realistic. Tsunokawa and Schofer based their functional form on the work of Watanatada and his colleagues at the World Bank (Watanatada et al., 1988). Through empirical work in developing countries, they found that the effect of current roughness on roughness reduction is mostly linear, while the effect of overlay thickness in non-linear, with a diminishing contribution of thickness. However, this function implies that even if the overlay thickness w is zero, there can be a reduction in roughness, which is unreasonable. In any case, to simplify comparison of our results to those in the literature, we decided to use the same model.

Our solution approach proceeds as follows:

(1) Minimise J_2, the total discounted cost incurred since the first resurfacing (including the cost of the first resurfacing), evaluated at t_1;

For each cycle starting with a resurfacing, the discounted cost evaluated at the beginning of the cycle is j_2

$$j_2 = m_1 w + m_2 + \int_0^{\tau} c_1 s_1 \mathrm{e}^{f_1 t} \mathrm{e}^{-rt}\, \mathrm{d}t. \tag{9}$$

From

$$s_1 \mathrm{e}^{f_1 \tau} = s_2, \tag{10}$$

$$s_2 - s_1 = g_1 \sqrt{w} + g_2 s_2 + g_3 \tag{11}$$

we obtain

$$w = \left[\frac{s_2(1 - g_2 - \mathrm{e}^{-f_1 \tau}) - g_3}{g_1} \right]^2. \tag{12}$$

Therefore, the discounted cost for all cycles, evaluated at t_1, is

$$J_2 = \frac{m_1 [(s_2(1 - g_2 - \mathrm{e}^{-f_1 \tau}) - g_3)/g_1]^2 + m_2 + (c_1 s_2/(f_1 - r))(\mathrm{e}^{-r\tau} - \mathrm{e}^{-f_1 \tau})}{1 - \mathrm{e}^{-r\tau}}. \tag{13}$$

It turns out that for fixed s_2, J_2 is decreasing with τ. Fig. 3 shows this relationship.

Therefore, the optimal strategy is to resurface the pavement to the best state achievable. This result is consistent with other researchers' finding (Darter et al., 1985) that "the least cost strategy is to repair to a practically new state and then perform maintenance before subsequent deterioration becomes very extensive" (Carnahan, 1988). Because $F(s) = f_1 s$, the deterioration rate would be zero and the cycle time would be infinite if the best state achievable were zero. That should not be the case in reality. Therefore, for the model to produce meaningful results, we must define a best achievable state s_1, which is greater than zero.

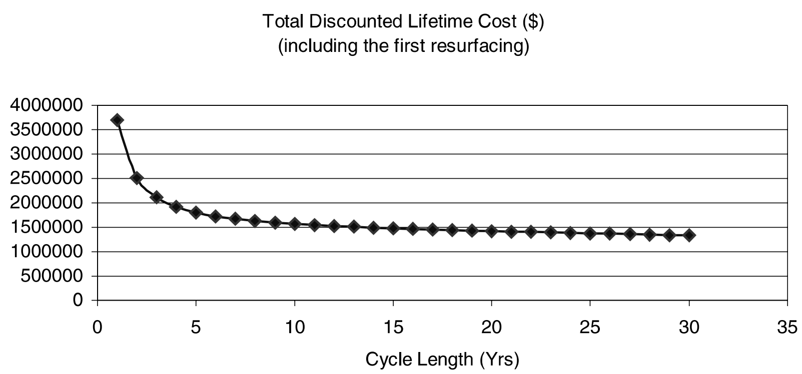

Fig. 3. Relationship between J_2 and τ for $s_2 = 100$ QI.

(2) Minimise J_1, the total discounted cost incurred after the first resurfacing (excluding the cost of the first resurfacing), evaluated at t_1.

For each cycle ending with a resurfacing, the discounted cost evaluated at the beginning of the cycle is j_1.

$$j_1 = (m_1 w + m_2)\mathrm{e}^{-r\tau} + \int_0^{\tau} c_1 s_1 \mathrm{e}^{f_1 t} \mathrm{e}^{-rt}\,\mathrm{d}t. \tag{14}$$

From

$$s_1 \mathrm{e}^{f_1 \tau} = s_2, \tag{15}$$

$$s_2 - s_1 = g_1\sqrt{w} + g_2 s_2 + g_3 \tag{16}$$

we obtain

$$w = \left[\frac{s_1[(1-g_2)\mathrm{e}^{f_1\tau} - 1)] - g_3}{g_1}\right]^2. \tag{17}$$

Therefore, the cost for all cycles, evaluated at t_1, is

$$J_1 = \frac{\left\{m_1[(s_1[(1-g_2)\mathrm{e}^{f_1\tau} - 1)] - g_3)/g_1]^2 + m_2\right\}\mathrm{e}^{-r\tau} + (c_1 s_1/(f_1 - r))\left(\mathrm{e}^{(f_1-r)\tau} - 1\right)}{1 - \mathrm{e}^{-r\tau}}. \tag{18}$$

The relationship between J_1 and τ is shown in Fig. 4.

Using the above equations, we solved for the maximum roughness level s_2, cycle time τ, resurfacing intensity w, and total discounted cost J_2 for different values of best achievable roughness level s_1. Our results are presented in Table 1. It can be seen that the optimal policy is highly dependent on s_1.

Tsunokawa and Schofer did not specify s_1 in their paper. Their solution is described as: "for all initial roughness values, the steady-state optimal strategy is to apply a 96 mm overlay every 10 years. Depending on the initial roughness, however, the optimal transitional strategies are to

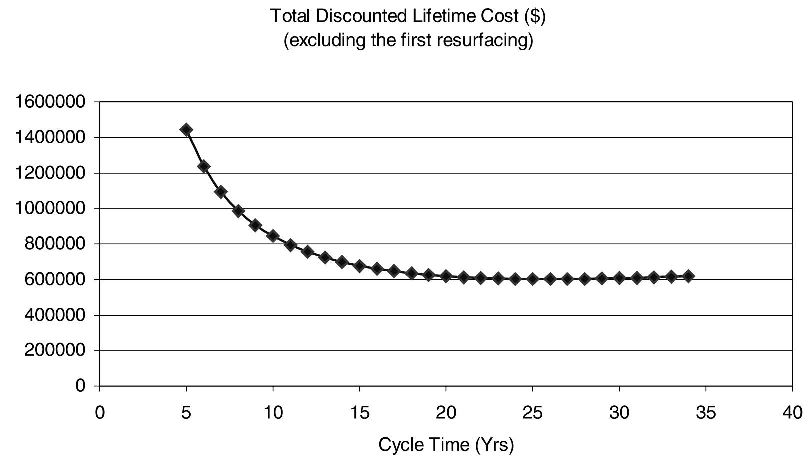

Fig. 4. The relationship between J_1 and τ for $s_1 = 20$ QI.

Table 1
Optimal resurfacing policy

s_1 (QI)	20	30	40	50	60
s_2 (QI)	73	82	89	96	104
$\tau_{(\text{years})}$	26	20	16	13	11
$w_{(\text{mm})}$	154	116	83	55	33
J_2 (million \$)	0.60	0.82	1.01	1.17	1.32

apply one or two overlays that are thinner than or as thick as those of steady-state strategy at shorter intervals. The exception is for the case where the initial roughness is 20 QI, in which no transitional overlay is needed." Table 2 shows the optimal overlay strategies given by their trend curve method.

Their solution is in agreement with ours in that, for all initial roughness values, the steady-state strategies are the same. Transient resurfacing cycles are used because their solution is stated in terms of a steady-state cycle, rather than a steady-state standard. On the other hand, it can be seen by comparison with Table 1 that their steady-state optimal resurfacing cycle times and intensities are not accurate. This is a consequence of using a continuous approximation to the saw-tooth curve, as explained below.

As shown in Fig. 4, the objective function is rather flat near the optimal solution. In fact, the discounted lifetime cost is not very sensitive to cycle time. For example, we find that, when $s_1 = 20$ QI, to resurface every 16 years or every 43 years results in costs that are less than 10% higher than when resurfacing at the optimal interval of 26 years. Therefore, moderate deviations in cycle times should not increase the costs substantially. However, the continuous approximation of the saw-tooth curve erases the discontinuity in the curve, which contains information about the best achievable roughness level. Unfortunately, the best achievable roughness level is what most influences the minimum total cost, as shown in Table 1. Using a continuous approximation to the saw-tooth curve leads to the loss of an important feature of the optimal solution, and the resulting policies are sub-optimal. The lesson here is that the frequency of resurfacing is less important than its intensity for minimising total cost. In other words, a good strategy is to rehabilitate the pavement to the best achievable level, even if this is not scheduled at the optimal time.

Table 2
Optimal overlay strategies given by trend curve method

Initial roughness s_0 (QI)	1st overlay		2nd overlay		Nth overlay ($N > 2$)
	t_1 (year)	w_1 (mm)	t_2 (year)	w_2 (mm)	
20.0	10.0	96	20.0	96	*
40.0	6.8	96	16.8	96	*
60.0	3.8	96	13.8	96	*
80.0	2.0	96	12.0	96	*
100.0	1.0	76	11.5	96	*
120.0	1.0	36	9.7	96	*

Source: Tsunokawa and Schofer (1994).
Note: $*$ = overlay of the same thickness as that of the previous overlay, to be applied ten years after the previous one.

6. Uncertainty in the deterioration process

In our model, Eq. (2) says that the deterioration rate depends only on the current condition of the pavement. Eq. (3) states that the reduction in roughness depends only on the resurfacing intensity and the condition of the pavement when resurfacing is performed. These assumptions are simplistic. For example, we would expect traffic loading and environmental factors such as precipitation and the number of freeze-thaw cycles, to have an impact on pavement deterioration. Therefore, the assumption of deterministic deterioration, i.e. of a perfectly predictable saw-tooth trajectory, is not defensible.

Environmental factors may vary during the course of a year, but over long periods of time, their average should remain constant. As for traffic loading, if the facility is operating near capacity, then traffic flows may change with time, but should not exhibit any systematic trends. As long as the expected values of traffic loading and environmental factors are constant, the optimal strategy is still to define a minimum serviceability level (or maximum roughness level s_2), and whenever the pavement deteriorates to that level, to resurface at a fixed intensity.

Unfortunately, the uncertainty in deterioration results in variable cycle times, so we cannot write out equations to determine the maximum roughness level or resurfacing intensity. A simple method to deal with this uncertainty is simulation. Recall that in the case study, the deterioration function is $F(s) = f_1 s$, and $f_1 = 0.05$. We represent the uncertainty in the deterioration process by assuming that f_1 for each year is a random variable uniformly distributed over the interval (0.03, 0.07). This analysis should give us some rough estimate of the effect of uncertainty on the optimal maximum roughness level s_2. Fig. 5 shows the average result of 30 simulations for $s_1 = 40$ QI.

In the case of deterministic deterioration, the optimal maximum roughness was $s_2 = 89$ QI. We do not see any significant change from that solution when the deterioration process is stochastic. In other words, the minimum serviceability level (or maximum roughness) strategy is robust to the uncertainty in the deterioration process.

The robustness of the optimal standard policy is a desirable property, because it allows decision-makers to obtain near-minimum lifecycle costs even if the cost, deterioration and improvement models that are used are not very precise. This is important in practice, because error-free prediction of pavement deterioration and rehabilitation effectiveness is not possible.

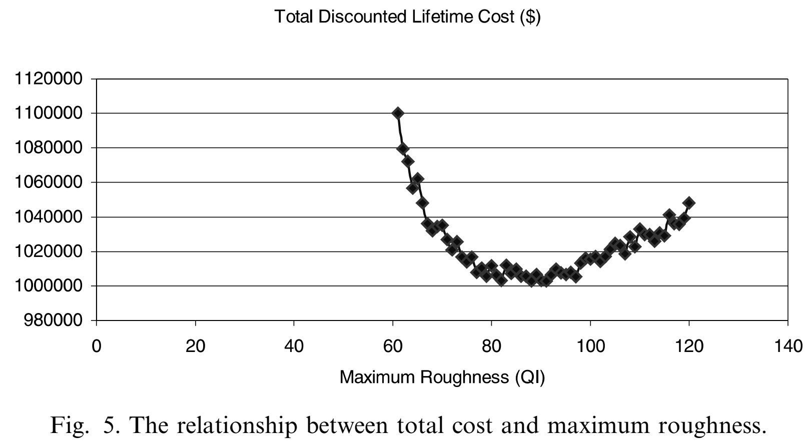

Fig. 5. The relationship between total cost and maximum roughness.

7. Conclusions

This paper presented a simple approach for optimising pavement resurfacing frequency and intensity for the continuous time, continuous state case. At optimality, the system enters the steady state at the time of the first resurfacing. The optimal resurfacing strategy is to define a minimum serviceability level (or maximum roughness level), and whenever the pavement deteriorates to that level, to resurface with a fixed intensity. The optimal strategy does not depend on the initial condition of the pavement, as long as the initial condition is better than the condition that triggers resurfacing. We developed a simple solution procedure and applied it to a case study, using data obtained from the literature. The results indicate that the discounted lifetime cost is not very sensitive to cycle time. What matters most is the best achievable roughness level. The minimum serviceability level strategy is robust in that, when there is uncertainty in the deterioration process, the level that triggers resurfacing does not change much.

One implication of the results shown in this paper is that explicitly accounting for prediction uncertainty in the optimal pavement resurfacing problem may not be needed, as the optimal solution is quite robust to such uncertainty. This result suggests that research in other decision problems in pavement management should focus on obtaining robust policies that can sustain a certain amount of modelling error.

References

Bryson, A.E., Ho, Y., 1975. Applied Optimal Control: Optimisation, Estimation and Control. Hemisphere, New York, NY.

Carnahan, J.V., 1988. Analytical framework for optimizing pavement maintenance. Journal of Transportation Engineering 114, 307–322.

Darter, M., Smith, R., Shahin, M., 1985. Use of life cycle cost analysis as the basis for determining cost-effectiveness of maintenance and rehabilitation treatments for developing a network level assignment procedure. In: Proceedings of the North American Pavement Management Conference, vol. 2, pp. 7.5–7.18.

Fernandez, J.E., Friesz, T.L., 1981. Influence of demand quality interrelationships in optimal policies for stage construction of transportation facilities. Transportation Science 15, 16–31.

Friesz, T.L., Fernandez, J.E., 1979. A model of optimal transport maintenance with demand responsiveness. Transportation Research 13B, 317–339.

Golabi, K., Kulkarni, R., Way, G., 1982. A statewide pavement management system. Interfaces 12, 5–21.

Madanat, S., Ben-Akiva, M., 1994. Optimal inspection and repair policies for transportation facilities. Transportation Science 28, 55–62.

Markow, M., Balta, W., 1985. Optimal rehabilitation frequencies for highway pavements. Transportation Research Record 1035, Transportation Research Board, National Research Council, Washington, DC.

Markow, M., Madanat, S., Gurenich, D., 1993. Optimal rehabilitation times for concrete bridge decks. Transportation Research Record 1392, Transportation Research Board, National Research Council, Washington, DC.

Paterson, W.D.O., 1987. Road Deterioration and Maintenance Effects: Models for Planning and Management. Johns Hopkins University Press, Baltimore, MD.

Tsunokawa, K., Schofer, J.L., 1994. Trend curve optimal control model for highway pavement maintenance: case study and evaluation. Transportation Research 28A, 151–166.

Watanatada, T., Harral, C.G., Paterson, W.D.D., Dhareshwar, A.M., Bhandari, A., Tsunokawa, K., 1988. The Highway Design and Maintenance Standard Model (1): Description of the HDM-III Model. Johns Hopkins University Press, Baltimore, MD.