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A bridge network maintenance framework for Pareto optimization of stakeholders/users costs

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ABSTRACT

For managing highway bridges, stakeholders require efficient and practical decision making techniques. In a context of limited bridge management budget, it is crucial to determine the most effective breakdown of financial resources over the different structures of a bridge network. Bridge management systems (BMSs) have been developed for such a purpose. However, they generally rely on an individual approach. The influence of the position of bridges in the transportation network, the consequences of inadequate service for the network users, due to maintenance actions or bridge failure, are not taken into consideration. Therefore, maintenance strategies obtained with current BMSs do not necessarily lead to an optimal level of service (LOS) of the bridge network for the users of the transportation network. Besides, the assessment of the structural performance of highway bridges usually requires the access to the geometrical and mechanical properties of its components. Such information might not be available for all structures in a bridge network for which managers try to schedule and prioritize maintenance strategies. On the contrary, visual inspections are performed regularly and information is generally available for all structures of the bridge network. The objective of this paper is threefold (i) propose an advanced network-level bridge management system considering the position of each bridge in the transportation network, (ii) use information obtained at visual inspections to assess the performance of bridges, and (iii) compare optimal maintenance strategies, obtained with a genetic algorithm, when considering interests of users and bridge owner either separately as conflicting criteria, or simultaneously as a common interest for the whole community. In each case, safety and serviceability aspects are taken into account in the model when determining optimal strategies. The theoretical and numerical developments are applied on a French bridge network.

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1. Introduction

Highway bridges may experience severe deterioration due to natural hazards, ageing, and increased structural performance demands. In a context of scarce financial resources, managers of highway bridges face increasing challenges in meeting public expectations and, in turn, have to be more efficient for their management strategies. A bridge network is part of a transportation infrastructure and, consequently, plays a major role in urban and economic development and quality of life. Unavailability of bridges due to maintenance and repair actions may have a negative social impact. Indeed, due to their critical location, their partial or total closure results in major disruption such as long diversions, additional congestion, use of the transportation

network not up to equal standards and even the total isolation of certain areas [1].

The aim of this paper is to provide stakeholders with a global decision making tool at the scale of a transportation network, using a probability-based formulation. Remarkable accomplishments have been achieved so far in the field of bridge network maintenance optimization. There have been several attempts to coordinate maintenance strategies at the scale of the transportation network and not for each structure considered individually. Adey et al. [2–4] propose a supply and demand system approach and focus on the ability of the transportation network as a whole to provide an adequate LOS. They optimize the maintenance strategies for a bridge network by balancing supply and demand costs. Also, the use of the reliability theory, which is an excellent candidate to capture uncertainties associated with structural degradation of bridges, has been investigated these last years in bridge network analyses. Liu and Frangopol [5,6] express the overall performance of a bridge network by a time-dependent reliability of connectivity between the origin and the destination

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locations. This enables to determine cost-effective maintenance strategies that are crucial to the network performance. Liu and Frangopol [7,8] use a time-dependent bridge network reliability and bridge reliability importance factor to take into account the importance of each bridge in the transportation network. Frangopol and Liu [9] use a stochastic dynamic program to optimize bridge network maintenance. Orcesi and Cremona [10] propose a reliability-based network-level framework to optimize maintenance strategies of a bridge network. In this model, an event-tree maintenance decisions, based on a structural performance margin considered at inspections, enables to assess the expected maintenance and failure costs (by using also a safety margin). In addition, the probability to have equipment maintenance (e.g., repair activities due to pavement distress, joint degradation) is calculated. Finally, a combination of structural and equipment maintenance is proposed.

However, a bridge network manager may not have access to geometric and material data, and may have very few information on each bridge except the results of visual inspections (e.g., condition rating or overall score of the bridge). In this context, proposing a bridge management framework based on visual inspection is of paramount importance for stakeholders.

The use of stochastic Markov chains for predicting the performance of a bridge network is investigated herein. It is reminded that a Markov chain is a special case of the Markov process whose development can be treated as a series of transitions between certain states. Morcous et al. [11] define environmental categories associated with different bridge elements adopted by existing Markov-chain models. Morcous and Lounis [12] minimize the life-cycle cost of an infrastructure network while fulfilling reliability and functionality requirements over a given planning horizon. Orcesi and Cremona [13] propose a Markov-chain framework applied to a reinforced concrete bridge stock. Their objective is to appraise the overall condition of the reinforced concrete bridge stock to assess the funding required at a national level. However, all the bridges of this bridge stock are considered in the same way, without taking into consideration differences in environmental factors, traffic or size. Also, Smilowitz and Madanat [14], Kuhn and Madanat [15], and Madanat et al. [16] propose bridge management frameworks very efficient in handling large number of facilities. However, there is a problem of heterogeneity among the facilities which is not addressed in these models. It is noted that Robelin and Madanat [17] quantify the heterogeneity at the facility level and propose an optimization maintenance framework based on a history-dependent Markov chain model of bridge deck deterioration.

In this paper, the objectives are to:

- (i) propose a network-level approach where the lifetime-based indicator is available for each bridge of the network (typically a visual inspection result). Users' costs are assessed by an advanced traffic assignment technique reaching an equilibrium, named Wardrop equilibrium [18], where the traffic volume is distributed among the different routes within the transportation network (see [10] for further details). This traffic assignment method is very efficient to model complex networks where the search of the best route is influenced by congestion and leads to a traffic distribution on the overall network.
- (ii) Optimize maintenance strategies of the bridge network by using a Markov chain model to quantify the performance of each bridge and also the uncertainties in the future decisions at inspection times. To include these uncertainties in the decisions made after each inspection, an event tree is built, based on each bridge condition that is quantified through a



Fig. 1. Bridge network.

Markov chain model. This event-tree enables to consider all possible outcomes in the future inspections and calculate expected maintenance and failure costs for users and the bridge owner. It is noted that a failure event is defined herein as the need to perform an unexpected rehabilitation of the bridge. Optimal solutions are determined using a genetic algorithm.

(iii) Compare optimal maintenance strategies, obtained when considering interests of users and bridge owner either separately as conflicting criteria, or simultaneously as a common interest for the whole community.

The proposed probability-based methodology is applied to a French bridge network (see Fig. 1) that plays a strategic role in the overall traffic distribution of the East Paris road network and is directly concerned by possible traffic disruption with quick tremendous consequences. This bridge network is managed by the quality picture of structures—image qualité des ouvrages d'art (IQOA) scoring system [19]. There are five scores in this scoring system which are 1 for a good overall condition, 2 and 2E for non-urgent and urgent maintenance needed, respectively, in case of equipment failures or minor structural damage, 3 and 3U for non-urgent and urgent maintenance needed, respectively, in case of serious structural distress.

2. A Markov model for bridge scoring evolution

As mentioned previously, the objective is to quantify the performance of each facility in the bridge network through the use of an adequate lifetime indicator. This indicator is determined herein by the probability for a bridge to be scored in a certain condition with time. The scoring probability can be perfectly forecasted if the transition matrix and the initial probability vector are known. Indeed, if (i) the probability of a bridge b to be quoted in any score is known at year i (for example, after a visual inspection of the bridge) and stored in a vector $\mathbf{q}_b(i)$ and (ii) the associated homogeneous Markov chain, associated with a transition matrix P_b , is determined, the probability at year i+1 is given by the equation $q_b(i+1) = q_b(i)P_b$. The description of this approach is described in the following, first by investigating if the homogeneous Markovian assumption is justified in the case of the French bridge stock, and, second, by determining the transition matrices for several types of bridges.

2.1. Homogeneous Markovian assumption

The purpose of this section is to check the validity of the homogeneous Markovian assumption in the case of the French national bridge stock (when considering different types of bridges separately). The use of the Markovian assumption was justified in a qualitative way by Orcesi and Cremona [13] in the case of the French

assumed herein to be the total surface of bridges that were scored q_1 at year i and q_2 at year i+1 divided by the total surface of bridges of nature x that were in q_1 at year i, for i between 1996 and 2004 [10]. It is noted that some transitions between IQOA scores are not observed in the database during the time period considered (1996–2004). The corresponding probabilities are then automatically fixed at 0. The obtained transition matrices are

$$\mathbf{P}_1 = \begin{pmatrix} 0.83 & 0.13 & 0.04 & 0 & 0 \\ 0.01 & 0.93 & 0.05 & 0.01 & 0 \\ 0.01 & 0.09 & 0.89 & 0.01 & 0 \\ 0 & 0.01 & 0.15 & 0.81 & 0.03 \\ 0 & 0.06 & 0.04 & 0.08 & 0.82 \end{pmatrix} \quad \mathbf{P}_2 = \begin{pmatrix} 0.84 & 0.12 & 0.04 & 0 & 0 \\ 0.01 & 0.89 & 0.09 & 0.01 & 0 \\ 0 & 0.14 & 0.85 & 0.01 & 0 \\ 0 & 0.06 & 0.08 & 0.86 & 0 \\ 0.06 & 0 & 0.03 & 0 & 0.91 \end{pmatrix} \quad \mathbf{P}_3 = \begin{pmatrix} 0.76 & 0.22 & 0.02 & 0 & 0 \\ 0 & 0.92 & 0.08 & 0 & 0 \\ 0 & 0.07 & 0.91 & 0.02 & 0 \\ 0 & 0 & 0.15 & 0.85 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 & 0 \\ 0 & 0 & 0 & 0.20 & 0.80 \end{pmatrix}$$

$$\mathbf{P}_4 = \begin{pmatrix} 0.88 & 0.12 & 0 & 0 & 0 \\ 0.02 & 0.94 & 0.03 & 0.01 & 0 \\ 0.01 & 0.10 & 0.85 & 0.04 & 0 \\ 0 & 0.05 & 0.09 & 0.86 & 0 \\ 0.11 & 0.16 & 0 & 0.30 & 0.43 \end{pmatrix}$$

reinforced concrete bridge stock only. The objective is herein to justify this assumption for various types of bridges. The IQOA database (using scores of around 9000 bridges between 1996 and 2004) enables to determine the probability for 1 m² of bridge to move from one score to another one within one year for several types of bridge: masonry, reinforced concrete, prestressed concrete, steel and composite.... Indeed, the condition of having at least two consecutive condition rating records for a large number of bridges at different condition levels [12] is fulfilled in case of this database. The Markov assumption states that the condition of a structure at time i depends only on its previous condition for time i-1. With this assumption, the present score is the only one which is taken into account to determine the future condition of a bridge [20]. An analysis of the state transition sequences (STS), as it was realized by Scherer and Glagola [21], is proposed herein to investigate this assumption for several types of bridges. For a given three-states transition sequence (m,j/i), three condition states are considered: past, present, and future condition ratings i, j, and m, respectively (occurring over a three year period). The number of times that this sequence appears in the IQOA database (State Sequence Occurrence—SSO) is determined. The same procedure is applied to the number of times that the sequence present/ future appears (Two Sequence Occurrence—TSO). Given a STS (m,j/i), the probability to move to condition rating *m* knowing the condition ratings *i* and *j* is given by SSO/TSO. Under the Markovian assumption, this probability has to be the same independently of i (if there is no discrepancy in the data). Table 1 presents some examples of STS. The results suggest a relative independence of state-to-state deterioration transition from past state even if some sequences slightly differ or are missing ((3U,3/1) or (3U,3/2) for instance). It is noted that a more exhaustive procedure should be carried out with recent IQOA results to further verify this Markovian assumption. Besides, the homogeneity of the database allows using homogeneous Markov chains in the prediction model. This assumption is justified herein since the French bridge network built during the past 30 years represents a homogeneous bridge stock in terms of design, building, and maintenance. In other words, the probability for a bridge to move from one score to another does not evolve with time in this model.

2.2. Transition matrices

The objective is to build the transition matrix P_x for bridges of type x directly from the IQOA collected scores (scores 1, 2, 2E, 3, and 3U). The probability for a bridge to move from score q_1 to q_2 is

for reinforced concrete slab bridges (P_1), prestressed concrete box girder (P_2), composite bridges (P_3), and frame bridges (P_4), respectively. This approach to determine P_x is practical and efficient for large scoring system database. It is noted that advanced methodologies exist to use a panel of data in the determination of continuous time Markov processes [22].

The matrices detailed in Eq. (1) have to be modified to be used in the optimization problem. Indeed they contain some terms under the diagonal line that are linked to actual maintenance actions and are not pure deterioration matrices. The objective is to determine new transition matrices \tilde{P}_k , for each bridge type, that are pure deterioration matrices without any maintenance (i.e., with no terms larger than 0 under the diagonal line). To eliminate the terms under the diagonal line (by considering that no maintenance action is performed), two options are proposed herein. First, for each line, it is possible to spread the terms under the diagonal line over the different terms according to their rate of change. Second, for each line, the terms under the diagonal line are summed up and added to the diagonal line terms, meaning that the bridge should at least keep its score for the following year if nothing is done. This latter approach is used in the following. If the probability of the bridge b (of type k) to be quoted in any score is known at year i and collected in the vector $\mathbf{q}_b(i)$

$$\boldsymbol{q_b}(i) = \begin{pmatrix} q_{b,1}^i & q_{b,2}^i & q_{b,2E}^i & q_{b,3}^i & q_{b,3U}^i \end{pmatrix} \tag{2}$$

the probability at year i+1 is given by the equation $q_b(i+1) = q_b(i)\tilde{P}_k$. As mentioned previously, if the initial probability vector is known after a visual inspection, the transition matrix enables to determine future scoring probability vectors.

3. Maintenance event-tree based on Markov chains

As mentioned in the introduction, to include uncertainties in the decisions made after a visual inspection, an event tree is built, based on scoring probability vector of each bridge forecasted by using the Markov process detailed in the previous section. For a bridge b with scoring probability vector $\mathbf{q}_b(T_{b,\chi}^{ins})$ at the χ th inspection, the objective is to determine the probability of having different types of maintenance, for example equipment maintenance such as patching pavement, waterproofing replacement, or preventive structural maintenance on deck, or rehabilitation.

Table 1Probabilities associated with STS for different types of bridges.

STS	Reinforced concrete slab bridge	Prestressed concrete slab bridge	Prestressed concrete box girder	Composite bridge
(2,2/1)	1	0.9663	1	1
(2,2/2)	0.9096	0.9148	0.8955	0.8929
(2E,2/1)	0^{a}	0.0637	0^{a}	0^a
(2E,2/2)	0.0694	0.0776	0.0845	0.1011
(3,2/1)	0^a	0^a	0^a	0^a
(3,2/2)	0.0201	0.0075	0.02	0.0060
(3U,2/1)	0^{a}	0^a	0^{a}	0^a
(3U,2/2)	8.99×10^{-4}	0^a	0^{a}	0^a
(2E,2E/1)	1	1	1	1
(2E,2E/2)	1	1	1	1
(2E,2E/2E)	0.9778	0.9740	0.9903	0.9758
(3,2E/1)	0^a	0^a	0^a	0^a
(3,2E/2)	0^a	0^a	0^a	0^a
(3,2E/2E)	0.0152	0.0260	0.0064	0.0242
(3U,2E/1)	0^{a}	0^a	0^a	0^a
(3U,2E/2)	0^{a}	0^a	0^a	0^a
(3U,2E/2E)	0.0070	0	0.0033	0
(3,3/1)	0^a	0^a	0^a	0^a
(3,3/2)	1	1	1	1
(3,3/2E)	1	1	1	1
(3,3/3)	0.9911	0.9753	1	1
(3U,3/1)	0^a	0^{a}	0^a	0^a
(3U,3/2)	0^a	0^{a}	0^a	0^a
(3U,3/2E)	0^a	0^{a}	0^a	0^a
(3U,3/3)	0.0089	0.0247	0^a	0^a

^a Probability is 0 because the corresponding sequence does not exist in the database.

It is reminded that the bridge owner is assumed to have no access to general laws to model and predict deterioration of different elements of the bridges. Therefore, the probability to have a particular maintenance action ζ is assessed in a non-parametric way by using the scoring probability vector and introducing maintenance strategy vectors, $\mathbf{s}_0, ..., \mathbf{s}_\zeta, ..., \mathbf{s}_n$, associated with maintenance strategies $S_0, ..., S_\zeta, ..., S_n$, as shown in a general way in Fig. 2. Such maintenance strategy vectors \mathbf{s}_ζ select scores for which the maintenance action ζ is performed. Elements of vector \mathbf{s}_ζ can have Boolean values 1 or 0. These values and their locations indicate that maintenance of type ζ is, or is not, performed for a bridge with the corresponding score (see Fig. 2). Finally, the probability for a bridge b to have the maintenance ζ is

$$P_{\zeta}^{b}(T_{h,k}^{\text{int}}) = \langle \mathbf{q}_{b}(T_{h,\chi}^{\text{ins}}), \mathbf{s}_{\zeta} \rangle$$
(3)

where $T_{b,k}^{\rm int}$ is the kth maintenance action time of bridge b and $\langle ... \rangle$ is the notation for scalar product of two vectors. It is noted that the inspection time $T_{b,\chi}^{\rm ins}$ and the associated intervention time $T_{b,k}^{\rm int}$ do not necessarily occur the same year.

The concept of probability of maintenance, assessed at each inspection, is applied to the IQOA scoring system, and, in particular to structural and equipment maintenance activities. As introduced in Eq. (3), different decisions can be made after an inspection. Dealing with such different physical phenomena, the probabilities to have structural and equipment maintenance are determined in a separate way [26]. The probabilities to decide of structural preventive maintenance, or structural rehabilitation are, respectively,

$$P_{R_{S}}^{b}(T_{b,k}^{\text{int}}) = \langle \boldsymbol{q}_{\boldsymbol{b}}(T_{b,\gamma}^{\text{ins}}), \boldsymbol{s}_{\boldsymbol{R}_{S}} \rangle \tag{4}$$

and

$$P_{\text{Reh}}^{b}(T_{b,k}^{\text{int}}) = \langle \mathbf{q_b}(T_{b,\gamma}^{\text{ins}}), \mathbf{s_{Reh}} \rangle$$
 (5)

where s_{R_S} and s_{Reh} is the strategy vectors for preventive structural maintenance and rehabilitation, respectively. In a similar way, the

probability to decide of equipment maintenance is

$$P_{R_{\text{eq}}}^{b}(T_{b,k}^{\text{int}}) = \langle \boldsymbol{q_b}(T_{b,\chi}^{\text{ins}}), \boldsymbol{s_{R_{\text{eq}}}} \rangle$$

$$\tag{6}$$

where $\mathbf{s}_{R_{eq}}$ is the strategy vectors for equipment maintenance. These decisions are illustrated in Fig. 3a and b, respectively, for a bridge b at inspection time $T_{b,\gamma}^{ins}$. They are directly related to the outcome of the bridge inspection and are statistically expressed. In the case of structural decisions, there can only be three possible actions: "do nothing" probability $\varepsilon_1 = 1 - (\langle \boldsymbol{q_b}(T_{b,\gamma}^{ins}),$ $s_{R_s} > + \langle q_b(T_{b,\gamma}^{ins}), s_{Reh} \rangle$) or "do preventive structural maintenance" (R_s) with probability $\varepsilon_2 = \langle \mathbf{q_b}(T_{b,\gamma}^{ins}), \mathbf{s_{R_s}} \rangle$ or "rehabilitate the bridge" (*Reh*) with probability $\varepsilon_3 = \langle \mathbf{q_b}(T_{b,\chi}^{ins}), \mathbf{s_{Reh}} \rangle$ (see Fig. 3a). Possible equipment maintenance decisions are "do nothing" (DN) with probability $\mu_1=1-\langle {\it q}_{\it b}(T_{\it b,\chi}^{\it ins}), {\it s}_{\it R_{\it eq}}
angle$ or "do equipment maintenance" (R_{eq}) with probability $\mu_2=\langle {\it q}_{\it b}(T_{b,\gamma}^{\rm ins}), {\it s}_{\it R_{eq}} \rangle$ (see Fig. 3b). Fig. 3c is a mixed event tree that splits the two first branches of the structural event tree of Fig. 3a. The first branch of the structural event tree that was "do nothing" becomes "do nothing" (DN) with probability $P_{DN}^{b}(T_{b,\gamma}^{ins}) = \varepsilon_1 \mu_1$ or "do only equipment maintenance" (R_{eq}) with probability $P_{\mathbf{R}_{eq}}^b(T_{b,\gamma}^{\text{ins}}) = \varepsilon_1 \mu_2$. In the same way, the second branch "preventive structural maintenance" can now be performed without or with equipment maintenance (R_s or R_{s+eq}), respectively, with probability $P_{R_s}^b(T_{b,\chi}^{\text{ins}}) = \varepsilon_2 \mu_1$ and probability $P_{R_{s+eq}}^b(T_{b,\chi}^{\text{ins}}) = \varepsilon_2 \mu_2$. Finally, the last branch (bridge rehabilitation Reh) remains the same with probability $P_{\text{Re}h}^b(T_{b,\gamma}^{\text{ins}}) = \varepsilon_3$ because it deals with large maintenance works including possible equipment repairs After an inspection, the probability to have each of the five decisions is shown in Fig. 3c.

The impact of maintenance works on the IQOA scoring system is introduced through maintenance transition matrices. Corresponding to the four types of maintenance decisions (R_{eq} , R_s , R_{s+eq} , R_s), four maintenance matrices are introduced in Eq. (7). The positions of the value 1 in these matrices are associated with a particular maintenance action. For example the transition matrix M_1 , for which a bridge scored in 2E moves to score 2, is associated

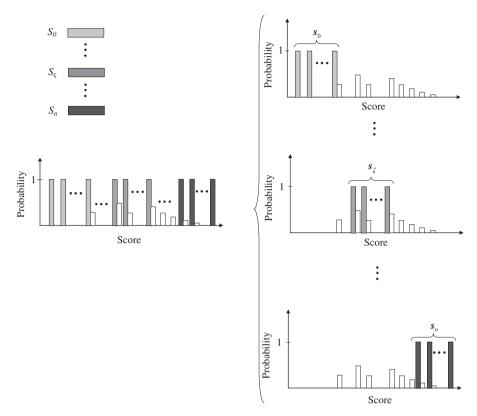


Fig. 2. Determination of the probability of maintenance by using strategy vectors, $\mathbf{s}_0, ..., \mathbf{s}_{\zeta}, ..., \mathbf{s}_{n}$

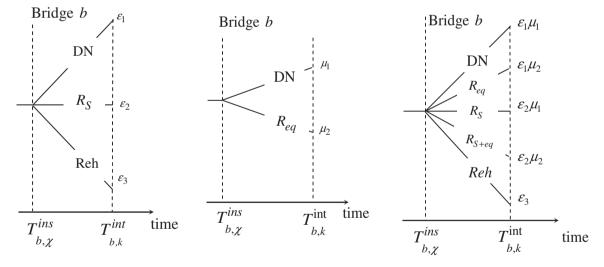


Fig. 3. Detail of (a) structural event tree decision, (b) equipment event tree decision, and (c) mixed reliability and serviceability event tree decision.

with an equipment maintenance (R_{eq}) .

$$\mathbf{M}_{1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{M}_{2} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 &$$

If equipment maintenance is performed on the bridge b at intervention time $T_{b,1}^{\rm int}$, the scoring probability vector after this maintenance is $\boldsymbol{q_b}(T_{b,1}^{\rm int}) = \boldsymbol{q_b}(0)\tilde{\boldsymbol{P}}_b^{T_b,1} {\rm int}\boldsymbol{M}_1$. Fig. 4 details the framework to build the event tree for all intervention times of the N bridges in the network. Such an event tree procedure enables to fully consider the impact of decisions made at one intervention time on the following intervention times and probabilities of maintenance/rehabilitation actions.

3.1. Probability of failure

The construction of an event tree enables to assess failure and maintenance probabilities at each inspection or intervention time.

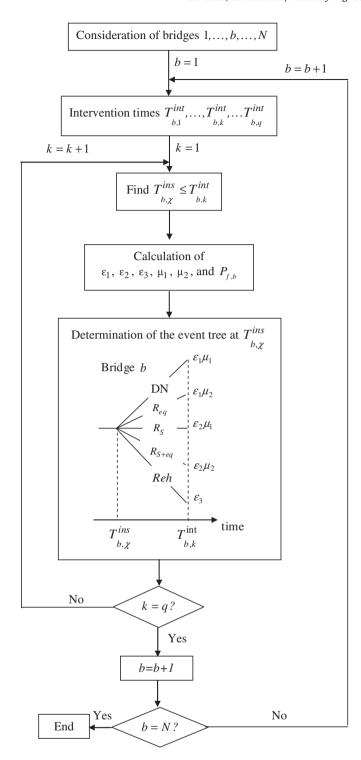


Fig. 4. Framework for determination of an event-tree maintenance decision for the bridge network.

The probability of failure for a bridge at time t is a function of all previous interventions. For a bridge b, with maintenance times $T_{b,1}^{\text{int}}$, $T_{b,2}^{\text{int}}$, $T_{b,3}^{\text{int}}$, ... the probability of failure for $t \le T_{b,1}^{\text{int}}$ is

$$P_{f,b}(t) = q_{b,3U}^t p_{dem} \tag{8}$$

where $q_{b,3U}^t$ is the probability for the bridge b to be scored in 3U at time t (see Eq. (2)), and p_{dem} is the probability that some rehabilitation is performed on a bridge scored in 3U. In France, around 5% of bridges in 3U are rehabilitated each year. This

percentage is used herein to assess the probability of rehabilitation p_{dem} for a bridge in 3U.

For $T_{b,1}^{\rm int} < t \le T_{b,2}^{\rm int}$, the probability of failure of bridge b depends now on all the possible maintenance actions that were carried out at time $T_{b,1}^{\rm int}$. The probability of failure for this bridge is then given by

$$\begin{cases}
P_{f,b}(t) = \left(P_{DN}^{b}(T_{b,1}^{ins})q_{b,3U|DN,T_{b,1}^{int}}^{t} + P_{Req}^{b}(T_{b,1}^{ins})q_{b,3U|R_{eq},T_{b,1}^{int}}^{t} + P_{R_{s}}^{b}(T_{b,1}^{ins})q_{b,3U|R_{s},T_{b,1}^{int}}^{t} + k \\
k + P_{R_{s+eq}}^{b}(T_{b,1}^{ins})q_{b,3U|R_{s+eq},T_{b,1}^{int}}^{t} + P_{Reh}^{b}(T_{b,1}^{ins})q \\
b,3U \middle| \text{Reh},T_{b,1}^{int} \middle| p_{dem}
\end{cases}$$
(9)

where $q_{b,3U|\zeta,T_{b,1}^{\rm int}}^t$ is the probability that the bridge b is scored in 3U at time $t \geq T_{b,1}^{\rm int}$ knowing that the maintenance ζ was performed at $T_{b,1}^{\rm int}$.

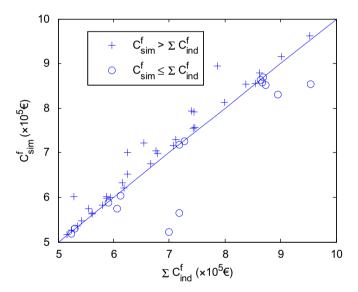


Fig. 5. Network effect.

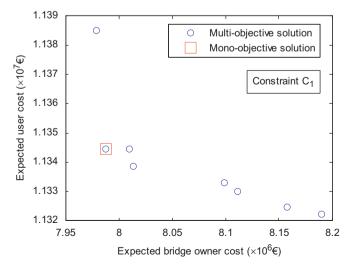


Fig. 6. Trade-off between expected user and bridge owner costs for solutions of maintenance times (constraint C_1).

4. Users and bridge owner costs

The road users are always assumed to take the less expensive way. As vehicle speed is limited by traffic volume, the larger the number of vehicles on a link is, the more the speed will be reduced. The speed, and consequently the travel time, depends both on the traffic congestion according to the Highway Capacity Manual [23]. The search of the best route combined with traffic congestion leads to an equilibrium, named Wardrop equilibrium, where the traffic volume is distributed among the different routes

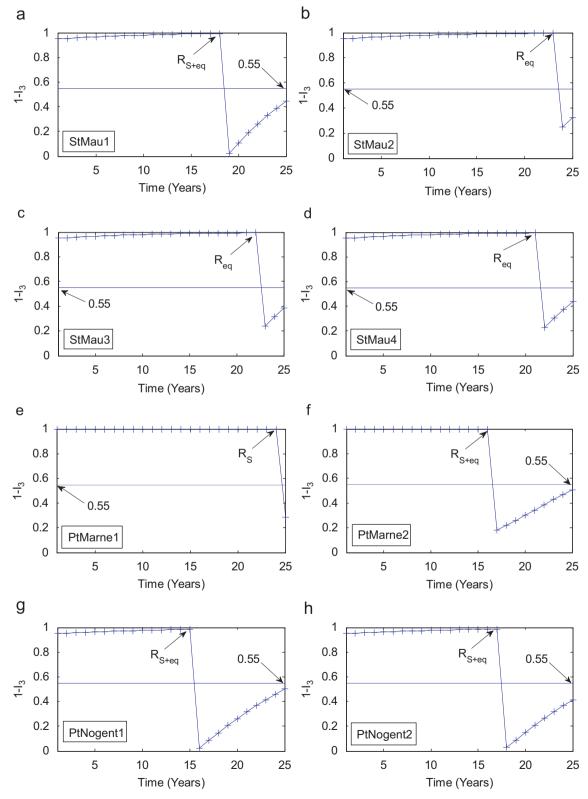


Fig. 7. Profile of $1-I_3$ (constraint C_1) for (a) StMau1, (b), StMau2 (c), StMau3, (d) StMau4, (e), PtMarne1 (f), PtMarne2, (g) PtNogent1, and (h) PtNogent2.

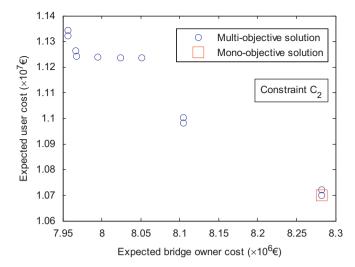


Fig. 8. Trade-off between expected user and bridge owner costs for solutions of maintenance times (constraint C_2).

within the network: When this equilibrium is reached, no user can change his route to improve his travel cost. The network failure cost is the difference between users' costs when an adequate level of service (LOS) is provided and the users' costs for an inadequate service [3,10]. If one or several bridges are partially operated due to maintenance actions or failure events, users' costs are determined again by a new Wardrop equilibrium. This model enables to quantify each event in a monetary value and express the ability of the network to provide an adequate LOS as a whole. It is assumed herein that equipment and preventive structural maintenances (including R_{eq} , R_s R_{s+eq}) need an hourly traffic capacity decrease of 30% and that rehabilitation (Reh) needs an hourly traffic capacity decrease of 60% because of scaffolding for instance. Failure events are associated with distresses that lead to important and unexpected rehabilitation works. In this case, the traffic capacity is assumed to be decreased by 90% because of safety measures. Two modes are considered: cars with an average travel time value between business, home-based and personal trips for travel distance under 50 km (8.94 €/h), and heavy goods vehicles (HGV) with a travel time value fixed at 31.4 €/h. The cost database QUADRO (QUADRO "QUeues And Delays at ROadworks" 2006) provides a method for assessing road users' costs. This database is used in this paper to determine vehicle operating costs (VOC) and to consider that the traffic disruption induced by maintenance works has a negative impact on road safety and, consequently, increases the accident rate on the road part affected by works. Further details concerning this traffic assignment model can be found in Ref. [10].

Maintenance costs for the bridge owner are assessed by using the statistical study realized by Binet [19]. In this study, an estimation of maintenance costs is calculated for a sample of bridges chosen at random and the results are extrapolated to the whole population. This study enables to assess the maintenance costs according to different bridge types and scores. Equipment cost is that usually induced by resealing and is assumed herein to be the cost of maintenance for a bridge scored in 2E. Preventive structural maintenance cost is associated with bridge strengthening and is assumed to be the cost for a bridge scored in 3. When preventive structural and equipment maintenance actions are performed simultaneously, the cost is the sum of those for equipment and preventive structural maintenance. Rehabilitation deals with large works to increase bridge reliability and the associated cost are assumed to be that for a bridge scored in 3U. Finally, the cost of

failure for the bridge owner is assumed to be the rehabilitation cost increased by 15%, to include unexpected charges.

A combination θ of two bridges linked by a maintenance combination ξ means that maintenance $\xi(k)$ on bridge $\theta(k)$ and maintenance $\xi(l)$ on bridge $\theta(l)$ are performed simultaneously. For the owner, maintenance $\cot C^{\theta}_{owner,REP,\xi}$ is assumed to be the sum of maintenance costs for each bridge. For users, repair cost $C^{\theta}_{users,REP,\xi}$ includes additional traffic congestion and travel time increase. In each case, additional users' costs due to maintenance works and/or bridge failure are obtained by assessing a new Wardrop equilibrium. These costs cannot be determined as easily as those for the owner, since bridges in the combination do not necessarily have maintenance performed at the same time. Traffic disruption is different if maintenance is performed only on one bridge or on both bridges of the combination. Similarly, costs for the owner $C^{\theta}_{owner,f}$ and the users $C^{\theta}_{users,f}$, when there are failure events (unexpected major rehabilitation works) for a bridge combination θ , are considered herein.

To highlight the network effect on users' costs, the bridge network shown in Fig. 1 is considered and the example of a failure event (90% decrease of traffic capacity) on a two bridge combination is studied. Fig. 5 opposes the costs if the lanes closure occurs simultaneously with the sum of costs if it occurs at different times. For an unfavourable bridge combination, the cross in Fig. 5 is above the median line. Unlikely, a favourable combination leads to a circle below the median line. Finally, if there is no difference between the two situations, the circle is on the median line. Depending on the bridge combination, users will take advantage or not, if lane closures occur simultaneously or at different times. The objective of the optimization framework, detailed in the following section, is to consider the network effect when determining optimal maintenance strategies.

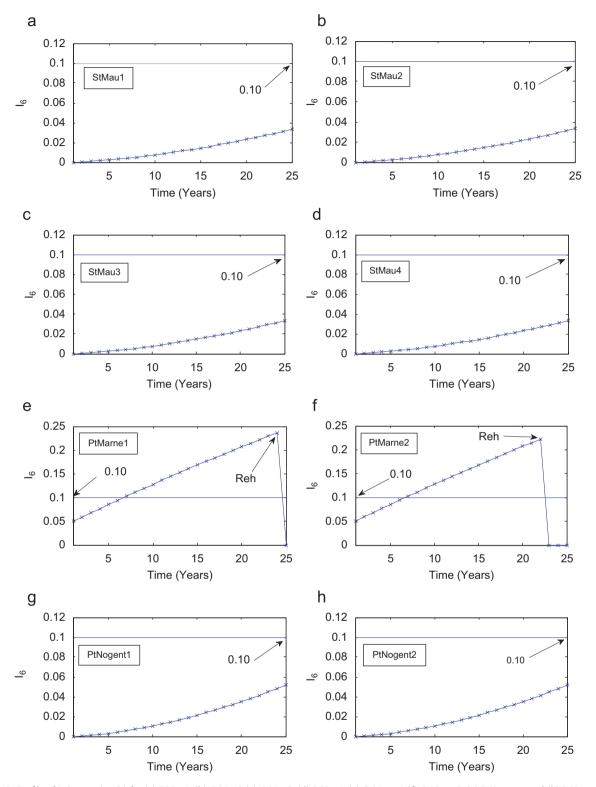
5. Optimization of maintenance strategies

The two-step optimization framework proposed in Ref. [10] is used herein. First, intervention times are determined by considering all possible inspection results. Obviously, those with largest expected costs (costs multiplied by the associated probability of occurrence) will drastically influence the choice of the intervention times. Second, intervention times previously found are fixed and maintenance actions that minimize maintenance costs are determined. A genetic algorithm based procedure [24] is used herein to solve this two-step optimization problem [25]. Each step of the optimization procedure can be handled either with only one objective function (i.e., the sum of expected user and bridge owner costs) or with two objective functions (i.e., the expected user bridge owner cost considered separately). In the first case, interests of users and bridge owner are considered as common (for the whole community). In the second case, they are considered separately as conflicting criteria.

5.1. Determination of optimal intervention times

Since the maintenance strategies depend on inspection times, optimal maintenance strategies should take into consideration all the possible inspection results. Consequently, all the possible maintenance actions are weighted by the probability that the corresponding distress is detected at the last inspection before the intervention time. Expected costs that are considered during this first optimization step are the expected inspection cost

$$E[C_{INSP}^{X}] = \sum_{m=1}^{N} \left\{ \sum_{n=1}^{n_{ms}^{m}} \left\{ \left(C_{X,insp}^{m} (1 - P_f^{m} (T_{m,n}^{ins})) \right) \frac{1}{(1 + \alpha)^{T_{m,n}^{ins}}} \right\} \right)$$
(10)



 $\textbf{Fig. 9.} \ \ Profile \ of \ \textit{I}_{6} \ (constraint \ \textit{C}_{2}) \ for \ (a) \ StMau1, \ (b), \ StMau2 \ (c), \ StMau3, \ (d) \ StMau4, \ (e), \ PtMarne1 \ (f), \ PtMarne2, \ (g) \ PtNogent1, \ and \ (h) \ PtNogent2.$

the expected cost of failure

and the expected maintenance cost

$$E\left[C_{f}^{X}\right] = \sum_{m=1}^{N} \left(\sum_{l=1}^{n_{\text{int}}} \left\{\sum_{\omega=1}^{\Omega} \sum_{\theta \in \Theta} C_{X,f}^{\theta} \frac{P_{f}^{\theta}(T_{m,l}^{\text{int}})}{(1+\alpha)^{T_{m,n}^{\text{int}}}}\right\}\right)$$

$$(11) \qquad E\left[C_{REP}^{X}\right] = \sum_{m=1}^{N} \left(\sum_{l=1}^{n_{\text{int}}} \left\{\sum_{\omega=1}^{\Omega} \sum_{\theta \in \Theta(m)} \sum_{\xi \in \Xi(\theta)} C_{X,\xi}^{\theta} \frac{P_{\xi}^{\theta}(T_{m,l}^{\text{int}})}{(1+\alpha)^{T_{m,l}^{\text{int}}}}\right\}\right)$$

$$(12)$$

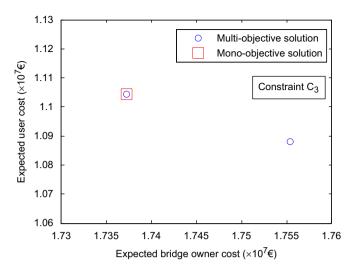


Fig. 10. Trade-off between expected user and bridge owner costs for solutions of maintenance times (constraint C_3).

where X is either the bridge owner or the users, N is the number of bridges, n_{ins}^m is the number of inspection times for bridge m, $P_f^m(T_{m,n}^{ins})$ is the probability of failure of bridge m at the nth inspection, ω is the size of the bridge combination, Θ is the set of combination bridge, $\Theta(m)$ is the set of bridge combination that includes bridge m, $\Xi(\theta)$ is the set of all events combination for each set θ of bridges (these events can be maintenance or failure events), n_{int}^m is the number of intervention times for bridge $m \in \theta$, $P_{\xi}^{\theta}(T_{m,l}^{\text{int}})$ is the probability of maintenance ξ for the bridge combination θ , at the *l*th intervention time for bridge $m \in \theta$, $P_f^{\theta}(T_{m,l}^{\text{int}})$ is the probability of failure of the bridge combination θ , at the *l*th intervention of bridge $m \in \Theta$, $C_{X,ins}^m$ is the inspection cost (for X) of bridge m, $C_{X,\xi}^{\theta}$ is the cost (for X) of maintenance combination ξ on bridge combination θ , $C_{X,F}^{\theta}$ is the cost of failure (for *X*) of bridge combination θ , and finally α is the yearly discount rate of money fixed at 4% herein.

The optimization problem for maintenance times is defined as

Find
$$T_{1,1}^{\text{int}}, \dots, T_{1,n_{\text{int}}}^{\text{int}}, \dots, T_{N,1}^{\text{int}}, \dots, T_{N,n_{\text{int}}}^{N}$$
 (13)

to minimize
$$\sum_{users,owner} E[C_{INSP}^{X}] + E[C_{f}^{X}] + E[C_{REP}^{X}]$$
 (14)

for a mono-objective procedure and

to minimize
$$\begin{cases} E[C_{INSP}^{users}] + E[C_{REP}^{users}] + E[C_f^{users}] \\ E[C_{INSP}^{owner}] + E[C_{REP}^{owner}] + E[C_f^{owner}] \end{cases}$$
 (15)

for a multi-objective procedure, such that

$$CI^m(t) < CI_0^m(t), \quad \forall m = 1...N$$
 (16)

where $Cl^m(t)$ is the condition index of bridge m at time t and $Cl_0^m(t)$ is the minimal condition index of bridge m at time t. In the following, condition index is associated with IQOA annual quality indicators $1-I_3(t)$ and $I_6(t)$, where $I_3(t)=q_{b,1}^t+q_{b,2}^t$ and $I_6(t)=q_{b,3U}^t$ (see Eq. (2)). These indicators enable to assess the probability of performing preventive and urgent structural maintenance to prevent disruption and ensure safety, respectively. The sampling period is one year and maintenance times can get integer values between 1 and 25 years (end of the intervention horizon). An initial population of 51 solutions is considered in the genetic algorithm, and crossover/mutation operations are used with

respective probabilities fixed at 85% and 10%. In case of a monoobjective procedure (Eq. (14)), optimal solution is that with the lowest expected intervention cost for the bridge owner and users as a whole. In case of a multi-objective procedure, Pareto solutions are obtained by considering the two conflicting objective functions in Eq. (15). Since the maintenance planning is a constraint optimization problem, the penalty method is used to ensure that solutions satisfy the constraints. The maximum number of iteration is fixed at 3000 iterations. It is noted that (i) larger population sizes were also considered in the optimization process, and (ii) the optimization process was performed several times, to check that the optimization results remain the same and avoid local optimum.

5.2. Determination of optimal maintenance actions

Once the optimal intervention times are obtained and fixed, optimal maintenance actions can be determined. The expected costs for X (X being either the owner or the users), considered in this second optimization step, are the expected cost of failure

$$E\left[\hat{C}_{f}^{X}\right] = \sum_{m=1}^{N} \left(\sum_{l=1}^{n_{\text{int}}^{m}} \sum_{\omega=1}^{\Omega} \sum_{\theta \in \Theta(\omega)} C_{X,f}^{\theta} P_{f}^{\theta}(T_{m,l}^{\text{int}}) \frac{1}{(1+\alpha)^{T_{m,n}^{\text{int}}}} \right)$$
(17)

and the expected maintenance cost

$$E\left[\hat{C}_{REP}^{X}\right] = \sum_{m=1}^{N} \left(\sum_{l=1}^{n_{\text{int}}^{m}} C_{X,r_{m,l}}^{m} (1 - P_{f,m}(T_{m,l}^{\text{int}})) \frac{1}{(1+\alpha)^{T_{m,l}^{\text{int}}}}\right)$$
(18)

where $r_{m,l}$ is the lth possible maintenance action for bridge m (see Eqs. (10), (11) and (12) for other notations). The maintenance actions optimization problem is formulated as

find
$$r_{1,1}, \dots, r_{1,n_1}, \dots, r_{N,1}, \dots, r_{N,n_N}$$
 (19)

to minimize
$$\sum_{users.owner} E[\hat{C}_{REP}^X] + E[\hat{C}_f^X]$$
 (20)

in case of a mono-objective procedure and

to minimize
$$\begin{cases} E[\hat{C}_{REP}^{users}] + E[\hat{C}_{f}^{users}] \\ E[\hat{C}_{REP}^{owner}] + E[\hat{C}_{f}^{owner}] \end{cases}$$
(21)

in case of a multi-objective procedure such that

$$CI^{m}(t) < CI_{0}^{m}(t), \quad \forall m = 1...N$$
 (22)

The genetic algorithm procedure, previously introduced to determine intervention times, is used herein in a similar way to determine optimal maintenance actions.

5.3. Case study: optimization of maintenance strategies for a bridge network

The optimization problem, detailed in the previous section, is used to determine optimal maintenance strategies for the eight bridges shown in Fig. 1. The bridges are prestressed concrete structures (denoted PtMarne1, PtMarne2, StMau1, StMau2, StMau3, and StMau4, respectively) located on the common trunk section of the A4 and A86 motorways (part of the network with the highest congestion level) and composite bridges (denoted PtNogent1 and PtNogent2, respectively). An initial scoring probability vector is introduced for each bridge to consider the uncertainty in the IQOA scoring process. Hence, the initial probability of being in one of the five IQOA scores is $\mathbf{q}_{0.1} = \begin{pmatrix} 0 & 0.05 & 0.95 & 0.05 & 0 \end{pmatrix}$ for bridges StMau1, StMau2, StMau3 and StMau4 and the bridges PtNogent1 and PtNogent2. This means that all these bridges are currently

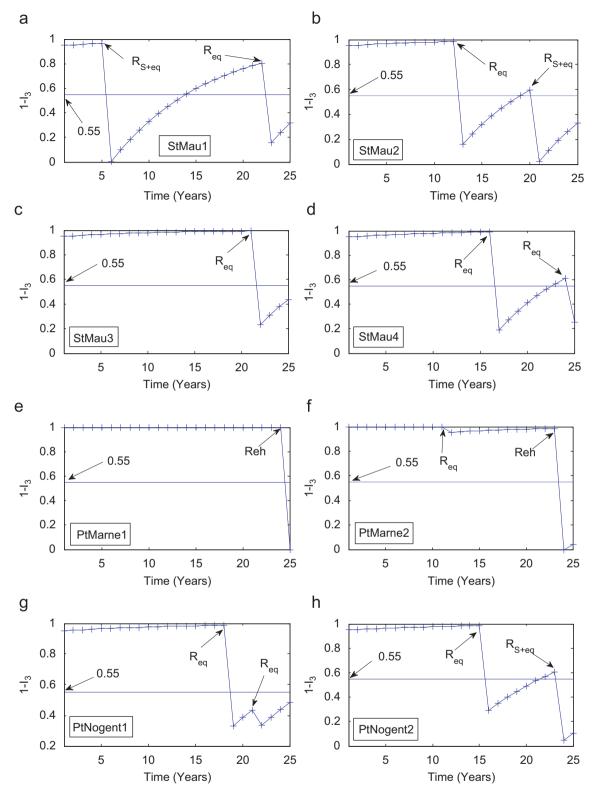


Fig. 11. Profile of 1-I₃ (constraint C₃) for (a) StMau1, (b), StMau2 (c), StMau3, (d) StMau4, (e), PtMarne1 (f), PtMarne2, (g) PtNogent1, and (h) PtNogent2.

scored in 2E with a degree of uncertainty assessed at 10% (there might be a probability of 5% to be scored either in 2 or in 3). Besides, the vector $\mathbf{q}_{0,2} = \begin{pmatrix} 0 & 0 & 0.05 & 0.90 & 0.05 \end{pmatrix}$ is the initial probability distribution for bridges PtMarne1 and PtMarne2 (with a probability of 5% to be scored either in 2E or in 3U). Concerning the maintenance strategy assumed to be fixed by the owner at each

inspection, the following strategy vectors, $\mathbf{s_{R_{eq}}} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \end{pmatrix}$, $\mathbf{s_{R_S}} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix}$, and $\mathbf{s_{Reh}} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \end{pmatrix}$, are used. They define IQOA scores that will lead, respectively, to an equipment maintenance, a preventive structural maintenance or rehabilitation after an inspection. For example, positions of 1 and 0 in $\mathbf{s_{R_{eq}}}$ indicate that a bridge scored 2E or 3 will receive an equipment maintenance.

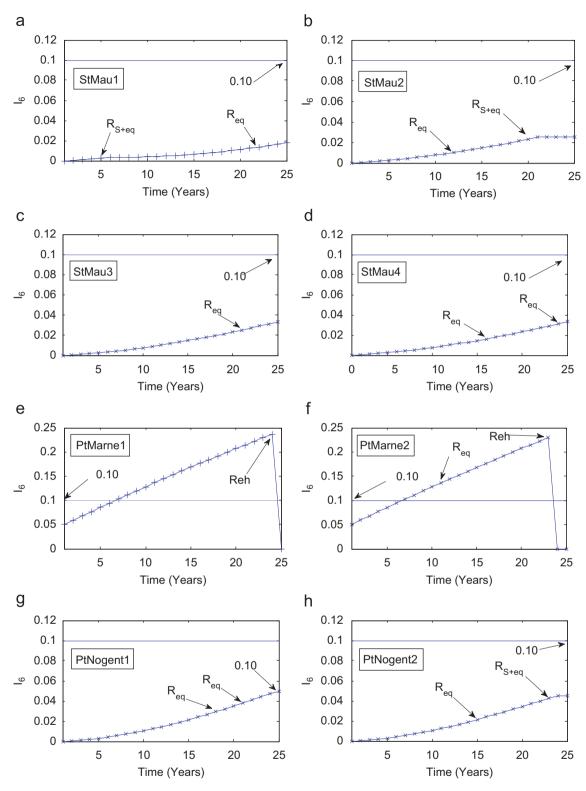


Fig. 12. Profile of I_6 (constraint C_3) for (a) StMau1, (b), StMau2 (c), StMau3, (d) StMau4, (e), PtMarne1 (f), PtMarne2, (g) PtNogent1, and (h) PtNogent2.

It is noted that $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix} - (s_{R_S} + s_{Reh})$ is the strategy vector for doing no structural maintenance (Fig. 3a). Similarly, $\begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix} - (s_{R_{eq}})$ is the strategy vector for doing no equipment maintenance (Fig. 3b). The optimal maintenance planning is determined for the three constraint cases

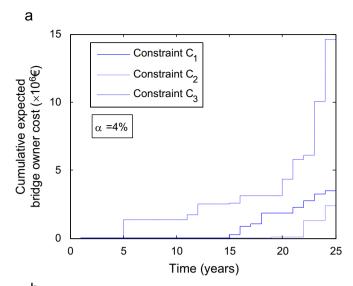
constraint
$$C_1$$
: $1 - I_3(T_f) < 55\%$ (23)

constraint
$$C_2$$
: $I_6(T_f) < 10\%$ (24)

and

constraint
$$C_3$$
: $1-I_3(T_f) < 55\%$ and $I_6(T_f) < 10\%$ (25)

where T_f is the end of the planning calendar. For constraint C_1 , the trade-off between the expected user and bridge owner costs for



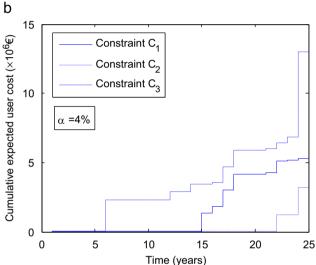


Fig. 13. Cumulative expected costs, when considering the mono-objective solutions for constraints C_1 , C_2 , and C_3 , respectively, for (a) the bridge owner, and (b) users.

solutions of maintenance times (first step in the optimization process) is shown in Fig. 6 (after 3000 iterations). The optimal mono-objective solution is also identified in Fig. 6. It is noted that the mono-objective solution provides a good compromise between reducing the expected user cost and not increasing the expected bridge owner cost in a significant way. The second optimization step enables to determine optimal maintenance actions at times previously determined. This second step is performed herein for intervention times obtained with the mono-objective solution. In this second step, only one solution is obtained either with the mono or the multi-objective optimization procedure. This shows that the interests of users and the bridge owner are not conflicting criteria (in this example) in the second optimization step. The profiles of indicator $1-I_3$ are illustrated in Fig. 7(a-h) for the eight bridges, respectively, and it is shown that the constraint C_1 is satisfied at T_6 .

The same procedure is performed for constraint C_2 . The tradeoff between the expected user and bridge owner costs for solutions of maintenance times (first step in the optimization process) is illustrated in Fig. 8 (after 3000 iterations). The optimal mono-objective solution, identified in Fig. 8, is a solution of the multi-objective process with the highest cost for the bridge owner. Therefore, the effort of the bridge owner has to be more

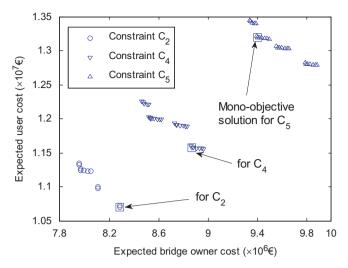


Fig. 14. Trade-off between expected user and bridge owner costs for solutions of maintenance times when considering constraints C_2 , C_4 , and C_5 .

important than that of the users to reach the optimal solution for the whole community, when considering constraint C_2 . Finally, the profiles of indicator I_6 are shown in Fig. 9(a–h) for the eight bridges, respectively, to check that the constraint C_2 is satisfied.

For constraint C_3 , there is no solution if there is only one intervention time for each bridge. Indeed, two intervention times for each bridge are needed to satisfy the constraint C_3 in Eq. (25). The trade-off between the expected user and bridge owner costs for solutions of maintenance times (first step in the optimization process) is illustrated in Fig. 10 (after 3000 iterations) and there are only two solutions for the multi-objective optimization. This can be justified since few solutions satisfy constraint C_3 (with thresholds on $1-I_3(T_f)$ and $I_6(T_f)$, simultaneously). The profiles of indicators $1-I_3$ and I_6 , shown in Figs. 11(a-h) and 12(a-h), respectively, for the eight bridges, satisfy constraint C_3 .

It is noted that the mono-objective solution is always one particular solution of the Pareto front in each of the three simulations (see Figs. 6, 8, 10) but not always located in the same area of the Pareto front. The optimal solution for the community (the mono-objective solution) is then not necessarily the most interesting for users or for the bridge owner.

The cumulative expected costs, for the mono-objective solutions when considering constraints C_1 , C_2 , and C_3 , respectively, are shown in Fig. 13a for the bridge owner and in Fig. 13b for users. It is obvious that constraint C_2 leads to the lowest expected cumulative costs for both users and the bridge owner, and that constraint C_3 leads to the highest expected cumulative costs for both users and the bridge owner.

In Eqs. (23)–(25), thresholds are applied only at the end of the maintenance planning T_f . Two additional constraints are considered to take into account the change of constraints with time t

constraint
$$C_4$$
: $I_6(t) < 0.30 - 0.008t$ (26)

and

constraint
$$C_5$$
: $I_6(t) < 0.15 - 0.002t$ (27)

The three Pareto fronts for solutions of maintenance times, obtained when considering constraints C_2 , C_4 , and C_5 , are provided in Fig. 14. It is shown that the constraint C_5 leads to the most "expensive" optimal solutions (both for the bridge owner and the users). Conversely, optimal solutions are less expensive when considering constraint C_2 . Also, the mono-objective solution for

intervention times is identified in Fig. 14 for each case of constraint. Obviously, constraint C_4 is more restrictive than C_2 , and C_5 is more restrictive than C_4 . Since the mono-objective solution shifts on the left of the Pareto front when considering successively C_2 , C_4 , and C_5 , it is shown that the optimal solution for the whole community requires larger efforts from the users and, consequently, less from the bridge manager. Finally, by comparing mono and multi-objective optimization frameworks, it is possible to provide the bridge manager with several optimal solutions, depending on his/her financial resources, and to indicate which solution is optimal for the whole community.

6. Conclusions

The objective of the methodology proposed in this paper is to help the bridge owner in scheduling its bridge maintenance strategies at the scale of the transportation network. The originality of this approach is to determine performance indicators through the use of Markov chains which, in turn, enables to determine an event tree decision at each inspection time. This allows quantifying the performance of a bridge network even if the only information available is the result of visual inspections. The homogeneous Markovian assumption to model the ageing of the bridge score is proposed and discussed for various types of bridges. The homogeneity of the database allows using homogeneous Markov chains in the prediction model. Then, the transition matrices for several types of bridges are estimated. This prediction model is combined with an advanced traffic assignment that considers congestion phenomenon and enables to quantify the LOS of the bridge network in an accurate way. Finally it is possible to determine optimal maintenance strategies at the scale of the transportation network when considering interests of the users and the bridge owner separately and/or simultaneously.

Future improvements require taking into account uncertainties of the transition matrix elements in a more accurate way. A more exhaustive procedure should be carried out to further justify the use of the Markovian assumption for each type of bridge. It is noted that new inspection results should be included in the database each year to be sure that the transition matrix always includes the best information for each bridge. Also, the same transition matrix is used before and after a maintenance action in this paper. The consideration of two different degradation processes (i.e., two different transition matrices), before and after a maintenance action in the optimization process, would increase the accuracy of the cost analysis. Finally, different geographical areas should be considered when determining transition matrices, to include the effect of the environment in the prediction model.

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