Estimation of Pavement Performance Deterioration Using Bayesian Approach

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Abstract: This paper investigates an incremental pavement performance model based on experimental data from the American Association of State Highway Officials road test. Structural properties, environmental effects, and traffic loading, the three main factors dominating the characteristic of pavement performance, are incorporated into the model. Due to the limited number of variables that can be controlled and observed, unobserved heterogeneity is almost inevitable. Most of the existing models did not fully account for the heterogeneity issue. In this paper, the Bayesian approach is adopted for its ability to address the issue of interest. The Bayesian approach aims to obtain probabilistic parameter distributions through a combination of existing knowledge (prior) and information from the data collected. The Markov chain Monte Carlo simulation is applied to estimate parameter distributions. Due to significant variability in the parameters, the need exists to address heterogeneity in modeling pavement performance. Furthermore, it is shown that not all the parameters are normally distributed. It is therefore suggested that the performance model developed in this research provides a more realistic forecast than most previous models. In addition, pavement deterioration forecast based on the Gibbs output is performed at different confidence levels with varying inspection frequencies, which can enhance the decision-making process in pavement management. In general, the Bayesian approach presented in this paper provides an effective and flexible alternative for model estimation and updating, which can be applied to both the road test data sites and other data sources of interest.

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Introduction

Performance prediction plays a central role in highway infrastructure system design and management. From the design perspective, highway service life is estimated by the projected performance as a function of structural properties, environmental conditions, and traffic loading. In the context of infrastructure management, a transportation agency's work plan, and budget allocation also relies on the pavement performance forecast.

Pavement performance can be expressed in terms of distresses including rutting, cracking, and roughness. It can also be evaluated through subjective indicators such as the present serviceability rating (PSR) as established by the American Association of State Highway Officials (AASHO) during the Road Test (HRB 1962).

To date, much effort has been given to developing state-of-the-art pavement performance models to address the deterioration process. Basically, either the empirical or mechanistic approach, or the two approaches combined, is utilized for performance modeling. For example, an empirical sigmoid curve was applied to fit the pavement deterioration process by Garcia-Diaz (Garcia-Diaz and Riggins 1984). A mechanistic approach was incorporated to develop the damage functions for rutting, fatigue cracking, and loss of pavement serviceability index (PSI) by Rauhut (Rauhut et al. 1983). After the World Bank road test in Brazil, Paterson (1987) established a series of empirical performance models on the basis of a comprehensive study of previous modeling efforts and the characteristic of the road test data. Currently, the most widely accepted model is the AASHTO design equation (AASHTO 1993).

In addition, in terms of model format, both linear and nonlinear models were examined in the previous studies. The nonlinear models were found to be more appropriate for determining performance deterioration due to traffic and environmental impacts. However, many of the nonlinear models lack physical explanation, statistical soundness, or are not suitable for the process. These problems have been improved through development of a nonlinear model with panel data (Archilla 2000; Archilla and Madanat 2001; Prozzi 2001; Prozzi and Madanat 2003). The nonlinear modeling is capable of better addressing performance deterioration process along time, while the panel data structure facilitates to differentiate performance characteristics across pavement sections (heterogeneity). In such cases, the unobserved heterogeneity was captured by the intercept term by means of random-effect models, while it is assumed that the other parameters were fixed. Although this approach produces efficient parameter estimates, heterogeneity is not sufficiently captured. The unobserved heterogeneity should potentially be reflected not only through the intercept but through the other regression parameters. Hence, it is more reasonable to relax the above assumption and let the relevant parameters be random (random parameters

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J. Infrastruct. Syst. 2006.12:77-86.

model), since each section may possess unique characteristics affecting the deterioration process. The tradeoff of adopting the more flexible model is the need for additional computational effort. Even for the random-effect nonlinear model, the process of searching for an iterative solution under the classical approaches such as generalized least square (GLS) and maximum likelihood estimation (MLE) is time consuming. It can be expected that the computational work required for a random parameter nonlinear model is more demanding.

As an alternative, this paper will describe how the Bayesian approach can be applied to effectively address the issue. The Bayesian approach offers the flexibility to incorporate existing knowledge so that previous experience can be utilized rather than ignored (Zellner 1971). In addition, obtaining the probabilistic distribution of the parameters to reflect the performance heterogeneity is straightforward, and the output is the density function, which can provide comprehensive statistics of the individual parameters. Another desired property of the Bayesian approach is that it avoids the maximization of any function, which, however, is required and numerically difficult in the aforementioned classical approaches (Train 2001). Asymptotically, the estimator by the Bayesian approach is equivalent to that by the MLE approach.

The following section provides a review of the data set used in this research as well as the background of the AASHO road test. "Model Specification" describes the proposed incremental pavement deterioration model. "Model Estimation through Bayesian Approach" presents the Bayesian approach for estimating the specified model. The Gibbs sampling algorithm is utilized to estimate the distribution of each parameter. In addition, the performance forecast procedure is examined based on Gibbs output and an illustrative example is given. The final section presents study conclusions and major findings.

AASHO Road Test

The AASHO road test was carried out from November 1958 to December 1960 near Ottawa, Ill. (HRB 1962). Both rigid and flexible pavement structures were tested; however, this paper will focus on flexible pavements. Six two-lane loop tracks were constructed for the test, with loops two through six subject to traffic loading. Loop one was used to test environmental effects. Approximately 1,114,000 axle repetitions were applied on the test sections using trucks with three types of axle configurations.

A total of 142 flexible pavement structures were distributed across the various loops. Taking into consideration the variation of traffic levels assigned to the adjacent lanes, the number of different test sections totaled 284, with 32 designed as duplicates. In this paper, the performance data from 252 sections were used to estimate the model while the remaining duplicates were used for model validation.

Model Specification

AASHO Model

Based on the experimental data from the AASHO road test, a state-of-the-art pavement performance model was established (HRB 1962), as is shown in the following equation

$$p_t = p_0 - (p_0 - p_f)\left(\frac{W}{\rho}\right)^{\alpha} \tag{1}$$

where p_t=serviceability at time t; p_0=serviceability at time $t=0$, i.e., initial serviceability; p_f=terminal serviceability; W=accumulated axle load repetitions until time t, ρ=accumulated axle load repetitions until failure; and α=regression parameter determining the curvature of performance model.

When estimating the AASHO model, ρ and α are further expressed as the function of the variables associated with traffic and pavement structures. The deficiency of the model was identified with regard to determining ρ and α in both specification and parameter estimation aspects (Rauhut et al. 1983; Prozzi and Madanat 2000). Consequently, an improved model is adopted by considering the factors affecting pavement performance, in particular those factors involving pavement structures, environment, and traffic.

Since the performance deterioration trend is described by the AASHO model, it can be utilized as the basis for an improved model; thus the proposed model is formed from the AASHO equation

$$p_t = f(N_t) + \mu = a - bN_t^c + \mu \tag{2}$$

where p_t=serviceability at time t; N_t=some measure of traffic until time t; μ=error term; a=parameter representing initial serviceability; b=parameter representing the deterioration rate; and c=parameter representing the curvature.

Improved Model Specification

The deterioration of pavement performance is dependent on the combined impact of traffic load and environment on the pavement system. Therefore, as a sound model, Eq. (2) should incorporate these relevant variables. Since traffic load is directly included in the term N_t, the remainder of the variables (environment and pavement-related input) are incorporated in the term b.

As proposed in the original AASHO model, the thickness index $1+\gamma_1 H_1+\gamma_2 H_2+\gamma_3 H_3$ denotes the contribution of each pavement layer to the resistance of the deterioration, where γ_1, γ_2, and γ_3 are the parameters of each pavement layer that denote the relative ability of the individual layer to support the traffic. The three layers are surface, base, and subbase with thicknesses of H_1, H_2, and H_3, respectively. The term "1" did not have a physical meaning, but it is used to avoid the possible mathematic indetermination when the thickness index is equal to zero since it appears in the denominator of the function for α (HRB 1962). In the current model, a different form is proposed to make the specification physically meaningful. This is based on the assumption that the pavement performance deterioration rate will decrease with the increase in the pavement structural strength and vice versa. Consequently, the contribution to the resistance of deterioration from the pavement structural strength is depicted as

$$b_S = \exp\{\lambda\}\exp\{\gamma_1 H_1 + \gamma_2 H_2 + \gamma_3 H_3\} \tag{3}$$

The implication is that γ_1, γ_2, and γ_3 will be expected with negative signs. Except for the three thicknesses reflecting the pavement surface layer, base, and subbase, it is shown that Eq. (3) includes one multiplicative term $\exp\{\lambda\}$. The reason for adding this term is to consider the contribution of the subgrade. Physically, $\exp\{\lambda\}$ denotes the pavement deterioration rate due to the first pass of traffic applied directly on top of the subgrade.

J. Infrastruct. Syst. 2006.12:77-86.

Environmental impact on pavement performance is another critical aspect since pavement is sensitive to temperature and moisture, which is not directly reflected in the AASHO model. It was found that the most significant environmental factor influencing pavement performance is the frost penetration gradient (Prozzi 2001). The frost penetration gradient, denoted as G_t, is defined as the ratio between the change of frost penetration depth during period t and the length of time. Hence, the environmental factor, denoted as b_E, is

$$b_E = \exp\{\varphi G_t\} \tag{4}$$

where φ=parameter to be estimated.

Intuitively, during the thawing period, with the drop in frost penetration depth (i.e., negative gradient), the serviceability loss will accelerate due to the sudden loss of base and subbase strength. Thus, the sign of φ is expected to be negative so that during this period φG_t will be larger than zero leading to b_E being larger than one, which implies more serviceability loss. On the other hand, during the freezing period, the sign of G_t is expected to be positive and thus b_E will be less than one, implying less serviceability loss.

For the complete form, an incremental model is adopted. The benefits of using an incremental model to forecast the next-period serviceability loss are as follows:

1. From the infrastructure management context, the one-period-forward (next-period) performance forecast is usually adopted in decision making, allowing for the adjustment of the schedule plan based on actual performance data; and
2. Only next-period traffic is included while traffic from the previous traffic levels have already been obtained.

Based on previous research (Archilla 2000; Prozzi 2001), the first order Taylor expansion on Eq. (2) based on the one-period-forward performance condition is

$$\Delta p_t = p_{t-1} - p_t = dN_{t-1}^{e}\Delta N + \varepsilon \tag{5}$$

where d,e=parameters to be estimated; N_{t-1}=some measure of accumulative traffic until the one period backward of time t; ΔN=projected incremental traffic during the time period between $t-1$ and t; and ε=error term.

The determination of d follows. From the first order Taylor expansion, it is shown that d [Eq. (5)] is equal to b multiplied by c [Eq. (2)]. b encompasses both the structural factor b_S and environmental factor b_E. In addition, considering the sign of c should be nonnegative, it is integrated with $\exp\{\lambda\}$ as $\exp\{\lambda'\}$. Hence, the denotation of d becomes

$$d = \exp\{\lambda'\}\exp\{\gamma_1 H_1 + \gamma_2 H_2 + \gamma_3 H_3\}\exp\{\varphi G_t\} \tag{6}$$

Consequently, the incremental model representing serviceability loss at certain pavement sections can be expressed in the full specification as

$$\Delta p_{it} = \exp\{\beta_{0i} + \beta_{1i}H_{1i} + \beta_{2i}H_{2i} + \beta_{3i}H_{3i} + \beta_{4i}G_t\}N_{i,t-1}^{\beta_{5i}}\Delta N_{it} + \varepsilon_{it} \tag{7}$$

where

$$N_{i,t-1} = \sum_{l=1}^{t-1} \Delta N_{il} \tag{8}$$

and

$$\Delta N_{il} = n_{il}\left[\left(\frac{\mathrm{FA}_i}{\beta_{6i}18}\right)^{\beta_{8i}} + A_i\left(\frac{\mathrm{SA}_i}{18}\right)^{\beta_{8i}} + B_i\left(\frac{\mathrm{TA}_i}{\beta_{7i}18}\right)^{\beta_{8i}}\right] \tag{9}$$

where Δp_{it}=serviceability loss in pavement section i during time period t; β_{0i}–β_{8i}=parameters to be estimated; H_{1i}=surface layer thickness at pavement section i; H_{2i}=base layer thickness at pavement section i; H_{3i}=subbase layer thickness at pavement section i; G_t=frost penetration gradient at time period t; $N_{i,t-1}$=cumulative traffic until the one-period backward of time t; ΔN_{il}=incremental traffic during the time period between $l-1$ and l; ε_{it}=error term; n_{il}=traffic volume during the time period between $l-1$ and l; FA_i=front axle load magnitude; SA_i=single axle load magnitude; TA_i=tandem axle load magnitude; A_i=number of single axles on one vehicle; and B_i=number of tandem axles on one vehicle.

Three types of axle configurations were applied during the AASHO road test: front steering axles (single axle with single wheels), single axle with dual wheels, and tandem axles with dual wheels. It is assumed that the impact on pavement performance with each pass of axle load can be converted into an equivalent value based on its configuration and magnitude. The standard load for single axle with dual wheels is 18 kips (80 kN), while two coefficients β_{6i} and β_{7i} are assigned to section i for obtaining the standard loads of the front axle and tandem axles, respectively. Considering both β_{6i} and β_{7i} are larger than zero in the underlying context, it is assumed that $\beta_{6i}=\exp(\beta'_{6i})$ and $\beta_{7i}=\exp(\beta'_{7i})$. The motivation to adopt the exponential form is to avoid random draws from truncated distribution, which is found to lead to slow simulation speed.

In addition, it is important to describe the parameter structures in detail. Denote the parameters for a given section i as

$$\boldsymbol{\beta}^i = [\beta_{0i},\beta_{1i},\beta_{2i},\beta_{3i},\beta_{4i},\beta_{5i},\beta'_{6i},\beta'_{7i},\beta_{8i}]^T$$

To capture the unobserved heterogeneity of pavement performance, a hierarchical parameter structure is applied in this study. It is assumed that $\boldsymbol{\beta}^i$ are drawn from a common normal distribution

$$\boldsymbol{\beta}^i \sim N(\boldsymbol{\beta},\Sigma)$$

where $\boldsymbol{\beta}=[\beta_0,\beta_1,\beta_2,\beta_3,\beta_4,\beta_5,\beta'_6,\beta'_7,\beta_8]^T$, denoting the mean of the normal distribution. Σ=variance-covariance matrix, assumed to be diagonal with its diagonal elements representing the variances of the individual parameters in $\boldsymbol{\beta}^i$.

Furthermore, for the matrix of Σ, its diagonal elements are categorized into two types. One group is involved with the parameters deemed differing across pavement sections. These parameters include $\beta_{1i},\beta_{2i},\beta_{3i},\beta'_{6i}$, and β'_{7i}. β_{1i},β_{2i}, and β_{3i} are considered varying because construction quality may vary across sections, which leads to the variation of layer coefficients. β'_{6i} and β'_{7i} are regarded as varying due to the fact that the load equivalency factor for a given axle load varies among different pavement structures (AASHTO 1993). Thus, their corresponding variance elements in Σ are assumed larger than zero. However, whether these values are significantly different from zero will be tested statistically in the subsequent text. The rest of the parameters are deemed to share the common values across sections, which means that their variance terms are zeros.

Customarily, it can be assumed that the prior distributions of $\boldsymbol{\beta}$ are independently and normally distributed (Gelfand et al. 1990)

J. Infrastruct. Syst. 2006.12:77-86.

$$\boldsymbol{\beta} \sim N(\boldsymbol{\beta}_u, \Lambda)$$

where $\boldsymbol{\beta}_u$=mean of $\boldsymbol{\beta}$, denoted as $[\beta_{u0}, \beta_{u1}, \beta_{u2}, \beta_{u3}, \beta_{u4}, \beta_{u5}, \beta'_{u6}, \beta'_{u7}, \beta_{u8}]^T$, and Λ=variance-covariance matrix of $\boldsymbol{\beta}$.

Due to the assumed independence among the parameters, Λ is a diagonal matrix with its diagonal elements equal to the variances of the individual parameters. The process for estimating the prior values of each parameter is presented in the next section.

Model Estimation through Bayesian Approach

Eq. (7) is nonlinear in the parameters. It will be shown that the Bayesian approach with Markov chain Monte Carlo (MCMC) simulation is remarkably powerful for estimating the statistics of model parameters. Bayesian inference combines the information from observed data with prior knowledge about the parameters (prior) to arrive at the updated distribution of the parameters (posterior) (Zellner 1971), which is described as

$$p(\boldsymbol{\psi}|\boldsymbol{X}, \mathbf{Y}) = \frac{p(\boldsymbol{X}, \mathbf{Y}|\boldsymbol{\psi})p(\boldsymbol{\psi})}{\int_{\boldsymbol{\psi}} p(\boldsymbol{X}, \mathbf{Y}|\boldsymbol{\psi})p(\boldsymbol{\psi})d\boldsymbol{\psi}} \propto p(\boldsymbol{X}, \mathbf{Y}|\boldsymbol{\psi})p(\boldsymbol{\psi}) \quad (10)$$

where $p(\boldsymbol{\psi}|\boldsymbol{X}, \boldsymbol{Y})$ denotes the posterior distribution of the set of parameters $\boldsymbol{\psi}$ given the observed data including $\boldsymbol{X}$ (explanatory variables) and $\boldsymbol{Y}$ (dependent variable); $\boldsymbol{\psi}$ consists of the parameters to be estimated; $p(\boldsymbol{X}, \mathbf{Y}|\boldsymbol{\psi})$ denotes the likelihood of the observed data given the parameters ψ; and $p(\boldsymbol{\psi})$ denotes the prior distribution of the parameter set $\boldsymbol{\psi}$.

Due to the integral in the denominator (i.e., total probability across the parameter space being a constant), the posterior can be written as proportional to the numerator, the multiplication of likelihood and prior distribution, as shown in the right-hand side of Eq. (10).

The next step consists of determining the prior and the likelihood of filling in the corresponding components in Eq. (10).

Prior Specification

When determining the statistics for the prior, previous research results are used as the knowledge learned on the issue of interest. However, if there is no existing information available, an estimate based on the authors' judgment is proposed.

First, the values for prior parameters $\boldsymbol{\beta}_u$ are established. Since $\exp\{\beta_0\}$ includes the information on serviceability loss by one standard axle load applied directly on top of the subgrade, it is deemed to be smaller than 1, so the sign of β_0 can be negative. A prior mean of $\beta_{u0}=-1$ is assumed, corresponding to the value of "1" in the denominator of α in the AASHO equation. For β_1, β_2, and β_3, according to the AASHO equation, the three parameters are assumed to be −0.44, −0.14, and −0.11, respectively. These values are used as the prior means for the three parameters. For β_4, the sign is expected to be negative since the decrease in frost penetration depth (negative gradient) contributes more to pavement serviceability loss, leading to the term $\exp\{\varphi G_t\}$ larger than 1. Using the information in Prozzi (2001), an estimated value is proposed as −0.1 for prior mean of β_4. For β_5, a similar estimate was made based on Prozzi's findings so that the prior mean is assumed to be −0.5. Since β_6 is used to convert the steering axle load into the equivalent load of a single axle with dual wheels, 18 kip (80 kN), its prior mean is assumed to be 0.5; thus $\beta'_{u6}=\ln(0.5)=-0.69$. Following similar reasoning, the prior mean for β_7 is assumed to be 2.0 since it is the case for the tandem axle; thus $\beta'_{u7}=0.69$. For β_8, considering the "4th power law" was used in estimating the load pavement impact (Huang 2003), the prior mean is assumed to be 4.0. In summary, the prior means for the parameters are chosen as

$$\boldsymbol{\beta}_u = [-1, -0.44, -0.14, -0.11, -0.1, -0.5, -0.69, 0.69, 4]^T$$

Second, concerning the uncertainty, the concept of precision is applied. In Bayesian approach, the precision—denoted as τ—is customarily defined as the reciprocal of variance, namely, $\tau=1/\sigma^2$. The implication of the precision is straightforward: the larger the precision, the less uncertainty a variable has. Hence, three representative precisions are selected in this study to examine the sensitivity of the posterior distribution to different assumptions on prior precisions. The first alternative adopts relatively smaller precision, 0.1, corresponding to the higher uncertainty of the parameters. The second alternative uses relatively larger precision, 1, corresponding to the lower uncertainty. For the third precision alternative, it is assumed that before the study the parameter of coefficient of variance (COV) for each parameter is 1 (i.e., $\sigma/\mu=1$). In this case, it can be shown that all parameters except β_8 have a precision larger than 1. In addition, the error term ε_{it} is also assumed to be independently and normally distributed

$$\varepsilon_{it} \sim N(0, 1/\eta)$$

where η=precision of the error term distribution. Considering that the majority of the model's uncertainty is incorporated in the error term and little is known about it before the study, the error term's precision is further reflected by a gamma distribution

$$\eta \sim \text{gamma}(\phi, \varsigma)$$

where ϕ, ς=parameters of the gamma distribution of η.

Finally, for the matrix of Σ, the precisions of those random parameters are each assumed to have gamma distributions with relatively diffuse prior, both shape and scale parameters equal to 10^{-3}.

As a result, by assuming that the priors are independent, the prior joint distribution including all the parameters $\boldsymbol{\theta}$ is obtained as

$$p(\boldsymbol{\theta}) \propto \prod_i p_N\left(\boldsymbol{\beta}^i|\boldsymbol{\beta}, \sum\right)p_N(\boldsymbol{\beta}|\boldsymbol{\beta}_u, \Lambda)\prod_j p_G(\omega_{j,j}|\vartheta_j, \varphi_j)p_G(\eta|\phi, \varsigma) \quad (11)$$

where i=pavement section number; $p_N(\bullet|\bullet)$=normal density function; $p_G(\bullet|\bullet)$=gamma density function; $\omega_{j,j}=j,j$ element of matrix Σ^{-1}, $\omega_{j,j}\sim\text{gamma}(\vartheta_j, \varphi_j)$, j=1, 2, 3, 6, and 7; and η=precision of the error term distribution.

Likelihood Function

As stated previously, the error term ε_{it} is assumed to be independently and normally distributed. Hence, the likelihood function is

$$p(\mathbf{X}, \mathbf{Y}|\boldsymbol{\theta}) = \prod_i \prod_t p_N[\Delta p_{it}|g_{it}(\mathbf{X}_{it}, \boldsymbol{\beta}^i), 1/\eta] \quad (12)$$

where Δp_{it}=observed serviceability loss during time period t on pavement section i; η=as in Eq. (11); and $g_{it}(\boldsymbol{X}_{it}, \boldsymbol{\beta}^i) = E(\Delta p_{it}|\boldsymbol{X}_{it}) = \exp\{\beta_{0i}+\beta_{1i}H_{1i}+\beta_{2i}H_{2i}+\beta_{3i}H_{3i}+\beta_{4i}G_t\}N_{i,t-1}^{\beta_{5i}}\Delta N_{it}$.

J. Infrastruct. Syst. 2006.12:77-86.

Posterior

With the prior and likelihood functions at hand, using Eq. (10), the posterior is obtained as

$$p(\boldsymbol{\theta}|\boldsymbol{X},\mathbf{Y}) \propto \prod_i p_N\left(\boldsymbol{\beta}^i|\boldsymbol{\beta},\sum\right)p_N(\boldsymbol{\beta}|\boldsymbol{\beta}_u,\Lambda)\prod_j p_G(\omega_{j,j}|\vartheta_j,\varphi_j)p_G(\eta|\phi,\varsigma) \tag{13}$$

$$\prod_i\prod_t p_N[\Delta p_{it}|g_{it}(\boldsymbol{X}_{it},\boldsymbol{\beta}^i),1/\eta]$$

As shown in the posterior distribution, there are nine regression parameters ($\beta_0-\beta_8$) to be estimated. Although the joint distribution of these variables conditional on the given data is obtained, the goal is to arrive at the marginal distribution of each variable, which requires the multi-dimensional integration of the right-hand side of Eq. (13). It is apparent that the multi-dimensional integration task for solving Eq. (13) is staggering. However, an alternative to avoid the complexity in obtaining the marginal distribution is available through the Gibbs sampler with MCMC simulation, which is presented in the following section.

Implementation of MCMC

Gibbs sampling is a Markovian updating scheme (Gelfand and Smith 1990). The process of the algorithm used in the Gibbs sampling is described as follows: for a set of random variables $U_1,U_2,\ldots,U_m$, the joint distribution is denoted as $f(U_1,U_2,\ldots,U_m)$. With given arbitrary starting values of U_s's, say $U_1^{(0)},U_2^{(0w)},\ldots,U_m^{(0)}$, the first iteration of random draws of U_s's is obtained as

$$U_1^{(1)} \text{ from } f(U_1|U_2^{(0)},U_3^{(0)},\ldots,U_m^{(0)})$$

$$U_2^{(1)} \text{ from } f(U_2|U_1^{(1)},U_3^{(0)},\ldots,U_m^{(0)})$$

$$\vdots$$

$$U_m^{(1)} \text{ from } f(U_m|U_1^{(1)},U_2^{(0)},\ldots,U_{m-1}^{(1)})$$

In a similar manner, the second set of random draws of U_s's is obtained through the update process. After r iterations as shown above, the series of U_s's is obtained as $(U_1^{(r)},U_2^{(r)},\ldots,U_k^{(r)})$. It is shown that under mild conditions for each variable $U_s^{(r)}\xrightarrow{d}U_s \sim f(U_s)$ as $r\rightarrow\infty$ (Geman and Geman 1984), which means that after enough iterations, r, $U_s^{(r)}$ can be regarded as a random draw from the distribution of $f(U_s)$.

Based on the above algorithm, the application of MCMC for obtaining the marginal distribution of each parameter conditional on observed data is straightforward. It is shown that the joint distribution by the given data set is available through the Bayesian approach, which is the posterior [Eq. (13)]. With the joint conditional distribution of the parameter set, the MCMC simulation is carried out, leading to the simulated distribution of each parameter of interest.

During the process of determining the marginal distributions, the notion of convergence is worth noting. As is shown in the Gibbs sampling, convergence to the marginal distribution is obtained when the number of iterations is large enough. This convergence is required for the sampled value to represent a random draw from the marginal distribution. To address the issue of convergence, two Markov chains are run simultaneously. After the traces of the two chains are found to be overlapping, convergence is confidently achieved. The random draws before the convergence are referred to as preconvergence or "burn-in" data sets. The simulated variables after the "burn-in" are used to determine the statistical inference of the underlying parameters.

Table 1. Estimation Results at Two Prior Precision Levels

Precision	$\tau=0.1$		$\tau=1$		COV=1	
Parameters	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
β_0	−6.882	0.535	−6.038	0.371	−6.096	0.372
β_1	−0.705	0.083	−0.762	0.077	−0.732	0.076
β_2	−0.208	0.028	−0.204	0.029	−0.203	0.026
β_3	−0.178	0.022	−0.179	0.023	−0.169	0.022
β_4	−0.112	0.005	−0.116	0.005	−0.116	0.005
β_5	−0.035	0.048	−0.089	0.044	−0.098	0.044
β_6'	−0.465	0.242	−0.478	0.183	−0.245	0.307
β_7'	0.866	0.196	0.829	0.108	0.785	0.082
β_8	2.960	0.313	3.295	0.306	3.089	0.315
σ_1	0.077	0.030	0.078	0.027	0.076	0.028
σ_2	0.084	0.024	0.073	0.024	0.079	0.023
σ_3	0.042	0.012	0.040	0.013	0.040	0.012
σ_6	0.251	0.137	0.293	0.128	0.284	0.071
σ_7	0.171	0.084	0.153	0.080	0.155	0.071

Parameter Estimation Results

The AASHO road test data set aforementioned is used for model estimation. After 10,000 "burn-in" iterations of each of the two chains, the parameters statistics are obtained based on 100,000 additional iterations (50,000 from each chain). At each of the three precision levels (in prior) the convergence is achieved since the two simulated chains of the individual variables overlap each other after burn-in iterations. It can be shown that through the hierarchical Bayesian model both the population and individual level parameters can be obtained. The basic sample statistics (mean and standard deviation) of the population level parameters are presented in Table 1. The statistics of each parameter are almost the same among the three precision levels. The results imply that the posterior is not significantly sensitive to the selected prior's precisions. Since the parameters are of interest, the following focuses on the parameters' inference in more detail.

As representatives, the regression parameter sets associated with the prior precision equal to 1 are chosen for further analysis (it is similar for the other two). The densities of these parameters are illustrated in Fig. 1. In addition to the means and standard deviations shown in Table 1, the following statistics are investigated in Table 2:

1. Coefficient of variance (COV);
2. Skewness; and
3. Kurtosis.

The COV is used to reflect the degree of variability. The larger the COV, the less uncertainty a variable has. Skewness is an indicator of the symmetry of a distribution around its mean. The negative skewness value indicates that the data are more spread out to the left of the mean, and vice versa. Kurtosis determines the extent to which the distribution is outlier prone. Normal distribution

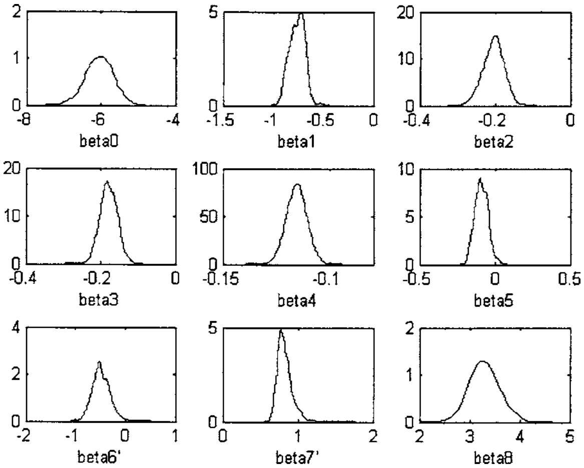

Fig. 1. Posterior distributions of parameters

has a kurtosis of 3. If the kurtosis is larger than 3, the underlying distribution is more outlier-prone than the normal distribution.

With regard to the parameters of three pavement thicknesses, it is shown that the largest mean relative performance deterioration resistance is denoted by $-\beta_1$ (0.762), followed by $-\beta_2$ (0.204), and then $-\beta_3$ (0.179). This result follows the practice in pavement engineering that the surface layer contributes more to the resistance of serviceability loss than the base and subbase. Moreover, it is implied that the relative contribution of the surface layer's unit thickness is 3.7 times that of the base or subbase. The base and subbase are close in terms of their ability to resist serviceability loss. In addition, Table 2 shows that skewness and kurtosis values of the three parameters being near 0 and 3, respectively, suggests that each of the distributions can be regarded as normal, which is also reflected in their densities as shown in Fig. 1.

Other parameters deserving special attention are those associated with traffic information, β_6, β_7, and β_8. β_6 and β_7 are the coefficients for estimating equivalent axle loads for the steering and tandem axles, respectively. It can be shown that the rth moment of β_6 is equal to $\exp(r\beta'_{u6}+r^2\sigma^2_{\beta'_6}/2)$, where, β'_{u6} and $\sigma_{\beta'_6}$ are mean and standard deviation estimates for β'_6 in Table 1. As a result, the mean of β_6 is obtained as 0.667, which means that the equivalent load for a steering axle is 12.0 kip (53.3 kN) on average. Similarly, the mean of β_7 equal to 2.343 implies that a tandem axle load is 42.2 kip (187.2 kN). Both distributions of β'_6 and β'_7 (especially the latter) are asymmetric, as shown by their densities in Fig. 1. Their positive skewness values indicate that the equivalent loads are more concentrated on lighter axle loads for the steering and tandem axle. Table 2 shows that the kurtosis values of β'_6 and β'_7 are larger than 3, particularly that for β'_7, indicating that the distributions of the two parameters are more outlier-prone than the normal distribution. Therefore, the assumption of normality of the two parameters is not recommended. Regarding β_8, its mean value of 3.3 is different from its counterpart obtained in the analysis from the original AASHO road test, which is around 4. The result implies that by applying the "4th power law" the impact of heavy loads on pavement might be overestimated, while the impact of a light load is underestimated. Additionally, both skewness (close to 0) and kurtosis (close to 3) confirm the normal assumption of β_8 as in the traditional analysis.

Table 2. Statistics on Variability of Parameters and Normality Check

Parameters	COV	Skewness	Kurtosis
β_0	−0.061	−0.136	2.941
β_1	−0.101	−0.024	2.941
β_2	−0.142	0.167	3.171
β_3	−0.128	−0.016	3.014
β_4	−0.043	−0.027	3.096
β_5	−0.494	0.212	2.907
β'_6	−0.382	0.496	3.805
β'_7	0.130	1.650	8.536
β_8	0.093	0.164	3.115

The statistics in Table 2 and the densities in Fig. 1 suggest that the remaining parameters are almost symmetric and normally distributed.

In addition, for testing heterogeneity, the estimates for the five assumed nonzero elements in $\Sigma^{1/2}$, denoted as σ_1, σ_2, σ_3, σ_6, and σ_7 (with $\sigma_j = 1/\sqrt{\omega_{j,j}}$) are also presented in Table 1. The highest posterior density (HPD) interval is used to test whether these individual parameters are significantly different from zero. The x percent HPD interval is the shortest interval in parameter space that contains x percent of the posterior probability. A 95% HPD interval is applied in this study to conduct the test. The results show that none of the 95% HPD interval of the above parameters includes zero, which implies the unobserved heterogeneity exits.

In summary, the data generation process (through simulation) shows interesting and reasonable results. First, not all the parameters follow the normal distribution, particularly those associated with estimating the equivalent axle loads for steering and tandem axles. Second, there is significant variability in the parameters and the statistical test reveals the existence of unobserved heterogeneity, which leads to the necessity of accounting for these issues.

Pavement Performance Forecast

The ultimate goal for model estimation is to forecast pavement performance. As shown in the following paragraphs, the performance forecast is a straightforward process. A simulation approach based on Gibbs output is adopted to achieve the predicted pavement performance.

Let Δp^f_{it} denote the serviceability loss of a pavement section number i during the time period t to be forecast, which is given by

$$\Delta p^f_{it} = \exp\{\beta_{0i} + \beta_{1i}H_{1i} + \beta_{2i}H_{2i} + \beta_{3i}H_{3i} + \beta_{4i}G_t\}N^{\beta_{5i}}_{i,t-1}\Delta N_{it} + \varepsilon_{it} \tag{14}$$

where the explanatory variables are known for the pavement section of interest and the parameters are from the postconvergence of the Gibbs output. The error term is drawn randomly from simulated errors of the Gibbs output.

In order to obtain the forecast serviceability loss for each time period, Monte Carlo simulation is again carried out as follows:

1. Obtain one set of random draws of $\boldsymbol{\beta}^i$, denoted as $\boldsymbol{\beta}^{i(m)}$, from the postconvergence Gibbs output, plug them into Eq. (14), and calculate $\Delta p^{f(m)}_{i,t}$, which has the normal density

$$\Delta p^{f(m)}_{it} \sim N[g_{it}(\boldsymbol{X}_i, \boldsymbol{\beta}^{i(m)}), 1/\tau^{(m)}] \tag{15}$$

   where $\boldsymbol{X}_i$ = vector of explanatory variables from the pavement section i to be forecast, and $\tau^{(m)}$ = precision.
2. Obtain one random draw from the density of $\Delta p^{f(m)}_{it}$.

J. Infrastruct. Syst. 2006.12:77-86.

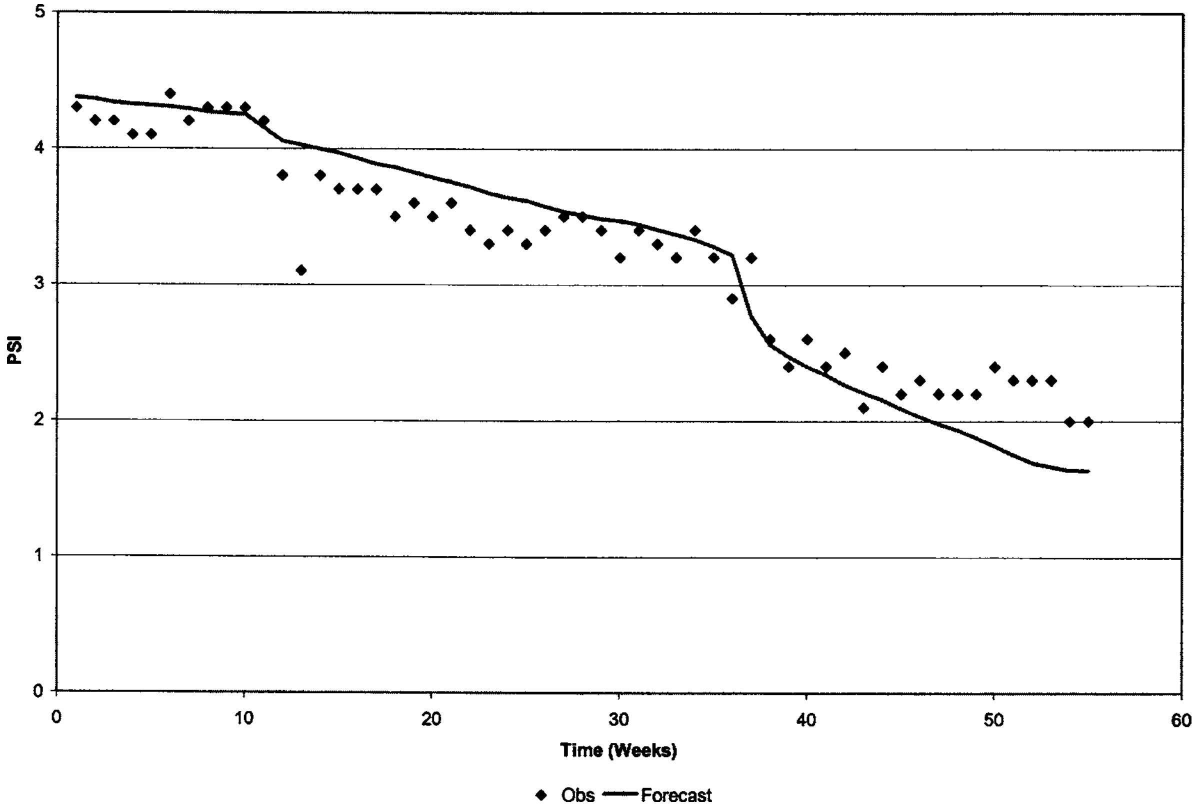

Fig. 2. Observed versus forecast mean performance curves on section not used in model estimation sample

3. Repeat steps 1 and 2 to obtain a total of M values.
4. Calculate the statistics of the forecast serviceability loss based on the sample of M random values obtained in steps 1, 2, and 3. For example, the predicted mean of serviceability loss can be estimated through the sample mean

$$E(\Delta p_{it}^{f}|X,\text{Data}) = \frac{1}{M}\sum_{m=1}^{M} \Delta p_{it}^{f(m)} \tag{16}$$

Case Study of Performance Forecast

To illustrate the effectiveness of the model, pavement section 271 from the AASHO road test is selected, a section that was not used during the development and estimation of the model. The parameters used for the performance forecast are from its duplicate section included in the parameter estimation sample, section 263.

The serviceability loss at each time period is calculated so that the performance deterioration curve can be obtained. Fig. 2 represents the forecast deterioration curve by simulated mean values at time points along the road test duration. It is demonstrated that the forecast line fits the observations accurately.

From a pavement management perspective, one-period-forward performance deterioration forecasting is of prime interest. In general, pavement condition data are collected on a regular basis at frequencies determined by the highway agency based on resource availability. Thus, pavement performance forecasts between two data collection intervals are required for decision-making. To illustrate the effect of survey frequency and forecast confidence, six possible scenarios are presented as a case study:

1. Three pavement condition survey frequencies: every 2 years, once a year, and every 6 months; and
2. Two prediction confidences: 50th and 60th percentile, e.g., the 50th percentile curve indicates that 50% of the forecast sample will have a higher serviceability value than that curve.

The performance forecast in the first interval is based on the initial condition and that for the second interval is based on the observed PSI at the beginning of that interval. In addition, the deterioration is categorized by different percentile levels to represent performance forecast uncertainty. The percentile-based incremental performance forecast results for the three groups of scenarios with increasing frequency are depicted in Figs. 3–5, respectively. The observed performance curve lies close to the 50th percentile lines. In addition, figures indicate that as the confidence level increases (higher percentile), forecast performance drops significantly, which leads to a profound implication for reliability-based pavement design. The reliability obtained from the median performance forecast line corresponds to the condition 50% of system reliability. The 10% increase in reliability level can be obtained but results in a significant drop in forecast PSI. Moreover, it is implied that with an increase in inspection frequency, the forecast variation decreases, leading to more confidence in the performance forecast. Therefore, it is implied that the balance between accurate performance forecasting and inspection frequency is of critical importance in order to realize optimal highway infrastructure system management. Figs. 3–5 indicate that, when all sources of uncertainty are properly accounted for, aiming at higher reliability levels (such as 90 or 95%) may be unrealistic. In-depth research into determining appropriate and realistic reliability levels as a function of inspection frequency (function of available resources at the highway agency) is imperative.

J. Infrastruct. Syst. 2006.12:77-86.

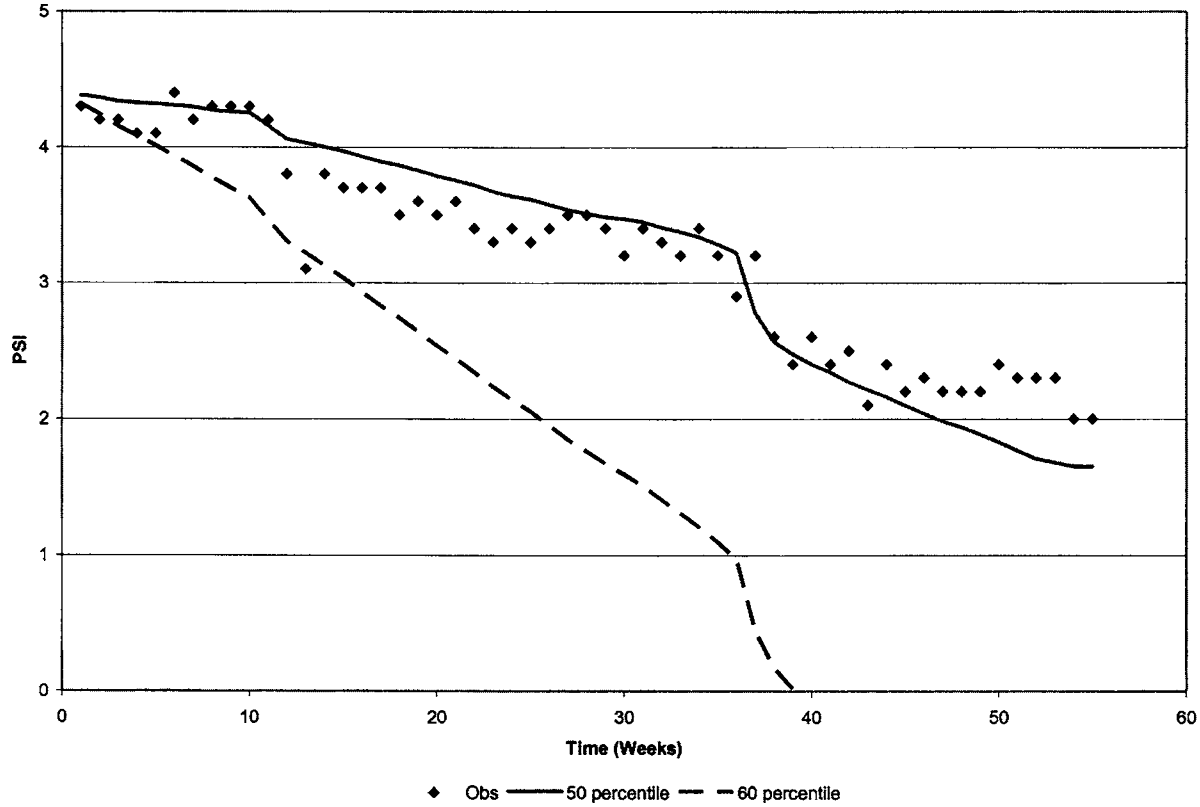

Fig. 3. Observed versus forecast performance at different percentile levels with inspection frequency of once every 2 years

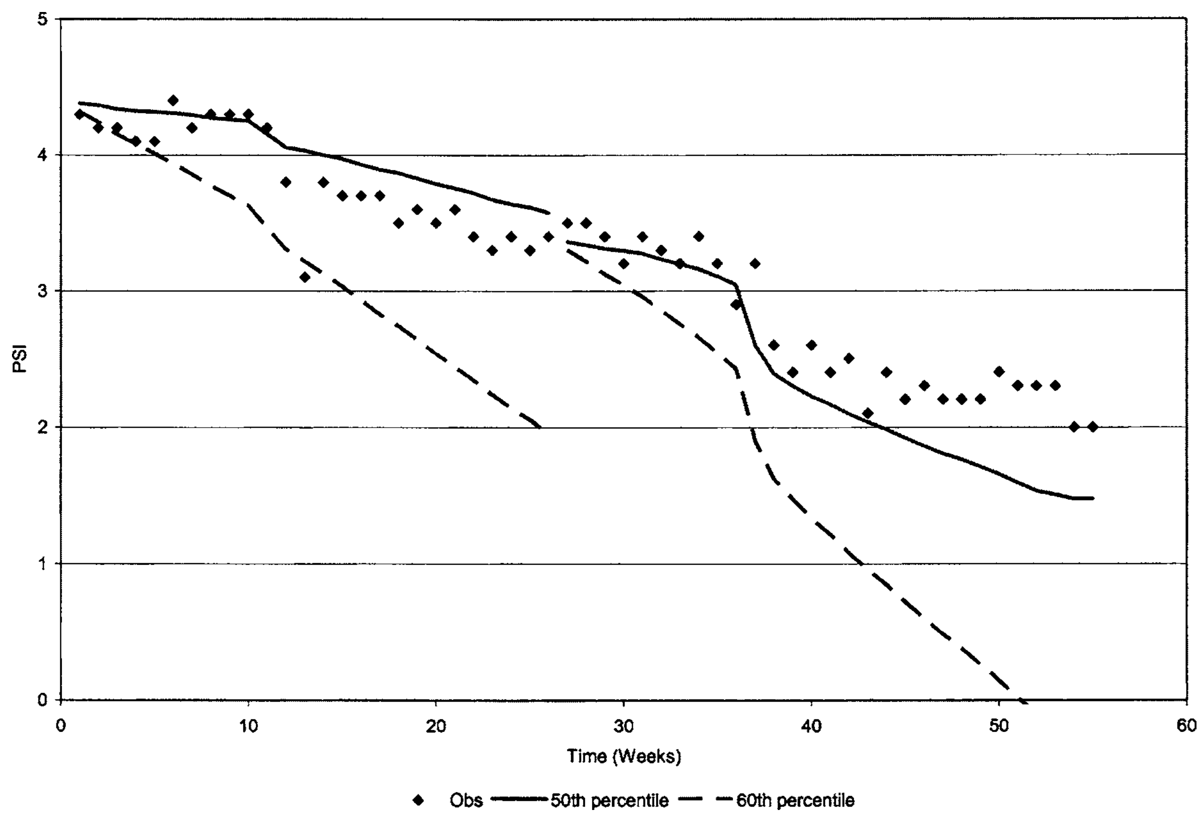

Fig. 4. Observed versus forecast performance at different percentile levels with inspection frequency of once per year

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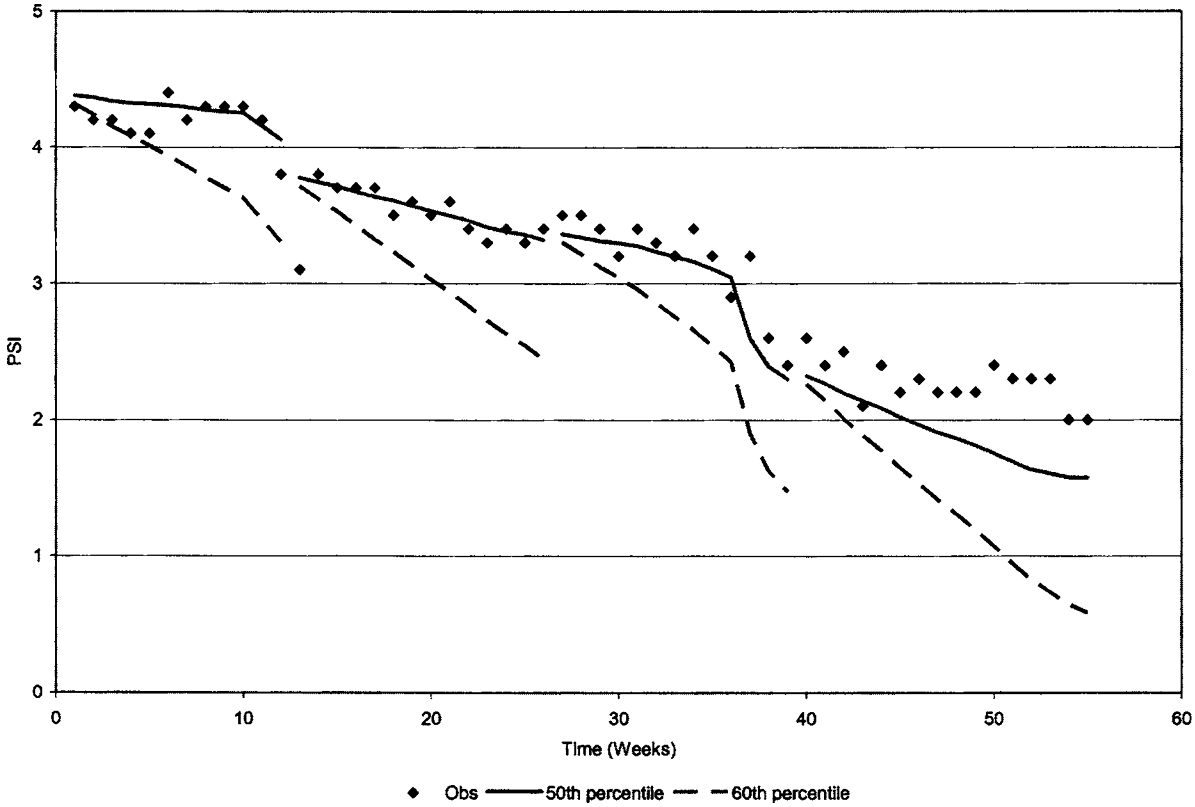

Fig. 5. Observed versus forecast performance at different percentile levels with inspection frequency of once per 6 months

Conclusions

An incremental pavement deterioration model was investigated in this paper based on the AASHO road test data. The model accounts for the fundamental factors associated with pavement performance: structural properties, environment, and traffic.

The Bayesian approach is used to estimate the model parameters. The purpose is to assess the effects of unobserved heterogeneity on the pavement performance model parameters by exploring the statistical characteristic of the parameters. The parameter information from previous studies and the authors' understanding of pavement deterioration process are used as the prior. The posterior is obtained by updating the prior with the observed data and reflects the characteristics of the actual deterioration process. The Gibbs sampling algorithm is used to estimate the distributions of the individual parameters. Through the application of this methodology and based on the critical analysis of the model and the prediction results, the following results should be considered:

1. The model was developed in an attempt to capture the physical process of deterioration and the estimated results match data and the validation set accurately. For instance, the means of β_1, β_2, and β_3 support the fact that the surface layer contributes most to pavement structural strength, followed by the base and subbase. The relative magnitudes are also consistent with previous research.
2. The normality assumptions seem reasonable for most parameters with the exception of β_6' and β_7', which are associated with calculating the equivalent axle load for steering and tandem axle, respectively.
3. The parameter β_8 is slightly lower than the counterpart in the "4th power law" obtained after the original analysis of the AASHO road test data, which suggests that the impact of heavier loads (>18 kips) on the pavement is currently overestimated, while the impact of the lighter axle loads is underestimated.
4. It is shown that there is significant variability in the individual parameters. This means that the model is subject to uncertainty through the variability of the parameters. The statistical test confirms that the unobserved heterogeneity exists. Consequently, it implies that addressing unobserved heterogeneity in pavement performance is a critical issue in modeling.

In addition, the Gibbs output is used for predicting pavement performance. As an advantage of the incremental model, pavement performance forecast at a given interval (corresponding to the inspection frequency) can be obtained. Furthermore, the varying percentiles of performance forecast are presented, which can aid the decision-making process based on the confidence in the forecast variables.

Following the philosophy of Bayesian updating, the research approach presented in this paper can be applied to enhance the current model as new data are collected. With the collected data from different sources (such as pavement management systems), the results obtained herein can serve as the prior. After integrating with the new data through the approach presented in this paper, the updated model can be used to facilitate pavement management at particular locations of interest. Model updating with a new data set deserves further investigation. In addition, further work will also be motivated towards the study of sequential data considering the fact that, in practice, the performance data are obtained sequentially over time.

J. Infrastruct. Syst. 2006.12:77-86.

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