Transportation Research Part B 46 (2012) 511–519

Contents lists available at SciVerse ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

# Joint optimization of pavement maintenance and resurfacing planning

Weihua Gu [a,*], Yanfeng Ouyang [b], Samer Madanat [c]

[a] Department of Civil and Environmental Engineering, University of California, 416 McLaughlin Hall, Berkeley, CA 94720, USA
[b] Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
[c] Department of Civil and Environmental Engineering, University of California, 109 McLaughlin Hall, Berkeley, CA 94720, USA

ARTICLE INFO

*Article history:*
Received 29 July 2011
Received in revised form 6 December 2011
Accepted 9 December 2011

*Keywords:*
Pavement maintenance
Pavement resurfacing
Lifecycle costs
Optimization

ABSTRACT

This paper presents an analytical approach for joint planning of pavement maintenance and resurfacing activities that minimizes pavement lifecycle costs, including user, maintenance and resurfacing costs, for an infinite time horizon. The optimization problem is formulated as a nonlinear mathematical program with continuous pavement state and continuous time, and optimality conditions are derived. Managerial insights and practical implications are drawn from two realistic application scenarios, where the maintenance cost is either independent of or linearly dependent on pavement condition, to address impacts of routine maintenance activities on pavement resurfacing planning decisions. Numerical examples demonstrate clear trade-offs between maintenance and resurfacing activities in terms of both pavement improvement effectiveness and costs. This paper shows that maintenance activities, if applied optimally, have the potential to significantly prolong pavement service life between consecutive rehabilitations and reduce overall pavement lifecycle costs.



## 1. Introduction

Current practice in pavement management usually involves multiple treatments; e.g., inexpensive and frequent maintenance activities such as crack or chip sealing, and costly and infrequent rehabilitation actions such as resurfacing. The effects of these treatment activities in terms of preserving pavement serviceability are interdependent. For example, it is well-known to researchers and practitioners that maintenance activities can effectively retard the deterioration process and prolong pavement service life (Chong, 1989; Ponniah, 1992; Ponniah and Kennepohl, 1996). Similarly, rehabilitation activities renew the pavement condition which temporarily reduces the need for maintenance treatments.

Due to the enormous expenditures on the management of transportation infrastructure systems and the large traffic volume in those systems, optimal planning of maintenance, rehabilitation, and reconstruction (MR&R) activities for infrastructure systems has been widely studied in literature. Representative works in this field usually fall in three categories with regard to their modeling features: (i) discrete time and finite pavement condition states (Golabi et al., 1982; Carnahan et al., 1987; Madanat, 1993; Madanat and Ben-Akiva, 1994); (ii) discrete time and continuous pavement state (Durango-Cohen, 2007; Ouyang and Madanat, 2004; Ouyang, 2007); and (iii) continuous time and pavement state (Tsunokawa and Schofer, 1994; Li and Madanat, 2002; Ouyang and Madanat, 2006). Each category can be further divided into subcategories based on the scope of the problem: single facility (e.g., Ouyang and Madanat, 2006) or multiple facilities (e.g., Sathaye and Madanat, 2011). The present work falls in the facility-level problem version of the third category of the literature.

Related problems in highway pavement stage construction and maintenance were first tackled by applying optimal control theory (Friesz and Fernandez, 1979; Fernandez and Friesz, 1981; Markow and Balta, 1985). Later with the help of a

* Corresponding author. Tel.: +1 510 931 6646; fax: +1 510 643 8919.
*E-mail address:* weihuagu@berkeley.edu (W. Gu).

0191-2615/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.trb.2011.12.002

"trend-curve" approximation, Tsunokawa and Schofer (1994) solved the problem of optimal pavement resurfacing. The exact solutions developed by Li and Madanat (2002), and by Ouyang and Madanat (2006) revealed the threshold structure of the optimal pavement resurfacing plan. They obtained the optimal solutions by utilizing a steady-state property for infinite planning horizon problems, and by using calculus of variations for finite horizon problems. These methods, in contrast to optimal control, can handle discontinuities that occur in the trajectories of pavement condition without having to use trend-curve approximations, and thus are suitable for the present problem.

As noted in the above studies, the threshold structure has yielded keen insights and important practical implications. It features an optimal trigger state: a pavement is resurfaced when its condition reaches this state. Further, analysis has shown that this trigger state (and associated planning variables) is invariant to the pavement's initial condition. As we shall see, this finding remains true when routine maintenance is incorporated into the planning process.

Most of the previous studies, however, have focused on only one treatment (usually rehabilitation). The only exception, to the best knowledge of the authors, is Rashid and Tsunokawa (2010), in which resealing and reconstruction activities were assumed as alternatives to resurfacing during the initial stage, instead of treatments that jointly affect the pavement condition with resurfacing over the planning horizon. Thus, the interaction between routine maintenance and rehabilitation has usually been overlooked.

In light of this, the present paper examines the effects of maintenance activities on the pavement resurfacing planning decisions and on the pavement lifecycle costs, including the user, maintenance, and resurfacing costs. To this end, a mathematical program is formulated that minimizes the pavement lifecycle costs over an infinite horizon. The present model, with continuous time, pavement state, and resurfacing intensity, extends those in the literature (Tsunokawa and Schofer, 1994; Ouyang and Madanat, 2004) by jointly optimizing resurfacing and maintenance planning. The latter part is modeled in a continuous fashion. This is because, as reported in the literature (Labi and Sinha, 2003; Ponniah and Kennepohl, 1996), a maintenance activity such as crack sealing has a long-term effect that slows down the deterioration of pavement, rather than an immediate effect that improves the pavement condition.

The remainder of the paper is organized as follows: the problem formulation and the associated cost and performance models are furnished in Section 2; Section 3 solves the optimization problem; a numerical example, followed by insights, is discussed in Section 4; Section 5 concludes the paper.

## 2. Problem formulation

### 2.1. General formulation

We denote the pavement state by $s(t)$, expressed in terms of quarter-car roughness index (QI), at time $t \in [0, \infty)$; a larger value of $s(t)$ indicates a worse pavement condition. $s(t)$ starts from an initial state, $s_0$, and increases continuously in $t$ except when the pavement is resurfaced. The slope of $s(t)$ at time $t$, i.e., $\dot{s}(t)$, is described by a deterioration model, $F(s(t), b)$. $b$ is termed the deterioration rate; a larger $b$ indicates more rapid deterioration. We assume $b = b_0$ if no routine maintenance is applied. $b$ decreases as the maintenance intensity increases, but only down to a minimum rate $\underline{b}$. For simplicity, in this paper we assume $b$ to be constant throughout the planning horizon. When the $i$-th ($i = 1, 2, \ldots$) resurfacing is applied, $s(t)$ is immediately reduced by $G(w_i, s(t_i^-))$, the resurfacing effectiveness, where $w_i$ is the intensity of the resurfacing, and $t_i$ the resurfacing time ($0 = t_0 \leqslant t_1 \leqslant t_2 \leqslant \cdots$). $t_i^-$ $(t_i^+)$ indicates the time immediately before (after) the resurfacing action. Each $w_i$ should not exceed the maximum effective resurfacing intensity, denoted by $R_i$. The total lifecycle cost, $J$, includes the following components: (i) the user cost, $C$, as a function of $s(t)$; (ii) the resurfacing cost, $M$, as a function of $w_i$; and (iii) the maintenance cost, $C_M$, as a function of the maintenance intensity, $z$. The maintenance intensity is often difficult to quantify because no measure is suitable for all maintenance treatments (e.g., crack sealing and micro-surfacing). However, note that the maintenance effectiveness in terms of the reduction in the deterioration rate, i.e., $\Delta b = b_0 - b$, is a function of $s(t)$ and $z$. $s(t)$ is involved in this function because in some cases (e.g., for crack sealing) the pavement condition will influence the effectiveness of maintenance activities. Further note that by inverting this function, we can write $z$ as a function of $s(t)$ and $\Delta b$. Thus we can eliminate $z$ from the maintenance cost-effectiveness model and write $C_M$ as a function of $s(t)$ and $\Delta b$. Finally, we define $r$ as the discount factor. Given the above, the problem can be formulated as:

$$\min\ J(\boldsymbol{t}, \boldsymbol{w}, b) = \sum_{i=0}^{\infty} \int_{t_i}^{t_{i+1}} [C(s(u)) + C_M(s(u), \Delta b)] e^{-ru} du + \sum_{i=1}^{\infty} M(w_i) e^{-rt_i} \tag{1a}$$

$$\text{subject to}\quad s(t_i^-) - s(t_i^+) = G(w_i, s(t_i^-)), \quad i = 1, 2, \ldots \tag{1b}$$

$$\dot{s}(t) = F(s(t), b) \tag{1c}$$

$$0 \leqslant w_i \leqslant R_i, \quad i = 1, 2, \ldots \tag{1d}$$

$$t_0 \equiv 0, t_i \geqslant t_{i-1}, \quad i = 1, 2, \ldots \tag{1e}$$

$$\Delta b = b_0 - b \tag{1f}$$

$$\underline{b} < b \leqslant b_0 \tag{1g}$$

$$s(0) = s_0 \tag{1h}$$

We seek to minimize the total pavement lifecycle cost, $J$, using the following decision variables: (i) the timing of resurfacing, $\boldsymbol{t} = (t_1, t_2, \ldots)$, that satisfies (1e); (ii) the intensities of resurfacing, $\boldsymbol{w} = (w_1, w_2, \ldots)$, that is constrained by (1d); and (iii) the reduction in the deterioration rate, $\Delta b$, that is defined by (1f) and further constrained by (1g). Constraint (1b) describes the effect of resurfacing actions on pavement condition; (1c) defines the pavement deterioration process; and the initial state of the pavement is given in (1h).

We can see from this formulation that, lowering $b$ slows down the deterioration process, and thus postpones the resurfacing action. This is associated with a decrease in the sum of user and resurfacing costs and an increase in maintenance cost. This trade-off explains the importance of accounting for the effect of maintenance on the pavement resurfacing planning, which has been ignored in previous studies.

The cost and performance models used in this formulation are described next.

### 2.2. Cost and performance models

We choose the cost and performance models from Ouyang and Madanat (2004) because they are realistic and simple enough to be incorporated into the mathematical model. These models are:

$$C(s(t)) = c_1 s(t) + c_2, \quad \forall t \tag{2}$$

$$M(w_i) = m_1 w_i + m_2, \quad \forall i \tag{3}$$

$$G\left(w_i, s\left(t_i^-\right)\right) = \frac{g_1 w_i}{g_2 s\left(t_i^-\right) + g_3} \cdot s\left(t_i^-\right), \quad \forall i, 0 \leqslant w_i \leqslant R_i = g_2 s\left(t_i^-\right) + g_3 \tag{4}$$

$$F(s(t), b) = b \cdot s(t) \tag{5}$$

where $c_1$, $c_2$, $m_1$, $m_2$, $g_1$, $g_2$, $g_3$ are constants that may be determined from empirical data. Note that the value of $c_2$ does not affect the solution to (1), so we set $c_2 = 0$.

To the best knowledge of the authors, no empirical model has been furnished in the literature that links the cost to the effectiveness of maintenance activities. However, given that this model has to satisfy some practical properties, the function form of $C_M(s(t), \Delta b)$ can be reasonably assumed. These properties are:

(i) $C_M$ is non-decreasing in $s$; e.g., for crack sealing, a larger roughness usually means more cracks to seal. We assume it is a linear function of $s$:

$$C_M(s(t), \Delta b) = C_{M,1}(\Delta b) \cdot s(t) + C_{M,2}(\Delta b) \tag{6}$$

where $C_{M,1}(\Delta b) \geqslant 0$ for all $\Delta b > 0$. Functions $C_{M,1}(\Delta b)$ and $C_{M,2}(\Delta b)$ indicate how much the maintenance cost is dependent and independent, respectively, of the pavement condition. These functions can vary across different types of maintenance activities.

(ii) $C_M$ is increasing in $\Delta b$. Further, this cost increases more rapidly as $\Delta b$ increases; i.e., $C_M(s(t), \Delta b)$ is a convex function of $\Delta b$ when $\Delta b > 0$.

(iii) When no maintenance is taken, $C_M(s(t),0) = C_{M,1}(s(t),0) = C_{M,2}(s(t),0) = 0$; but any maintenance activity will induce a non-zero fixed-cost, i.e., $\lim_{\Delta b \downarrow 0} C_M(s(t), \Delta b) > 0$ for any $s(t)$. Thus $C_M(s(t), \Delta b)$ is discontinuous in $\Delta b$ at $\Delta b = 0$.

For illustration, in this paper we assume the following function forms:

$$C_{M,i}(\Delta b) = \begin{cases} \alpha_i e^{\beta_i \Delta b} + \gamma_i, & \text{if } \Delta b > 0 \\ 0, & \text{if } \Delta b = 0 \end{cases}, \quad i = 1, 2 \tag{7}$$

where constants $\alpha_i \geqslant 0$, $\beta_i$, $\gamma_i > 0$ for $i = 1, 2$. One can easily check that the maintenance cost function defined by (6) and (7) satisfies the three properties described above.

Note that if $C_{M,1}(\Delta b) = 0$ for all $\Delta b$, then the maintenance cost is independent of the pavement condition. This special case might occur for some maintenance activities, e.g., chip-sealing, where the maintenance intensity required for achieving a certain effectiveness is independent of the pavement condition.

## 3. Solution

A two-step approach is used to find the optimal solution to problem (1). In Step 1, for any given $b$, we optimize the total lifecycle cost with regard to $\boldsymbol{t}$ and $\boldsymbol{w}$; in Step 2, we solve the optimization problem with regard to $b$.

In Step 1, we substitute (2) and (6) into (1a) and obtain

$$
\begin{aligned}
J(\boldsymbol{t},\boldsymbol{w},b) &= \sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}[c_1 s(u)+C_{M,1}(\Delta b)\cdot s(u)+C_{M,2}(\Delta b)]e^{-ru}du+\sum_{i=0}^{\infty}M(w_i)e^{-rt_i}\\
&= \sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}[c_1+C_{M,1}(\Delta b)]s(u)e^{-ru}du+\sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}C_{M,2}(\Delta b)e^{-ru}du+\sum_{i=0}^{\infty}M(w_i)e^{-rt_i}\\
&= \sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}[c_1+C_{M,1}(\Delta b)]s(u)e^{-ru}du+\int_0^{\infty}C_{M,2}(\Delta b)e^{-ru}du+\sum_{i=0}^{\infty}M(w_i)e^{-rt_i}
\end{aligned}
$$

We write $c_1'(\Delta b) = c_1 + C_{M,1}(\Delta b)$, and thus:

$$
J(\boldsymbol{t},\boldsymbol{w},b) = \sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}c_1'(\Delta b)\cdot s(u)e^{-ru}du+\sum_{i=0}^{\infty}M(w_i)e^{-rt_i}+C_{M,2}(\Delta b)\int_0^{\infty}e^{-ru}du = J_0(\boldsymbol{t},\boldsymbol{w},b)+\frac{1}{r}C_{M,2}(\Delta b) \tag{8}
$$

where $J_0(\boldsymbol{t},\boldsymbol{w},b) = \sum_{i=0}^{\infty}\int_{t_i}^{t_{i+1}}c_1'(\Delta b)\cdot s(u)e^{-ru}du+\sum_{i=0}^{\infty}M(w_i)e^{-rt_i}$.

The program is then reduced to minimizing $J_0(\boldsymbol{t},\boldsymbol{w},b)$ in the decision variables $\boldsymbol{t}$ and $\mathbf{w}$ subject to (1b)–(1f), (1h) and (3)–(5). Note that if we treat $c_1'(\Delta b)$ as the "user cost" per unit time, this program has the same form as the pavement resurfacing planning problem that is solved by Ouyang and Madanat (2006). Their solution has a "threshold structure", which involves a trigger roughness, $s^*$, defined as:

$$
s^* = \frac{r(m_1 g_3 + m_2)}{c_1'(\Delta b)g_1 + (b-r)m_1 g_2}. \tag{9}
$$

The solution can be described as following:

i) If the initial state $s_0 \geqslant s^*$, repeat resurfacing at $t = 0$ with maximal effective intensity until the roughness index falls below $s^*$; i.e., for $i = 1,2,\ldots,n_0$:

$$
\begin{aligned}
&t_i = 0\\
&w_i = R_i = g_2 s_{i-1} + g_3\\
&s_i = s_{i-1} - \frac{g_1 w_i}{g_2 s_{i-1}+g_3}s_{i-1} = (1-g_1)s_{i-1}\\
&s_{n_0} < s^* \leqslant s_{n_0-1}
\end{aligned} \tag{10}
$$

where $s_i (i = 1,2,\ldots,n_0)$ is defined as the pavement condition right after the $i$-th resurfacing.
Thus we have:

$$
n_0 = \left[\frac{\log s^* - \log s_0}{\log(1-g_1)} + 1\right]^-, \tag{11}
$$

where $[a]^-$ indicates the maximum integer that is no greater than $a$.
The total cost associated with these initial resurfacings is:

$$
\begin{aligned}
M_{INIT}(b) &= \sum_{i=1}^{n_0}(m_1 w_i + m_2) = m_1 g_2\sum_{i=1}^{n_0}s_{i-1} + n_0(m_1 g_3 + m_2) = m_1 g_2 s_0\sum_{i=1}^{n_0}(1-g_1)^{i-1} + n_0(m_1 g_3 + m_2)\\
&= \frac{m_1 g_2 s_0}{g_1}[1-(1-g_1)^{n_0}] + n_0(m_1 g_3 + m_2).
\end{aligned} \tag{12}
$$

And we define:

$$
s_0' = s_{n_0} = (1-g_1)^{n_0}s_0. \tag{13}
$$

Noting that $s_0' < s^*$, the remaining maintenance and resurfacing planning is identical to that of a pavement that starts from the initial condition $s_0'$, which is presented next.[1]

(ii) If $s_0 < s^*$, the optimal solution requires that, whenever $s(t)$ reaches $s^*$, resurface with the maximal effective intensity, i.e.,

$$
w^* = g_2 s^* + g_3; \tag{14}
$$

Thus we can easily find the time when the first resurfacing is taken, $t_1$:

$$
t_1 = \frac{1}{b}\log\frac{s^*}{s_0} \tag{15}
$$

And the optimal cycle length between two successive resurfacing actions, $\tau^*$:

[1] In practice, should pavement condition be so poor at $t = 0$ that multiple resurfacings are needed, the agency would likely perform a reconstruction instead. This case will be discussed later in Section 4.1.

$$\tau^* = -\frac{1}{b}\log(1-g_1) \tag{16}$$

Hence the minimum value of $J_0$ for a fixed $b$ can be calculated as:

$$\begin{aligned} J_0(b) = \min_{\boldsymbol{t},\boldsymbol{w}} J_0(\boldsymbol{t},\boldsymbol{w},b) &= \int_0^{t_1} c_1'(\Delta b)s_0 e^{(b-r)t}dt + \frac{e^{-rt_1}}{1-e^{-r\tau^*}}\left[\int_0^{\tau^*} c_1'(\Delta b)\cdot(1-g_1)s^* e^{(b-r)t}dt + m_1(g_2 s^* + g_3) + m_2\right] \\ &= \frac{c_1'(\Delta b)s_0}{b-r}(e^{(b-r)t_1}-1) + \frac{e^{-rt_1}}{1-e^{-r\tau^*}}\left[\frac{c_1'(\Delta b)\cdot(1-g_1)s^*}{b-r}(e^{(b-r)\tau^*}-1) + m_1(g_2 s^* + g_3) + m_2\right] \end{aligned} \tag{17}$$

Note that for case (ii), there is no resurfacing action taken at $t = 0$; i.e., $n_0 = 0$, $M_{INIT}(b) = 0$, and $s_0' = s_0$. Thus, we can replace (11) by

$$n_0 = max\left\{0, \left[\frac{\log s^* - \log s_0}{\log(1-g_1)} + 1\right]^-\right\}, \tag{18}$$

and then (12) and (13) will hold for both cases.

In light of all above, we can rewrite the original problem as an optimization in a single decision variable, $b$:

$$\min \quad J(b) = J_0(b) + \frac{1}{r}C_{M,2}(b_0 - b) \tag{19}$$

$$\begin{aligned} \text{subject to} \quad J_0(b) = M_{INIT}(b) &+ \frac{c_1'(\Delta b)s_0'}{b-r}(e^{(b-r)t_1}-1) \\ &+ \frac{e^{-rt_1}}{1-e^{-r\tau^*}}\left[\frac{c_1'(\Delta b)\cdot(1-g_1)s^*}{b-r}(e^{(b-r)\tau^*}-1) + m_1(g_2 s^* + g_3) + m_2\right] \end{aligned} \tag{20}$$

$$(1g) \tag{21}$$

where $s^*$, $M_{INIT}(b)$, $s_0'$, $t_1$, and $\tau^*$ are given by 9, 12, 13, 15, and 16, respectively.

This univariate optimization problem can be solved in Step 2 by numerical methods (such as the Newton–Raphson method).[2]

## 4. Numerical examples and insights

In the following two sections, we discuss the insights obtained from two numerical examples: i) a special case when the maintenance cost is independent of $s$ (Section 4.1); and ii) a general case when that cost depends on $s$ (Section 4.2).

### 4.1. A special case when $C_M$ is independent of $s(t)$

In this case, $C_{M,1}(\Delta b) = 0$, and the function form (7) is used for $C_{M,2}(\Delta b)$. The parameter values used for this example (for a one kilometer long highway lane) are:

$$\begin{aligned} &c_1 = 1000\ \$/\text{QI/year}, \quad m_1 = 11{,}000\ \$/\text{mm}, \quad m_2 = 150{,}000\ \$, \quad g_1 = 0.66, g_2 = 0.55, \quad g_3 = 18.3, \quad r = 0.07, \\ &\alpha_2 = 400, \quad \beta_2 = 120, \quad \gamma_2 = 2000, \quad b_0 = 0.08, \quad \underline{b} = 0.035. \end{aligned}$$

The lifecycle costs versus $b$ for $s_0 = 25$ QI are plotted in Fig. 1. In this figure, the solid curve that represents the total lifecycle cost reveals an interior minimum, $J(b^*) = 9.39 \times 10^5$ \$, that dwells at $b^* = 0.059$. Compared to the no-maintenance case (when $b = b_0 = 0.08$; note that the curve of the total lifecycle cost is discontinuous at this point), the maintenance associated with this $b^*$ can reduce the lifecycle cost by 6.1%. The optimal resurfacing plan is: $s^* = 41$ QI, $w^* = 41$ mm, and $\tau^* = 18$ years. Note that these values are very different from the case without maintenance, where the three planning variables would be 34 QI, 37 mm, and 13.5 years, respectively.

To better understand the cause-and-effect relations between the lifecycle cost and the maintenance effectiveness, the individual components of the lifecycle cost are also plotted in Fig. 1. These curves show that the total lifecycle cost save for maintenance cost (the dash-dot curve) increases almost linearly with $b$, while the maintenance cost (the shorter-dashed curve) declines with $b$ with a diminishing slope. The latter is a result of the convexity of the first expression of (7). This means that the maintenance cost required to achieve a certain amount of reduction in the user and resurfacing cost becomes higher as $b$ decreases. Thus no maintenance and too much maintenance are both unfavorable.

A further look at the cost components unveils that, as $\Delta b$ increases, the resurfacing cost (the dotted curve) decreases, while the user cost (the longer-dashed curve) increases; i.e., increasing the intensity of maintenance benefits the pavement management agency, but not the users. This can be explained by examining the changes of the optimal planning variables (i.e., $s^*$, $w^*$, and $\tau^*$) in response to the inclusion of maintenance, as described next.

[2] A note regarding the convexity of $J(b)$: the second term of the right-hand-side of (19) is convex (see the function form (7)), while this may not be true for the first term (i.e., $J_0(b)$). Thus $J(b)$ may not be convex in general, although it happens to be convex in the numerical examples in Section 4.

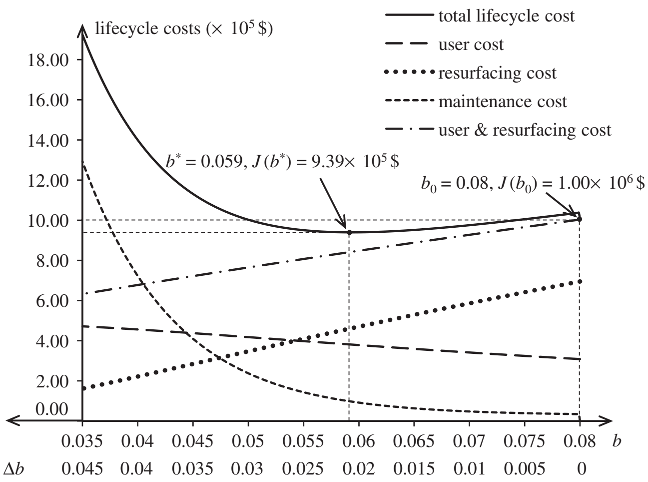


**Fig. 1.** Lifecycle costs versus $b$ for the special case.

Fig. 2 shows that $s^*$, $w^*$, and $\tau^*$ all increase as $b$ declines. This means if maintenance is applied, it is optimal to resurface the pavement less frequently, at a higher trigger roughness, and with greater intensity for each resurfacing. These findings are consistent with both practice and intuition. In particular, applying maintenance prolongs the service life of the pavement between two resurfacing actions (i.e., $\tau^*$ increases). This longer cycle length can potentially increase the user cost per cycle. Thus, the trade-off shifts between the user cost per cycle and the cost of the resurfacing action in a cycle, resulting in both a

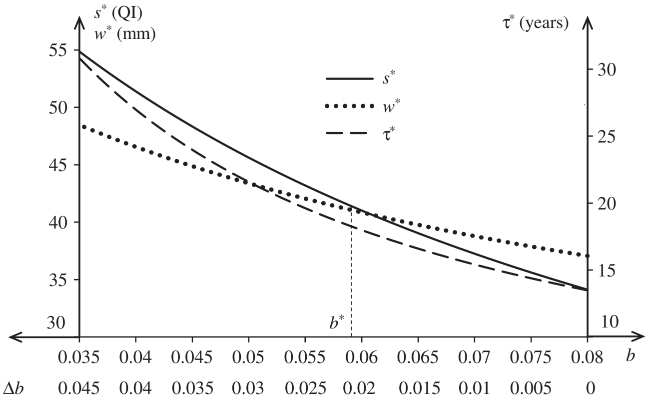


**Fig. 2.** Changes of $s^*$, $w^*$, and $\tau^*$ for the special case.

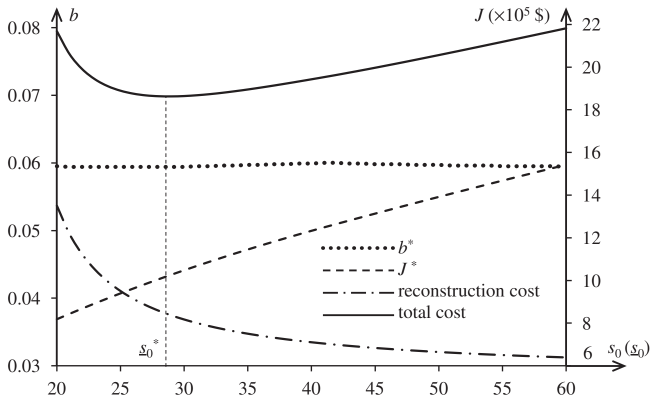


**Fig. 3.** The influence of $s_0$ and optimal planning with reconstruction (for the special case).

larger user cost per cycle and a larger cost of a single resurfacing action. The latter means higher $w^*$ and $s^*$. Note that $w^*$ is always a linear function of $s^*$ when $s_0 < s^*$ (which is the case of this example); see (14).

Further note that the increase in $s^*$ leads to an increase in the lifecycle user cost; and although $w^*$ increases as well, the lower resurfacing frequency (i.e., the increased $\tau^*$) explains the decrease in the lifecycle resurfacing cost, as we have seen in Fig. 1.

Fig. 3 exhibits the influence of the initial pavement state, $s_0$. It reveals that, although the total lifecycle cost (the dashed curve) increases significantly with $s_0$, the optimal deterioration rate (the dotted curve) is insensitive to $s_0$.

In practice, when the initial pavement condition is very poor, the agency would perform a reconstruction instead of multiple resurfacing actions. In this case, the cost and the parameters of the reconstruction activity are determined by the desired pavement condition after reconstruction, $\underline{s}_0$. An example of the reconstruction cost as a function of $\underline{s}_0$ is illustrated by the dash-dot curve in Fig. 3. Adding this reconstruction cost to the optimal lifecycle cost of maintenance and resurfacing for any given $\underline{s}_0$, we obtain the total cost (including reconstruction) as a function of $\underline{s}_0$. Then we can find the optimal reconstruction effectiveness, indicated by $\underline{s}_0^*$ in Fig. 3, and the corresponding parameters of the reconstruction activity.

### 4.2. General case: when $C_M$ is a linear function of $s(t)$

Now we examine the general case, i.e., when $C_{M,1}(\Delta b) \neq 0$. Recall that crack sealing is an example of this case. We use the same parameter values as above except for the following:

$$\alpha_1 = 12, \beta_1 = 120, \gamma_1 = 60, \alpha_2 = 100, \beta_2 = 120, \gamma_2 = 500.$$

Like Figs. 1, 4 shows how the lifecycle costs vary in $b$ when $s_0 = 25$ QI. The optimal deterioration rate, $b^*$, is found to be 0.059; and the associated total lifecycle cost, $J(b^*)$, is $9.36 \times 10^5$ \$. This optimal lifecycle cost is 6.4% lower than that in the absence of maintenance. These results are similar to the previous case. The optimal resurfacing plan is: $s^* = 34$ QI, $w^* = 37$ mm, and $\tau^* = 18$ years. If the effect of maintenance is ignored, the optimal values of these variables are 32 QI, 36 mm, and 13.5 years, respectively. Note that unlike the special case, $s^*$ and $w^*$ are close to those values without maintenance, for reasons that shall be made clear later in this section.

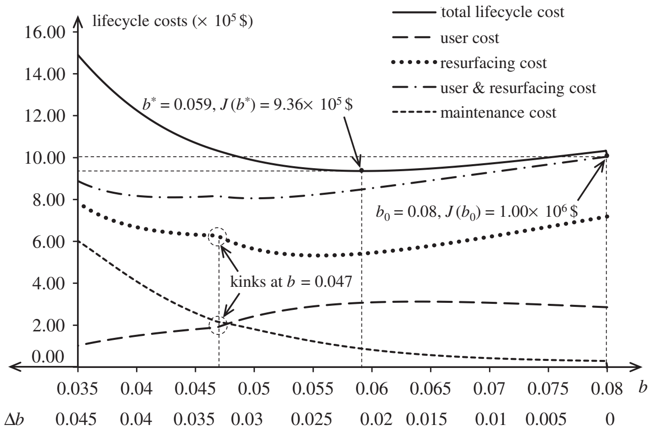


**Fig. 4.** Lifecycle costs versus $b$ for the general case.

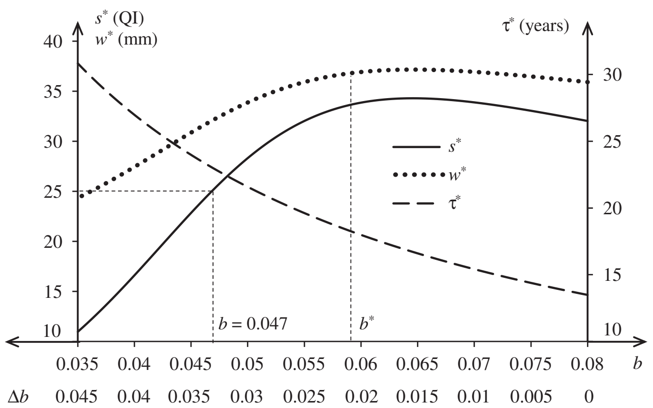


**Fig. 5.** Changes of $s^*$, $w^*$, and $\tau^*$ for the general case.

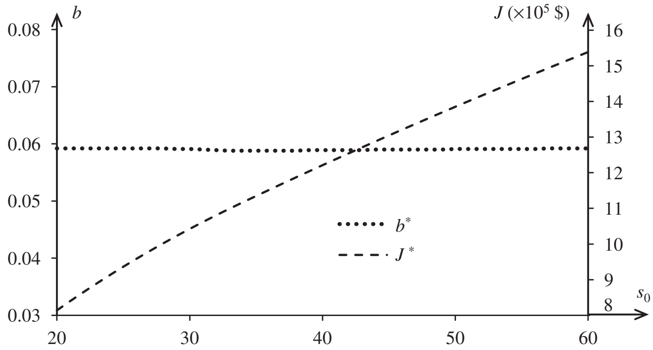


**Fig. 6.** The influence of $s_0$ for the general case.

The trends of the user cost and the resurfacing cost, however, are different from the previous case. As Fig. 4 shows, when $\Delta b$ increases, the user cost first increases and then decreases, while the resurfacing cost moves in the opposite direction. Hence, unlike the special case, too much maintenance benefits the users, while the agency is largely worse off. These trends are explained next.

Fig. 5 shows that, as $\Delta b$ increases, both $s^*$ and $w^*$ increase for high values of $b$, and decrease otherwise. The former can be explained by the cause-and-effect relations unveiled in Section 4.1. The latter trend can be explained as follows: when $b$ decreases to a low level, $C_{M,1}(\Delta b)$ increases rapidly, so that $s$ has to be low enough to avoid a too high maintenance cost. As a result, both $s^*$ and $w^*$ diminish as $\Delta b$ increases. This readily explains the decreasing segment of the user cost curve for a growing $\Delta b$ (see Fig. 4). The corresponding, increasing segment in the resurfacing cost curve can be explained by the timing of the first resurfacing action: as $s^*$ decreases, the first resurfacing is taken earlier and so the present value of its cost grows. This raises the lifecycle resurfacing cost despite the fact that both the resurfacing frequency and intensity are decreasing. Finally, when $s^*$ falls below $s_0$, an initial resurfacing has to be taken at $t = 0$, this causes sudden changes in the slopes of the user cost and resurfacing cost curves (the kinks on these curves in Fig. 4).

Similar to the special case, $b^*$ is almost invariant to $s_0$, as shown in Fig. 6.

## 5. Conclusions

This paper describes a mathematical formulation that incorporates the effects of maintenance activities in the pavement resurfacing planning problem. The analytical solution to this problem is developed, and realistic numerical examples are discussed to unveil managerial insights and practical implications.

For example, our numerical examples have shown the benefit of applying a moderate amount of maintenance activities to reduce the overall pavement lifecycle cost by more than 6%. Another interesting result for pavement management agencies is that a resurfacing plan with excessive maintenance may cost more than a plan with no maintenance (see again Figs. 1 and 4). Our results have also confirmed that maintenance can significantly prolong the pavement's service life between consecutive rehabilitations (recall that $\tau^*$ increases significantly in both examples). However, the optimal resurfacing plan depends on the maintenance model (recall that the changes in $s^*$ and $w^*$ are very different between the two examples).

Another important finding comes from the fact that $b^*$ is almost invariant to $s_0$ (see again Figs. 3 and 6). Recall that previous studies (e.g., Li and Madanat, 2002; Ouyang and Madanat, 2006) have shown that for a given $b$, the optimal resurfacing plan of a threshold structure is invariant to $s_0$ (see Section 1). Hence, $s^*$, $w^*$, and $\tau^*$ are all insensitive to $s_0$ since $b^*$ remains unchanged with $s_0$; i.e., the optimal resurfacing plan is still invariant to the initial pavement condition when the maintenance effect is incorporated.

While the results presented in this paper are quite general and realistic, we acknowledge that our findings are limited in that: i) the maintenance cost-effectiveness models used in this paper are not based on field data; and ii) we assume that the deterioration rate is constant throughout the planning horizon. Yet in our view, the work presented in this paper sheds light on better understanding the interdependence between pavement routine maintenance and rehabilitation activities. Further, our modeling approach can be used to solve similar problems with realistic maintenance cost and effectiveness models, as long as the maintenance cost is a linear function of the pavement state. Models with time-varying deterioration rate are currently being explored.

## References

Carnahan, J.V., Davis, W.J., Shahin, M.Y., Kean, P.L., Wu, M.I., 1987. Optimal maintenance decisions for pavement management. ASCE Journal of Transportation Engineering 113 (5), 554–572.

Chong, G.J., 1989. Rout and Seal Cracks in Flexible Pavement – A Cost-Effective Preventive Maintenance Procedure. Rep. No. 890412, Ontario Ministry of Transportation, Ontario, Canada.

Durango-Cohen, P., 2007. A time series analysis framework for transportation infrastructure management. Transportation Research Part B 41 (5), 493–505.

Fernandez, J.E., Friesz, T.L., 1981. Influence of demand-quality interrelationships on optimal policies for stage construction of transportation facilities. Transportation Science 15 (1), 16–31.

Friesz, T.L., Fernandez, J.E., 1979. A model of optimal transport maintenance with demand responsiveness. Transportation Research Part B 13 (4), 317–339.

Golabi, K., Kulkarni, R., Way, G., 1982. A statewide pavement management system. Interfaces 12 (6), 5–21.

Labi, S., Sinha, K.C., 2003. The Effectiveness of Maintenance and Its Impact on Capital Expenditures. Joint Transportation Research Program, Paper 208, Purdue University.

Li, Y., Madanat, S., 2002. A steady-state solution for the optimal pavement resurfacing problem. Transportation Research Part A 36 (6), 525–535.

Madanat, S., 1993. Incorporating inspection decisions in pavement management. Transportation Research Part B 27 (6), 427–438.

Madanat, S., Ben-Akiva, M., 1994. Optimal inspection and repair policies for transportation facilities. Transportation Science 28 (1), 55–62.

Markow, M., Balta, W., 1985. Optimal rehabilitation frequencies for highway pavements. Transportation Research Record 1035, Transportation Research Board, National Research Council, Washington, DC, pp. 31–43.

Ouyang, Y., 2007. Pavement resurfacing planning on highway networks: a parametric policy iteration approach. ASCE Journal of Infrastructure Systems 13 (1), 65–71.

Ouyang, Y., Madanat, S., 2004. Optimal scheduling of rehabilitation activities for multiple pavement facilities: exact and approximate solutions. Transportation Research Part A 38 (5), 347–365.

Ouyang, Y., Madanat, S., 2006. An analytical solution for the finite-horizon pavement resurfacing planning problem. Transportation Research Part B 40 (9), 767–778.

Ponniah, J.E., 1992. Crack sealing in flexible pavements: a life-cycle cost analysis. The Research and Development Branch, Ontario Ministry of Transportation.

Ponniah, J.E., Kennepohl, G.J., 1996. Crack sealing in flexible pavements: a life-cycle cost analysis. Transportation Research Record 1529, Transportation Research Board, National Research Council, Washington, DC, pp. 86–94.

Rashid, M.M., Tsunokawa, K., 2010. Trend curve optimal control model for optimizing pavement maintenance strategies consisting of various treatments. Computer-Aided Civil and Infrastructure Engineering, in press.

Sathaye, N., Madanat, S., 2011. A bottom-up solution for the multi-facility optimal pavement resurfacing problem. Transportation Research Part B 45 (7), 1004–1017.

Tsunokawa, K., Schofer, J.L., 1994. Trend curve optimal control model for highway pavement maintenance: case study and evaluation. Transportation Research Part A 28 (2), 151–166.