Optimal Scheduling of Replacement and Rehabilitation of Water Distribution Systems

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Abstract: Many municipal water distribution systems across North America are reaching or have exceeded their design lives and, therefore, require extensive upgrading through rehabilitation and/or replacement. However, these needs far surpass the available resources, and decision makers must prioritize their replacement/rehabilitation needs. One such approach is the determination of the optimal replacement time based on the minimization of the total or annual average cost during a predetermined service period. This paper describes a simple approach for the optimization of replacement/rehabilitation activities for a network of buried pipes, which is based on the assumption that the occurrence of breaks in a pipeline segment follows a nonhomogeneous Poisson process. Equations for evaluating the optimal replacement time are derived by minimizing the expected annual average cost during the service period of the pipeline segment. Predictions are compared with those obtained by minimizing the expected total (accumulated) cost during the service or planning period. The use of the proposed approach is illustrated via numerical examples. Optimal replacement time predictions, based on minimization of the annual average cost, were found to be significantly longer than those obtained based on minimization of the total cost for the case—where break occurrence rate follows an exponential function.

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Introduction

The deterioration of pipelines in municipal water distribution systems due to various corrosion mechanisms, such as soil corrosion, bimetallic corrosion, and stray currents, is a well-documented and widespread problem. Deterioration leads to leaks and breaks, which results in pressure head losses, higher repair costs, loss of valuable water, higher pumping costs, and erosion of roadways' subgrade. Other potential adverse consequences include flooding of roads and nearby basements, and damage to adjacent utilities and foundations. A recent study (Deb et al. 2003) indicates that approximately $325 billion are needed for replacement and rehabilitation of water distribution systems across the United States. In Canada, the cost of upgrading municipal water distribution systems is estimated to be about 11.5 billion Canadian dollars over the next 15 years (CWWA 1997).

The need for the renewal and rehabilitation of potable water distribution systems would not present a problem if plentiful resources were available for the retrofitting or replacement of these systems. Unfortunately, resources are scarce and decisions must be made regularly regarding the prioritization of repair and replacement needs as to accommodate budgetary constraints. A methodology that optimally prioritizes rehabilitation and replacement needs will enable agencies to maximize the benefit/cost ratio associated with capital expenditures in their water distribution systems, as well as accelerate the bridging of the infrastructure gap.

Several approaches have been proposed in the literature to facilitate the selection of optimal replacement strategies (e.g., Shamir and Howard 1979; Kleiner et al. 2001; Loganathan et al. 2002). Shamir and Howard (1979) suggested that the optimal replacement time for a particular segment of the water distribution system could be computed by minimizing the total cost that includes the cost of repair and replacement during the planning period. The planning or decision period was considered to be equal to the optimal replacement time. A simple equation for calculating the optimal replacement time, which assumes the break occurrence rate to follow an exponential curve with respect to time, was developed. The use of the minimum expected total cost and a limited planning period for decision making were also adopted by other researchers, but used different break occurrence rate models and a nonhomogeneous Poisson process (Andreou et al. 1987; Loganathan et al. 2002). If the break occurrence rate function is known, the break occurrence rate evaluated at the optimal replacement time is termed the "critical," or "threshold," break occurrence rate (Walski and Pelliccia 1982; Loganathan et al. 2002). If the break occurrence rate of a pipeline segment is greater than this critical rate, the pipeline segment should be replaced rather than repaired. Kleiner et al. (2001) extended the approach proposed by Shamir and Howard (1979) by considering the decision period to be infinity and the option of multiple replacements. In their extension, performance attributes of the

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water distribution network, such as the required water pressure head, were also considered. Further, Kleiner et al. (2004) assumed that, after the first replacement, the optimal time between two successive replacements is a constant and can be obtained independently of the optimal time for the first replacement.

However, optimal replacement time—obtained by minimizing the expected total cost (at present value) for a finite planning period—might not lead to the minimum annual average cost during the service period. In other words, a solution that minimizes the total cost might not provide the maximum benefit—in terms of minimizing the cost (at present value) per unit service period (or time) (i.e., maximizing the service period per dollar spent). Nowadays, replacement rates of water distribution systems in most North American municipalities rarely exceed 2% per year; implying a turnover of 50 years or more. Thus, the challenge faced by municipal engineers is to reach a sustainable status, where pipeline segments are replaced at an optimized time period with the definition of "optimized period" governed by criteria, such as agency costs, social costs, and performance criteria. The writers believe that the utilization of the minimum expected annual average cost—as a criterion for optimal scheduling—is equally, if not more, important than the utilization of the minimum expected total cost, because it focuses the attention of the decision makers on maximizing the service period per dollar spent; a desirable objective in a sustainability driven management approach.

In this study, a simple approach is developed and demonstrated for the prioritization of rehabilitation or replacement activities associated with buried pipe networks. The proposed approach is focused on the decision algorithm, and assumes a limited planning period, due to the belief that future pipeline materials, manufacturing, construction, and repair technologies may differ from current and near future practices. The developed approach assumes that break occurrences in a buried pipeline segment follow a nonhomogeneous Poisson process. Equations—for evaluating the optimal replacement time and its associated critical break occurrence rate—are derived by minimizing the expected annual average cost during the service period of the pipeline segment. These equations are compared with those derived by minimizing the expected total cost. Additionally, it is argued that a pipeline segment could remain in good condition beyond the end of the planning period. Thus, a maintenance-free service period could be gained until the occurrence of the first break following the planning period. The impact of this maintenance-free service period on the optimal decision is also investigated. A parametric analysis is carried out to examine the sensitivity of the optimal replacement period (T_1)—calculated based on the minimization of total and annual average costs—to the duration of the planning period (t), cost ratio (R_c), discount rate (γ), pipe age (t_0), model parameter (A), and the consideration of a maintenance-free period (U) following the conclusion of the planning period. The proposed approach is illustrated via numerical examples. Optimal replacement times for two hypothetical pipe segments are computed—based on the minimization of the expected annual average cost—and compared to those calculated—based on the minimization of the expected total cost.

Modeling the Break Occurrence and Evaluating Expected Cost

For the formulation presented in this study, it is assumed that the occurrence of the number of breaks, $N(t)$, in a pipeline segment—during a service period t—is a Poisson process. This occurrence process is considered to be nonhomogeneous, since the age of the pipeline segment affects the break occurrence rate. The difference between a homogeneous and a nonhomogeneous Poisson process is that the occurrence rate is a constant for the former, while it varies with time for the latter. For a nonhomogeneous Poisson process with an occurrence rate $\nu(t|t_0)$, where t_0 indicates that the pipeline segment has already been in service for t_0 years with necessary repairs and maintenance, the probability of the occurrence of n breaks within $(0,t)$, $P[N(t)=n]$, is given by the following equation (Parzen 1962):

$$P(N(t)=n)=\frac{1}{n!}\left(\int_0^t \nu(\tau|t_0)d\tau\right)^n \exp\left(-\int_0^t \nu(\tau|t_0)d\tau\right) \quad (1)$$

where the time t represents the future service period (i.e., the measure of t starts at present). By letting

$$s=V(t)=\int_0^t \nu(\tau|t_0)d\tau \quad (2)$$

it can be shown (Parzen 1962) that $N_S(s)=N(V^{-1}(s))$ is a normalized homogeneous Poisson process with an occurrence rate equal to 1.0, and the probability of n breaks within $(0,s]$, $P[N_S(s)=n]$, given by

$$P[N_S(s)=n]=(s)^n\exp(-s)/n! \quad (3)$$

where $V^{-1}(\bullet)$ denotes the inverse transformation of $V(\bullet)$. Note that since the break occurrence rate $\nu(t|t_0)$ is always positive, $V(\bullet)$ is a monotonically increasing function and its inverse exists.

Let $C_T(t)$ denote the present value of the maintenance cost (including the cost of repair and the cost of replacement) to be incurred over the next t years of service for an existing pipeline segment. Consider that the pipeline segment has already been in service for t_0 years, and that maintenance repairs were carried out immediately whenever a break occurred. Given the occurrence of n breaks within $(0,t]$, the cost $C_T(t)$ can be expressed as

$$C_T(t)=\sum_{i=1}^{n} C_R(i)\exp(-\gamma t_i) \quad (4)$$

or

$$C_T(t)=C(V^{-1}(s))=\sum_{i=1}^{n} C_R(i)\exp(-\gamma V^{-1}(s_i)) \quad (5)$$

where $C_R(i)$, $i=1,\cdots,n$, denotes the cost of repair for the ith break occurred at t_i; and γ represents the discount rate. It is important to note that $N_S(s)$ is a homogeneous Poisson process, and that an important theorem for the homogeneous Poisson process states that the times $s_1,\cdots,s_n$, considered to be unordered random variables, are independently and uniformly distributed in the interval 0 to s (Parzen 1962). Using this and assuming that $C_R(i)$ are independent and identically distributed with mean equal to m_{C_R}, and that $C_R(i)$ is independent of underlying Poisson process, Eq. (5) can be rewritten as

$$E(C_T(t)) = \sum_{j=1}^{\infty}\left[\left(\frac{1}{s}\right)^j \int_0^s \cdots \int_0^s \left(\sum_{i=1}^{j} m_{C_R}\exp(-\gamma V^{-1}(\tau_i))\right) d\tau_1\cdots d\tau_j\right] \times\left[\frac{1}{j!}s^j\exp(-s)\right] \quad (6)$$

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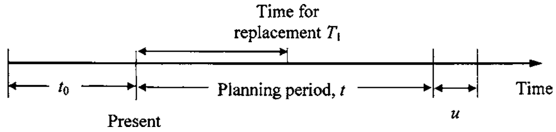

Fig. 1. Time line associated with proposed approach

where $E(C_T(t))$=expected total cost. Using algebraic manipulations, Eq. (6) could be simplified to the following form

$$E(C_T(t)) = m_{C_R}\int_0^t \exp(-\gamma\tau)\nu(\tau|t_0)d\tau \tag{7}$$

Consider that one is interested in finding the expected total cost $E(C_T(t))$ given that the replacement of the pipeline segment is scheduled at T_1, and that repairs are carried out for the future service period t whenever a break occurs (see Fig. 1). It is assumed that after the replacement, the occurrence of the breaks for the pipeline segment follows a nonhomogeneous Poisson process with an occurrence rate $\nu_N(t-T_1|0)$, thus the total cost $C_T(t)$ can be expressed as

$$C_T(t) = \left\{\sum_{i=1}^{n} C_R(i)\exp(-\gamma t_i) + C_N\exp(-rT_1) + \sum_{k=1}^{m} C_R(k)\exp(-\gamma t_k')\right\} \tag{8}$$

where C_N represents the replacement cost; t_i, $i=1,\cdots,n$=time associated with the ith break prior to the replacement; and t_k', $k=1,\cdots,m$, denotes the time of the kth break after the replacement was carried out at time T_1. By conducting algebraic manipulations similar to that carried out for Eqs. (6) and (7), Eq. (8) could be rewritten to express the expected total cost $E(C_T(t))$

$$E(C_T(t)) = m_{C_R}\left\{\int_0^{T_1}\exp(-\gamma\tau)\nu(\tau|t_0)d\tau + \frac{m_{C_N}}{m_{C_R}}\exp(-\gamma T_1) + \int_0^{t-T_1}\exp(-\gamma(\tau+T_1))\nu_N(\tau|0)d\tau\right\} \tag{9}$$

Optimal Replacement Time

Approach Based on Expected Total Cost

The expected total cost $E(C_T(t))$, shown in Eq. (9), was derived based on a planning period of t years. Provided that the functions describing the break occurrence rate $\nu(\tau|t_0)$ and $\nu_N(\tau|0)$—as well as the values of T_1 and t_0—are known, $E(C_T(t))$ can be calculated.

One of the commonly employed models for describing the break occurrence rate in a pipeline segment, $\lambda(\tau|t_0)$, is that given by Shamir and Howard (1979)

$$\lambda(\tau|t_0) = \lambda_0\exp[A(\tau+t_0)] \tag{10}$$

where A=model parameter; λ_0 represents the break occurrence rate for a new pipeline segment that is just commissioned for service; and t_0=duration of already sustained service period with necessary maintenance repairs. $\lambda(\tau|t_0)$ can also be viewed as the break occurrence rate at time $\tau+t_0$, where $\tau+t_0>0$. It is noteworthy that both $\nu(\tau|t_0)$ and $\nu_N(\tau|t_0)$ represent the break occurrence rate at time τ given that the pipeline segment has already been in service for t_0 years. Therefore, it is reasonable to assume that $\nu(\tau|t_0)$ and $\nu_N(\tau|t_0)$ are the same and equal to $\lambda(\tau|t_0)$, if the pipeline construction and manufacture practices—at present and in the past—are considered to be similar, and the model presented by Eq. (10) is assumed adequate. This assumption was adopted for the remainder of this study, unless otherwise indicated. It must be emphasized that this is an assumption used to simplify the analysis, although it is acknowledged that pipeline material and construction practices have varied over the past 120 years (which is the age of some of the oldest systems still in operation). By adopting this assumption, Eq. (9) becomes

$$E(C_T(t)) = m_{C_R}\lambda_0\left\{\frac{\exp(At_0)}{A-\gamma}(\exp((A-\gamma)T_1)-1) + \frac{m_{C_N}\exp(-\gamma T_1)}{m_{C_R}\lambda_0} + \frac{\exp(-\gamma T_1)}{A-\gamma}(\exp((A-\gamma)(t-T_1))-1)\right\} \tag{11}$$

Given an arbitrary planning period t, and by adopting the criterion that for a pipeline segment, the optimal replacement time T_1 can be obtained by minimizing $E(C_T(t))$, Eq. (11) can be used to find this optimal replacement time. This optimal time will depend on the planning period t.

It is noted that in some studies the planning period t is considered to be equal to the optimal replacement time (i.e., $t=T_1$) (e.g., Shamir and Howard 1979; Loganathan et al. 2002). This consideration causes the last term in Eq. (11) to vanish. The optimal value of T_1 is obtained by solving the equation defined by $dE(C_T(T_1))/dT_1=0$ [where $d^2E(C_T(T_1))/dT_1^2$ is clearly positive guaranteeing that the optimum value of T_1 results in a minimum at $E(C_T(t))$] that can be expressed as

$$\exp(A(t_0+T_1)) - \frac{m_{C_N}\gamma}{m_{C_R}\lambda_0} = 0 \tag{12}$$

This indicates that the sum t_0+T_1 satisfying Eq. (12) does not depend on how long the pipeline segment has already been in service, and that the solution of t_0+T_1 will depend only on A and $(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$. Solving Eq. (12) leads to the optimal replacement time T_1 given by

$$T_1 = -t_0 + \frac{1}{A}\ln\left(\frac{m_{C_N}\gamma}{m_{C_R}\lambda_0}\right) \tag{13}$$

It must be emphasized that the above solution was based on the considerations that the pipeline segment has already sustained a service period of t_0, and that the planning period t is set equal to T_1. Therefore, under such considerations, for this solution to be meaningful, it is necessary that $t_0 \leq t_0+T_1$ or $0 \leq T_1$ holds. Consequently, all numerical analyses to be carried out are focused on the cases when $0 \leq T_1 \leq t$.

The solution shown in Eq. (13) is slightly different than $T_1=-t_0+(1/A)\ln[(\ln(1+\gamma')m_{C_N})/(m_{C_R}\lambda_0)]$ given by Shamir and Howard (1979). This is due to the manner used for evaluating present value, where $\exp(-\gamma t)$ is used in the current derivation; while $1/(1+\gamma')^t$ was used by Shamir and Howard (1979). Both expressions give the same result for $\gamma=\ln(1+\gamma')$ and yield approximately the same result for $\gamma=\gamma'$, since the discount rate is typically a small value.

Note that use of the optimal replacement time T_1 in the break occurrence rate equation [Eq. (10)] leads to the critical break occurrence rate λ_{cr},

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$$\lambda_{cr} = \lambda_0 \exp[A(T_1 + t_0)] \tag{14}$$

If the break occurrence rate in a pipeline segment is greater than λ_{cr}, the replacement of the pipeline segment is overdue.

The consideration of the planning period t equal to T_1 leads to a simple equation for evaluating the optimal T_1. However, it does not take into account the extension of the pipe's useful service period due to the rehabilitation/replacement action. To take the impact of the replacement on future service into account, we note that t_0+T_1—that satisfies Eq. (12)—does not depend on how long the pipeline segment has been in service. This implies that the optimal total service time, or the optimal time between the replacements, is equal to t_0+T_1. Therefore, if a replacement is carried out for a pipeline segment, it is expected to remain in service for a period of t_0+T_1. In other words, after its replacement at time T_1, the pipeline segment is expected to remain in service, with necessary maintenance repairs, for a period equal to t_0+T_1. Thus, it is argued that the planning period t, should be set to t_0+2T_1 for the first replacement, and t_0+T_1 for subsequent replacements. Substituting the modified expression for the planning period into Eq. (11), one gets

$$E(C_T(t)) = m_{C_R}\lambda_0 \left\{ \frac{\exp(At_0)}{A-\gamma}(\exp((A-\gamma)T_1)-1) + \frac{m_{C_N}\exp(-\gamma T_1)}{m_{C_R}\lambda_0} + \frac{\exp(-\gamma T_1)}{A-\gamma}(\exp((A-\gamma)(t_0+T_1))-1) \right\} \tag{15}$$

The optimal value of T_1 is obtained by taking the first derivative of Eq. (15) with respect to T_1, $dE(C_T(t_0+2T_1))/dT_1=0$.

$$\exp(A(t_0+T_1)) - \frac{m_{C_N}\gamma}{m_{C_R}\lambda_0} + \frac{1}{A-\gamma} \times [(A-2\gamma)\exp((A-\gamma)(t_0+T_1))+\gamma] = 0 \tag{16}$$

Equation (16) indicates that, similar to the previous case, the sum t_0+T_1 does not depend on how long the pipeline segment has been in service. However, the solution depends not only on A and $(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$, but also on γ. Therefore, the critical break occurrence rate λ_{cr}, obtained by substituting t_0+T_1 into Eq. (10), will depend on A, $(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$, and γ.

Approach Based on Expected Annual Average Cost

In the previous section, the optimal replacement time T_1 was derived based on the minimization of the expected total cost $C_T(t)$ for a planning period of t years. Although the planning period and the expected total cost evaluation were terminated at time t, it does not mean that the pipeline segment should be deemed inadequate at time t, since it may still provide adequate service at no additional cost from time t to the time of the next break. In other words, there is a residual useful service period that needs to be considered in the analysis. Furthermore, as mentioned earlier, the optimal T_1 value—obtained by minimizing the expected total cost—may not represent the maximum service time per dollar spent or the minimum cost per unit service time period.

To evaluate the effect of using the minimum cost per unit service time period as a decision-making criterion on the value of T_1, the expected annual average cost $C_A(t)$ during a planning period t can be written as

$$E[C_A(t)] = E\left(\frac{1}{t+U}C_T(t)\right) \tag{17}$$

where U denotes the residual maintenance-free service time gained between the end of the planning period t to the time of the first break following it. Break occurrence after the original planning period, t, is a nonhomogeneous Poisson process in the original time scale, but a homogeneous Poisson process in the new time scale defined by s,

$$s = V_N(u) = \int_0^u \nu_N(\tau|t-T_1)d\tau = \int_0^u \lambda_0 \exp[A(\tau+t-T_1)]d\tau = \frac{\lambda_0\exp(A(t-T_1))}{A}[\exp(Au)-1] \tag{18}$$

This implies that in the new time scale, the time from the end of the planning period to the first break is exponentially distributed with a probability density function equal to $\exp(-s)$, and it is independent of the history of the break occurrences. Therefore, Eq. (17) can be written as

$$E(C_A(t)) = E\left(\frac{1}{t+U}\right)E(C_T(t)) = \left(\int_0^\infty \frac{1}{t+u}\exp(-s)ds\right)E(C_T(t)) \tag{19}$$

where u obtained from Eq. (18) is given by

$$u = \frac{1}{A}\ln\left(1+\frac{sA}{\lambda_0\exp(A(t-T_1))}\right) \tag{20}$$

and $E(C_T(t))$ is obtained from Eq. (11) for $t=t_0+T_1$ and from Eq. (15) for $t=t_0+2T_1$.

The solution T_1 that minimizes $E(C_T(t))$ can be obtained numerically by any available optimization algorithm.

Note that if the possible gain of a maintenance-free service period at the end of the planning period, U, is neglected, Eq. (18) reduces to

$$E(C_A(t)) = \frac{1}{t}E(C_T(t)) \tag{21}$$

Numerical Results

Comparison and Discussions

Equations derived in the previous section, for evaluating the optimal replacement time T_1, are employed to carry out a sensitivity analysis. For comparison purposes, the following criteria are considered for selecting the optimal T_1 values:

1a. Minimizing the expected total cost for $t=T_1$ [see Eq. (13)];
1b. Minimizing the expected total cost for $t=t_0+2T_1$ [see Eq. (16)];
2a. Minimizing the expected annual average cost, while neglecting possible gain in the form of a maintenance-free service time following the planning period, for $t=T_1$ [see Eq. (21)];
2b. Minimizing the expected annual average cost, while neglecting possible gain in the form of a maintenance-free service time following the planning period, for $t=t_0+2T_1$ [see Eq. (21)];

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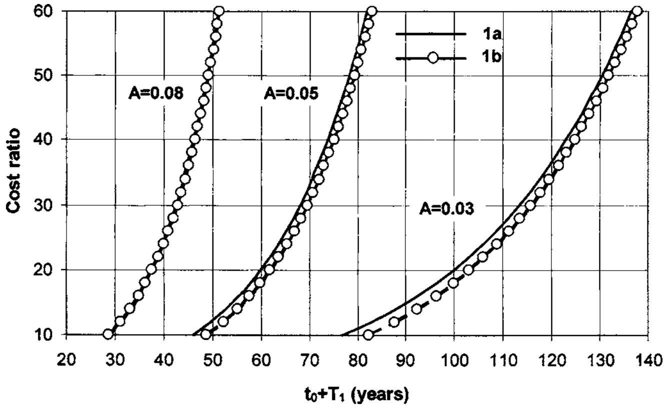

Fig. 2. Optimal replacement time obtained by minimizing the expected total cost

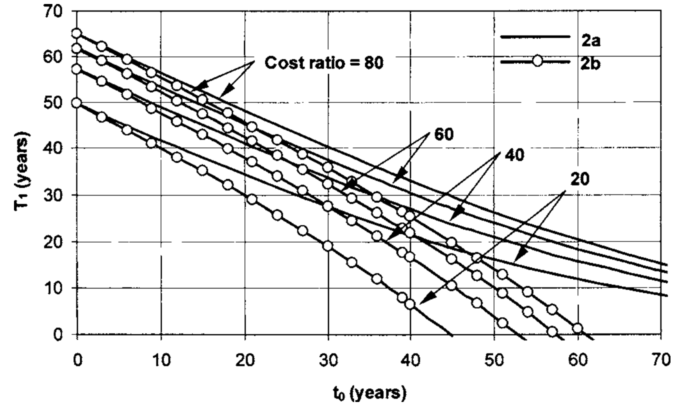

Fig. 4. Optimal replacement time obtained by minimizing the expected annual average cost given in Eq. (21) for $A=0.08$ and $\gamma=0.05$

3a. Minimizing the expected annual average cost, taking into account possible gain in service time following the completion of the planning period, for $t=T_1$ [see Eq. (19)]; and,
3b. Minimizing the expected annual average cost, taking into account possible gain in service time following the completion of the planning period, for $t=t_0+2T_1$ [see Eq. (19)].

The values of the parameters A, γ, $(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$, and λ_0 employed in the analysis, are within the ranges considered by Shamir and Howard (1979) and Kleiner and Rajani (1999).

Note that for Criteria 1a and 1b, t_0+T_1 is independent of the value of t_0 as discussed in the previous sections. Thus, the optimal time interval between successive replacements is considered to be independent of the already sustained service period t_0. The values of t_0+T_1 for a discount rate γ equal to 0.05 are shown in Fig. 2 for sets of values of the model parameter (A), the time discounted, and occurrence rate modified cost ratio R_c; where $R_c=(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$. The results shown in Fig. 2 suggest that the optimal values of t_0+T_1 are very sensitive to the values of A and R_c. However, a comparison of the results for Criteria 1a and 1b indicates that the value of t_0+T_1 is relatively insensitive to the planning period t, when the minimum expected total cost is employed as the basis for selecting the optimal replacement time.

The influence of the discount rate γ on the optimal value of t_0+T_1 for different cost ratios is shown in Fig. 3 for γ equal to 0.05 and 0.07. Note that in Fig. 3, the ordinate representing R_c/γ (i.e., $m_{C_N}/(m_{C_R}\lambda_0)$) is independent of the discount rate γ. Fig. 3 suggests that the greater the value of γ, the greater is the optimal value of t_0+T_1 (i.e., interval between successive replacements) for a given cost ratio.

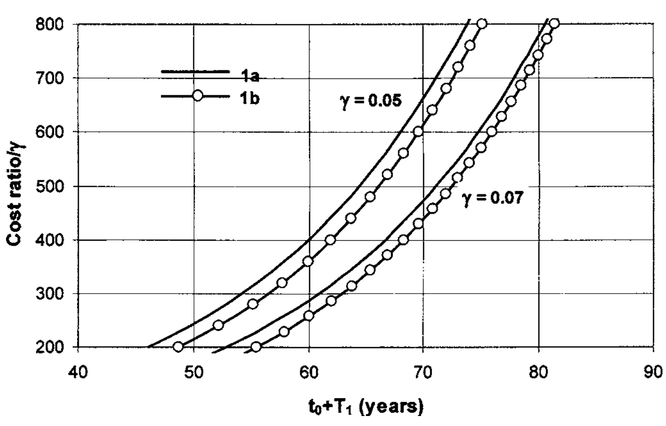

Fig. 3. Influence of discount rate on the optimal replacement time obtained by minimizing the expected total cost

Employing Criteria 2a and 2b, the results for $A=0.08$ and $\gamma=0.05$ were calculated for a set of R_c values, as shown in Fig. 4. It can be seen that the optimal value of T_1 is fairly sensitive to cost ratio, R_c. A comparison of the results depicted in Fig. 4 for Criteria 2a and 2b indicates that the optimal value of T_1 is highly dependent on the manner in which the planning period t is defined. For example, from Fig. 4, it can be observed that for $t_0=40$ years and $R_c=40$, T_1 equals approximately 27 years for Criterion 2a, while it is only about 17 years for Criterion 2b. That is, under the same conditions, the consideration of the service period after the replacement (i.e., Criterion 2b) leads to a significantly earlier replacement time than the one computed by ignoring the cost associated with maintenance repairs after the replacement. The relatively slow decline in the value of T_1, for large values of t_0 in Criterion 2a, reflects the fact that—in formulating this case—the cost of replacement at the end of the planning period is accounted for, while the service period to be provided by the new pipe is not.

Fig. 4 also shows that unlike Criteria 1a and 1b, the values of t_0+T_1 for Criteria 2a and 2b depend on the already sustained service period, t_0. To better appreciate the differences between the results obtained for Criteria 1a and 2a and those obtained for Criteria 1b and 2b, comparative plots are shown in Figs. 5(a and b), respectively. It can be seen that, for the considered cases, the optimal T_1 value obtained by minimizing the expected annual average cost is always greater than the one obtained by minimizing the expected total cost. The sensitivity of the expected total and annual average costs to the replacement time is illustrated in Figs. 6(a and b) for $t_0=40$ years and $R_c=40$. The optimal value of T_1 was found to be equal to 6.1, 6.4, 27, and 17 years for Criteria 1a, 1b, 2a, and 2b, respectively. The expected total costs for Criteria 1a and 1b, shown in Fig. 6(a), are nearly indistinguishable; while the expected annual average costs for Criteria 2a and 2b are clearly different. This could be explained by the fact that calculations using the expected total costs (Criteria 1a and 1b) are insensitive to different planning periods, while calculations using the annual average costs are sensitive to the duration of the planning period.

Generally speaking, the observed differences for the optimal replacement times, obtained by minimizing the expected total cost and by minimizing the expected annual average cost, may be

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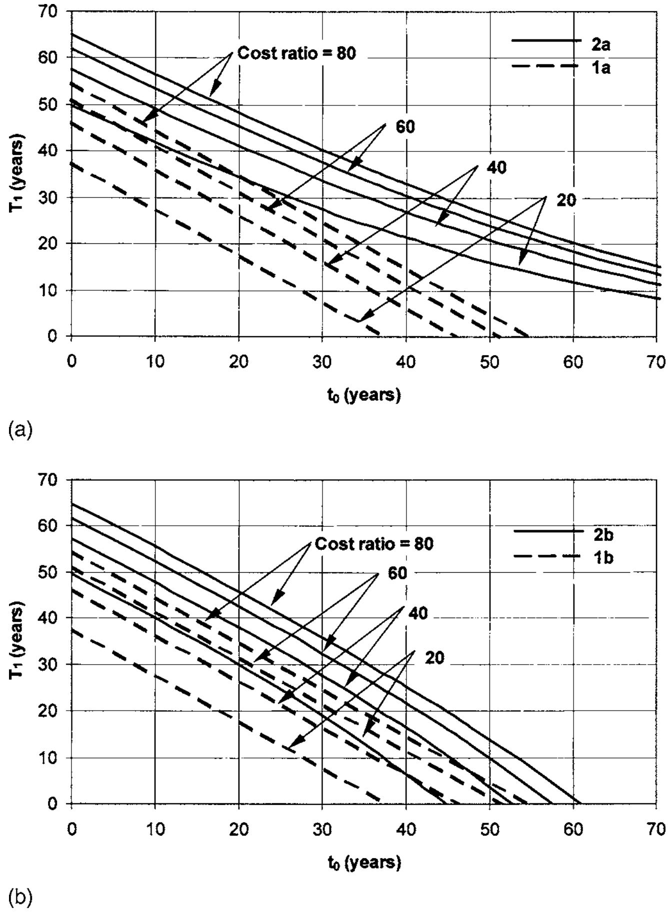

Fig. 5. Comparison of the optimal replacement time T_1 for $A=0.08$ and $\gamma=0.05$: (a) Criteria 1a and 2a (planning period $=T_1$); (b) Criteria 1b and 2b (planning period $=t_0+2T_1$)

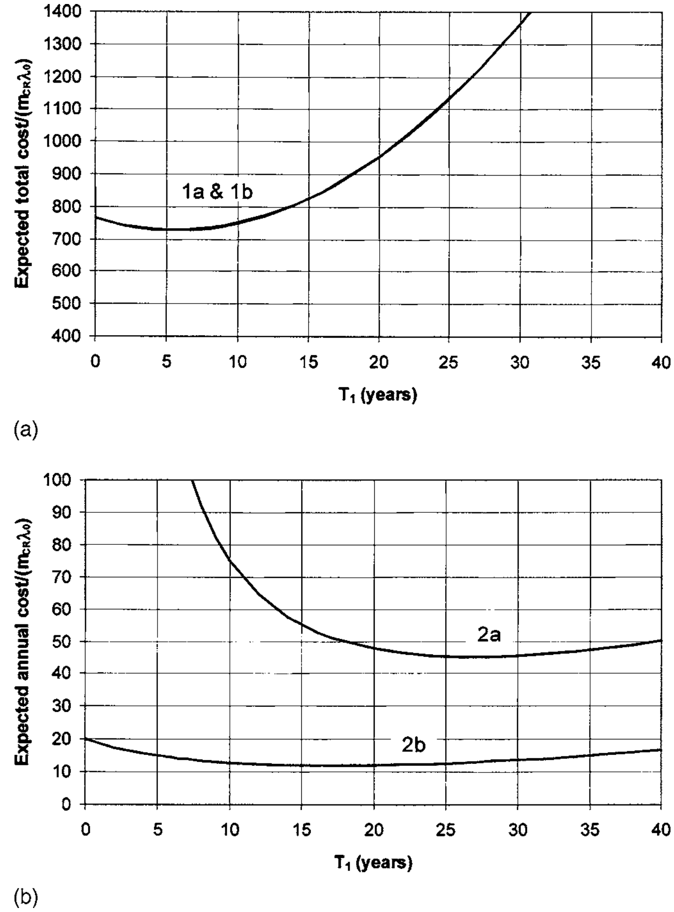

Fig. 6. Expected cost for $A=0.08$, $\gamma=0.05$, $t_0=40$ years and $R_c=40$: (a) expected total cost; (b) expected annual average cost

explained as follows. Using the minimum expected annual average cost as the decision-making criterion spreads the overall cost more evenly over the planning period, and an increase in T_1 will lead to a decrease in the expected annual average cost, provided that the expected total cost remains unchanged. Therefore, this criterion is likely to lead to an optimal value of T_1 that is greater than the one dictated by the minimum expected total cost criterion. Alternatively, one may say that the utilization of the minimum expected total cost as the criterion for decision making tends to shorten the replacement cycle (i.e., reduces the overall cost); leading to an optimal value of T_1 that is smaller than the one dictated by minimizing the expected annual average cost.

By incorporating the possibility of a maintenance-free service duration following the termination of the planning period t, the results obtained for Criteria 3a and 3b are shown in Fig. 7 for $\lambda_0=0.1$. Additional numerical results were computed by varying the value of λ_0 while maintaining the values of A, γ, and $(m_{C_N}\gamma)/(m_{C_R}\lambda_0)$ to be the same as the ones used for the results shown in Fig. 7. Differences among results for different values of λ_0 were found not to be significant, and thus only values for $\lambda_0=0.1$ are presented. Fig. 7 shows that for t_0 equal to zero, the optimal replacement time for Criterion 3a is slightly smaller than that for Criterion 3b. This could be explained by noting that, for $t_0=0$, the planning period t for Criterion 3a is one-half of that for Criterion 3b, and that the duration of the potential maintenance-free service period is independent of the length of the planning period. Therefore, the case with a smaller planning period stands to benefit more from the maintenance-free service period, since the relative reduction in expected annual average cost is more significant. This leads to a slightly greater decrease in the optimal replacement time T_1 for Criterion 3a.

A comparison of Figs. 4 and 7 indicates that, in general, observations drawn from results for Criteria 2a and 2b are also applicable for Criteria 3a and 3b. The optimal values of T_1 for Criterion 3a are only slightly smaller than those obtained for

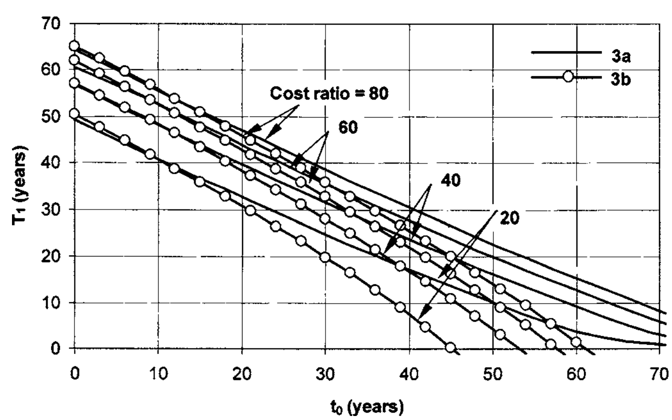

Fig. 7. Optimal replacement time obtained by minimizing the expected annual average cost given in Eq. (19) for $A=0.08$ and $\gamma=0.05$

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Criterion 2a, while the optimal values of T_1 for Criterion 3b are practically identical to those obtained for Criterion 2b. Based on these observations, it is concluded that the optimal T_1 value obtained from Criterion 3b could be adequately approximated by that obtained from Criterion 2b, thus avoiding the evaluation of the integration associated with Eq. (19).

The results presented in this section appear to suggest that, overall, the values of T_1 obtained by minimizing the expected total cost are smaller than those obtained by minimizing the expected annual average cost. The former emphasize the minimization of the overall costs, while the latter are concerned with minimizing the cost per unit service time. In the total cost approach, the optimal time interval between replacements, t_0+T_1, is not influenced by the pipe's current service time, t_0; while in the annual average cost approach, the value of t_0+T_1 is partially affected by t_0.

Perhaps the most significant criticism of using the minimum expected total cost—with a varying planning period—for decision making is that it does not take into account the duration of the service period received. Therefore, it is suggested that one should adopt the optimal replacement time obtained by minimizing the expected annual average cost. Furthermore, since the consideration of the planning period t equal to t_0+2T_1 takes into account the benefit received from the pipeline replacement, and the differences between the results obtained based on Criterion 2b and 3b are insignificant, it is suggested that one should select the optimal replacement time by minimizing the expected annual average cost, as defined in Eq. (21) (i.e., by ignoring the possible maintenance-free service period U).

It must be emphasized that the numerical results presented in this section were obtained by considering that the break occurrence rate can be adequately modeled by Eq. (10). If a different break occurrence rate function is employed, the differences between the optimal T_1 values obtained by minimizing the expected total and annual average costs could vary.

Illustrative Examples

The proposed approach is illustrated via two simple numerical examples. Values for break occurrence rate and cost of repair and replacement were obtained from Kleiner and Rajani (1999). Although it was suggested in the previous section that one should adopt Criterion 2b for selecting the optimal replacement time, results obtained by using Criterion 1a are included for comparison purposes.

The first example is concerned with a 5 km long pipeline segment installed in 1950. Based on historical data, it was found that the parameters for the break occurrence rate A and λ_0 are equal to 0.052 (/year) and 0.025 (/km), respectively. The discount rate γ is assumed to be 0.05. The expected cost of replacement is considered to be equal to \$350/m; the expected cost of repair, including some indirect costs, is assumed to be equal to \$6,000/break. The optimal scheduling for replacement is to be determined at present (in the year 2003).

Based on the above information, t_0 equals 53 years (=2003−1950) and λ_0 for 5 km of pipeline segment is equal to 0.125 (=0.025×5). Substituting these values into Eq. (13) yields

$$T_1 = -t_0 + \frac{1}{A}\ln\left(\frac{m_{C_N}\gamma}{m_{C_R}\lambda_0}\right) = -53 + \frac{1}{0.052}\ln\left(\frac{350\times 5{,}000\times 0.05}{6{,}000\times 0.125}\right) = 38.5 \quad (22)$$

By adopting Criterion 2b and substituting the same input values into Eq. (21), we obtain the following expression

$$E(C_A(T_1)) = \frac{6{,}000\times 0.125}{T_1} \times\left\{\frac{e^{2.756}}{0.002}(e^{0.002T_1}-1) + 2333.3e^{-0.05T_1} + \frac{e^{-0.05T_1}}{0.002}(e^{0.002(53+T_1)}-1)\right\} \quad (23)$$

By minimizing $E(C_A(T_1))$ in this equation, the obtained value of T_1 equals 57.7 (years), which is significantly greater than the one obtained from Eq. (22) for Criterion 1a.

The second example considers a pipeline segment installed in 1930, and a decision at present (in the year 2003) regarding the scheduling of a pipe replacement. The length of the pipeline segment is 3 km; A and λ_0 are 0.18 (/year) and 4×10^{-6} (/km), respectively. The expected cost of replacement equals \$450/m and the expected cost of repair per break equals \$6,000.

Again, by adopting Criterion 1a and using the given information, Eq. (13) yields

$$T_1 = -t_0 + \frac{1}{A}\ln\left(\frac{m_{C_N}\gamma}{m_{C_R}\lambda_0}\right) = -(2003-1930) + \frac{1}{0.18}\ln\left(\frac{450\times 3{,}000\times 0.05}{6{,}000\times 4\times 10^{-6}\times 3}\right) = 3.4 \quad (24)$$

However, if Criterion 2b is considered [Eq. (21)], the set of input data yields

$$E(C_A(T_1)) = 6{,}000\times 4\times 10^{-6}\times 3 \times\frac{1}{T_1}\left\{\frac{e^{13.14}}{0.13}(e^{0.13T_1}-1) + 1.875\times 10^{-7}e^{-0.05T_1} + \frac{e^{-0.05T_1}}{0.13}(e^{0.13(73+T_1)}-1)\right\} \quad (25)$$

By minimizing $E(C_A(T_1))$, as shown in Eq. (25), the obtained optimal replacement time T_1 is equal to 6.1 (years). This example again shows that different criteria lead to significantly different values for the optimal replacement time.

Conclusions

An approach is developed for selecting the optimal scheduling for prioritizing replacement/rehabilitation of pipeline segments. The approach is based on the minimization of the expected annual average cost—during the planning period of the pipeline segment—rather than the minimization of the expected total cost, an approach employed by several previous studies reported in the literature. The former is based on the notion that one is interested in maximizing the service time per dollar spent (i.e., minimizing the annual average cost during the planning period); while the latter focuses on minimizing the expected total cost, but overlooks the fact that optimized replacement times for competing alternatives might be based on different planning periods.

It should be noted that, although the approach presented in this study can cope with different break occurrence rate functions, the use of a break occurrence rate that follows an exponential curve as a function of time is accentuated. By adopting this rate

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function, numerical analyses were carried out by considering the service period and the cost of rehabilitation/replacement. The results indicate that the optimal replacement time, obtained by minimizing the expected annual average cost, could be significantly greater than that obtained by minimizing the expected total cost.

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