Reliability Importance of Components in a Complex System

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SUMMARY AND CONCLUSIONS

Reliability importance indices are valuable in establishing direction and prioritization of actions related to an upgrading effort (reliability improvement) in system design, or suggesting the most efficient way to operate and maintain system status. Existing indices are calculated through analytical approaches, and application of these indices to complex repairable systems may be intractable.

Complex repairable systems are being increasingly seen, and issues related to analytical system reliability and availability solutions are well known. To overcome this intractability, discrete event simulation, through the use of reliability block diagrams (RBD), is often used to obtain numerical system reliability characteristics. Traditional use of simulation results provides no easy way to compute reliability importance indices. To bridge this gap, several new reliability importance indices are proposed and defined in this paper. These indices can be directly calculated from the simulation results and their limiting values are traditional reliability importance indices. Examples are provided to illustrate the application of the proposed importance indices.

1. *INTRODUCTION*

One of the purposes of system reliability analysis is to identify the weakness in a system and to quantify the impact of component failures. The so-called "reliability importance" is used for this purpose. These importance measures provide a numerical rank to determine which components are more important to system reliability improvement or more critical to system failure.

In order to evaluate the importance of different aspects for a system, a set of importance measurements, including Structure Importance, Birnbaum Component Importance, Reliability Criticality Importance, Upgrading Function and Operational Criticality Importance [1][2], have been well defined and widely used in engineering practice. One index may aim at the greatest gain in system reliability improvement efforts, while others may offer the best assistance in system design and optimization or suggest the most efficient way to operate a system or to prevent system failure.

Generally, reliability importance is a function of operation time, of failure and repair characteristics (of all components in the system) and of the system structure. So far, all reliability importance indices are calculated through combinatorial approaches (such as RBD, or Fault Tree Analysis (FTA)), or structure function or Markov Modeling.

With modern technology and higher reliability requirements, systems are getting more complicated. Multi-level redundant, hot swappable, on-line repairable, multistage interconnection (network) designs have been seen increasingly in many practical systems, such as communication systems, computing systems, control systems, and critical power systems. These redundant techniques span a wide spectrum in the design space, and allow a system to achieve high reliability, at the expense of system complexity. In addition to this complexity, most of these "real world" systems are also repairable. In such practical applications, where one is dealing with large complex systems composed of many components that fail and get repaired based on different distributions and with additional "real world" constraints (such as spare parts availability, repair crew response time, etc.), exact analytical solutions become intractable. Thus, to solve such systems, one needs to resort to simulation (more specifically, discrete event simulation) to obtain metrics and results of interest [3][4]. Traditionally, such simulation results or metrics do not allow for the identification of the previously mentioned importance measures. To address this, new reliability importance indices (directly obtained from simulation) are introduced and presented in this paper.

2. *FAILURE CRITICALITY INDEX*

In general, importance ranks the components in a system using a numerical rank (relative importance), based on certain system characteristic of interest, such as the component's contribution to a system (failure) event occurrence. Birnbaum first introduced the concept of importance in 1969 [5], and one of the most widely used reliability importance indices is Birnbaum's component importance [2]. Analytically this is defined by

$$I_k^B(t) = \frac{\partial R_S(t)}{\partial R_k(t)} = \frac{\partial F_S(t)}{\partial F_k(t)}, \tag{1}$$

where

I_k^B is reliability importance of the k^{th} component,

$R_S(t)$ and $F_S(t)$ are the system reliability and unreliability at time t, respectively

and

$R_k(t)$ and $F_k(t)$ are the reliability and unreliability of component k at time t.

As an example, and by using a very simple non-repairable system, as shown in Fig. 1, the system reliability for this configuration can be expressed as

$$R_S = 1 - [(1 - R_1 R_2) \cdot (1 - R_3)],$$

or

$$R_S = R_1 R_2 + R_3 - R_1 R_2 R_3 .$$

Hence, according to Eq. (1), the Birnbaum importance of each component is

$$I_1^B(t) = \frac{\partial R_S(t)}{\partial R_1(t)} = R_2(t) - R_2(t) \cdot R_3(t) ,$$

$$I_2^B(t) = \frac{\partial R_S(t)}{\partial R_2(t)} = R_1(t) - R_1(t) \cdot R_3(t) ,$$

and

$$I_3^B(t) = \frac{\partial R_S(t)}{\partial R_3(t)} = 1 - R_1(t) \cdot R_2(t) .$$

The Birnbaum importance measure (metric) of a component is independent of the reliability of the component itself. By the definition, I_k^B is the rate of increase (at time t) of the system reliability with respect to the component's reliability increase. It also measures the probability of a component being responsible for system failure at time t.

Whereas the Birnbaum importance provided the probability that a given component would be responsible for the failure at time t, another measure, "component criticality importance" can be used to determine the probability that the given component was responsible for system failure before time t. This measure is given by

$$\begin{aligned} I_k^C(t) &= \frac{\partial R_S(t)}{\partial R_k(t)} \cdot \frac{F_k(t)}{F_S(t)}, \\ &= I_k^B(t) \cdot \frac{R_k(t)}{R_S(t)}. \end{aligned} \tag{2}$$

The following alternative definition has also been used:

$$\begin{aligned} I_k^C(t) &= \frac{\partial R_S(t)}{\partial R_k(t)} \cdot \frac{R_k(t)}{R_S(t)}, \\ &= I_k^B(t) \cdot \frac{R_k(t)}{R_S(t)}. \end{aligned}$$

The component criticality importance for the non-repairable system in Fig. 1, using the definition of Eq. (2), can be expressed as, respectively,

$$\begin{aligned} I_1^C(t) &= [R_2(t) - R_2(t) \cdot R_3(t)] \\ &\quad \times \frac{1 - R_1(t)}{1 - R_1(t)R_2(t) - R_3(t) + R_1(t)R_3(t)} , \end{aligned}$$

$$\begin{aligned} I_2^C(t) &= [R_1(t) - R_1(t) \cdot R_3(t)] \\ &\quad \times \frac{1 - R_2(t)}{1 - R_1(t)R_2(t) - R_3(t) + R_1(t)R_3(t)} , \end{aligned}$$

and

$$\begin{aligned} I_3^C(t) &= [1 - R_1(t) \cdot R_2(t)] \\ &\quad \times \frac{1 - R_3(t)}{1 - R_1(t)R_2(t) - R_3(t) + R_1(t)R_3(t)} . \end{aligned}$$

Additional reliability importance metrics exist and are similarly defined. Their end result is a rate of increase of the system reliability with respect to component reliability. In computing such a metric, an analytical relationship between system reliability and component reliability is required. That analytic relationship between a system and its component, albeit not trivial, can be generally obtained for complex non-repairable systems. However, when one is dealing with a repairable system, system reliability (and system availability) cannot be expressed in terms of component reliability. System reliability in this case depends on more than just the failure characteristics of the components. It is dependent on multiple component and system characteristics, such as component time-to-failure distributions and time-to-restore distributions, system behavior, maintenance practice, spare availabilities, etc. Given these contributory factors, no analytical expression (or even a governing equation) for complex repairable system reliability has been proposed or established to this date. Thus, the most widely used method to numerically obtain reliability values for a complex repairable system is still through simulation.

Such simulation methods provide an "exact" solution to system reliability and are fairly straightforward to implement once the system model (e.g., RBD) has been constructed. Simulation is a series of numerical experiments on the system model or RBD. An "actual" realization of states is simulated on each component. During the course of the simulation, component events (working or failing) are made to occur at times determined by random processes obeying failure or repair time distributions of the component. A system (state) realization is then composed according to all component realizations and the system design. After having observed many realizations ("histories") of the system, estimates are made of the desired reliability indices statistically. Fig. 2 shows an example of one trial of two repairable blocks in parallel.

The simulation outcomes are the system and component event histories over the time of the simulation. With such system simulation results available, a new index can now be defined. We will call this index the "failure criticality index" or FCI. For a given component k, this index is given by

$$I_k^{FCI}(t) = \frac{\text{Number of system failures caused by comp } k \text{ in } (0,t)}{\text{Number of system failures in } (0,t)} . \tag{3}$$

This index gives the percentage of times that a system failure event was caused (triggered) by a failure of this component over the simulation time (0, t). Intuitively, this index would have the same meaning, and therefore the same use, as the traditional component criticality importance, $I_k^C(t)$.

An alternative definition for FCI could be

$$I_k^{FCI}(t) = \frac{\text{Number of system failures caused by comp } k \text{ in } (0,t)}{\text{Number of this component failures in } (0,t)}, \quad (4)$$

which gives the percentage of times that a failure of this component will cause a system failure in (0, *t*).

To illustrate this, again consider the simple example system in Fig. 1 with the assumption that the system is repairable. Specifically, assume that all three components are on-line repairable and with the following failure/repair distributions.

Component	Time-to-failure distribution	Time-to-repair distribution
1	Exponential $MTTF = 1{,}000$ hrs	Normal $\mu = 24, \sigma = 4$ hrs
2	Weibull $\beta = 2.5, \eta = 1{,}500$ hrs	Exponential $MTTR = 10$ hrs
3	Weibull $\beta = 1.5, \eta = 800$ hrs	Normal $\mu = 12, \sigma = 2$ hrs

Using ReliaSoft's BlockSim 6 software for the analysis, the FCI for the components (as defined in Eq. (3)) was obtained and plotted. This is shown in Fig. 3. From this figure it can be clearly seen that component 3 has the highest importance.

The FCI metric can also be used to pinpoint weaknesses of the system. This analysis and metric were utilized for a distributed control system (DCS) designed for a power generation plant by the General Electric Company. The system is fairly complicated, and repairable as well. During the early design stages, the system's initial reliability was analyzed using BlockSim 6. The RBD of this complex system is shown in Fig. 4. During that initial analysis it was found that system *MTBF* was lower than expected, even though it had already met the customer's requirement. Given the complexity of this system, and to better understand component criticality, a component FCI index was computed for each item. Fig. 5 shows this index for the top twenty components. It can be easily seen that six devices were of similar importance level and that these six devices had a considerably higher FCI than the rest of the devices in the system. Based on these findings, the system was then re-designed, with a focus on these six items, while still using the same devices and the same maintenance plans. The revised system was then reanalyzed. The analysis of the revised design showed an FCI that was nearly uniformly distributed across all devices. This in turn resulted in a much higher system *MTBF* than the original version.

3. *RESTORE CRITICALITY INDEX*

The restore criticality index (RCI) for a component is defined as the percentage of times that system restoration results from the restoration of this component; that is,

$$I_k^{RCI}(t) = \frac{\text{\# of actions on comp } k \text{ that restored the system in } (0,t)}{\text{\# of times the system was restored in } (0,t)}, \quad (5)$$

or it may also be defined by

$$I_k^{RCI}(t) = \frac{\text{\# of actions on comp } k \text{ that restored the system in } (0,t)}{\text{\# of time component } k \text{ was restored in } (0,t)}. \quad (6)$$

This index is the percentage of times that a restoration of the component will result in restoring the system from a down state in (0, *t*).

4. *OPERATIONAL CRITICALITY INDEX*

The operational criticality index (OCI) can be defined as the percentage of a component's down time over the system down time; that is,

$$I_k^{OCI}(t) = \frac{\text{Total down time of comp } k \text{ when system down in } (0,t)}{\text{Total system down time in } (0,t)}, \quad (7)$$

or the percentage of a component's up time over the system up time; that is,

$$I_k^{OCI}(t) = \frac{\text{Total up time of comp } k \text{ when system up in } (0,t)}{\text{Total system up time in } (0,t)}. \quad (8)$$

Other indices can also be defined similarly in this category.

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BIOGRAPHY

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Wendai Wang works in the area of Electronic and Photonics Systems at the General Electric Global Research Center. He received his B.S. and M.S. in Mechanical Engineering from Shanghai Jiaotong University, China, and his Ph.D. in Reliability Engineering from University of Arizona. He has more than 15 years teaching and working experience in Reliability and Maintainability Engineering. His research interests include reliability prediction, reliability life testing, ALT/ADT/HALT/HAST, reliability data analysis, system reliability and availability, physics-of-failure, and probabilistic design.

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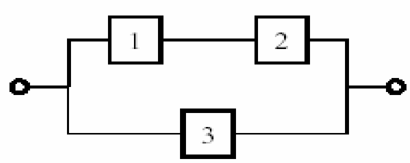

Fig. 1 – A Simple System

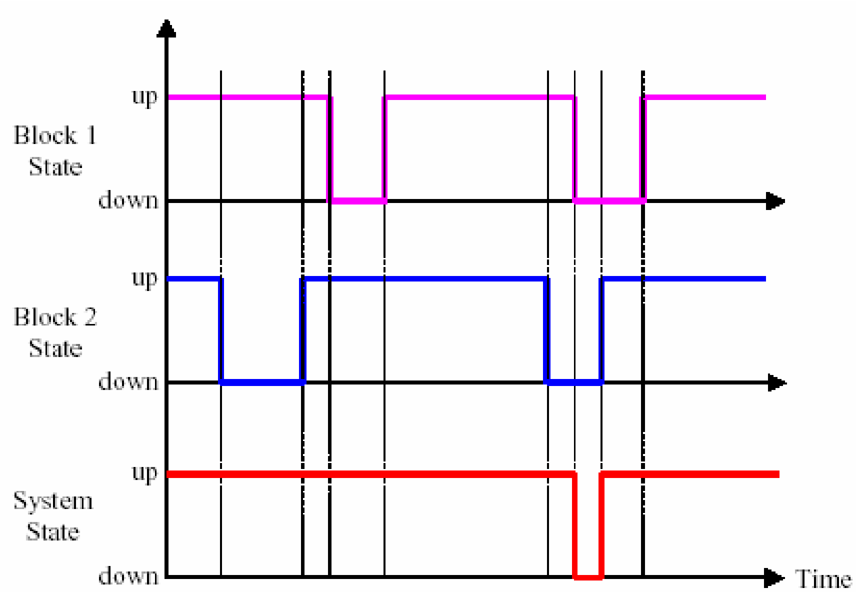

Fig. 2 – A simulated "history" of a two-unit parallel system

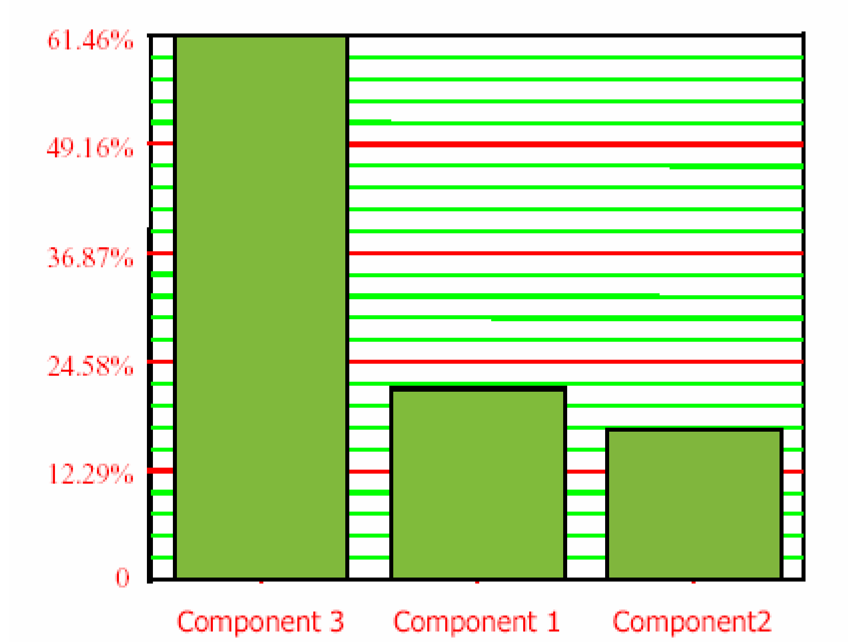

Fig. 3 – Failure Criticality Indices for components in a repairable system

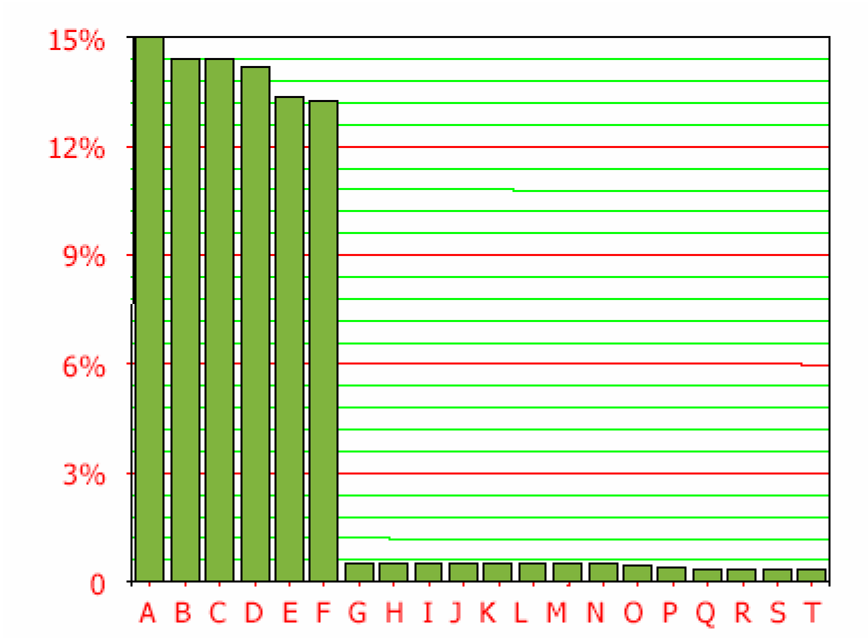

Fig. 5 – Partial list of FCI for components in a distributed control system

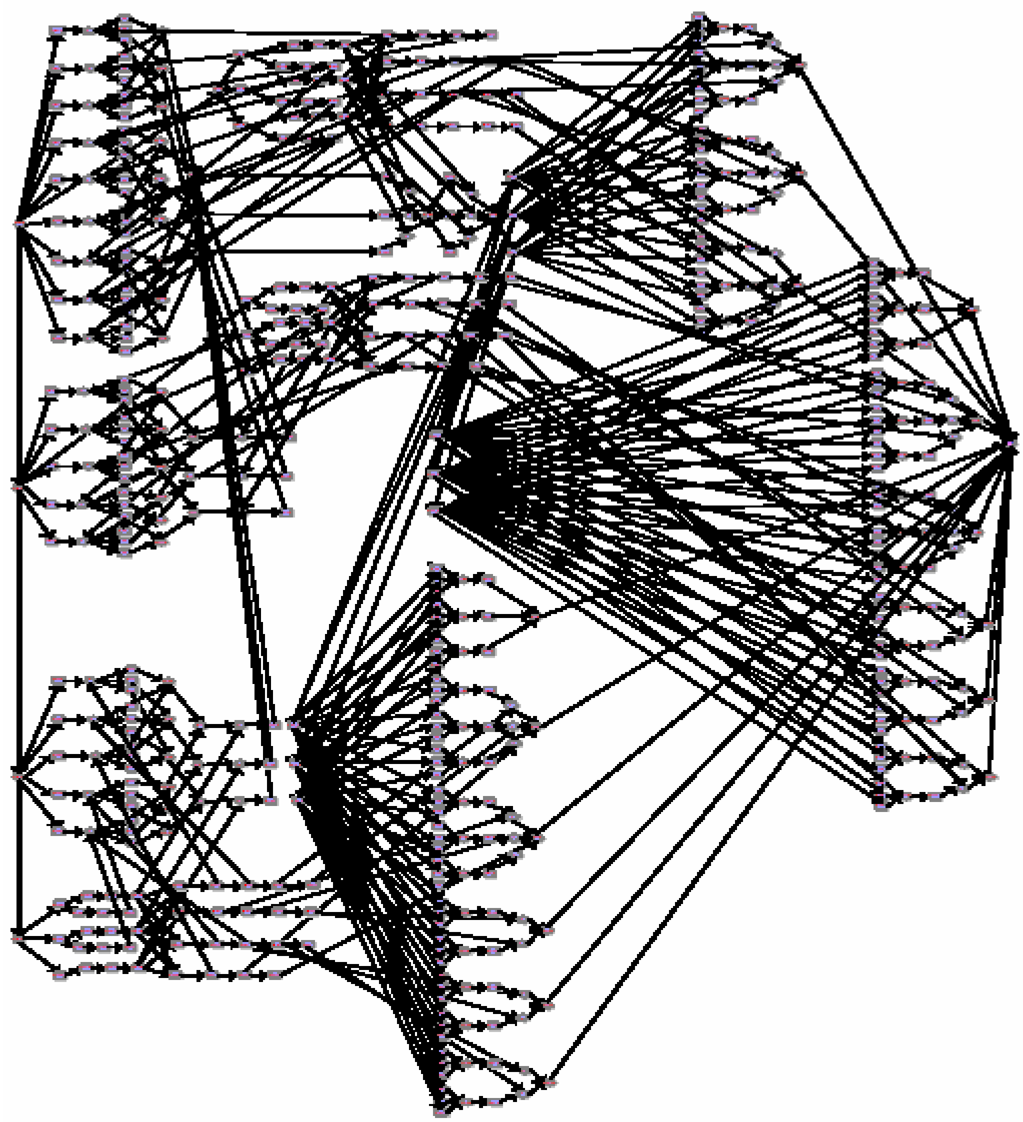

Fig. 4 – Reliability block diagram for the initial design of a power generation DCS