Pavement Resurfacing Planning for Highway Networks: Parametric Policy Iteration Approach

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Abstract: This paper presents a modeling framework for planning pavement resurfacing activities on highway networks in the case of continuous pavement state, discrete time, and infinite horizon. Optimal resurfacing policies that minimize discounted life-cycle costs are obtained by solving a multidimensional dynamic program, where travelers' route choices and the agency's resource allocation decisions are considered simultaneously. To reduce computational difficulty, policy iteration is used together with a parametric function approximation technique. Numerical examples show that the proposed approach solves the planning problem efficiently. The effect of various factors (travelers' route choice, agency budget, etc.) on the optimal policies is also analyzed.

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Introduction

This paper addresses the problem of planning pavement resurfacing activities for a highway network where traffic flows are responsive to pavement conditions. The objective is to minimize the discounted life-cycle costs, including agency and user costs, at the network level over an infinite planning horizon.

Discrete-time rehabilitation planning problems are often solved with Markov decision process (MDP) formulations that were originally proposed in Bellman (1955), Dreyfus (1960), and Klein (1962). Most studies on pavement rehabilitation focus on individual pavements (Golabi et al. 1982; Carnahan et al. 1987), and some analytical properties of optimal solutions, such as the threshold structure of optimal policies, are known (Ouyang and Madanat 2006). Other researchers have approached the problem for a system of pavement facilities by formulating mixed-integer mathematical programs in discrete time (Al-Subhi et al. 1990; Jacobs 1992; Ouyang and Madanat 2004). In all these studies, the traffic flow of each facility is assumed to be an exogenous factor that influences pavement deterioration.

A few studies have explored the effects of responsive traffic demand on highway facilities. Friesz and Fernandez (1979) proposed an optimal control model for one pavement facility where traffic flow varies with pavement condition. Network-level route choices, etc., are not considered. Intuitively, however, highway pavement conditions directly influence vehicle operating costs and driver comfort, and in turn affect drivers' route choices. Hawas (2004) collected empirical data to confirm that pavement condition is indeed one of several significant factors that influence traveler route choices. Therefore, it is reasonable to assume that on highway networks, not only does the deterioration of pavement facilities depend on the distribution of traffic flows, but also the traffic flows are influenced by pavement conditions. A recent study by Uchida and Kagaya (2006) considered this bidirectional effect, with formulation and solution techniques developed in a user equilibrium assignment framework.

The paper aims to address the network-level pavement management problem in a multidimensional dynamic programming framework. Each pavement facility has a continuous state, and the traffic assignment is responsive to the states of the facilities. Due to the curse of dimensionality, computational difficulty tends to increase dramatically for large networks, especially when pavements have continuous states. We propose an approximation method to overcome this challenge and efficiently solve the posed problem. The paper is organized as follows. The Network Problem section introduces the problem formulation and pavement performance models. The Solution Procedure section discusses suitable solution procedures. This is followed by numerical examples in the Numerical Example section, and conclusions in the Conclusion and Future Research section.

Network Problem

General Formulation

Consider n interconnected pavement facilities distributed as links on a highway network, each of which has approximately homogeneous physical (e.g., deterioration) properties. We are concerned about the net present value of life-cycle costs including agency investment and user costs.

Suppose all system parameters (e.g., cost coefficients) are constant in an infinite horizon $(0,\infty)$. The traffic demand over the entire highway network is represented by a set of origin/destination (O/D) flows, $\mathbf{D}(t), \forall t \in (0,\infty)$, which is assumed to be time varying but independent of the condition of the highway network. Each traveler perceives, and makes route choices based on, a user cost that depends on traffic times as well as pavement conditions. The traffic flows on highway links,

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$\mathbf{v}(t) := [v_1(t), v_2(t), \ldots, v_n(t)]$, is therefore influenced by the pavement condition (i.e., roughness), $\mathbf{s}(t) := [s_1(t), s_2(t), \ldots, s_n(t)]$. A larger value of $s_i(t)$ indicates a worse condition.

Pavement facilities deteriorate continuously through their service to traffic flows; the larger the traffic flow, the faster the deterioration. At discrete time points (i.e., beginning of fiscal years), $\tau = 1, 2, \ldots$, the highway agency selects non-negative intensities (e.g., thickness of overlay), $\mathbf{w}(\tau) := [w_1(\tau), w_2(\tau), \ldots, w_n(\tau)]$, for resurfacing activities on these facilities. If $w_i(\tau) = 0$, no resurfacing is conducted on facility i at time τ. The pavement condition on facility i improves instantaneously at τ if $w_i(\tau) > 0$.

In mathematical notations, the infinite horizon problem can be formulated as

$$\min \sum_{\tau=0}^{\infty} \sum_{i=0}^{n} \left\{ \int_{\tau}^{\tau+1} C_i[s_i(t), \mathbf{v}(t)] e^{-rt} dt + M_i[w_i(\tau)] e^{-r\tau} \right\} \quad (1a)$$

$$\text{Subject to } \frac{\mathrm{d}s_i(t)}{\mathrm{d}t} = F_i[s_i(t), v_i(t)], \quad t \in (\tau, \tau+1), \ \forall\, i, \tau \quad (1b)$$

$$s_i(\tau^-) - s_i(\tau^+) = G[w_i(\tau), s_i(\tau^-)], \quad \forall\, i, \tau \quad (1c)$$

$$\mathbf{v}(t) = H[\mathbf{s}(t), \mathbf{D}(t)], \quad \forall\, t \quad (1d)$$

$$\sum_{i=0}^{n} M_i[w_i(\tau)] \le B, \quad \forall\, \tau \quad (1e)$$

$$0 \le w_i(\tau) \le R_i[\mathbf{s}(\tau)], \quad \forall\, i, \tau \quad (1f)$$

In the objective function (1a), $C_i(s_i, \mathbf{v})$ = user cost per unit time as a function of roughness s_i and traffic flow $\mathbf{v}$; $M_i(w_i)$ = agency cost for a resurfacing of intensity w_i on facility i; and parameter r = discount rate. Constraint (1b) describes how each pavement deteriorates over time and how traffic flow partly contributes to the process. Constraint (1c) defines the effectiveness of resurfacing activities in reducing roughness. The amount of reduction $s_i(\tau^-) - s_i(\tau^+)$ depends on w_i and $s_i(\tau^-)$, where $s_i(\tau^-)$ and $s_i(\tau^+)$ denote the roughness before and after resurfacing. Constraint (1d) determines time-varying traffic flow distribution over the network given O/D demand $\mathbf{D}$ and road condition $\mathbf{s}$. Constraint (1e) poses a budget constraint on agency expenditures and constraint (1f), realistic feasibility bounds on the resurfacing intensity.

Performance Models

In this section, we discuss the specifications of the cost functions and pavement performance models, $C_i(s_i, \mathbf{v})$, $H(\mathbf{s}, \mathbf{D})$, $M_i(w_i)$, $F_i(s_i, v_i)$, and $G(w_i, s_i)$.

User costs consist of two major components: vehicle operating costs and travel time costs. Travel time, d_i, is a function of traffic flow on link i and relatively irresponsive to pavement roughness condition. A practical formula developed by the U.S. Bureau of Public Roads (BPR) is in the following form:

$$d_i[\mathbf{v}(t)] = d_{i0}\{1 + \alpha_i [v_i(t)]^{\beta}\}, \quad \forall\, i, t$$

where d_{i0} = free flow travel time, and parameter α_i decreases with the traffic flow capacity of facility i. For simplicity, we assume that the vehicle operating costs are approximately proportional to roughness (Li and Madanat 2002; Ouyang and Madanat 2004, 2006), such that the user cost per unit time per vehicle is in the form of $c[s_i(t), \mathbf{v}(t)] = c_0 + c_1 \cdot s_i(t) + c_2 \cdot d_i[\mathbf{v}(t)]$, where constants $c_0 \ge 0$, $c_1 \ge 0$, and $c_2 \ge 0$. The total user cost per unit time for all travelers on facility i is thus

$$C_i[s_i(t), \mathbf{v}(t)] = \{c_0 + c_1 \cdot s_i(t) + c_2 \cdot d_i[\mathbf{v}(t)]\} \cdot v_i(t), \quad \forall\, t \quad (2)$$

Traffic flows on the highway network depend on pavement conditions as well as traffic conditions. We assume that all travelers are well informed and make independent route choices based on the generalized link travel cost function $c[s_i(t), \mathbf{v}(t)]$, $\forall i$. Given $\mathbf{s}(t)$ and O/D demand $\mathbf{D}$(t), the traffic assignment function H yields link flows $\mathbf{v}(t)$. Without losing generality, we assume that function H follows deterministic user equilibrium. Extensive literature exists on this topic and various algorithms have been proposed; see Sheffi (1992) for a review.

The agency costs for pavement resurfacing consist of a fixed setup component (e.g., for machine rental, operation, and labor), and a variable component that is proportional to the intensity (e.g., material expenditures). Therefore, the undiscounted agency cost for a resurfacing of intensity w_i may be represented by the following function:

$$M_i(w_i) = \begin{cases} m_{i0} + m_{i1} \cdot w_i, & \text{if } w_i > 0 \\ 0, & \text{otherwise} \end{cases} \quad \forall\, i \quad (3)$$

where $m_{i0} \ge 0$, $m_{i1} \ge 0$ are facility-specific parameters.

Based on the empirical data reported in Paterson (1990), the following formula is proposed in Ouyang and Madanat (2004) for resurfacing effectiveness:

$$G[w_i, s_i(\tau^-)] = g_1 s_i(\tau^-) \cdot \min\left\{1, \frac{w_i}{g_2 s_i(\tau^-) + g_3}\right\}, \quad \forall\, i, \quad (4)$$

where $g_1 = 0.66$, $g_2 = 0.55$, and $g_3 = 18.3$. In Eq. (4), $G(w, s)$ and s are in quarter-car index (QI) and w_i in millimeter (mm). It is obvious from the truncation that any resurfacing is suboptimal if $w_i > g_2 s_i(\tau^-) + g_3$. Thus, we rewrite Eq. (4) into

$$G[w_i, s_i(\tau^-)] = g_1 s_i(\tau^-) \cdot \frac{w_i}{g_2 s_i(\tau^-) + g_3} = \frac{g_1 w_i}{g_2 + g_3 / s_i(\tau^-)}, \quad \forall\, i \quad (5)$$

and

$$0 \le w_i \le R_i[\mathbf{s}(\tau)] := g_2 s_i(\tau^-) + g_3 \quad (6)$$

In the rest of the paper, we will keep Eq. (6) as a separate constraint and let $G(w, s)$ refer to the expression in Eq. (5).

Pavement roughness deteriorates continuously between resurfacing. The process is often modeled as a simple exponential function of time (see Tsunokawa and Schofer 1994; Li and Madanat 2002; Ouyang and Madanat 2004, 2006). When traffic flows vary over time, we generalize the deterioration process by adding a linear traffic flow component into the deterioration rate, such that for $\forall i$, t

$$F_i[s_i(t), v_i(t)] = s_i(t) \cdot [b_{i0} + b_{i1} v_i(t)] \quad (7)$$

The term $b_{i0} + b_{i1} v_i(t) > 0$ (time^{-1}) is facility specific and shall increase with link traffic flow $v_i(t)$. From Eqs. (1b) and (7) we have

$$s_i(t) = s_i(\tau^+) \cdot \exp\left\{ \int_{\tau}^{t} [b_{i0} + b_{i1} v_i(t)] \mathrm{d}t \right\}, \quad t \in (\tau, \tau+1), \ \forall\, i, \tau$$

Fig. 1 illustrates some of these definitions.

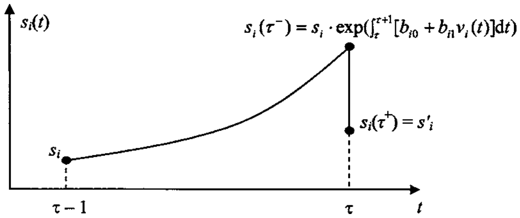

Fig. 1. Deterioration and resurfacing of a pavement facility

Solution Procedure

Problem formulation (1*a*)–(1*f*) is a deterministic infinite-horizon dynamic program with multidimensional continuous state variables $\mathbf{s}$ and $\mathbf{v}$, while $\mathbf{v}$ is directly determined by $\mathbf{s}$ from Eq. (1*d*). We now write the multidimensional control variable $\mathbf{w}$ as a policy $\mathbf{w}(\mathbf{s})$, and let $J(\mathbf{s})$ be the value function in an infinite horizon. The optimal policy can be found by solving the deterministic Bellman equation

$$J(\mathbf{s}) = \min_{\mathbf{w}(\mathbf{s})}\left\{\sum_{i=0}^{n}\left(\int_0^1 C_i[s_i(t),\mathbf{v}(t)]e^{-rt}dt + M_i(w_i)e^{-r}\right) + e^{-r}\cdot J(\mathbf{s}')\right\}, \quad \forall\, \mathbf{s} \tag{8a}$$

where

$$\mathbf{v}(t) = H[\mathbf{s}(t),\mathbf{D}(t)], \quad \forall\, t \in (0,1) \tag{8b}$$

$$s_i' = s_i \cdot \exp\left(\int_0^1 [b_{i0} + b_{i1}v_i(t)]\mathrm{d}t\right) - G\left\{w_i, s_i \cdot \exp\left(\int_0^1 [b_{i0} + b_{i1}v_i(t)]\mathrm{d}t\right)\right\}, \quad \forall\, i \tag{8c}$$

$$\sum_{i=0}^{n} M_i(w_i) \le B \tag{8d}$$

$$0 \le w_i \le g_2 s_i \cdot \exp\left\{\int_0^1 [b_{i0} + b_{i1}v_i(t)]\mathrm{d}t\right\} + g_3, \quad \forall\, i \tag{8e}$$

Note that we only need to focus on $t \in [0,1]$ from now on. The optimal policy $\mathbf{w}(\mathbf{s})$ and value function $J(\mathbf{s})$ should satisfy the following equation:

$$J(\mathbf{s}) = \sum_{i=0}^{n}\left\{\int_0^1 C_i[s_i(t),\mathbf{v}(t)]e^{-rt}dt + M_i(w_i)e^{-r}\right\} + e^{-r}\cdot J(\mathbf{s}'), \tag{9}$$

subject to Eqs. (8*b*)–(8*e*).

For problems with finite states, Eq. (9) can be solved numerically via either policy iteration or value iteration. For our case, both state and control variables are continuous. There is considerable difficulty regarding efficient algorithms to solve this type of high-dimensional dynamic program. A straightforward method is to discretize the continuous state space into a very large number of grid points and then solve the resulting finite-state dynamic program. This method obviously involves a lot of variables and is computationally expensive.

Another method involves a parametric approximation (Benitez-Silva et al. 2000; Howitt et al. 2002). The value function in the state space is approximated by a smooth function with only a few unknown parameters. The parameters should be chosen such that the resulting approximate function "best fits" the true function according to some metric (e.g., sum of squared errors). Benitez-Silva et al. (2000) argues that in many cases, one can obtain a good overall approximation using only a small number of parameters. With this approximation, the Bellman equation can be solved relatively easily.

Intuitively, this approximation approach is suitable for the pavement management problem because the expected loss function J should be a smooth function of its argument, $\mathbf{s}$. The algorithm we propose in this paper is a combination of parametric approximation and policy iterations, which is a slight modification of the parametric policy iteration (PPI) algorithm in Benitez-Silva et al. (2000) and Howitt et al. (2002). The proposed PPI algorithm for our pavement management problem involves the following major steps.

- Parametric approximation: We approximate the value function $J(\mathbf{s})$ with a finite-linear combination of basis functions, $\rho_k(\mathbf{s})$, $k=1,\ldots,K$, (e.g., a complete set of ordinary or Chebychev polynomials in each s_i); i.e.

$$J(\mathbf{s}) \approx [\rho_1(\mathbf{s}),\rho_2(\mathbf{s}),\ldots,\rho_K(\mathbf{s})]\cdot\boldsymbol{\theta} = \sum_{k=1}^{K}\theta_k\rho_k(\mathbf{s}) \tag{10}$$

  where, $\boldsymbol{\theta}=[\theta_1,\theta_2,\ldots,\theta_K]^{\mathrm{T}}$ is a column vector of real parameters.
- State-space sampling: To estimate $\boldsymbol{\theta}$, we take a finite sample (e.g., N grid points) of states from the state space, $\mathbf{s}^1,\ldots,\mathbf{s}^N$, where $N > K$. The number N does not have to be very large. Each sample state, $\mathbf{s}^i$, represents a possible initial pavement condition at time $t=0$.
- Initialization: Specify an initial parameter vector $\boldsymbol{\theta}$ and initial policies $\mathbf{w}(\mathbf{s}^i)=\mathbf{w}_0$ for all i.
- Parameter update: With the parametric approximation, Eq. (9) is transformed into the following:

$$\sum_{k=1}^{K}\theta_k\rho_k(\mathbf{s}) = \sum_{i=0}^{n}\left\{\int_0^1 C_i[s_i(t),\mathbf{v}(t)]e^{-rt}dt + M_i[w_i(\mathbf{s})]e^{-r}\right\} + e^{-r}\cdot\sum_{k=1}^{K}\theta_k\rho_k(\mathbf{s}')$$

  and thus

$$\sum_{i=0}^{n}\left\{\int_0^1 C_i[s_i(t),\mathbf{v}(t)]e^{-rt}dt + M_i[w_i(\mathbf{s})]e^{-r}\right\} = \sum_{k=1}^{K}[\rho_k(\mathbf{s}) - e^{-r}\cdot\rho_k(\mathbf{s}')]\cdot\theta_k. \tag{11}$$

  Within each iteration, we numerically calculate from Eqs. (8*b*) and (8*c*) the deterioration of pavement conditions over time, $\mathbf{s}(t)$, and the corresponding traffic flow distribution, $\mathbf{v}(t)=H[\mathbf{s}(t),\mathbf{D}(t)], \forall\, t \in (0,1)$, for each sample point [i.e., $\mathbf{s}(0)=\mathbf{s}^i$, $i=1,2,\ldots,N$]. This calculation also yields the final pavement conditions $\mathbf{s}'$ at time $t=1^+$, and the following variables for each sample point:

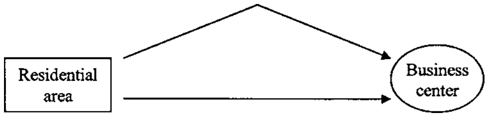

Fig. 2. Simple highway network with $n=2$ competing routes

$$y := \sum_{i=0}^{n} \left\{ \int_0^1 C_i[s_i(t), \mathbf{v}(t)] e^{-rt} dt + M_i[w_i(\mathbf{s})] e^{-r} \right\},$$

$$x_k := \rho_k(\mathbf{s}) - e^{-r} \cdot \rho_k(\mathbf{s}'), \quad \forall\, k$$

Regressing y on $\mathbf{x} := [x_1, x_2, \ldots, x_K]$ over the N observations gives a new estimation of the parameter vector $\boldsymbol{\theta}^{\text{new}}$. We update $\boldsymbol{\theta}$ by $\boldsymbol{\theta} \leftarrow (1-\alpha)\boldsymbol{\theta} + \alpha\boldsymbol{\theta}^{\text{new}}$, where $0 < \alpha < 1$.

Normally, small α improves asymptotic convergence, but too small α would slow down convergence speed. An alternative choice is to decrease α gradually, i.e., use $\alpha = \alpha_m$ in iteration m, such that $\alpha_m \to 0$ and $\Sigma\alpha_m \to \infty$.

- Policy update: Within each iteration, we also update the optimal resurfacing intensity at each sample point. The updated parameter vector $\boldsymbol{\theta}$ is used to obtain function $J(\mathbf{s}')$ on the right-hand side of Eq. (8*a*), which becomes the following minimization problem:

$$\min_{\mathbf{w}} \sum_{i=0}^{n} \left\{ \int_0^1 C_i[s_i(t), \mathbf{v}(t)] e^{-rt} dt \right\} + e^{-r} \left[\sum_{i=0}^{n} M_i(w_i) + J(\mathbf{s}') \right]$$

Note that $\mathbf{s}'$ is defined by Eqs. (8*b*) and (8*c*), and $\mathbf{w}$ is constrained within a convex polytope $\boldsymbol{W}(\mathbf{s})$ defined by Eqs. (8*d*) and (8*e*). Given $\mathbf{s}$, the first summation term is independent of $\mathbf{w}$; thus, the optimization simplifies to

$$\min_{\mathbf{w} \in W(\mathbf{s})} z(\mathbf{w}) = \left\{ \sum_{i=0}^{n} M_i(w_i) + J(\mathbf{s}') \right\} \tag{12}$$

At the interior of $\boldsymbol{W}(\mathbf{s})$, we find, by substituting Eq. (8*c*) into Eq. (12), that the gradients of z,

$$\begin{aligned} \frac{\partial z}{\partial w_i} &= m_{i1} + \sum_j \frac{\partial J}{\partial \mathbf{s}_j'} \frac{\partial \mathbf{s}_j'}{\partial w_i} \\ &= m_{i1} + \frac{\partial J}{\partial \mathbf{s}_i'} \frac{\partial \mathbf{s}_i'}{\partial w_i} \\ &= m_{i1} - \frac{\partial J}{\partial \mathbf{s}_i'} \\ &\quad \times \left(\frac{g_1}{g_2 + g_3 \Big/ \left\{ s_i \exp\left(\int_0^1 [b_{i0} + b_{i1} v_i(t)] \mathrm{d}t \right) \right\}} \right), \quad \forall\, i \end{aligned} \tag{13}$$

is independent of $\mathbf{w}$. Thus, an optimal solution always exists at an extreme point of $\boldsymbol{W}(\mathbf{s})$. Thus, the complexity of the policy updating step is similar to a simpler mixed-integer linear program.

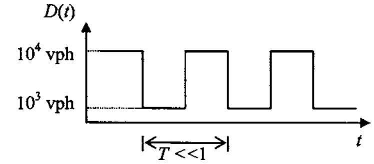

Fig. 3. Traffic demand

The PPI algorithm repeats the parameter update and policy update steps iteratively at the N sample points until the parameter vector $\boldsymbol{\theta}$ converges. Each iteration step involves N numerical integrations, one regression, and N optimizations. At convergence, $J(\mathbf{s})$ can be calculated from Eq. (10) and the optimal policy $\mathbf{w}(\mathbf{s})$ can be computed for any state $\mathbf{s}$ with one more round of policy update.

It should be noted that the parametric policy iteration algorithm does not always guarantee convergence (Benitez-Silva et al. 2000), especially for small K and N. However, it is obvious that the approximated function (10) should asymptotically converge to the true continuous value function $J(\mathbf{s})$ if (1) K goes to infinity; and (2) the basis functions, $\rho_k(\mathbf{s})$, $k=1,\ldots,K$, are properly chosen. Following this observation and the discussions in Bellman (1961), Howitt et al. (2002) hinted that the solution from the PPI algorithm should asymptotically converge to the true optimum under Conditions (1) and (2).

Numerical Example

For the convenience of illustration, we analyze a simplest network with $n=2$ pavement facilities connecting a residential area to a business center (see Fig. 2). Travelers commute from the residential area to the business center. The travel demand $D(t)$ is a scalar that alternates between a high value of 10,000 vehicles per hour and a low value of 1,000 vehicles per hour, each lasting for a period of $T/2$ (e.g., half a day, such that $T=1/365$ year) (see Fig. 3).

We extract from past literature (e.g., Tsunokawa and Schofer 1994; Ouyang and Madanat 2006) reasonable values for the parameters in the performance models: $\beta=4$, $c_0=0$, $c_1=2\times10^{-4}$ \$/QI/vehicle, $c_2=0.5$ \$/min, $g_1=0.66$, $g_2=0.55$, and $g_3=18.3$. Other parameters are listed in Table 1. Facility 1 has a larger traffic capacity and larger resurfacing costs; it is assumed to be "major."

Scenario 1: No Budget Constraint

We first consider a benchmark case with $B=\infty$. Constraint (1*e*) is essentially nonbinding. The pavement facilities are related only through the travelers' route choices.

We approximate the value function by a complete set of ordinary polynomials of total degree 5 in s_1 and s_2; i.e.

Table 1. Facility-Specific Parameter Values

i	Description	d_{i0} (min)	α_i	m_{i0} (dollars)	m_{i1} (dollars/mm)	b_{i0}	b_{i1}
1	Four lanes	1.0	3.662×10^{-17}	3×10^6	5×10^4	0.040	2×10^{-5}
2	Two lanes	0.5	5.859×10^{-16}	2×10^6	3×10^4	0.045	10^{-5}

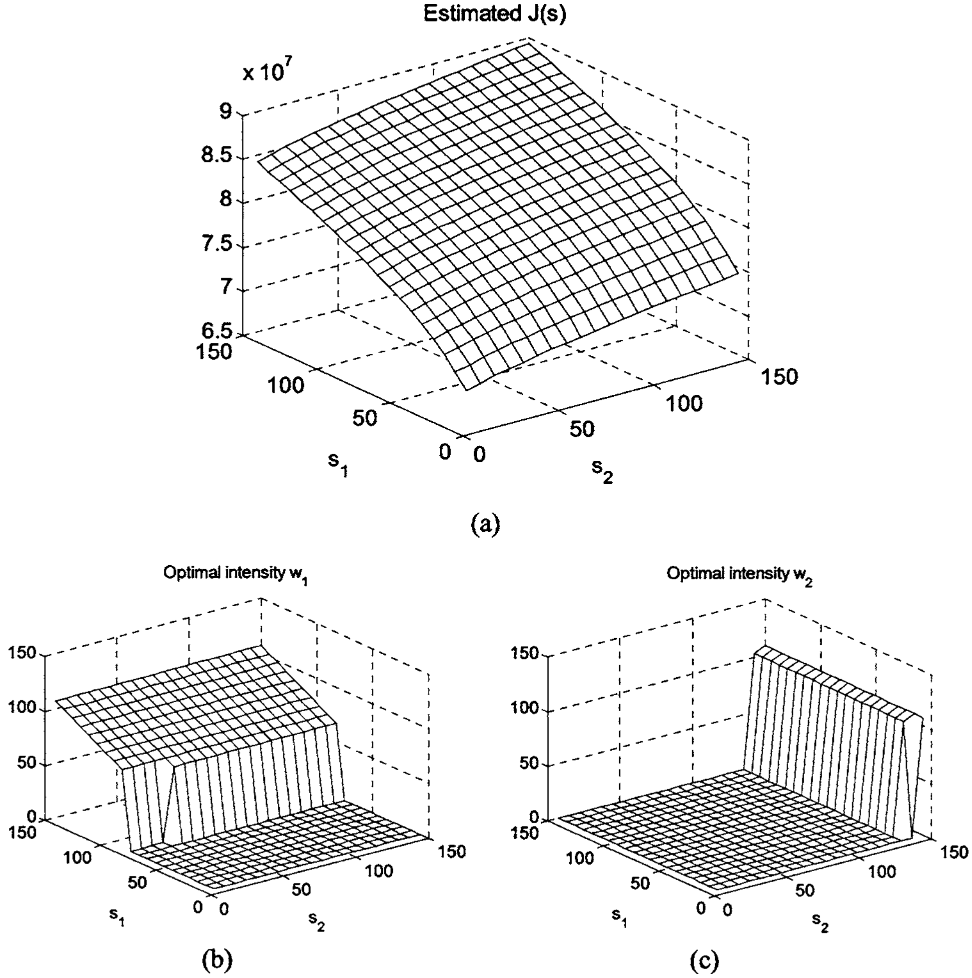

Fig. 4. Optimal solution with no budget constraint: (a) estimated $J(\mathbf{s})$; (b) optimal $w_1(\mathbf{s})$; and (c) optimal $w_2(\mathbf{s})$

$$\{s_1^{k_1}s_2^{k_2}: \forall\, k_1,k_2, 0 \le k_1+k_2 \le 5\}$$

The total number of parameters $K=21$. We then discretize the state space with $N=400$ grid points (20 each in the s_1 and s_2 dimensions, from 7.5 QI, 15 QI, ..., to 150 QI). Then we apply the iterative procedure with the following initial parameters:

$$\theta_k = 1,\ \forall\, k = 1,2,\ldots,K, \quad \text{and}$$

$$w_1(\mathbf{s}^i) = w_2(\mathbf{s}^i) = 0,\ \ \forall\, i = 1,2,\ldots,400$$

Benitez-Silva et al. (2000) reported that the parametric policy iteration algorithm does not necessarily guarantee convergence, but with our choice of initial parameter values and constant $\alpha = 0.5$, convergence is achieved within 50 iterations.

Figs. 4(a–c) show the value function $J(\mathbf{s})$ and the optimal policy $w_1(\mathbf{s})$, $w_2(\mathbf{s})$ at convergence. Obviously, $J(\mathbf{s})$ is monotonically and concavely increasing with both s_1 and s_2. In this case, the optimal policy for each pavement facility roughly has a threshold structure. Figs. 4(b and c) show that it is not optimal to resurface a facility unless its roughness reaches a certain value (i.e., about 70 QI for Facility 1 and about 130 QI for Facility 2). This is similar to the optimal policies for a single pavement facility (Ouyang and Madanat 2006). However, due to travelers' route choices, the threshold for one facility is dependent of the conditions of other facilities. For example, when Facility 2 is in a very good condition ($s_2 < 30$ QI), we tend to refrain from resurfacing Facility 1 unless s_1 deteriorates to a larger value ($s_1 > 80$ QI) [see Fig. 4(b)]. A similar pattern exists in Fig. 4(c). The reason is obvious. Since travelers tend to choose a "better" pavement facility, the traffic flow on one facility is smaller if other facilities are in better conditions. The user costs on this facility will slightly decrease. This makes "do nothing" a more appealing option.

Scenario 2: Budget Constraint

We then impose a finite budget $B = 6 \times 10^6$ \$ to see how limited resource affects the optimal policies. In this case, the pavement facilities are related not only through travelers' route choices but also through constraint (1*e*).

We use exactly the same PPI procedure to obtain the optimal solution to this problem. The value function $J(\mathbf{s})$ and optimal policies $w_1(\mathbf{s})$, $w_2(\mathbf{s})$ are plotted in Figs. 5(a–c). Again, $J(\mathbf{s})$ is monotonically and concavely increasing with s_1 and s_2, in a shape similar to the benchmark case in Fig. 4(a). Fig. 6 plots the change of $J(\mathbf{s})$ due to the imposition of the finite-budget constraint. As expected, the expected cost $J(\mathbf{s})$ increases for all $\mathbf{s}$, and the increase is most significant for large s_1 (i.e., when Facility 1 is in bad condition). This is intuitive. When the "major" facility is in bad condition, it is more costly to fully resurface it, and then the budget constraint becomes most restrictive.

As in Figs. 4(b and c), Facilities 1 and 2 shall not be resurfaced unless their conditions reach certain thresholds. When a

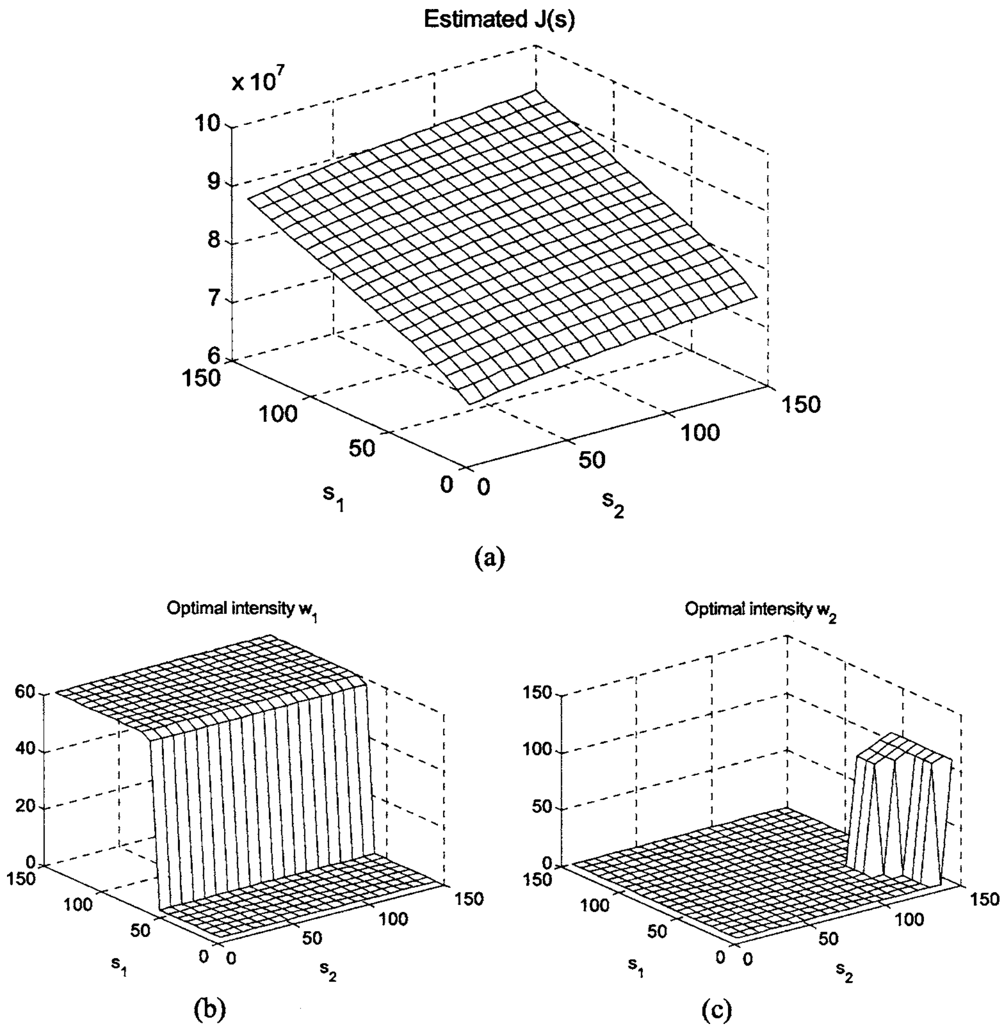

Fig. 5. Optimal solution with a finite budget: (a) estimated $J(\mathbf{s})$; (b) optimal $w_1(\mathbf{s})$; and (c) optimal $w_2(\mathbf{s})$

finite-budget constraint is imposed, facilities tend to have lower resurfacing thresholds. For example, the threshold for Facility 1 reduces to about 60 QI in Fig. 5(b). When $s_1 \approx 50$ QI, similarly, Facility 2 will be resurfaced as long as s_2 reaches 120 QI [see Fig. 5(c)].

Also clear from Figs. 5(b and c), major facilities are given resurfacing priority if resource (budget) is limited. In our case, the optimal policy for Facility 2 is very different from the one with infinite budget. Facility 2 shall never be resurfaced if that poses a competition of resource against Facility 1; i.e., when Facility 1 is in a moderate to bad condition ($s_1 > 60$ QI). This seemingly counterintuitive finding can be related to the concept of economy of scale—when resources are limited, investing all or nothing on alternative facilities may be the most cost-effective choice. In practice, however, highway agencies may pose additional constraints to prevent certain pavement facilities from deteriorating infinitely. Such constraints are not considered in this model.

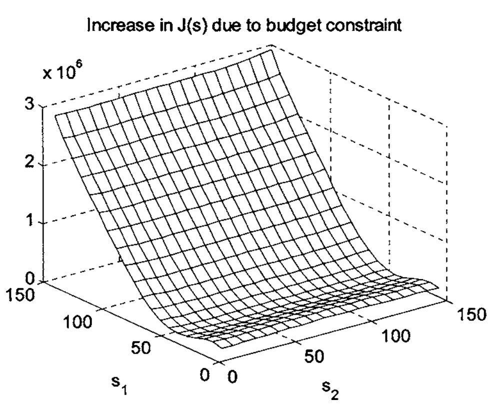

Fig. 6. Increase of $J(\mathbf{s})$ due to the imposition of the budget constraint

The change of optimal policies can be explained as follows. We know from Eq. (13) that the gradient of $z(\mathbf{w})$ satisfies $\partial z/\partial w_i = m_{i1} + (\partial J/\partial \mathbf{s}_i')(\partial \mathbf{s}_i'/\partial w_i), \forall i$, while $\partial \mathbf{s}_i'/\partial w_i$ is always negative. Observe that when the budget constraint is imposed, not only $J(\mathbf{s})$ but also $\partial J/\partial \mathbf{s}_i'$ increases for most of $\mathbf{s}'$. Therefore, when budget constraint is imposed, $\partial z/\partial w_i$ will have a smaller value during every policy update step even for the same $\mathbf{s}$ and $\mathbf{s}'$. This reduction in gradient will make a larger w_i, $\forall i$, more favorable. In our case, Facility 1 has priority for budget allocation because $\partial J/\partial \mathbf{s}_1'$ increases the most significantly. This makes increasing w_1 the most favorable.

Conclusions and Future Research

This paper presents a multidimensional dynamic program for planning pavement resurfacing activities on highway networks.

The objective is to find optimal resurfacing policies that minimize discounted life-cycle costs in the case of continuous pavement state, discrete time, and infinite horizon. The framework proposed in this paper addresses decision making on both the supply and demand sides of general highway networks; i.e., travelers' route choices and the agency's resource allocation decisions are considered simultaneously. To reduce computational difficulty, a parametric approximation technique is used together with the policy iteration approach to solve the problem. Numerical examples show that this parametric policy iteration algorithm solves the network level problem quite well. The optimal solution can be related to the optimal policy structure of known single-facility problems, while several interesting new results are observed and interpreted.

The framework and solution procedure are quite general. The state of pavement facilities can be represented by any continuous pavement unserviceability measures. The formulation can be easily extended to a finite-horizon case by defining $J_\tau(\mathbf{s})$ for every time lattice point τ. The parametric policy iteration method presented in this paper still applies. The traffic assignment function can also be replaced by more sophisticated techniques that address stochastic traveler behavior, time-varying network conditions, and extended periods of congestion on networks.

In the future, the convergence properties of the parametric policy iteration algorithm should be examined, either analytically or numerically. The appropriate choice for K and N has not been systematically studied in the literature; in this study, we determine these numbers through a few experiments. We also find that the convergence performance of the algorithm is more sensitive to K than to N. If K is too small, the algorithm may not converge, because $J(\mathbf{s})$ may not be accurately approximated by a small set of polynomials. Also, the initial choice of parameters (e.g., $\boldsymbol{\theta}$) influences the convergence, and we found in our experiments that it is helpful to assign positive initial values to all elements in $\boldsymbol{\theta}$. This should be further studied.

More computational tests shall also be conducted to address the scalability property of the parametric policy iteration algorithm. This study uses a simple network example to demonstrate the effect of driver route choices on pavement management decisions. However, the model formulation and solution technique apply to any networks. The proposed parametric policy iteration approach shall significantly reduce computational complexity, since the increases of K and N are expected to be much smaller than the expansion of the continuous pavement state space. This should be examined.

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