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Reliability analysis of maintenance operations for railway tracks

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ABSTRACT

Railway engineering is confronted with problems due to degradation of the railway network that requires important and costly maintenance work. However, because of the lack of knowledge on the geometrical and mechanical parameters of the track, it is difficult to optimize the maintenance management. In this context, this paper presents a new methodology to analyze the behavior of railway tracks. It combines new diagnostic devices which permit to obtain an important amount of data and thus to make statistics on the geometric and mechanical parameters and a non-intrusive stochastic approach which can be coupled with any mechanical model. Numerical results show the possibilities of this methodology for reliability analysis of different maintenance operations. In the future this approach will give important informations to railway managers to optimize maintenance operations using a reliability analysis.

1. Introduction

Due to the influence of many factors (frost, water drainage, soil structure and characteristics, number, weight and speed of trains), railway tracks present heterogeneities which are the cause of a high variability of the physical and mechanical parameters of these structures. This variability leads to geometrical modifications of the track and can be detrimental to the behavior of the track. Usually, the choice of the maintenance interventions, from the ballast tamping to the complete renewal of the track, is based on empirical methods from the measurement and the evolution of the global track stiffness and geometry, without taking into account the local state and the variability of the different components of the track.

In the last decade, many studies have focused on the maintenance management of railway networks and their safety. These works concerned several levels of analysis. Carretero et al. [1] presented a methodology of maintenance optimization applied to the railway networks based on Reliability Centered Maintenance techniques. Podofillini et al. [2] and Vatn and Aven [3], and more recently Macchi et al. [4], were interested in the maintenance management of railway infrastructures also using a reliability analysis. Garcia Marquez et al. [5] were interested on a specific problem related to switches and crossing using reliability centered approach based on monitoring.

This paper presents a new method based on a reliability analysis in order to provide to the railway track managers a quantitative methodology for choosing the best maintenance solution for the track. This method is based on a stochastic analysis of the track behavior carried out by taking into account the variability of the geometrical and mechanical parameters of the track obtained from in situ measurements. Then the comparison of the most common maintenance operations is realized using a reliability analysis.

This paper is organized as follows. The first section presents the usual methodology and the principal operations used by the railway track managers for tracks maintenance. Then, Section 2 focuses on the deterministic modeling of the track and Section 3 on the probabilistic modeling and the associated problem. The SFE method proposed is then detailed in Section 5 and the last section deals with a reliability analysis of the maintenance operations adapted to the problem of railway tracks in order to provide to the railway managers a quantitative method to compare the different maintenance operations.

2. Railway track maintenance

2.1. Track irregularities

A conventional railway track is a multi-components system (Fig. 1) whose components are the track superstructure (rails,

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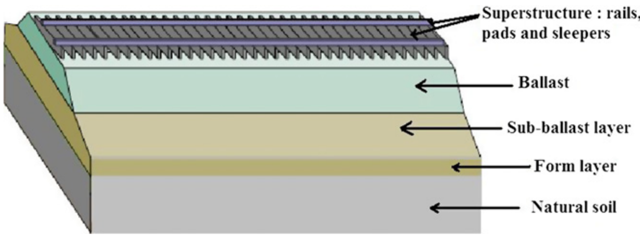

Fig. 1. Track structure.

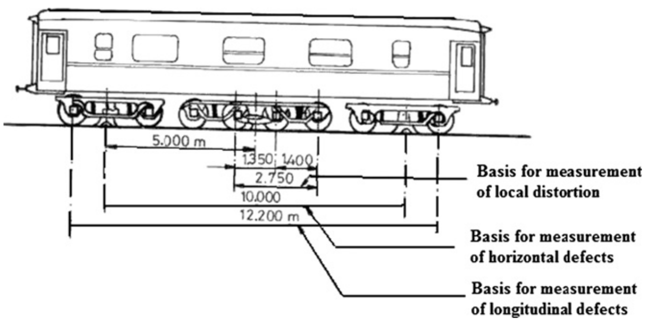

Fig. 2. Mauzin car.

Fig. 3. Track raising [7].

pads and sleepers), the ballast (granular material consisting in crushed rocks that maintain the superstructure intact at the required position by resisting the forces transmitted by the sleepers), the sub-ballast layer and the subgrade soil for supporting to the superstructure.

The track structure supports the rolling charge of train transmitted through the wheel–rail contact. This load is composed by a rolling static loading and a dynamic loading caused by changes in track modulus. The complexity of the track system and its heterogeneity are the cause of the high variability of physical and mechanical parameters of these structures. This variability leads to defects and changes in behavior of the track.

Defects detection and track state estimation are usually carried out with recording vehicles (such as Mauzin (Fig. 2) for French railway manager), traveling on the track at specified intervals. These vehicles measure the values of various track defects and conventional irregularities in accordance with a specific basis of measurements. Different parameters are defined to evaluate these irregularities. Among them, the *NL* (longitudinal levelling) and its evolution are the main indicators on which rests on the global track behavior analysis and the maintenance operations set off. The *NL* corresponds to the standard deviation of an experimental sample constituted by the variations, measured every 10 cm on a distance of 200 m, between the vertical displacement of a central side wheel of the Mauzin car and the average of the vertical displacements of all the corresponding side wheels [6]. When these irregularities exceed the allowed thresholds, maintenance operations have to be carried out.

2.2. Track maintenance operations

Maintenance operations can be either carried out on the ballast layer or on the other layers of the railway platform. The most practiced maintenance operations are:

Ballast tamping: Maintenance tamping [7] is the most effective way of restoring and adjusting track geometry. The process involves lifting the sleepers to a desired level and inserting tamping tines into the ballast with the lifted sleepers between each pair of tines. The tamping tines then squeeze the ballast to fill the void underneath the lifted sleepers.

Raising of the track: This process consists in adding a layer of new ballast under the sleepers after having lifted them (Fig. 3). It has for objective to improve the behavior of the track without removing the fouling ballast [7].

Sub-ballast layer treatment: A sub-ballast layer is required to reduce the stresses on the subgrade [7]. In case of degradation or in order to improve its mechanical performances, a treatment of this layer is required. It can consist on compacting, reinforcing or substituting the materials of this layer.

Fig. 4. Track drainage works.

Improvement of drainage of the track: Railway track drainage is of the first importance in track maintenance because it has a significant effect on both the response of a track to loading and the durability of the ballast and sub-ballast. When drainage deficiencies occur, it cannot be corrected by usual maintenance practices. Different drainage systems exist [7] to keep the subgrade soil as dry and stable as possible (Fig. 4).

Due to economical and exploitation reasons, the maintenance operations performed on the ballast layer are often preferred to those performed on the other track layers.

Anyway, the track investigation devices usually used cannot identify the exact nature and location of the defects in the railway platform. Currently, there is no method allowing to take into consideration the track variability and/or providing the real impact of a maintenance operation on the track behavior. It is therefore difficult for the railway track managers to determine the performance of a specific maintenance operation compared to others. As a result, railway engineering cannot establish a maintenance management policy appropriate to each type of defect and thus treat on a hierarchical basis the maintenance actions. Attempts of probabilistic analysis from the history of the maintenance operations of the track have so far not given conclusive practical results [8].

Therefore, we propose in this paper a methodology to identify the properties of a section of track, to quantify and take into account the variability of the parameters in order to study the impact of a maintenance operation on the behavior of the railway track through a reliability analysis.

This numerical and experimental methodology consists in three points:

- development of a method of diagnosis the track to characterize the physical and mechanical parameters of the different layers of the railway platform and their variability,
- development of a realistic stochastic model, representing a portion of track, and taking into account the variability of the input data,
- use of a reliability analysis to compare the impact of several maintenance operations on the behavior of the track. The efficiency of track maintenance operations is judged through the computing of probabilities of failure. The criteria of failure are related to the acceleration and/or the deflection at different points of sleepers and rail.

These different steps are presented later in this article and illustrated on a real example.

2.3. Experimental tools of diagnosis

In order to characterize the physical and mechanical parameters of the railway platform and to quantify their variability, a methodology of diagnosis [9] based on the use of geophysical tools (Ground Penetrating Radar) coupled with penetrometer (light dynamic penetrometer Panda [10] and geo-endoscopy [11]) tests has been developed. This methodology, which provides the elastic modulus and the thickness of the different layers of the platform and information concerning the soils classification and their hydric state, is well adapted to the railway track auscultation due to its easy and fast implementation.

The Ground Penetrating Radar (GPR) allows the rapid and continuous auscultation of the track and is a comprehensive way to identify heterogeneities and the thickness of layers. The light dynamic penetrometer Panda provides the cone resistance of each layer. Through the images recorded by the geoendoscopy, it is possible to identify the texture and type of materials composing the railway structure. Knowing the nature of the soils of the platform and their cone resistance, it is possible to estimate the elastic modulus of the layers using the relation linking the cone resistance to the oedometric modulus and the oedometric modulus to the elastic modulus as several authors showed it [12–15]:

$$E_{oed} = \alpha.q_d; \quad E = E_{oed}\frac{(1+\nu)(1-2\nu)}{(1-\nu)} \tag{1}$$

where α depends on the soil nature and ν is the Poisson ratio taken equal to 0.3.

It should be noticed that, because of its quick implementation, using this methodology, it is possible to achieve an important amount of data and consequently to realistically model the random variability of the uncertain parameters. Indeed, from large samples of values of these parameters, and using classical statistical tools, it is possible to obtain accurate estimates of the probability distributions and moments (mean, variance,…) of these parameters.

3. Deterministic modeling

A railway track is a heterogeneous system consisting of two steel rails supported at regular intervals by sleepers, filled in between with crib ballast. Sleepers are supported by compacted ballast layer based on the subgrade. Several numerical approaches, mainly based on the finite element (FE) method, have been proposed in the last decades to describe the global or local behavior of railway tracks. These models have been developed for static and/or dynamic analysis. Most of them are based on elastic or elastic–plastic constitutive laws [16] and concern the study of the vertical behavior of the track (settlements, rails deflection, vertical stiffness,…).

Due to the large variability in the subgrade soil properties and the costs involved in testing, the practitioners experience considerable difficulties in establishing the appropriate resilient modulus for design purposes. Subgrade soil modulus can be determined from laboratory testing such as repeated triaxial tests (destructive and time-consuming) or back-calculated from non-destructive testing data [17]. The values considered in this study for these modulus and for the thickness of the different layers result from Panda penetrometer measurements and endoscopy measurements (see Section 2.3).

To describe the dynamics of a portion of a railway track, we have developed a specific non-linear FE model, whose general form is

$$\begin{cases} \ddot{\mathbf{q}}(t) + \mathbb{H}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \xi(t), t) = \mathbf{0}, & t > 0 \\ \mathbf{q}(0) = \mathbf{q}_0, & \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0 \end{cases} \tag{2}$$

where t is the time, $\mathbf{q}$ is a function from $\mathbb{R}_+$ into $\mathbb{R}^l$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are its first and second derivatives with respect to t, ξ is a function from $\mathbb{R}_+$ into $\mathbb{R}^l$, $\mathbb{H}$ is a function from $\mathbb{R}^l \times \mathbb{R}^l \times \mathbb{R}^l \times \mathbb{R}_+$ into $\mathbb{R}^l$, $(\mathbf{q}_0, \dot{\mathbf{q}}_0)$ is a given element of $\mathbb{R}^l \times \mathbb{R}^l$ and l is a specified strictly positive integer which denotes the number of degrees-of-freedom of the oscillator defined by Eq. (2). The function $\mathbb{H}$ characterizes the non-linear dynamic behavior of the railway track. The functions ξ and $\mathbf{q}$ represent, respectively, the nodal excitations of the system (due to the traffic in this case) and the nodal displacements. ξ and $\mathbb{H}$ are known, $\mathbf{q}$ is unknown and must be determined numerically.

More precisely, this model has been developed with the FE code cast3m [18] and results from a 2D (plain stresses) multi-layer finite elements model representing a traditional railway section. In this representation, the railway section is reduced to a system composed of one rail (UIC60), of raid pads, of concrete sleepers, and of four layers making the support of the track (ballast, sub-ballast, form layer and natural ground). For the superstructure (rail, sleepers and pads) and the non-compacted ballast layer, the mechanical behavior is assumed elastic linear. For the ballast and the subgrade soil layers, because of the importance of irreversible strains, it is assumed elastic–plastic with linear strain hardening. In addition, in order to minimize the number of parameters, the strain hardening modulus H of each component is expressed as a function of elastic modulus E *via* the relation: $H=\frac{1}{4}E$.

Such a model is a compromise between accuracy and computations costs. A great attention was granted to some particular problems of numerical modeling such as the FE discretization or the size of the model's domain (in order to minimize the influence of boundary conditions). A parametric analysis showed that a length of about 30 m (50 sleepers) sufficiently limits the lateral boundary effect. The mesh of this portion is made up of 12,792 rectangular four nodes elements (see Fig. 5). Normal displacements of lateral and lower boundaries are prohibited. The train action is modelled as two vertical concentrated loads distant of 3 m and of magnitude 85 kN each, moving to a 160 km/h speed along the rail. This pair of loads represents the train boogie action.

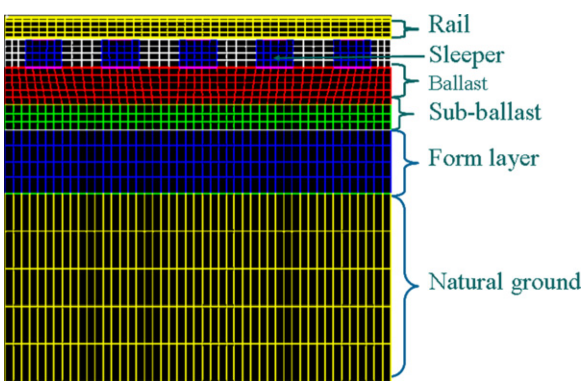

Fig. 5. Detail of FE model mesh.

Several numerical applications have been conducted to validate this model. As an illustration, Fig. 6 shows a comparison between the results provided by the latter and those obtained, on one hand, form experimental measurements on a traditional railway track [19] and, on the other hand, *via* another code (Dynavoie 2.0) [20], for the vertical acceleration and deflection under the 25th sleeper. We can observe that the results given by of the proposed model are in good agreement with the experimental results.

4. Probabilistic modeling and associated probabilistic problem

4.1. Modeling of uncertain parameters

The uncertain parameters of the FE model are:

- the thickness h_b and the Young modulus E_b of the ballast,
- the thickness h_{sb} and the Young modulus E_{sb} of the sub-ballast,
- the thickness h_{fl} and the Young modulus E_{fl} of the form layer,
- the thickness h_{gl} and the Young modulus E_{gl} of the ground layer.

A statistical analysis from results of in situ measurements carried out using the Panda penetrometer and geoendoscopic tests [21] showed that:

- each of these eight parameters can realistically be modeled as a lognormal random variable (r.v.),
- some of them are statistically correlated.

The experimental values of the means (m), the standard deviations (σ) and coefficients of variation ($\nu=\sigma/m$) of these parameters are listed in Table 1. We can see that the values of the coefficients of variation are significant, indicating that the

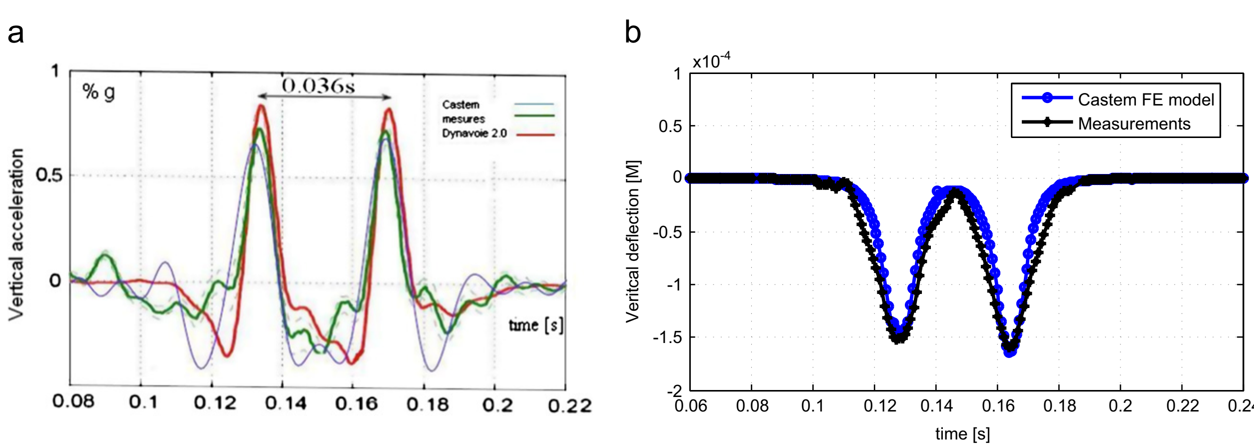

Fig. 6. Vertical acceleration (%g) and deflection (m) under the 25th sleeper.

Table 1
Experimental values of the means (m), standard deviations (σ) and coefficients of variation (ν) of the random parameters.

	h_b	h_{sb}	h_{fl}	h_{gl}	E_b	E_{sb}	E_{fl}	E_{gl}
m	0.26 m	0.64 m	0.97 m	1.62 m	38.91 MPa	26.55 MPa	9.21 MPa	10.74 MPa
σ	0.11 m	0.25 m	0.59 m	0.69 m	11.27 MPa	18.27 MPa	5.84 MPa	8.06 MPa
ν	0.42	0.39	0.61	0.43	0.29	0.69	0.63	0.75

considered parameters have a large random variability (*i.e.* have a strong scattering). The values of the deterministic parameters are given in Table 2.

Remark. In our study, the ground layer thickness h_{gl} corresponds to the maximum depth reached by the penetrometer and thus does not correspond to the thickness of this layer. In the numerical model, it is introduced with a boundary condition corresponding to zero vertical displacement of the bottom line: we have checked that no significant stresses exist at this level.

A preliminary study of the uncertainties propagation [22–24], showed that the random parameters having the greatest influence on the random response of the system are:

- the ballast thickness h_b,
- the Young modulus of the ballast E_b,
- the Young modulus of the sub-ballast E_{sb},
- the Young modulus of the form-layer E_{fl}.

Consequently, only these four r.v.'s have been considered in this study, the other parameters being assumed to be deterministic and equal to their mean values given in Table 1. From now on, we will use the following notations:

$$Y_1 = h_b, \quad Y_2 = E_b, \quad Y_3 = E_{sb}, \quad Y_4 = E_{fl} \tag{3}$$

$$\mathbf{Y} = (Y_1, Y_2, Y_3, Y_4)^T \tag{4}$$

The $\mathbb{R}^4$-valued r.v. $\mathbf{Y}$ is the probabilistic model of the uncertain parameters. It is assumed to be lognormal (hence its components Y_j are lognormal, which is in agreement with the above-mentioned experimental results) and its statistical characteristics are given in Tables 3 and 4 (Table 3 recalls the experimental values of the means, standard deviations and coefficients of variation of the components Y_j of $\mathbf{Y}$, while Table 4 gives the correlation coefficients of the couples (Y_i, Y_j), obtained from in situ measurements on the site of Chambéry [21]).

Note that the deterministic differential equation (2) can be rewritten

$$\begin{cases} \ddot{\mathbf{q}}(t) + \mathbf{H}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \xi(t), t, \mathbf{y}) = \mathbf{0}, & t > 0 \\ \mathbf{q}(0) = \mathbf{q}_0, & \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0 \end{cases} \tag{5}$$

where $\mathbf{y} \in \mathbb{R}^4$ is the vector gathering the uncertain parameters destined to be modeled by the r.v.'s $(Y_j)_{1 \le j \le 4}$ and $\mathbf{H}$ is the function from $\mathbb{R}^l \times \mathbb{R}^l \times \mathbb{R}^l \times \mathbb{R}_+ \times \mathbb{R}^4$ into $\mathbb{R}^l$, such that, $\forall t \in \mathbb{R}_+$:

$$\mathbf{H}(\mathbf{q}(t), \dot{\mathbf{q}}(\mathbf{t}), \xi(t), t, \mathbf{y}) = \mathbb{H}(\mathbf{q}(t), \dot{\mathbf{q}}(t), \xi(t), t) \tag{6}$$

The solution $\mathbf{q} = (\mathbf{q}(t), t \in \mathbb{R}_+)$ of Eq. (5) is a $\mathbb{R}^l$-valued deterministic function of t which depends on $\mathbf{y}$ (*i.e.* it is a function of t indexed by $\mathbf{y}$). We can therefore write formally, $\forall t \in \mathbb{R}_+$

$$\mathbf{q}(t) = \mathbf{R}(t; \mathbf{y}) \tag{7}$$

where for any fixed t in $\mathbb{R}_+$, $\mathbf{R}(t; .)$ is a function from $\mathbb{R}^4$ into $\mathbb{R}^l$ expressing the dependence of $q(t)$ with respect to $\mathbf{y}$.

Once the uncertain parameters modeled as the r.v.'s $(Y_j)_{1 \le j \le 4}$, and therefore $\mathbf{y}$ modeled as the random vector $\mathbf{Y}$, the solution $\mathbf{q}$ becomes a $\mathbb{R}^l$-valued stochastic process $\mathbf{Q} = (\mathbf{Q}(t), t \in \mathbb{R}_+)$ which satisfies the random differential equation

$$\begin{cases} \ddot{\mathbf{Q}}(t) + \mathbf{H}(\mathbf{Q}(t), \dot{\mathbf{Q}}(t), \xi(t), t, \mathbf{Y}) = \mathbf{0}, & t > 0 \\ \mathbf{Q}(0) = \mathbf{q}_0 \ a.s., & \dot{\mathbf{Q}}(0) = \dot{\mathbf{q}}_0 \ a.s. \end{cases} \tag{8}$$

where *a.s.* is the abbreviation of *almost surely*.

From Eq. (7) this process can be formally written, $\forall t \in \mathbb{R}_+$

$$\mathbf{Q}(t) = \mathbf{R}(t; \mathbf{Y}) \tag{9}$$

and can be interpreted as the probabilistic model of $\mathbf{q}$ associated with the probabilistic model $\mathbf{Y}$ of $\mathbf{y}$.

4.2. Control variables

A control variable is a r.v. M (generally scalar) associated with an observation $Z = (Z(t), t \in \mathbb{R}_+)$ (generally also scalar) of the response process $\mathbf{Q} = (\mathbf{Q}(t), t \in \mathbb{R}_+)$, and which plays an important role in the control of the reliability of the structure. It is therefore essential to know its probability distribution or at least some of its statistical characteristics, as its mean, its variance, its skewness, its kurtosis, *etc.* In this study, three control variables are considered:

- the absolute maximum M_1 over T of the vertical acceleration $Z_1 = (Z_1(t), t \in \mathbb{R}_+)$ under the 25th sleeper (located in the middle of the studied railway track section),
- the absolute maximum M_2 over T of the vertical rail deflection $Z_2 = (Z_2(t), t \in \mathbb{R}_+)$ in the middle of the studied railway track section,
- the absolute maximum M_3 over T of the vertical deflection $Z_3 = (Z_3(t), t \in \mathbb{R}_+)$ under the 25th sleeper,

where $T = [0, \tau], \tau > 0$, is the time interval during which the dynamical behavior of the considered railway track section is studied.

Denoting by $(g_i)_{1 \le i \le 3}$ the functions linking the response process $\mathbf{Q}$ to the observation processes $(Z_i)_{1 \le i \le 3}$, that is the functions from $\mathbb{R}^l$ into $\mathbb{R}_+$ such that, $\forall t \in \mathbb{R}_+$

$$Z_i(t) = g_i(\mathbf{Q}(t)) \tag{10}$$

we can write, $\forall i \in \{1,2,3\}$

$$M_i = \max_{t \in T} |Z_i(t)| = \max_{t \in T} |g_i(\mathbf{Q}(t))| \tag{11}$$

Table 2
Values of the deterministic parameters.

	Young's modulus (E) (MPa)	Poisson's ratio (ν)	Thickness (h)	Voluminal mass (ρ) (kg m^{-3})
Non-compacted ballast	20	0.3	0.21 m	1300
Rails	2×10^5	0.3	UIC60 profile	7850
Pads	40	0.25	9×10^3 m	900
Sleepers	3×10^4	0.25	0.21 m	2400

Table 3
Experimental values of the means (m_{Y_j}), standard deviations (σ_{Y_j}) and coefficients of variation (ν_{Y_j}) of the components Y_j of $\mathbf{Y}$.

	$Y_1 = h_b$	$Y_2 = E_b$	$Y_3 = E_{sb}$	$Y_4 = E_{fl}$
m_{Y_i}	0.26 m	38.91 MPa	26.55 MPa	9.21 MPa
σ_{Y_i}	0.11 m	11.27 MPa	18.27 MPa	5.84 MPa
ν_{Y_i}	0.42	0.29	0.69	0.63

Table 4
Experimental values of the correlation coefficients ρ_{Y_i,Y_j} of the couples (Y_i, Y_j).

	Y_1	Y_2	Y_3	Y_4
Y_1	1.00	0.20	0.50	0.85
Y_2	–	1.00	0.15	0.17
Y_3	–	–	1.00	0.44
Y_4	–	–	–	1.00

where it should be noted that the g_i's are known and numerically computable functions.

4.3. Probabilistic problem

We are interested here in the mean $m_{M_i}=\mathbb{E}[M_i]$, the variance $\sigma^2_{M_i}=\mathbb{E}[M_i^2]-m^2_{M_i}$ and the cumulative distribution function (CDF) $F_{M_i}(u)=\mathbb{P}(M_i<u)$, $u\in\mathbb{R}$, of each control variable M_i. Consequently, the probabilistic problem to solve is the following: for any $i\in\{1,2,3\}$, estimate the statistical characteristics m_{M_i}, $\sigma^2_{M_i}$ and F_{M_i} of the r.v. M_i given by Eq. (11) in which $\mathbf{Q}=(\mathbf{Q}(t),t\in\mathbb{R}_+)$ is the $\mathbb{R}^l$-valued stochastic process solution of the second order random differential equation (8). By gathering Eqs. (8) and (11), the complete formulation of this problem is therefore: for any $i\in\{1,2,3\}$, estimate the triplet $(m_{M_i},\ \sigma^2_{M_i},\ F_{M_i})$ such that

$$\begin{cases}\ddot{\mathbf{Q}}(t)+\mathbf{H}(\mathbf{Q}(t),\dot{\mathbf{Q}}(t),\xi(t),t,\mathbf{Y})=\mathbf{0}, & t>0\\ \mathbf{Q}(0)=\mathbf{q}_0\ a.s., & \dot{\mathbf{Q}}(0)=\dot{\mathbf{q}}_0\ a.s.\\ M_i=\max\limits_{t\in T}|g_i(\mathbf{Q}(t))| & \\ m_{M_i}=\mathbb{E}[M_i] & \\ \sigma^2_{M_i}=\mathbb{E}[M_i^2]-m^2_{M_i} & \\ F_{M_i}(u)=\mathbb{P}(M_i<u),u\in\mathbb{R} & \end{cases} \tag{12}$$

where the random vector $\mathbf{Y}$, the deterministic vectors $\mathbf{q}_0$, $\dot{\mathbf{q}}_0$ and the deterministic functions $\mathbf{H}$, ξ and g_i are given.

5. Solution of the probabilistic problem

To solve the problem (12), use was made of a non-intrusive stochastic finite element (NISFE) method based on a stochastic collocation procedure [22–25]. Moreover, this method was coupled with a Monte-Carlo procedure for estimating the CDF F_{M_i}.

5.1. Standardization of the problem formulation

The first step of the proposed solution method consists in rewriting the probabilistic problem (12) in Gaussian context by expressing the $\mathbb{R}^4$-valued lognormal r.v. $\mathbf{Y}=(Y_1,\ldots,Y_4)^T$ as a function of a $\mathbb{R}^4$-valued Gaussian r.v. $\mathbf{X}=(X_1,\ldots,X_4)^T$ according to the classical relationship:

$$\mathbf{Y}=\mathbf{T}(\mathbf{X})\Leftrightarrow\begin{cases}Y_1=T_1(\mathbf{X})=\exp\left(\mu_1+\sum\limits_{q=1}^{4}L_{1q}X_q\right)\\ \vdots\\ Y_4=T_4(\mathbf{X})=\exp\left(\mu_4+\sum\limits_{q=1}^{4}L_{4q}X_q\right)\end{cases} \tag{13}$$

where

- $\mathbf{L}=(L_{kq})_{1\le k,q\le 4}$ is a lower triangular (4×4) matrix such that $\boldsymbol{\Lambda}=\mathbf{L}\mathbf{L}^T$ (Cholesky factorization of $\boldsymbol{\Lambda}$);
- $\boldsymbol{\Lambda}=(\Lambda_{kq})_{1\le k,q\le 4}$ is a positive-definite (4×4) matrix such that

$$\Lambda_{kq}=\ln\left(1+\frac{C_{Y_kY_q}}{m_{Y_k}m_{Y_q}}\right),\quad 1\le k,\ q\le 4 \tag{14}$$

- $m_{Y_k}=\mathbb{E}[Y_k]$, $1\le k\le4$, is the mean of the component Y_k of $\mathbf{Y}$;
- $C_{Y_kY_q}=\mathbb{E}[(Y_k-m_{Y_k})(Y_q-m_{Y_q})]=\rho_{Y_kY_q}\sigma_{Y_k}\sigma_{Y_q}$, $1\le k,q\le4$, is the covariance of the couple $(Y_k,\ Y_q)$, with $\rho_{Y_kY_q}$ the correlation coefficient of this couple and σ_{Y_k} (resp. σ_{Y_q}) the standard deviation of Y_k (resp. Y_q);
- $\mu=(\mu_1,\ldots,\mu_4)^T$ is a vector of $\mathbb{R}^4$ such that

$$\mu_k=\ln\left(\frac{m_{Y_k}}{\sqrt{1+\upsilon^2_{Y_k}}}\right),\quad 1\le k\le4 \tag{15}$$

- $\upsilon_{Y_k}=\sigma_{Y_k}/m_{Y_k}=C^{1/2}_{Y_kY_k}/m_{Y_k}$, $1\le k\le4$, is the coefficient of variation of the component Y_k of $\mathbf{Y}$.

Once implemented the transformation (13), the problem (12) takes the new form: for any $i\in\{1,2,3\}$, estimate the statistics m_{M_i}, $\sigma^2_{M_i}$ and F_{M_i} such that

$$\begin{cases}\ddot{\mathbf{Q}}(t)+\mathbf{h}(\mathbf{Q}(t),\dot{\mathbf{Q}}(t),\xi(t),t,\mathbf{X})=\mathbf{0},\ t>0\\ \mathbf{Q}(0)=\mathbf{q}_0\ a.s.,\ \dot{\mathbf{Q}}(0)=\dot{\mathbf{q}}_0\ a.s.\\ M_i=\max\limits_{t\in T}|g_i(\mathbf{Q}(t))|\\ m_{M_i}=\mathbb{E}[M_i]\\ \sigma^2_{M_i}=\mathbb{E}[M_i^2]-m^2_{M_i}\\ F_{M_i}(u)=\mathbb{P}(M_i<u),\ u\in\mathbb{R}\end{cases} \tag{16}$$

where $\forall t\in\mathbb{R}_+$

$$\mathbf{h}(\mathbf{Q}(t),\dot{\mathbf{Q}}(t),\xi(t),t,\mathbf{X})=\mathbf{H}(\mathbf{Q}(t),\dot{\mathbf{Q}}(t),\xi(t),t,\mathbf{T}(\mathbf{X}))\ a.s. \tag{17}$$

Eq. (16) is the required standard Gaussian formulation of the problem (12). Note that the stochastic process $\mathbf{Q}=(\mathbf{Q}(t),t\in\mathbb{R}_+)$ solution of Eq. (16) can be regarded as a function of t and $\mathbf{X}$, and more precisely as a function of t indexed by $\mathbf{X}$. Therefore, we can write formally, $\forall t\in\mathbb{R}_+$

$$\mathbf{Q}(t)=\mathbf{r}(t;\mathbf{X}) \tag{18}$$

where according to Eqs. (9) and (13), $\mathbf{r}(t;\mathbf{X})=\mathbf{R}(t;\mathbf{T}(\mathbf{X}))$.

This result shows that the control variable M_i defined in Eq. (16) can be rewritten formally

$$M_i=s^i(\mathbf{X}) \tag{19}$$

where s^i is a function from $\mathbb{R}^4$ into $\mathbb{R}_+$ such that, $\forall\mathbf{x}\in\mathbb{R}^4$

$$s^i(\mathbf{x})=\max_{t\in T}|g_i(\mathbf{r}(t;\mathbf{x}))| \tag{20}$$

The idea of the proposed method is then to approximate s^i by a function $\tilde{s}^i$ possessing the following properties:

P1 It is easy to implement and its construction do not requires to modify the deterministic FE model described in Section 3.
P2 $\forall\mathbf{x}\in\mathbb{R}^4$, the calculation of $\tilde{s}^i(\mathbf{x})$ is much simpler and fastener than that of $s^i(\mathbf{x})$.
P3 The mean $m_{\tilde{M}_i}=\mathbb{E}[\tilde{M}_i]$, the variance $\sigma^2_{\tilde{M}_i}=\mathbb{E}[\tilde{M}_i^2]-m^2_{\tilde{M}_i}$ and the CDF $F_{\tilde{M}_i}(u)=\mathbb{P}(\tilde{M}_i<u)$, $u\in\mathbb{R}$, of the r.v. $\tilde{M}_i=\tilde{s}^i(\mathbf{X})$ are easy to calculate and are good approximations of the target statistics m_{M_i}, $\sigma^2_{M_i}$ and $F_{M_i}(u)=\mathbb{P}(M_i<u)$, $u\in\mathbb{R}$, respectively.

It is therefore a method of *response surface* type [26]. It is non-intrusive (because it uses the existing FE model without any modification of the latter) and its originality in this context comes from the fact that to construct the response surface $\tilde{s}^i(\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^4$, and to estimate the target statistics use is made of a stochastic collocation procedure.

5.2. Application of the stochastic collocation procedure

The stochastic collocation procedure described below is detailed in reference [25] and used in references [22–24] to study the random dynamical behavior of a railway track section.

5.2.1. Preliminaries, notations

To each component X_k of the standard Gaussian vector $\mathbf{X}=(X_1,\dots,X_4)^T$ are associated an integer $n_k>0$ and n_k+1 pairs $(x_{k,j_k},\omega_{k,j_k})_{0\le j_k\le n_k}$ such that:

- the $(x_{k,j_k})_{0\le j_k\le n_k}$ are the roots of the Gauss-Hermite polynomial $H_{n_k+1}(u)$, $u\in\mathbb{R}$,
- the $(\omega_{k,j_k})_{0\le j_k\le n_k}$ are the solutions of the linear system

$$\sum_{j_k=0}^{n_k}\omega_{k,j_k}H_{i_k}(x_{k,j_k})=\delta_{i_k0},\quad i_k=0,\dots,n_k \tag{21}$$

where δ_{ij} denotes the Kronecker symbol.

The n_k+1 reals $(x_{k,j_k})_{0\le j_k\le n_k}$ are the collocation points associated with the pair (X_k,n_k), or equivalently with the pair (φ,n_k), where φ is the standard Gaussian density on $\mathbb{R}$, such that

$$\varphi(u)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{u^2}{2}\right),\quad u\in\mathbb{R} \tag{22}$$

The n_k+1 reals $(\omega_{k,j_k})_{0\le j_k\le n_k}$ are the Gaussian weights associated with these points. Let us note that the integer n_k has to be chosen for each component X_k of $\mathbf{X}$.

In the following, we denote by L_{n_k,j_k} the n_k-order Lagrange polynomial based on the collocation point x_{k,j_k}, that is the polynomial:

$$L_{n_k,j_k}(u)=\prod_{\substack{i_k=0\\ i_k\neq j_k}}^{n_k}\frac{u-x_{k,i_k}}{x_{k,j_k}-x_{k,i_k}},\quad u\in\mathbb{R} \tag{23}$$

To each component X_k of $\mathbf{X}$ is associated the family $(L_{n_k,j_k})_{0\le j_k\le n_k}$. These Lagrange polynomials verify the following very important properties:

$$L_{n_k,j_k}(x_{k,i_k})=\delta_{i_kj_k} \tag{24}$$

$$\begin{aligned}\mathbb{E}[L_{n_k,j_k}(X_k)]&=\int_{\mathbb{R}}L_{n_k,j_k}(u)\varphi(u)\,du\\ &=\sum_{i_k=0}^{n_k}\omega_{k,i_k}L_{n_k,j_k}(x_{k,i_k})=\sum_{i_k=0}^{n_k}\omega_{k,i_k}\delta_{i_kj_k}=\omega_{k,j_k}\end{aligned} \tag{25}$$

$$\begin{aligned}\mathbb{E}[L_{n_k,i_k}(X_k)L_{n_k,j_k}(X_k)]&=\int_{\mathbb{R}}L_{n_k,i_k}(u)L_{n_k,j_k}(u)\varphi(u)\,du\\ &=\sum_{l_k=0}^{n_k}\omega_{k,l_k}L_{n_k,i_k}(x_{k,l_k})L_{n_k,j_k}(x_{k,l_k})\\ &=\sum_{l_k=0}^{n_k}\omega_{k,l_k}\delta_{i_kl_k}\delta_{j_kl_k}=\omega_{k,j_k}\delta_{i_kj_k}\end{aligned} \tag{26}$$

5.2.2. Approximation of the response surface

The method developed to construct this approximation consists of two steps.

Step 1 For any fixed $t\in\mathbb{R}_+$, the mapping $\mathbf{x}\to\mathbf{r}(t;\mathbf{x})$ defined in Eq. (18) is approximated as follows:

$$\mathbf{r}(t;\mathbf{x})\simeq\mathbf{r}_{appr.}(t;\mathbf{x})=\sum_{j_1=0}^{n_1}\dots\sum_{j_4=0}^{n_4}\mathbf{r}_{j_1\dots j_4}(t)L_{n_1,j_1}(x_1)\times\cdots\times L_{n_4,j_4}(x_4) \tag{27}$$

where $\mathbf{x}=(x_1,\dots,x_4)^T$ is the generic element of $\mathbb{R}^4$, $n_1,\dots,n_4$ are four given positive integers and, for any fixed $k\in\{1,\dots,4\}$, $(L_{n_k,j_k})_{0\le j_k\le n_k}$ are the n_k-order Lagrange polynomials based on the collocation points $(x_{k,j_k})_{0\le j_k\le n_k}$ associated with the kth component X_k of the standard Gaussian vector $\mathbf{X}$. The $(n_1+1)\times\cdots\times(n_4+1)$ $\mathbb{R}^l$-valued functions $\mathbf{r}_{j_1\dots j_4}=(\mathbf{r}_{j_1\dots j_4}(t),t\in\mathbb{R}_+)$, $0\le j_k\le n_k$, $1\le k\le 4$, are obtained using the following strategy:

(a) Consider the deterministic differential equation

$$\begin{cases}\ddot{\mathbf{q}}(t)+\mathbf{h}(\mathbf{q}(t),\dot{\mathbf{q}}(t),\xi(t),t,\mathbf{x})=\mathbf{0}, & t>0\\ \mathbf{q}(0)=\mathbf{q}_0, & \dot{\mathbf{q}}(0)=\dot{\mathbf{q}}_0\end{cases} \tag{28}$$

derived from the two first equations of the system (16) by replacing $\mathbf{X}$ by the deterministic variable $\mathbf{x}\in\mathbb{R}^4$ (hence the solution process $\mathbf{Q}$ becomes a deterministic $\mathbb{R}^l$-valued function $\mathbf{q}$).

(b) For this equation, we seek a solution of the form

$$\tilde{\mathbf{q}}(t;\mathbf{x})=\sum_{j_1=0}^{n_1}\dots\sum_{j_4=0}^{n_4}\mathbf{r}_{j_1\dots j_4}(t)L_{n_1,j_1}(x_1)\times\cdots\times L_{n_4,j_4}(x_4) \tag{29}$$

(c) Inserting Eq. (29) into Eq. (28), then setting $\mathbf{x}=\mathbf{x}_{j_1,\dots,j_4}=(x_{1j_1},\dots,x_{4j_4})^T$ and using the property (24) of the Lagrange polynomials, yields:

$$\begin{cases}\ddot{\mathbf{r}}_{j_1,\dots,j_4}(t)+\mathbf{h}(\mathbf{r}_{j_1,\dots,j_4}(t),\dot{\mathbf{r}}_{j_1,\dots,j_4}(t),\xi(t),t,\mathbf{x}_{j_1,\dots,j_4})=\mathbf{0}, & t>0\\ \mathbf{r}_{j_1,\dots,j_4}(0)=\mathbf{q}_0, & \dot{\mathbf{r}}_{j_1,\dots,j_4}(0)=\dot{\mathbf{q}}_0\end{cases} \tag{30}$$

It is the differential equation satisfied by each function $\mathbf{r}_{j_1,\dots,j_4}$, with $0\le j_k\le n_k$ and $1\le k\le 4$.

(d) The $(n_1+1)\times\cdots\times(n_4+1)$ functions $\mathbf{r}_{j_1\dots j_4}$ are obtained by solving each deterministic differential equation (30), for $0\le j_k\le n_k$, $1\le k\le 4$.

Step 2 The response surface $s^i(\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^4$, defined by Eq. (20) is approximated by

$$s^i(\mathbf{x})\simeq s^i_{appr.}(\mathbf{x})=\max_{t\in T}|g_i(\mathbf{r}_{appr.}(t;\mathbf{x}))|,\quad \mathbf{x}\in\mathbb{R}^4 \tag{31}$$

with $\mathbf{r}_{appr.}(t;\mathbf{x})$ given by Eq. (27), and $s^i_{appr.}(\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^4$, is approximated in turn by

$$s^i_{appr.}(\mathbf{x})\simeq\tilde{s}^i(\mathbf{x})=\sum_{j_1=0}^{n_1}\dots\sum_{j_4=0}^{n_4}s_{j_1\dots j_4}L_{n_1j_1}(x_1)\times\cdots\times L_{n_4j_4}(x_4) \tag{32}$$

The $(n_1+1)\times\cdots\times(n_4+1)$ coefficients $s_{j_1\dots j_4}$, $0\le j_k\le n_k$, $1\le k\le 4$, are calculated by writing that the functions $s^i_{appr.}$ and $\tilde{s}^i$ satisfy the equality $s^i_{appr.}(\mathbf{x}_{j_1,\dots,j_4})=\tilde{s}^i(\mathbf{x}_{j_1,\dots,j_4})$ at each collocation point $\mathbf{x}_{j_1,\dots,j_4}=(x_{1j_1},\dots,x_{4j_4})^T$ and using the property (24) of the Lagrange polynomial. We obtain

$$s_{j_1,\dots,j_4}=\max_{t\in T}|g_i(\mathbf{r}_{j_1,\dots,j_4}(t)|,\quad 0\le j_k\le n_k,\ 1\le k\le 4 \tag{33}$$

Once calculated these coefficients, the response surface $\tilde{s}^i(\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^4$, is known and it is possible, from the latter and using again the stochastic collocation technique, to construct approximations for m_{M_i}, $\sigma^2_{M_i}$ and F_{M_i}.

5.3. Estimation of the target statistics

The basic idea to estimate the target statistics m_{M_i}, $\sigma^2_{M_i}$ and F_{M_i} is to approximate the r.v. M_i given by Eq. (19) by the r.v.

$$\tilde{M}_i=\tilde{s}^i(\mathbf{X})=\sum_{j_1=0}^{n_1}\cdots\sum_{j_4=0}^{n_4}s_{j_1\dots j_4}L_{n_1,j_1}(X_1)\times\cdots\times L_{n_4,j_4}(X_4) \tag{34}$$

and to take $m_{\tilde{M}_i}=\mathbb{E}[\tilde{M}_i]$, $\sigma^2_{\tilde{M}_i}=\mathbb{E}[\tilde{M}_i^2]-m^2_{\tilde{M}_i}$ and $F_{\tilde{M}_i}(u)=\mathbb{P}(\tilde{M}_i<u)$, $u\in\mathbb{R}$, as approximations of m_{M_i}, $\sigma^2_{M_i}$ and $F_{M_i}(u)=\mathbb{P}(M_i<u)$,

$u \in \mathbb{R}$, respectively

$$m_{M_i} \simeq m_{\tilde{M}_i} = \mathbb{E}[\tilde{M}_i] \tag{35}$$

$$\sigma^2_{M_i} \simeq \sigma^2_{\tilde{M}_i} = \mathbb{E}[\tilde{M}_i^2] - m^2_{\tilde{M}_i} \tag{36}$$

$$F_{M_i}(u) \simeq F_{\tilde{M}_i}(u) = \mathbb{P}(\tilde{M}_i < u), \quad u \in \mathbb{R} \tag{37}$$

5.3.1. Estimation of the mean m_{M_i}

From Eqs. (34) and (35), and taking into consideration the mutually independence of the r.v. $X_1, \ldots, X_4$, we can write

$$m_{M_i} \simeq \mathbb{E}[\tilde{M}_i] = \sum_{j_1=0}^{n_1} \cdots \sum_{j_4=0}^{n_4} s_{j_1 \ldots j_4} \mathbb{E}[L_{n_1,j_1}(X_1)] \times \cdots \times \mathbb{E}[L_{n_4,j_4}(X_4)]$$

Hence, from Eq. (25)

$$m_{M_i} \simeq \sum_{j_1=0}^{n_1} \cdots \sum_{j_4=0}^{n_4} s_{j_1 \ldots j_4} \omega_{1,j_1} \times \cdots \times \omega_{4,j_4} \tag{38}$$

5.3.2. Estimation of the variance $\sigma^2_{M_i}$

From Eq. (34), and given the mutually independence of the r.v. $X_1, \ldots, X_4$, we can write

$$\mathbb{E}[\tilde{M}_i^2] = \sum_{i_1=0}^{n_1} \cdots \sum_{i_4=0}^{n_4} \sum_{j_1=0}^{n_1} \cdots \sum_{j_4=0}^{n_4} s_{i_1 \ldots i_4} s_{j_1 \ldots j_4} \mathbb{E}[L_{n_1,i_1}(X_1) L_{n_1,j_1}(X_1)] \times \cdots \times \mathbb{E}[L_{n_4,i_4}(X_4) L_{n_4,j_4}(X_4)]$$

From the property (26), this equation becomes

$$\mathbb{E}[\tilde{M}_i^2] = \sum_{j_1=0}^{n_1} \cdots \sum_{j_4=0}^{n_4} s^2_{j_1 \ldots j_4} \omega_{1,j_1} \times \cdots \times \omega_{4,j_4}$$

Hence, from Eq. (36)

$$\sigma^2_{M_i} \simeq \sum_{j_1=0}^{n_1} \cdots \sum_{j_4=0}^{n_4} s^2_{j_1 \ldots j_4} \omega_{1,j_1} \times \cdots \times \omega_{4,j_4} - m^2_{M_i} \tag{39}$$

with m_{M_i} given by Eq. (38).

5.3.3. Estimation of the CDF F_{M_i}

To estimate F_{M_i}, we used the approximation (37) in which the CFD $F_{\tilde{M}_i}$ was estimated *via* a Monte-Carlo procedure based on the probabilistic simulation of the r.v. $\tilde{M}_i$ defined by Eq. (34). Such a simulation comes down to simulate the standard Gaussian vector $\mathbf{X}$ and to apply the formula (34) to each simulated realization of $\mathbf{X}$. Once obtained by this way a sufficiently large numerical sample of $\tilde{M}_i$, $F_{\tilde{M}_i}$ can then be estimated statistically using a suitable estimator of the CDF. The relationship (34) linking $\mathbf{X}$ to $\tilde{M}_i$ being explicit and polynomial, this procedure is simple and not time-consuming.

6. Reliability analysis

The management of the maintenance of the infrastructure in the form of customer-supplier contract is based on a set of standards constituting the base of the reference quality frame of track maintenance. This normative framework allows a righter interpretation of the actions to be undertaken. For each observation variable, it defines thresholds whose values depend on the severity of the critical situations to avoid:

- slackening values (SV) requiring speed limitations,
- intervention values (IV) imposing an intervention within a relatively short time so that SV critical threshold is not reached,
- alarms values (AV) requiring to program a monitoring or an intervention in the medium term.

In this paper, two observation variables are concerned by the reliability analysis: the vertical acceleration $Z_1 = (Z_1(t), t \in \mathbb{R}_+)$ under the 25th sleeper and the vertical rail deflection $Z_2 = (Z_2(t), t \in \mathbb{R}_+)$ in the middle of the studied railway track section. The threshold values chosen for these observation variables are, respectively, denoted $\overline{z}_1$ and $\overline{z}_2$ ($\overline{z}_1, \overline{z}_2 \in \mathbb{R}^*_+$).

To each pair $(Z_i, \overline{z}_i)$, $i \in \{1,2\}$, are associated:

(a) the *failure criterion* C_i defined as follows: the dynamical behavior of the studied railway track section is failing if $\max_{t \in T} |Z_i(t)| \geq \overline{z}_i$, that is, according to Eqs. (11) and (19), if $M_i = s^i(\mathbf{X}) \geq \overline{z}_i$,

(b) the function $\Gamma_i : \mathbb{R}^4 \to \mathbb{R}$ such that, $\forall \mathbf{x} \in \mathbb{R}^4$

$$\Gamma_i(\mathbf{x}) = \overline{z}_i - s^i(\mathbf{x}) \tag{40}$$

(c) the event

$$E^i_f = \{\Gamma_i(\mathbf{X}) \leq 0\} \tag{41}$$

The failure criterion C_i is characterized by the inequality $\Gamma_i(\mathbf{X}) \leq 0$. The function Γ_i and the event E^i_f are, respectively, the *limit state function* and the *failure event* associated with C_i.

The reliability of the railway track section (with respect to the criterion C_i) is quantified by the probability $P^i_f = \mathbb{P}(E^i_f)$, called *probability of failure* [26]. This probability is given by

$$P^i_f = \int_{\mathbb{R}^4} \mathbb{1}_{D^i_f}(\mathbf{x}) \varphi_4(\mathbf{x}) d\mathbf{x} = \mathbb{E}[\mathbb{1}_{D^i_f}(\mathbf{X})] \tag{42}$$

where D^i_f is the subset of $\mathbb{R}^4$ such that

$$D^i_f = \{\mathbf{x} \in \mathbb{R}^4 : \Gamma_i(\mathbf{x}) \leq 0\} \tag{43}$$

$\mathbb{1}_{D^i_f}$ is the indicator function of D^i_f (*i.e.* $\mathbb{1}_{D^i_f}(\mathbf{x}) = 1$ if $\mathbf{x} \in D^i_f$ and $\mathbb{1}_{D^i_f}(\mathbf{x}) = 0$ if $\mathbf{x} \notin D^i_f$) and φ_4 is the standard Gaussian density on $\mathbb{R}^4$ given by

$$\varphi_4(\mathbf{x}) = \frac{1}{(2\pi)^2} \exp\left(-\frac{\|\mathbf{x}\|^2}{2}\right), \quad \mathbf{x} \in \mathbb{R}^4 \tag{44}$$

in which $\|\cdot\|$ denotes the canonical Euclidian norm.

To estimate P^i_f, this probability is first approximated by

$$P^i_f \simeq \tilde{P}^i_f = \int_{\mathbb{R}^4} \mathbb{1}_{\tilde{D}^i_f}(\mathbf{x}) \varphi_4(\mathbf{x})\, d\mathbf{x} = \mathbb{E}[\mathbb{1}_{\tilde{D}^i_f}(\mathbf{X})] \tag{45}$$

where

$$\tilde{D}^i_f = \{\mathbf{x} \in \mathbb{R}^4 : \tilde{\Gamma}_i(\mathbf{x}) \leq 0\} \tag{46}$$

and $\tilde{\Gamma}_i$ is the function from $\mathbb{R}^4$ into $\mathbb{R}$ such that $\forall \mathbf{x} \in \mathbb{R}^4$

$$\tilde{\Gamma}_i(\mathbf{x}) = \overline{z}_i - \tilde{s}^i(\mathbf{x}) \tag{47}$$

where $\tilde{s}^i$ is the approximation of s^i given by Eq. (32).

The probability $\tilde{P}^i_f$ is then estimated using the crude Monte-Carlo method [26–28]. According to this method, a N-estimate $\tilde{P}^{i,N}_f$ of $\tilde{P}^i_f$ is given by:

$$\tilde{P}^{i,N}_f = \frac{1}{N} \sum_{k=1}^{N} \mathbb{1}_{\tilde{D}^i_f}(\mathbf{x}^k) \tag{48}$$

where $\{\mathbf{x}^1, \ldots, \mathbf{x}^N\}$ is a N-sample of independent realizations of the standard Gaussian vector $\mathbf{X} = (X_1, \ldots, X_4)^T$. Rigorously, this estimate must be complemented by a confidence interval at a given confidence level [27,28]. For a confidence level $1-\alpha$, $(0 \leq \alpha \leq 1)$, such an interval is given by:

$$I_{1-\alpha}^{i,N} = \left[\tilde{P}_f^{i,N} - \eta_{1-\alpha/2}\sqrt{\frac{\tilde{P}_f^{i,N}(1-\tilde{P}_f^{i,N})}{N}},\quad \tilde{P}_f^{i,N} + \eta_{1-\alpha/2}\sqrt{\frac{\tilde{P}_f^{i,N}(1-\tilde{P}_f^{i,N})}{N}} \right] \tag{49}$$

where $\eta_{1-\alpha/2}$ is the $(1-\alpha/2)$-quantile of the standard Gaussian distribution (*i.e.* $\eta_{1-\alpha/2}$ is the solution of $\Phi(\eta_{1-\alpha/2}) = 1-\alpha/2$, where Φ is the standard Gaussian CDF); for example, for $\alpha = 0.05$, $\eta_{1-\alpha/2} \simeq 1.96$ and for $\alpha = 0.10$, $\eta_{1-\alpha/2} \simeq 1.645$.

Note that the probability $\tilde{P}_f^i$, which results from approximating $\Gamma_i(\mathbf{x})$ by the polynomial function $\tilde{\Gamma}_i(\mathbf{x})$, cannot be a very accurate approximation of P_f^i. However, many numerical experiments showed that this approximation always conserves the magnitude order of P_f^i.

7. Numerical applications

The purpose of the numerical applications presented in this section is to show, through the example described in Sections 3 and 4, that the proposed probabilistic approach is a tool well suited for evaluating the effects of maintenance operations on railway track sections. In the considered applications, these effects are observed on the second order statistics of the control variables M_1, M_2, M_3 and on the failure probabilities linked to M_1 and M_2 *via* the failure criteria C_1 and C_2. We recall below the meaning of these control variables and criteria.

- M_1 = absolute maximum over T of the vertical acceleration $Z_1 = (Z_1(t), t \in \mathbb{R}_+)$ under the 25th sleeper (located in the middle of the considered railway track section);
- M_2 = absolute maximum over T of the vertical rail deflection $Z_2 = (Z_2(t), t \in \mathbb{R}_+)$ in the middle of the considered railway track section;
- M_3 = absolute maximum over T of the vertical deflection $Z_3 = (Z_3(t), t \in \mathbb{R}_+)$ under the 25th sleeper;
- C_1: failure if $M_1 \geq \overline{z}_1$, not failure if $M_1 < \overline{z}_1$, with $\overline{z}_1$ = threshold value of Z_1;
- C_2: failure if $M_2 \geq \overline{z}_2$, not failure if $M_2 < \overline{z}_2$, with $\overline{z}_2$ = threshold value of Z_2.

The proposed applications concern three maintenance operations:

1. a raising of the track,
2. a treatment of the ballast and sub-ballast layers,
3. a drainage operation.

Before presenting them, we give some numerical results deriving from a preliminary reliability analysis of the studied railway track section, before any maintenance operation.

7.1. Some preliminary numerical results

The railway track section considered in this study (whose mechanical and probabilistic models are described in Sections 3 and 4) has been the subject of a complete reliability analysis in its initial configuration (*i.e.* before any maintenance operation). Three results derived from this study are presented below. The first concerns the effect of the statistical correlation of the random parameters on the failure probability of the system, the second shows how this probability evolves according to the value of the failure threshold of a given failure criterion, and the third gives a reliability characterization of the track section through the second order statistics of the control variables M_1, M_2, M_3 and the failure probabilities associated with the failure criteria C_1 and C_2. In these applications, the failure probabilities were estimated using the crude Monte-Carlo method described in Section 6, with $N = 100\,00$, and the second order statistics of the control variables $(M_i)_{1 \leq i \leq 3}$ were calculated by means of the collocation procedure detailed in Section 5.3, with $n_1 = n_2 = n_3 = n_4 = 3$ and $\tau = 0.7425$ s. These numerical characteristics were also used in the three applications presented in Section 7.2.

7.1.1. Effect of the statistical correlation

Recall that the random vector $\mathbf{Y} = (Y_1, \ldots, Y_4)^T$ modeling the uncertain parameters of the mechanical model follows a lognormal distribution. Two cases are then considered:

1. the components $Y_1, \ldots, Y_4$ of $\mathbf{Y}$ are mutually independent and therefore uncorrelated,
2. these components are correlated with correlation coefficients given in Table 4.

The effect of the correlation is observed on the failure probability P_f^2 associated with the failure criterion C_2, with $\overline{z}_2 = 6.8$ mm. The obtained estimated values of P_f^2 and the associated 95%-confidence intervals (see Section 6) are given in Table 5. We can see that the correlation of random parameters increases by 25% the failure probability. Consequently, the correlated case is more harmful to

Table 5
Probability of failure in uncorrelated and correlated cases.

	Estimated values of P_f^2	95% confidence intervals
Uncorrelated case	0.0451	[0.0417, 0.0485]
Correlated case	0.0625	[0.0585, 0.0665]

Table 6
Evolution of P_f^1 according to $\overline{z}_1$.

$\overline{z}_1$ (m/s^2)	Estimated values of P_f^1	95% confidence intervals
8	0.3233	[0.3156, 0.3310]
10	0.2013	[0.1947, 0.2079]
12	0.0762	[0.0719, 0.0806]
15	0.0211	[0.0188, 0.0235]
20	0.0040	[0.0030, 0.0050]
25	0.0003	[0.0001, 0.0006]

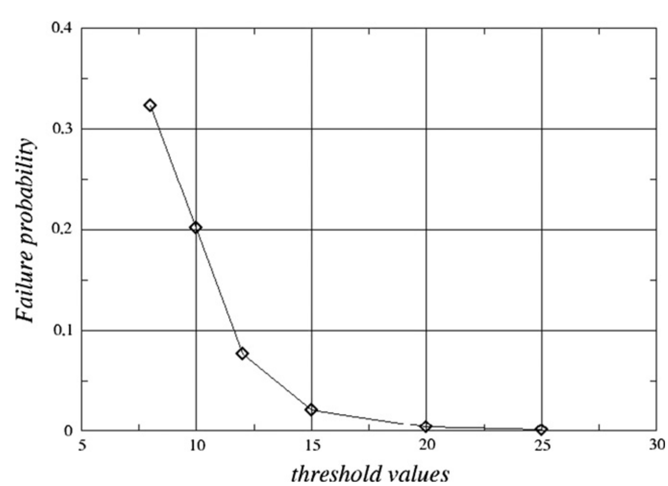

Fig. 7. Graphic representation of the evolution of P_f^1 according to $\overline{z}_1$.

the track safety than the uncorrelated case. For this reason, only the correlated case will be considered in the sequel.

7.1.2. Evolution of the failure probability

The failure probability considered here is the probability P_f^1 associated with the failure criterion C_1 in which the threshold $\bar{z}_1$ is variable and takes successively the values: 8, 10, 12, 15, 20, and 25 (m/s^2). Table 6 gives the estimated value of P_f^1 and the associated 95%-confidence interval, for each value of $\bar{z}_1$. The evolution of P_f^1 according to $\bar{z}_1$ is also illustrated graphically in Fig. 7. We can observe that P_f^1 logically decreases when $\bar{z}_1$ decreases and, moreover, that this decreasing is exponential. The interest of this kind of parametric study is to provide a rational method for choosing the value of the failure threshold $\bar{z}_1$: it is chosen such that P_f^1 is equal to a given probability (*i.e.* to a probability fixed *a priori*). On the basis of this method, we have chosen the value $\bar{z}_1 = 12$ m/s^2. This value has been used in all the other applications.

7.1.3. Reliability characterization of track in its initial configuration

The initial configuration of the track is the configuration for which the values of the deterministic parameters and of the second order characteristics of the lognormal random vector **Y** are ones given in Tables 1–4 (and resulting from statistical estimation of experimental measurements for the second order characteristics of **Y**). For this configuration, we have focused our attention on:

(a) the means and standard deviations of the control variables M_1, M_2 and M_3,
(b) the failure probabilities P_f^1 and P_f^2 associated, respectively, with the failure criteria C_1 and C_2, with $\bar{z}_1 = 12$ m/s^2 and $\bar{z}_2 = 6.8$ mm.

The obtained results are summarized in Tables 7 and 8. They will be used as reference values in the next applications.

Table 7
Means and standard deviations of the control variables M_1, M_2 and M_3.

	M_1 (m/s^2)	M_2 (mm)	M_3 (mm)
Mean	10.611	3.354	2.997
Standard deviation	2.891	1.099	1.064

Table 8
Estimated values of P_f^1 and P_f^2 and associated 95% confidence intervals.

	Estimated values	95% confidence intervals
P_f^1	0.0762	[0.0719, 0.0806]
P_f^2	0.0625	[0.0585, 0.0665]

Table 9
Mean values and standard deviations of the random parameters $(Y_i)_{1 \le j \le 4}$ before and after the raising of the track.

	Random parameters	Mean values	Standard deviations
Before the raising of the track	$Y_1 = h_b$	0.26 m	0.11 m
	$Y_2 = E_b$	38.91 MPa	11.27 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa
Raising of 5 cm	$Y_1 = h_b$	0.31 m	0.05 m
	$Y_2 = E_b$	45.00 MPa	8.00 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa
Raising of 8 cm	$Y_1 = h_b$	0.34 m	0.05 m
	$Y_2 = E_b$	45.00 MPa	8.00 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa
Raising of 10 cm	$Y_1 = h_b$	0.36 m	0.05 m
	$Y_2 = E_b$	45.00 MPa	8.00 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa

7.2. Reliability analysis of maintenance operations

As mentioned previously, we are interested here in the effect of the maintenance operations (raising of the track, treatment of the ballast and sub-ballast layers, drainage) on the reliability of the studied track section. The gain resulting from each of these operations will be judged by comparison with the initial reliability state of the track section, that is to say the reliability state of the latter in its initial configuration. The reliability state will be characterized by the second order moments and the probability density functions (PDF's) of the control variables $(M_i)_{1 \le i \le 3}$ and by the failure probabilities P_f^1 and P_f^2 associated with the criteria C_1 and C_2, respectively.

7.2.1. Raising of the track

This operation consists in increasing the thickness of the ballast layer. The preliminary estimate of the variation of the

Table 10
Mean values and standard deviations of the control variables $(M_i)_{1 \le j \le 3}$ before and after the raising of the track.

	Control variables	Mean values	Standard deviations
Before the raising of the track	M_1	10.611 m/s^2	2.891 m/s^2
	M_2	3.35 mm	1.10 mm
	M_3	3.00 mm	1.06 mm
Raising of 5 cm	M_1	9.480 m/s^2 (−10.7%)	1.881 m/s^2 (−34.9%)
	M_2	3.33 mm (−0.6%)	1.02 mm (−7.3%)
	M_3	2.94 mm (−2.0%)	1.04 mm (−1.9%)
Raising of 8 cm	M_1	9.378 m/s^2 (−11.6%)	1.743 m/s^2 (−39.7%)
	M_2	3.31 mm (−1.2%)	1.02 mm (−7.3%)
	M_3	2.93 mm (−2.3%)	1.03 mm (−2.8%)
Raising of 10 cm	M_1	9.035 m/s^2 (−14.9%)	1.077 m/s^2 (−62.7%)
	M_2	3.28 mm (−2.1%)	1.04 mm (−5.4%)
	M_3	2.91 mm (−3.0%)	1.03 mm (−2.8%)

mechanical and geometrical characteristics generated by the various raising operations is difficult. From a statistical point of view, it has two consequences for the ballast layer:

(1) it increases the mean and decreases the standard deviation of its thickness,
(2) it increases the mean and decreases the standard deviation of its Young modulus.

These two consequences are drawn from a statistical analysis based on experimental measurements performed before and after the raising operation. The second results from the stuffing operation that always accompanies the raising operation.

Three raising thicknesses are considered here: 5 cm, 8 cm and 10 cm. In Table 9 are summarized, for each of these thicknesses, the values of the means and standard deviations of the random parameters $(Y_i)_{1 \le j \le 4}$, before and after the raising of the track.

The effect of each considered raising thickness on the reliability of the studied track section is analyzed through, on the one hand, the mean values and the standard deviations of the control variables $(M_i)_{1 \le i \le 3}$ and, on the other hand, the failure probabilities P_f^1 and P_f^2 associated with the criteria C_1 and C_2, respectively. The obtained numerical results are given in Table 10 for the first analysis and in Table 11 for the second. In these tables, the gain due to the raising operation is mentioned between parentheses.

From these results, we can note that the profit brought by the raising of the track is weak, whatever the level of raising considered here. These operations have a limited effect on the control variables compared to initial state, except for the control variable M_1, corresponding to the sleeper's acceleration. The improvement observed is of 11% on the mean and 35% on the standard deviation for the injection of 5 cm of ballast and of 15% on the mean and 62% on the standard deviation for an injection of 10 cm of ballast. This operation thus makes it possible to decrease the dispersion of accelerations: the coefficient of variation of this control variable passes from 27% (initial state) to 20% after raising of 5 cm.

These remarks are confirmed on the probability of failure of each analyzed case, notably for P_f^2 (criterion C_2). For criterion C_1, the variation on P_f^1 is more important: this point is related to the fact that sleepers deflection is more sensitive to the thickness and the Young's modulus of the ballast layer.

7.2.2. Ballast and sub-ballast layers treatment

Among the operations performed to improve the quality of railway tracks at the end of their life cycle, one of the most important is known as "Renewal of Track and Ballast" (RTB). Such an operation may be activated when the preventive maintenance actions are not effective any more, *i.e.* when degradation cannot be reduced any more. It is intended to replace the track structure as a whole (*i.e.* the rails, the sleepers and the ballast and sub-ballast layers), or only a part of this one. We are interested only here in the replacement of the layers of ballast and sub-ballast.

When realizing this maintenance operation, the implementation and the compaction of the considered layers must comply with precise regulations. The level of compaction depends on the nature and on the mechanical and geometrical characteristics of the layers. In this application, two levels of compaction are considered for the sub-ballast layer, referred to as option 1 and option 2.

The replacement of the layers of ballast and sub-ballast is reflected on the random parameters $(Y_i)_{1 \le i \le 4}$ by a change in their statistical characteristics. In this application, the characteristics concerned are the means and the standard deviations (the random vector $\mathbf{Y} = (Y_1, \ldots, Y_4)^T$ remaining lognormal with unchanged correlation matrix given in Table 4). The values of

Table 11
Failure probabilities P_f^1 and P_f^2 before and after the raising of the track.

	P_f^1 (criterion C_1)	P_f^2 (criterion C_2)
Before the raising of the track	0.0762	0.0625
Raising of 5 cm	0.0382 (−49.9%)	0.0614 (−1.8%)
Raising of 8 cm	0.0319 (−58.1%)	0.0593 (−5.1%)
Raising of 10 cm	0.0205 (−73.1%)	0.0573 (−8.3%)

Table 12
Mean values and standard deviations of the random parameters $(Y_i)_{1 \le j \le 4}$ before and after the treatment of the track section (options 1 and 2).

	Random parameters	Mean values	Standard deviations
Before the treatment of the track	$Y_1 = h_b$	0.26 m	0.11 m
	$Y_2 = E_b$	38.91 MPa	11.27 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa
After the treatment: option 1	$Y_1 = h_b$	0.26 m	0.03 m
	$Y_2 = E_b$	50.00 MPa	2.50 MPa
	$Y_3 = E_{sb}$	50.00 MPa	7.00 MPa
	$Y_4 = E_{fl}$	19.00 MPa	5.84 MPa
After the treatment: option 2	$Y_1 = h_b$	0.26 m	0.03 m
	$Y_2 = E_b$	50.00 MPa	2.50 MPa
	$Y_3 = E_{sb}$	70.00 MPa	7.00 MPa
	$Y_4 = E_{fl}$	19.00 MPa	5.84 MPa

Table 14
Failure probabilities P_f^1 and P_f^2 before and after the treatment of the track (options 1 and 2).

	P_f^1 (criterion C_1)	P_f^2 (criterion C_2)
Before the treatment	0.0762	0.0625
After the treatment: option 1	0.0466 (−38.8%)	0.0522 (−16.5%)
After the treatment: option 2	0.0193 (−74.4%)	0.0341 (−45.4%)

Table 13
Mean values and standard deviations of the control variables $(M_i)_{1 \le j \le 3}$ before and after the treatment of the track section (options 1 and 2).

	Control variables	Mean values	Standard deviations
Before the treatment	M_1	10.611 m/s^2	2.891 m/s^2
	M_2	3.35 mm	1.10 mm
	M_3	3.00 mm	1.06 mm
After the treatment: option 1	M_1	8.851 m/s^2 (−16.6%)	1.999 m/s^2 (−30.9%)
	M_2	3.16 mm (−5.7%)	0.96 mm (−12.7%)
	M_3	2.74 mm (−8.7%)	0.96 mm (−9.4%)
After the treatment: option 2	M_1	8.200 m/s^2 (−22.7%)	1.527 m/s^2 (−47.2%)
	M_2	2.91 mm (−13.1%)	0.88 mm (−20.0%)
	M_3	2.54 mm (−15.3%)	0.86 mm (−18.9%)

these characteristics before and after treatment (according to option 1 or 2 for the compaction of the sub-ballast layer) are given is Table 12.

The contribution of the implemented treatment (with the two options considered for the sub-ballast compaction) to the reliability of the studied track section is analyzed through, on the one hand, the mean values and the standard deviations of the control variables $(M_i)_{1 \le i \le 3}$ and, on the other hand, the failure probabilities P_f^1 and P_f^2 associated with the criteria C_1 and C_2, respectively. Tables 13 and 14 summarize the numerical values obtained for these quantities in each case. In addition, on Fig. 8 are compared, for M_1 and M_3, the PDF's obtained before (*i.e.* in the initial state) and after the treatment. In Tables 13 and 14, the gain due to the treatment of the track section is mentioned between parentheses.

In Table 13, we note that a higher level of compaction of the sub-ballast layer (option 2) reduces the mean and the dispersion of the deflection of the rail (13% and 20%) as well as of the sleepers. Thus, a notable reduction in the dynamic effects will be obtained. The first alternative (option 1), which corresponds to a middle level of compaction, makes it possible to reduce by 6% the mean of the deflection of the rail and by 13% its standard deviation. The PDF's of M_1 and M_3 represented in Fig. 8 illustrate these results graphically.

All in all, the results obtained show that the treatment of the sub-ballast layer makes it possible to reduce the means and dispersions of the control variables of considerable manner when compared to the initial state and even after the raising of the track. The contribution of this kind of maintenance action is increased with the level of compaction imposed. In a complementary way, the results presented in Table 14 show that the treatment of the sub-ballast layer accompanied by a renewal of the ballast layer makes it possible to reduce the probability of failure for criterion C_2, dependant on the deflection of the rail, of 15% with a medium level of compaction of the underlayer and of more than 45% for a level of more important compaction (option 2). This is due to the fact that the acceleration of the sleepers is more sensitive to the variations accompanying this kind of maintenance action, as previously noticed. Generally, for the model of track analyzed, the treatment of the sub-ballast layer is more beneficial than the operations of raising of the track.

However, it should be pointed out that it is difficult to impose a level of compaction making it possible to have a mean of the Young's modulus equal to 70 MPa for the sub-ballast layer (option 2) without modifying the characteristics of the subgrade not being modified.

7.2.3. Drainage operation

The presence of water is detrimental to the behavior of the track and can generate risks related to the sensitivity at the hydric state of the constituent materials. One of the possible improvements of the quality of the track is the collection and the drainage of internal waters. Its role consists in avoiding the stagnation of rainwaters and of surface waters.

Improving drainage conditions involves improving the quality of the railway platform and more particularly its mechanical characteristics. We assume here that only the form layer and the sub-ballast layer require a drainage operation. Such an operation results in a significant improvement of the statistical characteristics of the Young moduli of these layers (increase of their means and decrease of their standard deviations). Table 15 gives the values of these characteristics before and after the drainage operation.

The effect of this drainage operation on the reliability of the studied track section is analyzed through, on the one hand, the mean values and the standard deviations of the control variables $(M_i)_{1 \le i \le 3}$ and, on the other hand, the failure probabilities P_f^1 and P_f^2 associated with the criteria C_1 and C_2, respectively. The obtained numerical results are given in Tables 16 and 17. In each of them, the gain due to the drainage operation is mentioned between parentheses.

From the numerical values of Table 16, we notice that the drainage is the maintenance action leading to an important improvement of the various control variables (mainly those related to the deflection of the rail and of the sleepers and especially on the acceleration of the sleepers). Moreover, we can notice that an operation of treatment of the platform, by an

Table 15
Mean values and standard deviations of the random parameters $(Y_i)_{1 \le j \le 4}$ before and after the drainage operation.

	Random parameters	Mean values	Standard deviations
Before the drainage operation	$Y_1 = h_b$	0.26 m	0.11 m
	$Y_2 = E_b$	38.91 MPa	11.27 MPa
	$Y_3 = E_{sb}$	26.55 MPa	18.27 MPa
	$Y_4 = E_{fl}$	9.21 MPa	5.84 MPa
After the drainage operation	$Y_1 = h_b$	0.26 m	0.11 m
	$Y_2 = E_b$	38.91 MPa	11.27 MPa
	$Y_3 = E_{sb}$	40.00 MPa	8.00 MPa
	$Y_4 = E_{fl}$	30.00 MPa	3.00 MPa

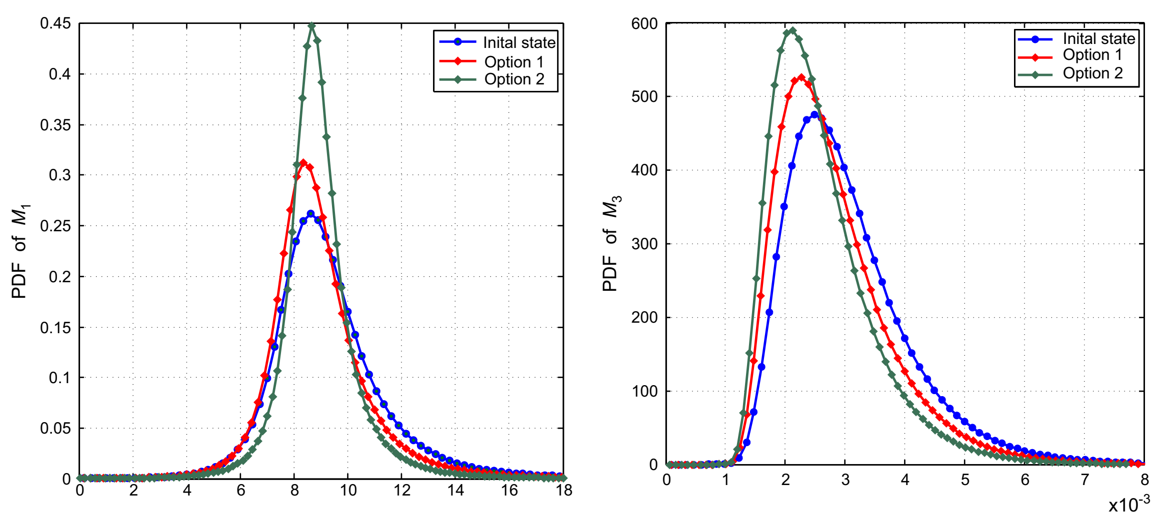

Fig. 8. Comparison, for M_1 and M_3, of the PDF's obtained before (*i.e.* in the initial state) and after the treatment of the track section (options 1 and 2).

Table 16
Mean values and standard deviations of the control variables $(M_i)_{1 \le j \le 3}$ before and after the drainage operation.

	Control variables	Mean values	Standard deviations
Before the drainage operation	M_1	10.611 m/s^2	2.891 m/s^2
	M_2	3.35 mm	1.10 mm
	M_3	3.00 mm	1.06 mm
After the drainage operation	M_1	8.397 m/s^2 (−20.9%)	2.173 m/s^2 (−24.8%)
	M_2	2.39 mm (−28.7%)	0.62 mm (−43.6%)
	M_3	2.04 mm (−32.0%)	0.61 mm (−42.4%)

Table 17
Failure probabilities P_f^1 and P_f^2 before and after the drainage operation.

	P_f^1 (criterion C_1)	P_f^2 (criterion C_2)
Before the drainage operation	0.0762	0.0625
After the drainage operation	0.0023 (−97.0%)	0.0018 (−97.1%)

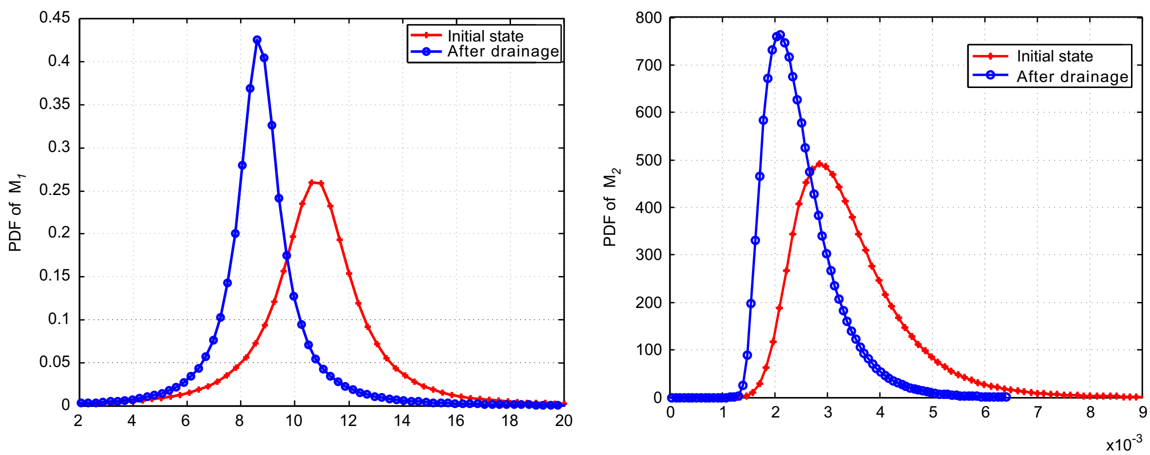

Fig. 9. Comparison of the PDF's of M_1 and M_2 before and after the drainage operation.

operation of drainage, makes it possible to reduce of more than 95% (see Table 17), for the criteria of failure selected here, the probability of exceeding the fixed thresholds. The contribution of this kind of operation in term of safety is obvious. It is primarily due to the fact that this kind of operation improves the quality of the most influential parameters on the behavior of the track. The PDF's of M_1 and M_2 confirm these results (see Fig. 9).

7.2.4. *Conclusions on the reliability analysis of maintenance operations effects*

In this section, we have examined the influence of different operations of treatment of the railway platform on the response of the track in term of probability of failure. In this kind of analysis, the main difficulty lies in estimating the impact of a maintenance actions on the characteristics of the railway track. From the presented results, we can note that an operation of drainage, represented by an increase in the Young's modulus of the intermediate layers, involves a significantly improvement of the analyzed parameters. This effect is beneficial because it reduces in all the cases the dispersion of these variables by improving their mean. We also could notice that this operation had an effect on the control variables more important than that of the raising operation. Indeed, in the case of an operation of drainage, the quality of the mechanical characteristics of the sub-ballast layer and of the sub-grade are improved, which are parameters having a great influence on the response of the system. However, an operation of drainage is technically more complicated than an operation of raising, and especially more expensive.

8. Conclusions

Currently, railway engineering has no method allowing the integration of the track variability and/or providing the real impact of maintenance operations on the track behavior. It is therefore difficult for the railway track managers to determine the performance of a specific maintenance operation compared to others.

Usually, the choice of the maintenance interventions is based on empirical methods from the measurement and the evolution of the global track stiffness and geometry without taking into account the local state and the variability of the different components of the track. As a result, railway engineering cannot establish a maintenance management policy appropriate to each type of defect and thus treat the maintenance actions on a hierarchical basis.

A reliability analysis of different maintenance operations, adapted to the problem of railway tracks has been proposed in this paper. This analysis is based both on a methodology of track diagnosis which provides a reliable evaluation of the mechanical and geometric variables and of their variability of the different track components and on a probabilistic modeling of the track based on a SFE method. This method allows the estimation of statistical moments of any control variable related to the response process. Finally the reliability analysis is based on a parametric study to quantify probabilistically the contribution of various maintenance operations: raising of the track, sub-ballast layer treatment and drainage.

The different operations have been compared from their contribution to reduce the probability of failure defined from the probability of exceeding allowable threshold values of the rail deflection (criteria 1) and of the peak sleeper acceleration (criteria 2). This analysis has shown the great contribution of the latter two types of treatment in terms of reducing the dispersion of the model response and especially to reduce the risk of default. The contribution of drainage operations remains more beneficial for the behavior of the track system in terms of safety.

Finally, a better knowledge of the impact of maintenance actions on the characteristics of the railway track will still improve this analysis.

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