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The risk of vulnerable failure

J.T. Pinto*, D.I. Blockley, N.J. Woodman

Department of Civil Engineering, University of Bristol, Bristol BS8 1TR, UK

Abstract

There is, as yet, no generally accepted theory of structural robustness or of its corollary, structural vulnerability. The theory of structural vulnerability developed at the University of Bristol is a theory of form that seeks to identify failure scenarios where small damage can lead to disproportionate consequences. In this paper the previously reported theory is combined with a standard response analysis to produce a measure of structural risk which includes the chance of a vulnerable loss of functionality. A simple example of a pin jointed structure is presented.

Keywords: Vulnerability; Reliability; Risk; Progressive failure; Structure

1. Introduction

Structural vulnerability theory, as developed at Bristol [1], is a theory of structural form and connectivity. The purpose of the theory is to identify the most vulnerable parts of a structure. A structure, or part of it, is vulnerable when some damage to the structure causes disproportionate consequences. Vulnerability is different from reliability but is one aspect of risk. In vulnerability analysis the damage can come from any action and the analysis addresses those consequences that are disproportionate to that damage. The purpose of article is to begin to integrate structural vulnerability analysis with standard structural response and risk analysis.

2. Structural vulnerability theory

A detailed description of the methodology for structural vulnerability analysis has been given in [1–4]. In the theory the vulnerable parts of a structure are described in terms of five vulnerable scenarios. These are the *total*, the *maximum*, the *minimum*, the *minimum demand* and the '*interesting*' failure scenarios. The interesting failure scenarios are the ones in which the designer is

* Corresponding author. Tel.: +44-117-928-8262; fax: +44-117-928-7783.

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specifically interested for local reasons such as a sensitivity to a particular usage or piece of equipment. The *total* and the *maximum failure scenarios* are the most relevant here. The *total failure scenario* is the one where least effort is required for the whole structure to become a mechanism and to become separated from a reference structure (usually the ground). The *maximum failure scenario* is the one that results in maximum damage from least effort and may not be total.

In brief, the analysis for the identification of the vulnerable scenarios consists in two main stages: the *clustering process* and the *unzipping process*. These processes have been reported previously [1] and so a detailed explanation is not included here.

2.1. The clustering process

In the first stage clustering a structure is represented as a hierarchy of interconnected structural rings (i.e. hierarchical model of a structure). These rings form clusters which form the whole structure. The theoretical basis for this process is graph theory. A structural ring is the basic unit of structure. It is a load path which, by its configuration, is capable of resisting an arbitrary set of applied forces. A structural cluster is a subset of the graph model of the whole structure in which the objects are in some sense defined more tightly connected to each other than to other objects outside of the cluster (e.g. a *branch cluster* is any cluster comprising sub-clusters). In the simplest case, a cluster can be a set of two joint objects and one member object (e.g. a *leaf cluster* is the most primitive cluster) and in a complex case, it can be a large number of joint objects and member objects including the ground. A *reference cluster* is the cluster from which the structure is separated in a total failure scenario and is normally the ground.

A hierarchical model of a structure (in which fault trees are a special case) is built using a previously reported clustering algorithm [1] with five clustering criteria. The first and most important of these is the *well-formedness* (Q) of a piece of structure. The others include the *minimum damage demand* ($D_{\min}$), the *nodal connectivity* (N) and the *distance from the reference* (D_{is}). If none of the above criteria apply then a free choice is made (F_{C}). *Well-formedness* is a measure of the quality of the form of a joint that is independent of the co-ordinate system but depends on the principal stiffness coefficients of the joint, the type of joint and the stiffness and configuration of the members framing into it. The *well-formedness* of a ring or a cluster (Q) is the average of the well-formedness of the all joints within that cluster [Eq. (1)].

$$Q = \frac{\sum q_i}{n} \text{ summed over all i} \tag{1}$$

where n is the total number of joints in the ring and q_i [Eq. (2)] is the well-formedness of the ith joint in the ring.

$$q_i = \det(K_{ii}) = \lambda_1 \times \lambda_2 \times \lambda_3 \tag{2}$$

where $\det(K_{ii})$ is the determinant of the stiffness submatrix associated with a joint i. K_{ii} is the sum of the stiffness submatrices of all members contained in the ring that meet at joint i. Eigenvalues λ_1, λ_2 are the principal translational stiffness coefficients of a joint i, and λ_3 is the rotational stiffness coefficient.

The *damage demand* of a cluster is a measure of the effort required to cause damage. It is assumed to be directly proportional to the loss of the principal stiffness caused by a deteriorating event. The *damage demand* of a failure scenario is the sum of the damage demands for each deteriorating event in that failure scenario.

2.2. The unzipping process

The second stage 'unzipping' process is to use the cluster hierarchy of the structure to search for various failure scenarios. The search process for the total failure scenario and for the maximum failure scenario are quite similar except that the scale of the search may be different. The unzipping process starts at the very top of the hierarchy and checks each ring until a cluster is found that can be damaged by inserting a damage event such as a pin or cut. This defines the first deteriorating event (E_1). This, of course, changes the form of the structure and a new hierarchy is built. This then is unzipped in the same way to produce a second deteriorating event (E_2). This process continues until the structure or part of it becomes a mechanism.

When unzipping a cluster the ring forming that cluster (one level down) is examined and the sub-cluster within that ring is chosen, using the following criteria in order, which:

- is not a reference cluster (N_R)
- forms a ring with the reference cluster (F_R)
- connects directly to the reference cluster (but does not form a ring with the reference cluster) (C_D)
- is a leaf cluster rather than a branch cluster (L)
- has the smallest well-formedness (S_Q)
- has the smallest minimum damage demand (S_D)
- was clustered the latest (C_L).

If none of the above criteria apply then a free choice (F_C) is made. As stated above this process of choosing sub-clusters continues down the hierarchy until a ring is found at the lowest level in that part of the hierarchy. That ring is then damaged by a deteriorating event. The structure is then re-clustered and the whole process repeated. This continues until the structure or part of it becomes a mechanism. The sequence of damage events is a failure scenario and is a candidate for one of the important scenarios we are looking for.

The measures of the failure scenarios used are as follows:

(a) S*eparateness* γ is a measure of failure consequence and is the ratio of the loss in structural well-formedness of the deteriorated structure to the well-formedness of the intact structure, i.e.

$$\gamma = \frac{Q(S) - Q(S')}{Q(S)} \tag{3}$$

where $Q(S)$ is the well-formedness of the intact structure S and $Q(S')$ is the well-formedness of the deteriorated structure S'.

(b) *Relative damage demand* D_r is the ratio of the damage demand of the failure scenario to the maximum possible damage demand of a failure scenario in the structural system.

$$D_r = \frac{D}{D_{max}} \tag{4}$$

where D is the damage demand and D_{max} is the sum of the damage demand of all members.

(c) *Vulnerability index* φ is a measure of the vulnerability of a structure and is the ratio of the separateness to the relative damage demand for a given failure scenario [Eq. (5)]. Thus, the vulnerability index is a measure of the "disproportionateness" of the consequences (the separateness) to the damage (the damage demand).

$$\varphi = \frac{\gamma}{D_r} \tag{5}$$

The total failure scenario is the one, amongst all of the possible failure scenarios, that has a separateness equal to one and the highest vulnerability index. The maximum failure scenario is the one that has highest vulnerability index.

A vulnerable scenario is an ordered set of deteriorating events (E_i). Here we restrict these events to the insertion of a pin or cut in a member.

3. Consequence of a failure scenario

As described above, separateness is a measure of the failure consequence. When the value of separateness γ is equal to zero there is no failure scenario and hence no failure consequence. When γ is equal to one then the failure scenario will cause complete separation (i.e. total collapse).

The emphasis of structural vulnerability analysis is not the usual one of structural response analysis where a structure is examined under specific loading conditions—the nature of the action causing the damage is not considered. The cause of the deteriorating events can be any kind of action including relatively unexpected ones such as vehicle impact, explosion or sabotage.

In this preliminary consideration a variable X is used as a simple measure of the ultimate consequences of a failure scenario in a response analysis. X is equal to zero when the structure, after some damage to a ring, still functions and X is equal to one when the damaged structure no longer functions (i.e. a limit state is exceeded).

The functional consequence of a failure scenario C should include both a degree of separateness and a loss of functionality. We define it as:

$$C = \max\{\gamma, X\} \tag{6}$$

where γ is the separateness of the failure scenario and X is a measure of the functionality of the structure.

At this stage we will consider only the dead load since it is ever present. The occurrence of an unexpected action with a simultaneously strong windy condition or an excessive imposed load is less likely and will be considered later. Furthermore, if the damaged structure still satisfies the limit states under the dead load then there may well be time to carry out some repair.

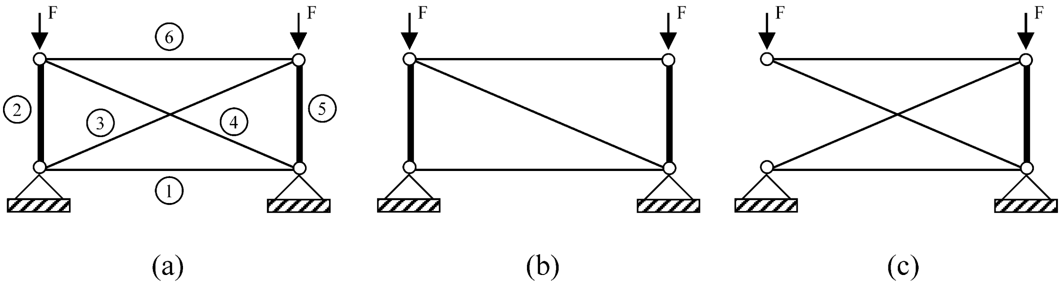


Fig. 1. An example structure.

Fig. 1(a) illustrates a very simple pinned structure with six members. The dead load is F. If member 3 is damaged by inserting a pin or a cut, Fig. 1(b), then under no load, the structure remains a structure. It may remain a structure under dead load if all of the other members still satisfy the limit states (of course this may be simply checked by a response analysis). In this case the structure keeps its functionality and so $X = 0$. However, there will be a loss in structural well-formedness through the loss of member 3. Thus according to Eq. (6), $C = \max\{\gamma_3, X\} = \max\{\gamma_3, 0\} = \gamma_3$.

If member 2 were to be similarly damaged [Fig. 1(c)], then again under no load the structure remains. However it is most likely that under dead load, the forces will redistribute and the structure will collapse progressively. If this were to happen then there is a loss of functionality with $X = 1$. The functional consequence of this failure scenario is $C = \max\{\gamma_2, X\} = 1$. It is quite possible for the separateness of this failure scenario to be not equal to one because member 1 may still be connected to the supports.

4. Risk of a failure scenario

We define risk as being the combined effect of the chances of occurrence of some failure or disaster and its consequences **in a given context** [5].

$$R(\text{context}) = p \times \text{Con} \tag{7}$$

where: R(context) is the risk of a failure in a given context;

p is the chance of a failure;
Con is the consequence of a failure.

Within the totality of all possible modes of failure (including business and financial failure) we define the structural risk, SR, as **one aspect** (structural functionality) of the total project risk as:

$$SR = p_{\mathrm{f}} \times C \tag{8}$$

where p_{f} is the probability of failure which is calculated using the techniques of structural reliability analysis and C is as defined in Eq. (6).

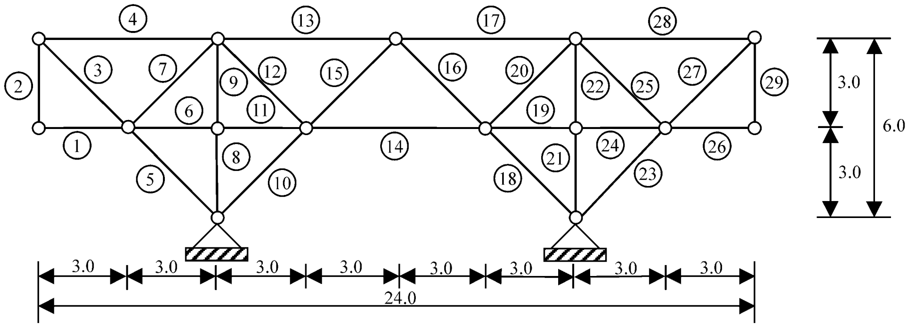


Fig. 2. The structure used as an example.

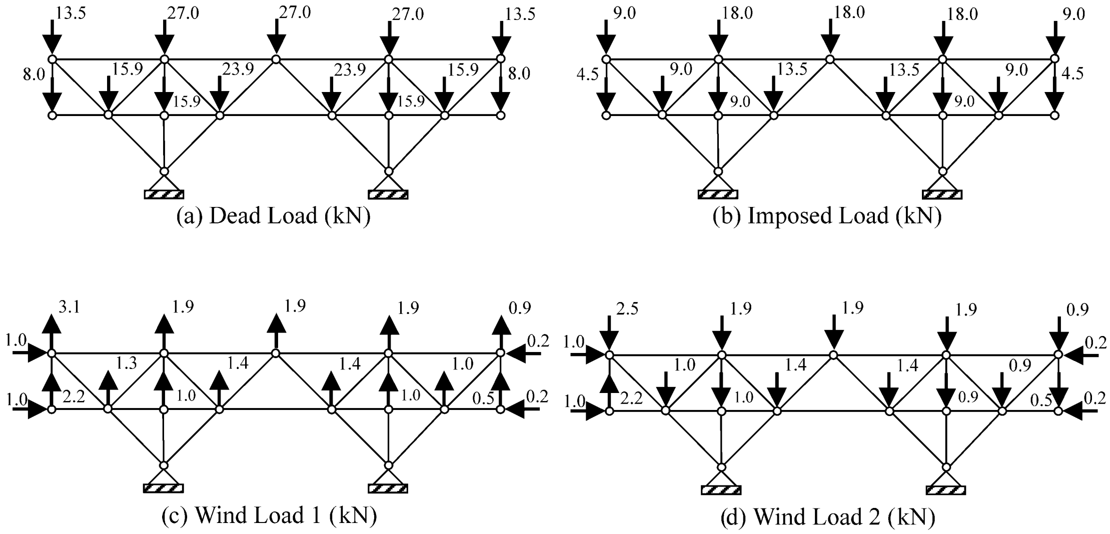


Fig. 3. Loads (a) dead load (kN), (b) imposed load (kN); (c) wind load 1 (kN), (d) wind load 2 (kN).

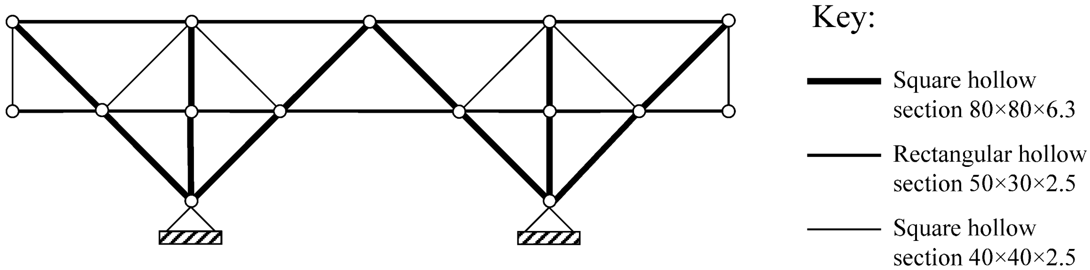
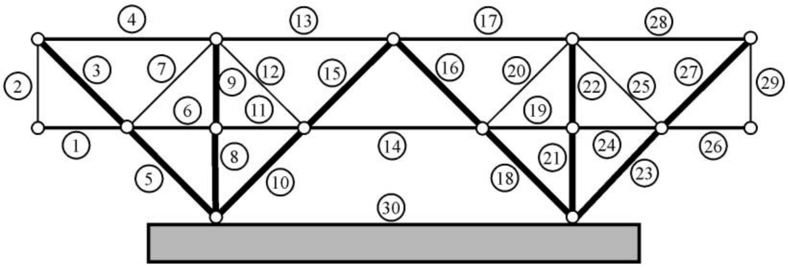


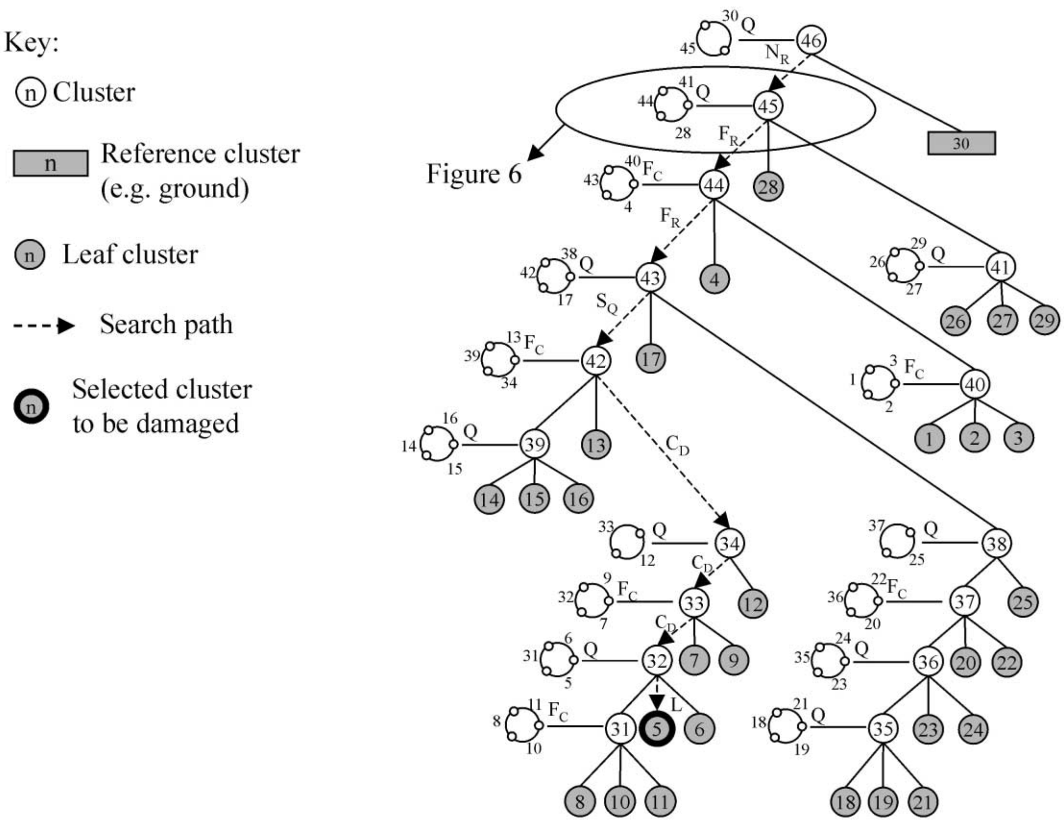
Fig. 4. Hot rolled steel beams adopted for each member.

Table 1
Search for the first deterioration event (E_1) within a total failure of Cluster 46

Cluster 46



The hierarchical model of Cluster 46



• Start from cluster 46 with two child clusters 30 and 45. Cluster 30 is a reference cluster so by criteria N_R choose cluster 45 to search down

• Cluster 45 has child clusters 28, 41 and 44. None is a reference cluster. Cluster 44 forms a ring with the reference cluster. Cluster 44 is chosen to search down by the unzipping criteria F_R. Focus on cluster 44 and bypass clusters 28 and 41

Table 1 (*continued*)

Cluster 46
• Search down cluster 44. Cluster 44 has child clusters 4, 40 and 43. None is a reference cluster. Cluster 43 forms a ring with the reference cluster. Cluster 43 is chosen to search down by the unzipping criteria F_R. Focus on cluster 43 and bypass clusters 4 and 40
• Search down cluster 43. Cluster 43 has child clusters 17, 38 and 42. None is a reference cluster. Clusters 38 and 42 form a ring with the reference cluster. Both are branch clusters. Cluster 42 has smaller well-formedness than cluster 38 thus, focus on cluster 42 and bypass clusters 17 and 38 (S_Q)
• Search down cluster 42. Cluster 42 has child clusters 13, 34 and 39. None is a reference cluster. Cluster 34 connects directly to the reference cluster (C_D). Focus on cluster 34 and bypass clusters 13 and 39
• Search down cluster 34. Cluster 34 has child clusters 12 and 33. None is a reference cluster. Cluster 33 connects directly to the reference cluster (C_D). Focus on cluster 33 and bypass cluster 12
• Search down cluster 33. Cluster 33 has child clusters 7, 9 and 32. None is a reference cluster. Cluster 32 connects directly to the reference cluster (C_D). Focus on cluster 32 and bypass clusters 7 and 9
• Search down cluster 32. Cluster 32 has child clusters 5, 6 and 31. None is a reference cluster. Clusters 5 and 31 connect directly to the reference cluster. Select cluster 5 to damage because it is a leaf (L). The deterioration event required to cause failure in cluster 5 is to insert a pin. This is the first event (E_1) in the failure scenario
• The failure of leaf cluster 5 does not result in the total failure of cluster 46 or in total failure of the whole structure (under no load). The search of cluster 46 continues to find a second deteriorating event

5. An example

5.1. The structure

Fig. 2 shows an example pin jointed structure with 29 members.

For the design of the structure four types of loads were considered (Fig. 3). These loads are calculated according to BS 6399, they are simulated by concentrated forces and measured in kN.

Fig. 4 shows the type of steel sections designed for each member of the structure in order to satisfy the ultimate limit states and the serviceability limit states according to BS5950 and for different load combinations.

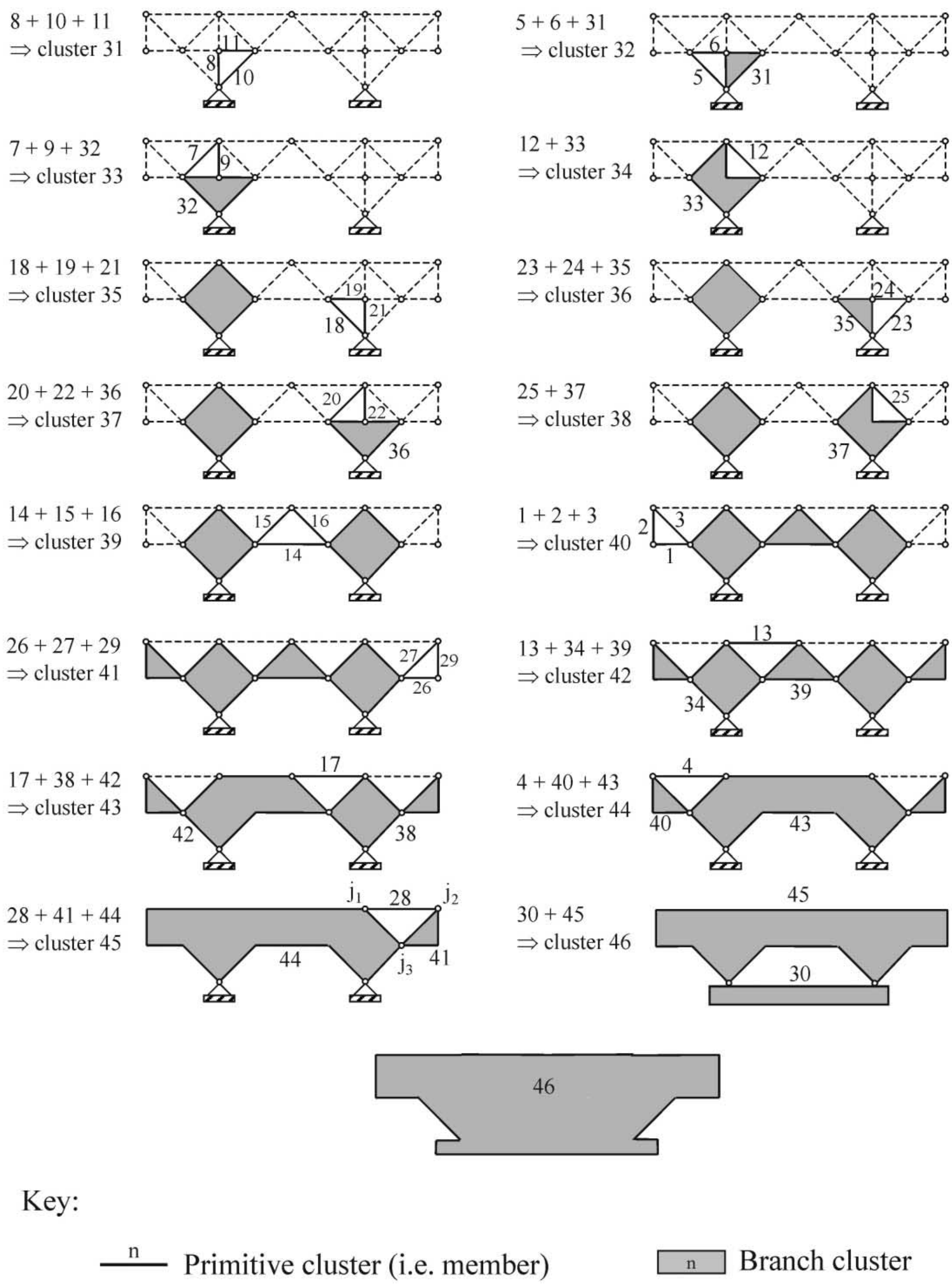
5.2. The clustering and unzipping

The first stage of the vulnerability analysis is the clustering process as shown in Fig. 5. This process finishes when the whole structure (including the ground) is the single cluster 46. The hierarchical model is shown in Table 1. This table together with Tables 2 and 3 also sets out, step by step, the unzipping of cluster 46.

The resulting failure scenario is to insert pins in members 5, 10 and 8 (by symmetry also 23, 18 and 21) and is shown as failure scenario No. 32 (33) in Table 4.

5.3. Searching branch clusters

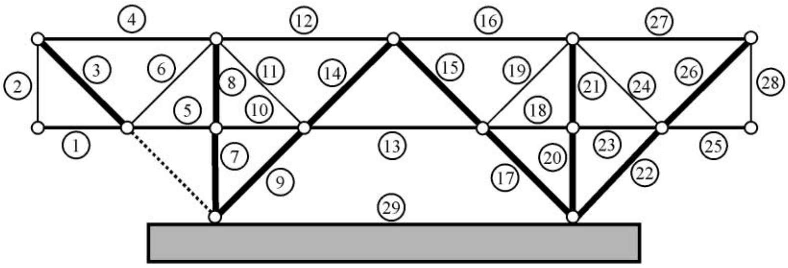
As well as searching the total structure as described above it is also necessary to search all of the branch clusters for local failures. We will illustrate this process using cluster 42, Fig. 7(a). When



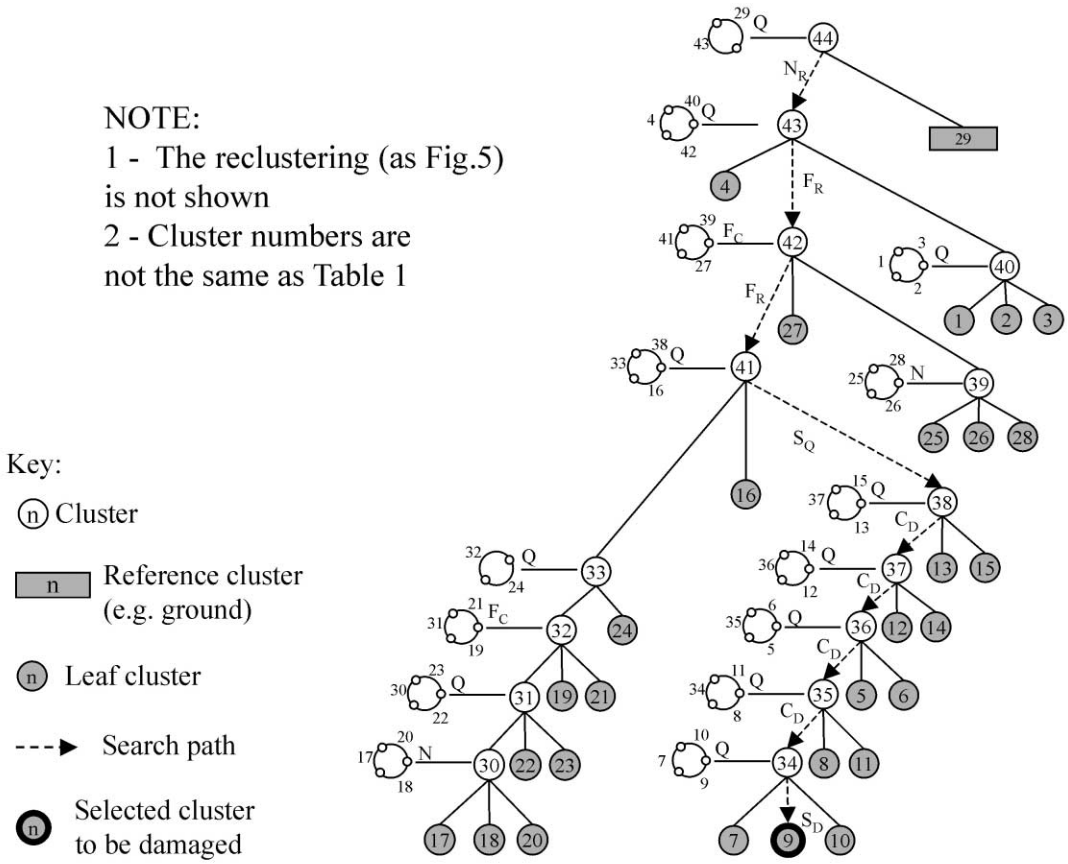
[Fig. 5. Clustering process (read left to right).]

Table 2
Search for the second deteriorating event (E_2) within a total failure of Cluster 46

New cluster 44 (i.e. cluster 46 of Table 1 without member 5)



The hierarchical model of new cluster 44 (i.e. cluster 46 of Table 1 without member 5)



• Start from cluster 44 with two child clusters 29 and 43. Cluster 29 is a reference so by the criteria N_R choose cluster 43 to search down

• Cluster 43 has child clusters 4, 40 and 42. None is a reference cluster. Cluster 42 forms a ring with the reference cluster. Cluster 42 is chosen to search down by the unzipping criteria F_R. Focus on cluster 42 and bypass clusters 4 and 40

Table 2 (*continued*)

New cluster 44 (i.e. cluster 46 of Table 1 without member 5)
• Search down cluster 42. Cluster 42 has child clusters 27, 39 and 41. None is a reference cluster. Cluster 41 forms a ring with the reference cluster. Cluster 41 is chosen to search down by the unzipping criteria F_R. Focus on cluster 41 and bypass clusters 27 and 39
• Search down cluster 41. Cluster 41 has child clusters 16, 33 and 38. None is a reference cluster. Clusters 33 and 38 form a ring with the reference cluster. Both are branch clusters. Cluster 38 has smaller well-formedness than cluster 33 thus, focus on cluster 38 and bypass clusters 16 and 33 (S_Q)
• Search down cluster 38. Cluster 38 has child clusters 13, 15 and 37. None is a reference cluster. Cluster 37 connects directly to the reference cluster (C_D). Focus on cluster 37 and bypass clusters 13 and 15
• Search down cluster 37. Cluster 37 has child clusters 12, 14 and 36. None is a reference cluster. Cluster 36 connects directly to the reference cluster (C_D). Focus on cluster 36 and bypass clusters 12 and 14
• Search down cluster 36. Cluster 36 has child clusters 5, 6 and 35. None is a reference cluster. Cluster 35 connects directly to the reference cluster (C_D). Focus on cluster 35 and bypass clusters 5 and 6
• Search down cluster 35. Cluster 35 has child clusters 8, 11 and 34. None is a reference cluster. Cluster 34 connects directly to the reference cluster (C_D). Focus on cluster 34 and bypass clusters 8 and 11
• Search down cluster 34. Cluster 34 has child clusters 7, 9 and 10. None is a reference cluster. Clusters 7 and 9 connect directly to the reference cluster. Clusters 7 and 9 are leaf clusters. Cluster 9 has smaller minimum damage demand than cluster 7 thus, select cluster 9 to damage (S_D) and bypass cluster 7. The deterioration event required to cause failure in cluster 9 is to insert a pin. This is the second event (E_2) in the total failure scenario. Cluster 9 is cluster 10 in the original structure of Table 1.
• The failure of leaf clusters 5 and 10 (of the original structure) does not result in the total failure of cluster 46 or in total failure of the whole structure (under no load). The search of cluster 46 continues to find a third deteriorating event

isolated, as in Fig. 7(b), cluster 42 is attached to the reference cluster (the ground) through the other parts of the structure. For the search we absorb these other parts into the reference cluster to create an extended reference cluster [e.g. 13 and 14 in Fig. 7(b)]. The search then proceeds as before and results in failure scenario 25 in Table 4.

5.4. The results

Table 4 shows the 45 possible failure scenarios for this example. In 26 of them the whole structure collapses (i.e. has a separateness equal to one e.g. Nos. 10, 11). The total failure scenario has a separateness of one and the highest vulnerability index. Thus failure scenario 25 is total with a pin inserted into member 13 followed by a pin inserted into member 14. Likewise failure 26 is total with pins inserted in members 17 and 14.

The total failure scenario is also the maximum failure scenario in this example because it has the highest vulnerability index regardless of the separateness.

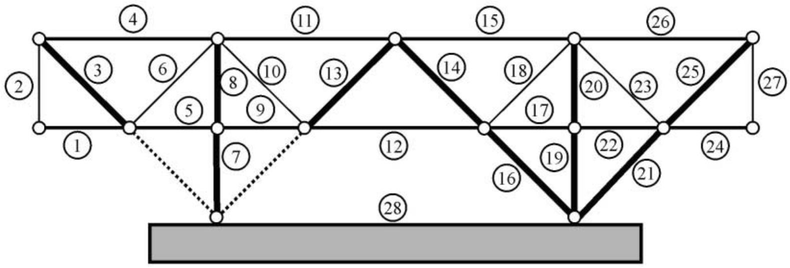
Let us assume that the probability of accidental damage to member 13 (E_1) is $P[E_1] = 1 \times 10^{-3}$.

A second order reliability method (SORM) produces a probability of failure of member 14 (E_2) under dead and imposed loads, given failure of member 13, of

$$P[E_2|E_1] = 2.2 \times 10^{-6}$$

Table 3
Search for the third deteriorating event (E_3) within a total failure of Cluster 46

New cluster 42 (i.e. cluster 46 of Table 1 without members 5 and 10)



The hierarchical model of new cluster 42 (i.e. cluster 46 of Table 1 without members 5 and 10)

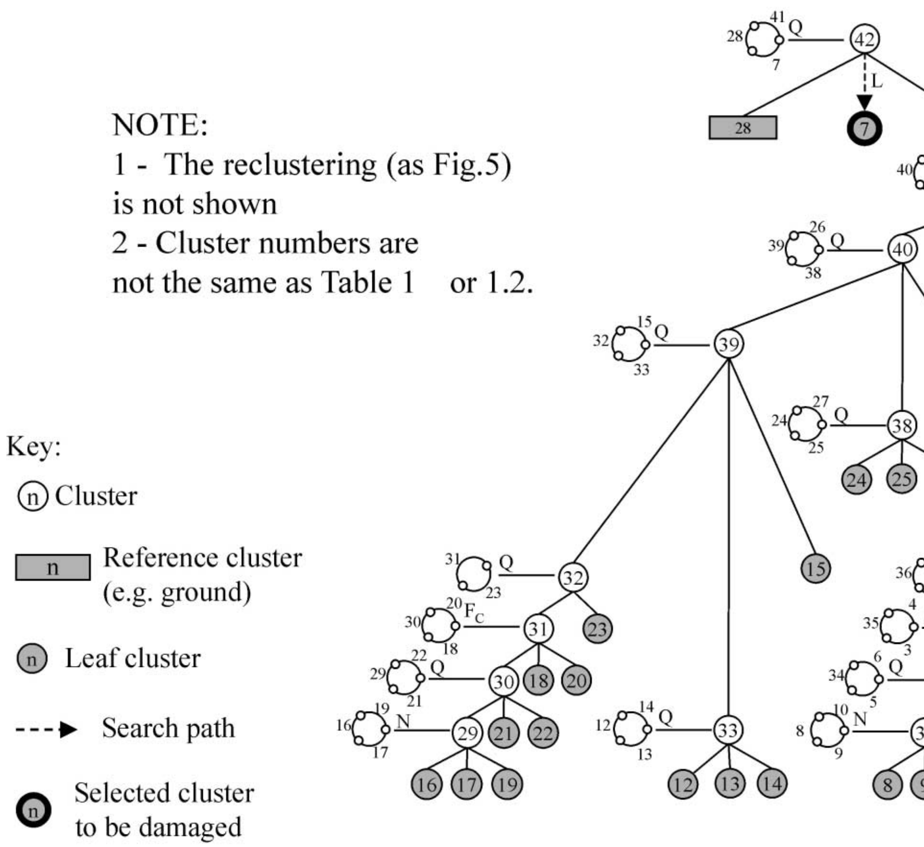
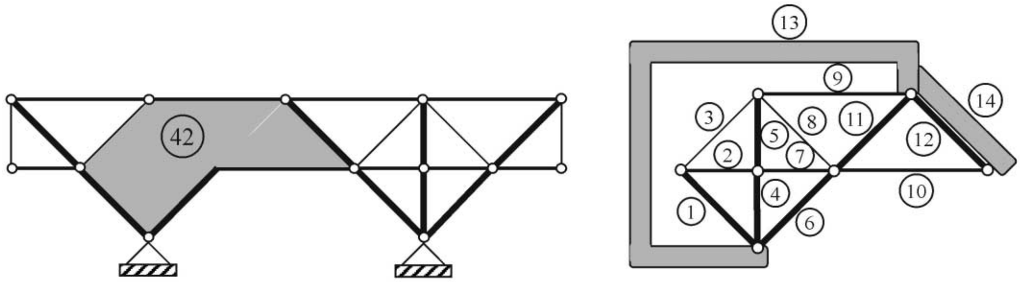


Table 3 (*continued*)

New cluster 42 (i.e. cluster 46 of Table 1 without members 5 and 10)

• Start from cluster 42 with three child clusters 7, 28 and 41. Clusters 7 and 41 are not reference clusters. They form a ring with the reference cluster. Cluster 7 is a leaf thus, select cluster 7 by the unzipping criteria L and bypass clusters 28 and 41. The deterioration event required to cause failure in cluster 7 is to insert a pin in. This is the third event (E_3) in the failure scenario. Cluster 7 is cluster 8 of the original structure in Table 1

• The failure of clusters 5, 10 and 8 (in the original structure) results in the total failure of cluster 46 and hence the whole structure. The search finishes



(a) (b)

This is a mechanism and a possible failure scenario (E_1, E_2, E_3) and by symmetry (E'_1, E'_2, E'_3) with

Relative damage demand (D_r) = 0.23

Separateness (γ) = 1.0

Vulnerability index (φ) = 4.36

For simplicity assume that scenarios 25 and 26 are totally dependent. The probability of failure (p_f) of the total and maximum failure scenarios is

$$p_f = \mathrm{MAX}(p_{f25}, p_{f26}) = 2.2 \times 10^{-9} \text{ in which}$$

$$p_{f25} = P\left[E_1 \bigcap E_2\right] = P[E_1]P[E_2|E_1] = 2.2 \times 10^{-9};$$

$$p_{f26} = p_{f25} = 2.2 \times 10^{-9}.$$

Because the collapse is total, $C = 1$.

and $SR = p_f \times C = 2.2 \times 10^{-9} \times 1 = 2.2 \times 10^{-9}$

Of course this risk is a nominal one and is not to be interpreted statistically. It indicates that although the probability of accidental damage is quite high the probability of vulnerable system failure under dead and imposed loads is very low. This would form evidence in an analysis of the total risk to the total construction process [6].

Table 4
A summary of the possible failure scenarios

No.	Searched cluster[a]	Failure scenario[b]	Relative damage demand (D_r)	Separateness (γ)	Vulnerability index (φ)
1	31	m_8	0.12	0.21	1.76
2	31	m_{10}	0.06	0.26	4.69
3	31	m_{11}	0.01	0.07	8.57
4	34	m_{12}	<0.01	0.03	10.64
5	35, 37	m_{18}	0.06	0.26	4.69
6	35	m_{19}	0.01	0.07	8.57
7	35	m_{21}	0.12	0.21	1.76
8	37	m_{20}	<0.01	0.03	10.64
9	39	m_{14}	<0.01	0.14	99.41
10	39	m_{15}	0.06	1.00	17.98
11	39	m_{16}	0.06	1.00	17.98
12	40	m_3	0.06	0.05	0.83
13	41	m_{27}	0.06	0.05	0.83
14	31	m_8+m_{10}	0.17	0.29	1.45
15	31	m_8+m_{11}	0.13	0.31	2.41
16	31	$m_{10}+m_{11}$	0.06	0.23	3.51
17	34	$m_{10}+m_{12}$	0.06	1.00	17.08
18	35	$m_{18}+m_{19}$	0.06	0.23	3.51
19	35	$m_{18}+m_{21}$	0.17	0.29	1.45
20	35	$m_{19}+m_{21}$	0.13	0.31	2.41
21	37	$m_{18}+m_{20}$	0.06	1.00	17.08
22	39	$m_{14}+m_{15}$	0.06	1.00	17.52
23	39	$m_{14}+m_{16}$	0.06	1.00	17.52
24	39	$m_{15}+m_{16}$	0.11	1.00	8.99
25	42	$m_{13}+m_{14}$	<0.01	1.00	349.01
26	42	$m_{17}+m_{14}$	<0.01	1.00	349.01
27	41, 40	$m_{29}+m_{27}$	0.06	0.05	0.75
28	41, 40	m_2+m_3	0.06	0.05	0.75
29	35, 31	$m_{11}+m_{10}+m_8$	0.18	1.00	5.49
30	35, 31	$m_{19}+m_{18}+m_{21}$	0.18	1.00	5.49
31	39	$m_{14}+m_{15}+m_{16}$	0.11	1.00	8.87
32	46, 45, 44, 43	$m_5+m_{10}+m_8$	0.23	1.00	4.36
33	46, 45, 44, 43	$m_{23}+m_{18}+m_{21}$	0.23	1.00	4.36
34	33	$m_7+m_9+m_{10}$	0.18	1.00	5.67
35	33	$m_{25}+m_{22}+m_{18}$	0.18	1.00	5.67
36	32	$m_{11}+m_{10}+m_6$	0.07	1.00	13.73
37	32	$m_{19}+m_{18}+m_{24}$	0.07	1.00	13.73
38	37	$m_{20}+m_{22}+m_{19}+m_{18}$	0.19	1.00	5.40
39	37	$m_{12}+m_9+m_{11}+m_{10}$	0.19	1.00	5.40
40	36	$m_{24}+m_{23}+m_{19}+m_{18}$	0.13	1.00	7.78
41	36	$m_6+m_5+m_{11}+m_{10}$	0.13	1.00	7.78
42	34	$m_{12}+m_7+m_9+m_{10}$	0.18	1.00	5.57
43	34	$m_{20}+m_{25}+m_{22}+m_{18}$	0.18	1.00	5.57
44	38	$m_{25}+m_{20}+m_{22}+m_{19}+m_{18}$	0.19	1.00	5.32
45	38	$m_7+m_{12}+m_9+m_{11}+m_{10}$	0.19	1.00	5.32

[a] Cluster numbers are the same as Table 1.
[b] A pin inserted in members (m_i) as defined in Fig. 2.

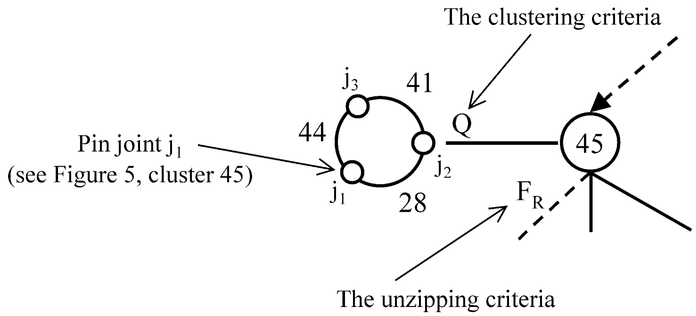


Fig. 6. Abstract representation of cluster 45 with a 3-link-ring.

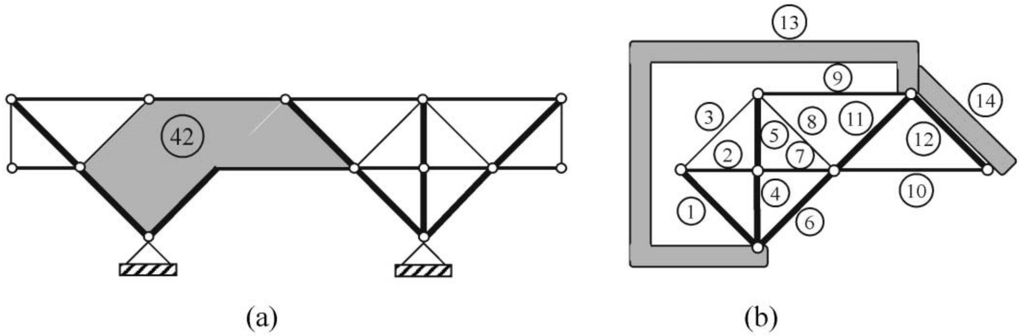


Fig. 7. Branch cluster 42.

6. Conclusions

1. Structural vulnerability theory as developed at the University of Bristol has been used together with a simple standard response analysis to identify important failure risks.
2. Structural risk has been defined in a specific way as one aspect of total risk. It is the product of the chance of a specific failure scenario and the specific consequence C. C is unity if functionality is lost but is otherwise the separateness of that part of the damaged structure from that which is left when functionality is maintained.
3. The search process described has been programmed to cope with simple but practical 2D structures.
4. Combining structural vulnerability analysis with standard structural response analysis offers a potentially richer analysis of structural risk. The information will help to decide if an existing structure is susceptible to progressive collapse and whether it should be suitably protected and monitored. The methodology might also be applicable to the retrofitting or the demolition of structures.

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