TRACK QUALITY INDEX FOR HIGH SPEED TRACK

By Alfred E. Fazio[1] and J. L. Corbin[2]

(Reviewed by the Urban Transportation Division)

ABSTRACT: Railroads have long faced the problem of formulating an objective measure of track quality, termed a Track Quality Index (TQI). However, the industry has yet to formulate a standardized TQI. The Federal Railroad Administration has mandated track safety standards. These could be interpreted as the lower bound of track condition, with track design standards representing the upper bound. A Track Quality Index addresses the condition of track between these bounds. Since track geometry serves as the forcing function on a free rolling rail car, it is the logical basis for formulating a TQI. Another advantage of track geometry is its ability to be automatically measured and recorded at high speeds. Research has shown that track geometry can be represented as a periodically modulated random process. In good quality, i.e., high speed track, a significant percentage of the maintenance performed is directed towards the deterministic components. It is therefore essential to include these in a TQI for high speed track.

INTRODUCTION: WHAT IS TQI?

A Track Quality Index (TQI) is a numerical representation of the ability of railroad track to perform its design function, or more precisely, to support the train movements required of it. It is an attempt to quantify track condition, i.e., to objectively distinguish between "good" and "bad" track.

There are currently a variety of TQI's in use in North America. However, none are universally accepted, and none are officially endorsed by the American Railway Engineering Association. The implementation of scientific methods of programming track maintenance, is predicated on the development of a TQI. Attempting to formulate track maintenance standards and an associated track maintenance program in the absence of a means of quantifying track condition would be analogous to attempting to formulate the science of thermodynamics without a temperature scale. An object is defined to be "hot" or "cold" according to its temperature; so must a means be developed for quantifying track condition in order to quantify the return on each unit resource expended on track maintenance.

The approach to quantifying track condition recommended here draws upon the results of research into the statistical characterization of track geometry (3,7), which shows that track, when considered as a dynamic forcing function on a rail vehicle, is not a purely random process.

[1]Asst. General Superintendent-Track, New York City Transit Authority, Brooklyn, NY; formerly Staff Engr., Amtrak, Pennsylvania Sta., New York, NY.
[2]Pres., BRM Technologies, McLean, VA.

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Applications of TQI

A TQI has direct application to the development of track maintenance standards. Maintenance standards must be distinguished from design standards for track, which are the nominal specifications to which track should be ideally constructed. Clearly, variations to these design specifications are acceptable even in high speed passenger track. These design specifications, therefore, represent the upper bound of track maintenance standards. Few, if any, North American railroads have the financial resources to maintain track to these standards. Even those companies that could afford to maintain track at near perfect geometry would realize a very poor return on their Maintenance of Way budgets if track were maintained to design standards, since rail vehicles will safely operate over track maintained to something less than design standard.

The Federal Railroad Administration publishes (6) and mandates safety standards for various classes of track. Class designations are formulated in terms of geometric and construction (e.g., maximum number of defective cross ties per segment of track) defects, with track class related to maximum legal speeds for passenger and freight trains. However, the Federal Railroad Administration (FRA) Track Safety Standards should not be considered maintenance standards for two reasons:

1. They represent the *minimum* legal standards for a given track class.
2. Not all of the existing standards are based on vehicle performance, i.e., they are not necessarily derivatives of vehicle response to a particular track deviation.

In addition, the existing FRA standards do not address operation of passenger trains at speeds in excess of 110 mph (176 km/h).

Role of Geometric TQI

A variety of methods, other than those based on loaded track geometry, are currently being used for measuring track quality. One simple technique involves the direct assessment of physical characteristics; e.g., a count of failed cross ties constitutes a measure of track quality. However, the formulation of a TQI comprised solely of loaded track geometry is advantageous in that:

1. Track geometry is the forcing function that directly determines response of a free rolling vehicle. While track structural parameters such as defective tie counts or subgrade type functionally determine instantaneous track geometry, they are not directly related to vehicle response since their influence on vehicle performance is through their effect on geometry. Structural parameters are more appropriately related to the change in track quality than to vehicle response at a particular instant.
2. Track geometry can be accurately measured and recorded automatically at high speeds using automatic track geometry cars with onboard computers. Measurement of other track parameters such as rail wear or defective tie counts require laborious and sometimes subjective measurement.

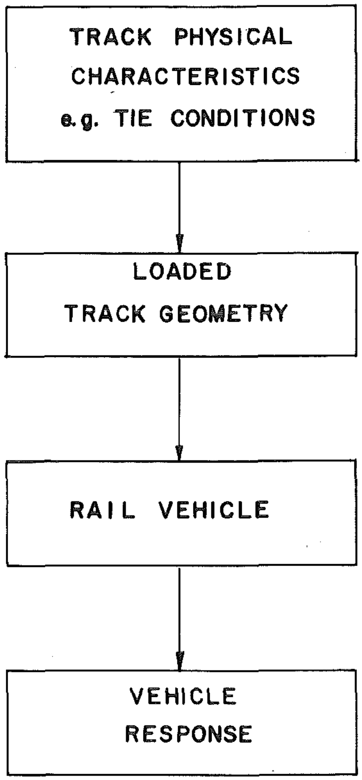

FIG. 1.—Relation between Track Geometry and Other Track Characteristics

A TQI based solely on geometry fits well into the context of an overall track maintenance planning model if the geometric TQI is supplemented by other data that pertains to the structural components of the track and to the track's usage, as shown in Fig. 1.

For example, tie condition is extremely important in determining the rate of change of track geometry ("track surface life"). Assume two hypothetical tracks, A and B, are identical in all respects, and each has 25% defective ties. If Track A receives both cross tie renewal and geometry restoration ("surfacing"), and Track B is "surfaced" only, a purely geometric TQI will show approximately the same track quality for each track subsequent to surfacing. Assuming both tracks receive the same traffic, Fig. 2 shows what might be expected to happen to the TQI over some period of time. The implicit difference in the structure of Tracks A and B, which is not apparent if the geometric TQI's for both tracks are recorded at time t_0, will be accounted for in a functional relationship that expresses the rate of track degradation.

Although it is not advantageous to include track's structural characteristics in the TQI, these factors must be included in a separate mathematical model that is capable of predicting the rate of change of a TQI,

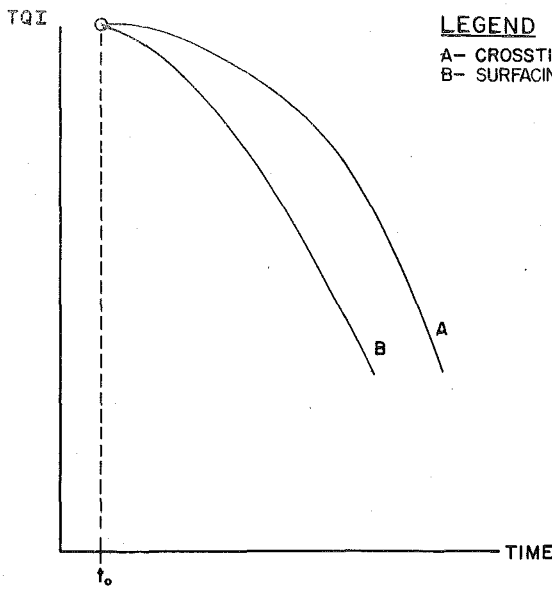

FIG. 2.—Degradation of Two Hypothetical Tracks

if the TQI is to be used for long range planning. The degradation equation will be of the form:

$$\Delta TQI = (TQI_{t_1} - TQI_{t_0}) = f(TQI_{t_0}, S, T, M) \quad (1)$$

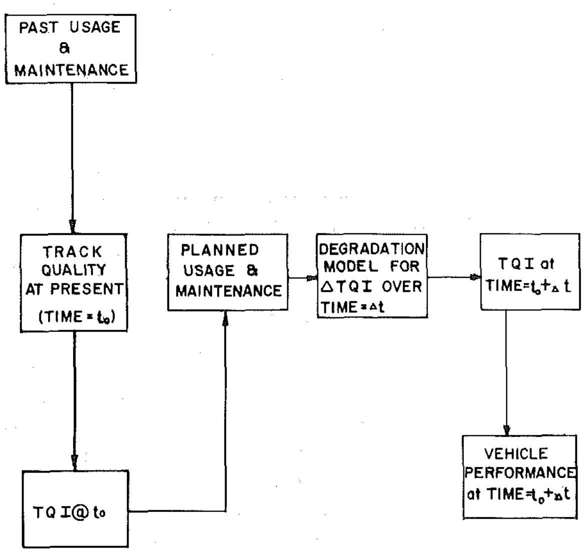

FIG. 3.—Role of TQI in MOW Planning

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where S = structural characteristics, e.g., tie and rail condition; T = usage factors, e.g., annual tonnage; and M = miscellaneous factors such as drainage conditions.

As Fig. 3 shows, the primary function of a geometric TQI in the context of a Maintenance of Way (MOW) planning system is to predict the future condition of the track as a result of a certain usage and maintenance program. This future track condition, or maintenance goal, is quantified in terms of the TQI. Track Maintenance standards are then expressible in terms of a range of acceptable TQI values.

Performance Based TQI's

As mentioned, the primary justification for limiting a TQI to a measure of track geometry is that geometry acts as a displacement forcing function to directly determine the performance of a free rolling rail vehicle. In order for a TQI to be "performance-based," it should be derived from vehicle response characteristics. It is impossible to formulate a performance-based TQI without considering the track and vehicle as one operating system. Although a number of studies have led to a variety of proposed geometric TQI's, many of these TQI's have considered the track as a self-contained system, independent of the rail car (2). For example, one study (1) advocates the use of the sample standard deviation of track guage, S^2, as a TQI. Although S^2 represents a fairly comprehensible TQI, the standard deviation statistic is not particularly relevant to the performance of individual rail vehicles and results in a TQI that is, at best, only remotely related to vehicle performance.

Track surface represents a continuous forcing function on a rail car; one in which the amplitudes and phasing of geometry deviations are critical to vehicle response. A research program that investigated typical track geometry signatures as recorded by a track geometry car concluded that track could be classified as a semi-random (displacement) forcing function (3). Track was observed to be a fundamentally random process in which certain deterministic processes are superposed. Fig. 4 shows a typical plot of the profile of bolted rail as a function of distance along the track. In this profile are superimposed two deterministic (nonrandom) processes, a rectified sine wave, which is a result of the joints (or in the case of continuous welded rail, of joint "memory"), and a transient signature, which represents a highway crossing at grade. Figs. 5(a–c) show each component of the total track geometry signature. In order to formulate a TQI, each of these component signatures must be investigated within the context of the type of vehicle response that it evokes.

Random Component.—The random component of the track forcing function can best be described by its mean Power Spectral Density (PSD). This statistic can be interpreted as the average value of the amplitude of the track geometry variation at each frequency. In Fig. 6, Curve A represents a typical PSD for FRA Class 6 (maximum authorized passenger train speeds of 110 mph) track (3). The ordinate, $I(f)$, indicates the magnitude of the geometry defects which have a frequency of f. The PSD represented by curve A is a random variable. For example, Curve A might represent mean values over 100 miles of track. However, a one mile sample with particularly severe rail corrugations might be repre-

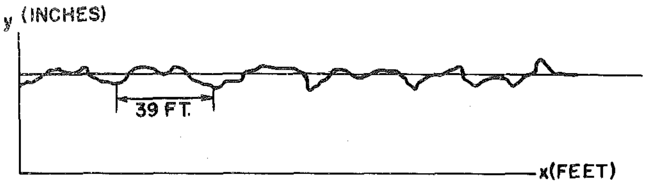

FIG. 4.—Typical Track Geometry Signature

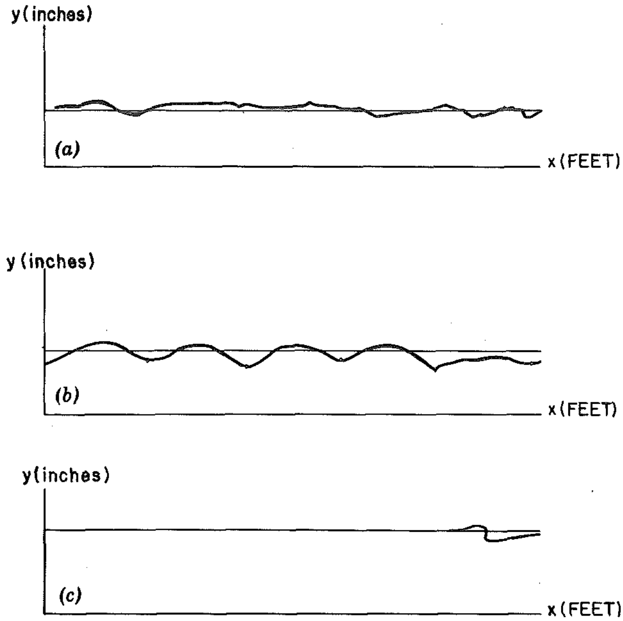

FIG. 5.—Track Profile: (*a*) Random Component; (*b*) Periodic Component; (*c*) Transient Component

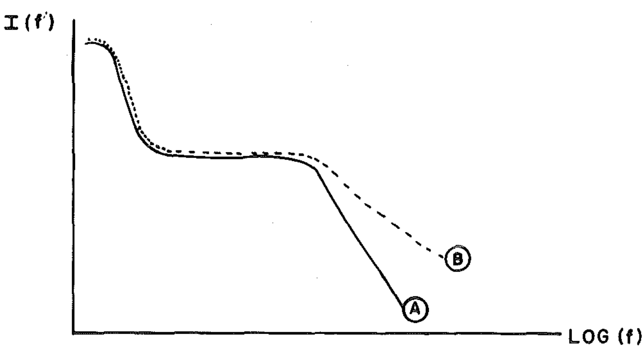

FIG. 6.—Typical PSD for Class 6 Track

sented by Curve B. As this figure shows, the rail with severe corrugations shows a higher occurrence of high frequency deviations.

Given a PSD, $I(f)$, which could represent a displacement forcing function on a rail vehicle, the PSD of a particular vehicle response mode, $g(w)$, can be determined from the equation:

$$g(w) = [(f)(v)]\, F(iw)\, F(-iw) \qquad (2)$$

where $F(iw)$ = the appropriate response characteristic of the vehicle; and (v) = the vehicle velocity. The mean square value of the response can be calculated as the area under the response PSD (6). That is

$$\sigma^2 = \int g(w)dw \qquad (3)$$

where σ^2 = the standard deviation of the displacement that occurs in the vehicle for the particular response mode under consideration. While the PSD has been used extensively in vehicle design (8), it and other associated root mean square statistics such as standard deviations, appear to have limited use as TQI's for two reasons:

1. Phase information of the track geometry, which is important in determining vehicle response, is suppressed by the PSD. For example, consider the hypothetical track profiles given by Eqs. 4 and 5 and shown in Figs. 7(*a*–*b*), respectively.

$$y = \sin\frac{\pi x}{l} + \frac{1}{4}\sin\frac{2\pi x}{l} \qquad (4)$$

$$y' = \sin\frac{\pi x}{l} + \frac{1}{4}\sin\frac{2\pi x}{l} + \frac{\pi}{2} \qquad (5)$$

The profile y will elicit a different vehicle response than y', however, the PSD's for both are identical as is shown in Fig. 8. The standard deviation of the track profile is also identical for both tracks.

2. Information regarding extreme values of track geometry is suppressed. This is not a serious limitation when using the PSD for vehicle design since the vehicle is designed to operate over "typical" track, not the one or two percent of track which requires repair. It is a severe limitation when developing a TQI that is to be utilized to program track maintenance since it is extreme values of geometry that lead to derailment or other types of system failure. When viewing the problem from the prospective of track maintenance it is unwise for a TQI to submerge this portion of the defect population.

Eq. 3 describes how vehicle response can be generated from a PSD. As the equation shows, it is possible to generate only root mean square responses. In order to make more specific determinations of vehicle response, e.g., to estimate the probability of the displacement response exceeding some threshold value, it is necessary to make assumptions regarding the probability distribution of the track geometry.

In the case of a forcing function for which the most complete possible description is that of a stationary random process, there exists no choice

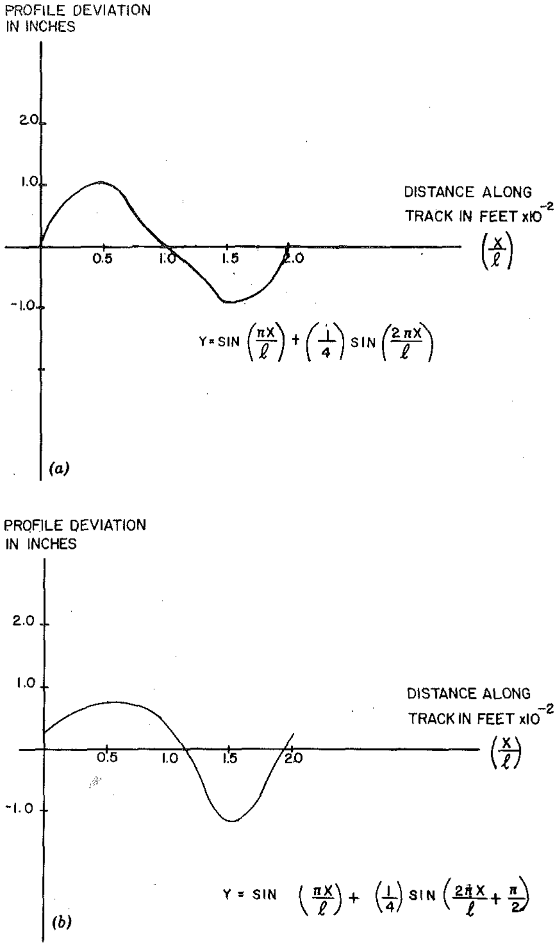

FIG. 7.—Two Hypothetical Track Profiles

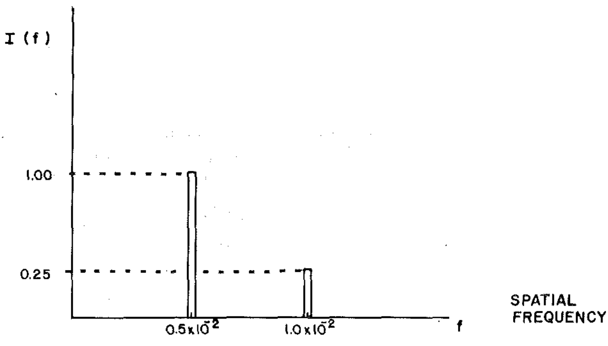

FIG. 8.—PSD's of Hypothetical Track Profiles

other than to describe vehicle response in terms of root mean square statistics. However, where more information is available regarding the forcing function, it is desirable to incorporate as much of the additional information as is known. In the case of track, this requires a review of the deterministic components of the geometry.

Deterministic Components.—The deterministic component is comprised of a combination of transient and periodic signatures. The transient portion of this component can be investigated on an individual basis and a geometric specification defined for each signature (7). For example, the track quality at highway crossings should be evaluated on the basis of their track geometry signature as well as by the physical inspection of the structure. Fig. 9 shows a typical profile signature for highway grade crossing. A performance specification for a grade crossing could be defined in terms of the acceptable value of parameter "h" for a given track class.

These transient geometry signatures are generally associated with special track fixtures such as turnouts, road crossing, and bridges. As such they are not relevant to the formulation of a TQI for nonspecial track, but must be included in developing track maintenance standards. The other deterministic signature in track geometry is the periodic component. This signature stems from the presence of rail joints. In track comprised of continuous welded rail this periodicity is also observed, although at smaller amplitudes. The retention of this periodicity in continuous welded rail track is known as joint "memory" and is attributable to ballast and subgrade deformation and imperfect geometry at the weld locations. The periodicity is most important at the resonance frequency of the rail vehicle. Since the rail car perceives geometry as a forcing function at a temporal frequency (i.e., cycles per second), while track geometry is physically described in terms of spatial frequency (cycles per feet), it is necessary to convert track geometry to a temporal frequency. The vehicle speed determines this conversion. The significance of this conversion from spatial to temporal frequencies to track maintenance can best be illustrated by the example using the simplified vehicle shown in Fig. 10.

Assume that the vehicle's resonance response frequency occurs at $w_0 = 0.5$ Hz (cycles/sec). Assume also that the vehicle is moving over a track whose profile is a pure sine wave of wave length equal to 40 ft (i.e., whose spatial frequency is 0.025 cyc/ft). If the vehicle is moving 40 ft/sec (27 miles/hr) it "sees" a forcing function of: $w = (f)(v) = (1/40 \text{ cyc/sec})(40 \text{ ft/sec}) = 1.0$ cyc/sec, which is well out of the resonance range. However, if the vehicle speed is halved, the forcing function on the vehicle occurs at the resonance frequency: $w = (f)(v) = (1/40 \text{ cyc/sec})(20 \text{ ft/sec}) = 0.5$ cyc/sec. Thus, the vehicle "sees" this particular geometry deviation at different frequencies depending on its speed.

The track in this simple example is safer at the *higher* of the two speeds. This phenomena is well-known to track maintenance personnel who avoid speed regimes of 15–25 mph (24–40 km/h) when placing temporary speed restrictions on track because of a freight car's tendency to rock at these speeds on 39 ft rails. The implication of the spatial/temporal conversion to formulating a TQI is that all speeds less than or equal to maximum authorized speed must be included in the analysis of track quality, since

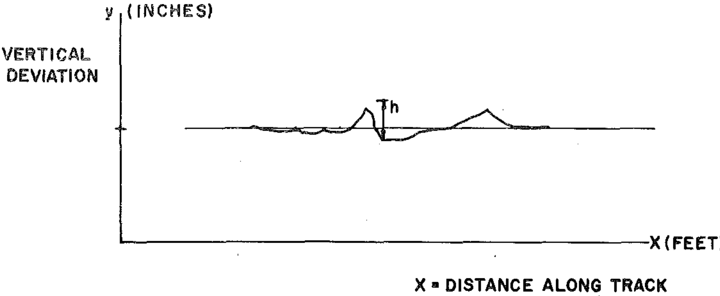

FIG. 9.—Response Signature for Highway Grade Crossing

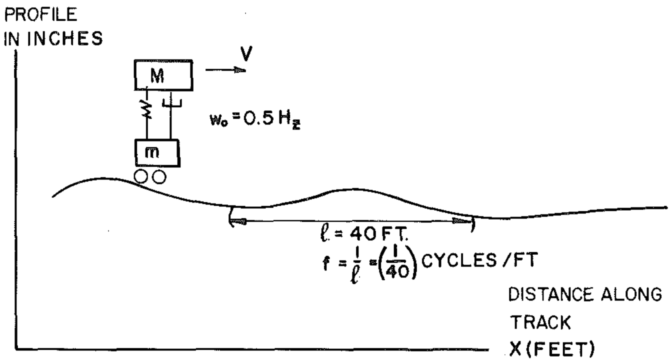

FIG. 10.—Relation between Temporal and Spatial Frequency

a train may travel at any speed up to the authorized speed. It is therefore necessary to investigate geometry deviations at a continuum of spatial frequencies, since each may, depending upon the speed of the traversing vehicle, constitute a forcing function at the vehicle's resonance frequency. Consider the vehicle shown in Fig. 10, traversing two different geometry defects. In the first case, Fig. 11(*a*), the simplified vehicle is traveling at 20 ft/sec; in the second case, shown in Fig. 11(*b*), it is moving at 40 ft/sec.

The 80 ft wavelength geometry deviation causes the identical response in the sample vehicle traveling at 40 ft/sec as does the 40 ft wavelength defect in the sample vehicle traveling at 20 ft/sec. Fig. 12 shows the response as a function of the temporal frequency of the forcing function. As shown in this figure, a track geometry deviation with an amplitude of one inch will elicit a response of two inches if it occurs at the resonant frequency.

Fig. 13, representing the inversion of the response signature of Fig. 12, gives the amplitude of the geometry deviations of different wavelengths that provide a relative vehicle response of 1.0. As Fig. 13 shows, the excitation that occurs at the vehicle's resonant frequency is the most critical. Since the vehicle can travel at any speed less than or equal to maximum authorized track speed, it is necessary to develop such a curve

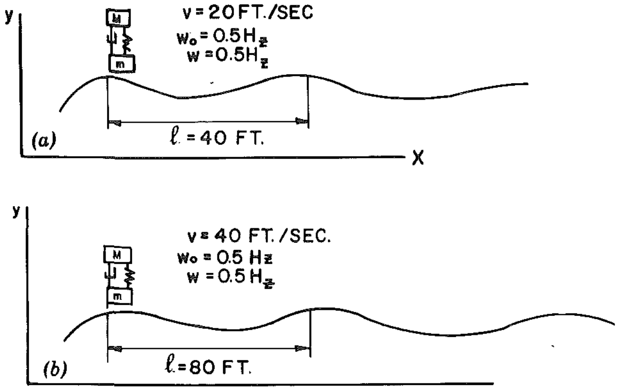

FIG. 11.—(*a*) Profile Deviation of l = 40 ft Traversed at 20 ft/sec; (*b*) Profile Deviation of l = 80 ft Traversed at 40 ft/sec

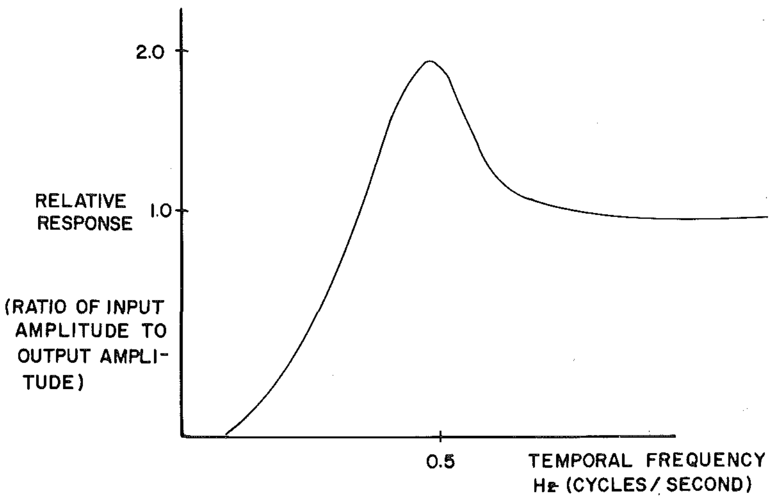

FIG. 12.—Response Signature for Simple Vehicle Model

for each possible speed. This leads to an envelope, shown in Fig. 14, which describes the amplitudes of permissible geometry deviations. This enveloping process results in standards for track geometry that are comprised of constant amplitude thresholds defined over a variety of wavelengths (i.e., a fixed amplitude, variable wavelength standard). This standard is somewhat different from the current FRA Track Safety Standards, which are primarily based on measurements taken at a fixed wavelength and which vary the amplitude of the permissible defect according to a track's maximum speed. The FRA standards are, for the most part, variable-amplitude, fixed wavelength standards.

A review of some existing geometric standards for high speed passenger track will illustrate the difference between fixed and variable

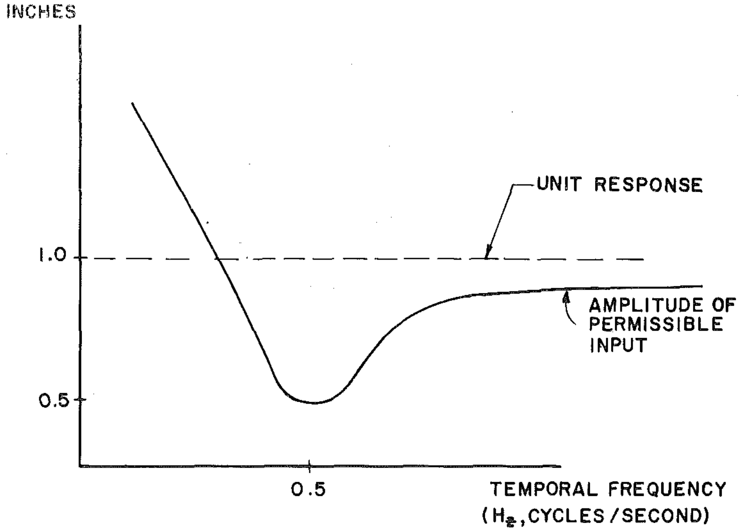

FIG. 13.—Inversion of Response Signature

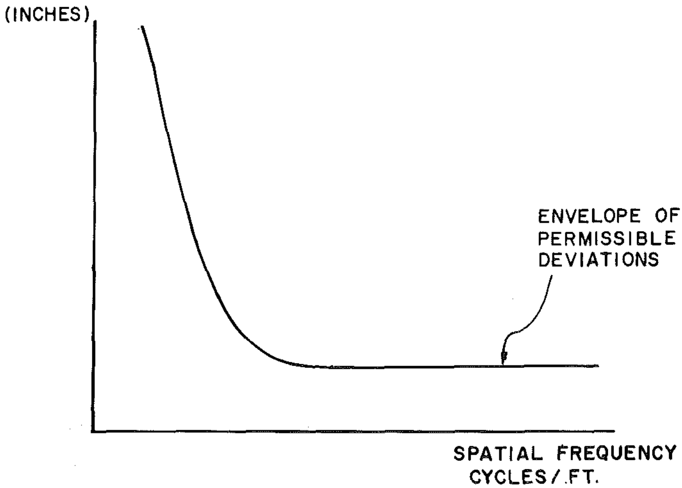

FIG. 14.—Envelope of Permissible Amplitudes

wavelength standards. The FRA Safety Standards prescribe the maximum limit for profile deviations for Class 4 track, on which passenger trains may operate at speeds of up to 80 mph, as two inches (5.1 cm) as measured by a 62 ft Mid-Chord Offset (MCO). For Class 5 track, on which maximum passenger train speed is 90 mph, the maximum rail profile deviation is 1-1/4 in. (3.2 cm), again as measured by a 62 ft MCO. This safety standard for rail profile, shown in Fig. 15, is developed on the basis of a constant wavelength measurement of 124 ft. Conversely,

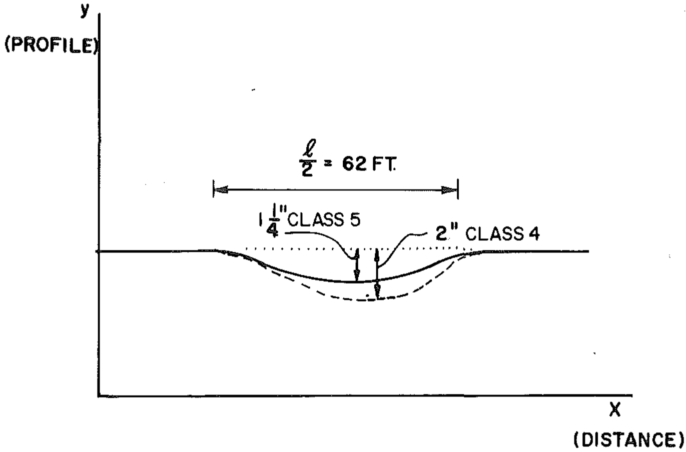

FIG. 15.—Current Safety Standards for Rail Profile

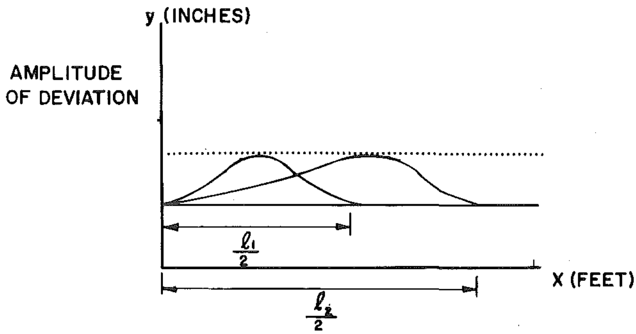

FIG. 16.—Application of Fixed Amplitude; Variable Wavelength Control to Track

an application of a fixed-amplitude, variable-wavelength standard is shown in Fig. 16. In this case, as a result of the enveloping procedures, it is the maximum wavelength of measurement which increases with an increase in track speed. The maximum permissible amplitude of the deviation is constant.

Formulating TQI

Consideration of deterministic components in the track geometry signature eliminates root mean square statistics (mean value of amplitude, S^2, PSD) as viable TQI's.

It is advisable to consider transient, deterministic components separately; with each category of special track items such as switches and highway crossings defined by an acceptable signature. The periodic component should be monitored by a simple count of the number of exceedences to the variable wavelength envelope. RMS statistics are useful as a final check of overall track quality.

More research is required in order to define a TQI that properly accounts for the nonrandom components of track geometry. Three specific problems that must be addressed are:

1. Defining the number of consecutive geometry deviations of given amplitude and wavelength that are unacceptable. It is certainly possible to fix a conservative standard whose limit would be a single geometry deviation above the threshold. However, a single wave of a forcing function, as shown in Fig. 16, will generally not cause unacceptable vehicle behavior since the suspension system will dampen the response. The threshold value of amplitude was developed assuming that the unacceptable vehicle behavior results from a periodic forcing function occurring at the vehicle's resonance frequency. At a given wavelength two or three perturbations in sequence might be necessary to cause undesirable vehicle response.

2. The worst case vehicle response to a periodic forcing function occurs, for a linear vehicle model, when the forcing function is a pure sine wave. Prototype vehicles all exhibit nonlinearities, however, and it is assumed in this development that these nonlinearities will not alter this response characteristic.

3. Further study is required regarding the combined effects of different types of geometry deviations. The comments in this paper have been purposely restricted to a single geometric parameter (profile) acting on a simple model with only one response mode. This simplification is useful in illustrating the differences in the treatment of the deterministic and random components of geometry. However, in practice, all of the critical response modes in the vehicle must be evaluated with respect to the particular geometry parameters, or combinations of parameters, which elicit them. A combination of geometry defects, such as the simultaneous occurrence of alinement and profile deviations, is more dangerous to a rail car than are the individual deviations considered separately. Although some experimental (10) and analytical (5) work has been performed in this area, widely applicable results have not been achieved.

In summary, research indicates that:

1. Any technique that proposes to define track quality for purposes of allocating maintenance resources must consider the rail vehicle and track as a unified system.

2. Such an approach logically leads to loaded track geometry as the basis for a TQI, since this is the forcing function which determines vehicle response.

3. Studies regarding the statistical character of track geometry, coupled with the dynamics of vehicle response, indicate that root mean square statistics are not adequate as Track Quality Indices. The track geometry's deterministic components, both periodic and transient, must be considered in assessing track quality. Root mean square statistics suppress information relating to deterministic components.

Applications to Maintenance

The inclusion of the periodic component of track geometry leads di-

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rectly to the formulation of a TQI, which is simply a count of the number of geometry defects per unit of track length. In the microscopic case (daily or routing track maintenance), *each* geometry deviation is remedied as soon as possible without regard to the rate of defect formation. At a more macroscopic planning level, the TQI can be used in conjunction with data describing the track's physical and usage characteristics to formulate a degradation model. Such a model can assess the need for track reconstruction, or for additional allocations for routine maintenance, by monitoring the rate of defect formation as well as the total number of defects on a track segment (4).

Maintenance standards developed using the approach described in this paper, would, by necessity, be closely related to the vehicle type and operating speed. The standards would require revision should either change significantly. To the knowledge of these writers, only one American railroad has established vehicle performance based track maintenance standards (10), and these were developed for relatively slow speed freight train operation in which probability of derailment was the primary indicator of vehicle performance. For a high speed passenger railroad on which the probability of track-caused derailment approaches zero, some other criteria, such as acceptable magnitudes of car body acceleration, must be defined.

AMTRAK has defined a "ride quality standard" for its Northeast Corridor operation of 0.35 g vertical and 0.30 g horizontal accelerations as measured in a standard passenger coach. These accelerations are currently measured by an accelerometer on the coach floor. Because there is, as yet, no correlation between accelerations and track geometry, line maintenance personnel must rely on experience in converting acceleration data to a track maintenance program. However, analysis of the response of the standard AMFLEET passenger coach to the deterministic components of track geometry would allow their existing ride quality standard to be expressed directly in terms of track geometry. Track maintenance standards would then be defined in terms of acceptable *values* of the TQI for nonspecial track and would be supplemented by specific amplitudes for the transient signatures associated with special track fixtures.

Conclusion

A quantitative measure of track quality (TQI) is required to scientifically plan maintenance on high speed track. A method has been described that leads to purely geometric TQI's capable of relating track geometry to vehicle performance. A geometric TQI was selected since track geometry is the forcing function that determines the response of a free-rolling rail car. It has the further advantage of being capable of automatic and objective measurement at high speed. An RMS-type TQI, which, by inference, treats track as a totally random process, results in the inefficient allocation maintenance resources. It is more appropriate to assign maintenance on the basis of a TQI derived from the number of exceptions to a performance based threshold. However, an RMS statistic, such as power spectrum, could be used as a supplement to the TQI in order to exercise control over random geometry defects. Track main-

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tenance standards must be distinguished from track safety standards, particularly for high speed passenger track.

The problem of formulating a TQI is distinct from that of studying track surface degradation. The latter may be best expressed in terms of the change in the value of the TQI and must consider a wide variety of track usage and structural characteristics. Given the complexities of track, its degradation can probably best be studied experimentally. However, a quantitative study of track degradation is predicated upon the development of a performance based TQI.

Appendix I.—References

1. Bing, A., and Gross, A., "Development of a Track Quality Index," presented at the annual meeting of the Transportation Research Board, Jan., 1983, Washington, DC.
2. Bradley, K., et al., *Acquisition and Use of Track Geometry Data in Maintenance of Way Planning, Technical Report FRA-ORDED-75-27,* NTIS: PB-24 1196/AS, Mar., 1975.
3. Corbin, J., *Statistical Representation of Track Geometry, Vol. I and II,* USDOT Report No. FRA-ORD-80-22-2, Mar., 1980.
4. Corbin, J., and Fazio, A., "Development of Performanced Based Track Quality Indices," *Transportation Research Board Record No. 802,* 1980, Washington, DC.
5. Dimasi, F., and Weinstock, H., "A Parametric Study to Relate Railcar Speed to Permissible Combinations of Track Geometry Deviations," *Measurement and Control Journal of ASME,* Vol. 100, Dec., 1978, pp. 252–259.
6. FRA Track Safety Standards, *Congressional Record,* 1971.
7. Hamid, A., et al., *Analytical Description of Dynamically Severe Track Geometry,* FRA Report No. RTE-80-10, Oct., 1979.
8. Rinehart, A. E., "Locomotive Response to Random Track Surface Irregularities," *Proceedings of the ASME,* 78-WA/RT-12, Dec., 1978.
9. Tsien, H. S., *Engineering Cybernetics,* McGraw-Hill, New York, NY, 1954.
10. Tuve, R. F., "An Application of Track Geometry Data to Track Maintenance Planning on the Southern Railway," presented at 59th Annual Meeting of Transportation Research Board, Washington, DC, Jan., 1980.

Appendix II.—Notation

The following symbols are used in this paper:

f = spatial frequency in cycles/ft,
i = $\sqrt{-1}$,
l = spatial wavelength, ft/cycle, $l = 1/f$;
v = vehicle velocity in ft/sec,
w = temporal frequency in cycles/sec,
w_0 = resonance frequency for a particular response made in cycles/sec, and
ϕ = temporal wavelength, sec/cycle, $\phi = 1/w$.