Potential Use of Inventory Theory to Bundle Interventions in Bridge Management Systems

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Before an optimal bridge management strategy can be determined in existing bridge management systems, there must first be a determination of the type and schedule of interventions to be performed on each element of the bridge to achieve a minimal-cost long-term steady state. Once these element-level strategies are known, they are grouped together, or bundled, to develop a management strategy for the bridge. This is currently done with a set of agency rules without regard to whether the bundle of interventions is optimal. Interventions are bundled because it is not desirable to perform interventions on the same bridge in successive years, principally because of user costs. The work discussed here uses inventory theory to bundle interventions on bridges optimally with multiple elements. Two examples are presented. Example 1 shows how a basic joint replenishment model can be used on a multielement bridge to determine optimal management strategies when interventions are bundled and a single-stage policy is used. Example 2 shows how a joint replenishment model with quantity discounts can be used to alleviate some of the problems with a basic joint replenishment model. The optimal management strategy determined in Example 2 is compared with the optimal management strategy determined when interventions are not bundled. Direction for future work is given.

Bridge management systems (BMS) help infrastructure managers determine optimal management strategies for their bridges. In existing BMSs, such as Pontis (*1*) and KUBA (*2*), the first step is to determine the optimal interventions to be performed on each of the elements of the bridge to achieve a minimal-cost long-term steady state. For example, the optimal management strategy for a bridge deck may be to have it replaced every 30 years and to do no maintenance between replacements. Once the optimal management strategies for the individual elements are known, they are bundled to develop a management strategy for the entire bridge. Interventions are bundled because it is realized, principally due to user costs, that it is not desirable to have interventions done on the same bridge in successive years. For example, the federal roads authority of Switzerland has implemented a law stating that each 15-km section of road can have an intervention only once every 10 years (*3*).

The bundling of interventions in existing BMSs is currently done by using a set of agency rules without regard to the optimality of the resulting management strategy. Some of the rules that are currently used to group interventions are as follows (*1*):

- Paint rules. Rules that recognize practical considerations of painting bridge components that did not need painting when the optimal policy was followed;
- Agency policy rules. Rules that allow an agency to determine the actions to be taken for a given element-condition state regardless of the least-cost option;
- Scoping rules. Rules that recognize the practical consideration that work on some elements requires work on other elements;
- Look-ahead rules. Rules that recognize work scheduled prior to performing a program simulation; and
- Major rehabilitation rules. Rules that recognize the practical considerations of performing rehabilitation work when following the optimal policy. Specifically, agency practices and work-staging considerations often dictate that when a rehabilitation project is performed, all needs on the bridge must be addressed.

Although these rules do succeed in bundling interventions, there has not been any investigation, to the author's knowledge, to determine if the resulting strategies are optimal. The work discussed herein is a first effort to optimally bundle element-level interventions using inventory theory. This work illustrates how inventory theory can be used on a multielement bridge to bundle interventions using a single-stage policy. The results are compared with an optimal element-level policy in which the same interventions are not bundled. The main reference used for information on inventory theory is Zipkin (*4*).

USE OF INVENTORY THEORY TO BUNDLE INTERVENTIONS

Inventory theory is a science that has been developed to model and optimize the amount of time for and the cost of holding items that are (*a*) in demand and (*b*) being supplied. Inventory theory has been predominately used in the business community to stock stores and plan distribution. There appears to be potential, however, to use inventory theory to bundle interventions in BMSs. This investigation of the use of inventory theory in bridge management uses the following analogy.

A bridge is similar to a warehouse. A bridge stores element-condition units as a warehouse stores items. Each element-condition unit can be thought of as an element-condition state. For example, if a bridge deck has *x* element-condition units, it is in condition state *x*. The demand for the element-condition units stored in the bridge comes from deterioration, as the demand for the items in the warehouse

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Transportation Research Record: Journal of the Transportation Research Board, No. 1933, Transportation Research Board of the National Academies, Washington, D.C., 2005, pp. 44–49.

comes from sales. Interventions on the bridge replenish the stock of element-condition units, as the ordering of items from suppliers replenishes the stock of items in the warehouse.

The use of inventory theory to optimally bundle interventions in bridge management is investigated using this analogy. The inventory theory terminology used in this report and its meaning with respect to the management of bridges is given in Table 1.

JOINT REPLENISHMENT MODEL WITHOUT QUANTITY DISCOUNTS

Formulation

Depending on agency policy the objective when determining optimal bridge-level management strategies may be to do all necessary interventions each time that one intervention is required on a bridge. The most promising inventory theory model to do this bundling of interventions is the joint replenishment model using the analogy described previously. In multielement systems that have all element-condition units supplied by the same source, joint replenishment offers advantages, such as savings on setup costs. These situations have what are known as "economies of scope"; that is, money can be saved by bundling diverse activities, such as performing different types of interventions on bridges.

Using the basic joint replenishment model, the joint replenishment model in its simplest form, the optimal time between interventions is the value that minimizes the total costs per unit time equation, $C_{\text{tot/unit time}}$, Equation 1 (*4*).

$$C_{\text{tot/unit time}} = \frac{k}{u} + \frac{1}{2} \cdot g \cdot u \tag{1}$$

where k is the sum of the fixed setup cost of performing an intervention regardless of the type of intervention and the setup costs of performing each intervention

$$k = k_0 \sum_{j=1}^{J} k_j \tag{2}$$

where

k_0 = the fixed setup cost of performing an intervention regardless of the type of intervention;
k_j = the setup cost of performing intervention *j;*
u = the intervention interval; and
g = the total holding costs or the sum of the cost of an element having one element-condition unit more than required for one unit of time longer than required multiplied by the rate of deterioration or how long they will have to be held.

$$g = \sum_{j=1}^{J} g_j \tag{3}$$

$$g_j = h_j \lambda \tag{4}$$

where h is the cost of an element having one element-condition unit more than required for one unit of time longer, and λ is the rate of deterioration.

Example 1

To illustrate the use of the basic joint replenishment model to determine optimal management strategies for bridges, a bridge with four different elements is used. It is assumed that there is a large setup cost for closing the bridge in order to perform the interventions, k_0, which may be thought of as the cost to users of the associated traffic restrictions, and that there are smaller fixed intervention costs for performing an intervention on each element, k_i, which may be thought of as the costs of mobilizing the equipment to perform the specific

TABLE 1 Interpretation of Commonly Used Expressions in Inventory Theory for Bridge Management

Variable	Common Inventory Theory Definition	Interpretation for Bridge Management
c	Cost of producing each item	Cost of each element condition unit added during an intervention, e.g., cost of manual labor required to replace a concrete girder or the cost of a one-lane traffic restriction to the users in the form of increased travel time.
h	Holding cost per item per unit of time	Cost of an element having one element condition unit more than required for one unit of time longer than required. This may be thought of as the cost to society of having more money invested in bridges than required. Holding costs are not considered to include discount rate.
k_0	Fixed setup cost of ordering a batch of items, regardless of the type of item	Fixed setup cost of performing an intervention, regardless of the type of intervention, e.g., mobilization of men to go to the bridge, or closing of the road to perform the intervention.
k_i	Setup cost of ordering one batch of item *i*	Setup cost of performing intervention *i*, mobilization of the equipment required for specific intervention.
p_i	Shortage cost per unit of time short	Additional cost of having an element having fewer element condition units than acceptable, e.g., traffic restrictions, or increase in expected failure costs.
λ	Rate of demand	Rate of deterioration
u	Time between orders	Time between interventions

intervention. It is also assumed that there are variable intervention costs for performing interventions on each element, c_i, which may be thought of as an additional cost that varies due to some variable parameter once all equipment is mobilized (e.g., the thickness of a new layer of concrete added to a bridge deck). The assumed values are shown in Table 2.

Using the values shown in Table 2 and Equations 1 through 4, the optimal management strategies can be found. The optimal management strategies are determined assuming four different holding costs (Table 3 and Figure 1). It can be seen in Figure 1 that as the holding cost increases, the optimal time between interventions decreases.

TABLE 2 Assumed Values for Example 1

Variables	Values	Definitions
k_0	15	Major setup cost related to the closing of the bridge to do one or many interventions
k_1, k_2, k_3, k_4	5, 2, 3, 4	Setup cost related to an intervention on element 1, 2, 3, 4, respectively
h_i	0.25	Cost of element *i* being in each condition state lower than its initial condition state immediately after intervention
c_i	2	Cost of adding an element condition unit to element *i*
λ_1, λ_2, λ_3, λ_4	0.25, 0.12, 0.13, 0.1	Average deterioration rate of element 1, 2, 3, 4, respectively

Discussion of Issues

As shown in example 1, the basic joint replenishment model can be used to bundle interventions; however, there are some problems. These problems are as follows:

- The cost of element-condition units varies depending on how many are added. For example the addition of two element-condition units that are added through the resurfacing of a bridge deck will not cost the same as the addition of four element-condition units that are added by the replacement of the bridge deck. It is possible to remedy this problem by using quantity discounts, as shown in the next section.
- There are no costs associated with the increase in expected failure cost with worsening condition states. Expected failure cost is defined herein as the probability of having a failure in the upcoming year multiplied by the cost of this failure. It is possible to remedy this problem by restructuring the optimization equation as shown in the following section.
- It is not always possible to add a precise fraction of element-condition units. The exact number of element-condition units depends on the condition state before the intervention, the condition state after the intervention, and the type of intervention. This problem, although not dealt with in this paper, can most likely be overcome by defining ranges that signify that a specific intervention is to be performed. For example, if an element has only 2.22 element-condition units, a new layer of concrete should be added to the bridge deck to bring the element-condition state to four. Further investigation of this issue is required.
- Deterioration is assumed to be deterministic. Although not dealt with in this paper, the deterministic assumption may not be a problem when dealing with long-term steady-state strategies. Further investigation of this issue is required.
- There are no costs associated with the element falling below the lowest acceptable condition state, because it is not considered. In the determination of the long-term optimal policy, this may also be appropriate, because in the long term bridges should be managed so that they do not fall into an unacceptable condition state. Further investigation of this issue is required.
- Only a single-stage policy is used. Better results may be possible with a multiple-stage policy. Further investigation of this issue is required.
- There is no consideration of the changing time value of money (i.e., the discount rate). It is simple to introduce discount rates into the discussed equations if desired; however, the addition of discount rates in this paper would only complicate the discussion.

In addition to these specific problems, a general disadvantage of using the joint replenishment model to bundle interventions in bridge management is the loss of flexibility to optimize each element individually by forcing all element-condition units to be restored at regular intervals. For example, if k_0 is small compared to the others, k_j, joint interventions make little sense, and a less rigid approach that recognizes both the advantages and disadvantages of synchronization would be better. Further investigation is required to determine whether or not this is a problem with bridge interventions.

MODIFIED JOINT REPLENISHMENT MODEL

Formulation

To address the two principal problems cited in the previous section, namely, that there is variation in the cost of element-condition units when different interventions are performed and that expected failure

TABLE 3 Optimal Management Strategies with Varying Holding Costs for Example 1

	Optimal Management Strategy				
		Interventions (number of element condition units to add)			
h_i (μ)	Interval Between Interventions (years)	Element 1	Element 2	Element 3	Element 4
5	4.4	1.1	1.73	2.46	4.91
2	6.95	0.53	0.83	1.17	2.35
1	9.83	0.57	0.90	1.28	2.56
0.25	19.66	0.44	0.70	0.98	1.97

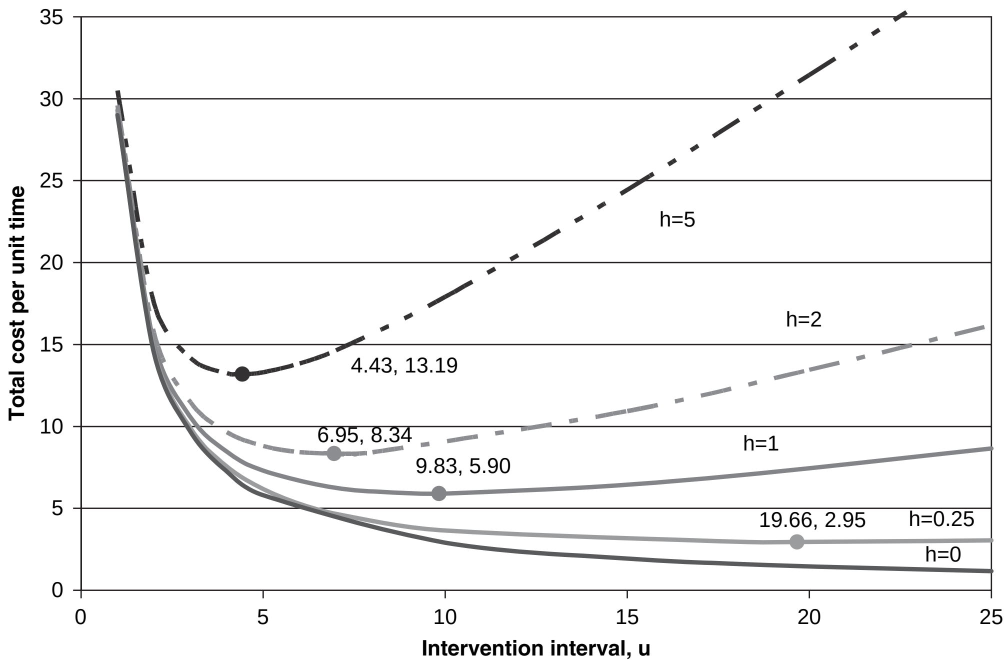

FIGURE 1 Total cost per unit time versus intervention interval.

costs increase with each loss of an element-condition unit below five element-condition units, the optimization equation given by the basic joint replenishment model is restructured. Quantity discounts are added to take the cost variation into consideration, and shortage costs are introduced to take expected failure costs into consideration. It is assumed that expected failure costs increase linearly with the loss of element-condition units and that the expected cost of failure when an element has five element-condition units acts as a benchmark and can therefore be neglected. The restructured optimization equation is

$$\text{minimize } C_{\text{tot/unit time}} = \left(\frac{k}{u}\right) + \left(\frac{1}{2}\cdot g\cdot u\right) + \left[\frac{\sum_{i=1}^{I}(5-\lambda_i\cdot u)\cdot p_i}{u}\right] + \left(\sum_{i=1}^{I}\lambda_i\cdot c_i\right) \qquad (5)$$

where p is the additional cost of having an element with fewer element-condition units than acceptable and k_i and c_i are given by 7 and 8, respectively, subject to the constraint that an intervention must be performed before an element deteriorates to the point where no element-condition units are remaining.

$$\lambda_i \cdot u \le 5 \qquad i = 1, 2, 3, 4 \qquad (6)$$

Example 2

The joint replenishment model with quantity discounts is explained using the same example given in Example 1, except that quantity discounts are introduced and shortage costs are added (5 μ per element-condition unit less than 0 element-condition units) (Table 4). It is assumed that there are five possible interventions per element (all elements are assumed to be the same). The fixed intervention costs are 20, 4, 3, 2, and 1 μ for interventions 1, 2, 3, 4, and 5, respectively. The variable intervention costs per element-condition unit added are 0, 0.2, 0.4, 0.6, and 0.8 μ for interventions 1, 2, 3, 4, and 5, respectively. The values are given in Equations 7 and 8. The k values chosen reflect the idea that a full replacement is very expensive and that it is better to do small maintenance activities on a regular basis. The

TABLE 4 Assumed Values for Example 2

Variables	Values	Definitions
k_0	15	Major setup cost related to the closing of the bridge to do one or many interventions
h_i	5	Cost of element i being in each condition state lower than its initial condition state immediately after intervention
p_i	5	Expected cost of failure of element i being in a lower condition state than condition state 1
λ_1, λ_2, λ_3, λ_4	0.25, 0.12, 0.13, 0.1	Average deterioration rate of element 1, 2, 3, 4, respectively

c values chosen reflect the idea that for some interventions (in this case, intervention 1) there may be no variation, whereas for some interventions (in this case, interventions 2 through 5) there may be variation. An example of where there may be no variation is the replacement of an element; in this case, the replacement may cost the same regardless of the initial condition state. An example of where there may be variation is the addition of a new layer of concrete; in this case, there is a certain cost for hydroblasting the old layer of concrete and a variable cost that depends on the amount of new concrete added.

$$k_i = \begin{cases} 20, \text{ ecu added } = 5 \\ 4, \text{ ecu added } = 4 \\ 3, \text{ ecu added } = 3 \\ 2, \text{ ecu added } = 2 \\ 1, \text{ ecu added } = 1 \end{cases} \qquad i = 1, 2, 3, 4 \tag{7}$$

$$c_i = \begin{cases} 0, \text{ ecu added } = 5 \\ 0.2, \text{ ecu added } = 4 \\ 0.4, \text{ ecu added } = 3 \\ 0.6, \text{ ecu added } = 2 \\ 0.8, \text{ ecu added } = 1 \end{cases} \qquad i = 1, 2, 3, 4 \tag{8}$$

where ecu is the element-condition unit.

Using Equations 5 and 6, the optimal management strategy for the example bridge would involve intervening every 9 years to add 2.25, 1.08, 1.17, and 0.9 element-condition units to elements 1, 2, 3, and 4, respectively. The $C_{\text{tot/unit time}}$ is 24.5 µ. Figure 2 shows the $C_{\text{tot/unit time}}$ versus different intervention intervals for Example 2. As the time interval increases initially (from 1 to 9), the $C_{\text{tot/unit time}}$ decreases. After year 9 more expensive interventions are required, and increasing the intervention interval further becomes more expensive. The least-expensive intervention interval is 9 years.

Comparison with Unbundled Interventions

To compare the benefit of bundling interventions in an optimal bridge-level management strategy with the four optimal element-level management strategies, the optimal management strategies are determined for the bridge in example 2 without the bundling restriction. Using Equations 5 and 6 without the bundling restriction, the optimal management strategy is determined to be to

- Add 2.0 element-condition units to element 1 every 8 years at 9.0 µ per unit time;
- Add 1.4 element-condition units to element 2 every 12 years at 6.5 µ per unit time;
- Add 1.4 element-condition units to element 3 every 11 years at 6.8 µ per unit time; and
- Add 1.3 element-condition units to element 4 every 13 years at 6.0 µ per unit time.

The variation of $C_{\text{tot/unit time}}$ and intervention interval for each element is shown in Figure 3. If the four interventions occur in different years for the period of time to be investigated, the $C_{\text{tot/unit time}}$ is 28.27 µ or 15.3% more expensive than when the interventions are bundled. This can be viewed as an upper bound for the costs per unit time for intervention on the example bridge.

Discussion of Issues

It can be seen in Example 2 that the modified joint replenishment model with quantity discounts and consideration of expected failure costs can be used to bundle interventions so that the optimal bridge-level management strategy can be determined.

The two specific previously cited problems remaining are that it is not always possible to add a precise fraction of element-condition units and that multiple-stage policies may give better results than single-stage policies. Both of these problems can most likely be remedied

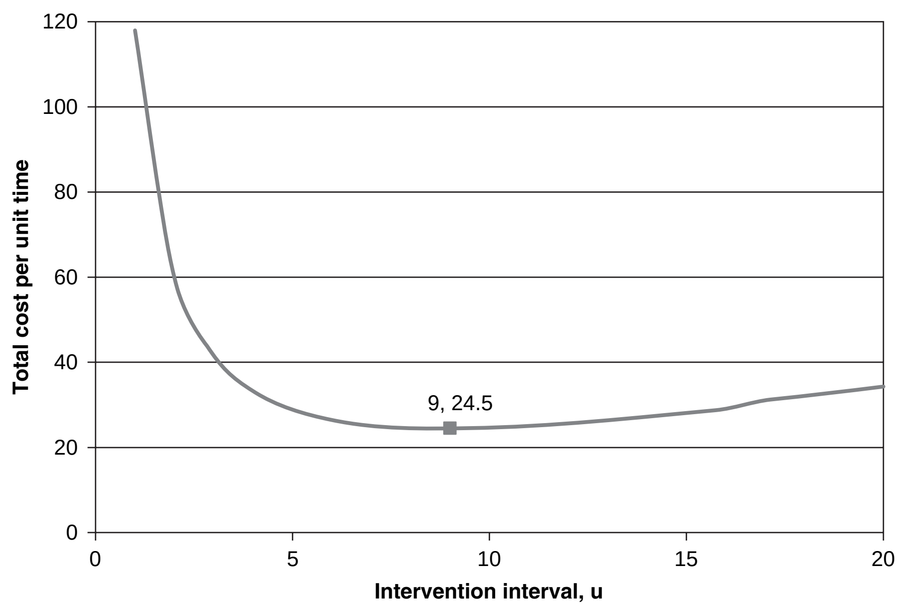

FIGURE 2 Total cost per unit time versus intervention interval for grouped elements.

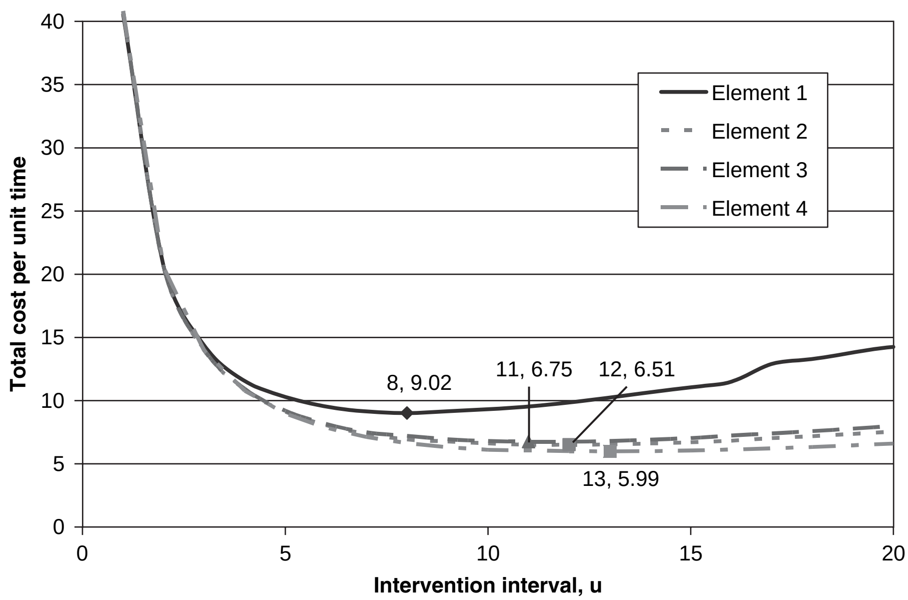

FIGURE 3 Total cost per unit time versus intervention interval for ungrouped elements.

by reformulating the optimization problem. In general, however, there is still the disadvantage that the use of the joint replenishment model results in the loss of flexibility to optimize each element individually by forcing all element-condition units to be restored at regular intervals. This may not, however, be the case with interventions on bridges because there are normally relatively large setup costs.

CONCLUSIONS

This investigation has shown that in general, there is potential to use inventory theory to bundle interventions on bridges so that optimal bridge level management strategies can be determined. More specifically, the joint replenishment problem with quantity discounts and consideration of expected failure costs presented herein appears to be a good starting point, as it was used successfully to estimate long-term optimal management strategies for bridges with multiple elements for the simple examples used.

Comparison of (*a*) the optimal management strategy determined in Example 2 with bundled interventions with (*b*) the management strategy developed for the same bridge without bundled interventions shows that savings can be made by bundling interventions. The magnitude of the savings will depend on the exact costs of the interventions, especially in terms of setup costs.

FUTURE WORK

This work is a preliminary investigation into the potential use of inventory theory to optimally bundle interventions. Many topics of research still remain. One of these areas is stochastic demand and lead times. There is need for further investigation into whether or not deterioration can be treated deterministically for the determination of long-term management strategies. Another of these areas is the incorporation of lead times into the optimization problem, although this problem is not currently addressed in existing BMSs. It seems that when the capacity of the supply system is limited (which might be applicable in the case of interventions; i.e., only so many element-condition units can be supplied), deterioration uncertainty induces longer and more variable lead times.

An immediate next step is to determine optimal management strategies for real bridges with real interventions. This effort will allow exploration of the strengths and weaknesses of using inventory theory to determine optimal management strategies that might not have been found here with fictive examples. It would also be interesting to compare the optimal management strategies determined for these bridges using inventory theory and those determined using the rules in existing BMSs.

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The Bridge Management Committee sponsored publication of this paper.