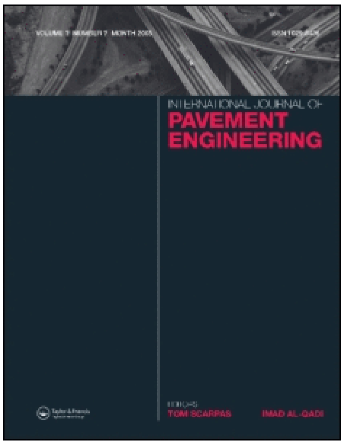

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Khaled A. Abaza[a]; Suleiman A. Ashur[b]
[a] Department of Civil Engineering, Birzeit University, Birzeit, Palestine [b] Indiana University-Purdue University Fort Wayne, Department of Engineering, Fort Wayne, IN, USA

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Optimum microscopic pavement management model using constrained integer linear programming

Khaled A. Abaza[a] and Suleiman A. Ashur[b]*

[a]*Department of Civil Engineering, Birzeit University, Birzeit, Palestine;* [b]*Indiana University-Purdue University Fort Wayne, Department of Engineering, Fort Wayne, IN, USA*

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A pavement management model has been developed using a microscopic approach to yield optimum pavement conditions for a given pavement system. The microscopic pavement management problem is formulated as a constrained integer linear programming model subjected to budget and improvement requirement constraints. The developed optimum microscopic model incorporates integer variables representing the number of pavement sections to be treated by the applicable maintenance and rehabilitation actions. The objective of yielding optimum pavement conditions is achieved by either considering the net pavement condition rating gain or age–gain applied to a given pavement system. In either approach, the optimisation model can be formulated to maximise pavement conditions or minimise maintenance and rehabilitation costs.

Data requirements for the microscopic model include identification of each pavement section and rating its distress condition, providing cost rates and performance parameters for the deployed maintenance and rehabilitation actions and specifying anticipated budget and pavement improvement requirements. Sample results from a case study have indicated that the developed model is very effective in yielding optimum pavement conditions.

Keywords: pavement maintenance; pavement rehabilitation; pavement management; pavement management system

Introduction

Several pavement management models were developed in the last two decades with one common objective of yielding the best maintenance and rehabilitation (M&R) plan. They have used different approaches in modelling the pavement management problem considered to be extremely complex. The complexity of the pavement management problem stems from the resulting large number of M&R variables typically associated with a given pavement system (Hass *et al.* 1994, Shahin 1994, Pilson *et al.* 1999, Abaza *et al.* 2001). A decision-making policy is typically formulated that incorporates the M&R variables and the feedback from pavement distress surveys. An appropriate optimisation approach is then selected to solve the formulated model to yield optimum pavement conditions. Several optimisation approaches have been used in pavement management ranging from the very simple node and branch algorithms to the very complex ones such as the application of genetic algorithms and efficient surfaces (Harper and Majidzadeh 1991, Tavakoli *et al.* 1992, Mbwana and Turnquist 1996, Abaza and Ashur 1999, Pilson *et al.* 1999, Ferreira *et al.* 2002).

The complexity of the pavement management problem can be substantially reduced if the M&R variables are assigned to pavement classes rather than to individual pavement sections. A pavement class is one that contains pavement sections with relatively similar pavement distress conditions, therefore, the sections qualify for the same M&R action(s). A pavement section is typically selected small in length for accurate assessment of pavement distress condition. The microscopic approach requires the identification, inspection and rating of each pavement section whereas the macroscopic one requires only obtaining the pavement proportions in the various deployed classes. Pavement management models that used the macroscopic approach define the M&R variables to represent the pavement proportions that should be treated by the corresponding M&R actions (Grivas *et al.* 1993, Chen *et al.* 1996, Liu *et al.* 1996, Abaza and Ashur 1999, Abaza *et al.* 2004). The microscopic approach defines the M&R variables as integer variables representing the number of pavement sections that should be treated by the applicable M&R actions. Several pavement management models have deployed the microscopic concept using different modelling and optimisation approaches (Harper and Majidzadeh 1991, Hill *et al.* 1991, Mbwana and Turnquist 1996, Pilson *et al.* 1999, Ferreira *et al.* 2002). The microscopic approach can

*Corresponding author. Email: ashurs@ipfw.edu

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generate an improved M&R plan compared to the macroscopic one as it relies on a more detailed assessment of pavement distress.

Most advanced pavement management systems usually require the incorporation of an effective pavement performance prediction model in case a long-term M&R programme is needed. The M&R programme is derived to provide an M&R plan for every time interval within a specified study period (Pilson *et al.* 1999, Abaza *et al.* 2001, Ferreira *et al.* 2002, Abaza *et al.* 2004). Pavement performance prediction models are mostly stochastic in nature and require extensive historical records of pavement distress to be effective in meeting the local condition requirements (Butt *et al.* 1987, Shahin *et al.* 1987, George *et al.* 1989). The developed pavement management model generates a single M&R plan for implementation during a short-time interval (1 or 2 years) without requiring the incorporation of a performance prediction model. Developing countries can greatly benefit from such model since they do not typically possess the resources and technical expertise needed to develop and maintain reliable performance prediction models. Therefore, the developed model can be classified as a short-term management model since it does not incorporate a performance prediction model.

Methodology

The developed microscopic pavement management model (MPMM) defines the improvement resulting from M&R actions using either pavement condition rating (PCR) or age gain (AG). Therefore, two distinct models are formulated deploying the PCR and age as the main improvement indicators. Each model is then designed as a maximisation or minimisation problem with the objective of yielding the best M&R improvement plan for a particular pavement system. A pavement system is divided into (n) pavement classes with each class qualifying for (m) M&R actions. The M&R variables (I_{ij}) that are defined as integer variables representing the numbers of pavement sections in the various deployed classes to be treated by the corresponding M&R actions. The two deployed improvement indicators, PCR and age, are two closely related performance parameters typically used to define a pavement performance curve.

PCR-based optimisation model

The first formulated MPMM model deploys the PCR as the main pavement improvement indicator. A PCR value is typically assigned to each pavement section based on conducting a field assessment of pavement distress. The net gain in PCR, as applied to a pavement class, is then determined from the difference between the expected initial PCR value associated with a particular M&R action and the average PCR value for the pavement sections in that class. This approach is similar to the one used by the American Association of State Highway and Transportation Officials (AASHTO) in the design of flexible pavements (AASHTO 1993) wherein an initial present serviceability index value is required. Therefore, an expected initial PCR value is required for each considered M&R action.

PCR-gain maximisation model

The pavement system improvement maximisation model is formulated as a constrained integer linear programming model. The associated linear objective function represents the net system improvement in PCR-gain ($\mathrm{RG_s}$) as applied to the pavement sections in all deployed pavement classes. Equation (1) defines the system PCR-gain as the sum of class PCR gains. The class PCR-gain is obtained as the product sum of PCR changes and the corresponding M&R variable values. The class PCR change is defined as the difference between the expected initial PCR ($\mathrm{PCR_0}$) associated with applying a particular M&R action and the class terminal PCR ($\mathrm{PCR_t}$). The $\mathrm{PCR_t}$ is estimated as the average PCR value ($\overline{\mathrm{PCR_t}}$) associated with pavement sections in that class. It is assumed that all pavement sections in the same class will have the same $\mathrm{PCR_0}$ once they receive the same M&R treatment.

The maximisation model is subjected to three mandatory and two optional sets of linear constraints. The two mandatory sets 1 and 5 are used to place upper and lower limits on the M&R variables with mandatory set 2 representing the budget constraint. The budget constraint requires an average cost rate that accounts for the degree of scatterness of M&R works within a pavement system. An M&R cost model is later presented to provide a procedure for estimating the required average M&R cost rates. Constraint sets 3 and 4 are optional class improvement constraints used to establish improvement relationships amongst the deployed pavement classes. Only one of the two sets can be used to ensure that all qualifying classes will receive their fair shares of improvements. Constraint set 3 requires improvement relations amongst the classes in proportion to their sizes as represented by the numbers of pavement sections in the various deployed classes. Whereas constraint set 4 requires a specified average PCR-gain for each pavement class to be multiplied by the corresponding number of pavement sections.

$$Maximise: \mathrm{RG_s} = \sum_{i=1}^{n}\sum_{j=1}^{m}\left[(\mathrm{PCR_0})_{ij} - \left(\overline{\mathrm{PCR_t}}\right)_i\right] \times I_{ij}. \tag{1}$$

$$\textit{Subject to}: 1)\ \sum_{j=1}^{m} I_{ij} \leq N_i \quad (i=1,2,\ldots,n)$$

$$2)\ \sum_{i=1}^{n}\sum_{j=1}^{m} A\times\bar{C}_{ij}\times I_{ij} \leq B_s$$

$$3)\ \mathrm{RG}_i = b_{i,i+1}\times\mathrm{RG}_{i+1}\ (i=1,2,\ldots,n-1)$$

$$\text{where, } \mathrm{RG}_i = \sum_{j=1}^{m}\left[(\mathrm{PCR}_0)_{ij} - \left(\overline{\mathrm{PCR}}_t\right)_i\right] \times I_{ij}, b_{i,i+1} = \frac{N_i}{N_{i+1}}$$

$$4)\ \mathrm{RG}_i = \mathrm{SRG}_i\times N_i$$

$$5)\ I_{ij} \geq 0$$

where:

RG_s, net PCR-gain applied to a pavement system as a result of M&R actions;
n, number of deployed pavement classes qualifying for M&R works;
m, number of deployed M&R actions;
i, an index for deployed pavment class;
j, an index for deployed M&R action;
$(\mathrm{PCR}_0)_{ij}$, expected initial PCR value resulting from the jth M&R action as applied to the ith class;
$(\overline{\mathrm{PCR}}_t)_i$, average terminal PCR value associated with the untreated pavement sections in the ith class;
I_{ij}, an integer M&R variable representing the number of pavement sections in the ith class to be treated by the jth M&R action;
N_i, total number of pavement sections in the ith class;
A, surface area in square meter associated with each pavement section obtained as the product of travel lane width and section length;
$\bar{C}_{ij}$, average cost rate per square meter associated with the jth M&R action as applied to the ith class;
B_s, anticipated M&R budget for a given pavement system;
RG_i, net PCR-gain applied to the ith class;
$b_{i,i+1}$, class improvement proportionality constant defined as the ratio of pavement sections in the ith class to the sections in the $(i+1)$th class; and
SRG_i, specified average PCR-gain for pavement sections in the ith class.

M&R cost minimisation model

The M&R cost minimisation model is formulated such that the objective function is mainly the budget constraint associated with the system improvement maximisation model. The objective is obtaining the best M&R plan that minimises the net M&R cost (C_s) associated with the net PCR-gain as applied to a given pavement system as indicated by Equation (2). The minimisation model is subjected to four constraint sets similar to those associated with the maximisation model. Constraint set 1, placing upper limits, is stated as equality to yield a feasible solution other than the absolute feasible one wherein all variables are assigned zero values (i.e. do-nothing alternative). The do-nothing alternative is obtained even if optional constraint set 2 is used. However, constraint set 1 shall be stated as inequality in conjunction with using optional constraint set 3. The latter case represents a more realistic one as it does not require the treatment of all pavements in a given system.

$$\textit{Minimise}: C_s = \sum_{i=1}^{n}\sum_{j=1}^{m} A\times\bar{C}_{ij}\times I_{ij} \tag{2}$$

$$\textit{Subject to}: \quad 1)\ \sum_{j=1}^{m} I_{ij} = N_i \quad (i=1,2,\ldots,n)$$

$$2)\ \mathrm{RG}_i = b_{i,i+1}\times\mathrm{RG}_{i+1} \Leftrightarrow \mathrm{RG}_i - b_{i,i+1}\times\mathrm{RG}_{i+1} = 0$$

$$3)\ \mathrm{RG}_i = \mathrm{SRG}_i\times N_i = \text{constant}$$

$$4)\ I_{ij} \geq 0$$

where all variables are defined as before.

Age-based optimisation model

The second MPMM model deploys the expected age associated with potential M&R actions as the main pavement improvement indicator. The expected age for each maintenance action can be estimated based on experience and/or referring to an agency historical maintenance records. Whereas for rehabilitation actions, the expected age is typically specified as a main design input parameter. The objective in the second MPMM model is to yield the best M&R plan that is associated with the optimum improvement for a particular pavement system by either maximising the net age–gain or minimising the net M&R cost.

Age–gain maximisation model

The system age–gain maximisation model is formulated as a constrained integer linear programming model subjected to linear constraints as indicated in Equation (3). The objective function is defined as the sum of age–gains applied to all qualifying pavement classes. The age–gain associated with a particular class is the product sum of the expected ages associated with M&R actions and the corresponding M&R variable values. Similar to the first MPMM model, the M&R variables represent the numbers of pavement sections in the various pavement classes to be treated by potential M&R

actions. Also, the five constraint sets are the same except the age–gain has replaced the PCR-gain in optional sets 3 and 4. Optional set 4 still requires a specified level of improvement for each pavement class but in terms of average age–gain using unit of years.

$$\textit{Minimise}: \mathrm{AG_s} = \sum_{i=1}^{n}\sum_{j=1}^{m} \mathrm{EA}_{ij} \times I_{ij}. \tag{3}$$

$$\begin{aligned} \textit{Subject to}: \quad & 1) \quad \sum_{j=1}^{m} I_{ij} \le N_i \quad (i = 1, 2, \ldots, n) \\ & 2) \quad \sum_{i=1}^{n}\sum_{j=1}^{m} A \times \bar{C}_{ij} \times I_{ij} \le B_s \\ & 3) \quad \mathrm{AG}_i = b_{i,i+1} \times \mathrm{AG}_{i+1} \\ & \qquad (i = 1, 2, \ldots, n-1) \\ & \qquad \mathrm{AG}_i = \sum_{j=1}^{m} \mathrm{EA}_{ij} \times I_{ij} \\ & 4) \quad \mathrm{AG}_i = \mathrm{SAG}_i \times N_i \\ & 5) \quad I_{ij} \ge 0 \end{aligned}$$

where:

$\mathrm{AG_s}$, net pavement age–gain in years applied to a pavement system as a result of M&R actions;
EA_{ij}, expected age in years associated with the jth M&R action as applied to pavement sections in the ith class,
AG_i, net pavement age–gain in years applied to the ith class, and
SAG_i, specified average age–gain in years for pavement sections in the ith class.

M&R cost minimisation model

The integer linear optimisation model for minimising the net M&R cost (C_s) for a particular pavement system is presented in Equation (4). The objective function and the four constraint sets are similar to the ones provided in the first MPMM cost minimisation model with again the age–gain improvement indicator is replacing the PCR-gain indicator. A specified average age–gain for each class, as required by optional constraint set 3, provides a vital alternative for allocating M&R funds wherein all qualifying pavement classes receive their fair shares of improvements as indicated by a given highway agency. Global system optimisation that excludes any class improvement constraints may not always yield a rational M&R plan as will be demonstrated in the sample presentation.

$$\textit{Minimise}: C_s = \sum_{i=1}^{n}\sum_{j=1}^{m} A \times \bar{C}_{ij} \times I_{ij}. \tag{4}$$

$$\begin{aligned} \textit{Subject to}: \quad & 1) \quad \sum_{j=1}^{m} I_{ij} \le N_i \quad (i = 1, 2, \ldots, n) \\ & 2) \quad \mathrm{AG}_i = b_{i,i+1} \times \mathrm{AG}_{i+1} \\ & \qquad (i = 1, 2, \ldots, n-1) \\ & 3) \quad \mathrm{AG}_i = \mathrm{SAG}_i \times N_i \\ & 4) \quad I_{ij} \ge 0. \end{aligned}$$

Constrained integer linear programming

The presented optimum MPMM models are to be solved as pure integer programming problems wherein all the M&R variables are restricted to be integer valued. In resolving such problems, one could simply solve the constrained integer linear problem as a linear program while ignoring the integer restrictions, and then simply rounding off the fractional values of the linear programming optimal solution to get an integer solution. But one must be careful so that the selected optimal solution remains a feasible one. Such an approach is frequently used in practice especially when the values of the variables are relatively large (Phillips *et al.* 1987, Bazaraa *et al.* 2006).

Microscopic M&R cost model

In the microscopic approach, individual pavement sections can receive different M&R actions, therefore, the corresponding M&R cost rates depend on the degree of location scatterness of pavement sections with similar distress conditions. The presented optimum MPMM models require the use of an average cost rate for each M&R action as applied to a particular pavement class. The average cost rate mainly depends on the degree of scatterness of pavement distress within a given pavement system (network). The degree of scatterness is defined using a histogram that shows the size distribution of pavement segments with similar degree of pavement distress as shown in Figure 1. A pavement segment is comprised of a number of interconnected pavement sections that are ranked in the same pavement class. Figure 2 shows that a highest cost rate (HC_{ij}) is applied when pavement class segments are comprised of single isolated sections, thus, indicating the highest degree of distress scatterness. A lowest cost rate (LC_{ij}) is used when a pavement segment is comprised of a maximum size of (m_{ij}) interconnected sections that can be treated in a 1-day work load. Figure 2 shows an assumed linear relationship between cost rate and segment size. Therefore, the cost rate associated with any pavement segment size (PS_{ik}) within the two extremes can be estimated from the slope of the depicted straight line as indicated by Equation (5a).

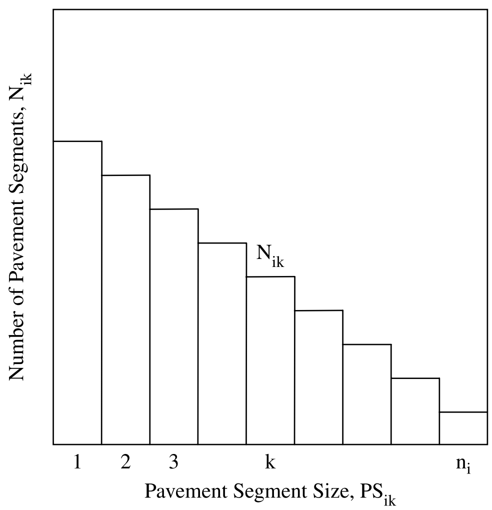

Figure 1. Pavement segment size distribution histogram.

The segment cost rate (C_{ijk}) as applied to certain pavement class and M&R action can then be estimated using Equation (5a) for various existing pavement segment sizes. An average cost rate is estimated as a weighted average wherein the weights are the numbers of pavement class segments as obtained from the histogram shown in Figure 1. The presented histogram takes on the general shape of a skewed distribution as supported by the field data provided in the sample presentation section. The average cost rate obtained from Equation (5b) for specified pavement class and M&R action is to be used in the outlined optimum MPMM models. A contractor can inversely relate the highest and lowest cost rates using the reduction in daily productivity rate for a particular M&R action as presented in Equation (5c).

$$C_{ijk} = \mathrm{HC}_{ij} - \left(\frac{(\mathrm{HC}_{ij} - \mathrm{LC}_{ij})(\mathrm{PS}_{ik} - 1)}{(m_{ij} - 1)}\right) \tag{5a}$$

$$\bar{C}_{ij} = \frac{\sum_{k=1}^{n_i} N_{ik} \times \mathrm{PS}_{ik} \times C_{ijk}}{\sum_{k=1}^{n_i} N_{ik} \times \mathrm{PS}_{ik}} \tag{5b}$$

where,

$$N_i = \sum_{k=1}^{n_i} N_{ik} \times \mathrm{PS}_{ik} \quad \text{where } n_i \leq m_{ij}$$

$$\mathrm{HC}_{ij} = \frac{\mathrm{LC}_{ij}}{P_{ij}}, \quad \text{where } 0 \leq P_{ij} \leq 1 \tag{5c}$$

where:

PS_{ik}, pavement segment comprised of (k) interconnected pavement sections in the ith class;
N_{ik}, number of pavement segments comprised of (k) interconnected pavement sections in the ith class;
C_{ijk}, cost rate per square meter associated with the jth M&R action as applied to the pavement segments comprised of (k) interconnected pavement sections in the ith class;
m_{ij}, maximum size of a pavement segment comprised of (m_{ij}) interconnected pavement sections that can be daily treated by the jth M&R action in the ith class;
LC_{ij}, lowest cost rate associated with the jth M&R action for a pavement segment comprised of (m_{ij}) interconnected pavement sections in the ith class;
HC_{ij}, highest cost rate associated with the jth M&R action for pavement segments comprised of only single isolated sections in the ith class;
n_i, maximum size of an existing pavement segment comprised of (n_i) interconnected pavement sections in the ith class; and
P_{ij}, reduction in the daily M&R productivity rate as a percentage, in decimal form, associated with the jth M&R action for pavement segments comprised of only single isolated sections in the ith class.

A pavement section identification system is required in order to establish a pavement segment size distribution for a given pavement system. A pavement section coding procedure based on 12-digits is used as demonstrated in Figure 3. Each pavement section has its unique reference number comprised of the highway number, direction number, lane number, beginning station number and PCR number. A computer system can easily generate a pavement segment size distribution by searching an inventory database containing the reference numbers of all pavement sections making up a given pavement system. The computer system needs to search for interconnected pavement sections with similar PCR values as defined

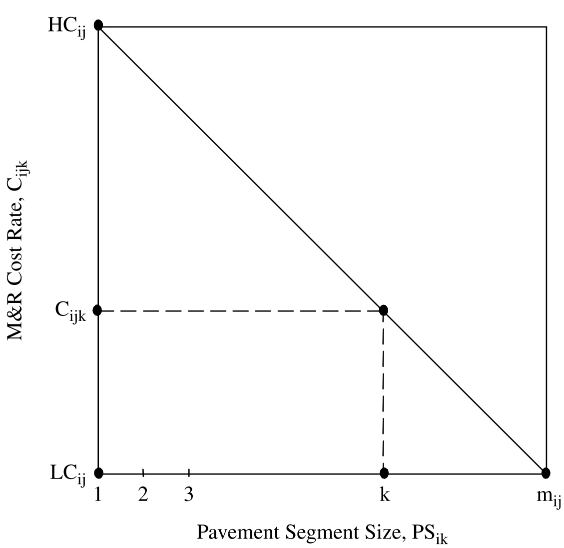

Figure 2. Pavement segment size vs. M&R cost rate.

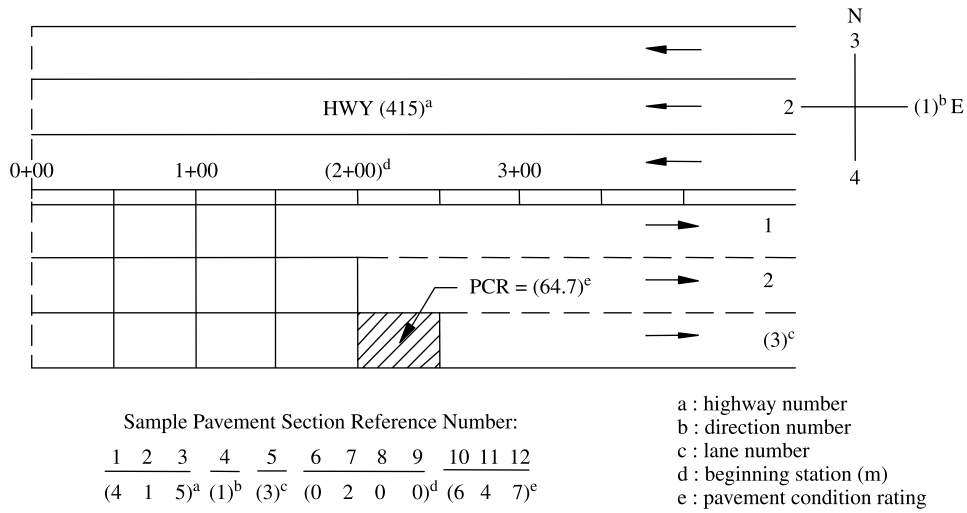

Figure 3. Pavement section identification system.

in the establishment of pavement classes. Pavement sections are typically selected 50-m in lane length. Therefore, longitudinal interconnection between two pavement sections is identified when there is a 50-m difference in their beginning station numbers assuming the same highway, direction and lane numbers. Transverse interconnection can also be identified when two sections have the same beginning station numbers.

Optimum pavement segment selection model

The optimum M&R budget allocation amongst the various applicable pavement classes can be determined once an optimal solution is generated as provided in Equation (6). The allocated optimum class budget can be used to treat a number of pavement sections as indicated by the M&R variable values. However, identification of pavement segment candidates can be established by assigning priorities to the longest pavement segments, all in the same class, as they are associated with the lowest M&R cost rates.

$$B_i^* = \sum_{j=1}^{m} A \times \bar{C}_{ij} \times I_{ij}^* \tag{6}$$

where:

B_i^*, optimum budget allocated to pavement in the ith class; and
I_{ij}^*, optimum value associated with the jth M&R variable in the ith class.

An alternative optimum constrained integer linear model for the selection of pavement segment candidates is presented in Equation (7) that is based on the prevailing pavement segment size distribution. The model is individually applied to the pavement segments in each class using the allocated optimum class budget. The model applies the PCR-gain indicator similar to what has been presented in Equation (1). An M&R variable represents the number of pavement segments all with the same size that should be treated by the corresponding M&R action. The average terminal PCR values and M&R cost rates associated with the pavement segment sizes are the main parameters that will identify the pavement segment candidates.

$$\textit{Minimise}: \mathrm{RG}_i = \sum_{j=1}^{m} \sum_{k=1}^{n_i} [(\mathrm{PCR}_0)_{ij} - (\overline{\mathrm{PCR}}_t)_{ik}] \times I_{ijk} \tag{7}$$

$$\textit{Subject to}: \quad 1) \quad \sum_{j=1}^{m} \sum_{k=1}^{n_i} A \times C_{ijk} \times I_{ijk} \le B_i^*$$

$$2) \quad \sum_{j=1}^{m} \sum_{k=1}^{n_i} I_{ijk} \le N_{ik}$$

$$3) \quad I_{ijk} \ge 0$$

where:

$(\overline{\mathrm{PCR}}_t)_{ik}$, average terminal PCR associated with the pavement segments in the ith class with the same kth size; and
I_{ijk}, an integer M&R variable representing the number of pavement segments in the ith class with the same kth size to be treated by the jth M&R action.

Optimum MPMM model requirements

The presented optimum MPMM models can be applied to a given pavement system with similar pavement structures and loading conditions. Pavement classes can be defined using the PCR as the main indicator of pavement distress. Figure 4 shows an example of pavement class definitions that utilises ten classes with each class defined using a PCR interval of 10 points. Maintenance actions are assumed to improve pavements by only one class, which is equivalent to an average of 10 points PCR upgrade. Rehabilitation actions as applied to potential pavement classes can place the pavements in class 10 with a 95 average PCR value. The deployed M&R actions must provide treatments capable of producing the outcomes indicated by Figure 4.

Rehabilitation actions typically include treatments such as plain overlay with variable thickness applicable to pavement classes generally designated as good to fair for instance classes 7 and 6, skin patch (overlay after milling) applicable to classes denoted as fair to poor such as classes 6 and 5, partial reconstruction (placement of new asphalt concrete after removal of existing one) applicable to very poor classes, for example, classes 4 and 3, and complete reconstruction up to subgrade applicable to very bad pavement classes such as classes 2 and 1. Rehabilitation actions are assumed to upgrade the pavement from its current class to the best class (class 10). Maintenance actions consist of treatments such as crack sealing, pothole patching, and localised surface treatments for instance sand and slurry seals. Maintenance actions can be applied to any pavement class with an anticipated outcome of only one-class upgrade as shown in Figure 4. Therefore, it is clear that the application of maintenance actions will be associated with different treatment intensities and cost rates as applied to different pavement classes. Also, the presented MPMM models can be readily used to incorporate class improvement outcomes different from those indicated by Figure 4 using appropriate M&R actions.

The presented MPMM models also require other input parameters as outlined in the presentation of these models including a survey of pavement distress so that each pavement section can be assigned a PCR value. A pavement segment size distribution can be established once all pavement sections are assigned reference numbers. The lowest cost rate and daily production rate are required for each M&R action to determine the corresponding average cost rate using the presented microscopic M&R cost model. In addition, the performance of each M&R action is required using either the expected age in years or the initial PCR value.

Sample presentation

A total of 1500 pavement sections were surveyed for pavement distress in the city of Nablus, West Bank, Palestine. The selected sections are part of the arterial roadway system. A decision was made that the sections would be surveyed for cracking as it is the dominating defect in a city mainly built on rock foundation. A PCR was assigned to each 50-m lane length section as defined in Equation (8). The distinction between the three types of Alligator cracking was made based on severity as indicated by the crack width. The deployed pavement distress assessment is mainly a low-cost procedure that depends on visual inspection and simple related measurements appropriate for use in developing countries.

$$\mathrm{PCR} = 100 - \sum_{i=1}^{4} w_i \times L_i \tag{8}$$

where,

$$L = \sum_{i=1}^{4} L_i \le 50,$$

$$w_1 = 0.5,\ \ w_2 = 1,\ \ w_3 = 1.5,\ \ w_4 = 2$$

where:

PCR, pavement condition rating estimated for a particular pavement section;
w_i, relative weight assigned to the ith cracking type;
L_1, length of pavement section with transverse; longitudinal; and block crackings;
L_2, length of pavement section with Alligator Type A cracking;
L_3, length of pavement section with Alligator Type B cracking;

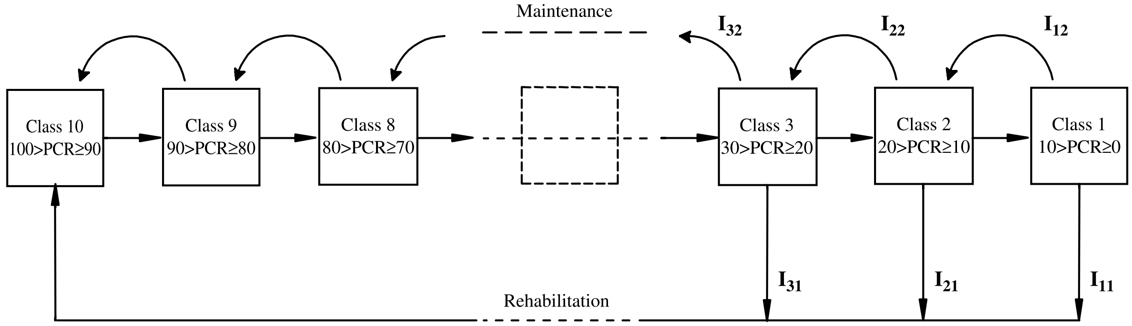

Figure 4. Sample pavement class definitions and M&R improvement outcomes.

Table 1. Pavement segment size distribution and class terminal PCR.

	Number of pavement segments (N_{ik})													
	Pavement segment size (PS_{ik}) ($k = 1, 2, \ldots, 11$)													
Class no. (i)	1	2	3	4	5	6	7	8	9	10	11	n_i	N_i	$(\overline{PSR}_t)_i$
1	3	4	1	1	4	2	1	3	0	0	0	8	81	6.3
2	6	3	4	5	2	3	0	1	2	0	1	11	109	14.8
3	7	5	8	4	6	3	4	2	1	0	0	9	158	27.1
4	11	8	6	5	7	6	4	3	2	3	0	10	236	33.9
5	8	9	7	5	3	4	2	5	1	3	2	11	221	44.7
6	13	9	10	7	8	4	6	4	2	0	2	11	267	56.4

L_4, length of pavement section with Alligator Type C cracking; and

L, lane length of pavement section selected to be 50 m.

The surveyed sections were assigned reference numbers as outlined earlier and stored in an inventory database. Then, a specially designed computer system was used to assign the sections to the ten deployed pavement classes according to their PCR values as outlined in Figure 4. The computer system also performed the search for longitudinal interconnection amongst the coded pavement sections and generated the pavement segment size distribution as provided in Table 1. Table 1 provides the summary information for only six classes, namely, those assumed to qualify for M&R works and to be considered in the sample presentation. The segment size distributions associated with the six classes tend to be skewed to the left as earlier assumed in Figure 1. The presented six classes contained 1072 pavement sections out of 1500 surveyed ones, which means only about 28.5% of the surveyed sections can be considered as 'good' pavements.

The inspected pavement sections were mainly constructed using 10 cm hot-mix asphalt concrete surface and crushed rock aggregate base with variable thickness (30–40 cm). Two M&R actions are assigned to each pavement class consisting of one rehabilitation action and one maintenance action. The corresponding treatments need to be appropriately selected as outlined earlier in the section titled 'Optimum MPMM Model Requirements' so that the intended outcomes can be achieved as defined in Figure 4. Typical maintenance treatments consist of crack sealing and pothole patching, localised surface treatments, and treatment of localised failures in the pavement structure. The extent and severity of pavement distresses vary from one pavement class to another, therefore, the intensity and cost of maintenance works will vary accordingly. Sample rehabilitation plans are proposed for the six qualifying pavement classes with treatment details as defined in Table 2.

Table 2. Definition of sample rehabilitation plans for flexible pavement.

Pavement class no. (i)	Rehabilitation variable	Rehabilitation plan details
6	I_{61}	Crack sealing and pothole patching, treatment of localised failures in the pavement structure, and placement of new 5 cm hot-mix high stability asphalt concrete overlay
5	I_{51}	Cold milling (5 cm thick) of existing asphalt surface, treatment of localised failures in the pavement structure, and placement of new 5 cm hot-mix high stability asphalt concrete overlay
4	I_{41}	Cold milling (5 cm thick) of existing asphalt surface, treatment of localised failures in the pavement structure, and placement of new 7 cm hot-mix high stability asphalt concrete overlay
3	I_{31}	Removal of the existing 10 cm asphalt concrete pavement, treatment of localised failures in the base and subgrade, and placement of new 10 cm hot-mix high stability asphalt concrete surface
2	I_{21}	Removal of the existing 10 cm asphalt concrete pavement, treatment of localised failures in the base and subgrade, and placement of new 5 cm leveling aggregate layer made of dense-graded crushed rock and placement of new 10 cm hot-mix high stability asphalt concrete surface
1	I_{11}	Removal of the existing 10 cm asphalt concrete pavement, treatment of localised failures in the base and subgrade, and placement of new 10 cm leveling aggregate layer made of dense-graded crushed rock and placement of new 10 cm hot-mix high stability asphalt concrete surface

Table 3. Cost rates and improvement indicators of M&R actions.

Class no. (i)	Rehabilitation action ($j = 1$)						Maintenance action ($j = 2$)					
	$(\mathrm{PCR}_0)_{ij}$	EA_{ij}	LC_{ij}	HC_{ij}	m_{ij}	$\bar{C}_{ij}$	$(\mathrm{PCR}_0)_{ij}$	EA_{ij}	LC_{ij}	HC_{ij}	m_{ij}	$\bar{C}_{ij}$
1	95	15	20	40	10	29.7	15	2	3	6	20	5.3
2	95	15	17	34	10	25	25	2	2.5	5	20	4.3
3	95	12	14	28	15	21.2	35	1.5	2	4	30	3.7
4	95	12	12	24	15	19.7	45	1.5	1.5	3	30	2.7
5	95	10	10	20	20	17.1	55	1	1	2	40	1.9
6	95	10	8	16	20	14	65	1	0.5	1	40	0.9

Therefore, the two considered M&R actions are associated with different cost rates and improvement indicators as applied to the six deployed classes. Table 3 provides the cost rates and improvement indicators for the six pavement classes. The lowest cost rates (LC_{ij}) are estimated based on a maximum segment size (m_{ij}) that can be treated in a one-day work load as shown in Figure 2. The highest cost rates are determined using Equation (5c) assuming 50% reduction in productivity rate when pavement segments consist of single isolated sections. The average cost rates in US dollars per square metre, to be used in the optimum MPMM models, are then determined using Equations (5a) and (5b). Generally, the M&R cost rates are inversely related to the class average terminal PCR values. Also, Table 3 provides estimates for the improvement indicators associated with the two outlined M&R actions consistent with the assumptions made in establishing Figure 4. The expected initial PCR value and expected age (EA_{ij}) in years are estimated for each M&R action. The anticipated improvements are much lower for

Maximize: $\mathrm{RG_s} = 88.7\,I_{11} + 8.7\,I_{12} + 80.2\,I_{21} + 10.2\,I_{22} + 67.9\,I_{31} + 7.9\,I_{32} + 61.1\,I_{41} + 11.1\,I_{42} + 50.3\,I_{51} + 10.3\,I_{52} + 38.6\,I_{61} + 8.6\,I_{62}$

Subject to:

1) $I_{11} + I_{12} \le 81;\quad I_{21} + I_{22} \le 109;\quad I_{31} + I_{32} \le 158$

$I_{41} + I_{42} \le 236;\quad I_{51} + I_{52} \le 221;\quad I_{61} + I_{62} \le 267$

2) $175\,[(29.7\,I_{11} + 5.3\,I_{12}) + (25\,I_{21} + 4.3\,I_{22}) + (21.2\,I_{31} + 3.7\,I_{32}) + (19.7\,I_{41} + 2.7\,I_{42}) + (17.1\,I_{51} + 1.9\,I_{52}) + (14\,I_{61} + 0.9\,I_{62})] \le 1 \times 10^6$

3) $88.7\,I_{11} + 8.7\,I_{12} = 0.7431\,(80.2\,I_{21} + 10.2\,I_{22});\quad 80.2\,I_{21} + 10.2\,I_{22} = 0.6899\,(67.9\,I_{31} + 7.9\,I_{32});\quad 67.9\,I_{31} + 7.9\,I_{32} = 0.6695\,(61.1\,I_{41} + 11.1\,I_{42});$

$61.1\,I_{41} + 11.1\,I_{42} = 1.0679\,(50.3\,I_{51} + 10.3\,I_{52});\quad 50.3\,I_{51} + 10.3\,I_{52} = 0.8277\,(38.6\,I_{61} + 8.6\,I_{62})$

4) $I_{ij} \ge 0$ (for $i = 1, 2, \ldots, 6$; and $j = 1, 2$)

Minimize: $C_s = 175\,[(29.7\,I_{11} + 5.3\,I_{12}) + (25\,I_{21} + 4.3\,I_{22}) + (21.2\,I_{31} + 3.7\,I_{32}) + (19.7\,I_{41} + 2.7\,I_{42}) + (17.1\,I_{51} + 1.9\,I_{52}) + (14\,I_{61} + 0.9\,I_{62})]$

Subject to:

1) Same as above

2) $88.7\,I_{11} + 8.7\,I_{12} = 3240;\quad 80.2\,I_{21} + 10.2\,I_{22} = 3815;\quad 67.9\,I_{31} + 7.9\,I_{32} = 4740$

$61.1\,I_{41} + 11.1\,I_{42} = 5900;\quad 50.3\,I_{51} + 10.3\,I_{52} = 4420;\quad 38.6\,I_{61} + 8.6\,I_{62} = 4005$

3) $I_{ij} \ge 0$ (for $i = 1, 2, \ldots, 6$; and $j = 1, 2$)

Figure 5. Sample optimum pavement condition rating gain model.

Maximize: $AG_s = 15\,I_{11} + 2\,I_{12} + 15\,I_{21} + 2\,I_{22} + 12\,I_{31} + 1.5\,I_{32}$
$+ 12\,I_{41} + 1.5\,I_{42} + 10\,I_{51} + 1\,I_{52} + 10\,I_{61} + 1\,I_{62}$

Subject to: 1) $I_{11} + I_{12} \leq 81;\quad I_{21} + I_{22} \leq 109;\quad I_{31} + I_{32} \leq 158$
$I_{41} + I_{42} \leq 236;\quad I_{51} + I_{52} \leq 221;\quad I_{61} + I_{62} \leq 267$

2) $175\,[(29.7\,I_{11} + 5.3\,I_{12}) + (25\,I_{21} + 4.3\,I_{22}) + (21.2\,I_{31} + 3.7\,I_{32})$
$+ (19.7\,I_{41} + 2.7\,I_{42}) + (17.1\,I_{51} + 1.9\,I_{52}) + (14\,I_{61} + 0.9\,I_{62})] \leq 1 \times 10^6$

3) $15\,I_{11} + 2\,I_{12} = 0.7431\,(15\,I_{21} + 2\,I_{22});\quad 15\,I_{21} + 2\,I_{22} = 0.6899$
$(12\,I_{31} + 1.5\,I_{32});\quad 12\,I_{31} + 1.5\,I_{32} = 0.6695\,(12\,I_{41} + 1.5\,I_{42});$
$12\,I_{41} + 1.5\,I_{42} = 1.0679\,(10\,I_{51} + 1\,I_{52});\quad 10\,I_{51} + 1\,I_{52} = 0.8277$
$(10\,I_{61} + 1\,I_{62})$

4) $I_{ij} \geq 0$ (for $i = 1, 2, \ldots, 6$; and $j = 1, 2$)

Minimize: $C_s = 175\,[(29.7\,I_{11} + 5.3\,I_{12}) + (25\,I_{21} + 4.3\,I_{22}) + (21.2\,I_{31} + 3.7\,I_{32})$
$+ (19.7\,I_{41} \quad 2.7\,I_{42}) + (17.1\,I_{51} + 1.9\,I_{52}) + (14\,I_{61} + 0.9\,I_{62})]$

Subjcct to: 1) Same as above

2) $15\,I_{11} + 2\,I_{12} = 810;\quad 15\,I_{21} + 2\,I_{22} = 981;\quad 12\,I_{31} + 1.5\,I_{32} = 1264$
$12\,I_{41} + 1.5\,I_{42} = 1652;\quad 10\,I_{51} + 1\,I_{52} = 1326;\quad 10\,I_{61} + 1\,I_{62} = 1335$

3) $I_{ij} \geq 0$ (for $i = 1, 2, \ldots, 6$; and $j = 1, 2$)

Figure 6. Sample optimum AG model.

maintenance works when compared to rehabilitation actions. The pavement section width is considered to be 3.5 m.

The two optimum MPMM models have been formulated using the relevant input parameters provided in Tables 1 and 3 with a total of 12 M&R variables. The first model is based on the net PCR-gains as applied to the six applicable classes with both corresponding maximisation and minimisation problems are being demonstrated in Figure 5. The maximisation problem used the first optional constraint set requiring class improvement relations proportional to the numbers of pavement sections in the six deployed classes as defined in Equation (1). Whereas, the minimisation problem applied the second optional constraint set requiring a specified average PCR-gain for each class as presented in Equation (2). The specified average PCR-gains for classes 1, 2, 3, 4, 5 and 6 are 40, 35, 30, 25, 20 and 15, respectively. The second model is formulated using the net expected age–gains considering both the maximisation and minimisation problems as indicated by Equations (3) and (4), respectively. Figure 6 shows the two corresponding problems wherein the maximisation problem used the first optional constraint set presented in Equation (3), and the minimisation problem assumed the specified average age–gains for classes 1, 2, 3, 4, 5 and 6 to be 10, 9, 8, 7, 6 and 5 years, respectively, as required by the second optional constraint set outlined in Equation (4).

The optimal solutions associated with the two sample MPMM models are obtained using a software package called Maple 8 developed by the Waterloo Maple Inc., Waterloo, Canada, with solutions provided in Table 4. The maximisation problems associated with the two models have generated different optimal solutions using a specified budget of \$1 million. The optimal M&R variable values are very much the same for classes 1, 2 and 3, but different for

Table 4. Optimum M&R solution plans for models presented in Figures 5 and 6.

M&R variable	Optimisation model: Optimum PCR-gain model: Maximum RG_s	B_i^* (\$)	Minimum C_s	B_i^* (\$)	Optimum age–gain model: Maximum AG_s	B_i^* (\$)	Minimum C_s	B_i^* (\$)
I_{11}	17	88,358	37	192,308	18	93,555	54	280,665
I_{12}	0		0		0		0	
I_{21}	25	109,375	48	210,000	24	105,000	65	284,375
I_{22}	00		0		0		0	
I_{31}	43	159,530	70	259,700	43	159,530	105	389,550
I_{32}	0		0		0		0	
I_{41}	34	212,660	66	307,860	65	224,088	138	475,755
I_{42}	202		170		0		0	
I_{51}	44	190,523	54	217,123	73	218,453	133	398,003
I_{52}	177		167		0		0	
I_{61}	87	241,500	57	172,725	68	197,943	119	314,860
I_{62}	180		210		199		148	
Total	809	1×10^6	879	1.36×10^6	490	1×10^6	708	2.14×10^6

classes 4, 5 and 6, however, the corresponding class budgets are relatively similar. A total of 809 pavement sections are to be treated according to the first model maximisation solution whereas the number is 490 sections for the second model. This means that more rehabilitation works are to be done using the second model solution plan, which is evidenced from the values of the corresponding rehabilitation variables, those with second subscript of 1. It is also to be noted that both maximisation solutions applied only rehabilitation works to the worst classes, namely, classes 1, 2 and 3.

The optimal solutions associated with the minimisation problems are also provided in Table 4. The table shows that more pavement sections are being treated using the first model by making more use of the maintenance variables at a total cost of \$1.36 million compared to \$2.14 million for the second model. The higher cost for the second model can be attributed to the fact that the specified average age–gains require higher class improvement standards compared to the specified average PCR-gains used in the first model. The optimum total cost associated with the first model has increased to \$2.16 million when the specified average PCR-gains are raised by 50% to better match the specified average age–gains used in the second model. Table 5 provides similar optimal solutions as in Table 4 but excluding the class constraint sets. The associated maximisation problems have generated two different solutions with unreasonable allocation of M&R funds amongst the six deployed classes. Whereas the two minimisation problems, essentially becoming identical problems, have resulted in optimal solutions that require the treatment of all

Table 5. Optimum M&R solution plans for models presented in Figures 5 and 6 excluding class constraints.

M&R variable	Optimisation model: Optimum PCR-gain model: Maximum RG_s	B_i^* (\$)	Minimum C_s	B_i^* (\$)	Optimum age–gain model: Maximum AG_s	B_i^* (\$)	Minimum C_s	B_i^* (\$)
I_{11}	0	0000	0	75,128	0	0000	0	75,128
I_{12}	0		81		0		81	
I_{21}	109	476,875	0	82,023	0	0000	0	82,023
I_{22}	0		109		0		109	
I_{31}	80	296,800	0	102,305	0	0000	0	102,305
I_{32}	0		158		0		158	
I_{41}	0	111,510	0	111,510	100	344,750	0	111,510
I_{42}	236		236		0		236	
I_{51}	0	73,483	0	73,483	0	0000	0	73,483
I_{52}	221		221		0		221	
I_{61}	0	42,053	0	42,053	267	654,150	0	42,053
I_{62}	267		267		0		267	
Total	913	1×10^6	1072	0.49×10^6	367	1×10^6	1072	0.49×10^6

pavement sections at a total cost of \$0.49 million using only maintenance variables.

Conclusions and recommendations

The presented sample results have indicated the effectiveness of the developed optimum MPMM models in yielding an optimum M&R plan for a given pavement system. The sample results have also indicated the significance role of class constraints in providing rational allocation of M&R funds wherein each potential class receives its fair share. The exclusion of class constraints has resulted in optimal solutions that have either denied some classes from any M&R funds as for the maximisation problems or deployed only maintenance actions as associated with the minimisation problems. The two proposed pavement improvement indicators, namely, PCR-gain and age–gain, have resulted in somewhat different outcomes considering the presented optimal solutions. This can be attributed to the fact that the estimated values for the two deployed improvement indicators are not quite compatible. The two MPMM models would have probably yielded very much similar solutions had the improvement indicators been estimated with a higher degree of compatibility. Therefore, the indicator estimated with a higher degree of confidence, probably the age–gain, can be used as a calibration tool against the other indicator. Highly correlated PCR and AGs can then be used to generate the corresponding performance curves frequently used in several pavement management applications.

The presented optimum MPMM models require minimal input data and can be easily formulated and solved for a given pavement system. A highway agency needs first to decide on which one of the two models it is interested in using, a decision mainly made based on which improvement indicator is to be used. The agency can then apply the maximisation model if an anticipated budget is available, which is typically the case. However, if budget is not available, then the minimisation model can be applied to provide an estimate of the required budget that would meet the agency improvement requirements, which can be very helpful in fund raising campaigns. The required input parameters are mainly the cost rates and improvement indicators associated with the deployed M&R actions. The outlined microscopic M&R cost model can be used to generate average cost rates that account for the scatterness degree of pavement distress based on establishing a segment size distribution, otherwise, average cost rates can be estimated based on experience and engineering judgment. The presented sample problems have deployed only 12 M&R variables as two M&R actions are assigned to each class. Practitioners can use more than two M&R actions for each class and choose to formulate separate models for maintenance and rehabilitation works. The commercially available linear programming software packages can handle problems with a much larger number of variables, and are simple to operate and generally yield very reliable optimal solutions.

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