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Evolutionary finance: introduction to the special issue[☆]

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Abstract

This paper introduces the special issue of the *Journal of Mathematical Economics* on Evolutionary Finance.

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This special issue of the *Journal of Mathematical Economics* contains papers at the cutting edge of research in *evolutionary finance*, a subfield of financial economics. Evolutionary finance provides a synthesis between the traditional and the behavioral finance point of view.

Recent empirical and experimental work has quite successfully challenged the traditional view of efficient markets and the long-sustained belief in market rationality, see for example the excellent surveys by Campbell (2000) and Hirshleifer (2001). Indeed a new paradigm

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based on behavioral models of decision under risk and uncertainty is beginning to crowd out the traditional view based on complete rationality of all market participants. The traditional and the behavioral finance model however share one important feature: They are both based on the notion of a representative agent—though this mythological figure is dressed differently. While traditionally he had rational preferences, expectations and beliefs, he is currently a prospect theory maximizer, unable to carry out Bayesian updating and likely to fall into framing traps.

Instead evolutionary finance suggests a model of portfolio selection and asset price dynamics that is explicitly based on the idea of heterogeneity of investors. As a single individual has little (or negligible) weight, evolutionary finance suggests to think of the financial market not in terms of individuals but in terms of strategies. For the market it is irrelevant who is investing according to, say, P/E-ratios. The only thing that matters is how much money is invested according to such a criterion. The investors' strategies may result from rational maximization of some intertemporal expected utility function, simple heuristics, behavioral finance or principal-agent models describing incentive problems in institutions. For evolutionary finance it is irrelevant *how* strategies are generated because it analyzes their *performance* once they are in the market.

Evolutionary finance is descriptive and normative as well, answering which set of strategies one would expect to be present in a market and how to find the best response to any such market. The first observation is that there is nothing like "the best strategy" because the performance of any strategy will depend on all strategies that are in the market. Rationality therefore is to be seen as conditional on the market ecology.

Whereas traditional finance—based on optimization and equilibrium—borrows a lot from classical mechanics, behavioral finance borrows from psychology. Instead evolutionary finance borrows from biology, in particular from biological models based on evolutionary dynamics. The principles of natural selection and mutation, as formulated by Charles Darwin, are two fruitful analogies that led evolutionary finance surprisingly far until today. While in biology resources like food are fought for, in finance strategies fight for market capital. Selection changes the relative weight of strategies in the market and may lead to extinction for some. On the other hand, mutation enriches the market's ecology (at least temporarily).

The evolutionary finance perspective also leads to a revival of the classical Law of Demand according to which prices are determined by demand and supply. According to traditional finance, prices are exclusively determined by expectations because agents are able to borrow without limits. Behavioral Finance, using the term "limits to arbitrage", has convincingly argued that unlimited borrowing is impossible so that prices are also determined by demand from noise traders, for example. Evolutionary finance takes this point serious and shows how the flow of wealth between strategies is the driving force of price formation. Ultimately, evolutionary finance models try to explain how the ecology of the market evolves over time, i.e. how the distribution of wealth across strategies as well as the strategies themselves change over time. This should also result in better prediction models for asset prices.

From its very nature, evolutionary finance models are better suited for empirical research than those in traditional finance that is based on the notion of expectations. Data on strategies that are pursued and the flow of wealth between these strategies can be collected on actual markets (for example in the mutual fund and the hedge fund sectors), while it is considerably harder to quantify traditional models by finding data on expectations. This is so because most capital is managed by delegation and in this process the principal (the investor) wants the agent (the wealth manager) to commit to some strategy in order to simplify monitoring and reduce verifiability problems. Indeed many banks compete for investors' money by advertising investment styles or strategies they want to commit to.

While it is too early at the present stage to assess the potential impact of evolutionary finance on our understanding of financial markets, it might be encouraging to recall Theodosius Dobzhansky's dictum that "Nothing in biology makes sense except in the light of evolution". A similar statement may well be true for the dynamics of financial markets.

This special issue of the *Journal of Mathematical Economics* comprises nine papers on topics within the emerging field of evolutionary finance. The papers aim to explore the above ideas in consistent and adequate models with the goal to contribute to a better understanding of the dynamics of financial markets. The collection of papers in this issue reflects the diversity of evolutionary approaches in terms of both conceptual and methodological aspects.[1]

On the conceptual level, the reader will notice several different modeling approaches pursued in the papers. Temporary and general equilibrium models are considered. Dynamical systems theory as well as game-theoretic reasoning is applied. Agents' behavior originates from expected utility maximization, genetic learning, or is only restricted by being adapted to the information filtration. Fundamentalists and noise traders also enter the stage. On the methodological level, a variety of treatments, analytical, empirical and numerical, have been applied by the authors. In the best tradition of the *Journal of Mathematical Economics* all papers share a thorough and formal treatment of the issue under consideration.

Brock et al. (2005) analyze in their paper "Evolutionary dynamics in markets with many trader types" the dynamical behavior of an asset market as the number of buyer and seller's types tends to infinity. A large type limit result is established in a rigorous mathematical fashion. It turns out that the limit dynamics, which provides an approximation of an economy with a large but finite number of traders, is deterministic. A specific asset market model with heterogenous beliefs (inspired by Brock and Hommes (1997, 1998)) is studied in detail. The dynamical properties of the limit dynamics in dependence of structural parameters of the economy, which can include different bifurcation scenarios, does generically reflect the dynamics in the original market.

In Thorsten Hens and Klaus Reiner Schenk-Hoppé's paper "Evolutionary stability of portfolio rules in incomplete markets", the endogenous wealth dynamics between different investment strategies is studied. It is shown that among all stationary investment strategies there is only one which is not destabilized by the introduction of any "mutant" strategy. This investment rule has an explicit representation: it prescribes to allocate one's budget across assets according to the expected value of their relative dividend payoffs. In the long run, relative asset prices are thus equal to their relative fundamental values. The main result has a game-theoretic interpretation—who wants to play a dominated strategy?

[1] It is tempting to remark that this special issue on evolutionary finance is an element of our strategy in competing for the professions' attention in a truly heterogenous population of approaches to the understanding of financial markets.

In “The asset market game” Alós-Ferrer and Ania (2005) elaborate this game-theoretic perspective in a simple financial market model which was first presented in Blume and Easley (1992). The weight, or frequency, of different investment strategies evolves through a process of imitation based on expected returns (but not according to an endogenous wealth dynamics). The authors also study a proportional imitation rule based on realized returns. It turns out that a modified Kelly rule is the unique evolutionary stable strategy à la Schaffer in this game.

Sandroni (2005)’s paper “Market selection when markets are incomplete” aims to generalize selection results by Sandroni (2000) and Blume and Easley (2000) to incomplete markets. To this end a general equilibrium model of an asset market in which consumers maximize expected discounted utility is studied. The question is analyzed whether the utility function and/or the accuracy of beliefs matter for survival. In complete markets it is known that only the latter matters. Sandroni’s results show that in incomplete markets a concept of entropy relative to the market’s belief is needed.

The long-run behavior of arbitrary incomplete markets of short-lived assets is analyzed in “Market selection and survival of investment strategies” by Amir et al. (2005). In this paper it is proved that asymptotically either the market portfolio is held by all investors or the investment strategy discovered in Hens and Schenk-Hoppé (2005) (the expected value of the relative dividends rule) will dominate the market. Since every adapted investment strategy is permitted, this result is in particular true in a general equilibrium asset market model.

Föllmer et al. (2005) in their paper “Equilibria in financial markets with heterogeneous agents: a probabilistic perspective” analyze financial market models in which agents form their demand for an asset on the basis of their forecasts of future prices and where their forecasting rules may change over time, as a result of the influence of other traders. Prices can exhibit transient behavior when chartists predominate. However, if the probability that an agent will switch to being a “chartist” is not too high then the process does not explode and the limit distribution of the price process exists and is unique.

At the heart of the models discussed above lies the long-run performance of investment strategies—this has normative appeal. However, the papers also acknowledge that the degree of risk aversion matters considerably for the evolution of the wealth distribution. Levy (2005) in his paper “Is risk-aversion hereditary?”, attempts to shed more light to what extent our preferences are learned or inherited. Empirical observations are shown to provide an upper bound on the role of heredity in determining risk-aversion.

Lux and Schornstein (2005) approach the problem of the explanatory power of heterogenous agent models in their thorough complex-adaptive-systems paper “Genetic learning as an explanation of stylized facts of foreign exchange markets”. It turns out that many regularities in actual currency markets can be observed in artificial markets with adaptive investment behavior that stems from genetic learning algorithms.

The importance and effect of different trading protocols on the outcome of market interaction is analyzed by Bottazzi et al. (2005) in their paper “Institutional architectures and behavioral ecologies in the dynamics of financial markets”. In particular, they show that market architectures bear a central influence upon the time series properties of market dynamics. Conversely, the revealed allocative efficiency of different market settings is strongly influenced by the trading behavior of the agents.

It is our hope that this collection of papers, which encompasses several directions of current developments in *Evolutionary Finance*, serves the interested reader and stimulates further research in this exciting field.

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