Development of Fatigue Cracking Prediction Models Using Long-Term Pavement Performance Database

Hsiang-Wei Ker[1]; Ying-Haur Lee[2]; and Pei-Hwa Wu[3]

Abstract: This study strives to develop improved fatigue cracking models using the long-term pavement performance database. The prediction accuracy of the existing models was found to be inadequate. Several modern regression techniques including generalized linear model and generalized additive model along with the assumption of Poisson distribution and quasi-likelihood estimation method were adopted for the modeling process. After many trials in eliminating insignificant and inappropriate parameters, the resulting model included several variables such as yearly KESALs), pavement age, annual precipitation, annual temperature, critical tensile strain under the asphalt-concerete surface layer, and freeze-thaw cycle for the prediction of fatigue cracking. The proposed model appeared to have substantial improvements over the existing models although their further enhancements are possible and recommended.

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Introduction

Fatigue cracking is one of the major flexible pavement distress types primarily caused by the accumulated traffic loads. Extensive research has been conducted to predict the occurrence of this distress type using various empirical and mechanistic-empirical approaches. Conventional predictive models usually correlate fatigue damage to the critical tensile strain and the stiffness of asphalt-concrete (AC) surface layer (Shell 1978).

Brief Literature Review

Since fatigue cracking is primarily caused by accumulated traffic loads various predictive models, as shown in Table 1, based on the following expressions have been proposed to estimate the maximum allowable number of repetitions (N_f) using the critical tensile strain (ϵ_t) and the dynamic modulus (E^*) of AC surface layer (Asphalt Institute 1982; Lin 2003; Huang 2004; FHwA 1998):

$$N_f = k_1(\varepsilon_t)^{-k_2}|E^*|^{-k_3} \tag{1}$$

in which k_1, k_2, and k_3=regression coefficients. The pavement is considered to be failed when there exists 20% of fatigue cracking in the entire lane area (or equivalent to 45% in the wheel path area) (Simpson et al. 1993). Cumulative fatigue damage (D_f) is then calculated by adding the damage caused by each individual load application based on Miner's hypothesis

$$D_f = \sum_{i=1}^{k} \frac{n_i}{N_{fi}} \tag{2}$$

where k=number of axle load type; n_i=number of axle applications; and N_{fi}=corresponding maximum allowable number of repetitions.

In the current *Mechanistic-Empirical Pavement Design Guide* (MEPDG), the revised MS-1 fatigue cracking model, which was originally developed by the Asphalt Institute for bottom up alligator cracking, is shown as follows (ARA 2004, Appendix II-1):

$$N_f = 0.00432 \times \beta_{f1} \times C \times \left(\frac{1}{\varepsilon_t}\right)^{3.291\beta_{f2}} \left(\frac{1}{E}\right)^{0.854\beta_{f3}}$$

$$C = 10^{4.84*[V_b/(V_a+V_b)-0.69]} \tag{3}$$

where, β_{f1},β_{f2}, β_{f3}=calibration factors; $\beta_{f1}=\beta'_{f1}\times k'_1$; β'_{f1}=numerical value; k'_1=function of the AC layer thickness; C=laboratory to field adjustment factor; ϵ_t=critical tensile strain; E=stiffness of the AC surface layer; V_a=air voids (%); and V_b=effective binder content (%). The alligator cracking model calibration process included the following steps: estimation of coefficients β_{f2} and β_{f3} for the MS-1 number of load repetitions; finding the fatigue cracking damage transfer function by correlating fatigue cracking with the damage using only sections with AC layer thickness greater than 4 in.; and then shifting the thin sections using the k'_1 parameter. The results showed that choosing β_{f2} equal to 1.2 and β_{f3} equal to 1.5 provided a more realistic prediction. The final transfer function to calculate fatigue cracking from cumulative fatigue damage (D_f) is based on the assumption that the alligator cracking of the total lane area would be 50% at a fatigue damage of 100%. The calibrated model for the bottom up fatigue cracking [(F.C.)(%)] is as follows:

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Table 1. Models for Predicting Allowable Load Repetitions

Organization	Author (year)	k_1	k_2	k_3
Asphalt Institute	Asphalt Institute (1981)	0.0796	3.291	0.854
Shell Oil	Shook et al. (1982)	0.0685	5.671	2.363
U.S. Army	Department of Defense (1988)	478.63	5	2.66
Mn/Road	Timm and Newcomb (2003)	2.83	3.21	0

$$\text{F.C.} = \left(\frac{6{,}000}{1+e^{(C_1\times C_1'+C_2\times C_2'\times\log(D_f\times 100))}}\right)\times\left(\frac{1}{60}\right) \tag{4}$$

in which $C_1=1.0$; $C_2=1.0$; $C_1'=-2\times C_2'$; $C_2'=-2.40874-39.748(1+h_{ac})^{-2.856}$; and h_{ac}=total thickness of the asphalt concrete layer (in.). The fatigue damage is calculated in a similar way using Eq. (2) based on Miner's hypothesis and more complex axle load spectra (ALS) concept.

Investigation of Existing Models

To investigate the goodness of prediction, cumulative fatigue damage (D_f) was calculated and plotted against the actual fatigue cracking based on Eqs. (1) and (2) and the coefficients given in Table 1 for AI, Shell Oil, U.S. Army, and Mn/Road models using the long-term pavement performance (LTPP) data to be discussed later in the "Data Preparation" section. Except for the Mn/Road model, the results of this analysis are quite similar as depicted in Figs. 1(a–d), respectively.

Together with the aforementioned AI model, the following relationship developed by Ali and Tayabji (1998) and FHwA (1998) was adopted to illustrate the goodness of fatigue cracking predictions using LTPP GPS-1 data as shown in Fig. 2(a), where

$$\%\text{Fatigue Cracking} = \frac{0.021}{0.027+e^{(-0.85D_f)}} \tag{5}$$

The prediction accuracy of the proposed models implemented in the current MEPDG was further investigated. To avoid undesirable misunderstanding of the new guide's prediction algorithm due to the complexity involved, it was decided to directly use the beta version of the MEPDG software for the prediction of alligator cracking. The goodness of fatigue cracking prediction using the recommended MEPDG models is shown in Fig. 2(b). Even though the use of cumulative fatigue damage based on Miner's hypothesis and more complicated ALS concept as recommended by the MEPDG seems to be a logical approach, the integration of monthly or seasonal environmental factors such as humidity and temperature changes often resulted in more variations in the predictions of fatigue cracking due to many uncertainties involved in the field. Unfortunately, the prediction accuracy of the existing fatigue cracking models was found to be inadequate and greatly in need of improvement.

Objectives of This Study

As pavement design evolves from traditional empirically based methods toward mechanistic-empirical, the equivalent single axle

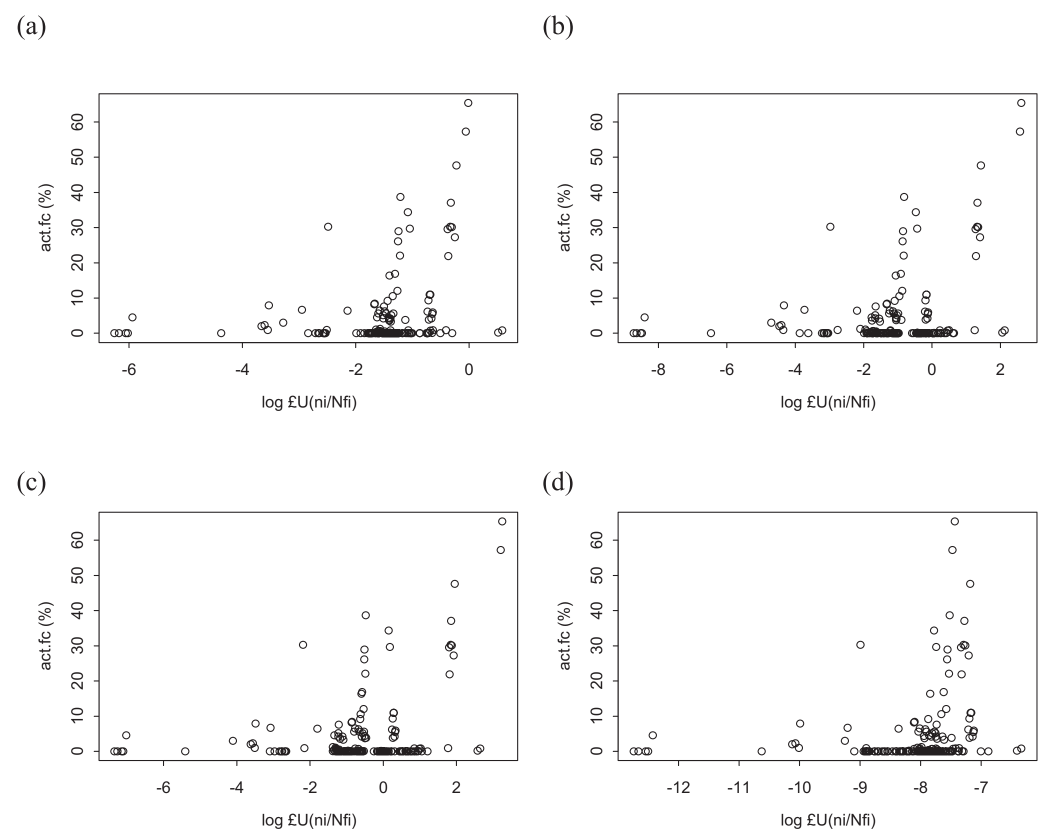

Fig. 1. Comparison of prediction results using: (a) AI model; (b) Shell Oil model; (c) U.S. Army model; and (d) Mn/Road model

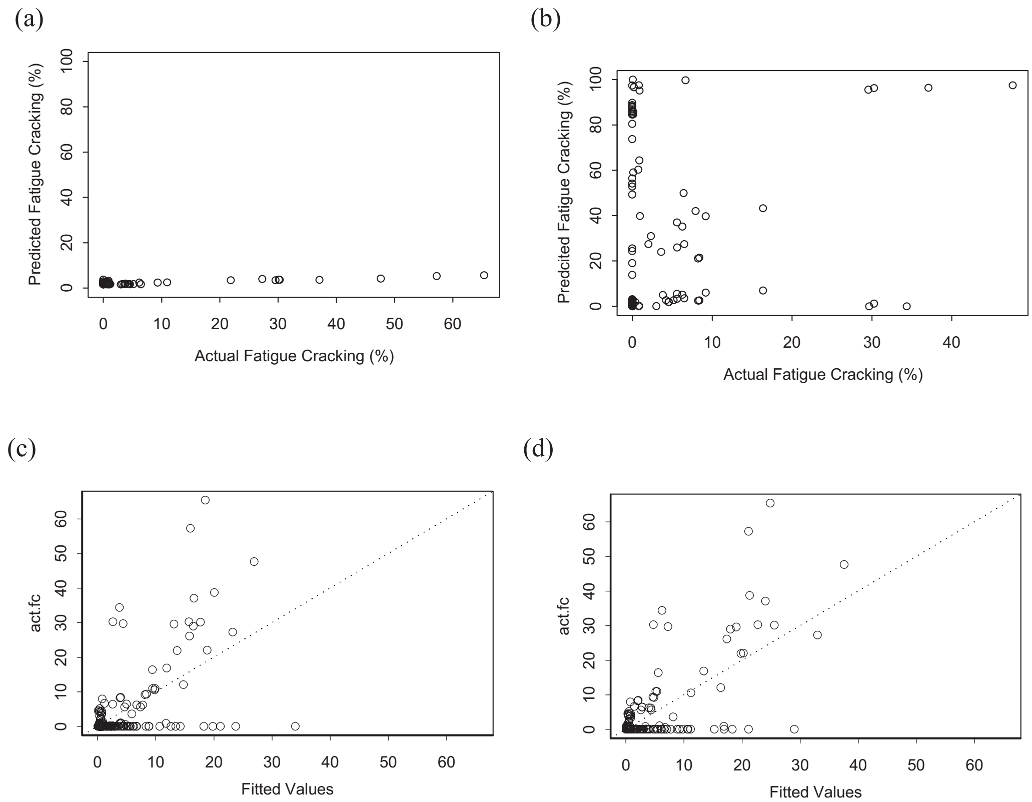

Fig. 2. Goodness of fatigue cracking prediction using: (a) AI model and Eq. (5); (b) MEPDG models; (c) preliminary Poisson loglinear model; and (d) proposed model

load (ESAL) concept used for traffic loads estimation is no longer recommended in the current MEPDG (ARA 2004). The success of the new design guide considerably depends upon the accuracy of pavement performance predictions. Thus, this study will first investigate its goodness of fit and strive to develop improved fatigue cracking prediction models for flexible pavements using the LTPP database (Wu 2006).

Data Preparation

Initially, the DataPave 3.0 program was used to prepare a database for this study. However, in order to obtain additional variables and the latest updates of the data, the LTPP database (Release 18.0) became the main source for this study. Starting from 1987, the LTPP program has been monitoring more than 2,400 asphalt and Portland cement concrete pavement test sections across North America. Very detailed information about original construction, pavement inventory data, materials and testing, historical traffic counts, performance data, maintenance and rehabilitation records, and climatic information have been collected. There are eight general pavement studies (GPS) and nine specific pavement studies (SPS) in the LTPP program, of which only AC pavements on granular base (GPS1) and on bound base (GPS2) were used for this study.

The standard release database is currently implemented in a relational database management system or an information management system (IMS). Automatic summary reports of the pavement information may be generated from different IMS modules, tables, and data elements. The thickness of pavement layers was obtained from the IMS Testing module rather than the IMS Inventory module to be consistent with the results of the Section Presentation module in the DataPave 3.0 program. Several other material properties such as air voids, effective binder content, etc. were queried from the Inventory module. Detailed traffic counts and ESALs were obtained from the Traffic module. The cumulated ESAL during the performance analysis period was calculated by multiplying pavement age with mean yearly ESAL (or kesal) which could be easily estimated from the database. Environmental data were retrieved from the IMS Climate module and the associated Virtual Weather Station (VWS) link. The alligator cracking data (including low, medium, and high severities) used in this study were obtained from the MON_DIS_AC_REV table in the IMS Monitoring module. Maintenance and rehabilitation activities could effectively reduce the distress quantities. Thus, the records in both Maintenance and Rehabilitation modules were used to assure that this study only chose the performance data of those sections without or before major improvements.

For the purpose of this study, a Microsoft Excel summary table containing the pavement inventory, material and testing, traffic, climatic, and distress data was created using the relational database features of the Access program. The Excel table was then stored as S-Plus datasets for subsequent analysis. The summary, table, cor, plot, pairs, and coplot functions were heavily utilized to summarize the information of interest and to provide more reliable data for this study. To estimate the critical tensile strain (ϵ_t) of the AC surface layer, a systematic approach was utilized and implemented in a Visual Basic software package to automatically read in the pavement inventory data from the summary table, generate the BISAR input files, conduct the batch runs, as well as summarize the results (Wu 2006) in which the static (or laboratory tested) elastic modulus data recorded in the IMS Testing module and a single wheel load of 40 kN(9,000 lb) with a tire pressure of 0.482 MPa(70 psi) were used for the analysis.

Furthermore, the aforementioned mechanistic-empirical models also require the dynamic Young's modulus of the AC surface layer. The LTPP program utilized the MODCOMP4 program (FHwA 2002) to backcalculate the dynamic modulus of each pavement layer which could be retrieved from the IMS Monitoring module. Thus, it would be interesting to compare the laboratory tested layer moduli versus the back-calculated dynamic Young's moduli to have a better understanding of their associated variability. The variability of the relationship between the dynamic and the static (or laboratory tested) moduli could not be ignored, the average ratios of which are approximately 2.6, 2.7, 7.3, and 3.4 by eliminating some apparent outliers for AC surface, base, subbase, and subgrade layers, respectively (Wu 2006).

A data cleaning process must be conducted before any preliminary analysis or regression analysis can be performed. With the help of graphical representation, fatigue cracking data were plotted against surveyed years for each section in the database with additional information displayed to examine the distress trends in order to identify possible data errors. Each section was carefully examined. Data correction and preparation were made in a systematic way that could be easily traced back. By doing so, different subsets of the final database providing more reliable data might be analyzed for different purposes. Of the 185 observations (40 sections), nine data points were identified as possibly having some maintenance or rehabilitation activities, although they were not recorded in the database. Thus, the remaining 176 data points were used in the subsequent analysis.

Statistical Analysis and Improved Models

The occurrence of fatigue cracking in the field depends on various factors namely traffic, environment, structure, construction, maintenance, and rehabilitation. Wang et al. (2005) developed a relationship to predict the median failure time due to fatigue cracking using the following explanatory parameters: traffic (different ESAL levels), thickness of AC layer, thickness of base layer, mean annual precipitation, and freeze-thaw cycles per year. Thus, it is prudent to develop improved fatigue cracking prediction models using not only the critical strain and the stiffness of the AC layer but the aforementioned parameters as well.

To develop a more reliable predictive model for practical engineering problems, Lee and Darter (1995) proposed a predictive modeling approach to incorporate robust (least median squared) regression, alternating conditional expectations, and additivity and variance stabilization algorithms into the modeling process. The robust regression is proposed due to its favorable feature of analyzing highly contaminated data by detecting outliers from both dependent and independent variables. Through the iterative use of the combination of these outlier detection and nonparametric transformation techniques, it is believed that some potential outliers and proper functional forms may be identified. Subsequently, traditional regression techniques can be more easily utilized to develop the final predictive model. Nevertheless, many preliminary trials using these regression techniques have shown extreme difficulty in achieving a satisfactory predictive model for this set of data (Wu 2006).

Currently, two methods are widely used based on the assumption of fatigue damage being normally distributed or lognormally distributed. Nevertheless, Sun et al. (2003) found that pavement fatigue cracking damage is neither normally nor lognormally distributed; much effort should be taken to characterize and predict fatigue cracking in terms of damage distribution. Exploratory data analysis of the response variable has also indicated that the normality assumption with random errors and constant variance using conventional regression techniques might not be appropriate for prediction modeling. The Shapiro–Wilk W-statistic for testing for departures from normality was also used to test the distribution of fatigue cracking (Venables and Ripley 2002; Insightful Corp. 2003). Apparently, the logarithm of fatigue cracking has better data scatter, although the W-statistic still indicated that the distribution of fatigue cracking is not lognormally distributed. Furthermore, since various distribution functions have been assumed for fatigue cracking analysis in the literature (ARA 2004, Appendix II-1; Wang et al. 2005), the Kolmogorov–Smirnov goodness-of-fit test was also used to test whether the fatigue cracking could be characterized by normal, exponential, gamma, lognormal, or Poisson distribution (Venables and Ripley 2002). Unfortunately, no apparent distribution function could be selected for this dataset.

Preliminary Models Using Poisson Regression Techniques

"When events of a certain type occur over time, space, or some other index of size, it is often relevant to model the rate at which events occur (Agresti 1996)." Due to the data collecting nature of fatigue cracking, fatigue cracking data could be treated as rate data, i.e., percent of the entire lane area. Agresti (1996) also suggested that using Poisson regression for rate data is an appropriate decision. Therefore, a generalized linear model (GLM) (Nelder and Wedderburn 1972; Insightful Corp. 2003) along with the assumption of Poisson distribution was adopted in this analysis in which a Poisson loglinear model is a GLM that assumes a Poisson distribution for the response variable and uses the log link. After going through several trials in eliminating insignificant and/or inappropriate parameters, the following model was obtained:

$$\begin{aligned}\ln(\mathrm{FC}) = &-7.455 + 0.121 \times \mathrm{age} + 0.00168 \times \mathrm{kesal} + 0.00269 \\ &\times \mathrm{precip} + 0.0473 \times \mathrm{temp} + 12{,}319.5 \times \mathrm{epsilon}\,.\,t \\ &+ 0.0133 \times \mathrm{ft}\end{aligned} \tag{6}$$

in which dispersion parameter for Poisson family taken to be 1; null deviance=2536.613 on 175 degrees of freedom; residual deviance=1403.364 on 169 degrees of freedom; age stands for pavement age (years); kesal is the yearly ESALs (thousands); precip=mean annual precipitation (mm); temp=mean annual temperature (°C); epsilon.t (ϵ_t)=critical tensile strain; ft=yearly freeze-thaw cycle; and FC=fatigue cracking in percent of entire lane area (%). The results of significant testing including the standard error estimate of each parameter and its corresponding t-value can be found in the literature (Wu 2006).

Fig. 2(c) depicts the goodness of fit of the above model. Since the main objective is to predict the rate of fatigue cracking, it is desirable to rearrange the above equation into the following expression and obtain new regression summary statistics. Note that R^2=coefficient of determination; SEE=standard error of estimate; and n=number of observations:

$$\begin{aligned}\mathrm{FC} = \exp(&-7.455 + 0.121 \times \mathrm{age} + 0.00168 \times \mathrm{kesal} + 0.00269 \\ &\times \mathrm{precip} + 0.0473 \times \mathrm{temp} + 12{,}319.5 \\ &\times \mathrm{epsilon}\,.\,t + 0.0133 \times \mathrm{ft})\end{aligned} \tag{7}$$

with the statistics: $R^2=0.3352$, SEE=8.741, $n=176$.

To improve the model fits, it is possible to develop separate models for different climatic zones to account for other factors not considered in the above model implicitly. Due to the unbalanced data structure of which 38, 85, 48, and five data points were obtained from Wet-Freeze, Wet-Nonfreeze, Dry-Freeze, and Dry-Nonfreeze zones, respectively, the following models were developed by regrouping the data into either Wet or Dry, or Freeze or Nonfreeze zones:

$$(FC)_{wet} = \exp(-6.539 + 0.078 \times age + 0.00187 \times kesal + 0.000673 \times precip + 0.0914 \times temp + 15{,}097 \times epsilon\,.\,t + 0.0272 \times ft) \tag{8}$$

with statistics: R^2=0.452, SEE=3.137, n=123

$$(FC)_{dry} = \exp(-48.411 + 0.119 \times age + 0.025 \times precip + 1.774 \times temp + 2{,}729 \times epsilon\,.\,t + 0.0272 \times ft) \tag{9}$$

with statistics: R^2=0.421, SEE=1.117, n=53

$$(FC)_{freeze} = \exp(-5.944 + 0.00583 \times precip + 41.768 \times epsilon\,.\,t - 0.002 \times visco + 0.4 \times trange) \tag{10}$$

with statistics: R^2=0.498, SEE=1.624, n=86

$$(FC)_{nonfreeze} = \exp(-7.87 + 0.102 \times age + 0.00219 \times kesal + 0.00102 \times precip + 0.0472 \times temp + 15{,}172 \times epsilon\,.\,t + 0.0476 \times ft) \tag{11}$$

with statistics: R^2=0.577, SEE=2.99, n=90.

Also note that new variables such as the viscosity of the AC layer (visco) and temperature range (trange, °C) were included to improve the model fits after eliminating some insignificant and inappropriate parameters in which trange is defined as the difference of maximum and minimum mean annual temperature.

Proposed Model Using Additional Modern Regression Techniques

Since the primary assumption of the above preliminary GLM models is that a linear function of the parameters was used in the model. The generalized additive model (GAM) extends GLM by fitting nonparametric functions using data smoothing techniques to estimate the relationship between the response and the predictors (Venables and Ripley 2002). To further enhance the model fits, GAM techniques were adopted in this analysis. The Box–Cox power transformation technique was routinely utilized to estimate a proper, monotonic transformation for each variable based on the resulting preliminary GAM model. The fatigue cracking data were refitted with these transformed predictors using GLM techniques. To alleviate the assumption of Poisson distribution, the quasi-likelihood estimation method was also used to estimate regression relationships without fully knowing the error distribution of the response variable. Visual graphical techniques as well as the systematic statistical and engineering approach proposed in the literature (Lee 1993; Lee and Darter 1995) were frequently adopted during the modeling process.

After a considerable amount of trials, the quasi-family with the same link and variance functions from the Poisson family appeared to be the best choice among several different distribution functions conducted in this analysis, i.e., normal/Gaussian, gamma, Poisson, and quasi- (Venables and Ripley 2002; Insightful Corp. 2003). Note that the Poisson family is useful for modeling count or rate data that typically follow a Poisson distribution. Consequently, the proposed model for predicting the fatigue cracking of AC pavements (in percent of entire lane area) is given as follows:

$$FC = \exp(-18.08 + 0.943 \times \sqrt{age} + 0.832 \times \log(kesal) + 0.121 \times \sqrt{precip} + 0.869 \times \sqrt{temp} + 31.489 \times (epsilon\,.\,t \times 1{,}000)^2 + 3.242 \times \log(ft)) \tag{12}$$

with statistics: R^2=0.4967, SEE=7.605, n=176, in which the dispersion parameter for the quasi-likelihood family was taken to be 7.701441, suggesting overdispersion; null deviance=2,536.613 on 175 degrees of freedom; and residual deviance=1,160.759 on 169 degrees of freedom. The results of significant testing including the standard error estimate of each parameter and its corresponding t-value can be found in the literature (Wu 2006). Fig. 2(d) displays the goodness of the fit of the proposed model, which is considered to have significant improvements over the existing models in terms of conciseness and the coefficient of determination (R^2). Sensitivity analysis of the explanatory variables indicated that their general trends seem to be fairly reasonable (Wu 2006).

Conclusions

Even though the use of cumulative fatigue damage based on Miner's hypothesis and to more complicated ALS concept as recommended by the MEPDG seems to be a logical approach, the integration of monthly or seasonal environmental factors such as humidity and temperature changes often resulted in more variations in the predictions of fatigue cracking due to many uncertainties involved in the field. Unfortunately, the prediction accuracy of the existing fatigue cracking models was found to be inadequate and greatly in need of improvement. A relatively skewed distribution for actual fatigue cracking was identified, which also indicated that normality assumption using conventional regression techniques might not be appropriate for this study. Thus, GLM and GAM along with the assumption of the Poisson distribution and quasi-likelihood estimation method were adopted for the modeling process.

After many trials in eliminating insignificant and inappropriate parameters, the resulting proposed model included several variables such as yearly KESALs, pavement age, annual precipitation, annual temperature, critical tensile strain under the AC surface layer, and freeze-thaw cycle for the prediction of fatigue cracking. A plot of the goodness of the model fit has indicated that the proposed model has substantial improvements over the existing models. The proposed models appeared to reasonably agree with the pavement performance data although their further enhancements are possible and recommended.

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