Available online at www.sciencedirect.com
ScienceDirect
International Journal of Project Management 29 (2011) 706 – 717
International Journal of Project Management
www.elsevier.com/locate/ijproman

Cost simulation in an item-based project involving construction engineering and management

Jui-Sheng Chou *

Department of Construction Engineering, National Taiwan University of Science and Technology, 43 Sec.4, Keelung Rd., Taipei, 106, Taiwan

Received 30 March 2010; received in revised form 1 July 2010; accepted 27 July 2010

Abstract

Despite the extensive use of simulation in management, the continuous simulation model for cost estimation remains unexploited, especially for construction engineering and management. This study introduces streamlining Monte Carlo simulation procedures with evaluation of stochastic processes and input probability distribution selection via hypothesis testing, and specification of correlations between simulated variates. By using self-developed algorithms and a spreadsheet-add-on program, this investigation uses historical construction projects as case study data to create an early-stage cost distribution for budget allocation. While establishing the applicability of the proposed simulation procedures, this study demonstrates that the simulated cost results present superior simulation accuracy in addition to separating the principal work items and unit price component model. Generally, the precision and absolute error rates fall into acceptable ranges when the proposed systematic simulation procedures are adopted. The cost simulation approach offers a simplified decision tool for fairly assessing construction cost and uncertainties based on the experienced judgment of project managers.

Keywords: Simulation; Cost estimation; Decision making; Project management; Engineering and construction

1. Introduction

Project estimation is largely performed using two approaches: probabilistic and deterministic. The deterministic approach is commonly used for detailed estimation when specific information permits reasonable accuracy. Estimation procedures generally perform single-value estimation based on historical data or employee professional experience. Associated techniques utilizing this approach include definitive formulation, linear programming, and optimization. However, in the current environment of rapid change, costs are typically subject to fluctuations owing to construction project uncertainty. As a result, deterministic value methods do not sufficiently consider potential risk.

Probabilistic estimation can compensate for deficiencies in traditional early estimates and clarify the likelihood and degree of cost overruns, thus helping determine the size of the project and management reserve or contingency funds with assessed uncertainties. Alternatively, probabilistic estimation can be implemented early in project development when little information is available. Monte Carlo simulation (MCS) is one of the widespread probabilistic techniques for conceptual cost estimation and decision making. The primary simulation processes include data collection, random-number generation, model formulation, analysis, and visual presentation.

Abbreviations: BCIS, Building Cost Information Service; CDF, cumulative distribution function; K–S, Kolmogorov–Smirnov; LCGs, linear congruential generators; MAPE, mean absolute percentage error; MCS, Monte Carlo simulation; MLNRS, multivariate lognormal random simulation; MNRS, multivariate normal random simulation; MPE, mean percentage error; NORTA, NORmal To Anything; NTD, New Taiwan Dollar; PC, Pearson's Chi-square; PDF, probability density function; PEM, probabilistic estimation method; PWI, principal work items; RICS, Royal Institute of Chartered Surveyors; SBS, stochastic budget simulation; SD, standard deviation; SDPE, standard deviation percentage error; TPC, total project cost derived by summing work items costs; TPCS, total project cost derived from sum of item quantity multiplied by unit price

* Tel.: +886 2 27376321; fax: +886 2 27376606.
E-mail address: jschou@mail.ntust.edu.tw.

0263-7863/$ - see front matter © 2010 Elsevier Ltd. and IPMA. All rights reserved.
doi:10.1016/j.ijproman.2010.07.010

Construction engineering projects involve the three stages of planning, design, and execution. Each stage is equally important, and carelessness during any stage can cause budget overruns, improper design and construction, and work delays. If the routine planned procedures can be simulated using reliable data, construction costs can be reasonably estimated, thus providing feedback that can help control the annual budget, increasing resource allocation efficiency. When the proposed budget is smaller than the required costs, planned construction may be delayed or may fail to meet expectations. If the proposed budget significantly exceeds actual cost requirements, budget may sit idle, denying funds to other more worthy construction projects.

However, in practice detailed information generally cannot be accessed in time for use in budget proposals during initial stages. In situations involving inadequate information during the initial stages of the project, construction authorities typically use subjective experience judgments or draw analogies with similar projects to perform deterministic cost single-value estimates. As judgment capabilities differ among professional employees, differences with no scientific basis occur in estimation results.

Although risk simulation analysis was promoted intermittently in project management based on the literature findings, adequate risk analysis and management techniques are rarely used due to a lack of knowledge and doubts regarding their suitability for construction industry activities (Akintoye and MacLeod, 1997). Simulation is one of the most widely used techniques in management sciences, while the continuous simulation model for cost estimation has not yet been systematically exploited, particularly for construction maintenance projects.

To promote the applicability of MCS to project management professionals involved in construction engineering, this article delineates streamlining MCS procedures including stochastic processes and input probability distribution selection through hypothesis testing, specifying correlations between simulated variates, and simulation precision. Furthermore, a case study using real project data is presented via various stochastic simulation procedures of the proposed mathematical cost models.

The remainder of this paper is organized as follows. Section 2 reviews the literature on critical issues involving probabilistic estimation. Based on the findings and suggestions from the literature, Section 3 outlines the adequate simulation methodology and stochastic procedures adapted for construction cost modeling. Section 4 then illustrates the case investigation and explains analytical simulation results by applying various stochastic procedures to the estimating models. Section 5 contains concluding remarks, cautions regarding modeling, and possibilities for improvement.

2. Reviews of probabilistic estimation and simulation issues

This section reviews general estimation techniques and probabilistic estimation. Additionally, this section discusses the potential influence of and concerns surrounding dependency between simulated variates to achieve a simulation process that closely approximates reality.

2.1. Cost estimation approaches

Cost estimation techniques can be classified into qualitative and quantitative techniques (Niazi et al., 2006). Qualitative cost estimation techniques utilize past historical cost data and expert experience to subjectively estimate project costs (PMI, 2008). Since relevant past historical information shares characteristics with the project to be estimated in terms of design, process, data, and knowledge, it can help in forecasting project costs. However, this approach suffers two difficulties: first, the degree of similarity is difficult to measure and heavily influenced by subjectivity; second, the impact on project costs of changes in external environmental factors and updates to technological processes cannot be adequately considered (Chou, 2009). Despite these shortcomings, qualitative assessments offer a useful reference for experienced users.

In contrast, quantitative techniques not only rely on previous data and expert knowledge, but can also analyze project designs, processes, and distinctive attributes. Analytic methods are used to explore cost functions and total the costs of resources used in project activities (items) to determine the approximate project costs (Artto et al., 2001; Berny and Townsend, 1993; Chou et al., 2006; Cooper et al., 1985; Franke, 1987; Laufer, 1991; Niazi et al., 2006; Touran, 2003; Wang and Huang, 2000; Wang, 2004). Although these techniques obtain estimates that closely approximate actual costs, they require time to gather sufficient data or obtain relevant information during initial project stages. The probabilistic estimation method (PEM) belongs to the quantitative category (Cooper et al., 2004), and furthermore achieves range estimates with additional probability information on event likelihood under repetitive simulation realizations.

2.2. Probabilistic estimation method

Risk analysis has attracted increasing attention in modern project assessments, because single-value estimation approaches do not adequately help project personnel understand potential risk (Hull, 1990; Rezaie et al., 2007). Typically, only limited information can be obtained during initial project stages; projects are subject to the greatest uncertainty during these initial stages, and decision behaviors of project personnel are also easily misdirected. If costs can be reasonably added up with certain contingency, project risk can be covered throughout its life cycle (Chou et al., 2009; Elkjaer, 2000; Yang, 2005). Consequently, seeking other estimation methods to compensate for inadequacies in single-value estimations is important.

Recently, increasing numbers of studies have utilized PEM to establish possible ranges of project costs, helping relevant personnel to apply PEM to assess project risks (Chou et al., 2005, 2009; Touran and Wiser, 1992; Yang, 2008). According to Diekmann (1983), the simulation method is more flexible than the other methods (Diekmann, 1983). Moreover, the time required for simulations has reduced with improvements in computer calculation capabilities, increasing the convenience of the simulation approach and allowing it to be applied more frequently in risk analysis and decision making (Chou et al., 2009; Elkjaer, 2000; Herbold, 2000; Khedr, 2006; Papagiannakis and Delwar, 2001; Rezaie et al., 2007; Salem et al., 2003; Tighe, 2001; Yang, 2005).

MCS has been applied to examine numerous project risk analyses regarding cost estimates. For example, Wall (1997) simulated office-building costs using data from the Building Cost Information Service (BCIS) of the Royal Institute of Chartered Surveyors (RICS) (Wall, 1997). Furthermore, Touran and Wiser (1992) illustrated methodologies for estimating office-building projects obtained from R.S. Means, Inc., Kingston, Massachusetts (Touran and Wiser, 1992). Additionally, Wang (2002a) proposed a model for determining a reasonable project ceiling price using a cost simulation approach (Wang, 2002a). These studies have focused on the complex applicability of MCS to building construction projects. Elkjaer (2000) proposed a simplified stochastic budget simulation (SBS) method, which unraveled the process of simulating estimating cost using a spreadsheet application (Elkjaer, 2000). Although SBS is relatively easy to use and quickly presents a virtual overview of the total cost, it neglects the correlations between simulated variates and input probability density function examination.

Rather than estimating actual construction costs, Herbold (2000), Tighe (2001) and Salem et al. (2003) presented an economical technique for determining virtual life-cycle costs of civil infrastructure construction and rehabilitation alternatives (Herbold, 2000; Salem et al., 2003; Tighe, 2001). Their work focused mainly on decision making processes involving heavy information input regarding the selection of pavement construction and rehabilitation alternatives in life-cycle scenarios. The deterministic life-cycle cost models are built first, after which the user must determine the values of the input variables (Salem et al., 2003). Computer simulation is used to generate the cumulative distribution functions of the model outcome, and offers valuable information regarding the probability of executing a construction/rehabilitation alternative at or below a certain budget. However, their life-cycle models do not consider stochastic processes and complex dependency. Besides, this approach focuses on the present value in selecting the best alternative long-term scenario, and is inappropriate for determining the actual construction cost with uncertainties.

2.3. Interactive effects between construction activities

In considering interactions between different variables, the sample values of each variable are independently and randomly extracted from marginal probability density functions. In practice, correlations may exist between variables, and may be positive or negative. Overlooking the mutual interactions between projects can lead to overestimation or underestimation of project risks (Chau, 1995a; Touran, 1993; Touran and Wiser, 1992; Wall, 1997).

Correlations between construction activities are generally one of four types (Chou et al., 2009; Cooper et al., 1985, 2004): (a) Two activities may be correlated because a common cause or driver affects each activity; for example, the need to expand a right-of-way may influence an excavated area and increase both cost items, and *vice versa*. Such correlations are very common in construction projects; (b) Two activities may be correlated when a problem in one activity leads to a problem in another; for example, an equipment breakdown in an earlier activity may cause delays, and replacing or repairing equipment may be less efficient than using the original, causing ripple effects on other activities; (c) If two risks emerge, their combined effects may exceed their individual effect, and may exponentially influence overall risk. In this case, the correlation may no longer be linear; (d) The last type refers to statistical dependencies, which are generally poorly defined and understood.

Correlation coefficients quantify the mutual impacts of uncertain factors between projects. When considering the correlation of multivariate random variables, the ranges of random extraction processes can be limited to prevent the extraction of inappropriate values (Wall, 1997). The Pearson correlation coefficient and Spearman rank correlation are frequently used to measure inter-variable correlations (Ghosh and Henderson, 2003; Touran, 1993; Wall, 1997).

2.4. Cholesky decomposition

While considering the association between projects helps improve simulation results compared with the consideration of the selection of variate probability distributions (Wall, 1997), correlation analysis is difficult to implement when data is insufficient (Moselhi and Dimitrov, 1993). Consequently, Touran (1993) observed that expert experience can be used to subjectively set inter-project correlation strength. Provided care is taken in subjectively setting correlations, simulation results should be reasonably accurate.

Stochastic processes that consider project correlation typically use Cholesky decomposition matrices. This decomposition method has a prerequisite in that any given matrix must be positive definite or positive semi-definite. Suppose that $\mathbf{A}$ is any $n \times n$ positive definite matrix (covariance or correlation matrix) and satisfies $\mathbf{X}^{\mathbf{T}}\mathbf{A}\mathbf{X} > \mathbf{0}$ for all nonzero eigenvectors $\mathbf{X}$, where $\mathbf{X}^{\mathbf{T}}\mathbf{A}\mathbf{X}$ denotes a 1×1 matrix, namely, a real number. Cholesky decomposition can be used when the matrix is positive definite to decompose the matrix into a lower triangular matrix $\mathbf{C}$ and an upper triangular matrix $\mathbf{C}^{\mathbf{T}}$ ($\mathbf{A} = \mathbf{C}\mathbf{C}^{\mathbf{T}}$). The coefficients ($a_{ij}$) in matrix $\mathbf{A}$ can then be obtained by solving n equations via back substitution, as follows.

$$
\begin{aligned}
a_{ii} &= \sqrt{\mathrm{COV}(X_i, X_i) - \sum_{j=1}^{i-1} a_{ij}^2} \quad \text{for } i = j = 2, 3, \ldots, n \\
a_{ji} &= \frac{\mathrm{COV}(X_i, X_j) - \sum_{x=1}^{j-1} a_{ix} a_{jx}}{a_{jj}} \quad \text{for } i \neq j, i = 2, 3, 4, \ldots, n
\end{aligned} \tag{1}
$$

where, COV (.) represents the covariance calculation.

2.5. Implementing issues and feasibility of stochastic process

In simulating correlated random variables, imaginary numbers are produced during decomposition if Cholesky decomposition is used to decompose a non-positive definite matrix, preventing calculations from proceeding further. This problem worsens when the rank of covariance or correlation matrix exceeds five. Additionally, several studies have suggested that human input errors or subjective estimation methods also lead to the occurrence of non-positive definite matrices (Touran, 1993; Touran and Wiser, 1992; Wang, 2002b; Yang, 2005). To alleviate this concern, Touran (1993) suggested that users can reduce the correlation coefficients of non-diagonal values by 1% in each realization (Touran, 1993). However, this method may alter the original correlation coefficient matrix significantly, rendering it inconsistent with actual project conditions.

Ghosh and Henderson (2003) suggested that users convert negative eigenvalues in a correlation matrix to positive values or 0^+ and then constitute a similar correlation matrix with the corrected eigenvalues (Ghosh and Henderson, 2003). Yang (2005) and Chou et al. (2009) followed this approach to resolve problems involving non-positive definite matrices, and case testing in both studies demonstrated no excessive variation between the similar and original correlation matrixes (Chou et al., 2009; Yang, 2005). Yang briefly presents steps involved in matrix adjustment and discrepancy examination (Yang, 2005).

Most of previous works focused on examining one of the following assumptions: the influence of correlated cost elements (*e.g.*, independency versus dependency with concern of non-positive definite correlation matrix) and probability distribution determination (*e.g.*, normal, lognormal, beta, Weibull, or triangular distribution); examining the sensitivity of hypothesis testing of the simulated results (*e.g.*, Pearson's Chi-square and Kolmogorov–Smirnov tests); or generating random variates (*e.g.*, multivariate normal random simulation, multivariate lognormal random simulation, or normal to anything stochastic process). Meanwhile, none of the previous studies has explored the in-serial compound effects of those simulation procedures, particularly in roadway repair and rehabilitation projects, in promoting risk management and informed decision making among construction professionals. Notably, the minimum threshold of realizations for reaching expected simulation error tolerance and confidence level is often not reported in the literature or set with no basis. Thus the following sections aim to fill these gaps by first presenting the building components which should be considered in the stochastic simulation together with a case study.

3. Methodology and building blocks of stochastic modeling

Monte Carlo simulation is a simulation technique designed to facilitate cost management decisions. Generally, MCS is based on random sampling of variable probability distribution patterns within the historical data, using computers to produce pseudorandom numbers and convert them to real-world responses via inverse functions under existing mathematical equations and assumed probability distributions. Repetitive sampling is performed to simulate possible future events, providing a distribution of cost range versus its likelihood of occurrence for risk analysis and project management. Fig. 1 presents the above simulation process.

3.1. Simulation repetitions and randomizer

Random numbers produced during simulation are not actually randomly produced. Correlation exists between numbers, and numbers can be termed pseudorandom numbers extracted from uniform distribution $U(0, 1)$. Although many alternatives exist in generating random numbers (Law and Kelton, 2000), the most common extraction method is the use of linear congruential generators (LCGs) producing each random number based on the previous one. Consequently, provided the first random number I_0 is determined, the following number series can be known. Restated, obtained number series are identical, as in situations with identical I_0 values (the seed or starting value). A sequence of integers $I_1, I_2, \ldots$ is defined using the recursive formula (Law and Kelton, 2000).

$$I_i = (aI_{i-1} + c)(\mathrm{mod}\, m) \tag{2}$$

where, a denotes the multiplier; c represents the increment; m is the modulus; mod denotes the mode.

3.2. Common probability distributions

Another key issue in the simulation process is to determine variable probability distributions. Wall's (1997) research suggested that lognormal distribution statistically determined based on Chi-square goodness-of-fit is more representative of actual building project costs than the beta distribution (Wall, 1997). Furthermore, Touran and Wiser (1992) asserted that, where costs are always positive, representing their probability distributions using a lognormal distribution is more appropriate than using a normal distribution. Alternatively, Chau (1995b) proposed that triangular distribution offers a more appropriate method of eliciting experience from construction personnel than other distribution methods, and that expert opinion can compensate for insufficient data (Chau, 1995b). However, in the field of engineering, extremely high cost values are typically associated with unexpected events or megaprojects and have lower occurrence probabilities than do normal projects. Using triangular distribution to represent these variables might lead to overestimation of risk and should be used with caution.

This article thus examines probability functions employed or proposed by previous literature (Chau, 1995b; Chou et al., 2005; Elkjaer, 2000; Robinson, 2004; Salem et al., 2003; Wall, 1997; Wang, 2002a,b; Yang, 2005) as marginal distribution functions used exclusively in construction engineering and management, including normal, lognormal, triangular, Weibull distribution, and beta distributions. Particularly, the lognormal and Weibull distributions are frequently used to represent physical quantities that cannot have a value less than zero, while the beta distribution holds the distinction of only being able to

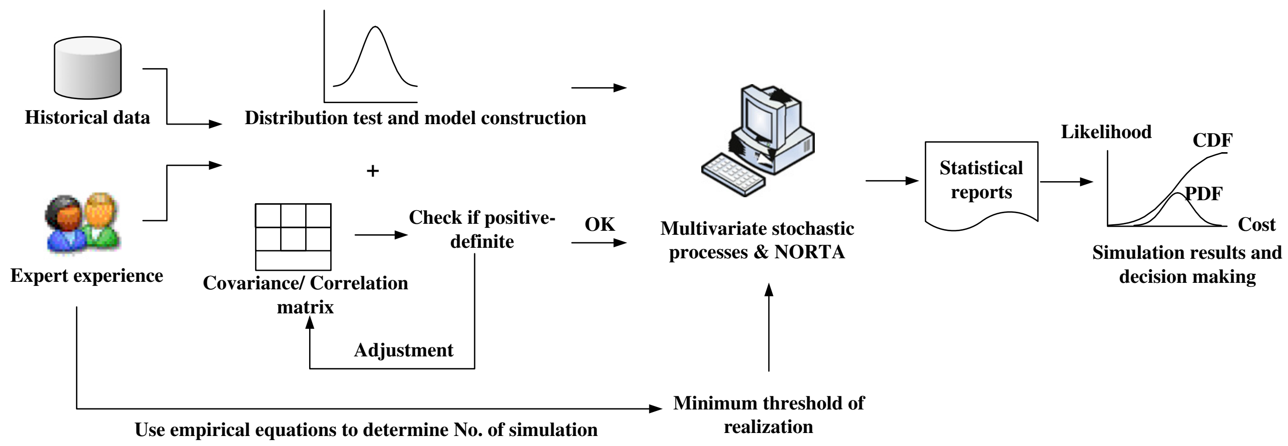

Fig. 1. The proposed Monte Carlo simulation process.

cover the range between zero and one. If the practitioner suspects that the data have a normal distribution, the triangular distribution may provide a good first approximation based on its simplicity. In practice, triangular parameters can be easier to acquire than the other distributions since they involve asking a manager to provide estimates.

3.3. Goodness-of-fit testing

Four methods exist for selecting probability distributions (Chung, 2004; Wall, 1997). The most fundamental method of fitting input data is the straightforward graphic approach, which creates histograms of observed and theoretical distribution and then visually compares them for similarity. The second method is expert variance, in which experts set the most appropriate probability distribution for random variables based on personal experience. The third method involves subjectively specific numbers of small samples and uses statistical testing to optimize the variable distribution. Finally, the fourth method involves collecting past historical project data and performing statistical testing to objectively optimize the distribution. This test is based on comparison between the observed data distribution and a corresponding theoretical distribution for the project data.

This investigation recommends using the last objective method, and employs the following goodness-of-fit testing methods to determine the appropriate probability distribution: Pearson's Chi-square (PC) test and the Kolmogorov–Smirnov (K–S) test, for both of which Chung (2004) presents steps for test execution together with examples. The test process involves determining the underlying theoretical distribution for a set of representative data. Although no set sample size exists that fits the proposed distribution, it is suggested that the minimum data set should exceed 25 to 30 data points (Chung, 2004) to prevent poor practitioner execution of a goodness-of-fit test.

3.4. Random vectors generation and stochastic processes

This section illustrates the methods employed to examine the possible influence of random variate generation on the simulation results. The most direct stochastic approach involves establishing a joint probability distribution for variates under multivariate normal distribution curves or simple custom distribution functions (Law and Kelton, 2000). Random vectors generated from multivariate normal distribution use the variance matrices between variables and consider the influences of variates on one another (Fishman, 1996). The following algorithm is used to generate the random multivariate normal vector **X**.

- Generate $Z_1, Z_2, \ldots, Z_m$ as IID (independent and identically distributed) $N(0, 1)$ random variates.
- For $i = 1, 2, \ldots, m$, let $\mathbf{X} = \boldsymbol{\mu} + \mathbf{C} \cdot \mathbf{Z}$ and return $\mathbf{X} = (X_1, X_2, \ldots, X_m)^T$, where $\boldsymbol{\mu}$ denotes a mean vector and **C** represents the Cholesky lower triangular matrix.

A similar method was proposed for obtaining random vectors from multivariate lognormal distribution (Fishman, 1996) by assuming $\mathbf{X} = (e^{Y_1}, e^{Y_2}, \ldots, e^{Y_m})^T$. The procedure thus involves generating $Y = (Y_1, Y_2, \ldots, Y_m)^T \sim N_m(\mu, \Sigma)$ and then returning $\mathbf{X} = (e^{Y_1}, e^{Y_2}, \ldots, e^{Y_m})^T$.

For complex or unknown multivariate distributions, Cario and Nelson (1997) devised random vectors with arbitrary marginal distributions and correlation matrices, named NORmal To Anything (NORTA) (Cario and Nelson, 1997). This method generally utilizes Cholesky decomposition matrices and requires confirmation of whether variance or correlation matrices are positive and definite to proceed with correlated random variate calculation. The NORTA procedure includes the following steps:

- Compute covariance matrix $\sum$ so that $\sum = \mathbf{C} \cdot \mathbf{C}^{\mathbf{T}}$.
- Generate independent standard-normal multivariate **Z**.
- Transform **Z** into a correlated standard-normal random vector $\mathbf{R} = (R_1, R_2, \ldots, R_n)$, where $\mathbf{R} = \mathbf{C} \cdot \mathbf{Z}$.
- Compute $U_i = \Phi(Z_i)$ $i = 1, 2, \ldots, n$, where Φ denotes the standard-normal cumulative distribution function (CDF).
- Return $x_i = F_i^{-1}(U_i)$ $i = 1, 2, \ldots, n$, where F_i^{-1} represents the inverse of marginal CDF.

3.5. Simulation precision

Although simulation results approach real-world results with increasing number of simulations, excessive repetitions may waste time and increase operating costs. Consequently, the following formula can be utilized, together with a 95% confidence level, to calculate the minimum necessary number of simulation repetitions. Eq. (3) calculates the minimum number of simulation realizations based on the error tolerance rate. By definition, the rate of tolerance for error mean with a 95% confidence level $= \left(\frac{\varepsilon}{\mu}\right)100\% = 196\frac{\delta}{\sqrt{N_1}}\%$; where, δ denotes the coefficient of variation (σ/μ). Thus, the required number of simulations, N_1, considering mean percentage error, is as follows.

$$N_1 = \left(\frac{196}{\%Error\ in\ \mu}\right)^2 \delta^2 \tag{3}$$

where, $\%Error\ in\ \mu$ denotes mean percentage error, and δ represents coefficient of variation $\left(\frac{\sigma}{\mu}\right)$.

Eq. (4) calculates the minimum number of simulation repetitions based on the rate of tolerance for error in variance, which can be defined as $\left(\frac{\varepsilon}{\sigma^2}\right)\%$. Through a series of substitutions, the variance percentage error is equal to $196\sqrt{\frac{2}{N_2-1}}\%$.

$$N_2 \approx 2\left(\frac{196}{\%Error\ in\ \sigma^2}\right)^2 + 1 \tag{4}$$

where, N_2 denotes the required number of simulations that consider percentage error in variance ($\%Error\ in\ \sigma^2$). The threshold of simulation realizations, $N_{threshold}$, can be determined using Eq. (5).

$$N_{threshold} = max(N_1, N_2). \tag{5}$$

4. Project description and simulation analysis

This section employs an item-based construction maintenance project as a case study to demonstrate the sound stochastic simulation processes proposed in this study for use in construction cost estimation. Moreover, this section proposes alternative mathematical models to examine the appropriateness of project cost simulation using various experimental designs. Finally, the findings are discussed based on the analytical simulation results.

4.1. Case background

As the Taiwanese economy has become fully industrialized, the scale and volume of construction maintenance projects have grown. Statistics from the Taiwanese Ministry of Transportation and Communication (MOTC, 2010) show that during the last two decades the density of public roads per hundred square kilometers in Taiwan increased from 47.7% to 57%, while mileage of new public roads increased by 19.5%. Additionally, since Taiwan is located in a high-temperature, high-precipitation sub-tropical zone, road overloading is another challenge, and road surfaces are easily damaged. The Taiwanese government reports that an average of 170 million square meters of road must be repaired and maintained annually. Effectively using limited annual budgets to maximize repair and maintenance benefits and thus maximize public road network quality to provide road users with safe and comfortable driving conditions is an important domestic development issue.

The study sample constituted randomly distributed paving construction cases. Roads are paved to provide a navigable surface for vehicles, and routine maintenance is necessary to retain surface quality, paving structure completeness, and driving safety. When paved surfaces exhibit signs of damage and related information can be obtained in time, appropriate repair measures can be formulated to ensure road user rights and benefits. Thus, efficient cost range estimation with risk contingency must be performed before fiscal budget review to ensure effective decision making and subsequent project tender process.

For illustration, this investigation randomly gathered 72 typical road surface repair projects performed by the transportation agency, representing a total of NTD 268 million of construction. Monte Carlo simulation was applied for cost range estimation and risk analysis. Spreadsheet and related software were employed as simulation tools together with custom-written algorithms as introduced above to conduct subsequent experimental simulation designs.

4.2. Mathematical estimation model

In engineering cost estimation, the most common method of deterministic calculation is the project cost bottom-up method (PMI, 2008). This study assumes costs indirectly related to construction to be fixed and in practice usually a percentage of direct costs. Therefore, the mathematical equation to be simulated can be represented using the following simplified model.

$$TPC = \sum_{j=1}^{n} ItemCost_j \tag{6}$$

In this formula: TPC denotes total project cost derived by summing work item costs; $ItemCost_j$ represents the cost of the jth work item; and n is the number of work items in the project.

Since item prices may be subject to external factors that cannot be controlled and vary with time, economic conditions, or region, the accuracy of cost estimation may increase if item quantity is estimated instead of item cost and published or average unit price is employed based on construction market information. To further examine the separate influences of quantity and unit price on TPC, estimated engineering costs are obtained from quantity multiplied by item unit price (quantity and unit price respectively; Eq. (7)) following the well-known bill-of-quantity tendering method.

$$TPCS = \sum_{j=1}^{n} ItemQty_j * UnitPrice_j \tag{7}$$

where *TPCS* denotes total project cost derived from the sum of item quantity multiplied by unit price; $ItemQty_j$ represents the quantity of the *j*th item of the project; $UnitPrice_j$ is the unit price of the *j*th item; and *n* denotes the number of work items in the project.

In reality, distinct detailed work items are difficult to grasp during initial project planning, and estimating each item and then calculating the total project cost is infeasible in situations involving insufficient information. Probabilistic estimation involves no attempt to precisely estimate the cost during the early stage of construction project. However, the estimation efficiency and effective management decisions can be improved by estimating principal work items (PWI) and then inferring total project cost by introducing the proportion *P*, defined as PWI costs over the TPC or TPCS. Consequently, Eqs. (6) and (7) can be rewritten in Eq. (8) (where unit price and quantity are not separated) and Eq. (9) (where unit price and quantity are separated). Thus, stochastic simulation can be applied to the distinct mathematical models for comparison.

$$TPC = \frac{\sum_{i=1}^{m} PrincipalWorkItemCost_i}{P} \tag{8}$$

$$TPCS = \frac{\sum_{i=1}^{m} PrincipalWorkItemQty_i * UnitPrice_i}{P} \tag{9}$$

In these formulas: $PrincipalWorkItemCost_i$ denotes the cost of the *i*th PWI; $PrincipalWorkItemQty_i$ represents the quantity of the *i*th PWI; $UnitPrice_i$ is the unit price of the *i*th PWI; and *m* denotes the number of PWIs.

4.3. Assortment of construction work items

Based on the historical construction items involved in roadway projects, this study proposes a simplified range of frequent appearing work items with large cost proportions. As a case study, Table 1 illustrates that the five PWI comprise an average of over 80% of total project costs, including the work items: (A) laying reclaimed dense-grade asphalt concrete; (B) salvaging, hauling, and stockpiling reclaimable asphalt pavement; (C) laying reflectorized pavement markings; (D) sprinkling tack coat; and (E) reclaimed asphalt concrete deductible from engineering cost. Although the PWI proportion *P* was not exactly 0.8 in each case in the study database, it fell between 66% and 89% in every case and had an average value of 82%, thus exceeding the 80/20 management rule.

Table 1
Percentage average cost and number of appearances.

Item description	Occurrence frequency	Item cost percentage
Laying reclaimed dense-grade asphalt concrete	72	70.33
Salvaging, hauling, and stockpiling reclaimable asphalt pavement	72	15.42
Laying reflectorized pavement markings	72	6.27
Sprinkling tack coat	72	4.28
Reclaimed asphalt concrete deductible from engineering cost	72	−13.99
Pavement of MC-1 prime coat	29	0.40
Asphalt concrete surface rehabilitation	25	3.29
Tempered glass retroreflective road markers	22	3.14
Adjusting road surface	19	1.59
Residual soil disposal	17	2.54
Removing stabilized base and asphalt pavement	15	4.96
Temporary markings	3	0.75
The remaining work items are omitted hereafter		

4.4. Stochastic configuration and experimental settings

Before initiating the experimental simulation, a minimum threshold of realizations for reaching expected simulation error tolerance and confidence level is determined using Eqs. (3), (4), and (5). Fig. 2 shows the settings configured in the streamline experiments, including: examining the sensitivity of the simulated historical data via hypothesis testing (*e.g.,* PC and KS tests); probability distribution determination (*e.g.,* normal, lognormal, beta, Weibull, or triangular distribution); the influence of correlated cost elements (*e.g.,* independency versus dependency; concern of non-positive definite correlation matrix); and generating random variates (*e.g.*, multivariate normal random simulation, multivariate lognormal random simulation, or normal to anything stochastic process). The experimental settings are such that each mathematical model has six paths.

The TPC model is constructed using PWI cost and proportion *P* of the total project cost, both of which are simulated as random variables. Considering NORTA and multivariate random variate generation, correlation of PWI, simulation precision, and differences in goodness-of-fit tests (PC testing and K–S testing), this study adopts the proposed Monte Carlo simulation procedure described above to yield the distribution curve of unit construction area cost. The same procedure is applied to TPCS, with the major difference being that it first simulates the PWI unit construction quantity (quantity/square meters) multiplied by historical average item unit costs (Table 2), then uses *P* to determine item-based construction project costs.

Table 3 shows that, the optimal distribution forms obtained from PC testing and K–S testing were not necessarily identical, whether in terms of item unit cost or quantity. The distribution form determined the sampling range of random values, which potentially affects the simulation results. Consequently, simulation experiments were conducted for the distribution forms of each item resulting from the statistical testing.

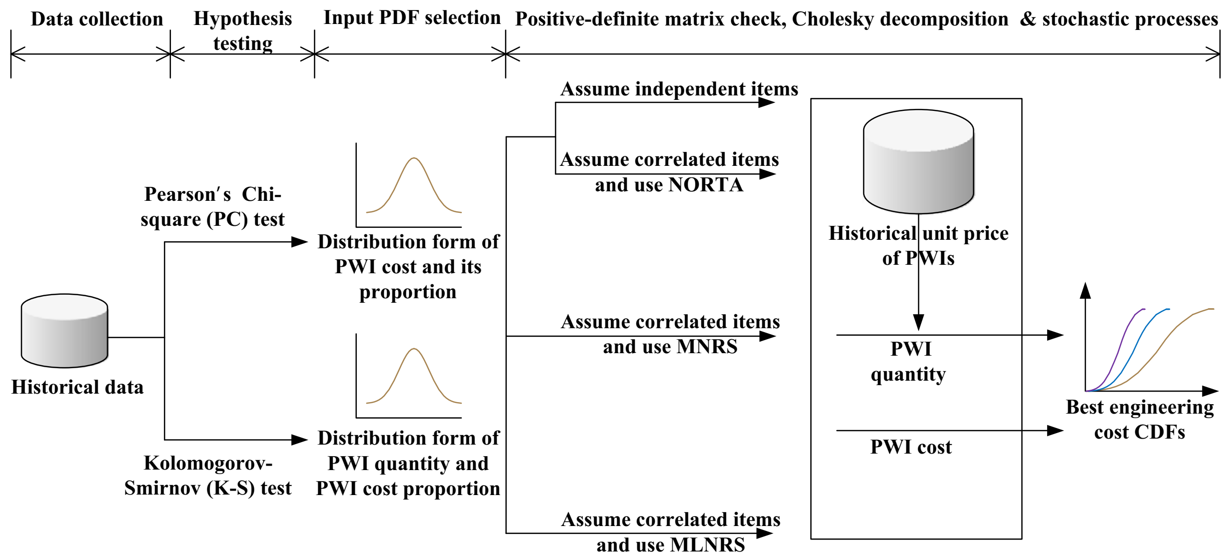

Fig. 2. Experimental engineering cost simulation for TPC/TPCS.

To consider the correlation between work items and the possible impact of correlation on the simulation results, this investigation analyzed the influence of multivariate normal random simulation (MNRS), multivariate lognormal random simulation (MLNRS) and NORTA assuming item independence and considering actual correlation between items in the cost simulation.

Prior to performing MNRS and MLNRS, it is necessary to obtain the covariance matrix for variables. Additionally, it is necessary to obtain the rank correlation matrix prior to using NORTA, before using Cholesky decomposition for all simulation methods to decompose the matrix and continue the simulation. This work used the Cholesky factorization function of Matlab software to perform the decomposition. In this case, the matrices must be positive-definite matrices following testing. If not, an adjustment as described in Section 2.5 must be conducted. The covariance matrices, rank correlation matrices, and decomposed matrices for PWI quantity and cost are listed in Tables 4 and 5, respectively.

4.5. Simulation results and implications

For the case simulation, at least 76,833 repetitions of calculations must be performed to achieve simulation error of ±1% with a 95% confidence level. Table 6 lists the results of 80,000 simulation realizations for both the TPC and TPCS models. Analysis of the results reveals that PC testing provides less cost variance than the K–S testing method. Implicitly PC testing thus offers a better choice for risk aversion decision-makers owing to its narrower simulation variance. Except in MLNRS in the case of the TPC model, the obtained standard deviation absolute error rate reduced when variable correlation was considered. Fig. 3 shows the best cumulative distribution functions along with the actual historical distribution.

To summarize, the following inferences can be obtained based on the simulation experiments.

With the TPC model:

- Assuming item independence, the error rates of the PC and K–S tests differ significantly in terms of average simulation results, and moreover the K–S test obtains more accurate results (PC: 1.9%; K–S: 0.6%).
- Using NORTA and assuming item correlation, the PC test performs better in terms of standard deviation (SD) (PC: 5.4%; K–S: 7.3%), while the K–S test performs better in terms of averages (PC: 2.0%; K–S: 0.6%).
- Whether with item independence or correlation, the error rate of the K–S test in terms of averages is less than that of the PC test.
- Assuming items are correlated, the standard deviation simulated value is greater than under items are independent, meaning that failure to account for inter-item association leads to underestimation of construction risk.

With the TPCS model:

- Assuming item correlation, performance in terms of SD does not differ significantly among MNRS, MLNRS and NORTA.
- Assuming item independence, the degree to which the K–S test underestimated SD exceeds that to which the PC test underestimated SD (PC: 10.7%; K–S: 12%).

Table 2
Average unit price of PWIs.

PWI	Historical average unit price (TWD/m^2)
Laying reclaimed dense-grade asphalt concrete	18.80
Salvaging, hauling, and stockpiling reclaimable asphalt pavement	4.95
Laying reflectorized pavement markings	658.26
Sprinkling tack coat	137.40
Reclaimed asphalt concrete deductible from engineering cost [a]	−308.17

[a] The reclaimed asphalt concrete materials can be recycled and is deductible from the cost.

Table 3
Empirical distribution form of PWI and cost percentage.

No.	PWI	Probability distribution of PWI cost	
		PC test	K–S test
A	Laying reclaimed dense-grade asphalt concrete	Beta (2.08, 2.93, 1.06, 33.81)	Normal (14.69, 6.53)
B	Salvaging, hauling, and stockpiling reclaimable asphalt pavement	Lognormal (6.62, 1.86, −2.64)	Lognormal (6.62, 1.86, −2.64)
C	Laying reflectorized pavement markings	Triang (12.75, 155.51, 33.03)	Beta (1.56, 2.68, 14.1, 162.52)
D	Sprinkling tack coat	Lognormal (7.16, 2.85, −1.33)	Weibull (1.68, 5.08, 1.27)
E	Reclaimed asphalt concrete deductible from engineering cost	Weibull (1.53, 13.1, 1.34)	Triang (1.65, 37.51, 1.89)
		Probability distribution of PWI quantity	
		PC test	K–S test
a	Laying reclaimed dense-grade asphalt concrete	Beta (2.7, 4.02, 0, 1.95)	Normal (0.78, 0.34)
b	Salvaging, hauling, and stockpiling reclaimable asphalt pavement	Triang (0.07, 1.84, 0.47)	Weibull (2.6, 0.92, 0)
c	Laying reflectorized pavement markings	Triang (0.02, 0.24, 0.05)	Weibull (1.9, 0.1, 0.01)
d	Sprinkling tack coat	Weibull (1.69, 0.04, 0)	Normal (0.04, 0.02)
e	Reclaimed asphalt concrete deductible from engineering cost	Weibull (2.11, 0.05, 0)	Beta (2.22, 4.11,0, 0.12)
		PC test	K–S test
	PWI cost percentage (*P*)	Beta (8.56, 2, 0.49, 0.9)	Beta (8.56, 2, 0.49, 0.9)

- Assuming item correlation and using NORTA, the PC test performs better in terms of SD (PC: 4%, K–S: 7.1%).
- Assuming item quantity correlation, using MNRS and PC testing leads to far better performance in terms of SD than with other methods.
- Overall, both mathematical models perform satisfactorily in terms of mean percentage error and SD, while TPCS outperforms TPC in most of the cases.

In terms of road surface repair construction, assuming that a parametric or empirical method is used to calculate the cost of such construction as the excerpted portion of the cumulative probability distribution graph for project cost best determined by Monte Carlo simulation (S3 in Table 4) is as shown in Fig. 3; for an initial price of 85 NTD/m^2, for example, based on historical experience or preliminary single estimate, there is only 40% confidence of successful project completion. For engineering personnel, this 40% probability is lower than the expected even odds of 50%. Consequently, project management personnel can increase construction reserve funds to increase the likelihood of eventual project completion. The likelihood increases from 40% to 50% as the cost per m^2 increases from NTD 85 to NTD 97. Furthermore, for experienced project managers, an even chance may be insufficient to compromise for the associated uncertainties, so project managers can increase price per m^2 (*e.g.*, NTD 97 to NTD 108 or even NTD 155) to increase the probability of successful completion with moderate or high confidence (*e.g.*, from 50% to 60% or

Table 4
The covariance, rank correlation matrices, and their Cholesky decomposition for PWI cost.

	A	B	C	D	E			A	B	C	D	E
A	42.04					Cholesky decomposition	A	6.48				
B	9.44	3.46				⟶	B	1.46	1.16			
C	173.24	44.17	992.36				C	26.72	4.55	16.05		
D	14.80	3.76	62.26	7.69			D	2.28	.38	−0.03	1.53	
E	30.54	8.14	156.42	8.99	57.01		E	4.71	1.11	1.59	−1.40	5.39
Covariance matrix for multivariate normal random simulation.												
A	0.298					Cholesky decomposition	A	0.546				
B	0.226	0.269				⟶	B	0.413	0.314			
C	0.237	0.223	0.248				C	0.434	0.140	0.199		
D	0.214	0.202	0.191	0.245			D	0.392	0.126	0.016	0.274	
E	0.259	0.261	0.249	0.189	0.484		E	0.474	0.207	0.072	−0.089	0.451
Covariance matrix for multivariate lognormal random simulation.												
A	1					Cholesky decomposition	A	1				
B	0.858	1				⟶	B	0.858	0.513			
C	0.876	0.862	1				C	0.876	0.216	0.432		
D	0.786	0.760	0.743	1			D	0.786	0.165	0.043	0.594	
E	0.640	0.660	0.685	0.524	1		E	0.640	0.215	0.181	−0.037	0.714
Rank correlation matrix for NORTA.												

Table 5
The covariance, rank correlation matrices, and their Cholesky decomposition for PWI quantity.

	a	b	c	d	e			a	b	c	d	e
a	0.1140					Cholesky decomposition	a	0.3377				
b	0.1126	0.1159				⟶	b	0.3334	0.0689			
c	0.0144	0.0152	0.0024				c	0.0426	0.0141	0.0199		
d	0.0062	0.0062	0.0008	0.0005			d	0.0184	0.0010	−0.0011	0.0111	
e	0.0061	0.0062	0.0009	0.0003	0.0004		e	0.0181	0.0020	0.0058	−0.0010	0.006
Covariance matrix for multivariate normal random simulation.												
a	0.27					Cholesky decomposition	a	0.52				
b	0.26	0.26				⟶	b	0.50	0.09			
c	0.25	0.25	0.26				c	0.47	0.13	0.15		
d	0.23	0.23	0.22	0.27			d	0.45	0.06	0.01	0.27	
e	0.28	0.27	0.27	0.24	0.3		e	0.53	0.05	0.08	−0.02	0.11
Covariance matrix for multivariate lognormal random simulation.												
a	1					Cholesky decomposition	a	1				
b	0.98	1				⟶	b	0.98	0.20			
c	0.87	0.91	1				c	0.87	0.29	0.41		
d	0.86	0.85	0.73	1			d	0.86	0.05	−0.05	0.51	
e	0.90	0.90	0.93	0.74	1		e	0.90	0.10	0.29	−0.05	0.3
Rank correlation matrix for NORTA.												

90%). The graph provides an easy quantitative tool to convert confidence level into monetary value, and *vice versa*, to facilitate decision making.

5. Conclusions

This work presents comprehensive stochastic processes involving the examination of a series of simulation building blocks for conceptual cost range estimates. This study examines experimental simulation of alternative mathematical equations using different testing methods and techniques of handling correlation random variables. The research results demonstrate that considering correlations in item costs or item quantities can increase the simulation accuracy. The main contribution of the proposed stochastic process is that project management professionals can apply it to make rational decisions regarding cost estimation and quantify its likelihood of success in situations with specific degrees of simulation precision.

In particular, this study demonstrates Pearson Chi-square testing to have superior error rate in simulating SD. In item probability distribution selection, NORTA and MNRS outperform other combinations in terms of SD error rates. Furthermore, individually identifying item quantity and unit costs and summing the two improves accuracy, enabling the TPCS model to outperform the TPC model. Notably, these results indirectly indicate that item costs are more appropriate than item quantities for use in MLNRS.

Cost estimation requires considering engineering reserve funds which intend to compensate for shortcomings in the accuracy, quality, or quantity of information gathered during project planning and detailed design or for other unexpected events. Another item is management reserve funds, which denote spending by decision-makers to improve management

Table 6
Simulation results of the TPC and TPCS models.

Goodness-of-fit testing methods for PWIs (A, B, C, D, E)	Correlation or independence between work items	Simulation process	TPC no.	MPE [a] (%)	SDPE [b] (%)	TPCS no.	MPE (%)	SDPE (%)
PC (Beta, Lognormal, Triang, Lognormal, Weibull)	Independence	Independence sampling by respective marginal distribution	T1	1.9	13.0	S1	0.8	10.7
K–S (Normal, Lognormal, Beta, Weibull, Triang)	Independence	Independence sampling by respective marginal distribution	T2	0.6	13.3	S2	1.0	12.0
PC (Beta, Lognormal, Triang, Lognormal, Weibull)	Correlation	NORTA	T3	2.0	5.4	S3	0.9	4.0
K–S (Normal, Lognormal, Beta, Weibull, Triang)	Correlation	NORTA	T4	0.6	7.3	S4	0.9	7.1
Normal	Correlation	Multivariate normal random simulation	T5	0.5	6.0	S5	0.8	5.6
Lognormal	Correlation	Multivariate lognormal random simulation	T6	0.3	7.4	S6	1.6	12.2
MAPE [c]				1.0	8.7		1.0	8.6

[a] MPE (mean percentage error) = absolute value (simulation mean − true mean) / true mean * 100%.
[b] SDPE (standard deviation percentage error) = absolute value (estimate standard deviation − true standard deviation) / true standard deviation * 100%.
[c] Mean absolute percentage error.

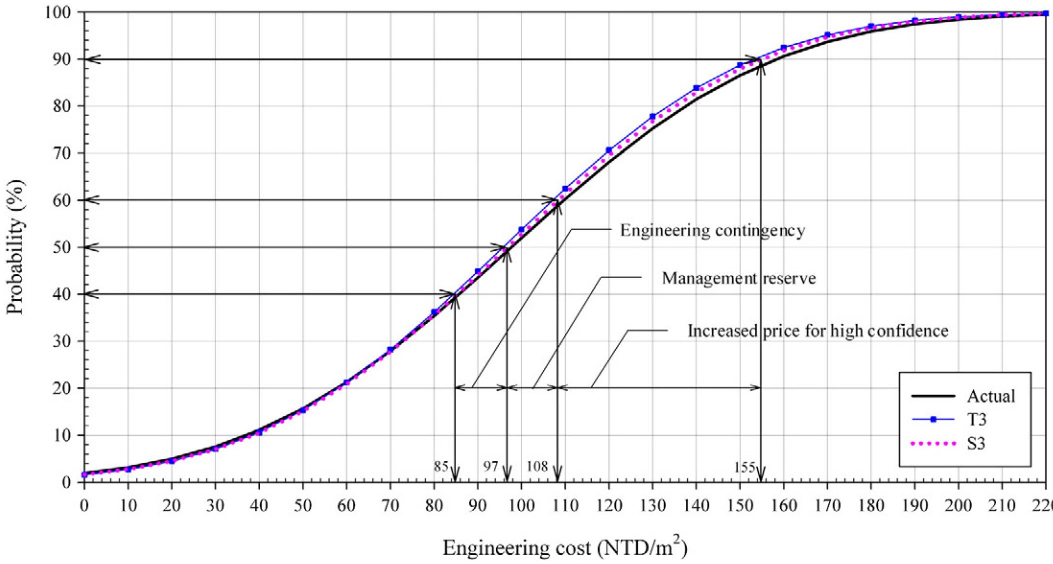

Fig. 3. Cumulative distribution curves of actual value, T3, and S3.

technologies applied to a project and thus reduce the likelihood of project failure. From the perspective of the bidder, a bid-no-bid decision can be examined cursorily based on the management expertise of the bidder.

Finally, the cost average absolute error rates of all the stochastic experiments fall between 0.3% and 2.0% and the cost SD ranges from 4.0% to 13.3%. During initial project planning these discrepancies are reasonable. The estimation model developed here used road repair and construction in Taiwan as a case study. Future research can apply the systematic model procedure to other types of construction to perform cost estimation and even expand it to simulate project life-cycle cost. How well the proposed models are tested across project types and how the simulation application related to real-world situations will be another potential content for further work on this research. Notably, as project construction parameters are not necessarily consistent, for example in terms of traffic flow, road type and original road surface damage, considering the variance of construction parameters in cost simulation can improve simulation accuracy.

Acknowledgements

The author would like to thank the National Science Council of the Republic of China, Taiwan for financially supporting this research under contract No. NSC 96-2416-H-194-011-MY3, which made this research work possible. Additional gratitude extends to Mr. M.-C. Yen for his assistance in data processing.

References

Akintoye, A.S., MacLeod, M.J., 1997. Risk analysis and management in construction. International Journal of Project Management 15, 31–38.

Artto, K.A., Lehtonen, J.M., Saranen, J., 2001. Managing projects front-end: incorporating a strategic early view to project management with simulation. International Journal of Project Management 19, 255–264.

Berny, J., Townsend, P., 1993. Macrosimulation of project risks — a practical way forward. International Journal of Project Management 11, 201–208.

Cario, M.C., Nelson, B.L., 1997. Modeling and generating random vectors with arbitrary marginal distributions and correlation matrix. Department of Industrial Engineering and Management Sciences. Northwestern University, Evanston, Illinois.

Chau, K.W., 1995a. Monte Carlo simulation of construction costs using subjective data. Construction Management and Economics 13, 369–383.

Chau, K.W., 1995b. The validity of the triangular distribution assumption in Monte Carlo simulation of construction costs. Empirical evidence from Hong Kong. Construction Management and Economics 13, 15–21.

Chou, J.-S., 2009. Web-based CBR system applied to early cost budgeting for pavement maintenance project. Expert Systems with Applications 36, 2947–2960.

Chou, J.-S., Wang, L., Chong, W.K., O'Connor, J.T., 2005. Preliminary cost estimates using probabilistic simulation for highway bridge replacement projects. American Society of Civil Engineers, Reston, VA 20191-4400, United States, San Diego, CA, United States, pp. 939–948.

Chou, J.-S., Peng, M., Persad, K.R., O'Connor, J.T., 2006. Quantity-based approach to preliminary cost estimates for highway projects. Transportation Research Record 22–30.

Chou, J.-S., Yang, I.-T., Chong, W.K., 2009. Probabilistic simulation for developing likelihood distribution of engineering project cost. Automation in Construction 18, 570–577.

Chung, C.A., 2004. Simulation Modeling Handbook: a Practical Approach Boca Raton New York.

Cooper, D., MacDonald, D., Chapman, C., 1985. Risk analysis of a construction cost estimate. International Journal of Project Management 3, 141–149.

Cooper, D.F., Grey, S., Raymond, G., Walker, P., 2004. Project Risk Management Guidelines: Managing Risk in Large Projects and Complex Procurements. Wiley.

Diekmann, J.E., 1983. Probabilistic estimating: mathematics and applications. Journal of Construction Engineering Management 109, 297–308.

Elkjaer, M., 2000. Stochastic budget simulation. International Journal of Project Management 18, 139–147.

Fishman, G.S., 1996. Monte Carlo: Concepts, Algorithms, and Applications. Springer-Verlag, New York.

Franke, A., 1987. Risk analysis in project management. International Journal of Project Management 5, 29–34.

Ghosh, S., Henderson, S.G., 2003. Behavior of the NORTA method for correlated random vector generation as the dimension increases. ACM Transactions on Modeling and Computer Simulation 13, 276–294.

Herbold, K.D., 2000. Using Monte Carlo simulation for pavement cost analysis. Public Roads 64, 2–6.

Hull, J.K., 1990. Application of risk analysis techniques in proposal assessment. International Journal of Project Management 8, 152–157.

Khedr, M.K., 2006. Project Risk Management Using Monte Carlo Simulation. 2006, p RISK.02.01-RISK.02.10.

Laufer, A., 1991. Construction planning in uncertain environments. International Journal of Project Management 9, 53–60.

Law, A.M., Kelton, W.D., 2000. Simulation Modeling and Analysis, 3rd ed. McGraw-Hill.

Moselhi, O., Dimitrov, B., 1993. Monte Carlo technique with correlated random variables. Journal of Construction Engineering Management 119, 658–661.

MOTC, 2010. http://www.thb.gov.tw/, accessed in Jan. 2010.

Niazi, A., Dai, J.S., Balabani, S., Seneviratne, L., 2006. Product cost estimation: technique classification and methodology review. Journal of Manufacturing Science and Engineering, Transactions of the ASME 128, 563–575.

Papagiannakis, T., Delwar, M., 2001. Computer model for life-cycle cost analysis of roadway pavements. Journal of Computing in Civil Engineering 15, 152–156.

PMI, 2008. A Guide to the Project Management Body of Knowledge, 4th ed. Project Management Institute.

Rezaie, K., Amalnik, M.S., Gereie, A., Ostadi, B., Shakhseniaee, M., 2007. Using extended Monte Carlo simulation method for the improvement of risk management: consideration of relationships between uncertainties. Applied Mathematics and Computation 190, 1492–1501.

Robinson, S., 2004. Simulation: the Practice of Model Development and Use. John Wiley & Sons, Ltd, Chichester, West Sussex, England.

Salem, O., AbouRizk, S., Ariaratnam, S., 2003. Risk-based life-cycle costing of infrastructure rehabilitation and construction alternatives. Journal of Infrastructure Systems 9, 6–15.

Tighe, S., 2001. Guidelines for probabilistic pavement life cycle cost analysis. Transportation Research Record 28–38.

Touran, A., 1993. Probabilistic cost estimating with subjective correlations. Journal of Construction Engineering Management 119, 58–71.

Touran, A., 2003. Probabilistic model for cost contingency. Journal of Construction Engineering Management 129, 280–284.

Touran, A., Wiser, E.P., 1992. Monte Carlo technique with correlated random variables. Journal of Construction Engineering Management 118, 258–272.

Wall, D.M., 1997. Distributions and correlations in Monte-Carlo simulation. Construction Management and Economics 15, 241–258.

Wang, W.-C., 2002a. SIM-UTILITY: model for project ceiling price determination. Journal of Construction Engineering Management 128, 76–84.

Wang, W.-C., 2002b. Simulation-facilitated model for assessing cost correlations. Computer-Aided Civil and Infrastructure Engineering 17, 368–380.

Wang, W.C., 2004. Supporting project cost threshold decisions via a mathematical cost model. International Journal of Project Management 22, 99–108.

Wang, C.H., Huang, Y.C., 2000. A new approach to calculating project cost variance. International Journal of Project Management 18, 131–138.

Yang, I.-T., 2005. Simulation-based estimation for correlated cost elements. International Journal of Project Management 23, 275–282.

Yang, I.-T., 2008. Distribution-free Monte Carlo simulation: premise and refinement. Journal of Construction Engineering Management 134, 352–360.