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INCORPORATING INSPECTION DECISIONS IN PAVEMENT MANAGEMENT

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Abstract — Pavement management systems need to address not only maintenance and rehabilitation (M&R) decisions, but also facility inspection decisions. The state of the art in pavement management is lacking of any consistent methodology for making such decisions on a cost-effectiveness basis. Such a methodology must recognize the presence of interactions between M&R and inspection decisions. These interactions argue for a joint decision-making approach, where the sum of inspection and M&R costs is minimized. This paper reviews different possible mathematical formulations to such a joint decision-making model, having various levels of restriction and computational complexity. These formulations are then compared and the effect of the forecast uncertainty on the minimum expected costs produced by each of them is investigated empirically. It is concluded that optimizing inspection decisions jointly with M&R decisions can lead to substantial cost savings, especially for high precisions of forecasting.

1. INTRODUCTION

Traditionally, pavement management has focused on maintenance and rehabilitation (M&R) decisions, including the selection of optimal activities to apply to different highway segments during each year of a given planning horizon. In addition to M&R activity selection decisions, highway agencies have to make decisions regarding facility inspections. There are basically two dimensions to the inspection decision. These are: (a) whether to inspect a facility in a given year, and (b) how to inspect it (that is, using which technology).

The second decision assumes that highway agencies have several measurement technologies available, of varying precisions and costs, and can select which one to use in any given year. In this paper, the emphasis will be on the first decision, because the models reviewed do not deal with the effect of imprecise measurements and thus assume that all inspections are perfectly precise. This eliminates any meaningful difference between inspection technologies. An improved model, which does recognize the presence of uncertainty in condition measurement, is presented in Madanat (1991; 1993).

The role of inspection in the context of pavement management is to provide the information required for making M&R decisions. Thus, inspections are usually scheduled just prior to the time when M&R decisions are made. Usually, M&R decisions are made at the beginning of every year. However, most agencies do not have the resources to perform yearly detailed inspections of all their facilities. Rather, they restrict their attention each year to "critical" facilities (where the definition of "critical" varies from one agency to another) while performing only minimal inspections, at most, of other facilities.

A representative example of the current practice in inspection decision-making is found in PAVER, the pavement management system designed by the Construction Engineering Research Laboratory (Shahin and Kohn, 1981). PAVER triggers the inspection of a highway segment if either of the following conditions has been met:

1. The predicted PCI (the condition measure used in PAVER) falls below the specified minimum standard, so that an overlay is required, or,
2. The time elapsed since last inspection equals the specified maximum standard.

Both of the above standards depend on the type of highway (primary, secondary or local street), and have to be specified by the agency, based on specific experience and judgement. No objective or consistent methodology for selecting the values of these standards is given.

To establish such a methodology, it is necessary to quantify the impacts associated with different inspection alternatives. For example, a delay in facility inspection may lead to a delay in performing a M&R activity, causing an increase in user costs or/and making it necessary to perform more expensive activities in the future. This is an example where a savings in inspection costs can lead to an increase in M&R costs. On the other hand, increasing the frequency of inspections beyond a certain point may lead to an increase in the total costs of managing a facility, if the information provided by these inspections does not improve the quality of the M&R decisions made by the agency.

The required methodology must, therefore, take into account the impacts of inspection decisions on lifecycle costs. It must also account for the fact that the inspection decision is affected by the history of M&R activities. For example, a facility that has received a major rehabilitation in the previous year may not need an inspection as critically as one that has not received any maintenance activity in the last five years.

These interactions between inspection and M&R activities make it logical to approach the inspection decision problem jointly with the M&R decision problem. More specifically, the alternatives that must be evaluated within the joint decision methodology are joint inspection and M&R activity alternatives, and the costs to be minimized must be the total lifecycle costs, including the costs of inspection.

The preceding discussion assumes that inspections are a source of information about facility condition. However, not all decision-making methods require this to be the case. Deterministic decision-making models assume that the condition of a highway pavement can be forecasted without error into any time in the future, for any sequence of M&R activities (Fernandez, 1979; Tsunokawa and Schofer, 1986). Under such an assumption, there is no benefit to be gained from performing inspections, because the information they would provide is assumed to be already known to the decision-maker. Hence, in addition to being unrealistic in their representation of pavement deterioration, deterministic models do not recognize the role of inspection.

On the other hand, stochastic decision models, generally based on Markov Decision Processes, or MDP, do recognize the uncertainty in facility condition forecasting and the role of inspections in reducing it. Markov Decision Processes now constitute the methodological basis for state-of-the-art pavement management systems (Golabi, *et al.*, 1982; Carnahan, *et al.*, 1987; Feighan, *et al.*, 1988) as well as bridge management systems (Harper, *et al.*, 1990). The following section will review this methodology in more detail.

2. OVERVIEW OF MARKOV DECISION PROCESSES

The central assumption of Markov Decision Processes is that the "system" being modeled (in our case, facility condition) evolves over time according to a Finite State Markov Chain. The mathematical structure of Markov Chains can be summarized as follows:

1. The system can be, at any discrete point in time t, in a state i out of n possible states:

$$\chi_t = i; \qquad i = 1,2, \ldots ,n; \qquad \forall t, \tag{1}$$

where:

χ_t = state of the system at time t.

2. The system transitions from one state to another with known probability:

$$p(\chi_{t+1} = j \mid \chi_t = i); \qquad i,j = 1, \ldots ,n; \qquad \forall t, \tag{2}$$

where:

$p(.\mid.)$ = a discrete transition probability.

3. The transition probability from one state to another is independent of the previous states of the process:

$$p(\chi_{t+1} = j | \chi_t = i, \chi_{t-1} = k, \ldots) = p(\chi_{t+1} = j | \chi_t = i);$$
$$i,j,k, \ldots = 1, \ldots, n; \qquad \forall t. \tag{3}$$

The above mathematical structure implies a rich array of properties (Ross, 1983). However, eqns (1-3) are sufficient to express all the characteristics of interest to us in this paper.

A Markov Chain with Rewards is a Markov Chain for which there exists a cost (or reward) associated with every state (or every transition between states). Mathematically, this cost is denoted by:

$$g(\chi_t) = \text{cost obtained for being in state } \chi_t = 1, \ldots, n \text{ at time } t.$$

A Markov Decision Process is a Markov Chain with Rewards where there is a decision-maker who, at every t, can select one of several activities. Each activity has associated with it a different cost (for every state), and a different transition probability distribution. Mathematically, at every t, the decision-maker selects one activity a_t, out of m possible activities, for which he incurs a cost $g(\chi_t, a_t)$. As a result, the system evolves according to the probabilities $p(\chi_{t+1} = j | \chi_t = i, a_t)$ during time period t. The number of time periods during which the decision-maker can control the system is called the planning horizon T, that is, decisions are made at $t = 0, 1, \ldots, T - 1$. The objective of the decision-maker in controlling the system is to minimize the total expected costs (or maximize total expected rewards).

Mathematically, the objective is:

$$\min H(\chi_0) = \min_{a_0,\ldots,a_{T-1}} \; E_{\chi_1,\ldots,\chi_T} \left\{ \sum_{t=0}^{T-1} \alpha^t g(\chi_t, a_t) + \alpha^T g(\chi_T) \right\}, \tag{4}$$

subject to:
the transition probabilities $p(\chi_{t+1} = j | \chi_t = i, a_t)$; $i,j = 1, \ldots, n$; $t = 0, \ldots, T - 1$,

where:

α = discount amount factor,
χ_0 is given, and there is a terminal cost $g(\chi_T)$ associated with the state of the system at the end of the last period.

The total expected cost is used as a criterion in program (4) because the costs per stage $g(\chi_t, a_t)$ depend on the value of χ_t, which is known only probabilistically, for $t > 0$, at the start of the planning horizon. The solution of program (4) is an optimal policy

$$\pi^* = \{\mu_0^*(\chi_0), \mu_1^*(\chi_1), \ldots, \mu_{T-1}^*(\chi_{T-1})\},$$

where:

$\mu_t^*(\chi_t)$ = a function which assigns an optimal activity $a_t^* = 1, \ldots, m$;
$t = 0, \ldots T - 1$ to each state $\chi_t = 1, \ldots, n$.

This function is itself a function of time because the optimal activity for a given facility state may vary from year to year. In addition, the solution provides the value of the minimum total expected cost $J^*(\chi_0) = \min H(\chi_0)$.

Such a problem may look formidable in size for even small values of T. Fortunately, by using the Dynamic Programming algorithm, it is possible not to evaluate all the combinations of activities and states at every time period.

The mathematical formulation of the Dynamic Programming algorithm is:

$$J_T(\chi_T) = g(\chi_T); \qquad \chi_T = 1, \ldots, n,$$

and

$$J_t(\chi_t) = \min_{a_t} \left\{ g(\chi_t,a_t) + \alpha \sum_{j=1}^{n} p(\chi_{t+1} = j | \chi_t,a_t) J_{t+1}(\chi_{t+1} = j) \right\}$$

$$\chi_t = 1, \ldots, n; \qquad t = 0, \ldots, T-1, \tag{5}$$

where:

$J_t(\chi_t)$ = minimum expected cost-to-go from t until T, given that the system is in state χ_t at time t,
$J_T(\chi_T)$ = terminal cost at T, given the system is in state χ_T.

Note that the cost-to-go is itself a function of t.

Program (5) is solved recursively for every $\chi_t = 1, \ldots, n$, from $t = T$ to $t = 0$ (where χ_0 is assumed given). The optimal policy $\pi^* = \{\mu_0^*(\chi_0), \ldots, \mu_{T-1}^*(\chi_{T-1})\}$ is obtained by tracing the optimum activities a_t^* that were selected for every state, for all time periods. The minimum total expected cost is given by the cost-to-go from $t = 0$ to $t = T$, that is:

$$J^*(\chi_0) = J_0(\chi_0) \tag{6}$$

Having reviewed the basic theory of Markov Decision Processes, we now turn to the question of how this methodology can be used to address the joint M&R and inspection problem. In the following sections, different approaches to the use of MDP to the joint problem will be reviewed. They can be broadly divided into models with temporal linkage between inspections and M&R decisions and models without temporal linkage. A joint decision model with temporal linkage assumes that an inspection has to be performed before an M&R decision can be made. These models, which are more limited in capability, will be reviewed first.

3. JOINT DECISION MODELS WITH TEMPORAL LINKAGE

The central assumption of MDP theory is that, since the state of the system at $t + 1$ is known only probabilistically at t, the decision-maker can select activity a_{t+1} only after the state χ_{t+1} becomes known (that is, at the beginning of period $t + 1$). Inherent in this assumption is the recognition that an inspection is performed before making M&R decisions at the beginning of every time period, since inspection is the process by which the state of the system is revealed. In other words, there is a forced temporal linkage between inspections and repair decisions.

In addition to this linkage, the MDP formulation of program (5) has other limitations. From the perspective of the joint inspection and M&R problem, a Markov Decision Process can be viewed as a joint decision model with a predetermined inspection frequency decision (once per time period).

The final limitation of MDP stems from the assumption that the condition state of the system is revealed without uncertainty after each inspection, which is equivalent to assuming that the inspection technology used is perfectly precise (free of random errors) and accurate (free of biases). The assumption of no bias is not as critical as the assumption of no random errors. This is because it may be possible to obtain estimates of the biases in measurement, which can be used to correct the measurements for their presence. A methodology which can be used to obtain estimates of the measurement biases can be found in Humplick (1992). This is not the case, however, with random errors, of which we know only the probabilistic distribution; see for example Ben-Akiva *et al.* (1991; 1993).

If the MDP model is viewed as a joint decision model subject to a constrained inspection frequency, the last limitation can be removed by formulating the model without the inspection constraint. Such models have been formulated in the field of industrial engineering, where the focus of the problem is on machine inspection and repair. The difference between these models and that of the previous section is that inspections are not constrained to take place at regular, equally spaced points in time. Rather, the decision of when to inspect is endogenous, that is, it is one of the outputs of the MDP algorithm.

Because the state of the facility prior to inspection is probabilistic, and because it is still assumed that knowledge of the state of the system is required for decision-making, inspection and repair decisions can only be made after an inspection. This implies that, at the beginning of the planning horizon, the state of the facility must be known, either through an inspection or some other means (for example, if we start with a new facility, which is known to be in its best state). At that time, a repair decision is made, as well as a decision on when to schedule the next inspection. Once this "next" inspection is performed, the decision-maker can again make a repair decision and schedule another inspection, and so on. Between inspections, no repair decisions can be made, and it is assumed that some default activity is performed, for example do-nothing. This joint approach to inspection and M&R decision-making guarantees that an inspection is made only when its cost is offset by its effect on the reduction in expected future costs.

In order to formulate the problem mathematically, we need to first specify the cost structure in a more detailed manner than was done so far. The costs included in this model are: (a) maintenance, repair and replacement costs, $ca(\chi_t,a_t)$, a function of the state of the system and of the type of maintenance activity; (b) inspection costs, cm; (c) user costs, $cu(\chi_t)$, a function of the state of the system.

In this model, the decision-maker, at the beginning of every time period t, jointly selects a repair activity a_t and the length of time until the next inspection Δt. Since the next decision will be done at the beginning of year $t + \Delta t$, the probabilities of the possible states of the facility at that time must be known to the decision-maker. These probabilities are denoted by $p(\chi_{t+\Delta t} = j\,|\,\chi_t,a_t)$.

To evaluate the expected user costs associated with the facility between year t and year $t + \Delta t$, the decision-maker also needs to know the probabilities of the possible states of the facility during these years, $p(\chi_{t+s} = j\,|\,\chi_t,a_t)$, $s \leq \Delta t$ (note that these include the state probabilities at the beginning of year $t + \Delta t$).

To evaluate $p(\chi_{t+s} = j\,|\,\chi_t,a_t), s \leq \Delta t$, basic Markov Chain principles are used. The transition probabilities of a Markov Chain, $p(\chi_{t+1} = j\,|\,x_t = i,a_t)$ can be arranged in square matrices $\mathbf{P_a}$, with one matrix for each activity "a":

$$\mathbf{P_a} = \begin{bmatrix} p_{11}^{(a)} & p_{12}^{(a)} & \cdots & p_{1n}^{(a)} \\ p_{21}^{(a)} & p_{22}^{(a)} & \cdots & p_{2n}^{(a)} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ p_{n1}^{(a)} & p_{n2}^{(a)} & \cdots & p_{nn}^{(a)} \end{bmatrix},$$

where:

$$\mathrm{p}_{ij}^{(a)} = \mathrm{p}(\chi_{t+1} = \mathrm{j}\,|\,\chi_t = \mathrm{i},\mathrm{a}_t). \tag{7}$$

Using the notation $a = 0$ for do-nothing, it can be seen that, for $s = 2 \leq \Delta t$:

$$\begin{aligned} p(\chi_{t+2} = k\,|\,\chi_t = i,a_t) &= \sum_{j=1}^{n} p(\chi_{t+2} = k\,|\,\chi_{t+1} = j,0)p(\chi_{t+1} = j\,|\,\chi_t = i,a_t) \\ &= (P_a * P_0)_{ik}, \end{aligned} \tag{8}$$

where * refers to the outer, or cross product of two matrices.

Similarly, for any $s \leq \Delta t$:

$$p(\chi_{t+s} = k\,|\,\chi_t = i,a_t) = (P_a * P_0^{s-1})_{ik}. \tag{9}$$

Using this information, we can write a Dynamic Programming formulation for this problem. It takes the form:

$$J_T(\chi_T) = 0, \qquad \chi_T = 1, \ldots, n,$$

and,

$$J_t(\chi_t) = \min_{(a_t,\Delta t)} \left\{ ca(\chi_t,a_t) + \sum_{s=1}^{\Delta t} \sum_{j=1}^{n} \alpha^s p(\chi_{t+s} = j \mid \chi_t,a_t) cu(j) + \alpha^{\Delta t} \left(cm + \sum_{k=1}^{n} p(\chi_{t+\Delta t} = k \mid \chi_t,a_t) J_{t+\Delta t}(\chi_{t+\Delta t} = k) \right) \right\}$$

$$\chi_t = 1, \ldots, n \quad t = 0, \ldots, T-1. \tag{10}$$

Note that it is assumed that the terminal cost at T is zero; this is not a necessary assumption for this model, but is justified in typical infrastructure applications, where the terminal cost represents the salvage value of the facility at the end of the planning horizon.

The solution of program (10) proceeds in a basically similar manner as that of the normal MDP of program (5). The first difference is in the number of alternatives that have to be evaluated for each state x_t. In program (5), the number of alternatives is the number of M&R activities, m, assuming that the choice set is fixed across states x_t and time periods t. In program (10) the number of alternatives for each state x_t is equal to:

$$m^*(T - t), \tag{11}$$

where:

T = length of planning horizon.

The other difference is that the optimal policy that is produced by algorithm (10), π^*, now consists of functions $\mu_t^*(\chi_t) = (a_t,\Delta t)$, which specify both the optimal repair activity and the length of time to the next inspection, for a given state of the facility, only for the years at which an inspection takes place. For other years, the state of the facility is not known with certainty, and the default repair activity is performed (do-nothing).

Several variations of this formulation to address the joint repair and inspection problem have been developed. Some examples are Klein (1962) and Mine and Kawai (1982). These examples differ in the specific algorithm used to solve for the optimal policies, and in the form of these policies (randomized or nonrandomized). However, the basic components of the model are the same.

Two elements of this formulation deserve attention. First, there is an assumption of no uncertainty in the inspection process, which, at least in the context of highway pavement inspection, is unrealistic. Second, although program (10) relaxes the restriction of inspection in every time period, it still requires that an inspection be performed before a repair activity can be selected. This is a result of the requirement that the system state be known before decisions can be made, which is central to Markov Decision Processes, as was mentioned earlier. The requirement still places a constraint on the optimization although it is a less restrictive one than the requirement of inspection at every t. In Section 5, a model formulation that does not depend on this requirement will be presented for the joint decision problem. However, before doing this, the technique of state augmentation (Bertsekas, 1987) must be introduced.

4. STATE AUGMENTATION (APPLIED TO THE PURE M&R PROBLEM)

The "state of the system" at the start of a time period in Dynamic Programming is defined as the variable which summarizes all the information about the system, which is known to the decision-maker at that point in time and is relevant to future decisions.

In the case where the decision-maker obtains the true condition state of the facility, through inspection, and where the facility condition evolves according to a Finite Markov Chain, which was the case so far, the "state of the system" at the start of t is the condition state of the facility at that point in time. Although the decision-maker also knows the state of the facility and the decisions made in previous time periods, this information is irrelevant to future decisions because it does not affect the future states of the facility, and hence future costs. This fact follows from the basic property of a Markov Chain, which, mathematically, was stated as:

$$p(\chi_{t+1} = j \mid a_t,\chi_t,a_{t-1},\chi_{t-1}, \ldots ,a_0,\chi_0) = p(\chi_{t+1} = j \mid a_t,\chi_t). \tag{12}$$

When the condition state of the facility is unknown to the decision-maker, the "state of the system" can no longer be the condition state of the facility. In order to apply Dynamic Programming, we need to define a new state, which is done using state augmentation. The technique of state augmentation consists of redefining the state of the system to include all the information relevant to future decisions. Dynamic Programming is then applied to the augmented states of the system.

To illustrate the application of state augmentation, a hypothetical case is presented. It is assumed that the condition state of the facility is never measured throughout the planning horizon, so we are dealing with a pure M&R problem with no inspection. It is also assumed that performing an M&R activity does not reveal any information about what the condition state of the facility was prior to the activity. Hence, the information available at the beginning of t includes the entire history of decisions made up to $t - 1$, in addition to the state of the facility at the beginning of the planning horizon, χ_0, which is assumed to be known.

The augmented state, denoted by I_t, for information state, is:

$$I_t = \{\chi_0,a_0,a_1, \ldots ,a_{t-1}\} \quad t = 1,2, \ldots ,T \tag{13}$$
$$I_0 = \{\chi_0\}.$$

It can be seen that:

$$I_t = \{I_{t-1},a_{t-1}\} \quad t = 1, \ldots ,T, \tag{14}$$

from which it follows that:

$$P(I_t \mid I_{t-1},a_{t-1}) = 1 \text{ if } I_t = \{I_{t-1},a_{t-1}\}, \qquad 0 \text{ otherwise.} \tag{15}$$

This means that the augmented state, I_t, evolves in a deterministic manner, given by (14). We can now write a Dynamic Programming formulation over the space of the information states. This will be done in terms of the generic cost function $g(\chi_t, a_t)$ for simplicity of presentation. To do that, the cost function must be rewritten in terms of the new variable I_t:

$$\tilde{g}(I_t,a_t) = \underset{\chi_t}{E} \{g(\chi_t,a_t) \mid I_t\}, \tag{16}$$

where:

$\underset{\chi_t}{E} \{g \mid I_t\}$ is the conditional expectation of the cost per stage over the condition state distribution, conditional on the state of the information.

The Dynamic Programming formulation is given by:

$$J_T(I_T) = \underset{\chi_T}{E} \{g(\chi_T) \mid I_T\} \qquad \forall I_T$$

and

$$J_t(I_t) = \min_{a_t} (\underset{\chi_t}{E} \{g(\chi_t,a_t) \mid I_t\} + \alpha J_{t+1}(I_{t+1})) \qquad \forall I_t \; t = 0, \ldots ,T - 1. \tag{17}$$

To solve program (17), we need to evaluate expression (16), which requires us to establish the relation between I_t and the probabilistic distribution of χ_t, that is:

$$p_t(\chi_t = j \mid I_t), j = 1, \ldots, n; \qquad \forall I_t; \qquad \forall t, \tag{18}$$

or, in vector form:

$$P_t \mid I_t, \ \forall I_t, \ \forall t. \tag{18a}$$

As was done in Section 3, the transition probabilities $p(\chi_{t+1} = j \mid \chi_t = i, a_t)$ can be arranged in square matrices, with one matrix for each activity "a":

$$\mathbf{P_a} = \begin{bmatrix} p_{11}^{(a)} & p_{12}^{(a)} & \ldots & p_{1n}^{(a)} \\ p_{21}^{(a)} & p_{22}^{(a)} & \ldots & p_{2n}^{(a)} \\ . & . & \ldots . \\ . & . & \ldots . \\ p_{n1}^{(a)} & p_{n2}^{(a)} & \ldots & p_{nn}^{(a)} \end{bmatrix} \quad ; \quad \forall a, \tag{19}$$

where:

$$p_{ij}(a) = p(\chi_{t+1} = j \mid \chi_t = i, a).$$

This representation is useful for calculating the state probabilities for any time t, as follows:

$$p_2(\chi_2 = k \mid \chi_0 = i, a_0, a_1) = \sum_{j=1}^{n} p(\chi_2 = k \mid \chi_1 = j, a_1) p(\chi_1 = j \mid \chi_0 = i, a_0)$$

$$= \sum_{j=1}^{n} p_{ij}(a_0) p_{jk}(a_1)$$

$$= (P_{a_0} * P_{a_1})_{ik}. \tag{20}$$

Similarly, for any t:

$$p_t(\chi_t = k \mid \chi_0 = i, a_0, a_1, \ldots, a_{t-1}) = (P_{a_0} * P_{a_1} * \ldots * P_{a_{t-1}})_{ik} \ k = 1, \ldots, n. \tag{21}$$

Using the elements $p_t(\chi_t = k \mid I_t)$, $k = 1, \ldots, n$, calculated above, we can rewrite (17) as:

$$J_T(I_T) = \sum_{i=1}^{n} p_T(\chi_T = i \mid I_T) g(\chi_T); \qquad \forall I_T$$

and

$$J_t(I_t) = \min_{a_1} \left(\sum_{i=1}^{n} p_t(\chi_t = i \mid I_t) g(\chi_t, a_t) + \alpha J_{t+1}(I_{t+1}) \right) \quad \forall I_t, \ t = 0, \ldots, T-1. \tag{22}$$

Program (22) can be solved recursively from $t = T$ to $t = 0$, and for all states of the information I_t at each year t, to obtain the minimum expected costs $J^*(I_0)$ and the optimal policy $\pi^* = \{\mu_0^*(I_0), \ldots, \mu_{t-1}^*(I_{T-1})\}$.

The unavoidable price that was paid for transforming the problem from the space of x_t to the space of I_t is an increase in the size of the problem to be solved. The original MDP involved n states per year (the n states of the facility), whereas (22) involves, at year t, m^t states, where m is the number of possible activities which can be applied to the facility. To see why, consider that at each year, the decision-maker can select one of m activities. Since at time 0, the state of information is given by x_0, which is known, and each selected activity enters in the information state, there can be m^t different information states at time t.

5. JOINT DECISION MODELS WITHOUT TEMPORAL LINKAGE

In this section the formulation of Section 4 will be extended to the joint inspection and M&R decision problem. In terms of the different approaches to the joint problem, the formulation of this section differs from that of Section 3 in the fact that inspections are no longer required to take place prior to making an M&R decision, that is, there is no temporal linkage between inspections and M&R decisions. The only major limitation of the traditional MDP which still holds in this section is that of no uncertainty in the inspection output.

To allow for the possibility of an inspection at the beginning of every time period, we introduce the variable r_τ, where $r_\tau = 1$ if an inspection was performed at time τ, and 0 otherwise. The state of the information now includes the history of selected inspection activities and M&R activities, in addition to the states that are measured after every inspection. The state of information at t becomes:

$$I_t = \{\chi_0, a_0, r_1 * \chi_1, a_1, \ldots, a_{t-1}, r_t * \chi_t\},\ t = 1, \ldots, T$$
$$I_0 = \{\chi_0\}. \tag{23}$$

It can be seen that the term $r_t * \chi_t$ is equal to 0 if no inspection is performed, and to the true state of the facility if an inspection is performed. As in Section 4, it is assumed that χ_0 is known.

If the last inspection was performed at time τ, then χ_τ becomes the new starting point to use in eqn (21). The probabilistic distribution of χ_t, $t > \tau$, becomes:

$$\begin{aligned} p_t(\chi_t = j \mid I_t) &= p_t(\chi_t = j \mid \chi_0, a_0, \ldots, r_\tau * \chi_\tau = i, a_\tau, \ldots, a_{t-1}, r_t * \chi_t = 0) \\ &= (P_{a_\tau} * \ldots * P_{a_{t-1}})_{ij} \end{aligned} \tag{24}$$

For $t = \tau$, after the inspection has been performed, we have:

$$p_\tau(\chi_\tau = j \mid I_\tau) = p_\tau(\chi_\tau = j \mid \chi_0, a_0, \ldots, r_\tau * \chi_\tau = i) = 1 \text{ if } j = i, \quad 0 \text{ otherwise} \tag{25}$$

We can now formulate the joint inspection and M&R decision model where the decision-maker, at the beginning of every time period t, selects an activity a_t and decides whether to inspect in the next time period ($r_{t+1} = 1$) or not ($r_{t+1} = 0$). This model will be written in terms of the detailed costs introduced in Section 3. It takes the form:

$$J_T(I_T) = 0, \qquad \forall I_T,$$

and

$$\begin{aligned} J_t(I_t) = \min_{(a_t, r_{t+1})} \Bigg\{ & \alpha r_{t+1} cm + \sum_{i=1}^{n} p_t(\chi_t = i \mid I_t) * \Bigg(ca(\chi_t, a_t) + \alpha \sum_{j=1}^{n} p(\chi_{t+1} \\ & = j \mid \chi_t = i, a_t)(cu(\chi_{t+1}) + r_{t+1} J_{t+1}(I_{t+1}))\Bigg) \\ & + \alpha(1 - r_{t+1}) J_{t+1}(I_{t+1}) \Bigg\} \qquad \forall I_t,\ t = 0, \ldots, T-1, \end{aligned} \tag{26}$$

where:

I_t is given by (23),
$I_{t+1} = \{I_t, a_t, r_{t+1} * \chi_{t+1}\}$, and
$p_t(\chi_t = i \mid I_t)$ is given by either (24) or (25) depending on the value of r_t; that is, on whether or not an inspection was performed at the beginning of t.

If $r_{t+1} = 0$, then (26) is the same as (22), the formulation of the problem with no measurement. This can be verified by substituting $r_{t+1} = 0$ in the expression for I_{t+1}, in the right hand side of (26), to obtain $I_{t+1} = \{I_t, a_t, 0\}$. In this case, the decision at $t + 1$ will be made based on the probabilistic distribution of x_{t+1}, as given by (24).

If $r_{t+1} = 1$, then the state of information at $t + 1$ is given by $I_{t+1} = \{I_t, a_t, \chi_{t+1}\}$, which includes knowledge of the condition state x_{t+1}. Since this condition state is not known at t, there are up to n distinct states of the information at $t + 1$ in this case, one for each possible condition state. Therefore, the decision at t must be based on the probability mass function of x_{t+1}. At the beginning of year $t + 1$, once the inspection has taken place and according to (25), the state of the information will include the true condition state of the facility, on which the decision at $t + 1$ will be based.

From a computational perspective, program (26) is more expensive than (22), which is expected since we have added a second dimension to the decision process, namely the inspection decision. Given a particular M&R activity at time $t - 1$, there are n possible outcomes of the inspection process (if an inspection is performed), which lead to n different states of the information at time t. For that same M&R activity, if no inspection is performed, only one state of the information is created. Hence, there is a total of $m^*(1 + n)$ possible states of the information created at time t from each state of the information at time $t - 1$, where m is the number of possible M&R activities. Hence, the number of states to which the recursion of (26) has to be applied, at time t, is equal to $(m^*(1 + n))^t$. The number of alternatives that have to be evaluated at every state of the information is 2^*m.

Program (26) is clearly more expensive to solve than the joint inspection and M&R decision problem of (10), which forced a temporal linkage between inspections and M&R decisions, because of the larger number of states to which the Dynamic Programming recursion must be applied, and because of the need to compute the true state probabilities using (24). However, this may be a small price to pay since the problem solved is less restricted.

Several variations of this formulation have been developed to address the joint repair and inspection problem. Some of these are given in Rosenfield (1976) and Tijms and Duyn-Schouten (1985).

The only limitation of this formulation is that it assumes that inspections are error-free. A model formulation that relaxes this assumption is the Latent Markov Decision Process. This specific aspect of the problem is not pursued further in this paper; the interested reader is referred to Madanat (1993) and Madanat and Ben-Akiva (1993) for more details. In the following section, an empirical analysis is presented in which the formulations of the previous sections are compared.

6. EMPIRICAL COMPARISON

An empirical comparison of the minimum expected lifecycle loss, incurred each of the three models described in this paper, is presented in this section. The objective of this comparison is to quantify the lifecycle savings that can be achieved by jointly selecting M&R and inspection activities for realistic cases. Moreover, the variation of these savings with the level of uncertainty inherent in the forecast of facility condition is investigated. This is done by varying the standard deviation of the transition probability distribution and observing the effect of this variation on the minimum expected loss. Expected loss is the difference between the minimum expected cost achieved by a given model and that achieved by the least constrained model at perfect forecasting precision.

6.1. Data

The condition scale of a pavement section was described by the PCI (Pavement Condition Index, Shahin and Kohn, 1981) which has values in the range 0 to 100. This range was divided into 8 states, that is: $i = 0, 1, \ldots, 7$ (each state represents a range of 12.5 units on the PCI scale). Hypothetical transition probabilities were selected, for four

Table 1. Do-Nothing Transition Matrix

X(t)\X(t+1)	7	6	5	4	3	2	1	0
7		1						
6			1					
5				1				
4					1			
3						1		
2							1	
1								1
0								1

different activities: do nothing, routine maintenance, rehabilitation, and reconstruction. These transition matrices are shown in Tables 1–4, for the case when the standard deviation of the condition forecast error is zero. The conditional forecast error was assumed to follow a normal distribution. A truncated normal distribution was used for larger standard deviations to guarantee that the forecasted condition did not violate some realistic constraints; for example, in the case of do-nothing, no improvement in condition may occur. The standard deviation of the conditional forecast error was varied parametrically to the following: 0.0, 2.5, 5.0, 7.5, 10.0 and 12.5 PCI units. These distributions were then discretized, using standard principles of probability theory, in order to obtain the corresponding transition probabilities.

The costs associated with the different M&R activities were taken from Carnahan *et al.* (1987), and are shown in Table 5. As a proxy for user costs, a condition constraint (minimum allowable state) was used. Such a constraint is introduced by specifying a penalty function, which takes a very large positive value for states below the minimum allowable state, and a value of zero for states above or at the minimum. The minimum allowable state for the facility was set to $i = 3$ (i.e. the penalty has a value of infinity for states 0, 1 and 2 and a value of 0 elsewhere). The cost of inspection was assumed to be 0.65 \$/sqyd, which is an estimate of the cost of detailed mapping by human inspectors (Hudson, *et al.*, 1987). Detailed mapping is the closest technology to a perfectly precise measurement technology, which is what the three models presented all assumed.

The planning horizon (T) for the study was set to 5 years and the interest rate to 5%, which corresponds to a discount amount factor (α) of 0.9524. The initial condition state was set to $x_0 = 7$.

Table 2. Routine Maintenance Transition Matrix

X(t)\X(t+1)	7	6	5	4	3	2	1	0
7	1							
6		1						
5			1					
4				1				
3					1			
2						1		
1							1	
0								1

Table 3. Rehabilitation Transition Matrix

X(t)\X(t+1)	7	6	5	4	3	2	1	0
7	1							
6	1							
5	1							
4	1							
3		1						
2			1					
1				1				
0					1			

6.2. *Results*

The results of the empirical comparison are summarized in Fig. 1. As can be seen in the figure, increasing the uncertainty in the forecast of condition leads to an increase in the value of the minimum expected loss, irrespective of the decision model used. This is an intuitive result, which can be attributed to the nonlinearity of the minimum cost-to-go functions (Madanat, 1993). This observation demonstrates the economic benefits of improving the precision of the deterioration models used in Pavement Management.

The second observation from Fig. 1 is that, as expected, relaxing the restrictions placed on the timing of inspections leads to reduced expected lifecycle loss: the curve corresponding to the joint decision model without linkage lies below that of the joint decision model with linkage, which itself lies below that of the classical MDP model. The difference between corresponding points on the three curves represents the benefits of the more complex decision-making models.

It can also be observed that the maximum savings achieved by the joint decision models over the classical MDP occur when the standard deviation of forecasting is zero. The intuitive explanation for this is that when the model forecasts are error-free, it is optimal never to inspect, since inspections would not contribute to reducing expected lifecycle costs. Thus, the joint decision models prescribe zero inspection for the duration of the planning horizon, whereas the classical MDP is constrained to select an inspection at every year. As the uncertainty in the model forecast increases, the optimal number of inspections increases accordingly, until it approaches the maximum possible number of inspections (one for every year of the planning horizon), at which point the minimum expected costs approach those achieved by the classical MDP, as can be seen at the right

Table 4. Reconstruction Transition Matrix

X(t)\X(t+1)	7	6	5	4	3	2	1	0
7	1							
6	1							
5	1							
4	1							
3	1							
2	1							
1	1							
0	1							

Table 5. Costs of Different Activities

condition state	do-nothing	routine maintenance	rehabilitation	reconstruction
7	0	0.04	3.81	25.97
6	0	0.15	3.91	25.97
5	0	0.31	4.11	25.97
4	0	0.65	6.64	25.97
3	0	0.83	9.06	25.97
2	0	1.4	10.69	25.97
1	0	2	12.31	25.97
0	0	6.9	21.81	25.97

of Fig. 1. This is an intuitive result: for deterioration models having poor forecasting precision, it is optimal to rely more on inspections than on the models forecasts.

7. CONCLUSIONS

In this paper, various formulations for incorporating inspection decisions within pavement M&R decision models were presented. The models reviewed ranged from highly restrictive, computationally simple models to fully unrestricted, computationally expensive models. An empirical comparison of these different models was presented, and demonstrated that jointly optimizing inspection and M&R decisions can lead to substantial cost savings, over a simple M&R optimization with a predetermined inspection frequency. It can be expected that the benefits achieved by the joint optimization methods will be substantial when an entire network of highway pavements is considered.

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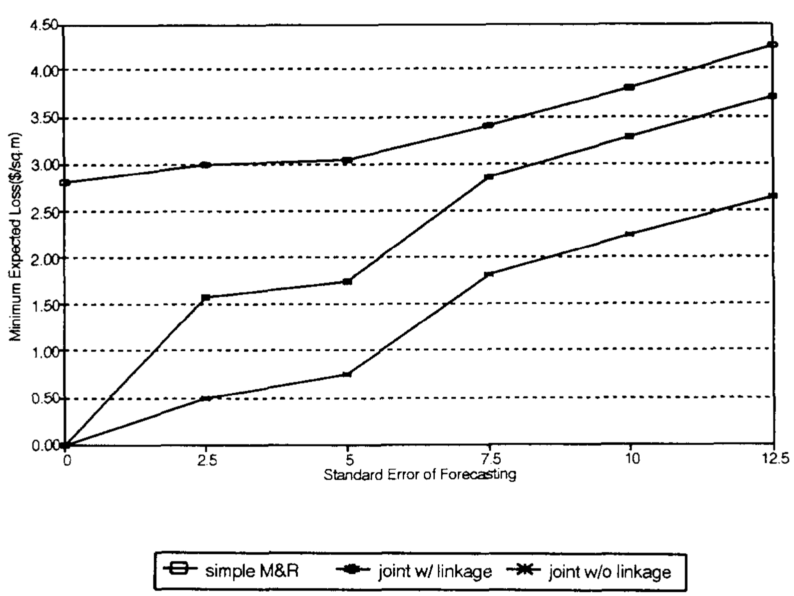

Fig. 1. Minimum expected lifecycle cost for different decision models.

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