Pavement Rehabilitation Project Ranking Approach Using Probabilistic Long-Term Performance Indicators

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A probabilistic performance-based approach for generating a priority ranking of pavement rehabilitation project candidates is presented. The deployed probabilistic approach uses a discrete-time Markovian model to predict pavement conditions at the network level. The expected future distress rating associated with a particular pavement project is determined as the mean of a compound uniform probability density function derived from the corresponding future state probabilities. The expected future distress ratings for a particular pavement project are then used to construct the corresponding performance curve. The generated performance curves form the basis for developing an effective mechanism for prioritizing potential rehabilitation projects. A priority ranking system is presented with two alternatives. The first priority ranking alternative requires a fixed analysis period wherein project candidates are evaluated by using three different long-term performance indicators derived from the corresponding performance curves. The deployed performance indicators include the area under the performance curve, the average distress rating, and the terminal distress rating. The second priority ranking alternative requires a fixed terminal distress rating for the pavement network under consideration. The time required for each project to reach the terminal distress rating is determined as the rehabilitation scheduling time, which is used to establish project priority ranking. Pavement rehabilitation project candidates are then scheduled according to their priority rankings and available budget. The developed probabilistic approach is demonstrated by considering a project sample from the two-lane highway system in Ohio.

Several pavement rehabilitation and management models were developed in the past two decades in an attempt to provide the best pavement rehabilitation plan at the network level. A major requirement for any potential pavement rehabilitation and management model is the ability to generate a long-term solution to the pavement management problem, recognized to be a very complex one (*1–6*). A long-term solution requires the incorporation of a pavement performance prediction model to estimate the future pavement conditions. Pavement performance prediction models used in pavement management are either deterministic or probabilistic in nature (*7–9*). Deterministic models are mainly empirical models generated from pavement testing and experience and used to predict future pavement conditions with certainty. Probabilistic models are developed assuming the outcomes of future pavement conditions to be associated with specified uncertainty levels. Both deterministic and probabilistic models have been extensively used in pavement rehabilitation and management (*1–3, 6, 10–12*).

The probabilistic model that has been widely used in predicting pavement conditions is the Markovian model with discrete-time chains. Several pavement management models have incorporated the Markovian model to predict future pavement conditions for the purpose of developing a long-term pavement rehabilitation plan. The effectiveness of the discrete-time Markovian model in predicting future pavement conditions has been recognized by several researchers (*1, 2, 13–16*). The Markovian model has been used to predict pavement conditions both in the absence and presence of pavement maintenance and rehabilitation (M&R) works. The predicted future pavement condition ratings can be used to construct the corresponding performance curves, which serve as invaluable tools in pavement design, rehabilitation, and management (*10, 12, 13, 17, 18*).

Pavement rehabilitation and management models have mostly defined the pavement network using pavement sections, segments, or projects. Pavement sections are typically selected smaller in length compared with segments. A pavement project is composed of a number of pavement sections or segments. The derivation of an optimum M&R plan at the network level requires dealing with a large number of pavement segments. This requirement combined with the large number of potential M&R options applicable to each segment has extremely complicated solving the pavement management problem even with the use of the most efficient optimization methods and the fastest available computers (*1–5*). Therefore, several researchers have used M&R variables to represent fractions (percentages) of pavements in the various deployed condition states or classes to be treated by the applicable M&R actions (*1, 2, 19–21*). This approach has greatly simplified the pavement management problem by avoiding the need to deal with individual pavement segments.

PROBLEM STATEMENT AND RESEARCH OBJECTIVES

The pavement management problem is typically formulated as a nonlinear optimization model. Generally, there are three major interrelated difficulties that faced most nonlinear pavement management models at the network level (*2–5, 19–21*). The first difficulty is the incorporation of an appropriate optimization method that can generate a global optimum solution. Most deployed optimization methods have their own drawbacks in terms of efficiency and convergence to optimal solutions and cannot be easily used and interpreted by

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Transportation Research Record: Journal of the Transportation Research Board, No. 2153, Transportation Research Board of the National Academies, Washington, D.C., 2010, pp. 3–12.
DOI: 10.3141/2153-01

practitioners. The second difficulty is related to the computer time required to solve the pavement management problem. Most used optimization methods are mainly search algorithms. Even a medium-size pavement management problem, using the fastest available personal computers, requires considerable time to solve. The third difficulty is the necessity to deal with a large number of pavement segments, time intervals, and M&R options, which would further complicate the optimization process and substantially affect computer time.

Roadway rehabilitation project ranking procedures have been used by several researchers (*22, 23*). Ranking procedures are generally faster and easier to apply, but are associated with some limitations. They would most likely result in nonoptimal allocation of resources. The effectiveness of a ranking approach in yielding an optimal solution can be investigated by comparing its results with those derived from an equivalent optimization model. An optimization model equivalent to the proposed ranking approach will be presented in this paper. Also, ranking procedures are not typically designed to incorporate a performance prediction model, which hinders their effectiveness in developing a long-term M&R program. The developed ranking approach incorporates a performance prediction model to generate a long-term pavement rehabilitation program. The discrete-time Markovian model is applied to predict the future pavement distress ratings used to construct the corresponding project performance curve. The generated performance curves are used to obtain certain long-term performance indicators, which form the basis for prioritization of potential rehabilitation projects. Potential rehabilitation projects are preliminarily selected depending on their present pavement distress ratings and the effectiveness of applicable rehabilitation strategies. Therefore, according to the developed ranking approach, prioritization of potential rehabilitation projects depends only on the deployed long-term performance indicators.

METHODOLOGY

The discrete-time Markovian model used to predict pavement distress ratings is presented for the purpose of generating the corresponding pavement performance curves. Long-term performance measures derived from the performance curves are presented for the purpose of developing an effective priority ranking system with two alternatives. The first priority ranking alternative requires a fixed analysis period and provides three long-term performance indicators, which are the area under the performance curve, the average distress rating, and the terminal distress rating. The second priority ranking alternative provides the rehabilitation scheduling time for each project according to a fixed terminal distress rating.

Discrete-Time Markovian Model

The main elements of the discrete-time Markovian model are the condition states, state probabilities, transition probabilities, and number of transitions (time intervals). A pavement system consists of a number of condition states equal to m. A condition state is assumed to contain pavements with similar distress ratings as defined using an appropriate pavement distress indicator. A transition matrix (P_r) is used to incorporate all applicable transition probabilities associated with the rth pavement system as presented in Equation 1. Each system is to be represented by a different transition matrix reflecting different loading conditions and pavement structures. The transition probabilities ($P^{(r)}_{i,i}$) provided along the matrix main diagonal represent the probabilities of pavements remaining in the same condition state after one transition. The transition probabilities ($P^{(r)}_{i,i+1}$) according to Equation 1 symbolize the deterioration rates from state i to state $i + 1$ in one time interval. Therefore, deterioration of pavements is assumed to take place only from a present state to the next worst state in one transition. This assumption is valid if the number of condition states (m) is reasonably large and the time interval is adequately small (*1, 2, 13, 16*). A matrix size of 10 states and a 1-year time interval (transition) would justify this assumption. The matrix entries below the main diagonal are all assumed to vanish in the absence of M&R works. Therefore, the transition matrix indicated by Equation 1 represents only the deterioration mechanism of a given pavement system with similar loading conditions and pavement structures. The sum of any row in the transition matrix must add up to one ($P^{(r)}_{i,i} + P^{(r)}_{i,i+1} = 1.0$) and ($P^{(r)}_{m,m} = 1.0$).

$$P_r = \begin{pmatrix} P^{(r)}_{1,1} & P^{(r)}_{1,2} & 0 & 0 & 0 & \cdots & 0 \\ 0 & P^{(r)}_{2,2} & P^{(r)}_{2,3} & 0 & 0 & \cdots & 0 \\ 0 & 0 & P^{(r)}_{3,3} & P^{(r)}_{3,4} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \cdots & P^{(r)}_{m-1,m-1} & & P^{(r)}_{m-1,m} \\ 0 & 0 & 0 & 0 & 0 & \cdots & P^{(r)}_{m,m} \end{pmatrix} \tag{1}$$

The basic discrete-time Markovian model is presented in Equation 2. The main objective of using the indicated Markovian model is to obtain the state probabilities after k transitions as the product of the initial state probability vector and the transition matrix raised to the power k. The pavement network contains a number of pavement systems equal to s. The rth pavement system is considered to include a number of pavement projects equal to p_r. All pavement projects in the same system are assumed to possess the same deterioration rates (transition matrix); however, they have different present distress conditions as indicated by the corresponding initial state probabilities. The initial state probability vector ($S^{(0)}_{j,r}$) associated with the jth project and rth system is used to predict the corresponding state probability vector ($S^{(k)}_{j,r}$) after k transitions. The initial state probability vector contains the state probabilities representing pavement proportions that exist in the various deployed states at any present time. The future pavement proportions are represented by the state probability vector ($S^{(k)}_{j,r}$) obtained from Equation 2 using a specified number of transitions (n). The sum of the state probabilities at anytime must add up to one.

$$S^{(k)}_{j,r} = S^{(0)}_{j,r} P^{(k)}_r \qquad k = 1, 2, \ldots, n; j = 1, 2, \ldots, p_r; r = 1, 2, \ldots, s \tag{2}$$

where

$$(S^{(0)}_{j,r}) = (S^{(0)}_{1,j,r}, S^{(0)}_{2,j,r}, S^{(0)}_{3,j,r}, \ldots, S^{(0)}_{i,j,r}, \ldots, S^{(0)}_{m,j,r})$$

$$(S^{(k)}_{j,r}) = (S^{(k)}_{1,j,r}, S^{(k)}_{2,j,r}, S^{(k)}_{3,j,r}, \ldots, S^{(k)}_{i,j,r}, \ldots, S^{(k)}_{m,j,r})$$

$$\sum_{i=1}^{m} S^{(k)}_{i,j,r} = 1.0$$

and

$$0.0 \le S^{(k)}_{i,j,r} \le 1.0$$

Compound Uniform Probability Density Function

The state probabilities estimated at the kth transition can be used to define a compound uniform probability density function that relates the state probabilities to the pavement distress indicator used to define the various deployed condition states. The state probability ($S^{(k)}_{i,j,r}$) associated with the ith state, jth project, and rth system is defined using the pavement distress rating (DR) as presented in Equation 3. A uniform probability density function is used because state probabilities are not explicit functions of the pavement distress rating; however, they are indirectly related. The pavement distress rating (DR) associated with the ith state probability is defined over a range of values as represented by the lower and upper distress ratings (LDR_i) and (UDR_i), respectively.

$$S^{(k)}_{j,r} = \begin{cases} S^{(k)}_{1,j,r}, & \text{LDR}_1 < \text{DR} \le \text{UDR}_1 \\ S^{(k)}_{(2,j,r)}, & \text{LDR}_2 < \text{DR} \le \text{UDR}_2 \\ \vdots & \vdots \\ S^{(k)}_{i,j,r}, & \text{LDR}_i < \text{DR} \le \text{UDR}_i \\ \vdots & \vdots \\ S^{(k)}_{m,j,r}, & \text{LDR}_m < \text{DR} \le \text{UDR}_m \end{cases} \tag{3}$$

Figure 1 shows a schematic presentation of the outlined compound uniform probability density function with equal band width. The band width is the difference between the state upper and lower distress ratings. The mean of each band width is called the state mean distress rating (B_i).

Prediction of Pavement Distress Rating

The future pavement distress rating associated with the jth project, rth system, and kth transition can be estimated as the expected value (mean) of the compound uniform probability density function indicated by Equation 3. The expected distress rating ($\text{EDR}^{(k)}_{j,r}$) is obtained as the product sum of multiplying the state mean distress ratings (B_i) by the corresponding future state probabilities ($S^{(k)}_{i,j,r}$) as presented in Equation 4. The state mean distress ratings (B_i) are constants representing the average distress ratings for pavements in the corresponding states. The state probabilities vary over time as determined from the Markovian model presented in Equation 2. The expected distress rating is determined for each transition number (k) over a specified number of transitions (n) representing the desired analysis period. The expected distress rating takes on its maximum value (B_1) when all pavements are in State 1 (best state) and undertakes its minimum value (B_m) when all pavements are in state m (worst state).

FIGURE 1 Compound uniform probability density function.

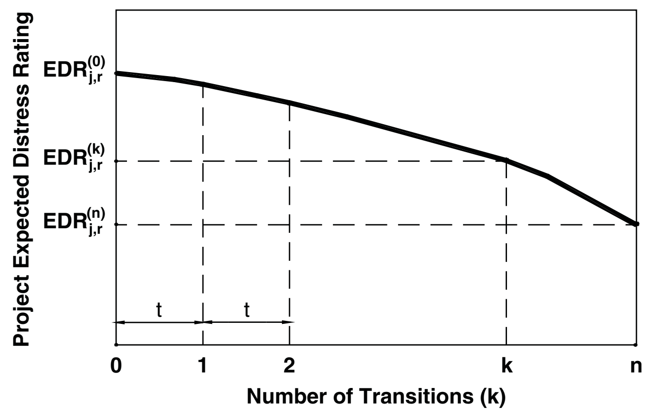

FIGURE 2 Project pavement performance curve for specified number of transitions (n).

$$\text{EDR}^{(k)}_{j,r} = \sum_{i=1}^{m} B_i S^{(k)}_{i,j,r} \qquad k = 0, 1, 2, \ldots, n \tag{4}$$

where

$$B_i = \frac{(\text{LDR}_i + \text{UDR}_i)}{2}$$

$$B_m \le \text{EDR}^{(k)}_{j,r} \le B_1$$

The expected distress ratings associated with the jth project in the rth system as obtained from Equation 4 can be used to construct the corresponding performance curve as shown in Figure 2. The depicted performance curve is simply a plot of the expected distress rating ($\text{EDR}^{(k)}_{j,r}$) versus the corresponding transition number (k). Performance curves generated to represent all potential pavement rehabilitation projects will be used to establish priority ranking systems as presented in the next section.

LONG-TERM PERFORMANCE-BASED PROJECT RANKING SYSTEM

Presented in this section is a probabilistic long-term performance-based priority ranking system based on the generated project performance curves. The priority ranking system provides the pavement engineer with two alternatives. The first priority ranking alternative requires specifying a fixed analysis period consisting of n transitions. The second priority ranking alternative requires specifying a fixed terminal distress rating for all projects in a given pavement network. Both alternatives take into account the long-term performance of pavement rehabilitation project candidates.

First Priority Ranking Alternative with Fixed Analysis Period

The first priority ranking alternative directly applies the expected pavement distress ratings ($\text{EDR}^{(k)}_{j,r}$) to derive three long-term performance indicators: the performance curve area, average expected distress rating, and terminal expected distress rating. All three performance indicators are to be estimated over an analysis period of n transitions. Each transition is typically considered to represent a discrete-time interval of 1 year or 2 years. The three performance indicators are defined in the following subsections.

Performance Curve Area Method

The area falling under the performance curve is recognized to provide a direct measure of the pavement relative structural capacity (*10, 17, 24*). Therefore, the performance curve area provides a direct measure of the long-term performance of pavement projects. The area under a particular project performance curve can be directly estimated from the corresponding expected distress ratings using the trapezoidal rule. The total performance area under the generated curve segment consists of trapezoidal strips, with each strip defined using two consecutive distress ratings separated by a time interval (t) as shown in Figure 2. The total performance area ($PA_{j,r}^{(T)}$) associated with the jth project in the rth system over a fixed analysis period of T years is determined as defined in Equation 5. The number of transitions (n) is equal to the analysis period (T) divided by the time interval (t). The number of transitions equals the analysis period if a 1-year time interval is used.

$$PA_{j,r}^{(T)} = \frac{t}{2}\left(EDR_{j,r}^{(0)} + EDR_{j,r}^{(n)} + 2 \times \sum_{k=1}^{n-1} EDR_{j,r}^{(k)}\right) \tag{5}$$

Optimal selection of rehabilitation projects, to be shown later, can be achieved if priority rating (PR_{PA}) is defined as the multiplication product of the project surface area ($SA_{j,r}$) and total performance area ($PA_{j,r}^{(T)}$). Priority listing of rehabilitation project candidates can be established such that projects with lower PR_{PA} will be given priority over ones with higher PR_{PA}.

Average Expected Distress Rating Method

The average expected distress rating method provides another reliable long-term performance indicator for evaluating potential pavement rehabilitation projects. The expected distress ratings obtained over n transitions are averaged to yield the average expected distress rating ($\overline{EDR}_{j,r}^{(T)}$) for the jth project in the rth system as indicated by Equation 6. Pavement rehabilitation project candidates can be assigned priority ratings ($PR_{\overline{EDR}}$), determined as the product of the project surface area and average expected distress rating. A project with lower $PR_{\overline{EDR}}$ will have priority over a project with higher $PR_{\overline{EDR}}$.

$$\overline{EDR}_{j,r}^{(T)} = \frac{\sum_{k=0}^{n} EDR_{j,r}^{(k)}}{n+1} \tag{6}$$

The average expected distress rating method is directly related to the performance curve area method. Sample results presented later will show that the two methods yield similar results.

Terminal Expected Distress Rating Method

The terminal performance of pavements has been used in the AASHTO design method of pavements as part of the serviceability concept (*17*). The terminal expected distress rating ($EDR_{j,r}^{(T)}$) associated with the jth project is simply the expected distress rating determined at the nth transition ($EDR_{j,r}^{(n)}$) as presented in Equation 7. Pavement rehabilitation project candidates can similarly be assigned priority ratings set equal to the project surface area multiplied by the terminal distress rating with priority given to projects with lower priority ratings.

$$EDR_{j,r}^{(T)} = EDR_{j,r}^{(n)} \tag{7}$$

The terminal expected distress rating method may not be equivalent to the other two methods. The performance curve area method and the average expected distress rating method provide more reliable long-term measures of pavement performance because they take into consideration the performance at each transition.

Second Priority Ranking Alternative with Fixed Terminal Distress Rating

The second priority ranking alternative is based on specifying a fixed terminal distress rating (TDR_N) for all projects in the same pavement network. The performance curves associated with all potential pavement rehabilitation projects are constructed using an adequate number of data points. The last data point must have a corresponding expected distress rating that is less than the fixed network terminal distress rating. The project rehabilitation scheduling time ($T_{j,r}$) is then determined from the corresponding performance curve such that the project expected distress rating ($EDR_{j,r}^{(T_{j,r})}$) at time $T_{j,r}$ is equal to the fixed terminal distress rating (TDR_N) as stated by Equation 8. Figure 3 shows a sample of project performance curves, along with their anticipated rehabilitation scheduling times. A priority rating (PR_T) is defined as the product of project surface area and rehabilitation scheduling time. Projects with lower PR_T will have priority over ones with higher PR_T.

$$EDR_{j,r}^{(T_{j,r})} = TDR_N \tag{8}$$

The sample projects shown in Figure 3 have different initial expected distress ratings ($EDR_{j,r}^{(0)}$) and different rehabilitation scheduling times ($T_{j,r}$). The project priorities assigned according to the rehabilitation scheduling times will be different from the priorities established using the initial expected distress ratings, assuming projects with lower $EDR_{j,r}^{(0)}$ are given higher priority. This indicates the

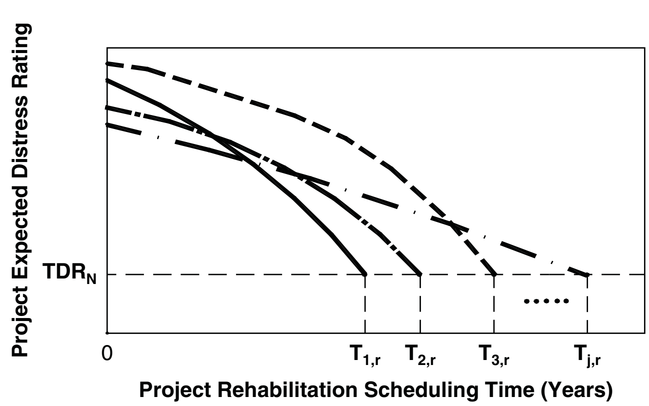

FIGURE 3 Project scheduling mechanism based on fixed network terminal distress rating (TDR_N).

significance of using the presented long-term performance indicators in developing an effective priority ranking system.

Establishing a Feasible Rehabilitation Project Schedule

Once pavement rehabilitation project candidates are assigned priority rankings using either one of the presented long-term performance indicators, the network budget available for the major rehabilitation program ($B_N^{(k)}$) during the kth transition is used to establish a feasible rehabilitation project schedule. The network budget ($B_N^{(k)}$) is the budget allocated for a specified rehabilitation cycle, typically 1 year or 2 years. Projects not selected in the first cycle according to Equation 9 can be assigned to the second cycle, and so on. The kth transition project rehabilitation cost ($\mathrm{PRC}_{j,r,l}^{(k)}$) associated with the jth project in the rth system using the lth rehabilitation strategy needs to be estimated. Estimation of project rehabilitation cost requires first the identification of the appropriate rehabilitation strategy, typically determined on the basis of the initial (present) pavement distress condition. Typical rehabilitation strategies, such as plain overlay, skin patch (milling and overlay), and reconstruction can be recommended according to the project initial distress rating ($\mathrm{EDR}_{j,r}^{(0)}$).

$$\max \sum_{r=1}^{s} \sum_{j=1}^{P_r} \mathrm{PRC}_{j,r,1}^{(k)} \leq B_N^{(k)} \qquad k = 1, 2, \ldots, n \tag{9}$$

The budget constraint presented in Equation 9 needs to be maximized while projects are selected according to their priority rankings. However, if the money left is inadequate to select the project next in the priority list, then a search is made among all remaining projects to select the ones with rehabilitation costs less than or equal to the remaining money, and the project highest in priority ranking is selected. This process is continued until the remaining money is less than the minimum project rehabilitation cost remaining in the list.

Identification of Pavement Rehabilitation Project Candidates

The identification of pavement rehabilitation project candidates is recommended before the presented long-term performance indicators are used to prioritize rehabilitation projects. It requires selecting potential rehabilitation strategies in relation to the initial (present) expected distress ratings ($\mathrm{EDR}_{j,r}^{(0)}$). Also, it requires establishing a mechanism to evaluate the effectiveness of selected rehabilitation strategies so that potential rehabilitation projects can be identified. Potential rehabilitation strategies are mainly plain overlay, skin patch, and reconstruction designed as appropriate for the pavement system (or individual projects) under consideration. The rehabilitation effectiveness ($\mathrm{RE}_{j,r,l}$) as applied to the jth project in the rth system is defined as the ratio of the cost rate ($\mathrm{CR}_{j,r,l}$) associated with the lth rehabilitation strategy to the change in the project expected distress rating as indicated by Equation 10. The change in the project expected distress rating ($\Delta\mathrm{EDR}_{j,r,l}$) is defined as the new expected distress rating after rehabilitation ($\mathrm{NEDR}_{j,r,l}$) minus the initial distress rating ($\mathrm{EDR}_{j,r}^{(0)}$) just before rehabilitation.

$$\mathrm{RE}_{j,r,l} = \frac{\mathrm{CR}_{j,r,l}}{\Delta\mathrm{EDR}_{j,r,l}} = \frac{\mathrm{CR}_{j,r,l}}{\mathrm{NEDR}_{j,r,1} - \mathrm{EDR}_{j,r}^{(0)}} \tag{10}$$

Therefore, the rehabilitation effectiveness ratio (RE) is basically a measure of rehabilitation cost to the corresponding improvement expected from applying a particular rehabilitation strategy. Network projects identified as rehabilitation project candidates are those associated with the lowest rehabilitation effectiveness ratios. This step simply helps shorten the list of potential rehabilitation projects for prioritization purposes using the presented long-term performance indicators.

DISCRETE-TIME MARKOVIAN MODEL REQUIREMENTS

The main components associated with the discrete-time Markovian model are the condition states, initial state probabilities, transition probabilities, length of transition (time interval), and number of transitions. It is recommended that 10 condition states and a 1-year time interval be used in the presented model for prioritization of pavement rehabilitation projects. It is also recommended that the maximum length of the analysis period be 5 years (five transitions), a requirement that guarantees the transition probabilities remain unchanged overtime (*2, 9, 13*). However, a longer analysis period can be used, provided that a revised transition matrix is deployed for the transitions following the first five transitions. The state and transition probabilities are to be estimated based on field assessment of pavement distress. Pavement sections with 50-m lane length are typically chosen for this purpose and surveyed for pavement defects. Using Equation 11, a distress rating (DR) can be assigned to each pavement section on the basis of the defect rating (d_i) and the corresponding assigned weight (w_i) to yield a section rating on a scale of 100 points (*1*).

$$\mathrm{DR} = 100 - \sum_i w_i d_i \tag{11}$$

A pavement project is composed of a number of pavement sections to be surveyed for pavement defects and assigned distress ratings. The initial state probabilities associated with a particular project can be directly estimated according to the number of pavement sections in the 10 deployed states as determined from conducting one cycle of distress survey. The 10 deployed states are to be defined as presented earlier using ranges of pavement distress ratings. Equation 12 provides estimates of the initial state probabilities ($S_{i,j,r}^{(0)}$) based on the ratio of the number of sections ($\mathrm{NS}_{i,j,r}^{(0)}$) belonging to state i to the total number of sections in the 10 states.

$$S_{i,j,r}^{(0)} = \frac{\mathrm{NS}_{i,j,r}^{(0)}}{\sum_{i=1}^{10} \mathrm{NS}_{i,j,r}^{(0)}} \qquad 1, 2, \ldots, p_r; r = 1, 2, \ldots, s \tag{12}$$

The transition probabilities associated with the rth pavement system can be estimated as indicated by Equation 13 using the basic definition of transition probabilities (*1, 2*). It is assumed that all projects belonging to the same pavement system have the same transition probabilities. It is required to conduct two cycles of distress survey separated by one time interval (transition) to obtain estimates of the transition probabilities. The number of pavement sections ($\mathrm{NS}_i^{(1,r)}$) that exist in state i in the first cycle is compared with the number of sections ($\mathrm{NS}_i^{(2,r)}$) that exist in the second cycle in the same state to obtain an estimate of the corresponding transition probability, as defined in Equation 13.

$$P_{i,i+1}^{(r)} = \frac{\mathrm{NS}_i^{(1,r)} - \mathrm{NS}_i^{(2,r)}}{\mathrm{NS}_i^{(1,r)}} \qquad i = 1, 2, \ldots, m$$

$$P_{i,1}^{(r)} = 1 - P_{i,i+1}^{(r)} \tag{13}$$

Alternatively, transition probabilities can be estimated using the method of least squares for residual minimization (*9, 13*).

SAMPLE PRESENTATION

The presented probabilistic performance-based approach for priority ranking of pavement rehabilitation project candidates is illustrated using a pavement network of three pavement systems. The selected sample pavement network belongs to the two-lane highway system in Ohio. The average daily traffic (ADT) is used to classify the three pavement systems as follows: System 1 (ADT < 4,000), System 2 (4,000 ≤ ADT ≤ 8,000), System 3 (ADT > 8,000). A sample of 50 segments (projects) is selected from the state two-lane highway system to be considered for priority ranking using the presented long-term performance indicators. Three major rehabilitation strategies are applied according to the initial (present) distress ratings $\mathrm{EDR}_{j,r}^{(0)}$ associated with the selected projects. They include plain overlay, skin patch, and reconstruction to be applied as follows: plain overlay ($40 < \mathrm{EDR}_{j,r}^{(0)} \le 50$), skin patch ($30 < \mathrm{EDR}_{j,r}^{(0)} \le 40$), and reconstruction ($\mathrm{EDR}_{j,r}^{(0)} \le 30$). Reconstruction mainly includes complete removal of existing asphalt surface, placement of leveling granular base, and placement of new asphalt surface. The cost rates ($\mathrm{CR}_{j,r,l}$) associated with plain overlay, skin patch, and reconstruction are estimated to be \$12, \$17, and \$26 per square meter for System 1; \$14, \$20, and \$31 per square meter for System 2; and \$16, \$23, and \$36 per square meter for System 3, respectively.

A distinct transition matrix with 10 states is applied to represent the deterioration mechanism of each pavement system using the form presented in Equation 1. The 10 states are defined using the distress rating (DR), based on a scale of 100 points. Each state is represented by an equal 10-point distress rating range with States 1 and 10 having the ranges 90–100 and 0–10, respectively, denoting the best and worst states. Pavement sections with 50-m lane length were surveyed for defects to assign a distress rating (DR) for each section using Equation 11. Table 1 provides the initial state probabilities estimated using Equation 12. The transition probabilities $P_{1,2}$ through $P_{9,10}$ are estimated for existing pavements to be for System 1 0.23, 0.30, 0.37, 0.41, 0.46, 0.50, 0.56, 0.59, and 0.62; for System 2 0.19, 0.25, 0.31, 0.34, 0.38, 0.42, 0.47, 0.49, and 0.52; and for System 3 0.16, 0.21, 0.26, 0.28, 0.32, 0.35, 0.39, 0.41, and 0.43. Projects in the *r*th system are represented by the same transition matrix. Equation 4 is then used to obtain the expected distress rating ($\mathrm{EDR}_{j,r}^{(k)}$) for a specified number of transitions.

Table 2 provides a priority listing of 18 projects based on the rehabilitation effectiveness ratio ($\mathrm{RE}_{j,r,l}$) defined in Equation 10. Only 18 projects out of the 50 selected ones have resulted in a present expected distress rating that qualifies for major rehabilitation ($\mathrm{EDR}_{j,r}^{(0)} \le 50$). The cost rate ($\mathrm{CR}_{j,r,l}$), the present expected distress rating ($\mathrm{EDR}_{j,r}^{(0)}$), and the new expected distress rating ($\mathrm{NEDR}_{j,r,l}$) are used to estimate the rehabilitation effectiveness ratio. The new expected distress ratings after rehabilitation are assumed to be 75, 85, and 95 for plain overlay, skin patch, and reconstruction, respectively. These values can be estimated from experience or using the strength associated with rehabilitated pavements, as will be outlined later. The value of 95 is the maximum value that can be predicted using Equation 4. The initial (present) distress rating ($\mathrm{EDR}_{j,r}^{(0)}$) is estimated from Equation 4 using the initial state probabilities. Table 2 shows the 18 projects listed in an increasing order with respect to the RE values as projects with lower RE values have priority over ones with higher RE values. The first-year project rehabilitation cost ($\mathrm{PRC}_{j,r,l}^{(1)}$) is determined as the product of surface area ($\mathrm{SA}_{j,r}$) and cost rate. The project surface

TABLE 1 Initial State Probabilities Associated with Selected Highway Project Sample

County	Route	r	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
Allen (ALL)	309R	2	.02	.05	.07	.06	.11	.22	.14	.13	.09	.11
Butler (BUT)	747R	3	.00	.02	.05	.06	.08	.07	.12	.16	.20	.24
Columbiana (COL)	164R	1	.01	.01	.02	.01	.12	.13	.07	.21	.14	.28
COL	014R	2	.07	.04	.03	.12	.14	.15	.16	.12	.08	.09
COL	062R	3	.05	.04	.09	.10	.22	.18	.14	.06	.07	.05
Defiance (DEF)	024R	2	.09	.14	.08	.04	.06	.08	.11	.14	.11	.15
Delaware (DEL)	003R	3	.04	.08	.11	.10	.08	.17	.05	.23	.11	.03
Erie (ERI)	006R	2	.00	.02	.03	.09	.04	.12	.13	.17	.22	.18
Gallia (GAL)	035R	3	.04	.03	.11	.10	.08	.23	.05	.17	.11	.08
Hardin (HAR)	068R	1	.00	.03	.01	.08	.05	.11	.13	.15	.21	.23
Henry (HEN)	024R	2	.00	.00	.05	.04	.11	.09	.12	.17	.19	.23
Highland (HIG)	050R	1	.07	.10	.08	.09	.07	.17	.13	.12	.09	.08
Logan (LOG)	068R	2	.06	.09	.09	.12	.09	.13	.11	.16	.12	.03
Lorain (LOR)	082R	3	.00	.02	.05	.07	.08	.07	.18	.22	.15	.16
Mahoning (MAH)	014R	1	.03	.05	.07	.13	.09	.11	.17	.12	.13	.10
MAH	045R	1	.00	.05	.08	.11	.08	.07	.15	.22	.15	.09
Marion (MAR)	004R	2	.02	.05	.09	.12	.13	.18	.00	.15	.14	.12
Wayne (WAY)	030R	3	.02	.01	.09	.13	.08	.15	.08	.11	.16	.17

TABLE 2 Projects Prioritized by Rehabilitation Effectiveness Ratio

County	Route	BLOG	ELOG	ADT	r	$CR_{j,r,l}$	$EDR^{(0)}_{j,r}$	$RE_{j,r,l}$	$PRC^{(1)}_{j,r,l}$($)
MAH	014R	0	677	2,300	1	12	42.1	0.364	56,868
MAH	045R	0	654	3,881	1	17	38.4	0.365	77,826
ERI	006R	2,235	2,700	5,920	2	20	30.4	0.366	65,100
COL	164R	1,799	2,301	3,561	1	26	27.6	0.386	91,364
HAR	068R	1,066	1,923	3,688	1	26	28.8	0.393	155,974
ALL	309R	1,721	2,506	5,393	2	14	41.3	0.415	76,930
MAR	004R	363	838	4,155	2	14	42.4	0.429	46,550
LOR	082R	0	768	8,073	3	23	32.4	0.437	123,648
HIG	050R	571	1,243	3,772	1	12	47.8	0.441	56,448
COL	014R	1,174	1,688	6,856	2	14	44.8	0.464	50,372
HEN	024R	1,091	1,947	7,370	2	31	28.9	0.469	185,752
DEF	024R	1,257	1,933	7,504	2	14	45.7	0.478	66,248
WAY	030R	1,255	1,681	19,770	3	23	37.9	0.488	68,586
GAL	035R	659	1,314	10,790	3	16	43.9	0.514	73,360
LOG	068R	204	629	5,490	2	14	48.6	0.530	41,650
BUT	747R	237	870	8,293	3	36	29.3	0.548	107,100
DEL	003R	129	721	8,802	3	16	46.7	0.565	66,304
COL	062R	486	1,002	10,830	3	16	49.3	0.622	57,792

NOTE: BLOG = project beginning station (meters) and ELOG = project ending station (meters).

area equals the 7-m highway width multiplied by the difference in the ELOG and BLOG. The BLOG and ELOG represent the project beginning and ending stations in meters. The generated priority listing is not directly related to the present expected distress rating.

Priority Ranking Using Long-Term Performance Indicators

Table 3 provides two project priority listings generated using the priority ratings PR_{PA} and $PR_{\overline{EDR}}$ determined equal to the performance curve area ($PA^{(T)}_{j,r}$) and average expected distress rating ($\overline{EDR}^{(T)}_{j,r}$) multiplied by the surface area ($SA_{j,r}$), respectively. The deployed fixed analysis period (T) is 5 years with 1-year time interval (t), thus requiring five transitions (n). The project performance area and average expected distress rating are estimated from Equations 5 and 6, respectively, using the future expected distress ratings obtained from Equation 4. The projects are listed in increasing order with respect to the two priority ratings, as projects with lower ratings are given priority over ones with higher ratings. Table 3 shows that both priority ratings have yielded the

TABLE 3 Projects Prioritized by Performance Curve Area and Average Expected Distress Rating

County	Route	$CR_{j,r,l}$	$EDR^{(0)}_{j,r}$	$PA^{(T)}_{j,r}$	$\overline{EDR}^{(T)}_{j,r}$	PR_{PA}	$PR_{\overline{EDR}}$	$PRC^{(1)}_{j,r,l}$($)
COL	164R	26	27.6	101.2	20.4	356	72	91,364
ERI	006R	20	30.4	112.8	22.7	367	74	65,100
WAY	030R	23	37.9	154.8	31.0	462	92	68,586
BUT	747R	36	29.3	115.6	23.2	512	103	107,100
MAR	004R	14	42.4	171.2	34.3	569	114	46,550
HAR	068R	26	28.8	96.7	19.5	580	117	155,974
LOG	068R	14	48.6	198.4	39.8	590	118	41,650
HEN	024R	31	28.9	107.2	21.6	642	129	185,752
COL	014R	14	44.8	180.3	36.1	649	130	50,372
MAH	045R	17	38.4	143.8	28.9	658	132	77,826
LOR	082R	23	32.4	126.6	25.4	681	137	123,648
COL	062R	16	49.3	208.4	41.7	753	151	57,792
MAH	014R	12	42.1	160.2	32.2	759	153	56,868
DEL	003R	16	46.7	195.2	39.1	809	162	66,304
GAL	035R	16	43.9	182.8	36.6	838	168	73,360
HIG	050R	12	47.8	188.6	37.8	887	178	56,448
ALL	309R	14	41.3	162.8	32.6	895	179	76,930
DEF	024R	14	45.7	191.1	38.3	904	181	66,248

same priority listings, indicating that the two deployed long-term performance indicators are compatible. However, the priority listings that can be generated using merely the performance curve area, and average expected distress rating would be different from those provided in Table 3. This indicates the effectiveness of the two deployed priority ratings in yielding project listings that are not dependent on a "worst first" selection criterion.

Table 4 provides similar results from the second priority alternative with fixed terminal distress rating. The projects are ranked using the priority rating (PR_T), determined as the product of rehabilitation scheduling time ($T_{j,r}$) and surface area. The fixed terminal distress rating is assumed to be 30 in this sample presentation. The performance curve associated with each project has been constructed using sufficient data points such that the last point has a distress rating value less than 30. A best-fit mathematical model is obtained to represent the constructed performance curve. The mathematical model is then used to estimate the time corresponding to a distress rating of 30, as required by Equation 8. The derived mathematical models are mostly polynomials with second degree and are associated with almost 1.0 coefficient of determination (R-square).

Table 4 provides a project priority listing in increasing order for the priority rating (PR_T), as projects with lower ratings have priority over those with higher ratings. The generated priority listing is different from the listing that would be obtained using only the rehabilitation scheduling time. The negative rehabilitation scheduling times are obtained for projects associated with a terminal expected distress rating less than 30, indicating that their rehabilitation is overdue. The absolute value of the negative rehabilitation scheduling time provided in Table 4 indicates the extent in years by which a rehabilitation project is overdue. The rehabilitation scheduling time can serve another purpose by indicating the exact time for scheduling project rehabilitation. It also allows establishing a long-term annual fund requirement plan, which is crucial for budgeting purposes. The three priority listings provided in Tables 3 and 4 are not directly related to the present distress ratings. This means that priority can be given to projects with higher present distress ratings compared with ones with lower present distress ratings as outlined earlier in relation to Figure 3. Therefore, this demonstrates again the effectiveness of the deployed long-term performance indicators in generating a priority listing that is not necessarily based on a "worst first" selection criterion.

Pavement rehabilitation project candidates can now be selected from any of the three presented priority listings according to the available budget, as shown in Tables 3 and 4. Alternatively, candidates can be first selected from the first priority listing (Table 2) according to available budget, and selected projects are then assigned new priority ratings according to the estimated long-term performance indicators. The second approach is recommended because it takes into consideration both the cost-effectiveness of applicable rehabilitation strategies and the long-term performance of existing pavement structures.

Optimal Selection of Rehabilitation Projects

Optimal selection of rehabilitation projects at the network level can be formulated as an optimization model using constrained binary integer programming as indicated by Equation 14. In binary programming, each binary variable ($X^{(k)}_{j,r,l}$) can take on the value of 0 or 1, representing the rejection or selection of the corresponding rehabilitation project. The objective function (Z) is formulated to maximize the overall pavement condition at the network level over an analysis period of n transitions. The overall pavement condition is defined as product sum of the project surface area ($SA_{j,r}$) multiplied by an appropriate long-term performance indicator ($LPI^{(n)}_{j,r,l}$). The long-term performance indicator can be represented by the performance curve area ($PAI^{(n)}_{j,r,l}$) or average expected distress rating ($\overline{EDR}^{(n)}_{j,r,l}$). These sample long-term performance indicators are to be derived from the Markov model using transition matrices that represent the deterioration mechanism of rehabilitated pavement projects. Therefore, the transition matrix can be different for the jth project in the rth system to be treated by the lth rehabilitation strategy.

TABLE 4 Projects Prioritized by Rehabilitation Scheduling Time

County	Route	$CR_{j,r,l}$	$EDR^{(0)}_{j,r}$	$T_{j,r}$(year)	PR_T	$PRC^{(1)}_{j,r,l}$($)
HAR	068R	26	28.8	–0.38	–2.28	155,974
HEN	024R	31	28.9	–0.24	–1.44	185,752
BUT	747R	36	29.3	–0.20	–0.89	107,100
COL	164R	26	27.6	–0.21	–0.74	91,364
ERI	006R	20	30.4	0.14	0.46	65,100
LOR	082R	23	32.4	0.54	2.90	123,648
WAY	030R	23	37.9	2.72	8.11	68,586
MAH	045R	17	38.4	2.04	9.34	77,826
MAR	004R	14	42.4	3.88	12.90	46,550
MAH	014R	12	42.1	2.98	14.12	56,868
COL	014R	14	44.8	4.34	15.62	50,372
LOG	068R	14	48.6	5.57	16.57	41,650
ALL	309R	14	41.3	3.20	17.58	76,930
HIG	050R	12	47.8	4.61	21.68	56,448
GAL	035R	16	43.9	4.92	22.56	73,360
COL	062R	16	49.3	6.53	23.59	57,792
DEL	003R	16	46.7	5.91	24.49	66,304
DEF	024R	14	45.7	5.71	27.02	66,248

$$\text{maximize } Z = \sum_{k=1}^{n}\sum_{l=1}^{q}\sum_{r=1}^{s}\sum_{j=1}^{P_r}\left(SA_{j,r}\right)\left(\text{LPI}_{j,r,l}^{(n)}\right)\left(X_{j,r,l}^{(k)}\right) \tag{14}$$

subject to

$$\sum_{l=1}^{q}\sum_{r=1}^{s}\sum_{j=1}^{P_r}\left(\text{PRC}_{j,r,l}^{(k)}\right)\left(X_{j,r,l}^{k} \leq B_N^{(k)}\right) \qquad k = 1, 2, \ldots, n$$

$$\sum_{k=1}^{n}\sum_{l=1}^{q} X_{j,r,l}^{(k)} \leq 1 \qquad j = 1, 2, \ldots, p_r;\ r = 1, 2, \ldots, s$$

$$X_{j,r,l}^{(k)} = 0 \text{ or } 1$$

The first constraint set associated with Equation 14 enforces the total rehabilitation cost associated with the kth transition to be less than or equal to the corresponding network budget ($B_N^{(k)}$). The second constraint set requires that each project can only be treated once during the analysis period using one optimal rehabilitation strategy. The binary integer model can be solved using the branch and bound method. However, the computer time required to solve a problem substantially increases with the increase in problem size. The binary integer model has been applied to the sample project list provided in Table 3 using the average distress rating ($\overline{\text{EDR}}_{j,r,l}^{(T)}$) as the long-term performance indicator. The optimal project list generated from the optimization approach is similar to the list obtained using the priority rating ($\text{PR}_{\overline{\text{EDR}}}$). The binary integer model in this case has been minimized to yield the optimal project list because the deployed transition matrices represent the performance of existing pavements and not the rehabilitated ones.

Managerial Implications

The presented ranking approach aims to provide managerial staff with an effective tool to assist in scheduling and budgeting pavement rehabilitation projects at the network level. Most highway agencies maintain a separate budgeting program for pavement rehabilitation, and they seek to develop long-term rehabilitation programs based on a solid engineering approach. The presented ranking approach provides highway agencies with several long-term rehabilitation program alternatives that are typically associated with different cost estimates. These cost estimates can help managerial staff in developing long-term budgeting plans to meet the cost estimates associated with the derived long-term rehabilitation programs. It can also help managerial staff using the corresponding long-term cost estimates in fund raising campaigns. The presented ranking approach can be applied separately for individual pavement systems if managerial staff decide to allocate separate budgets for different roadway systems. It is not too unusual for highway agencies to allocate rehabilitation funding according to the importance of the different roadway systems serving the public. The presented ranking approach can be applied regularly to obtain revised long-term rehabilitation programs based on newly estimated pavement performance data.

SUMMARY AND CONCLUSIONS

A probabilistic long-term performance-based approach has been presented for priority ranking of pavement rehabilitation project candidates at the network level. The probabilistic approach applies the discrete-time Markovian model to predict pavement distress ratings, which are used to construct the corresponding project performance curves. Project performance curves (or corresponding data points) are used to obtain four reliable long-term performance indicators, namely, the area under the performance curve, the average expected distress rating, the terminal expected distress rating, and the rehabilitation scheduling time. The presented sample results have demonstrated the effectiveness of the deployed long-term performance indicators in generating reliable priority rankings of potential rehabilitation projects. Preliminary investigation of the equivalent optimization model has yielded optimal results similar to those derived from the presented ranking approach. In addition, the presented sample results have shown that the deployed long-term performance indicators are very much compatible, which means that a highway agency interested in using the presented approach can select only one appropriate indicator. However, the rehabilitation scheduling time offers two other advantages: indicating the exact project rehabilitation time and providing a long-term fund requirement plan.

The major requirement for using the presented priority ranking approach is the project performance curves, which can be obtained using different approaches than the one presented in this paper. However, all deterministic and probabilistic approaches used to generate performance curves require conducting periodic pavement distress assessment. Many highway agencies have developed and maintained pavement distress assessment programs, which resulted in massive historical databases. These databases can be retrieved for relevant pavement distress records for the purpose of generating the needed performance curves. The effectiveness of any pavement management approach greatly depends on the reliability of the deployed assessment program in yielding accurate representation of actual pavement distress conditions. The presented project priority ranking approach has greatly simplified the very complex pavement management problem in the case of major rehabilitation while still applying sound engineering principles.

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The Pavement Management Systems Committee peer-reviewed this paper.