International Journal of Architecture, Engineering and Construction
Vol 3, No 3, September 2014, 153-167

Comparison of Three Preventive Maintenance Models to Determine Optimal Intervention Strategies for Transportation Infrastructures in Alpine Regions

Bryan T. Adey, Nam Lethanh* and Clemens Kielhauser

Institute of Construction and Infrastructure Management, Swiss Federal Institute of Technology (ETH), IBI ETH Zürich, Stefano-Franscini-Platz 5, 8093 Zürich, Switzerland

Abstract: The determination of optimal intervention strategies for alpine road infrastructure requires consideration of manifest (gradual) and latent (sudden) deterioration processes, such as chloride-induced corrosion of steel reinforcement and landslides. Although many models can be used to determine these strategies, it is unclear which should be used. Advantages and disadvantages are associated with each model in real-world situations. In this paper, an age replacement model, a block replacement model and a Markov model are compared. The models are used to identify the optimal intervention strategies on a road link composed of 11 objects (three reinforced concrete bridges, five asphalt road sections, and three culverts) that are affected by both manifest and latent deterioration processes. The performance of each model is discussed.

Keywords: Road infrastructure, intervention strategies, age replacement model, block replacement model, Markov model, infrastructure management

DOI: http://dx.doi.org/10.7492/IJAEC.2014.013

1 INTRODUCTION

Road infrastructure is important to the well-being of people who live in alpine regions as it is for people in other regions. Road infrastructure in alpine regions, however, is more likely to be affected by natural hazard events, such as avalanches, rock falls, floods and landslides. When this happens, the consequences related to infrastructure failure, which include the execution of corrective interventions (CIs) to restore the infrastructure so that it once again provides an adequate level of service (LOS), are often large (Hilker et al. 2009; Swissinfo 2006).

In addition to these natural hazard events, which can be seen to be occurring due to latent deterioration processes (LDPs), road infrastructure in alpine regions is also affected by manifest deterioration processes (MDPs). LDPs are processes whose progression over time is not followed in a way that a condition of the object triggers the execution of an intervention early enough so that it can be assumed that the object will never provide an unexpected inadequate LOS. An example of a LDP is an avalanche. MDPs are processes whose progression over time is followed in a way that a condition of the object triggers the execution of an intervention early enough so that it can be assumed that the object will never provide an unexpected inadequate LOS. An example of a MDP is chloride induced corrosion of reinforced concrete, which produces an expansive rust product. To counteract the effects of MDPs, preventive interventions (PIs) are planned before the infrastructure provides an inadequate LOS. Road managers in alpine regions must consider both types of processes when determining optimal intervention strategies (OISs).

Existing state-of-the-art computerized infrastructure management systems, such as PONTIS (Pontis is a Markov-based bridge management system widely used in the USA), the KUBA (KUBA is a Markov-based bridge management system used in Switzerland), the HDM-4 (HDM-4 is a deterministic-based pavement management system developed by the World Bank group), are used to help determine OISs. They currently perform relatively well in determining these stra-

*Corresponding Author. Email: lethanh@ibi.baug.ethz.ch

tegies and the resulting work programs with respect to MDPs (Golabi and Shepard 1997; Thompson et al. 1998; Hajdin 2001), but not with respect to LDPs.

Significant research has been conducted on the determination of OISs. Much of this work has been concentrated on objects affected by MDPs, for example, deterministic approaches in which the MDPs were modeled using exponential distributions (Ouyang and Madanat 2004; Ouyang and Madanat 2006; Sathaye and Madanat 2011; Ferreira et al. 2002; Zhang et al. 2013; FWA et al. 2000). Such models, although acceptable for objects affected by MDPs, are not useful for LDPs due to the high level of uncertainty regarding the occurrence of natural hazards, such as earthquakes, avalanches, and rock falls.

To deal with objects affected by LDPs, it is necessary to address the concept of risk. In recent years, a substantial amount of work has focused on the estimation of the risk related to infrastructure. For example, more effort has been focused on developing and applying probabilistic methods to model the occurrences of hazard events and the expected impacts (Castelli and Scavia 2008; Korup and Clague 2009; Faber and Stewart 2003; Schubert 2010). In the most common models used to quantify the occurrence of hazard events, it is normally assumed that the occurrence of the event can be modeled using certain probabilistic distributions, such as Poisson models (Kingman 1963), negative binomial models and zero-inflated models (Cameron and Trivedi 2013). These models are coupled with emerging technologies, such as the use of tools developed in geographic information system (GIS) (Baruffini 2010) or the use of Bayesian network modeling techniques (Bayraktarli and Faber 2011; Graf et al. 2009), to perform risk assessments and calculate the probabilistic occurrence of natural hazard events and impacts.

After the probabilistic occurrence of natural hazard events is estimated, the probability of failure of the civil infrastructures is estimated. Two popular methodologies that are used to estimate the failure probability of infrastructure objects due to hazard risks are 1) the use of fragility curves, which were developed in the field of structural analysis (Karim and Yamazaki 2001; Choi et al. 2004; Graf et al. 2009) and 2) the use of survival analysis methods, which were developed voluminously in the fields of biology, epidemiology, and social science (Lancaster 1990; Ibrahim et al. 2001). Among the survival analysis models, uncertainty is often modeled with the Weibull distribution because it can account for system memory and can effectively address historical data (Dodson 2006; Lethanh and Adey 2013).

Existing infrastructure management systems often use discrete Markov chain models to predict the progression of conditions due to MDPs and to determine the OIS. In the models, the physical condition of objects is expressed as a scale of discrete condition states (e.g., from 1 to 5). By using data collected through periodic inspections, transition probabilities among condition states can be estimated and used with the optimization model to trigger OISs. Researchers have recently attempted to modify the Markov model to consider both MDPs and LDPs in the determination of OISs. For example, Mayet and Madanat (2002) introduced the use of fragility curves to calculate the failure probability of an object outside of its manifest transition probabilities. The cited authors further developed a linear optimization model for a system of similar objects. Lethanh et al. (2013) later extended the model of Mayet and Madanat (2002) to a case of multiple objects with ranked condition states. However, the models of Mayet and Madanat (2002) and Lethanh et al. (2013) are limited in that the use of fragility curves can only be used to estimate physical failure and not functional failure of an object. This limitation can be overcome by using survival analysis methods based on historical data (i.e., physical and functional failures of objects can be estimated using the Weibull analysis given historical data on both MDPs and LDPs).

In addition to Markov models, there are two other classes of probabilistic preventive models: 1) the block replacement model and 2) the age replacement model. These models have traditionally been used in the management of equipment (Gertsbakh 2000). In these models, either a PI is executed when a certain time is reached or a CI is executed if the object fails before the time is reached. BR and AR models could be suitable in the management of road infrastructure, where both MDPs and LDPs need to be taken into consideration (i.e., by using the PI and CI time of occurrence to model the time to intervention with respect to the MDP and LDP, respectively).

The use of BR and AR models is closely linked to survival analysis. Because it originated from the field of facility management, the BR and AR models are generally built for systems with object conditions that are usually described in binary states (e.g., 0 is non-operational and 1 is operational). Physical failures, functional failures (Failures can be classified into physical and functional. Physical failure is directly linked to the stress and strain behaviors of structures. It denotes a state in which a structure cannot carry a certain load (e.g., partially or completely collapsed). Functional failure refers to a structure's serviceability and availability) and the amount of time an object remains in a state where an adequate LOS is provided (the MDP process) can be monitored over a long period of time; thus, the two models can address objects that are affected by both MDPs and LDPs. One of the main limitations of BR and AR models is the oversimplification of the condition state, when a binary state is used, especially for infrastructure objects (e.g., roads and bridges) (Lethanh and Adey 2013).

The research presented in this paper was conducted to investigate the potential use of the BR, AR, and Markov models in determining the OISs for road infrastructure that are affected by both MDPs and LDPs. A clear demonstration of the use of each model is provided and the advantages and disadvantages of each are discussed. Section 1.1 presents the mathematical formulation of each model. Section 2 describes some aspects of the model setup (i.e., how important aspects of the real world were considered in each of the models). Section 3 presents the case study for which each model simulated the OIS for a road link comprised of multiple objects that are affected by both MDPs and LDPs. Section 4 provides the results of the case study. Section 5 contains a discussion of the effects of the model setup on the determination of the OISs. Section 6 offers a discussion of the suitability of the models (i.e., the advantages and disadvantages of each model). Section 7 presents the conclusions and suggestions for future research.

1.1 Models

The three types of models investigated were BR, AR and Markov models. In the descriptions of these models, a denotes a type of intervention, n denotes an object and l denotes a road link.

1.2 Block Replacement Model

When BR models are used to determine the optimal PI interval, the following are assumed:

i the best option is to execute a PI on the object in question at regular intervals, whether or not the object fails during the PI interval
ii if the object fails in the PI interval, then a CI is executed immediately and is considered to take a negligible amount of time,
iii the execution of a PI takes a negligible amount of time
iv the optimal PI interval is identified by minimizing the impacts associated with the execution of interventions

The expected impacts per time interval are given by Eq. (1), and the mean cost per unit time is given in Eq. (2).

$$E_n[R] = C_n^{pi} + m_n(T_n)C_n^{ci} \tag{1}$$

$$\eta_n^A(T_n) = \frac{E_n[R]}{T_n} \tag{2}$$

where C_n^{pi} is the impacts related to the execution of a PI; C_n^{ci} is the impacts related to the execution of a CI; $m_n(T_n)$ is the mean number of CIs in time interval T, which depends on the failure probability of object n within $m_n(T_n)$ and is assumed to follow a distribution function $F_n(t) = \int_0^T F(t)dt$.

1.3 Age Replacement Model

When AR models are used to determine the optimal PI interval, the following are assumed:

i the best option is to execute a PI on the object in question at regular intervals following the construction or execution of an intervention on the object
ii if the object fails in the PI time interval, then a CI is executed immediately and is considered to take a negligible amount of time and a new time interval is started
iii the execution of a PI takes a negligible amount of time (The time of execution can take weeks or months. However, if the time is compared to the life-cycle time, then it can be considered negligible)
iv the optimal PI interval is identified by minimizing the impacts associated with the execution of interventions

Because the probability of failure is assumed to be represented as a distribution function $F_n(t)$, the first intervention will be executed at time Z_n, which is estimated by Eq. (3). The mean impact per unit time is given by in Eq. (4):

$$E(Z_n) = \int_0^{T_n} [1 - F_n(t)]dt \tag{3}$$

$$\eta_n^{age}(T) = \frac{F_n(T)C_n^{ci} + [1 - F_n(T)]C_n^{pi}}{E(Z_n)} \tag{4}$$

where t_n is any arbitrary time within the period $[0, T_n]$; $F_n(T)C_n^{ci} + [1 - F_n(T)]C_n^{pi}$ is the mean impact between interventions; and T_n is the time at which an intervention will be executed if no failure occurs before this time.

1.4 Markov Model

When Markov models are used to determine the optimal PI interval, the following are assumed:

i the best option is to execute a PI on the object when a specific CS is reached
ii if the object fails in the PI time interval, then a CI is executed that restores the object to a like new condition
iii the execution of a PI takes a negligible amount of time
iv the optimal PI interval is identified by minimizing the impacts associated with the execution of interventions, taking into consideration the probability and consequences of the failure associated with each CS

The annual failure probability due to latent processes is dependent on CS (Lethanh et al. 2013) and is calculated using a probabilistic distribution function with the assumption that the annual average failure probability will be equal to those in the AR and BR models.

The mean impacts per unit time for an intervention strategy are determined by Eq. (5).

$$\eta^{markov}(t) = \sum_{n=1}^{N} \sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} S_{n,a_n,i_n} \tag{5}$$

The mean time between the interventions of the MDPs is given by Eq. (6):

$$E_{i_n}(T) = \sum_{j=1}^{i} \frac{1}{\theta_j} \tag{6}$$

where t is the unit of time considered in the Markov model (e.g., one year interval); $n = (1, ..., N)$ represents the objects; $a_n = (1, ..., A_n)$ represents the interventions on object n; $i_n, j_n = (1, ..., I_n + L_n)$ are indexes of object n, if $i_n = (1, ..., I_n)$, then i_n represents the non-failure CSs, if $i_n = (I_n + 1, ..., I_n + L_n)$, then i_n represents the failure CS; π_{n,a_n,i_n} represents the steady state probability of object n, i.e., if an IS is consistently followed the probability of object n being in CS i_n in one time interval at an infinite point of time in the future, and intervention a_n being executed; and S_{n,a_n,i_n} represents the total impacts incurred due to the execution of intervention a_n on object n when it is in CS i_n.

Subject to the following constraints:

$$S_{n,a_n,i_n} = \sum_{s=1}^{S} c^s_{n,a_n,i_n} \tag{7}$$

where s is an index of stakeholders (e.g., owner, users, or the public), c^s_{n,a_n,i_n} is the impact incurred by stakeholder s when intervention a_n is executed on object n when it is in state i_n.

$$\sum_{n=1}^{N} \sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} c^s_{n,a_n,i_n} \leq B^s \tag{8}$$

where B^s is the maximum allowable impact to be incurred by stakeholder s (e.g., the limit of the amount of resources that can be used for the execution of interventions). The allowable impact is an expected average value within one interval of the investigated time period.

The steady state probability of object n is estimated as:

$$\pi_{n,a_n,i_n} \geq 0, \forall n, \forall a_n, \forall i_n \tag{9}$$

$$\sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} Q_{n,a_n,i_n,j_n} = \sum_{a_n=1}^{A_n} \pi_{n,a_n,i_n,j_n} \forall j_n \tag{10}$$

where Q_{n,a_n,i_n} is the transition probability matrix (from i to j) when the set of interventions a_n in the appropriate CSs are executed.

$$\sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} = 1 \tag{11}$$

$$\pi_{n,a_n,i_n} = 0 \text{ at } a_n = 1, \ldots, (A_n - 1) \text{ and } i_n = I_n \tag{12}$$

2 MODELING CONSIDERATIONS

2.1 Probability of Execution of Interventions

In determining the interventions to execute on a road link composed of multiple objects that are affected by both MDPs and LDPs, it is useful to be able to take into consideration that both are at work.

BR and AR Models

The BR and AR models are characterized by the following:

i the probability of execution of the interventions due to LDPs (i.e., CIs) is modeled with the Weibull distribution (Kobayashi et al. 2010; Lethanh and Adey 2013), which considers the amount of time since the last PI was executed on the object.

ii the probability of execution of interventions due to MDPs is

(a) not modeled in BR directly because they are expected to occur at regular intervals

(b) modeled in AR as 1 minus the probability of executing a corrective intervention over T.

The Weibull model is described with the hazard function in Eq. (13), probability density function in Eq. (14), and survival probability function in Eq. (15).

$$\lambda(\tau) = \alpha m \tau^{m-1} \tag{13}$$

$$f(\tau) = \alpha m \tau^{m-1} e^{-(\alpha\tau)^m} \tag{14}$$

$$\widetilde{F}(\tau) = e^{-(\alpha\tau)^m} \tag{15}$$

The values of the parameters used in the Weibull models, α and m, can be estimated using a regression analysis given the historical data on the occurrence of hazard events (Dodson 2006).

The annual failure probability from each condition state is estimated based on the Weibull distribution using Eq. (6):

$$\Omega^i = \frac{f^i(T)}{\int_{t=0}^{T} \widetilde{F}^i(t)dt} \tag{16}$$

where Ω^i is the annual failure probability of condition state i; $f^i(T)$ is the failure probability of condition state i during period T in which the object remains in CS i (refer to Eq. ((14)); and $\widetilde{F}^i(t)$ is the survival probability, which is defined in Eq. (15).

Markov Models

Markov models are characterized by the following:

i the probability of execution of the interventions due to LDPs (i.e., CIs) is also modeled with the Weibull distribution; however, the probability of execution of the CIs is estimated taking into consideration the amount of time that an object has been in each of the manifest CSs.

ii the probability of execution of the interventions due to MDPs is modeled as the probability of being in a manifest CS in which an intervention is required. This was done as described by Kobayashi et al. (2012) and Tsuda et al. (2006). The transition probabilities between the CSs due to the MDPs are estimated as follows:

$$p_{ij}(z) = Prob[h(\tau_B) = j | h(\tau_A) = i] = \sum_{k=i}^{j} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \exp(-\theta_k z) \tag{17}$$

where θ is the hazard rate of each CS; i, j, k, m are running indices of CS.

2.2 Grouping Objects

In determining the interventions to execute on a road link composed of multiple objects, it is useful to be able to take into consideration that multiple interventions can be executed simultaneously and that there are different amounts of impacts associated with these than interventions being executed on the same objects separately. This is done in all three of the models by modeling multiple objects as one. For example, bridge 1, bridge 2 and bridge 3 are modeled as a system of bridges.

BR and AR Models

In BR and AR models, the probability of executing CIs on grouped objects is modeled using a Weibull distribution function, where the parameters λ and m are weighted combinations of the λ and m parameters used for each individual object. The weighted parameters are estimated by multiplying the parameters of the individual objects with the ratio of impacts that are related to the execution of a CI on the object and the execution of CIs on all objects. For example, a CI for objects A and B cost 6 mu and 4 mu, respectively. The ratio for object A is 0.6 (6/(6+4)), and the ratio for object B is 0.4 (4/(6+4)).

The impacts and their evolutions over time, which are not related to the execution of interventions, are not modeled. For both models, it is assumed that the execution of PIs and CIs restores the objects to a like-new condition. This assumption is realistic enough to reflect the weighted average deterioration of the group of objects, especially if they share similar characteristics (e.g., a group of three bridges or a group of five road sections) and take into consideration the impacts contributed by the individual objects.

In both BR and AR models, the cost of PIs for grouped objects is determined by multiplying their original impacts (without grouping) with reduction factors.

Markov Models

The probability of executing CIs on grouped objects is modeled using a Weibull distribution function, where the parameters λ and m are weighted combinations of the λ and m parameters that are used for each individual object. This is performed similarly to the estimation of the probability of executing a CI on grouped objects in the AR and BR models, except that the failure probability is estimated from every CS.

In the Markov model, deterioration of grouped objects is modeled by estimating the transition probabilities using weighted hazard rates. The weighted parameters are estimated by multiplying the parameters of the individual objects with the ratio of impacts related to the execution of a PI on the object and the execution of PIs on all objects. This assumption is realistic enough to reflect the weighted average deterioration of the group of objects, especially if they share similar characteristics (e.g., a group of three bridges or a group of five road sections) and take into consideration the impacts contributed by individual objects.

2.3 Time Between Interventions

In determining the interventions to execute on a road link that is composed of multiple objects, it is useful to determine the average time between interventions.

BR and AR Models

In the BR and AR models,

i the average time between interventions due to LDPs (i.e., the average time between CIs) is estimated as the time until a failure occurs, which is estimated using the Weibull function (section 3.1.1).

ii the average time between interventions due to MDPs is set by determining T in the BR model and the age when the AR model is used.

Markov Models

In the Markov model,

i the average time between interventions due to LDPs is estimated as a Weibull function, which is similar to that used for the BR and AR models, except that it considers the failure probability from every CS.

ii the average time between interventions due to MDPs is estimated as the sum of the inverted value of the hazard rate (Eq. (6)).

2.4 Time Between Interventions

BR and AR Models

In the BR and AR models, impacts during the execution of PIs are estimated as the weighted average of the impacts related to the PIs that are executed in each of the CSs in the Markov model (e.g., Table 3 in the example section). During the execution of the CIs, these values are estimated as the average sum of all of the impacts incurred by stakeholders at the time of failure (both before and during the execution of the CI intervention. The impacts incurred when objects are grouped are determined using reduction factors. Impacts that may occur between the executions of interventions were not modeled with AR and BR.

Markov Models

In the Markov model, impacts during the execution of PIs are estimated as the sum of the impacts related to the PIs that are executed in each of the CSs (e.g., Table 3 in the example section). During the execution of the CIs, these values are estimated as the average sum of all of the impacts incurred by stakeholders at the time of failure (both before and during the execution of the CI intervention), as is done in the BR and AR models.

To ensure consistency between the models, the impacts related to the execution of the PIs were adjusted to take into consideration the probable PI at the time of execution. This was done by assuming that the impacts related to the PI executed at T in the BR model, and the AR model was the average of the impacts of the PIs executed in CS3, 4 and 5 of the Markov model.

3 CASE STUDY

3.1 Objects and Object Grouping

The three models were compared using a two-lane road section with a length of 760 m in a mountainous region comprised of 11 objects (Table 1). The detour, if the road is closed, is a 800-m-long farm road. It is considered that each object can be in one of five CSs. Because there are often lower impacts associated with the execution of interventions on multiple objects simultaneously, seven possible object groupings were investigated (Table 2). For example, in the first grouping (OG1), all of the objects are modeled separately. In OG2, all of bridges are modeled as one object, and all other objects are modeled separately.

3.2 Intervention Types and Intervention Strategies

The types of PIs and CIs and their associated impacts considered are listed in Table 3. The intervention strategies investigated were to determine the optimal time to intervene on the objects to stop an inadequate LOS from occurring due to the MDPs and to restore an adequate LOS as soon as it occurs due to the LDPs.

Table 1. Objects

Objects	Name	Description	Width (m)	Length (m)
1	Bridge 1	Steel-concrete hybrid	9.6	85
2	Bridge 2	Reinforced concrete	6.5	25
3	Bridge 3	Reinforced concrete	6.5	25
4	Road section 1	Asphalt concrete	5.7	290
5	Road section 2	Asphalt concrete	5.7	160
6	Road section 3	Asphalt concrete	5.7	45
7	Road section 4	Asphalt concrete	5.7	20
8	Road section 5	Asphalt concrete	5.7	65
9	Culvert 1	Reinforced concrete	6.5	10
10	Culvert 2	Reinforced concrete	6.5	10
11	Culvert 3	Reinforced concrete	6.5	25

Table 2. Investigated object groupings

OG	Bridge			Road sections					Culverts		
	1	2	3	4	5	6	7	8	9	10	11
1	-	-	-	-	-	-	-	-	-	-	-
2	x	x	x	-	-	-	-	-	-	-	-
3	-	-	-	x	x	x	x	x	-	-	-
4	x	x	x	x	x	x	x	x	-	-	-
5	x	x	x	-	-	-	-	-	x	x	x
6	-	-	-	x	x	x	x	x	x	x	x
7	x	x	x	x	x	x	x	x	x	x	x

Table 3. Types of preventive and corrective interventions

Object	Int. type	CS	Description	Impact	CS following int.
1	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	1'726'773	1
		4	Major Int.	2'299'001	1
		5	Renewal	2'871'685	1
	CI	1-5	Renewal	3'676'714	1
2	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	347'574	1
		4	Major Int	461'572	1
		5	Renewal	575'704	1
	CI	1-5	Renewal	736'451	1
3	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	347'574	1
		4	Major Int	461'572	1
		5	Renewal	575'704	1
	CI	1-5	Renewal	736'451	1
4	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	71'541	1
		4	Major Int	89'645	1
		5	Renewal	109'307	1
	CI	1-5	Renewal	137'971	1
5	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	41'530	1
		4	Major Int.	51'679	1
		5	Renewal	62'686	1
	CI	1-5	Renewal	78'648	1
6	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	14'227	1
		4	Major Int.	17'036	1
		5	Renewal	20'087	1
	CI	1-5	Renewal	24'535	1
7	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	8'379	1
		4	Major Int.	9'627	1
		5	Renewal	10'983	1
	CI	1-5	Renewal	12'839	1
8	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	18'906	1
		4	Major Int.	22'964	1
		5	Renewal	27'370	1
	CI	1-5	Renewal	33'795	1
9	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	86'595	1
		4	Major Int.	113'976	1
		5	Renewal	141'410	1
	CI	1-5	Renewal	174'222	1
10	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	86'595	1
		4	Major Int.	113'976	1
		5	Renewal	141'410	1
	CI	1-5	Renewal	174'222	1
11	PI	1	DN	0	N/A
		2	DN	0	N/A
		3	Minor Int.	210'937	1
		4	Major Int.	279'390	1
		5	Renewal	347'976	1
	CI	1-5	Renewal	430'005	1

Table 4. Average time in each CS

Object	Condition state				Average time to failure
	1	2	3	4	
1	6.13	16.36	14.04	11.44	22
2	6.02	16.13	13.66	11.36	21
3	5.88	15.87	13.85	11.19	20
4	3.19	8.61	8.25	5.34	14
5	3.3	8.69	8.33	5.45	14
6	3.12	8.33	8.06	5.39	14
7	3.06	8.26	7.46	5.34	14
8	3.02	8.19	7.35	5.29	14
9	5.85	10.1	10.2	7.46	17
10	5.71	9.52	10.1	6.8	17
11	5.62	8.85	9.71	6.58	17

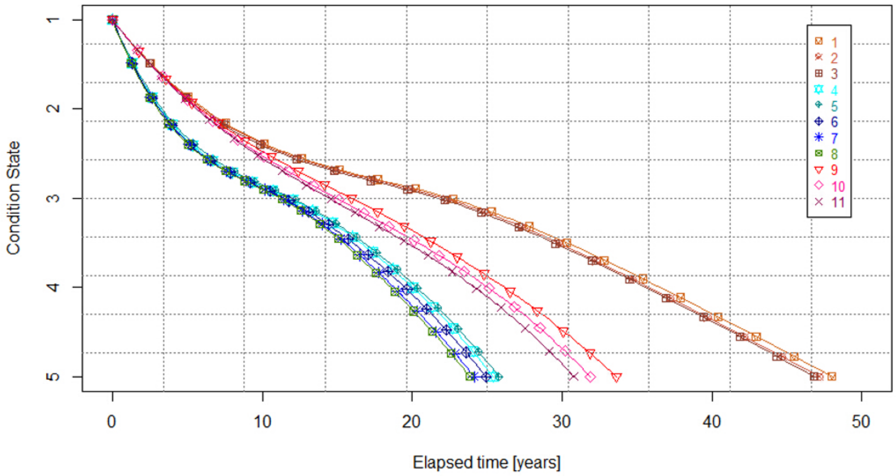

Figure 1. Manifest deterioration speed of the objects

3.3 Deterioration

The average amounts of time in each CS due to the MDPs of each object (i.e., the hazard rates) and the average time between failures (the execution of CIs) due to the LDPs of each object are shown in Table 4. The deterioration of the bridges due to the MDPs is slower than that of the road sections and the culverts.

In the BR and AR models, the deterioration due to the MDPs was modeled linearly between CSs, where the time of an object within each CS was obtained from Table 4. The deterioration due to the LDPs was modeled using the Weibull distribution function and the values of parameters α and m are provided in Table 6. The probabilities of failure of each object in each CS from the models are also presented in Table 6.

In the Markov models, the deterioration due to the MDPs was modeled by first determining the hazard rates θ_i to obtain the average time of being in each CS and then determining the transition probabilities to obtain these hazard rates (Table 5). The speeds of deterioration of the objects are illustrated in Figure 1.

The deterioration due to the LDPs was modeled by determining the values of the parameters of the Weibull function; the sum of the probabilities of failure when the object is in each CS is equal to the probability of failure that is correlated with the average time to failure.

3.4 Impacts

The impacts incurred by the four types of stakeholders (Owner, Users, DAP, and IAP) that are due to the execution of the PIs and CIs are listed in Tables 6 and 7. The impacts were assumed to vary proportionally to the surface area of the objects. It was assumed that the impacts related to the execution of the interventions on multiple objects simultaneously are lower than the impacts related to the execution of the interventions on the same objects separately. The impact reduction factors are provided in Table 6.

4 RESULTS

The average annual impacts and average annual time between interventions as estimated using the BR, AR and Markov models for each OG and each IS are presented in Tables 8 - 10. Each table indicates the following:

Table 5. Transition probabilities, hazard rate, Weibull parameter values and probabilities of failure

Object	CSs	Condition state					Hazard rate	Weibull parameter values		Probability
		1	2	3	4	5	θ_i	α	m	of failure
1	1	0.84951	0.14587	0.00451	0.00011	0	0.1631	0.041	2.7	0.00377
	2	0	0.94082	0.0571	0.00202	0.00006	0.061			0.04198
	3	0	0	0.93128	0.06577	0.00295	0.0712			0.14231
	4	0	0	0	0.91631	0.08369	0.0874			0.29503
	5	0	0	0	0	1	NA			0.42784
2	1	0.84696	0.14826	0.00466	0.00011	0	0.1661	0.043	2.7	0.00428
	2	0	0.93988	0.05795	0.00211	0.00006	0.062			0.04716
	3	0	0	0.92941	0.06753	0.00305	0.0732			0.16032
	4	0	0	0	0.91576	0.08424	0.088			0.33795
	5	0	0	0	0	1	NA			0.49936
3	1	0.84366	0.15138	0.00484	0.00012	0	0.17	0.045	2.7	0.00484
	2	0	0.93894	0.05888	0.00211	0.00006	0.063			0.05262
	3	0	0	0.93034	0.0666	0.00306	0.0722			0.1796
	4	0	0	0	0.91448	0.08552	0.0894			0.38546
	5	0	0	0	0	1	NA			0.58081
4	1	0.73118	0.25304	0.01514	0.00061	0.00003	0.3131	0.063	2.95	0.00244
	2	0	0.89039	0.10311	0.00611	0.0004	0.1161			0.04544
	3	0	0	0.88586	0.10389	0.01025	0.1212			0.18676
	4	0	0	0	0.82911	0.17089	0.1874			0.41449
	5	0	0	0	0	1	NA			0.59755
5	1	0.73853	0.24627	0.01459	0.00058	0.00003	0.3031	0.062	2.95	0.00233
	2	0	0.89128	0.10234	0.00601	0.00038	0.1151			0.04345
	3	0	0	0.88692	0.10313	0.00995	0.12			0.17839
	4	0	0	0	0.83244	0.16756	0.1834			0.39322
	5	0	0	0	0	1	NA			0.56341
6	1	0.72542	0.2579	0.01598	0.00066	0.00003	0.321	0.064	2.95	0.00256
	2	0	0.88683	0.1063	0.00645	0.00041	0.1201			0.04185
	3	0	0	0.88338	0.10624	0.01038	0.124			0.17204
	4	0	0	0	0.83077	0.16923	0.1854			0.39433
	5	0	0	0	0	1	NA			0.57448
7	1	0.72108	0.26182	0.01633	0.00073	0.00004	0.327	0.065	2.95	0.00268
	2	0	0.88595	0.1066	0.007	0.00045	0.1211			0.04373
	3	0	0	0.87459	0.11414	0.01127	0.134			0.17979
	4	0	0	0	0.82936	0.17064	0.1871			0.41492
	5	0	0	0	0	1	NA			0.60828
8	1	0.71821	0.26438	0.01662	0.00075	0.00004	0.331	0.066	2.95	0.0028
	2	0	0.88506	0.10732	0.00715	0.00047	0.1221			0.04566
	3	0	0	0.87284	0.11561	0.01155	0.136			0.18775
	4	0	0	0	0.8277	0.1723	0.1891			0.43637
	5	0	0	0	0	1	NA			0.64388
9	1	0.84282	0.14944	0.00749	0.00024	0.00001	0.171	0.051	2.76	0.00631
	2	0	0.90574	0.08971	0.00434	0.0002	0.099			0.05107
	3	0	0	0.90665	0.08727	0.00608	0.098			0.1557
	4	0	0	0	0.87459	0.12541	0.134			0.30726
	5	0	0	0	0	1	NA			0.40855
10	1	0.83946	0.15217	0.0081	0.00027	0.00001	0.175	0.052	2.76	0.00666
	2	0	0.90032	0.09482	0.00462	0.00023	0.105			0.05375
	3	0	0	0.90574	0.08755	0.00671	0.099			0.164
	4	0	0	0	0.86329	0.13671	0.147			0.31998
	5	0	0	0	0	1	NA			0.40787
11	1	0.83694	0.15392	0.00882	0.0003	0.00001	0.178	0.053	2.76	0.00701
	2	0	0.89315	0.10143	0.00515	0.00027	0.113			0.05256
	3	0	0	0.90213	0.09068	0.00719	0.103			0.1581
	4	0	0	0	0.85899	0.14101	0.152			0.32693
	5	0	0	0	0	1	NA			0.46365

i the average annual impacts incurred due to intervention on each object and the average time between interventions for an OG and an IS. For example, with OG-IS 1, it is expected that there will be an average annual impact of 3'171 mu related to object 1 and that the average time between interventions on object 1 is 10 years.

ii the average annual impact incurred due to all of the objects. For example, OG-IS 1 is expected to have 5'661 mu of impacts on average, and

iii the reduction in annual impacts when compared to the OG where no objects were bundled (OG-IS 1).

5 DISCUSSION

5.1 Impacts

The AR model estimated substantially lower total annual impacts for all of the OG-ISs than estimated by the BR and slightly lower total annual impacts for all

Table 6. Impact reduction factors due to the bundling of preventive interventions

OG	Bridge			Road sections					Culverts		
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	0.95	0.95	0.95	1	1	1	1	1	1	1	1
3	1	1	1	0.93	0.93	0.93	0.93	0.93	1	1	1
4	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	1	1	1
5	0.91	0.91	0.91	1	1	1	1	1	0.91	0.91	0.91
6	1	1	1	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
7	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

Table 7. Impact reduction factors due to the bundling of corrective interventions

OG	Bridge			Road sections					Culverts		
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	0.97	0.97	0.97	1	1	1	1	1	1	1	1
3	1	1	1	0.95	0.95	0.95	0.95	0.95	1	1	1
4	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	1	1	1
5	0.92	0.92	0.92	1	1	1	1	1	0.92	0.92	0.92
6	1	1	1	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
7	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91

Table 8. BR model: Average annual impacts and average annual time between PIs

OG-ISs	Bridge			Road sections					Culverts			Annual	Reduction
	1	2	3	4	5	6	7	8	9	10	11	Impact	
1	3'171	660	682	157	89	30	17	41	179	182	453	5'661	0
	10	10	9	8	8	8	8	8	9	9	9		
2	4'312	0	0	157	89	30	17	41	179	182	453	5'460	-201
	10	0	0	8	8	8	8	8	9	9	9		
3	3'171	660	682	313	0	0	0	0	179	182	453	5'639	-22
	10	10	9	8	8	8	8	8	9	9	9		
4	4'497	0	0	0	0	0	0	0	179	182	453	5'310	-350
	10	10	10	10	10	10	10	10	9	9	9		
5	4'870	0	0	157	89	30	17	41	0	0	0	5'204	-457
	10	10	10	8	8	8	8	8	10	10	10		
6	3'171	660	682	1ąŕ060	0	0	0	0	0	0	0	5'572	-89
	10	10	9	8	8	8	8	8	8	8	8		
7	5'130	0	0	0	0	0	0	0	0	0	0	5'130	-531
	10	10	10	10	10	10	10	10	10	10	10		

OG-ISs than estimated by the Markov models. For example, for the OG-IS1, the AR model estimates that the annual impacts are 2'987 vs. 5'661 and 3'307 *mu*, respectively.

These findings are due to the following:

i In the AR model, each time a CI is executed, it is assumed that the object is restored to a like-new condition. The probability of failure due to the LDPs is adjusted to take this into consideration, and it is assumed that the MDPs will restart. This situation reflects the real world situation where every time a failure occurs, the object is replaced with a new object with identical characteristics.

ii In the BR model, each time a CI is executed, it is assumed that the object (with respect to the LDPs) is restored to a like-new condition. Additionally, the probability of failure due to the LDPs is adjusted to take this into consideration, but it is assumed that the MDPs will continue as if nothing had been done to the object. This situation reflects the real world situation where at every time a failure occurs, the object is reassembled with the existing (i.e., deteriorated) elements. Thus, a BR model will estimate that substantially more PIs occur at much higher costs. A similar conclusion was reached by (Berg and Epstein 1978; Chen and Savits 1992).

Table 9. AR model: Average annual impacts and average annual time between PIs

OG-ISs	Bridge			Road sections					Culverts			Annual	Reduction
	1	2	3	4	5	6	7	8	9	10	11	Impact	
1	1'639	344	360	94	53	17	9	24	97	99	249	2'987	0
	25	24	23	16	17	17	19	16	21	21	20		
2	2'262	0	0	94	53	17	9	24	97	99	249	2'906	-82
	24	24	24	16	17	17	19	16	21	21	20		
3	1'639	344	360	187	0	0	0	0	97	99	249	2'977	-11
	25	24	23	16	16	16	16	16	21	21	20		
4	2'378	0	0	0	0	0	0	0	97	99	249	2'824	-163
	24	24	24	24	24	24	24	24	21	21	20		
5	2'560	0	0	94	53	17	9	24	0	0	0	2'758	-229
	24	24	24	16	17	17	19	16	24	24	24		
6	1'639	344	360	597	0	0	0	0	0	0	0	2'941	-46
	25	24	23	19	19	19	19	19	19	19	19		
7	2'714	0	0	0	0	0	0	0	0	0	0	2'714	-274
	23	23	23	23	23	23	23	23	23	23	23		

Table 10. Average annual impacts and average annual time between preventive interventions as estimated using the Markov replacement model

OG-ISs	Bridge			Road sections					Culverts			Annual	Reduction
	1	2	3	4	5	6	7	8	9	10	11	Impact	
1	1'773	381	407	114	63	21	11	29	110	114	285	3'307	0
	30	30	30	16	16	16	16	16	22	21	20		
2	2'468	0	0	114	63	21	11	29	110	114	285	3'215	-92
	30	30	30	16	16	16	16	16	22	21	20		
3	1'773	381	407	224	0	0	0	0	110	114	285	3'294	-13
	30	30	30	30	30	30	30	30	22	21	20		
4	2'610	0	0	0	0	0	0	0	110	114	285	3'118	-189
	29	29	29	29	29	29	29	29	22	21	20		
5	2'819	0	0	114	63	21	11	29	0	0	0	3'057	-251
	28	28	28	16	16	16	16	16	28	28	28		
6	1'773	381	407	699	0	0	0	0	0	0	0	3'260	-47
	30	30	30	19	19	19	19	19	19	19	19		
7	3'002	0	0	0	0	0	0	0	0	0	0	3'002	-305
	27	27	27	27	27	27	27	27	27	27	27		

iii In the Markov model, each time a CI is executed, it is assumed that the object is restored to a like-new condition. Additionally, the probability of failure due to the LDPs is adjusted to take this into consideration, and it is assumed that the MDPs will restart. This situation reflects the real world situation where at every time a failure occurs, the object is replaced with a new identical object.

The AR model estimated slightly lower total annual impacts for all of the OG-ISs as compared with the Markov model. For example, regarding OG-IS7, the Markov model estimates 3'002 *mu*, which is only 289 *mu* different from that of the AR model (2'714 *mu*).

These slightly lower total annual impacts occur because of the approximation of the probability of failure in each CS. Because the annual probability of failure modeled in the Markov model is based on the average amount of time the object is in each condition state and the amount of time that the object is in each condition state is modeled using an exponential distribution, an overall overestimation of the impacts associated with failure in each condition state occurs (there is approximately only a 37% chance that the object will survive longer than the average amount of time). This overestimation of the frequency of CIs indicates that a Markov model will predict fewer PIs than an AR model if the same IS is followed.

The Markov model will predict higher impacts related to an IS than the AR model. Thus, the impacts related to the CIs are higher with respect to the impacts related to the PIs. This will also result in longer average times between the executions of the PIs.

Based on the current ratios of PI and CI impacts, the Markov model estimates that the OIS will have slightly higher impacts and the average time between the executions of the PIs will be slightly greater than those of the AR model. These differences could be reduced if the probability of failure of an object in each CS was modeled more accurately (e.g., by modeling the probability of failure as weighted with respect to the amount of time that each object remains in each CS, instead of using the average amount of time).

5.2 Average Time Between Interventions

The average times between the PIs, which reflects the probability of executing PIs in each time interval, that are estimated by each model are significantly different.

The BR model always predicts a shorter time between the PIs than the AR and Markov models. For example, for OG-IS1 and object 1, the BR model predicts that the PIs will be executed on average every 10 years, whereas the AR and Markov models predict an execution every 25 and 30 years, respectively. This occurs because in both the AR model and the Markov model it is assumed that every CI restarts the MDPs, whereas in the BR they do not. This means that every increase in the probability of failure due to the LDPs will decrease the frequency of the PIs in the Markov and AR models, but not in the BR model.

The Markov model also predicts a slightly greater average time of intervention when compared to the AR model. This occurs because of the approximation of the probability of failure in each CS as described in the previous section.

5.3 Grouping Objects

The reference strategy (OG-IS-1) was estimated using all three models as having the highest annual impacts (BR model: 5'661 mu, AR model: 2'987 mu and Markov model: 3'307 mu). All strategies in which objects were grouped resulted in a reduction of the annual impacts when compared to the reference strategy (Tables 8 - 10). Thus, in all of the investigated cases, the additional impacts associated with executing interventions on some objects slightly earlier than the theoretically optimal times were outweighed by the reduction of the impacts due to executing interventions on multiple objects simultaneously.

The reduction in the annual impacts that occurs due to the grouping is similar. For example, the estimated annual impacts of following OG-IS6 were estimated to be lower than those when following the OG-IS1 by 1.57% with the BR model (89 mu), 1.54% with the AR model (46 mu), and 1.42% with the Markov model (47 mu).

6 SUITABILITY OF MODELS

6.1 General

The BR, AR, and Markov models can be used to determine OISs for a road link that is comprised of multiple objects. However, there are advantages and disadvantages associated with each model.

6.2 Markov Model

With respect to the AR and BR models, the advantages of the Markov model are that it allows the explicit consideration of

i the different types of PIs that will be executed on an object when it is in different condition states.
ii the different probabilities of failure that due to LDP that occur due to the changing condition of the object.

With respect to the AR and BR models, the disadvantages of the Markov model are:

i the OIS is determined based on the steady state probabilities. The acceptability of this is decreased when the probability of failure due to the LDPs cannot be ignored; in reality, if a failure due to a LDP occurred, it is highly likely that the object would be rebuilt in a different way than the original object.
ii the probability of failure is estimated with a series of exponential distributions.

6.3 BR Model

The only advantage that the BR model has over the AR model is related to modeling effort. If there were a real world situation in which an object is reassembled in the same way with the existing (i.e., deteriorated) elements every time a failure occurs, then it would be easier to model with the BR model than with the AR model; there should be no difference in the results. This savings in modeling effort increases with the reduction of the probability of failure.

The only advantage that the BR model has over the Markov model is related to modeling effort and accuracy. If there were a real world situation in which only one type of PI is executed, the impacts associated with the PI are always the same, and no impacts associated with the performance of the object occur between the executions of the interventions, then the situation could be modeled more easily and accurately with the BR model. In such a situation, the BR model would perform more accurately because it would not be forced to approximate the probability of failure with a series of exponential distributions.

A disadvantage of the BR model over the AR model is that if the interventions do cause the MDPs to restart, then it cannot be captured. This will lead to an overestimation of the number of PIs that are actually required.

A disadvantage of the BR model and the AR model with respect to the Markov model is that they do not explicitly consider the different types of PIs that will be executed on an object when it is in different condition states. The variation in the type of PI that would be executed due to the condition of the object is only considered by associating the average impact of all of the types of PIs that could be executed. Such approximations, however, will result in an overestimation of the impacts associated with ISs that include interventions on objects in relatively good condition and an underestimation of the impacts associated with interventions on objects in relatively poor condition. Another way could, however, be taken into consideration indirectly by assuming a variation over time of the impacts related to executing a PI. The over and underestimation of the impacts associated with intervention strategies would in this case depend on how the approximations are made.

6.4 AR Model

An advantage of the AR model over the BR model is that it can be used to take into consideration that a CI also restarts MDPs. This reflects more real world situations and will not result in the prediction of the execution of too many PIs.

The only advantage that the AR model has over the Markov model, like the BR model, is related to modeling effort and accuracy. If there were a real world situation where there was only one type of PI to be executed, and the impacts associated with the PI were always the same, and there were no impacts associated with the performance of the object between the executions of interventions, then the situation could be modeled more easily and more accurately with the AR model. In such a situation, the AR model would be more accurate because it would not be forced to approximate the probability of failure with a series of exponential distributions.

A disadvantage of the AR model with respect to the BR model is that if the real world situation exists where MDPs do not restart after a CI, then the modeling effort slightly increases.

A disadvantage of the AR model and the BR model with respect to the Markov model is that they do not explicitly consider the different types of PIs that will be executed on an object while it is in different condition states. The full explanation is provided in the previous section.

6.5 Additional Comments

Although not explicitly investigated in the case study, it is worthwhile to mention three other issues with respect to the suitability of the BR, AR and the Markov models: the impact constraints and the discount factor.

In the BR and AR models, impact constraints cannot be directly imposed; however, they can be imposed in the Markov model (Eq. (8)). For more information, please refer to (Mayet and Madanat 2002; Lethanh et al. 2013).

Although a non-zero discount factor was not considered in the case study, it can be accounted for in all three models. For more information, please refer to (Kaio and Osaki 1984; Chen and Savits 1992).

In this work, interventions were modeled with the assumption of 100% effectiveness. If this is not the case, then the Markov model has the additional advantage of easily considering other levels of effectiveness.

7 CONCLUSION

In this paper, an age replacement model, a block replacement model and a Markov model are compared. The models were used to determine the optimal intervention strategies on a road link that is composed of 11 objects (three reinforced concrete bridges, five asphalt road sections, and three culverts) that are affected by both manifest and latent deterioration processes. The advantages and disadvantages of each model are discussed.

The BR model was the most suitable under the following conditions:

- i there was only one type of PI executed,
- ii the impacts associated with the PI were always the same,
- iii there were no impacts associated with the performance of the object between the executions of interventions, and
- iv the execution of a CI does not change the progress of the MDPs.

The AR model was the most suitable under the following conditions:

- i there was only one type of PI executed,
- ii the impacts associated with the PI were always the same,
- iii there were no impacts associated with the performance of the object between the executions of interventions, and
- iv the execution of a CI completely resets the progress of the MDPs.

The Markov model is the most suitable for all other cases. The main limitations of the Markov model are as follows:

- i the effort that is required to estimate all of the appropriate parameters, especially the effort to estimate the annual transition probability of reaching failure from any manifest condition state. This could be improved with knowledge of reliability theory (e.g., the use of a fragility curve) or a maximum likelihood estimation to derive the appropriate failure probability from each condition state.

Further research should be focused on the investigation of the following:

- i other models of the same types, as there are many ways to build models of each type, (e.g., using the Markov model to model the performance of the link directly by representing the state of the link as combinations of the states of the objects that comprise the link).

ii other models of other types (e.g., periodic group repair).
iii different scales at which to use the models. For example, with the BR and AR models, an assumption may be that the interventions are executed on the link as a whole and the probabilities of executing the preventive and corrective interventions would be estimated from the probabilities of the objects whether they are likely to fail simultaneously or not. With the Markov model, the condition states in which the link could be, composed of all of the objects in the link could be used to estimate the OIS for the link.
iv the difficulty of determining the probabilities of failure or intervention of multiple objects simultaneously, in terms of accuracy and effort.

REFERENCES

Baruffini, M. (2010). "Systematic analysis of natural hazards along infrastructure networks using a GIS-tool for risk assessment." *Geophysical Research Abstracts*, EGU General Assembly.

Bayraktarli, Y. Y. and Faber, M. H. (2011). "Bayesian probabilistic network approach for managing earthquake risks of cities." *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 5(1), 2–24.

Berg, M. and Epstein, B. (1978). "Comparison of age, block, and failure replacement policies." *IEEE Transactions on Reliability*, R-27(1), 25–29.

Cameron, A. C. and Trivedi, P. K. (2013). *Regression Analysis of Count Data*. Cambridge University Press, United Kingdom.

Castelli, M. and Scavia, C. (2008). "A multidisciplinary methodology for hazard and risk assessment of rock avalanches." *Rock Mechanics and Rock Engineering*, 41(1), 3–36.

Chen, C. S. and Savits, T. H. (1992). "Optimal age and block replacement for a general maintenance model." *Probability in the Engineering and Informational Sciences*, 6(1), 81–98.

Choi, E., DesRoches, R., and Nielson, B., . p. (2004). "Seisemic fragality of typical bridges in moderate seismic zones." *Engineering Structures*, 26, 187–199.

Dodson, B. (2006). *The Weibull Analysis Handbook*. Quality Press, American Society for Quality, United States.

Faber, M. H. and Stewart, M. G. (2003). "Risk assessment for civil engineering facilities: Critical overview and discussion." *Reliability Engineering & System Safety*, 80(2), 173–184.

Ferreira, A., Antunes, A., and Picado-Santos, L. (2002). "Probabilistic segment-linked pavement management optimization model." *Journal of Transportation Engineering*, 128(6), 568.

FWA, T. F., Chan, W. T., and Hoque, K. Z. (2000). "Multiobjective optimization for pavement maintenance programing." *Journal of Transportation Engineering*, 126(5), 367–374.

Gertsbakh, I. (2000). *Reliability Theory With Applications to Preventive Maintenance*. Springer.

Golabi, K. and Shepard, R. (1997). "Pontis: A system for maintenance optimization and improvement of US bridge networks." *Interfaces*, 27(1), 71–88.

Graf, M., Nishijima, K., and Faber, M. H. (2009). "Bayesian updating in natural hazard risk assessment." *Australian Journal of Structural Engineering*, 9(1), 35–44.

Hajdin, R. (2001). *KUBA-MS: The Swiss Bridge Management System.* Available at: <http://link.aip.org/>.

Hilker, N., Badoux, A., and Hegg, C. (2009). "The Swiss flood and landslide damage database 1972-2007." *Natural Hazards and Earth System Sciences*, 9, 913–925.

Ibrahim, J. G., Chen, M.-H., and Sinha, D. (2001). *Bayesian Survival Analysis*. Springer Series in Statistics.

Kaio, N. and Osaki, S. (1984). "Extended block replacement models." *Operations Research*, 18(1), 59–70.

Karim, K. R. and Yamazaki, F. (2001). "Effect of earthquake ground motions on fragility curves of highway bridge piers based on numerical simulation." *Earthquake Engineering and Structural Dynamics*, 30, 1839–1856.

Kingman, J. F. C. (1963). "Poisson counts for random sequences of events." *The Annals of Mathematical Statistics*, 34(4), 1217–1232.

Kobayashi, K., Kaito, K., and Lethanh, N. (2010). "Deterioration forecasting model with multistage weibull hazard functions." *Journal of Infrastructure Systems*, 16(4), 282–291.

Kobayashi, K., Kaito, K., and Lethanh, N. (2012). "A Bayesian estimation method to improve deterioration prediction for infrastructure system with Markov chain model." *International Journal of Architecture, Engineering and Construction*, 1(1), 1–13.

Korup, O. and Clague, J. J. (2009). "Natural hazards, extreme events, and mountain topography." *Quaternary Science Reviews*, 28, 977–990.

Lancaster, T. (1990). *The Econometric Analysis of Transition Data*. Cambridge University Press, United Kingdom.

Lethanh, N. and Adey, B. (2013). "Investigation of the use of a weibull model for the deterioration of optimal road link intervention strategies." *Structure and Infrastructure Engineering*, 10(5), 684–696.

Lethanh, N., Adey, B., and Fernando, D. N. (2013). "A Markov model to determine optimal intervention strategies for multiple objects affected by uncorrelated manifest and latent processes." *IABSE Conference: Assessment, Upgrading and Refurbishment of Infrastructures.*

Mayet, J. and Madanat, S. (2002). "Incorporation of seismic considerations in bridge management systems." *Computer-Aided Civil and Infrastructure Engineering*, 17(3), 185–193.

Ouyang, Y. and Madanat, S. (2004). "Optimal scheduling of rehabilitation activities for multiple pavement facilities: Exact and approximate solutions." *Transportation Research Part A: Policy and Practice*, 38(5), 347–365.

Ouyang, Y. and Madanat, S. (2006). "An analytical solution for the finite-horizon pavement resurfacing planning problem." *Transportation Research Part B: Methodological*, 40(9), 767–778.

Sathaye, N. and Madanat, S. (2011). "A bottom-up solution for the multi-facility optimal pavement resurfacing problem." *Transportation Research Part B: Methodological*, 45(7), 1004–1017.

Schubert, M. (2010). *RiskNow-Falling Rocks: Excel-based Tool for the Risk Analysis of Rockfall Protection Galleries (In German).*

Swissinfo (2006). *Rockfall Cripples Transalpine Traffic.* Available at <http://www.swissinfo.ch>.

Thompson, P. D., Small, E. P., Johnson, M., and Marshall, A. R. (1998). "The pontis bridge management system." *Structural Engineering International*, 8(4), 303–308.

Tsuda, Y., Kaito, K., Aoki, K., and Kobayashi, K. (2006). "Estimating markovian transition probabilities for bridge deterioration forecasting." *Journal of Structural Engineering and Earthquake Engineering*, 23(2), 241–256.

Zhang, H., Keoleian, G., and Lepech, M. (2013). "Network-level pavement asset management system integrated with life-cycle analysis and life-cycle optimization." *Journal of Infrastructure Systems*, 19(1), 99–107.