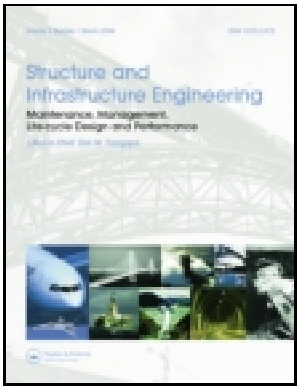

Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/nsie20

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Sandra Santa-Cruz [a] & Ernesto Heredia-Zavoni [b]

[a] Programa de Maestria y Doctorado en Ingenieria, UNAM, Mexico City, 04510, Mexico

[b] Instituto Mexicano del Petroleo, Eje Central Lazaro Cardenas Norte 152, Mexico City, 07730, Mexico

Available online: 16 Apr 2009

To cite this article: Sandra Santa-Cruz & Ernesto Heredia-Zavoni (2011): Maintenance and decommissioning real options models for life-cycle cost-benefit analysis of offshore platforms, Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance, 7:10, 733-745

To link to this article: http://dx.doi.org/10.1080/15732470902842903

Structure and Infrastructure Engineering
Vol. 7, No. 10, October 2011, 733–745

Taylor & Francis
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Maintenance and decommissioning real options models for life-cycle cost-benefit analysis of offshore platforms

Sandra Santa-Cruz[a] and Ernesto Heredia-Zavoni[b]*

[a]*Programa de Maestria y Doctorado en Ingenieria, UNAM, Mexico City 04510, Mexico;* [b]*Instituto Mexicano del Petroleo, Eje Central Lazaro Cardenas Norte 152, Mexico City 07730, Mexico*

(*Received 31 December 2007; final version received 23 February 2009; published online 16 April 2009*)

Maintenance and decommissioning real options (RO) models are developed for life-cycle cost-benefit (LCCB) analysis of offshore platforms. Uncertainties about hydrocarbon prices, maintenance costs, environmental loading, structural capacity and damage due to deterioration are taken into consideration in the RO modelling. Expressions are derived for expected costs and benefits in terms of the availability function of the platform, which depends on the hazard and restoring functions, and on the annual probability of failure of the structure. Results show that the use of the net present value (NPV) approach can significantly underestimate the LCCB. The RO formulation developed is shown to provide a proper framework to account for the value of the managerial flexibility to consider different options along the service lifetime of the structure, such as maintenance and decommissioning, and to make decisions adapting to future information.

Keywords: real options; life-cycle cost-benefit analysis; maintenance; decommissioning; net present value; offshore platforms

1. Introduction

Optimal design criteria, inspection plans and maintenance strategies for engineering infrastructure can be established based on life-cycle economic assessments aimed at maximising benefits or minimising costs and losses. In structural engineering, for instance, such an approach has been used to establish optimal design resistances for buildings in seismic areas (Rosenblueth and Mendoza 1971, Rosenblueth 1976) and for optimal inspection planning of offshore facilities (Faber *et al*. 2000, Heredia-Zavoni *et al*. 2008). Other applications can also be found in engineering fields dealing with foundations, water resources and the environment (Bouchart and Goulter 1998, Stansbury 1999, Whitman 2000). Life-cycle economic assessment of benefits and costs essentially involves the net present value (NPV) method, where total cash inflows and outflows are computed and compared at the present time, based on discounted future values distributed along the service lifetime of the structure. Then, various design, inspection or maintenance choices are assessed in order to select as optimal the ones that maximise the economic benefits.

Although the NPV analysis is a useful tool for life-cycle economic assessment, it does not provide a framework to properly account for managerial options along the service lifetime of structures. The NPV method presumes a determined scenario of incomes and costs, and does not account for flexibility in the management of the infrastructure in order to adapt to future conditions. In economic valuation of projects, it is convenient to account for the fact that, depending on future conditions that are not certain at the present time, a project can be deferred, or its scale expanded or reduced, or some planned actions can be interrupted and later on restarted, or that the project can be exchanged or eventually abandoned. The managerial flexibility to consider different options and make decisions along time, thus adapting to future conditions that, at the present time, are uncertain, increases the value of a project and should be considered in an economic valuation. These options can be found in high uncertainty investment projects, such as engineering projects related to infrastructure for exploitation and production of natural resources. In recent years, the real option (RO) method has become one of the most accepted methods for project valuation (Luehrman 1998). It provides a framework to account for managerial flexibility where options to make decisions and change plans along time are valuated based on the Black and Scholes formula for financial options (Black and Scholes 1973). The economic value of a project is then equal to the NPV plus the added value

*Corresponding author. Email: eheredia@imp.mx

ISSN 1573-2479 print/ISSN 1744-8980 online

DOI: 10.1080/15732470902842903
http://www.informaworld.com

of flexibility. The RO method has been applied to valuation of projects for offshore hydrocarbon production where uncertainty about economic and production variables has been considered (Paddock *et al.* 1988). The main differences between the NPV and RO methods are summarised in Table 1.

In this paper, a formulation is presented for life-cycle cost-benefit (LCCB) analysis of offshore platforms considering that maintenance and decommissioning are ROs for management along the service lifetime. The platform structure is subjected to deterioration, and thus the probability of structural failure due to extreme environmental events evolves with time. Maintenance can be provided, depending on findings from future inspections, and management will have the option to provide maintenance or not. At some point in time, management also has the option to consider decommissioning the platform. RO models are developed here for the economic value of both the maintenance and decommissioning options to assess the LCCB. The RO models are based on the Black and Scholes formulation for financial options. Uncertainties on the environmental hazards and structural capacity and deterioration are accounted through an availability function that is used in the RO models. Uncertainties about financial variables used for the assessment of economic benefits, such as hydrocarbon market price, are also accounted for in the RO formulation. An example is given to illustrate an application, and numerical results are used to compare the LCCB using the RO approach and the NPV method.

2. Maintenance and decommissioning ROs

Consider an offshore jacket platform with initial cost X_0 and design service lifetime L. Initial costs include planning, well drilling, design, construction, transportation and installation of the structure, as well as the equipment. The platform structure is subjected to progressive deterioration with time as a result of damage accumulation. Among causes of damage are corrosion, fatigue, dropped objects and ship impacts. The accumulation of damage produces a deterioration of the overall structural resistance of the platform, which increases the risk of failure due to extreme storm loading.

Table 1. Differences between the NPV and RO methods.

Feature	NPV	RO
Changing plans	Not possible	Possible. Decision to change depends on future conditions
Capital cost	Cost of opportunity	Risk-free rate
Monetary valuation	Expected value of cash flow	Expected value of cash flow plus value due to possibility of change

Periodic inspections are to be carried out during the design service lifetime L to detect if there is any structural damage and make decisions on maintenance so that the structure meets safety requirements. Suppose an inspection is scheduled at time T_1 in $[0, L]$. Depending on the findings after the inspection, the decision maker will consider whether to carry out some maintenance work in order to repair or upgrade the structure. If the decision is made not to repair or upgrade the structure, the probability of the platform not being in operation due to structural failure will continue to increase over time due to damage accumulation; therefore, the expected net cash flow (benefits minus costs) will decrease. The expected cash flow, F, is depicted schematically in Figure 1 for the cases where maintenance is provided or not at time T_1.

Consider the two alternatives at time T_1: (1) to carry out repair and maintenance work and continue operating the platform and (2) not to carry out any maintenance and continue operating the platform. Let us divide the service lifetime L into two phases: the first phase corresponds to time interval $[0, T_1]$ and the second phase to time interval $[T_1, L]$. Let S_0 denote the expected value of cash flow at time $t = 0$ from the first phase; S_1 denote the expected value of cash flow at $t = T_1$ from the second phase if maintenance is provided; s_1 denote the expected cash flow at $t = T_1$ from the second phase if there is no maintenance; and X_1 denote the maintenance costs, as illustrated in Figure 2. Alternative number 1 will be the most advantageous if $\underline{s}_1 = S_1 - s_1 > X_1$; on the contrary, the best alternative will be the second one. The expected value of the second phase at T_1 is:

$$V2 = s_1 + C_M*, \quad \text{where } C_M* = \max[(\underline{s}_1 - X_1), 0] \tag{1}$$

and the total LCCB at the present value, V_T, is:

$$V_T = S_0 - X_0 + s_1 \exp(-\delta T_1) + C_M, \tag{2}$$

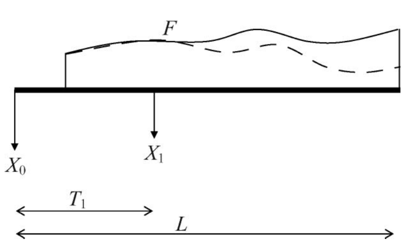

Figure 1. Expected cash flow of benefits and costs: with maintenance at time T_1 (continuous line) and without maintenance (dashed line).

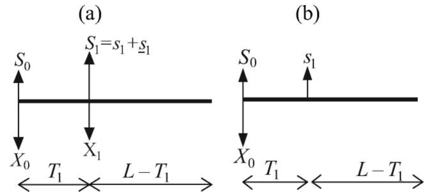

Figure 2. Concentrated cash flows: (a) with maintenance at time T_1 and (b) without maintenance.

where C_M is the present expected value of C_M* and δ is the discount rate. We define C_M as the 'maintenance option'. Note that the discount rate is generally greater than the risk-free rate of return, r. By definition, r is the percentage of return generated by investing in risk-free securities, such as government bonds.

Consider now the case where the decision has to be made at time T_1 ($0 < T_1 < L$) whether to continue with operation of the platform or to decommission it and sell the asset for some liquidation value, A_1. Note that T_1 is a decision point where the decision maker has two alternatives: (1) to continue operation and production until the end of the service lifetime and (2) to decommission the platform and sell it for a liquidation value. Alternative number 1 will be chosen if cash flow at T_1 from time interval $[T_1, L]$ is greater than the liquidation value, i.e. if $A_1 < S_1$. On the contrary, if $A_1 > S_1$, alternative number 2 will be chosen. The value of the second phase at time T_1 is:

$$V2 = A_1 + C_D*, \quad \text{where } C_D* = \max\left[(S_1 - A_1,)0\right] \tag{3}$$

and the total LCCB at the present value, V_T, is:

$$V_T = S_0 - X_1 + A_1 \exp(-rT_1) + C_D, \tag{4}$$

where C_D is the present expected value of C_D* and r is the risk-free rate. We define C_D as the 'decommissioning option'. Note that the risk-free rate r is used in Equation (4) since the opportunity to choose the best second phase makes it riskless.

In finances, a *call option* (Hull 1993) is a derivative security that gives the owner the right, but not the obligation, to buy an asset at a specified price (the exercise or strike price) up to a specified time (the expiration date). The formulation for the maintenance RO in Equation (1) is similar to the expression to estimate the value of a call option: $\underline{s}_1$ would be the asset value, X_1 the exercise price and C_M* the call option value just before the expiration date. In the same way, the formulation for the decommissioning RO in Equation (3) is similar to the expression for assessing the value of a call option: S_1 would be the asset value, A_1 the exercise price and C_D* the call option value just before the expiration date. This analogy is also used to evaluate corporate finances and investment projects when there are options to differ, abandon, expand or reduce the scale of projects at some time in the future. This approach is known as the RO method (Paddock *et al.* 1988, Bouchart and Goulter 1998, Luehrman 1998).

2.1. *Cash flow of benefits and costs*

The cash flow of benefits and costs at time t, $F(t)$, along the service lifetime L can be written as:

$$F(t) = \begin{cases} N(t)p(t) - G(t), & \text{when the structure is in an operational or safe state,} \\ -\text{CR}, & \text{when the structure is in a failure state.} \end{cases} \tag{5}$$

$N(t)$ is the volume of hydrocarbon produced per unit time; $p(t)$ is the hydrocarbon price; $G(t)$ is the cost of operation per unit time; and CR is the cost of failure per unit time, including costs to repair or rebuild the facility, injuries and loss of human lives and damages to the environment. Note that production is interrupted during the time the platform is in a failure state and there is no income flow from production sells. $N(t)$ and $G(t)$ are modelled here as continuous and deterministic functions and CR is taken to be a deterministic constant. The hydrocarbon price $p(t)$ is a random process that can be modelled as a geometric Brownian motion, as is carried out in finances for stock prices (Paddock *et al.* 1988, Hull 1993, Bouchart and Goulter 1998); thus:

$$\mathrm{d}p = \mu p \mathrm{d}t + \sigma p \mathrm{d}z, \tag{6}$$

where μ and σ are constants and z is a Wiener process. It is well known that $p(t)$ is lognormally distributed, and that the expected value of $p(t_j)$ given that $p(t_i)$ is known, $t_i < t_j$, is:

$$E\left[p(t_j)\right] = p(t_i)\exp\left[\mu\left(T_j - T_i\right)\right]. \tag{7}$$

The expected value of $F(t)$ is then:

$$E[F(t)] = [N(t)E[p(t)] - G(t)]D(t) - \text{CR}(1 - D(t)), \tag{8}$$

where $D(t)$ is the availability function of the platform structure, defined as the probability that the structure is in an operational or safe state at time t.

The expected value of cash flow at $t = 0$ from the first phase is then:

$$S_0 = \int_0^{T_1} E[F(t)][\exp(-\delta t)]\mathrm{d}t. \tag{9}$$

Substituting Equations (7) and (8) into (9), one obtains:

$$S_0 = p_0 n_0 - g_0 - m_0, \tag{10}$$

where $p_0 = p(0)$;

$$n_0 = \int_0^{T_1} N(t)D(t)[\exp(-(\delta-\mu)t)]\mathrm{d}t;$$
$$g_0 = \int_0^{T_1} G(t)D(t)[\exp(-\delta t)]\mathrm{d}t;$$

and

$$m_0 = \mathrm{CR}\int_0^{T_1} (1-D(t))[\exp(-\delta t)]\mathrm{d}t \tag{11}$$

Similarly, the expected cash flow at $t = T_1$ from the second phase if there is no maintenance, s_1, is:

$$s_1 = p_1 n_1 - g_1 - m_1, \tag{12}$$

where $p_1 = p(T_1)$;

$$n_1 = \int_{T_1}^{L} N(t)D(t)[\exp(-(\delta-\mu)(t-T_1))]\mathrm{d}t;$$
$$g_1 = \int_{T_1}^{L} G(t)D(t)[\exp(-\delta(t-T_1))]\mathrm{d}t; \text{ and}$$
$$m_1 = \mathrm{CR}\int_{T_1}^{L} (1-D(t))[\exp(-\delta(t-T_1))]\mathrm{d}t. \tag{13}$$

If maintenance is carried, the expected value of cash flow, S_1, is computed using an availability function of the structure that considers that maintenance has taken place, say D^{M}. Let $\underline{s}_1 = S_1 - s_1$ denote the difference between expected cash flows when maintenance is provided and when it is not. From Equations (10) to (12), it follows that:

$$\underline{s}_1 = S_1 - s_1 = \Delta n_1 p_1 - \Delta g_1 + \Delta m_1, \tag{14}$$

where:

$$\Delta n_1 = \int_{T_1}^{L} N_t \kappa(t)[\exp(-(\delta-\mu)(t-T_i))]\mathrm{d}t;$$
$$\Delta g_1 = \int_{T_1}^{L} G_t \kappa(t)[\exp(-\delta(t-T_i))]\mathrm{d}t;$$
$$\Delta m_1 = \mathrm{CR}\int_{T_1}^{L} \kappa(t)[\exp(-\delta(t-T_1))]\mathrm{d}t;$$
$$\text{and } \kappa(t) = D^{\mathrm{M}}(t) - D(t). \tag{15}$$

All parameters in Equations (11), (13) and (15) are deterministic and depend on the volume, cost of production and maintenance strategies. The parameter Δn_1 is a measure of the difference in expected production volume between the structure with and without maintenance; Δg_1 is the difference in expected operational costs due to the maintenance actions; and Δm_1 is the difference in expected failure costs between the structure with and without maintenance.

Since n_1 is positive, s_1 in Equation (12) is an increasing function of p_1; therefore, the probability density function (pdf) of s_1, f_s, can be obtained directly from the pdf of p_1, f_p, as follows:

$$f_S(x) = \frac{1}{n_1} f_p\left[\frac{x+g_1+m_1}{n_1}\right]. \tag{16}$$

It can be shown that (Santa-Cruz 2007):

(1) s_1 is lognormally distributed over the domain]–$(g_1 + m_1)$, $+\infty$ [with a pdf given by:

$$f_S(s_1) = \frac{1}{\Omega\sqrt{2\pi}(s_1+g_1+m_1)} \times \exp\left\{-\left[\ln\left(\frac{s_1+g_1+m_1}{n_1}\right)-M\right]^2 / 2\Omega^2\right\}, \tag{17}$$

Where:

$$M = \ln p_0 + \left(\mu - \frac{\sigma^2}{2}\right)T_1 \quad \text{and} \quad \Omega = \sigma\sqrt{T_1}. \tag{18}$$

(2) s_1 follows Ito's process:

$$\mathrm{d}s_1 = (s_1 + b_1)(\mu \mathrm{d}t + \sigma \mathrm{d}z), \quad \text{where } b_1 = g_1 + m_1. \tag{19}$$

Note that $\underline{s}_1$ in Equation (14) has the same functional form as s_1 in Equation (12); thus, it can be shown that $\underline{s}_1$ is also lognormally distributed over domain]–$(\Delta g_1 - \Delta m_1)$, $+\infty$ [, and also follows Ito's process:

$$\mathrm{d}\underline{s_1} = \left(\underline{s_1} + b_1\right)(\mu \mathrm{d}t + \sigma \mathrm{d}z), \quad \text{were } b_1 = \Delta g_1 - \Delta m_1. \tag{20}$$

A similar derivation can be made for S_1 (Santa-Cruz 2007).

2.2. *Black–Scholes formulation for RO valuation*

Black and Scholes (1973) obtained the analytical solution to evaluate call options in a market that follows the hypotheses listed in Table 2. We assume that all of the conditions of the market are valid for our analysis, except that, in our case, the assets s_1 and

Table 2. Hypotheses of the Black–Scholes formulation (1973).

1	Only market risks exist
2	The market is efficient
3	Market transactions are continuous
4	Asset prices follow a geometric Brownian behaviour
5	Risk rate and volatility are constants
6	Assets do not pay dividends
7	Asset prices are not necessarily an integer number
8	There is no *arbitrage*, meaning that the discount rate of a project with no risk is the riskless rate of the bank

S_1 follow Ito's process, rather than a Brownian geometric process (which is a particular case of Ito's process). Despite that difference, one can use Itos's lemma, as in the Black–Scholes formulation:

$$\mathrm{d}C = \left(\frac{\partial C}{\partial t} + \frac{1}{2}\frac{\partial^2 C}{\partial s^2}\sigma^2 (s+b)^2\right)\mathrm{d}t + \frac{\partial C}{\partial s}\mathrm{d}s, \tag{21}$$

where s and C are equal to $\underline{s}_1$ and C_{M} for the maintenance option, and to S_1 and C_{D} for the decommissioning option, respectively. Let V_{H} be a portfolio consisting of selling $\frac{\partial C}{\partial s}$ assets s and buying one call option C. Then:

$$V_{\mathrm{H}} = \left(\frac{\partial C}{\partial s}\right)(s) - (1)(C) \tag{22}$$

and

$$\mathrm{d}V_{\mathrm{H}} = \mathrm{d}s\frac{\partial C}{\partial s} - \mathrm{d}C. \tag{23}$$

Replacing Equation (21) in (23), we obtain:

$$\mathrm{d}v_{\mathrm{H}} = \left(-\frac{\partial C}{\partial t} - \frac{1}{2}\frac{\partial^2 C}{\partial s^2}\sigma^2 (s+b)^2\right)\mathrm{d}t. \tag{24}$$

Equation (24) is independent from dz and portfolio V_{H} can be considered to be risk-free in the time interval dt. In that case:

$$\mathrm{d}V_{\mathrm{H}} = rV_{\mathrm{H}}\mathrm{d}t, \tag{25}$$

where r is the risk-free interest rate. Replacing Equation (22) in (25), and from Equation (24):

$$\frac{\partial C}{\partial t} = rC - rs\frac{\partial C}{\partial t} - \frac{1}{2}\frac{\partial^2 C}{\partial s^2}\sigma^2 (s+b)^2. \tag{26}$$

The following boundary conditions are in place: $C = \max[(s\ -\ X,)\ 0]$, when $t = T_1$, just before the expiration date (X is equal to X_1 or A_1 for the maintenance or decommissioning options, respectively) and $C = 0$ when $s = 0$.

Notice that Equation (26) does not involve constant μ from Ito's process for oil price (Equation (6)); this is also the case in the Black–Scholes differential equation for call options (Black and Scholes 1973). Equation (26) depends only on the value of s, constant b, time t, constant σ and the risk-free interest rate, r. All of these parameters are independent from the risk preferences of the investors. This argument can be used to apply the neutral value theorem (Cox and Ross 1976), which states that, for obtaining the present value of an option, one can assume a riskless world; thus constant μ and the discount rate are both equal to the risk-free interest rate r. Therefore, the present expected value of the options in Equations (2) and (4) is obtained from:

$$C = \exp(-rT_1)\, E*\{\max[(s-X), 0]\}, \tag{27}$$

where E^* is the expected value in a risk-free world; hence:

$$C = [\exp(-rT_1)]\int_X^{\infty}(s-X)f_S^*(s)\mathrm{d}s. \tag{28}$$

The pdf, f_S^*, is the same as that in Equation (17), except that μ is replaced by r. The analytical solution of Equation (28) for the value of option C is given in Appendix 1. Following from Appendix 1, the maintenance option value is:

$$\begin{aligned} C_{\mathrm{M}} = {} & \Delta n_1[\exp(rT_1)]\left[\exp\left(\frac{2M+\Omega^2}{2}\right)\right] \\ & \times \Phi\left[\frac{-\ln\left(\frac{X_1+(\Delta g_1-\Delta m_1)}{\Delta n_1}\right)+(M+\Omega^2)}{\Omega}\right] \\ & -(X_1+\Delta g_1-\Delta m_1)[\exp(rT_1)] \\ & \times \Phi\left[-\frac{\ln(X_1+\Delta g_1-\Delta m_1)-(\ln(\Delta n_1)+M)}{\Omega}\right], \end{aligned} \tag{29}$$

where Φ is the normal standard distribution and M and Ω are given in Equation (18) with $r = \mu$. Replacing M and Ω in Equation (29):

$$\begin{aligned} C_{\mathrm{M}} = {} & \Delta n_1 p_0 \Phi[k1] - (X_1+\Delta g_1-\Delta m_1) \\ & \times[\exp(-rT_1)]\Phi[k2], \end{aligned} \tag{30}$$

where:

$$k1 = \frac{\ln\left(\frac{\Delta n_1 po}{X_1+\Delta g_1-\Delta m_1}\right)+(r+\sigma^2/2)T_1}{\sigma\sqrt{T_1}} \quad \text{and}$$
$$k2 = k1 - \sigma\sqrt{T_1}. \tag{31}$$

Equation (30) is valid for $X_1 > -\Delta g_1 + \Delta m_1$. Furthermore, $\underline{s}_1$ is always greater than X_1 and we can use the following approach. Consider a hypothetical portfolio Π, consisting of selling Δn_1 units of hydrocarbon and buying one option C_M, then:

$$\Pi(t) = \Delta n_1 p(t) - C_M. \tag{32}$$

Just before the expiration date, the portfolio value is:

$$\Pi(T_1) = \Delta n_1 p_1 - (\underline{s_1} - X_1) = \Delta g_1 - \Delta m_1 + X_1. \tag{33}$$

We can see that the portfolio value at T_1 is constant, so it gives a riskless benefit. Considering that the discount rate with no risk is equal to the riskless rate of the bank (i.e. the market is arbitrage-free), then:

$$\Pi(t=0) = (\Delta g_1 - \Delta m_1 + X_1)\exp(-rT_1.) \tag{34}$$

From Equations (34) and (32), we obtain the option value at $t = 0$. In summary, the value of the maintenance option is:

$$C_M = \Delta n_1 p_0 - (X_1 + \Delta g_1 - \Delta m_1) \times \exp(-\mathrm{r}T_1) \quad \text{if} \quad \Delta \mathrm{m}_1 - \mathrm{x}_1 - \Delta \mathrm{g}_1 \geq 0$$

and

$$C_M = \Delta n_1 p_0 \Phi[k1] - (X_1 + \Delta g_1 - \Delta m_1)[\exp(-rT_1)]\Phi[k2] \quad \text{if} \quad \Delta m_1 - x_1 - \Delta g_1 < 0, \tag{35}$$

where $k1$ and $k2$ are given in Equation (31). If there is no maintenance option, the LCCB at the present value is:

$$V_T = S_0 - X_0 + [\exp(-\delta T_1)](s_1), \tag{36}$$

where s_1 is the value of the second phase without maintenance.

Similar to C_M, one obtains that the present value of the decommissioning option C_D in Equation (3) is:

$$C_D = n_1 p_0 \Phi[k1] - (A_1 + g_1 + m_1)[\exp(-rT_1)]\Phi[k2], \tag{37}$$

where $k1$ is given in Equation (31) and $\Delta n_1 = n_1$, $\Delta g_1 = g_1$, $\Delta m_1 = m_1$ and $X_1 = A_1$.

2.3. Multiple ROs: decommissioning and maintenance

Suppose now that at T_1 ($0 < T_1 < L$), the decision maker has the following alternatives: (1) to carry out maintenance and continue production; (2) not to carry out maintenance and continue production; and (3) to decommission the platform and gain a liquidation value. In this case, it is said that the RO is multiple. Cash flows for these multiple options are shown in Figure 3. If the decision maker is to choose the most advantageous alternative, then the value of the second phase at T_1 is the maximum of $S_1 - X_1, s_1, A_1$. From Equations (10) and (14), it is seen that s_1 and S_1 are not uncorrelated. The value of the second phase is thus written in terms of s_1:

$$V2 = \max[(\alpha s_1 - \beta - X_1), s_1, A_1]; \quad \alpha = \frac{n_1 + \Delta n_1}{n_1} > 1 \quad \text{and} \quad \beta = \Delta g_1 - \Delta m_1 \tag{38}$$

The value $V2$ is shown schematically in Figure 4 with a solid line: $X_1{}^*$ is the value of s_1 when the second phases with and without maintenance are the same and $A_1{}^*$ is the value of s_1 when the second phase with maintenance is equal to the liquidation value A_1:

$$X_1{*} = \frac{X_1 + \beta}{\alpha - 1} \quad \text{and} \quad A_1{*} = \frac{A_1 + X_1 + \beta}{\alpha}. \tag{39}$$

If $X_1{}^* < A_1{}^* < A_1$, then:

$$V2 = \max[A_1, (\alpha s_1 - \beta - X_1)] = A_1 + \alpha C_1^*, \quad C_1^* = \max[0, (s_1 - A_1^*)], \tag{40}$$

where C^* is a decommissioning option with liquidation value equal to $A_1{}^*$. Note that if we express Equation (40) in terms of S_1 instead of s_1, we obtain Equation (3). This means that, if $X_1{}^* < A_1{}^* < A_1$, the multiple options transform into a single decommissioning option. On the other hand, if $A_1 < A_1{}^* < X_1{}^*$, then:

$$V2 = A_1 + C_2^* + (\alpha - 1)C_3^*, \; C_2^* = \max[(s_1 - A_1), 0] \text{ and } C_3^* = \max[(s_1 - X_1^*), 0], \tag{41}$$

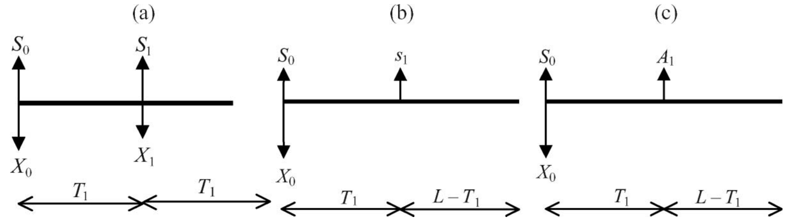

Figure 3. Concentrated cash flows: (a) with maintenance, (b) without maintenance and (c) decommissioning at time T_1 and gaining the liquidation value.

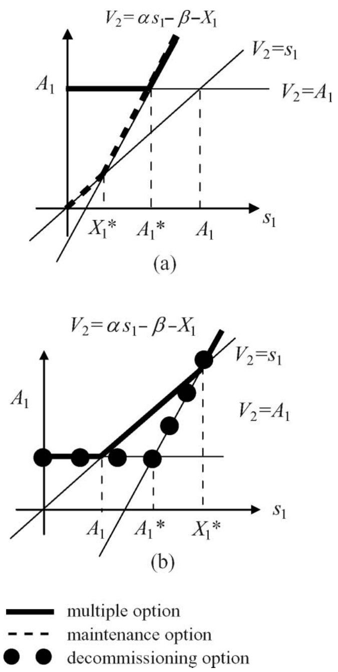

Figure 4. Value of the second phase with maintenance and decommissioning options.

where C_2* and C_3* are decommissioning options with liquidation values equal to A_1 and X_1*, respectively. The total LCCB at the present value is:

$$V_{\mathrm{T}} = S_0 - X_0 + A_1[\exp(-rT_1)] + \alpha C_1, \quad \text{if } X_1* < A_1* \tag{42}$$

and

$$V_{\mathrm{T}} = S_0 - X_0 + A_1[\exp(-rT_1)] + C_2 + (\alpha - 1)C_3, \quad \text{if } X_1* > A_1*. \tag{43}$$

From Figure 4, we can easily notice that the total LCCB considering multiple options is greater than that for single options. For instance, from Equation (2), the value of the second phase with a single maintenance option is max$[(S_1 - X_1), s_1]$ (see dashed line in Figure 4a). In the same way, the value of the second phase with a single decommissioning option is max$[S_1, A_1]$ (see dotted line in Figure 4b).

The formulation presented here for one decision point can be extended to the case of two or more decision points along the service lifetime of the structure. Following a procedure similar to the one given in this paper for the simple maintenance and decommissioning options, a formulation has been derived in Santa-Cruz (2007) for valuating decommissioning options with two decision points. It is shown that such a formulation results in expressions similar to those obtained by Geske (1979) for compound financial options. For more than two decisions points, expressions for valuating the options become more complex. Formulations for such cases are beyond the scope of this paper and should be the subject of further developments.

3. Application of the RO method: offshore jacket platforms

The availability function, $D(t)$, is the probability that the platform jacket will not be in a failure state at time t. For a structure that is repaired after failure and thus restored to an operating state, we have (Ang and Tang 1984):

$$D(t) = 1 - \{\exp[-Q(t)]\} \int_0^t \lambda(\tau)\{\exp[Q(\tau)]\}\mathrm{d}\tau, \quad Q(t) = \int_o^t [\lambda(\tau) + \gamma(\tau)]\mathrm{d}\tau, \tag{44}$$

where $\lambda(t)$ is the hazard function and $\gamma(t)$ is the restoring function. The hazard function is the rate at which a platform jacket in an operating or safety state would go into a failure state at time t. The restoring function is the rate at which a platform jacket in a failure state will be restored to an operating or safety state at time t.

Let $Pa(k)$ be the annual probability of failure of the jacket due to storm loading at the kth year:

$$Pa(k) = \sum_{i=0}^{Nd} P[\text{failure}\,|d(k) = d_i]\, P[d(k) = d_i], \tag{45}$$

where $P[\text{failure} \mid d(k) = d_i]$ is the annual conditional probability of failure of the jacket due to storm loading given damage state $d(k) = d_i$ in the kth year; $P[d(k) = d_i]$ is the probability that damage state d_i is reached in the kth year; and Nd is the number of damage states. Procedures for assessing $P[\text{failure} \mid d(k) = d_i]$ have been devised based on the reserve strength ratio formulation and on the ultimate base shear capacity of the jacket using Monte Carlo simulations (Bea *et al.* 1998, Ayala 2001, Silva and Heredia-Zavoni 2004). Equation (45) neglects correlation between damage states, which is taken to be conservative (Moan *et al.* 1999).

Without loss of generality, let us consider that a damage state is defined as a condition where fatigue failure occurs at one or more joints of the jacket. Let Nc be the total number of joints and d_i be the damage state where joints $\{J\} = \{\text{joint } i1, \text{joint } i2, \ldots, \text{joint } in\}$, $n < Nc$, fail because of fatigue. Extensive literature is available on the assessment of fatigue failure probabilities for joints in jacket platforms using fracture mechanics based on the Paris–Erdogan law (Paris and Erdogan 1963). Assuming independence among fatigue processes in the joints, the probability that damage state d_i is reached in the kth year can be estimated from:

$$P[d(k) = d_i] = P_{i1}(k)P_{i2}(k)\ldots P_{in}(k)\prod_{j=1}^{Nc}[1 - P_j(k)] \quad \text{for } j \neq i1, i2, \ldots in, \tag{46}$$

where $P_j(k)$ is the probability of fatigue damage in joint j in the kth year.

The failure probability of the jacket in $[0,t]$ years is:

$$Pf(t) = 1 - \prod_{j=1}^{t}[1 - Pa(j)]. \tag{47}$$

In terms of the hazard function, the probability of failure of the platform jacket can be expressed as:

$$Pf(t) = 1 - \exp\left[-\int_0^t \lambda(x)\mathrm{d}x\right]. \tag{48}$$

If λ(t) is given at discrete time intervals of 1 year, then:

$$Pf(t) = 1 - \exp\left[-\sum_{i=1}^{t}\lambda(i)\right] = 1 - \prod_{i=1}^{t}\exp[-\lambda(i)]. \tag{49}$$

From Equations (47) and (49) and using a Taylor's series expansion:

$$1 - Pa(i) = \exp[-\lambda(i)] = 1 - \lambda(i) + \frac{\lambda(i)^2}{2} - \frac{\lambda(i)^3}{6} + \ldots. \tag{50}$$

Under the assumption that $\lambda(i)$ is very small compared to 1, higher order terms can be neglected in the series and thus:

$$\lambda(i) \approx Pa(i). \tag{51}$$

Hence, the hazard function can be approximated by the annual probability of failure.

The restoring function $\gamma(t)$ depends on the pdf of repair time, Y. If Y is assumed to be exponentially distributed, the restoring function is constant and equal to the inverse of the mean repair time, $E[Y]$, (Ang and Tang 1984):

$$\gamma(t) = \gamma_0 = \frac{1}{E[Y]}. \tag{52}$$

If the repair time is considered to be a constant, then its density function is given by a Dirac delta function. It has been shown that the difference between these two probability distribution models for the repair time is not significant for most practical cases (Lewis 1987).

4. Illustrative example

Consider an eight-leg jacket platform with $L = 35$ years to be installed at a site in the Bay of Campeche in the Gulf of Mexico. The jacket has two longitudinal and four transverse frames. The platform is installed in a water depth of 40 m and is to produce 4.8×10^6 m^3 of gas per day. The jacket structure is exposed to fatigue deterioration due to stress cycles from wave loading, which may produce cracks in the welded joints of the jacket. For the purpose of the example, suppose that three joints of the jacket have been identified as critical for the structural integrity of the platform and therefore selected for future inspections. There are 8 (2^3) possible damage states defined by the combinations of fatigue failure of the three joints. Table 3 describes each damage state. The variation of the probability of fatigue joint failure with time and the conditional probabilities of platform failure given a fatigue damage state were taken from a detailed reliability analysis of steel jacket platforms in the Bay of Campeche (Ayala 2001, Heredia-Zavoni and Montes 2002, Silva and Heredia-Zavoni 2004, Heredia-Zavoni *et al.* 2008). In these studies, the platform capacity is characterised in terms of the ultimate base shear for each damage state. The conditional probabilities of jacket failure are then obtained by means of Monte Carlo simulations considering an extreme value probability distribution for wave heights according to hindcast metocean data for the site and a lognormal probability distribution for the base shear resistance. Fatigue failure probabilities of joints are obtained based on the Paris–Erdogan formulation for crack growth. Figure 5 shows the probabilities that damage states numbers 1, 4 and 7 develop, that no fatigue damage develops, as well as the annual probability of failure of the platform as a function of time.

Table 3. Damage states.

State	Description
*d*0	Intact structure
*d*1	Fatigue failure of joint 1
*d*2	Fatigue failure of joint 2
*d*3	Fatigue failure of joint 3
*d*4	Fatigue failure of joints 1 and 2
*d*5	Fatigue failure of joints 1 and 3
*d*6	Fatigue failure of joints 2 and 3
*d*7	Fatigue failure of joints 1, 2 and 3

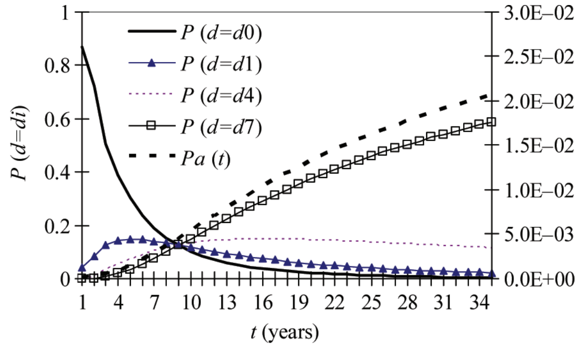

Figure 5. Probability of occurrence of damage states and annual probability of failure.

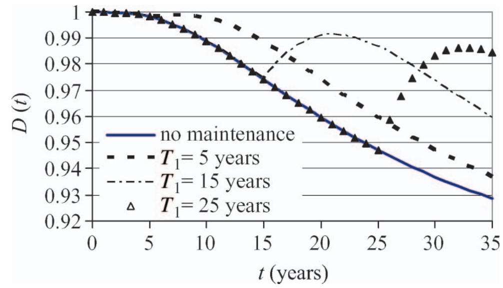

Figure 6. Platform availability with maintenance at T_1.

The probability of fatigue failure of joint i after ideal maintenance P_i^M(t), is:

$$P_i^{\mathrm{M}}(t) = P_i * (t - T_1) \quad \text{for} \quad t > T_1, \tag{53}$$

where P_i* is the probability of fatigue failure of the joint updated using the inspection findings at time T_1. A Bayesian formulation can be used for such updating procedure (Zhang and Mahadevan 2000, Heredia-Zavoni and Montes 2002, 2004). If no maintenance action is taken, then:

$$P_i^{\mathrm{M}}(t) = P_i * (t) \quad \text{for} \quad t > T_1. \tag{54}$$

For the purpose of this application and for the sake of simplicity, no Bayesian updating is performed for this example and it is simply taken that $P_i^{\mathrm{M}}(t) = P_i*(t - T_1) = P_i(t - T_1)$, for $t > T_1$.

The hazard function was obtained from the annual probability of failure shown in Figure 5. Suppose that the mean time for full reposition of the platform after a structural collapse is 4 years; thus $\gamma(t) = 0.25$. Figure 6 shows plots of the availability function of the structure when maintenance of the three critical joints is carried out at time T_1. It is interesting to note the increase in the availability of the platform structure after maintenance is performed. Suppose that a minimum availability of 0.97 was required, then first maintenance of the three critical joints should be performed in the 15th year. However, if an availability lower than 0.995 was not allowed, then first maintenance of the three joints should be carried out in the fifth year.

Suppose the decision maker is to assess the LCCB considering the option of providing maintenance at $T_1 = 15$ years, based on the findings of a planned fatigue inspection of the platform jacket. The maintenance cost, X_1, is uncertain, and its possible values depend on the fatigue crack size to be measured at a future inspection. For this example, we consider three possible repair actions and costs: (1) the crack size is less than the minimum detectable one and no repair action is taken, $X_1 = 0$; (2) the detected crack size is less than a repair threshold and the joint is grouted, $X_1 =$ US\$15,000; and (3) the detected crack size exceeds the repair threshold and a bracket is placed, $X_1 =$ US\$30,000. The probability that any of these repair actions is taken is equal to the probability that the fatigue crack size is less than the detectable one, or less (greater) than the repair threshold. Since there are three critical joints and three repair actions, then there are 27(3^3) possible maintenance programs. The total cost of each maintenance program and its corresponding probability are listed in Table 4. The annual probability of failure of the platform for some of the maintenance programs is shown in Figure 7. Ideal maintenance of the three joints brings the structure back to its original condition, hence, $Pa(t) = Pa(t - T_1)$ for $t > T_1$. Availability functions for some of these maintenance programs are shown in Figure 8. As seen, the maximum values of availability are obtained when the three critical joints are all given ideal maintenance.

The availability functions are used for the assessment of the maintenance option values and the computation of the LCCB for each possible maintenance program. Table 5 shows data and results for maintenance program number 27. The corresponding results for all maintenance programs are listed in Table 4. The total expected LCCB at the present value is then given by:

$$E[V_{\mathrm{T}}] = \sum E[V_{\mathrm{T}i}] P_p, \tag{55}$$

where $E[V_{\mathrm{T}i}]$ and P_p are the expected LCCB for each maintenance program and the corresponding probability of these programs, respectively. Using Equation (55), the LCCB with the maintenance option is found to be US\$4442 $\times$ 10^6. On the other hand, the NPV considering all possible maintenance programs is US\$4409 $\times$ 10^6. Thus, using the NPV method underestimates the total LCCB by US\$33 $\times$ 10^6.

Suppose now that at $T_1 = 15$ years, the decision maker is to decide whether to decommission or to

Table 4. Option values and LCCB for the various maintenance programs at $T_1 = 15$ years.

	Maintenance of joint 1		Maintenance of joint 2		Maintenance of joint 3					
Program	Cost ($\times 10^3$ \$US)	Probability	Cost ($\times 10^3$ \$US)	Probability	Cost ($\times 10^3$ \$US)	Probability	Total cost X_1 ($\times 10^3$ \$US)	Probability	Option value ($\times 10^6$ \$US)	LCCB ($\times 10^6$ \$US)
1	0	0.177	0	0.177	0	0.177	0	0.0055	0	4350.00
2	0	0.177	0	0.177	15	0.177	15	0.0055	58.34	4408.34
3	0	0.177	0	0.177	30	0.647	30	0.0202	58.34	4408.33
4	0	0.177	15	0.177	0	0.177	15	0.0055	58.35	4408.35
5	0	0.177	15	0.177	15	0.177	30	0.0055	85.54	4435.54
6	0	0.177	15	0.177	30	0.647	45	0.0202	85.54	4435.53
7	0	0.177	30	0.647	0	0.177	30	0.0202	58.34	4408.34
8	0	0.177	30	0.647	15	0.177	45	0.0202	85.54	4435.53
9	0	0.177	30	0.647	30	0.647	60	0.0739	85.53	4435.53
10	15	0.177	0	0.177	0	0.177	15	0.0055	58.37	4408.37
11	15	0.177	0	0.177	15	0.177	30	0.0055	85.61	4435.61
12	15	0.177	0	0.177	30	0.647	45	0.0202	85.60	4435.60
13	15	0.177	15	0.177	0	0.177	30	0.0055	85.62	4435.62
14	15	0.177	15	0.177	15	0.177	45	0.0055	101.86	4451.85
15	15	0.177	15	0.177	30	0.647	60	0.0202	101.85	4451.85
16	15	0.177	30	0.647	0	0.177	45	0.0202	85.62	4435.61
17	15	0.177	30	0.647	15	0.177	60	0.0202	101.85	4451.84
18	15	0.177	30	0.647	30	0.647	75	0.0739	101.84	4451.84
19	30	0.647	0	0.177	0	0.177	30	0.0202	58.37	4408.37
20	30	0.647	0	0.177	15	0.177	45	0.0202	85.60	4435.60
21	30	0.647	0	0.177	30	0.647	60	0.0739	85.59	4435.59
22	30	0.647	15	0.177	0	0.177	45	0.0202	85.62	4435.61
23	30	0.647	15	0.177	15	0.177	60	0.0202	101.85	4451.86
24	30	0.647	15	0.177	30	0.647	75	0.0739	101.84	4451.84
25	30	0.647	30	0.647	0	0.177	60	0.0739	85.61	4435.61
26	30	0.647	30	0.647	15	0.177	75	0.0739	101.84	4451.84
27	30	0.647	30	0.647	30	0.647	90	0.2704	101.83	4451.83

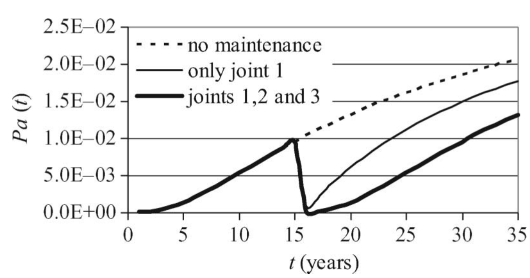

Figure 7. Annual probability of failure for some maintenance programs.

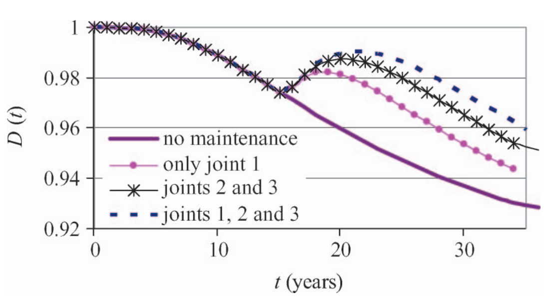

Figure 8. Platform availability for some maintenance programs at $T_1 = 15$ years.

carry out maintenance program number 27, continuing with operations until the end of the service time. Consider that, if decommissioned, the platform jacket can be re-used in a new site. At the present time, the decision is uncertain and depends on the conditions in the 15th year of production. Suppose the jacket will be worth US\$100 $\times$ 10^6 at that time. In this case, $A_1 =$ US\$100 $\times$ 10^6, $S_0 =$ US\$2579 $\times$ 10^6 and $X_1 =$ US\$90,000. The option value turns to be US\$3143 $\times$ 10^6 and the total LCCB is therefore US\$5569 $\times$ 10^6. The NPV corresponding to maintenance program 27 is only US\$4416 $\times$ 10^6, thus yielding a LCCB that is US\$1153 $\times$ 10^6 less than the one considering the decommissioning option. As seen, using the NPV method in this case will underestimate significantly the total LCCB when decommissioning is a RO in the future.

It is interesting to analyse the variation of the LCCB with decommissioning time, T_1. If the decision on whether to decommission the platform is made at the beginning of the service lifetime, $T_1 = 0$, then $S_0 = 0$ and S_1 is a deterministic variable since it depends on the hydrocarbon price at the present time, which is known. The value of C_D is simply $S_1 - A_1$.

Table 5. Valuation of LCCB with a maintenance option.

Variable	Value	Units
N	62,050,000 × 10^3	p^3
CR	50 × 10^6	US$
CI	10 × 10^3	US$
δ	0.1	
r	0.05	
μ	0.07	
P_0	3.5	US$
$E(p_1)$	10.00178	US$
S_0	2,579,116,976.30	US$
X_0	200 × 10^6	US$
s_1	8,832,877,124.43	US$
Δn_1	27,671,858.11	p^3
Δg_1	679,001.88	US$
Δm_1	11,316,698.03	US$
X_1	90,000.00	US$
$\underline{s}_1$	287,405,503.04	US$
S_1	9,120,282,627.46	US$

Hence, the total LCCB is $VTP = 0 - X_0 + A_1 + S_1 - A_1 = S_1 - X_0$, which is equal to the NPV. When the decision to continue operating the platform or to decommission it is made at the end of the service life, ($T_1 = L = 35$ years), S_0 becomes deterministic and $S_1 = 0$. The option value C_D is zero, and the total LCCB with the decommissioning option is $S_0 - X_0 + A_1\exp(-rL)$, which is equal to the NPV plus the present value of liquidation. Thus, at $T_1 = L$, the difference between the RO approach and the NPV method is the present value of liquidation. For the example, the total LCCB with the decommissioning option at 35 years is US$4433 × 10^6 and the NPV is US$4416 × 10^6; the difference being US$17 × 10^6, which is equal to the liquidation value at the present time discounted by the risk-free rate. Figure 9 shows the variation of the LCCB with the decommissioning option as a function of time T_1; the NPV is also shown. This plot indicates that the best time to decide whether to decommission or not is approximately at the 18th year.

We will now assess the LCCB considering both the maintenance and decommissioning options. In this case, the decision maker may choose to decommission, to continue operations with maintenance or to continue operations without maintenance. Consider X_1 = US$90,000, liquidation value A_1 = US$100 × 10^6 and $T_1 = 15$ years. Parameters and variables used for valuation are listed in Table 6. The LCCB considering the options is found to be equal to US$5500 × 10^6 and the NPV is US$4416 × 10^6. Thus, considering the managerial flexibility to carry out, or not carry out, maintenance or to decommission gives an additional value of US$1153 × 10^6 with respect to the NPV approach. A comparison of the LCCB for the different options analysed is shown in Table 7. For consistency, the LCCB for the

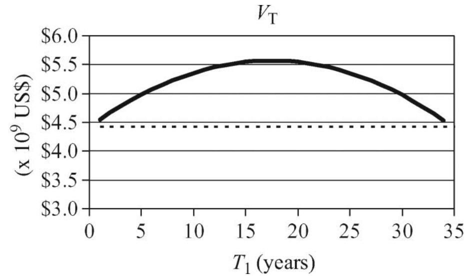

Figure 9. LCCB with decommissioning option at $t = T_1$. —, LCCB with deccommissioning option; - - -, NPV.

Table 6. Valuation of LCCB with decommissioning and maintenance options.

Variable	Value	Units
A_1	100,000,000.00	US$
n_1	25,924,670	m^3
n_1*	25,116,666	m^3
g_1	25,531,768.72	US$
g_1*	24,819,816.21	US$
m_1	7,163,096.64	US$
m_1*	19,028,971.81	US$
α	1.032170042	–
β	711,952.51	US$
γ	11,865,875.16	US$
X_1*	–343,920,055.47	US$
A_1*	86,164,172.30	US$
C_1	3,044,887,723.26	US$
LCCB	5,569,197,752.28	US$

Table 7. LCCB for the different options.

Options	Value (× 10^6 US$)
None: net present value	4416
Decommissioning option only	5569
Maintenance option only	4451
Maintenance and decommissioning options	5569

maintenance option in Table 7 has been assessed considering maintenance program number 27. The least LCCB is the one computed using the NPV method. The LCCB values for the decommissioning option only and for both maintenance and decommissioning options are, in this case, the same because $X_1^* < A_1^* < A_1$.

5. Conclusions

Maintenance and decommissioning real option (RO) models have been developed in this paper for life-cycle cost-benefit (LCCB) analysis of offshore platforms. The modelling of the maintenance and decommissioning options has been based on an analogy with financial options, and the Black and Scholes (1973) formulation for the value of call options has been used. Uncertainty about economic, as well as engineering, variables is taken into account in the maintenance and decommissioning RO models. Economic variables include hydrocarbon prices and maintenance costs of the platform structure, which depend on findings from future inspections. Engineering variables are related to environmental loading, structural capacity, and deterioration due to fatigue; these determine the probability of fatigue damage states being developed and the conditional probability of failure of the structure given damage states, which are used in the RO formulation. Expressions have been derived for the expected cash flow of benefits and costs to be used in the maintenance and decommissioning RO formulation; such expressions are given in terms of the availability function of the platform structure, which depends on the hazard and restoring functions, and the annual probability of failure of the platform jacket. The effect of maintenance is accounted for in the availability function and therefore in the expected cash flow estimation.

An application example has shown the advantage of considering the maintenance and decommissioning RO approach in the assessment of the LCCB as compared to the net present value (NPV) method. Numerical results have indicated that the total LCCB can be underestimated significantly with the NPV method. The RO approach has been shown to provide a framework to account properly for managerial options and flexibility to adapt to future conditions along the service lifetime of the structure. The RO models developed here for assessing the LCCB, which account for uncertainties in both financial and engineering variables, can also be used as a rational basis for decision making regarding maintenance and decommissioning for offshore platform structures. The formulation can also be extended to project valuation and decision making problems for other type of engineering infrastructure.

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Appendix 1. Call option valuation

We have the following equation:

$$E[c] = [\exp(-rT)] \int_X^{\infty} (S - X) f_S(S) \mathrm{d}S, \tag{A1}$$

where X is a constant and S is a random variable with pdf:

$$f_S(x) = \frac{1}{D\sqrt{2\pi}(x+g)} \exp\left\{ -\left[\ln\left(\frac{x+g}{n}\right) - M \right]^2 / 2D^2 \right\}, \quad -g < x < \infty, \tag{A2}$$

where M, D, g, and n are constants. From Equation (A1):

$$E[c] = [\exp(-rT)] \int_X^{\infty} S f_S(S) \mathrm{d}S - \exp[-rT] \int_X^{\infty} X f_S(S) \mathrm{d}S \tag{A3}$$

$$\begin{aligned} &= [\exp(-rT)] \int_X^{\infty} (S+g) \frac{1}{D\sqrt{2\pi}(S+g)} \\ &\quad \times \exp\left[-\left(\ln\left(\frac{S+g}{n}\right) - M \right)^2 / 2D^2 \right] \mathrm{d}S \\ &\quad - [\exp(-rT)] \int_X^{\infty} (g+X) \frac{1}{D\sqrt{2\pi}(S+g)} \\ &\quad \times \exp\left[-\left(\ln\left(\frac{S+g}{n}\right) - M \right)^2 / 2D^2 \right] \mathrm{d}S \end{aligned} \tag{A4}$$

Consider the change of variables:

$$\frac{S+g}{n} = \exp\omega \quad \text{and} \quad \frac{\mathrm{d}S}{n} = \exp\omega \, \mathrm{d}\omega, \tag{A5}$$

then:

$$z = \frac{\omega - \left(M + D^2\right)}{D} \quad \text{and} \quad \mathrm{d}z = \frac{\mathrm{d}\omega}{D}. \tag{A6}$$

For the first term on the right-hand side of Equation (A4), we obtain:

$$n[\exp(-rT)] \left[\exp\left(\frac{2M + D^2}{2} \right) \right] \Phi \left[\frac{-\ln\left(\frac{X+g}{n}\right) + \left(M + D^2\right)}{D} \right], \tag{A7}$$

where Φ is the standard normal probability distribution function. Using the change of variables:

$$S + g = \exp\omega, \quad \text{and} \quad \mathrm{d}S = \exp\omega \, \mathrm{d}\omega, \tag{A8}$$

then:

$$z = \frac{\omega - (\ln(n) + M)}{D} \quad \text{and} \quad \mathrm{d}Z = \frac{\mathrm{d}\omega}{D}, \tag{A9}$$

The second term on the right-hand side of Equation (A4) transforms into:

$$(g + X)[\exp(-rT)] \Phi \left[-\frac{\ln(X+g) - (\ln(n) + M)}{D} \right]. \tag{A10}$$

Then, replacing Equations (A7) and (A10) in (A4), we obtain:

$$\begin{aligned} E[c] &= n[\exp(-rT)] \left[\exp\left(\frac{2M + D^2}{2} \right) \right] \\ &\quad \times \Phi \left[\frac{-\ln\left(\frac{X+g}{n}\right) + \left(M + D^2\right)}{D} \right] \\ &\quad - (X+g)[\exp(-rT)] \Phi \left[-\frac{\ln(X+g) - (\ln(n) + M)}{D} \right]. \end{aligned} \tag{A11}$$