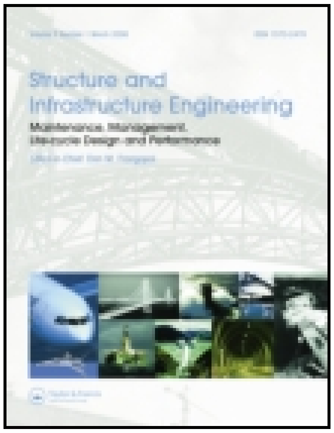

Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/nsie20

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Published online: 26 Nov 2014.

To cite this article: Dilum Fernando, Bryan T. Adey & Nam Lethanh (2015) A model for the evaluation of intervention strategies for bridges affected by manifest and latent deterioration processes, Structure and Infrastructure Engineering: Maintenance, Management, Life-Cycle Design and Performance, 11:11, 1466-1483, DOI: 10.1080/15732479.2014.976576

To link to this article: http://dx.doi.org/10.1080/15732479.2014.976576

Structure and Infrastructure Engineering, 2015
Vol. 11, No. 11, 1466–1483, http://dx.doi.org/10.1080/15732479.2014.976576

A model for the evaluation of intervention strategies for bridges affected by manifest and latent deterioration processes

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(*Received 18 March 2014; final version received 19 June 2014; accepted 23 July 2014; published online 26 November 2014*)

Markov models are often used in bridge management systems to evaluate intervention strategies (ISs) for bridges affected by manifest deterioration processes (MnDPs). These models do not directly take into consideration the effect of latent deterioration processes (LtDPs) on the object, i.e. the deterioration that might occur due to natural hazards (e.g. earthquakes and floods). In cases where there is a negligible probability of the occurrence of natural hazards, this is justified, otherwise it is not. In this paper, a model is proposed that can be used to evaluate ISs for bridge elements and bridges considering both MnDPs and LtDPs. The model is an extension of the Markov models, and includes condition states (CSs) that can occur due to both MnDPs and LtDPs, as well as the probabilities of transition (p.o.ts) between them. The contributions to the p.o.ts due to MnDPs are initially estimated using well-established methods and adjusted for the contributions to the p.o.ts due to LtDPs, which are estimated using fragility curves and adjusted considering element dependencies, i.e. how the elements of a bridge work together. The use of the model is demonstrated by predicting the future CSs of a bridge affected by both MnDPs and LtDPs.

Keywords: Markov models; infrastructure management; bridge management; hazard risks; optimal intervention strategy; fragility curves

1. Introduction

In bridge management, agencies are required to make intervention decisions for their bridges over the planning horizon. Such decisions are often based on optimisation, where expected costs, consisting of agency costs, user and public costs, are minimised and future expected costs are based on a set of bridge element deterioration models (Markow, Madanat, & Gurenich, 1993; Thompson, Small, Johnson, & Marshall, 1998). The deterioration models used for bridge elements can be either deterministic or probabilistic. Probabilistic models are preferred over the deterministic models, as the former allowed the uncertainties associated with deterioration of bridge elements to be considered.

A large fraction of the literature on bridge element deterioration has assumed that bridge element deterioration can be represented by a Markov process (Fruguglietti, Pasqualato, & Spallarossa, 2012; Golabi & Shepard, 1997; Hawk & Small, 1998; Madanat, Mishalani, & Ibrahim, 1995; Thompson et al., 1998). In discrete time Markov chain, the deterioration of a bridge element is modelled by determining the probability that the element will pass from one discrete condition state (CS) to another in a fixed period of time, where the CSs are defined using values of performance indicators (e.g. visual indicators of corrosion, roughness and cracking of the road surface) (Kobayashi, Kaito, & Lethanh, 2012a, 2012b; Kuhn, 2010). Markov models are currently used, however, only to model manifest deterioration processes (MnDPs)[3] (Kobayashi et al., 2012a, 2012b; Tsuda, Kaito, Aoki, & Kobayashi, 2006). Bridge elements can deteriorate due to both MnDPs and latent deterioration processes (LtDPs).[4]

Therefore, the optimal intervention strategies (OISs) derived from such models may not be the optimal one, especially for bridges elements and bridges located in regions with high probability of hazard occurrences. It is already shown that the failure probability of a bridge due to hazard occurrence depends on the condition of the bridge (Korup & Clague, 2009; Schubert, Faber, Jacquemoud, & Straub, 2010), which also means that the failure probability of a bridge due to hazard occurrence depends on how the bridge is maintained, thus the intervention strategies (ISs). Therefore, determination of the OISs must consider both MnDPs and LtDPs. This phenomenon has been demonstrated in other literature, particularly with the developments of block replacement models and age replacement models used in the field of facility management (Berg & Epstein, 1978; Chen & Savits, 1992; Gertsbakh, 1997, 2000; Kaio & Osaki, 1984).

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In recent years, attempting to address the need of our society to have proactive responses to natural hazards, bridge managers have put their focus on assessing and managing hazard risks to bridge structures and developing methods to incorporate hazards into existing bridge management systems (BMSs) (Biringer, Vugrin, & Warren, 2013; Taylor, Werner, & Graf, 2006; Thompson, Rogers, & Thomas, 2012). Work has been already carried out on risk assessment of hazard events using probabilistic methods (Castelli & Scavia, 2008; Faber & Stewart, 2003; Korup & Clague, 2009; Schubert et al., 2010). Existing studies have used probabilistic models to quantify the occurrence of hazards (Bayraktarli & Faber, 2011; Graf, Nishijima, & Faber, 2009; Korup & Clague, 2009; Zucchini & MacDonald, 2009). Once the probability of occurrence is known, different methodologies may be used to estimate the impact of a hazard on structures. Use of fragility curves, to date, is one of the most popular methodologies to estimate the failure probability of a structure due to hazard risks (Choi, DesRoches, & Nielson, 2004; Graf et al., 2009; Karim & Yamazaki, 2001).

Application of the fragility curves in structural analysis has been more to the design stage of the structures. Only few attempts have been made on application of fragility curves to the operational stage, where preventive and corrective interventions need to be executed at some point in time to reduce the negative impacts of both MnDPs and LtDPs. To the authors' knowledge, only two studies (Mayet & Madanat, 2002; Lethanh, Adey, & Fernando, 2014) have attempted to consider deterioration due to both MnDPs and LtDPs in determining OISs for bridges using the Markov model. The LtDPs were incorporated into the Markov model by adding a CS to represent the failure of the object. Fragility curves together with the hazard risk curve were used to estimate the failure probability. Even though both these models (Mayet & Madanat, 2002; Lethanh et al., 2014) significantly improved the existing knowledge on modelling hazard risks in BMSs, in both models the probability of transition (p.o.t) of an element from one CS to another is considered to be dependent only on the current CS of that element. This is not true for real bridges, where the failure of an element may cause the failure of other elements or the entire bridge.

In this paper, a model is proposed that can be used to evaluate ISs for bridge elements that are affected by both MnDPs and LtDPs. The model is developed based on the Markov models, which are widely used in existing state-of-the-art BMSs (Hawk & Small, 1998; Hajdin, 2006; Thompson et al., 1998). The three main differences between the proposed model and those currently used are as follows:

(1) Two sets of CSs, as opposed to one, are considered. Set 1 consists of the CSs in which it is not necessary to intervene immediately, which may occur due to either MnDPs or may or may not occur due to LtDPs, (called S1CSs hereafter). Set 2 consists of the CSs in which it is necessary to intervene, immediately. It is considered that these may occur only due to LtDPs (called S2CSs). This is because if only MnDPs were at work, these CSs would not be allowed to occur.
(2) The effect of the structural failure of one element on that of other elements, and, therefore, on the probability of bridge failure, is taken into consideration, as opposed to the current assumption that the performance of all elements is independent.
(3) The element p.o.ts change over time as a function of the failure probability of the other elements.

Although it is realised by the authors that dealing with LtDPs in the management of infrastructure properly requires consideration of all objects in the networks and how they are connected, the proposed methodology is seen as an incremental improvement on existing methodologies in BMSs.

The paper is structured as follows: in Section 2 the conventional Markov model is explained to give an overview of the state of the art. In Section 3, the new model to be used to evaluate ISs for objects affected by both MnDPs and LtDPs is presented. In Section 4, the methodology to be used to estimate the p.o.ts due to LtDPs is presented. In Section 5, the proposed model is demonstrated by evaluating ISs for a 29-m long two-lane bridge with an orthotropic deck supported on a steel box girder, supported on two abutments. The MnDPs to which the bridge elements are subjected are corrosion and cracking of reinforced concrete, paint deterioration and corrosion of the steel girders, and cracking of overlay. The LtDP to which the bridge elements are subjected is rock falls.

2. State-of-the-art

2.1 Evaluation of ISs for elements affected by MnDPs

2.1.1 General

One popular way to model deterioration in BMSs is to use a Markov model (Golabi & Shepard, 1997; Hajdin, 2006; Kobayashi et al., 2012a; Lethanh, 2009; Thompson et al., 1998; Tsuda et al., 2006), where the deterioration of an element is represented by the p.o.ts between S1CSs. In the current BMSs, such models are only used in modelling deterioration due to MnDPs and the p.o.ts are determined from past condition data, or in the absence of such data, by using mechanistic/empirical models or by expert opinion (Kobayashi, Kaito, & Lethanh, 2012b; Tsuda et al., 2006).

2.1.2 Condition states

The change in the condition of elements due to MnDPs is modelled as transitions between S1CSs. An example of

Table 1. CS descriptions for a painted steel girder.

CS	Description
1	Paint system sound and functioning as intended.
2	Surface or freckled rust has formed.
3	Paint system badly deteriorated/exposed metal but no significant section loss.
4	Significant section loss.
5	Alarming section loss.

S1CSs for a painted steel girder due to the MnDP steel corrosion is given in Table 1.

2.1.3 *Deterioration matrix*

The speed of deterioration is represented as the p.o.ts between CSs i, j $(i, j = 1, \ldots, I)$, when no interventions are to be executed, where $i = 1$ is the best CS and $i = I$ is the worst CS. The probability of an element being in *CS j* at time τ_B is conditional on *CS i* at τ_A:

$$\text{Prob}\,[\text{CS}\,(\tau_B) = j \quad | \quad CS\,(\tau_A) = i] = p_{ij}. \tag{1}$$

The p.o.ts due to deterioration are given in cardinal form as

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1I} \\ 0 & p_{22} & \cdots & p_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}, \tag{2}$$

subject to the following constraints:

$$\begin{cases} \sum_{j=1}^{I} p_{ij} = 1, & \forall i, j \\ p_{ij} = 0, \quad \text{when} & (i > j) \end{cases} \tag{3}$$

.

2.1.4 *Improvement matrix*

The improvement to be expected from the following IS d $(d \in D)$ is represented as the p.o.ts between CSs, when interventions are executed. The p.o.ts due to the execution of intervention, in cardinal form, are given by

$$R^d = \begin{pmatrix} r_{11}^d & r_{12}^d & \cdots & r_{1I}^d \\ r_{21}^d & r_{22}^d & \cdots & r_{1I}^d \\ \vdots & \vdots & \ddots & \vdots \\ r_{I1}^d & r_{I2}^d & \cdots & r_{II}^d \end{pmatrix}, \tag{4}$$

where r_{ij}^d is the p.o.t of an element in CS i to CS j in one unit of time when an IS d is followed.

2.1.5 *Deterioration–improvement matrix*

In order to evaluate the ISs, the deterioration and intervention matrices can be combined. The resulting deterioration–improvement matrix, Q^d, is given by

$$Q^d = P^d + R^d = p_{ij}^d + r_{ij}^d, \tag{5}$$

where P^d is the modified deterioration matrix (the values of all p.o.ts in which an intervention is to occur are set to 0), based on the IS d, P_{ij}^d are the p.o.ts in matrix P^d, i, j indicate the CSs in which the element can be and subject to the following constraints:

$$\begin{cases} r_{ij}^d \geq 0, \sum_{j=1}^{I} r_{ij}^d = 1, p_{ij}^d = 0 & \forall j, i = i' \\ r_{ij}^d = 0, p_{ij}^d = p_{ij} & \forall j, i \neq i' \end{cases}, \tag{6}$$

where i' indicates the CSs in which interventions are to be executed.

2.1.6 *Future expected cost*

The expected cost of following IS d in year t, $V^d(t)$, can be estimated by multiplying the CS probability vector $\phi_i(t)$ (i.e. the probability of being in CS i at time t) with the units costs of interventions, as represented in the cost vector c_i^d:

.

$$E[V^d(t)] = \sum_{i=1}^{I} \phi_i(t) c_i^d \tag{7}$$

2.1.7 *Total expected cost*

The total term expected cost of IS d, $TC(d \in D)$ is then defined as

$$\begin{aligned} TC(d \in D) &= \sum_{t=0}^{T} \frac{1}{(1+\gamma)^t} . E[V^d(t)] \\ &= \sum_{t=0}^{T} \frac{1}{(1+\gamma)^t} . \sum_{i=1}^{I} \phi_i(t) c_i^d, \end{aligned} \tag{8}$$

where γ represents the discount rate.

2.2 *Current research*

2.2.1 *Management systems*

Some initial work in considering the effects of LtDPs in the determination of OISs can be found in Mayet and Madanat (2002) and Adey, Bernard, and Gerard (2003). Other research, in which it has been attempted to develop a model to determine OISs for structures taking into consideration LtDPs can be found in Mayet and Madanat (2002) and Lethanh et al. (2014).

The model proposed by Lethanh et al. (2014) is based on the one initially proposed by Mayet and Madanat (2002).

It is based on the Markov model and two sets of CSs; Set 1 consists of the CSs (similar to S1CSs in this study) in which it is not necessary to intervene immediately, and Set 2 consists of CSs (similar to S2CSs in this study) in which it is necessary to intervene, immediately. It is assumed that the S2CSs can occur only due to LtDPs.

In the model proposed by Mayet and Madanat (2002), it is assumed that the p.o.ts between S1CSs can be determined from data and are exclusively used to model the change in condition of a bridge due to MnDPs. The p.o.ts from S1CSs to S2CSs are used to model the change in condition of a bridge due to LtDPs. There is no indication as to how the p.o.ts for a bridge should be estimated, taking into consideration how the elements within a structure are interrelated.

In both models the p.o.t of an element from one CS to another depends only on the current CS of that element. This is no longer a valid assumption if the probability of an element entering a S2CS depends on the condition of the other elements.

2.2.2 *Latent deterioration processes*

In order to evaluate ISs, and to determine OISs for bridges that are affected by both MnDPs and LtDPs, it is necessary to understand both the severity of the LtDPs, their probability of occurrence and how these processes can affect the element and bridge performance. There have been significant efforts made over the last few years, with respect to both of these items; the former resulting in the development of risk maps and risk curves (Frankel et al., 1997; Kron, 1997), and the later in development and use of fragility curves (Choi et al., 2004; Ghosn, Moses, & Wang, 2003; HAZUS, 1997; Karim & Yamazaki, 2001; Schultz, Gouldby, Simm, & Wibowo, 2010).

3. The proposed model

3.1 General

In order to consider how bridge elements are affected by LtDPs, in addition to MnDPs, the model to be used to evaluate ISs should include both the S1CSs of bridge elements that may occur due to MnDPs and may or may not occur due to LtDPs, the S2CSs of bridge elements that may occur only due to LtDPs. It is considered that if the element is in a S2CS, immediate intervention will be necessary and if the element is in a S1CS, then immediate intervention is not necessary. An example of the CSs for a steel-reinforced concrete abutment is given in Table 2. In this example, CSs 4 and 5 defined in Table 2 for an abutment cannot occur due to an earthquake, but CSs 6 and 7 could. CSs 6 and 7 would not normally be considered if the object was only affected by MnDPs as an intervention would always be executed in one of the CSs with less serious damage to ensure that an adequate level of service was provided (i.e. CSs 1–5 in Table 2).

Table 2. CS definition for an abutment.

CS		Description
S1CS	1	Any cracking that is present is only superficial. In good condition.
	2	Minor cracks (1/16”–1/8”). Minor spalling.
	3	Moderate cracks (1/8”–1/4”). Moderate spalling, but no exposed re-bars.
	4	Spalling with exposed re-bars and active corrosion.
	5	Severe spalling with advanced re-bar corrosion.
S2CS	6	Excessive spalling
	7	Failure

3.2 *Deterioration matrix*

The p.o.ts used to model MnDPs and LtDPs are given in Equation (9):

$$P = p_{ij}$$

$$= \left(\begin{array}{cccc|cccc} p_{11} & p_{12} & \cdots & p_{1I} & p_{1,I+1} & p_{1,I+2} & \cdots & p_{1,I+K} \\ 0 & p_{22} & \cdots & p_{2I} & p_{2,I+1} & p_{2,I+2} & \cdots & p_{2,I+K} \\ 0 & 0 & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & p_{II} & p_{I,I+1} & p_{I,I+2} & \cdots & p_{I,I+K} \\ \hline 0 & 0 & 0 & 0 & p_{I+1,I+1} & p_{I+1,I+2} & \cdots & p_{I+K,I+K} \\ 0 & 0 & 0 & 0 & 0 & p_{I+1,I+2} & \cdots & p_{I+K,I+K} \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right), \tag{9}$$

where p_{ij} represents the p.o.ts between CSs, $i = (1, \cdots, I)$ indicates the S1CSs, $k = (1, \cdots, K)$ indicates the S2CSs and $_K$ indicates the worst possible S2CS, which is typically taken as the failure of the element

subject to the following constraints:

$$\begin{cases} \sum_{j=1}^{I+K} p_{ij} = 1 & \forall \mathrm{i,j} \in [1, I+K] \\ p_{ij} = 0 \quad \text{when} & (i > j) \end{cases} \tag{10}$$

Equation (9) can be summarised as shown in Equation (11):

$$P = \left(\frac{P_1 | P_2}{P_3 | P_4}\right), \tag{11}$$

where the p.o.ts are represented in condensed form as follows: P_1 from S1CS to S1CSs, which can be non-zero due to MnDPs or LtDPs, P_2 from S1CS to S2CSs, which are zero due to MnDPs and can be non-zero due to LtDPs,

P_3 from S2CS to S1CSs, which are always 0 and P_4 from S2CS to S2CSs, which are zero due to MnDPs and can be non-zero due to LtDPs.

The deterioration matrix due to an MnDP alone (called MDM hereafter) has the values only in first quadrant and can be reduced to the form given by Equation (2), while the deterioration matrix due to LtDP alone (called LDM hereafter) takes the form of Equation (9) with satisfying the conditions given in Equation (10). The combined deterioration matrix can be obtained by the multiplication of the MDM (in its full form with all four quadrants) and the LDM. The p.o.ts due to LtDPs are determined using fragility curves as described in Section 4.

3.3 Improvement matrix

Improvement due to the execution of interventions when elements are in S1CSs, i.e. due to following IS d ($d \in D$) and elements are in S2CSs, i.e. IS a ($a \in A$), is modelled with the p.o.ts as shown in Equation (12) in cardinal form. It is assumed that an intervention executed when a bridge element is in any S2CS will improve the element to a S1CS. The probabilities that the element is in each CS at the beginning of the next time interval following the execution of the intervention are represented by R^d when the element is in an S1CS at the time of intervention and R^a when the element is in an S2CS at the time of intervention:

$$R^I = r_{ij}^I = \left[\begin{array}{cccc} r_{11}^d & r_{12}^d & \cdots & r_{1I}^d \\ r_{21}^d & r_{22}^d & \cdots & r_{2I}^d \\ \vdots & \vdots & \ddots & \vdots \\ r_{I1}^d & r_{I2}^d & \cdots & r_{II}^d \\ \hline r_{I+1,1}^a & r_{I+1,2}^a & \cdots & r_{I+1,I}^a \\ r_{I+2,1}^a & r_{I+2,2}^a & \cdots & r_{I+2,I}^a \\ \vdots & \vdots & \ddots & \vdots \\ r_{I+K,1}^a & r_{I+K,2}^a & \cdots & r_{I+K,I}^a \end{array}\right] = \left[\frac{R^d}{R^a}\right], \quad (12)$$

subject to the following constraints:

$$\begin{cases} r_{kj}^I \geq 0, & \forall k, j \\ \sum_{j=1}^I r_{kj}^I = 1, & \forall k, j \end{cases} \quad (13)$$

.

3.4 Combined deterioration–improvement matrix

Following an IS, it is considered that four events can occur in one unit of time: deterioration due to an MnDP or an LtDP, improvement due to the execution of an intervention when an element is in either an S2CS or an S1CS. Therefore, in order to determine the CS of the element after one time step, all these events are simultaneously considered, i.e. a deterioration–improvement matrix for MnDP and LtDP (called 'combined deterioration–improvement matrix' here after) is determined. In order to simplify the process of determining the combined deterioration–improvement matrix, two assumptions were made:

(1) It is assumed that deterioration due to an LtDP happens exactly at the end of each period. The acceptability of this assumption is inversely correlated with the length of the time intervals used.
(2) It is assumed that only one intervention is executed per time period, either an intervention because the element has entered an S1CS that requires the execution of an intervention, or because the element has entered an S2CS during the investigated time period. In the latter case, it is assumed that no other intervention would be executed on the element, even after improvement it is in an S1CS that requires an intervention at the end of the investigated time period.

The combined deterioration–improvement matrix, $\hat{Q}^{a(e)}(t)$, can be written as

$$\hat{Q}^{a(e)}(t) = \hat{P}^{a(e)}(t) + \hat{R}^{a(e)}(t) = \hat{p}_{ij}^{a(e)}(t) + \hat{r}_{ij}^{a(e)}(t), \quad (14)$$

subject to the following constraints:

$$\begin{cases} \hat{r}_{ij}^{a(e)}(\mathrm{t}) \geq 0, & \forall j, i = i' \\ \sum_{j=1}^I \hat{r}_{ij}^{a(e)}(t) = 1 - \sum_{h=1}^I p_{ih} \sum_{k=I+1}^{I+K} \bar{p}_{hk}^{(e)}(t), & \forall j, i = i' \\ \hat{r}_{ij}^{a(e)}(t) = 0, & \forall \mathrm{i} \neq \mathrm{i}' \\ \hat{p}_{ij}^{a(e)}(t) = \sum_{h=1}^I p_{ih} \cdot \left[\sum_{k=1}^K \bar{p}_{h,I+k}^{(e)}(t) . r_{kj}^a\right], & \forall i = i' \\ \hat{p}_{ij}^{a(e)}(t) = \sum_{h=1}^I p_{ih} \cdot \left[\bar{p}_{hj}^{(e)}(t) + \sum_{k=1}^K \bar{p}_{h,I+k}^{(e)}(t) . r_{kj}^a\right] & \forall i \neq i' \end{cases}, \quad (15)$$

where ˆ indicates both an MnDP and an LtDP are being considered, e indicates the element type. Three different element types, namely independent (IN), interdependent (ID) and dependent (DP) were defined (see Section 4.4 for more details). This was done as the probability of an element entering an S2CS, especially S2CS K can be dependent on the CS of the other elements of the bridge, and thus the estimation of p.o.ts will require consideration of these dependencies. $\hat{P}^{a(e)}(t)$ represents the p.o.ts due to the MnDP, the LtDP and IS a (Equation (16)):

$$\hat{P}^{a(e)}(t) = P \cdot \bar{Q}^{a(e)}(t), \tag{16}$$

where P represents the MDM. The probability of being in any S2CS at the beginning of time period t + 1 is 0, in this case. This happens because if an element enters an S2CS during t, it is assumed that an intervention, which improves that element to a S1CS and takes negligible time, is executed immediately. $\bar{Q}^{a(e)}(t)$ represents the deterioration–improvement matrix due to the LtDP and IS a (Equation (17)):

$$\begin{aligned}\bar{Q}^{a(e)}(t) &= \bar{P}_1^{(e)}(t) + \bar{P}_2^{(e)}(t).R^a = \bar{p}_{ij}^{(e)}(t) \\ &+ \sum_{k=1}^{K} \bar{p}_{i,I+k}^{(e)}(t) r_{kj}^a \quad \forall \quad 1 \le i,j \le I,\end{aligned} \tag{17}$$

where $\bar{}$ indicates that only the LtDP is being considered, $\bar{P}_1^{(e)}(t)$ and $\bar{P}_2^{(e)}(t)$ represent the first and second quadrants of the LDM. It is proposed that in determining $\bar{P}_1^{(e)}(t)$ and $\bar{P}_2^{(e)}(t)$, it be assumed that the elements are dependent only on the probability of the other elements in the bridge entering their highest S2CS, which varies over time. $\bar{p}_{ij}^{(e)}(t)$ represents the p.o.ts of the LDM and $\hat{R}^{a(e)}(t)$ represents the adjusted improvement matrix (Equation (18)), the one that results from the execution of the interventions in IS d, taking into consideration that the element might enter an S2CS, and an intervention a was executed:

$$\hat{R}^{a(e)}(t) = \hat{r}_{ij}^{a(e)}(t) = \begin{pmatrix} \hat{r}_{11}^{a(e)}(t) & \hat{r}_{12}^{a(e)}(t) & \cdots & \hat{r}_{1I}^{a(e)}(t) \\ \hat{r}_{21}^{a(e)}(t) & \hat{r}_{22}^{a(e)}(t) & \cdots & \hat{r}_{2I}^{a(e)}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{r}_{I1}^{a(e)}(t) & \hat{r}_{I2}^{a(e)}(t) & \cdots & \hat{r}_{II}^{a(e)}(t) \end{pmatrix}. \tag{18}$$

The mathematical relationship between the elements $\hat{r}_{ij}^{a(e)}(t)$ of matrix $\hat{R}^{a(e)}(t)$ and elements r_{ij}^d of matrix R^d is defined as follows:

$$\hat{r}_{ij}^{a(e)}(t) = \left(1 - \sum_{h=1}^{I} p_{ih} \sum_{k=I+1}^{I+K} \bar{p}_{hk}^{(e)}(t)\right) r_{ij}^d, \tag{19}$$

in which the term inside the bracket refers to the case where no interventions are executed on S2CSs, i.e. the total probability that the condition will not move to any S2CS. In other words, if the element is in CS i in time t-1, then the probability the element will be moved in to S2CS k is given by $\sum_{h=1}^{I} p_{ih} \bar{p}_{hk}^{(e)}(t)$. Taking the sum of this term over all S2CSs (i.e. $k = I + 1$ to K), and initial S1CSs (i.e. $i = 1$ to I) gives the total probability of the element moving to any S2CS within one time interval.

3.5 *Expected cost*

The expected cost for an element in year t, $V^{d,a(e)}(t)$, if ISs d and a are followed, is given by

$$\begin{aligned} E[V^{a(e)}(t)] = &\sum_{i=1}^{I} \left(1 - \sum_{h=1}^{I} p_{ih} \sum_{k=I+1}^{I+K} \bar{p}_{hk}^{(e)}(t)\right) \phi_i^{a(e)}(t-1) c_i^{d(e)} \\ &+ \sum_{i=1}^{I} \phi_i^{a(e)}(t-1) \sum_{h=1}^{I} p_{ih} \sum_{k=1}^{K-1} \bar{p}_{h,I+k}^{(e)}(t) c_k^{a(e)} \\ &+ \sum_{i=1}^{I} \left(\phi_i^{a(e)}(t-1) \sum_{h=1}^{I} p_{ih} \bar{p}_{h,K}^{(e)}(t) - P_K^{(n)}\right) c_K^{a(e)}, \end{aligned} \tag{20}$$

where $\phi_i^{a(e)}(t-1)$ is the probability of an element being in S1CS i in year $(t-1)$, $c_i^{d(e)}$ is the cost of entering S1CS i, $c_k^{a(e)}$ is the cost of entering S2CS k, $c_K^{a(e)}$ is the cost of the element which includes all direct and indirect costs due to failure (i.e. S2CS K), $P_K^{(n)}$ is the probability of element (n) entering S2CS K (failure CS), where element e failure is dependent on element n. If element e failure is independent, $P_K^{(n)}$ will be 0.

3.6 *Long-term expected costs*

The net present value of the total long-term expected cost of IS d and a on element e, $TC(d \in D, a \in A)$, is then defined as

$$\begin{aligned} TC(d \in D, a \in A) &= \sum_{t=0}^{T} \frac{1}{(1+\gamma)^t} . E[V^{d,a(e)}(t)] \\ &= \sum_{t=0}^{T} \frac{1}{(1+\gamma)^t} \\ &\cdot \begin{bmatrix} \sum_{i=1}^{I} \left(1 - \sum_{h=1}^{I} p_{ih} \sum_{k=I+1}^{I+K} \bar{p}_{hk}^{(e)}(t)\right) \phi_i^{a(e)}(t-1) c_i^{d(e)} \\ + \sum_{i=1}^{I} \phi_i^{a(e)}(t-1) \sum_{h=1}^{I} p_{ih} \sum_{k=1}^{K-1} \bar{p}_{h,I+k}^{(e)}(t) c_k^{a(e)} \\ \sum_{i=1}^{I} \left(\phi_i^{a(e)}(t-1) \sum_{h=1}^{I} p_{ih} \bar{p}_{h,K}^{(e)}(t) - P_K^{(n)}\right) c_K^{a(e)} \end{bmatrix}. \end{aligned} \tag{21}$$

4. Estimation of p.o.ts due to LDPs

4.1 General

P.o.ts, $\bar{p}_{ij}$, due to LtDPs, without considering any dependencies, can be calculated as

$$\bar{p}_{ij} = \begin{cases} P_{j-1}(CS > j-1/i) - P_j(CS > j/i) & K > j > i \\ 1 - P_j(CS > j/i) & j = i \\ P_{j-1}(CS > j-1/i) & j = K \end{cases}, \tag{22}$$

where P_j is the probability of being in a higher CS than CS j. The LDM takes the same form as Equation (9), with p_{ijj} replaced by $\bar{p}_{ij}$, which represents the p.o.ts between CSs due to the LtDP, subject to the following constraints:

$$\begin{cases} \sum_{j=1}^{I+K} \bar{p}_{ij} = 1 & \forall \mathrm{i,j} \in [1, I+K] \\ \bar{p}_{ij} = 0 \quad \text{when} & (i > j) \end{cases}, \tag{23}$$

where i and j are used to count through both S1CSs and S2CSs, assuming that they are consecutively numbered.

In addition, if it is not possible that an S1CS can occur due to the LtDP, then

$$\bar{p}_{ij} = 0 \quad \forall i \neq j \leq I \quad if \quad S1CS \otimes LP. \tag{24}$$

In this paper, only the general form of the LDM given in Equation (23) is discussed. Whenever it is not possible that an S1CS can occur due to the LtDP, similar adjustment as given in Equation (24) should be made.

In order to estimate $\bar{p}_{ij}$ shown in Equation (22), it is necessary to take into consideration the following:

- the CS that is likely to result if a hazard event occurs (Section 4.2) and
- the probability of occurrence of the hazard event (Section 4.3).

Once $\bar{P}$ is determined, then in order to determine $\bar{P}^{(e)}$, it is necessary to take into consideration the following:

- how elements within an object interact (Section 4.4).

4.2 Fragility curves

Fragility curves are commonly used to represent the relationship between the intensity of loading and the resulting CS (Choi et al., 2004; Karim & Yamazaki, 2001; Schultz et al., 2010). In this study, a fragility curve represents the probability of an element being in a CS worse than CS k for a load intensity s, if the element is in S1CS i at the time of being affected by the LtDP:

$$Y_k(s, i) = P(CS > k/s, i). \tag{25}$$

Example fragility curves for a bridge pier subjected to rock falls are shown in Figure 1. Each curve represents the probability that the damage level will exceed that required to enter S2CS k, for a specified rock volume for a given initial CS i. These probabilities may be calculated using various methods (Karim & Yamazaki, 2001; Schultz et al., 2010). The methods for generating these fragility curves are beyond the scope of this paper, thus not discussed.

4.3 Hazard curves

The probability of occurrence of hazard events of specified intensities, e.g. an earthquake of magnitude 5, is often presented as hazard curves, $H_S(s)$, which are determined as

$$H_S(s) = P(S > s, (0, 1)), \tag{26}$$

where S is the intensity of the event. The annual probability of being in a worse CS than CS k, if the object

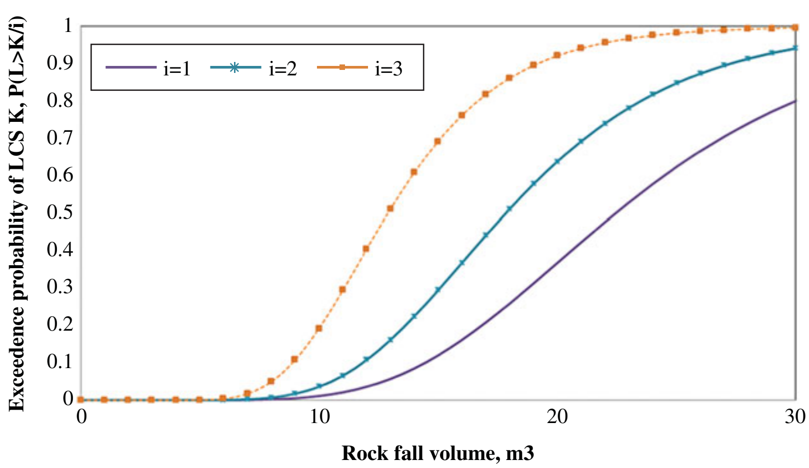


Figure 1. An example fragility curve for a bridge pier subjected to rock fall hazards, when bridge pier is in initial CS $i = 1,2,3$.

is in S1CS i, is

$$P_k(CS > k/i) = \int H_S(s)dY_k(s,i). \tag{27}$$

4.4 Element interactions

The failure of an element in an object may cause the failure of another element regardless of its CS (e.g. the failure of a girder may cause the failure of the deck irrespective of the CS of the deck). Therefore, the interactions between elements must be considered in estimating the p.o.ts due to an LtDP. Elements based on their interactions with other elements are classified as shown in Table 3. In Table 3, element type is defined for element x considering the interaction with element y. In the proceeding sections, the same element convention, i.e. elements x and y, will be used.

4.5 Independent elements

The deterioration matrix, $\bar{P}^{(\mathrm{IN})}(t)$, for an IN element is equal to $\bar{P}$, but is reproduced here for consistency:

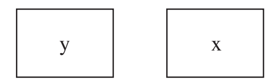
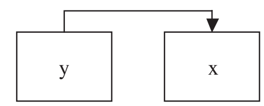
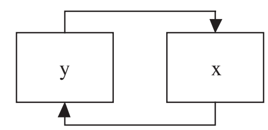
$$P^{(\mathrm{IN})}(t) = \begin{cases} \bar{P}_1^{(\mathrm{IN})}(t) = \bar{P}_1 \Rightarrow \bar{p}_{ij}^{(\mathrm{IN})} = \bar{p}_{ij} & \forall\ 1 \le i,j \le I \\ \bar{P}_2^{(\mathrm{IN})}(t) = \bar{P}_2 \Rightarrow \bar{p}_{i,k+I}^{(\mathrm{IN})} = \bar{p}_{i,k+I} & \forall\ 1 \le i \le I, 1 \le k \le K' \end{cases} \tag{28}$$

where IN is used to indicate that the element is independent. The total probability of entering S2CS k in t is given by

$$P_k^{(\mathrm{IN})}(t) = \sum_{i=1}^{I} \left[\sum_{j=1}^{I} \phi_j^{a(\mathrm{IN})}(t-1) p_{ji} \right] \bar{p}_{i,I+k}, \tag{29}$$

Since the S2CS K is failure of the element, the total failure probability of the element in t, $P_K^{IN}(t)$ can be estimated from Equation (29) when $k = K$.

Table 3. Element types and interaction types.

Element type	Definition: Failure of element y results in failure of element x	Definition: Failure of element x results in failure of element y	Picture in which element x is the element type
IN	Cannot	Cannot	
DP	Can	Cannot	
IID	Can	Can	

IN, independent; DP, dependent; ID, interdependent

4.6 Dependent elements

The deterioration matrix for a dependent element $\bar{P}_k^{(DP)}(t)$ can be calculated as

$$\begin{aligned} \bar{P}_1^{(DP)} &= \bar{p}_{ij}^{(DP)}(t) = \left(1 - P_K^{res}(t)\right).\bar{p}_{ij} \quad \forall \quad 1 \le i \le I, 1 \le j \le I \\ &= \begin{cases} \bar{p}_{ij}^{(DP)}(t) = \left(1 - P_K^{res}(t)\right).\bar{p}_{ij} & \forall \quad 1 \le i \le I, I+1 \le j \le I+K-1 \\ \bar{p}_{i,K}^{(DP)}(t) = \left(1 - P_K^{res}(t)\right).\bar{p}_{iK} + P_K^{res}(t) & \forall \quad 1 \le i \le I \end{cases}, \end{aligned} \tag{30}$$

where

DP is used to indicate that an element is dependent,

$P_K^{res}(t)$ represents the probability of failure of the other element/elements DP element is dependent on, i.e. failure probability of element y in DP element x given in Table 3.

The total probability of entering S2CS k in t is given by

$$P_k^{(DP)}(t) = \begin{cases} \sum_{i=1}^{I} \left[\sum_{j=1}^{I} \phi_j^{a(DP)}(t-1) p_{ji} \right] \left(1 - P_K^{res}(t)\right).\bar{p}_{iI+k} & \forall k < K \\ \sum_{i=1}^{I} \left[\sum_{j=1}^{I} \phi_j^{a(DP)}(t-1) p_{ji} \right] \left(1 - P_K^{res}(t)\right).\bar{p}_{iK} + P_K^{res}(t) & \forall k = K \end{cases} \tag{31}$$

$P_K^{res}(t)$ can be calculated by considering the element type of element y, i.e. IN, DP or ID, and also if element y is DP or IN, then number of elements element

Table 4. Examples of $P_K^{res}(t)$estimations for dependent elements for different types of elements *y*.

Type of elements *y*	Number of interactions	Formulas to estimate $P_K^{res}(t)$ (32)
IN	NA	$P_K^{res(y)}(t) = P_K^{(\mathrm{IN})}(t) = \sum_{i=1}^{I}\left[\sum_{j=1}^{I}\phi_j^{a(y)}(t-1)p_{ji}\right]\bar{p}_{i,I+K}^{(y)}$
DP	$n(\geq 1)$ interactions with other non-interacting elements	$P_K^{res(y)}(t) = P_K^{n_t}(t) + \sum_{i=1}^{I}\sum_{j=1}^{I}\phi_j^{y}(t-1)p_{ji}^{y}\bar{p}_{iK}^{y}\left(1 - P_K^{n_t}(t)\right)$, (33) where $P_K^{n_t}(t) = \prod_{h=1}^{n}\left(1 - P_K^{h-1}(t)\right)P_K^{h}(t)$ (34)
	Interactions with other $n(\geq 1)$ elements interdependent on each other	$P_K^{res(y)}(t) = P_K^{agg(n)}(t) + \sum_{i=1}^{I}\sum_{j=1}^{I}\phi_j^{y}(t-1)p_{ji}^{y}\bar{p}_{iK}^{y}\left(1 - P_K^{agg(n)}(t)\right)$, (35) where $P_K^{agg(n)}$ can be calculated from Equation (39).
ID	Interactions with other $n(\geq 1)$ IN elements	$P_K^{res(y)}(t) = P_K^{agg}(t)$, (36) where $P_K^{agg(n)}$ can be calculated from Equation (39).

y interacts with. Examples of $P_K^{res}(t)$ for different element types of element *y* and their interactions are given in Table 4.

4.7 Interdependent elements

The deterioration matrix for an interdependent element $\bar{P}^{(ID)}(t)$ can be calculated as

$$\begin{aligned} \bar{P}_1^{a(ID)} &= \bar{p}_{ij}^{a(ID)}(t) = N_i(t)\bar{p}_{ij} \quad \forall \quad 1 \leq i \leq I, 1 \leq j \leq I \\ \bar{P}_2^{a(ID)} &= \begin{cases} \bar{p}_{ij}^{a(ID)}(t) = N_i(t)\bar{p}ij \quad \forall \quad 1 \leq i \leq I, I+1 \leq j \leq I+K-1 \\ \bar{p}_{i,K}^{a(ID)}(t) = P_K^{agg}(t) \quad \forall \quad 1 \leq i \leq I \end{cases}, \end{aligned} \tag{37}$$

where

$$N_i = \frac{1 - P_K^{agg}(t)}{1 - \bar{p}_{i,I+K}}. \tag{38}$$

ID is used to indicate that an element is interdependent, $P_K^{agg}(t)$ represents the total aggregated probability of failure of *n* interconnected ID elements, which is

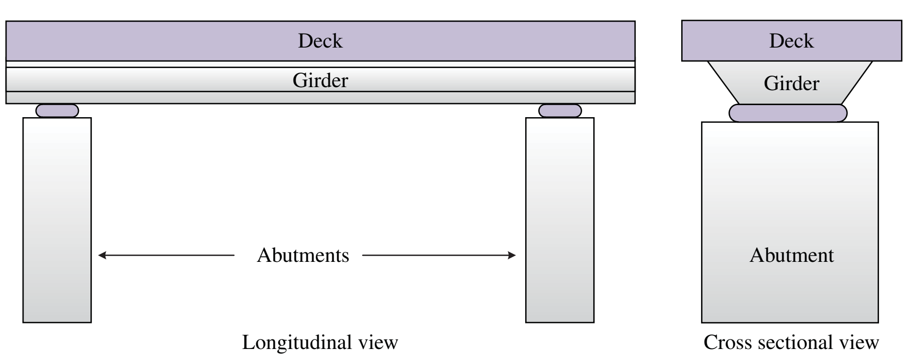


Figure 2. Example bridge.

Table 5. Element types and interactions, for example bridge.

Element interactions (element x–element y)	Element x type	Element y type	Interpretation
Abutment 1–abutment 2	ID	ID	An abutment failure will affect the other abutment
Girder–abutments	DP	ID	An abutment failure will affect the girder. But deck failure does not affect the abutments.
Deck–girder	DP	DP	Girder failure may affect the deck, but deck failure may not affect the girder.

calculated as

$$P_K^{agg}(t) = 1 - \prod_{n=1}^{N} \left(1 - P_K^n(t)\right), \tag{39}$$

where $P_K^n(t)$ is the failure probability of ID element n obtained assuming that n interconnected ID elements are independent from each other. $P_K^n(t)$ for each ID element is obtained from Equation (29) with $k = K$ if independent (other than on n ID elements), or from Equation (31) if dependent (other than on n ID elements).

The total probability of entering S2CS k in t is given by

$$P_k^{(ID)}(t) = \begin{cases} \sum_{i=1}^{I} \left[\sum_{j=1}^{I} \phi_j^{a(ID)}(t-1)\right] p_{ji} N_i(t) \bar{p}_{ij} & \forall k < K \\ P_K^{agg}(t) & \forall k = K \end{cases} \tag{40}$$

Table 6. Element S1CSs, and S2CSs definitions, interventions to be executed, intervention costs and intervention effectiveness.

Element	CS		Description	Feasible onterventions	Total cost of the intervention* (10^3 mu)	CS after the intervention
Deck	S1CS	1	No symptoms of distress. Any cracking that is present is only superficial.	DN		
		2	Light cracking. Distress area is 2% or less.	DN		
		3	Cracking is moderate. Distress area is between 2% and 10%.	DN		
		4	Heavy cracking. The distress area is between 10% and 25%.	Crack repair	215	1
		5	Severe cracking. Distressed area is more than 25%.	Replace	380	1
	S2CS	6	Failure	Replace	515	1
Girder	S1CS	1	Paint system sound and functioning as intended.	DN		
		2	Moderate deterioration of the paint system. Little exposed metal surface.	DN		
		3	Paint system badly damaged/exposed metal but no significant section loss.	Paint repair	56	1
		4	Significant section loss.	DN		
		5	Alarming section loss.	Replace	550	1
	S2CS	6	Failure	Replace	1215	1
Abutment	S1CS	1	Any cracking that is present is only superficial. In good condition.	DN		
		2	Minor cracks (1/16"–1/8"). Minor spalling.	DN		
		3	Moderate cracks (1/8"–1/4"). Moderate spalling, but no exposed re-bars.	DN		
		4	Severe spalling with exposed re-bars and active corrosion.	Major patching and repair of reinforcement.	280	1
		5	Excessive spalling with severe re-bar corrosion.	Replace	420	1
	S2CS	6	Excessive cracking/spalling	Major rehabilitation	320	1
		7	Failure	Replace	1631	1

Note: *mu, monetary units.

Table 7. Non-zero p_{ij} values for bridge elements.

p_{ij}	Abutment/pier	Girders	Deck
p_{11}	0.967	0.850	0.848
p_{12}	0.033	0.150	0.152
p_{22}	0.920	0.949	0.837
p_{23}	0.080	0.051	0.163
p_{33}	0.947	0.996	0.860
p_{34}	0.053	0.004	0.140
p_{44}	0.981	0.969	0.970
p_{45}	0.019	0.031	0.030

5. Example

5.1 General

To illustrate the use of the model, it is used to evaluate ISs for a 29-m long two-lane bridge with an orthotropic deck supported on a steel box girder, which is supported on two abutments, were considered (Figure 2). The element types are explained in Table 5. The MnDPs are corrosion and cracking of the reinforced concrete, paint deterioration and corrosion of the steel girders, and cracking of the overlay. The LtDP is rock falls. The ISs

Table 8. Fragility curve parameters for bridge elements.

		CSs									
		S1CS2		S1CS3		S1CS4		S1CS5		S2CS6	
Element	CSs	α	β	α	β	α	β	α	β	α	β
Deck	S1CS1	N/A	N/A	0.27	3.50	0.26	5.80	0.10	3.90	N/A	N/A
	S1CS2	N/A	N/A	0.28	3.40	0.26	5.70	0.10	3.80	N/A	N/A
	S1CS3	N/A	N/A	0.29	3.00	0.28	5.50	0.11	3.50	N/A	N/A
	S1CS4	N/A	N/A	N/A	N/A	0.30	5.30	0.13	3.30	N/A	N/A
	S1CS5	N/A	N/A	N/A	N/A	N/A	N/A	0.15	3.00	N/A	N/A
Girder	S1CS1	0.32	4.00	N/A	N/A	N/A	N/A	0.15	5.00	N/A	N/A
	S1CS2	0.34	3.90	N/A	N/A	N/A	N/A	0.15	5.00	N/A	N/A
	S1CS3	N/A	N/A	N/A	N/A	N/A	N/A	0.15	5.00	N/A	N/A
	S1CS4	N/A	N/A	N/A	N/A	N/A	N/A	0.18	4.50	N/A	N/A
	S1CS5	N/A	N/A	N/A	N/A	N/A	N/A	0.21	5.60	N/A	N/A
Abutments	S1CS1	0.30	3.50	N/A	N/A	N/A	N/A	0.25	6.00	0.10	4.00
	S1CS2	0.34	3.40	N/A	N/A	N/A	N/A	0.26	6.00	0.11	4.00
	S1CS3	N/A	N/A	N/A	N/A	N/A	N/A	0.27	5.90	0.12	3.90
	S1CS4	N/A	N/A	N/A	N/A	N/A	N/A	0.29	5.80	0.14	3.75
	S1CS5	N/A	N/A	N/A	N/A	N/A	N/A	0.33	5.60	0.16	3.60
	S2CS6	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.20	3.40

Table 9. Transition probabilities of LDM.

		CS after hazard event						
Element	CS before hazard event	S1CS1	S1CS2	S1CS3	S1CS4	S1CS5	S2CS6	S2CS7
Deck	S1CS1	0.981	0.000	0.000	0.009	0.006	0.004	N/A
	S1CS2	0.000	0.980	0.000	0.010	0.006	0.004	N/A
	S1CS3	0.000	0.000	0.980	0.009	0.006	0.005	N/A
	S1CS4	0.000	0.000	0.000	0.987	0.005	0.008	N/A
	S1CS5	0.000	0.000	0.000	0.000	0.989	0.011	N/A
	S2CS6	0.000	0.000	0.000	0.000	0.000	1.000	N/A
Girders	S1CS1	0.981	0.000	0.014	0.000	0.000	0.005	N/A
	S1CS2	0.000	0.979	0.016	0.000	0.000	0.005	N/A
	S1CS3	0.000	0.000	0.995	0.000	0.000	0.005	N/A
	S1CS4	0.000	0.000	0.000	0.992	0.000	0.008	N/A
	S1CS5	0.000	0.000	0.000	0.000	0.992	0.008	N/A
	S2CS6	0.000	0.000	0.000	0.000	0.000	1.000	N/A
Abutments	S1CS1	0.979	0.000	0.012	0.000	0.000	0.005	0.004
	S1CS2	0.000	0.975	0.016	0.000	0.000	0.004	0.005
	S1CS3	0.000	0.000	0.990	0.000	0.000	0.004	0.006
	S1CS4	0.000	0.000	0.000	0.989	0.000	0.004	0.007
	S1CS5	0.000	0.000	0.000	0.000	0.986	0.004	0.010
	S2CS6	0.000	0.000	0.000	0.000	0.000	0.986	0.014
	S2CS7	0.000	0.000	0.000	0.000	0.000	0.000	1.000

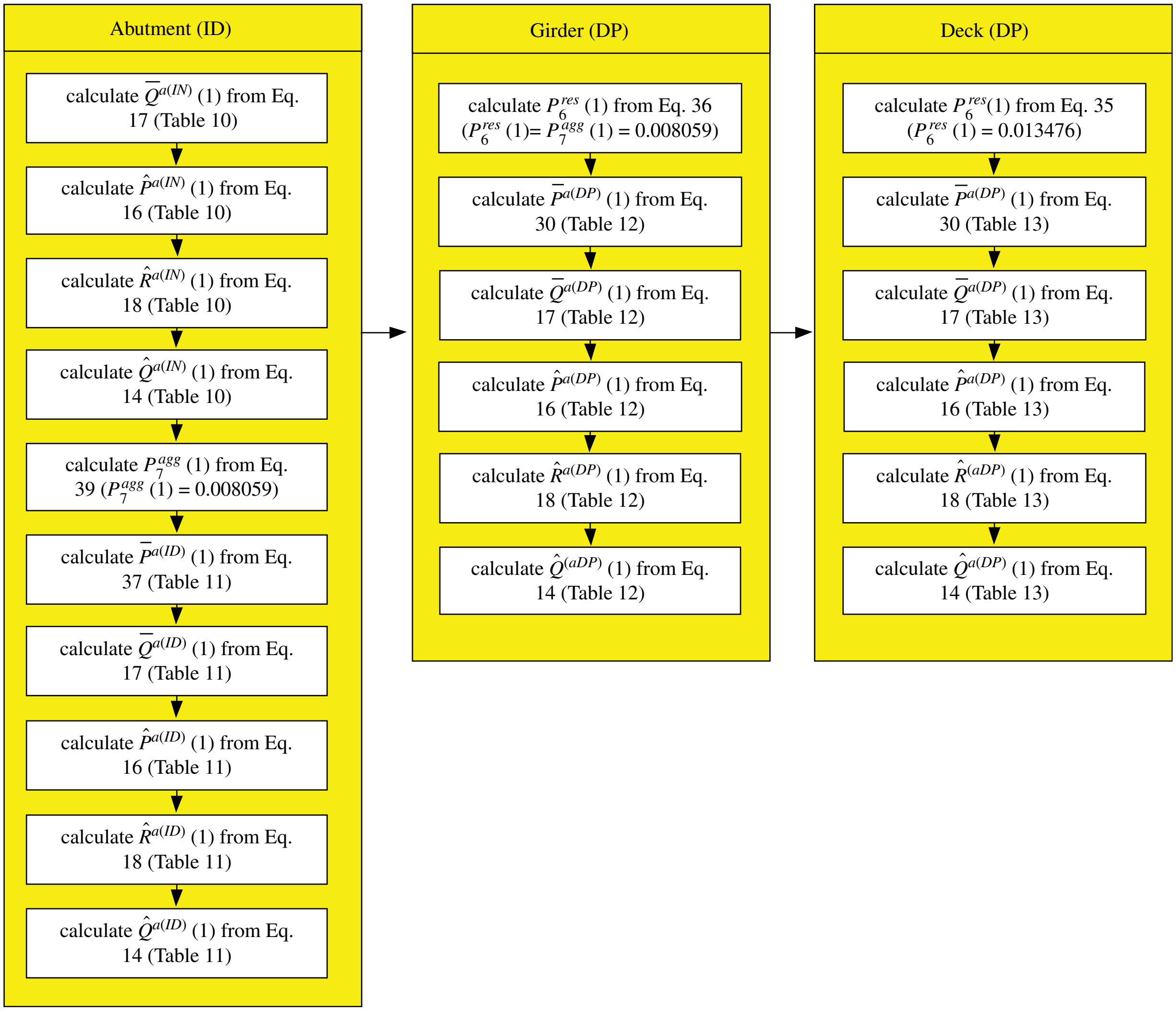


Figure 3. Steps for the calculation of p.o.ts of bridge elements.

are evaluated explicitly taking into consideration that rock falls may occur, ignoring the fact that rock falls may occur. The element types and interaction types are summarised in Table 5.

5.2 *Condition states and interventions*

The S1CSs and S2CSs for each element, the interventions to be executed if an element is in each of the CSs and their costs and effectiveness are given in Table 6. The CSs for the deck are defined so that the S1CSs could occur due to both MnDP and LtDP. In addition to the S1CSs, S2CS 6 is added to represent deck failure which is assumed to be caused only by LtDP. It is assumed that deck failure will not affect other elements. If the deck fails, it will be replaced with a new identical deck.

The CSs for the girders are defined so that the S1CSs 1–3 are possible due to both MnDP and LtDP. The S1CSs 4 and 5 are possible only due to MnDP, i.e. damage to the paint system is possible, whereas section loss is not possible due to rock falls. In addition to these S1CSs, S2CS 6 is added to represent girder failure. If the girders fail, it is assumed that the deck will also fail. If the girders fail, the girders will be replaced with new identical girders and the deck will be replaced with a new identical deck.

The CSs for the abutments are defined so that S1CSs 1–3 are possible due to both MnDP and LtDP. S1CSs 4 and 5 are possible due to only MnDP, i.e. cracking of concrete may occur, but corrosion of the re-bars cannot occur due to rock falls. In addition to these S1CSs, S2CSs 6 and 7 are defined, where S2CS 6 accounts for excessive cracking/spalling and S2CS 7 accounts for the failure of the abutments (hence the whole bridge) due to rock falls. If the abutments fail, it is assumed that the girders fail and the deck fails. This means that if the abutments fail, the entire bridge will be replaced with a new one consisting of identical abutments, girders and deck.

Table 10. Abutments $\bar{Q}^{a(IN)}$, $\hat{P}^{a(IN)}$, $\hat{R}^{a(IN)}$, $\hat{Q}^{a(IN)}$matrices for time interval 0–1 year.

Matrix	CS before hazard event	CS after hazard event				
		S1CS1	S1CS2	S1CS3	S1CS4	S1CS5
$\bar{Q}^{a(IN)}(1)$	S1CS1	0.99	0.00	0.01	0.00	0.00
	S1CS2	0.01	0.97	0.02	0.00	0.00
	S1CS3	0.01	0.00	0.99	0.00	0.00
	S1CS4	0.01	0.00	0.00	0.99	0.00
	S1CS5	0.01	0.00	0.00	0.00	0.99
$\hat{P}^{a(IN)}(1)$	S1CS1	0.955	0.032	0.013	0.000	0.000
	S1CS2	0.009	0.897	0.094	0.000	0.000
	S1CS3	0.010	0.000	0.938	0.052	0.000
	S1CS4	0.011	0.000	0.000	0.970	0.019
	S1CS5	0.014	0.000	0.000	0.000	0.986
$\hat{R}^{a(IN)}(1)$	S1CS1	0.000	0.000	0.000	0.000	0.000
	S1CS2	0.000	0.000	0.000	0.000	0.000
	S1CS3	0.000	0.000	0.000	0.000	0.000
	S1CS4	0.989	0.000	0.000	0.000	0.000
	S1CS5	0.986	0.000	0.000	0.000	0.000
$\hat{Q}^{a(IN)}(1)$	S1CS1	0.955	0.032	0.013	0.000	0.000
	S1CS2	0.009	0.897	0.094	0.000	0.000
	S1CS3	0.010	0.000	0.938	0.052	0.000
	S1CS4	1.000	0.000	0.000	0.000	0.000
	S1CS5	1.000	0.000	0.000	0.000	0.000

5.3 Transition probabilities due to MnDPs

The non-zero values of the p.o.ts due to MnDPs for abutments, girders and deck, taken from Walbridge, Fernando, and Adey (2012), are given in Table 7.

5.4 Transition probabilities due to LnDPs

A simple power law model as shown in Equation (41) was selected to model the probability of a rock fall (Schubert et al., 2010):

$$H_v(V) = \lambda v^{-\mu}, \tag{41}$$

where v is the rock fall volume in m^3, λ and μ are parameters which can be determined from the rock fall data (Schubert et al., 2010). The values of λ and μ were taken as 0.3 and 2.0, respectively. The fragility curves for the bridge elements were determined using

$$Y_d(a, i) = e^{-e^{-\alpha v + \beta}}, \tag{42}$$

where α and β for the bridge elements at different initial CSs, for possible S1CSs and S2CSs, are given in Table 8 for elements of the bridge. The resulting p.o.ts are given in Table 9.

Table 11. Abutments $\bar{P}^{a(ID)}$, $\bar{Q}^{a(ID)}$, $\hat{P}^{a(ID)}$, $\hat{R}^{a(ID)}$, $\hat{Q}^{a(ID)}$matrices for time interval 0–1 year.

Matrix	CS before hazard event	CS after hazard event						
		S1CS1	S1CS2	S1CS3	S1CS4	S1CS5	S2CS6	S2CS7
$\bar{P}^{a(ID)}(1)$	S1CS1	0.975	0.000	0.012	0.000	0.000	0.005	0.008
	S1CS2	0.000	0.971	0.016	0.000	0.000	0.005	0.008
	S1CS3	0.000	0.000	0.988	0.000	0.000	0.004	0.008
	S1CS4	0.000	0.000	0.000	0.988	0.000	0.004	0.008
	S1CS5	0.000	0.000	0.000	0.000	0.988	0.004	0.008
$\bar{Q}^{a(ID)}(1)$	S1CS1	0.988	0.000	0.012	0.000	0.000		
	S1CS2	0.013	0.971	0.016	0.000	0.000		
	S1CS3	0.012	0.000	0.988	0.000	0.000		
	S1CS4	0.012	0.000	0.000	0.988	0.000		
	S1CS5	0.012	0.000	0.000	0.000	0.988		
$\hat{P}^{a(ID)}(1)$	S1CS1	0.956	0.032	0.012	0.000	0.000		
	S1CS2	0.013	0.894	0.093	0.000	0.000		
	S1CS3	0.012	0.000	0.935	0.053	0.000		
	S1CS4	0.012	0.000	0.000	0.969	0.019		
	S1CS5	0.012	0.000	0.000	0.000	0.988		
$\hat{R}^{a(ID)}(1)$	S1CS1	0.000	0.000	0.000	0.000	0.000		
	S1CS2	0.000	0.000	0.000	0.000	0.000		
	S1CS3	0.000	0.000	0.000	0.000	0.000		
	S1CS4	0.988	0.000	0.000	0.000	0.000		
	S1CS5	0.988	0.000	0.000	0.000	0.000		
	S1CS1	0.956	0.032	0.012	0.000	0.000		
$\hat{Q}^{a(ID)}(1)$	S1CS2	0.013	0.894	0.093	0.000	0.000		
	S1CS3	0.012	0.000	0.935	0.053	0.000		
	S1CS4	1.000	0.000	0.000	0.000	0.000		
	S1CS5	1.000	0.000	0.000	0.000	0.000		

Table 12. Girder $\bar{P}^{a(DP)}$, $\bar{Q}^{a(DP)}$, $\hat{P}^{a(DP)}$, $\hat{R}^{a(DP)}$, $\hat{Q}^{a(DP)}$ matrices for time interval 0–1 year.

		CS after hazard event					
Matrix	CS before hazard event	S1CS1	S1CS2	S1CS3	S1CS4	S1CS5	S2CS6
$\bar{P}^{a(DP)}(1)$	S1CS1	0.973	0.000	0.014	0.000	0.000	0.013
	S1CS2	0.000	0.971	0.016	0.000	0.000	0.013
	S1CS3	0.000	0.000	0.987	0.000	0.000	0.013
	S1CS4	0.000	0.000	0.000	0.984	0.000	0.016
	S1CS5	0.000	0.000	0.000	0.000	0.984	0.016
$\bar{Q}^{a(DP)}(1)$	S1CS1	0.986	0.000	0.014	0.000	0.000	
	S1CS2	0.013	0.971	0.016	0.000	0.000	
	S1CS3	0.013	0.000	0.987	0.000	0.000	
	S1CS4	0.016	0.000	0.000	0.984	0.000	
	S1CS5	0.016	0.000	0.000	0.000	0.984	
$\hat{P}^{a(DP)}(1)$	S1CS1	0.840	0.146	0.014	0.000	0.000	
	S1CS2	0.014	0.921	0.065	0.000	0.000	
	S1CS3	0.013	0.000	0.983	0.004	0.000	
	S1CS4	0.016	0.000	0.000	0.954	0.030	
	S1CS5	0.016	0.000	0.000	0.000	0.984	
$\hat{R}^{a(DP)}(1)$	S1CS1	0.000	0.000	0.000	0.000	0.000	
	S1CS2	0.000	0.000	0.000	0.000	0.000	
	S1CS3	0.995	0.000	0.000	0.000	0.000	
	S1CS4	0.000	0.000	0.000	0.000	0.000	
	S1CS5	0.992	0.000	0.000	0.000	0.000	
	S1CS1	0.840	0.146	0.014	0.000	0.000	
$\hat{Q}^{a(DP)}(1)$	S1CS2	0.014	0.921	0.065	0.000	0.000	
	S1CS3	1.000	0.000	0.000	0.000	0.000	
	S1CS4	0.016	0.000	0.000	0.954	0.030	
	S1CS5	1.000	0.000	0.000	0.000	0.000	

Table 13. Deck $\bar{P}^{a(DP)}$, $\bar{Q}^{a(DP)}$, $\hat{P}^{a(DP)}$, $\hat{R}^{a(DP)}$, $\hat{Q}^{a(DP)}$ matrices for time interval 0–1 year.

		CS after hazard event					
Matrix	CS before hazard event	S1CS1	S1CS2	S1CS3	S1CS4	S1CS5	S2CS6
$\bar{P}^{a(DP)}(1)$	S1CS1	0.968	0.000	0.000	0.009	0.005	0.018
	S1CS2	0.000	0.967	0.000	0.010	0.005	0.018
	S1CS3	0.000	0.000	0.967	0.009	0.005	0.019
	S1CS4	0.000	0.000	0.000	0.974	0.005	0.021
	S1CS5	0.000	0.000	0.000	0.000	0.975	0.025
$\bar{Q}^{a(DP)}(1)$	S1CS1	0.986	0.000	0.000	0.009	0.005	
	S1CS2	0.018	0.967	0.000	0.010	0.005	
	S1CS3	0.019	0.000	0.967	0.009	0.005	
	S1CS4	0.021	0.000	0.000	0.974	0.005	
	S1CS5	0.025	0.000	0.000	0.000	0.975	
$\hat{P}^{a(DP)}(1)$	S1CS1	0.839	0.147	0.000	0.009	0.005	
	S1CS2	0.018	0.809	0.158	0.010	0.005	
	S1CS3	0.020	0.000	0.831	0.144	0.005	
	S1CS4	0.022	0.000	0.000	0.944	0.034	
	S1CS5	0.025	0.000	0.000	0.000	0.975	
$\hat{R}^{a(DP)}(1)$	S1CS1	0.000	0.000	0.000	0.000	0.000	
	S1CS2	0.000	0.000	0.000	0.000	0.000	
	S1CS3	0.000	0.000	0.000	0.000	0.000	
	S1CS4	0.992	0.000	0.000	0.000	0.000	
	S1CS5	0.989	0.000	0.000	0.000	0.000	
	S1CS1	0.839	0.147	0.000	0.009	0.005	
$\hat{Q}^{a(DP)}(1)$	S1CS2	0.018	0.809	0.158	0.010	0.005	
	S1CS3	0.020	0.000	0.831	0.144	0.005	
	S1CS4	1.000	0.000	0.000	0.000	0.000	
	S1CS5	1.000	0.000	0.000	0.000	0.000	

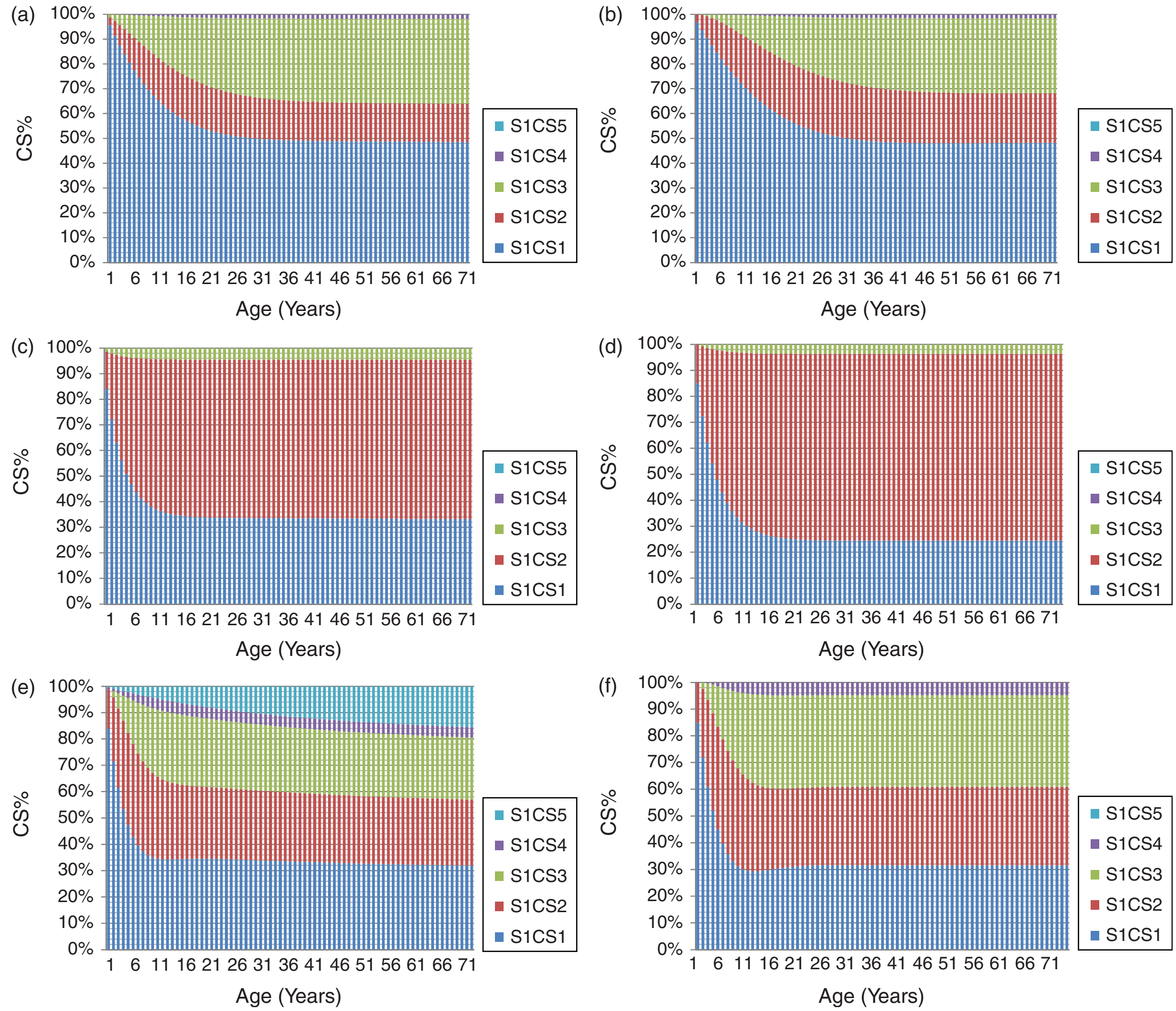


Figure 4. Distribution of element CSs. (a) Abutment element with LtDP; (b)Abutment element without LtDP; (c) Girder element with LtDP; (d) Girder element without LtDP(e) Deck element with LtDP(f) Deck element without LtDP.

5.5 *Adjustment of transition probabilites*

Once the p.o.ts due to deterioration for MnDPs and the LtDP alone have been determined, the p.o.ts can be calculated taking into consideration the dependencies between the elements of the bridge. As the failure of the deck is dependent on the girder failure, and the failure of the girder is dependent on the abutments failure (Table 5), first the p.o.ts of the abutments, then the p.o.ts of the girders and finally the p.o.ts of the deck should be calculated. The steps to follow are given in Figure 3 and the calculated p.o.ts for each element for time interval 0–1 year are given in Tables 10–13.

By multiplying the CS distribution of each element at year 0, by $\hat{Q}^{a(e)}(1)$ the element CS distribution at year 1 can be obtained. This procedure should be followed till the end of time period with updating the p.o.ts in $\bar{P}^{a(e)}(t)$ every year.

5.6 *Results and discussions*

5.6.1 *Condition*

The CS distribution of the elements and the expected cost of the bridge over 75 years were calculated. The CS distributions of each bridge element with and without hazard considerations are compared in Figure 4. Differences in the estimated CSs were observed for all the elements (Figure 4). The largest differences in the CSs when LtDP was considered were observed for the deck element. The total probability deck element will move into an S2CS is larger than that for abutments or for deck elements. This is obvious, as the failure of deck elements is dependent on the girders, and the failure of the girder will be dependent on the abutments, thus deck will always have a higher failure probability than the girder or the abutments.

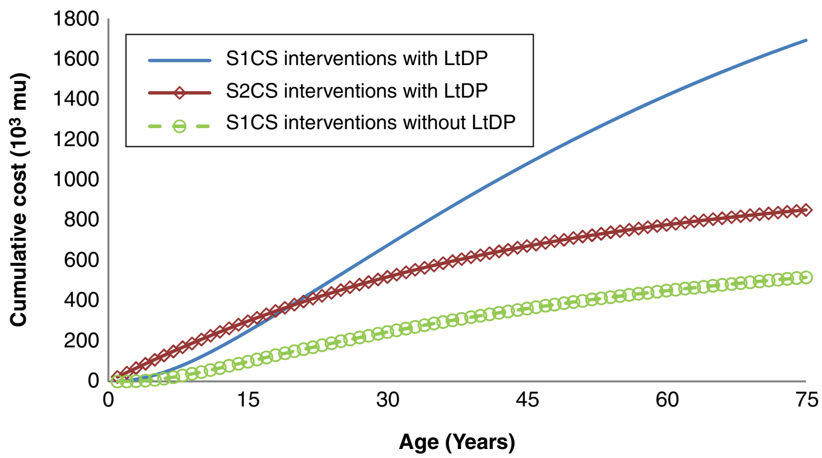


Figure 5. The comparison of cumulative S1CS intervention cost and S2CS intervention cost when LtDP was considered with the S1CS intervention cost without considering LtDP.

Therefore, when LtDPs were considered, the highest differences in the CSs could be expected for the element with the highest failure probability, i.e. the deck element in the current example. The agreement of the results from the proposed methodology with the obvious logic as explained above verifies the reliability of the proposed methodology. The proposed methodology is particularly useful, as it provides a means of quantifying the effects due to LtDPs, while obvious logic could only give a qualitative comparison.

5.6.2 Costs

The cumulative intervention costs of IS with and without considering LtDP are given in Figure 5. It is very clear that the total intervention costs have significantly increased when the LtDP was considered. This is not a surprise as one expects the LtDP to cause additional interventions, thus increasing the cost. It was also observed that the intervention costs on S1CSs increased when LtDP was considered (Figure 5). This means, more interventions were required in S1CSs due to LtDP. This could be explained by looking at LDM's of each element (Table 9), where it can be seen that LtDP also caused transition within S1CSs. Therefore, when LtDP was considered, increase in the intervention costs on S1CSs may result. Hence, if LtDP was ignored, intervention costs on S1CSs for a given IS will be significantly under-predicted. This under-prediction of the cost may yield inaccurate results when selecting the OIS based on minimum costs.

It is obvious that LtDPs will increase the intervention costs, and moreover LtDPs will increase the probability of failure, thus the costs associated with failure. However, in order to consider this obvious cost increase in intervention decision-making, it is important to be able to quantify the additional costs which may result due to LtDPs. The above example clearly demonstrates that the proposed methodology can effectively quantify the additional costs due to LtDPs, and allow the decision-makers to evaluate ISs for bridge elements that are affected by both MnDPs and LtDPs.

6. Conclusions

In this paper, a model is proposed that can be used to evaluate ISs for bridge elements and bridges, taking into consideration both MnDPs and LtDPs. The model is an extension of Markov models often used in existing BMSs.

The main difference between the proposed model and those currently used is that the new model makes it possible to take into consideration both MnDPs and LtDPs. This is done by using two sets of CSs, as opposed to one. Set 1 consists of the CSs in which it is not necessary to intervene immediately, which may occur due to either manifest processes or may or may not occur due to latent processes. Set 2 consists of the CSs in which it is necessary to intervene immediately. It is considered that these may occur only due to latent processes. The existing well-established methods were used to determine the p.o.ts due to MnDPs. The p.o.ts due to LtDPs were initially estimated using fragility curves and adjusted considering different types of element interactions of a bridge.

The proposed model is demonstrated through its use in the evaluation of an IS for a bridge that is affected by both MnDPs and LtDPs. The difference between consideration and no consideration of a LtDP on the evaluation of IS to follow is shown. In the example, the MnDPs considered are cracking of concrete, paint deterioration and corrosion of steel, and cracking of the overlay, and the LtDP considered is that which results in rock falls. Results demonstrated that non-consideration of LtDP significantly affects the evaluation of IS, thus may affect the selection of OIS for the bridge.

Notes

1. Email: lethanh@ibi.baug.ethz.ch
2. Email: adey@ibi.baug.ethz.ch
3. Manifest deterioration processes: Processes that result in changes in the condition of an object so that there is sufficient warning so that an intervention can be executed so that there is a negligible probability that an inadequate level of service is provided. An example of a typical MnDP is chloride-induced corrosion of reinforced concrete. MnDPs considered in this paper are limited to those that can be modelled as Markov chain processes.
4. Latent deterioration processes: Processes whose progression over time is *not* followed in a way that a condition of the object triggers the execution of an intervention early enough so that it can be assumed that the probability object will provide an unexpected inadequate level of service is negligible. An example of a typical LtDP is ground accelerations due to an earthquake.

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