Optimization of track renewal policy

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Introduction

RFF is the French infrastructure manager. The French network consists in about 45,000 Km of tracks, 27,000 Km of which being periodically renewed. Track expenses are approximately 1 billion euros per year. In 1999, RFF launched a study for the optimization of track renewal policies in order to set the best balance between renewal and maintenance, and to build its budget for the next five years.

This task was achieved in one year from 1999 to 2000, supported by the expertise and field data of the SNCF, and consultants from Bureau Veritas.

The problem was solved developing a specific software for optimization, based on a maintenance cost model derived from field data statistics.

This software can now also be used for the evaluation of maintenance costs and their variation according to traffic, and for spot renewal decisions.

Many issues were adressed within this task :

- finding information on tracks, traffic, maintenance costs, …
- modelling reliability,
- modelling maintenance and renewal costs,
- optimizing the decision in acceptable time.

In this paper we will focus on few of them, the most challenging from a theoretical point of view. But we will firts give an overview on data we found or did not find.

Finding information

A first approach showed that maintenance costs could be split into preventive maintenance, strongly dependent on traffic and slowly increasing in time, and repair, connected to reliability. As an example, repair accounts for two thirds of rail maintenance. It was thus decided to evaluate preventive maintenance cost by a statistical analysis of maintenance accounting, and repair costs by a statistical analysis of reliability field data.

It is important to pay attention to the fact that the renewal decision does not depend on maintenance costs, but on their rate of increase when the track gets worn out, ie. on the increase of repair costs. Such trends can not be observed through local information, but can only be derived from large scale surveys. In other words, the problem can not be solved without a suitable centralized maintenance information system, featuring information on costs and maintenance actions. Thus, this paper is an ode to statistics and data bases.

Such a system exists in France, features very detailed information for rails, and was provided by SNCF for this study’s purposes. This allowed to find quite easily reliability laws, which were already available for the rails. However, this information did not cover some aspects :

- for concrete sleepers, no accurate information was found on ageing : this equipment is quite young, and no ageing problems can be noticed yet. It was admitted that concrete sleepers will behave like wooden sleepers, but quite later,

- neither could age related problems be accurately observed on the ballast : the lifetime of wooden sleepers was slightly inferior to the ballast lifetime, and their usual common renewal had as a consequence the fact that few old ballast actually existed.

But maintenance information is not enough to solve maintenance problems. The wearing out of tracks also depends on the kind of traffic run on them. We could find and use information on recent yearly loads, but not on long past traffic, speeds and loads per axle. This is why our statistical analysis were limited to yearly loads, although we strongly feel that the other parameters are also important. It was thus decided to exclude from the study tracks with strong deviations of these parameters : very slow lines with huge traffic, very high speed lines, …

Modelling reliability

Modelling reliability was performed for rail and sleepers.

It was well known that rail reliability followed a Weibull law $R(T) = \exp((-T/\nu)^3)$ where T is the weight of cumulated traffic and R(T) the proportion of length of rail that has not been repaired. This means that the amount of repairs per length and traffic unit will be $I(T) = -R'(T) = k\, T^2 \exp((-T/\nu)^3)$.

> *Some documents mention the fact that the impact of traffic on fatigue has the form $I_m(T) = k\, T^2$. Although this has the same first order variation, it does not yield exactly the same results, as will be shown hereafter.*

It was found that sleepers reliability could be assumed to follow a quite similar law $R(t) = \exp((-t/\eta(A)^3)$ where t is time, A the traffic weight per time unit, and β a constant. $\eta(A)$ was found to follow different laws with the general shape $\eta(A) = k_1 - k_2 \log(A)$.
Rail reliability can be stated with the same formulas, with $\beta=3$, and $h(A)=\nu/A$. We shall further on use this more general expression.

The issues dealt with here are the following :
- what about the successive replacements of already replaced rails and sleepers ?
- how do figures change when the traffic changes in time ?
- what is the yearly amount of rail repairs for a certain length of tracks ?

Successive repairs

The idea of replacing twice the very same component might not seem important. It becomes important for secondary lines with little traffic where the wearing out is slow, and there is no reason for renewal. Of course, there is then no renewal decision problem. But one has to set the border, and this means going into details about the behaviour of very old tracks.
What would happen if there was no renewal at all ? The components would be in time entirely replaced piece by piece. What would then be in time the amount J(T) of repair by time and track length unit ?

It was proved in our study that J(T) is the solution of the equation $J = I + I*J$, where $I*J$ is the convolution of I and J, ie. :

$$I*J(t) = \int_0^t I(u)\, J(t-u)du$$

This equation can be obtained through (at least) two different approaches.

The first approach is to sum the series of first, second, third, … replacements. I(t) is the probability of first replacement at « time » t. Let $I_n(t)$ be the probability of n^{th} replacement at time t. Then

$$\begin{aligned} I_{n+1}(t) \quad &= \int_0^t \mathit{Prob}(n^{th} \text{ replacement at time } u)\ \mathit{Prob}\,(\text{first replacement at time } t-u)\, du \\ &= \int_0^t I_n(u)\, I(t-u)du \\ &= I_n * I(t) \end{aligned}$$

Hence $I_n=I^{*n}$, where I^{*n} is the n^{th} convolution of I.

Then

$$J = I + I^{*2} + I^{*3} + \ldots$$
$$I{*}J = I^{*2} + I^{*3} + I^{*4} + \ldots$$
$$= J - I$$

Quod erat demonstrandum.

A longer approach is to follow the age distribution of components in time.
Let D(t,u) be the probability for one component of having age u at time t>=u.
D(0,u) is a Dirac distibution $\delta_0(u)$, the age being certainly 0.
If t>0, J(t) = D(t,0) : being repaired at time t means having age 0 at time t.
J(0) is assumed to be 0.

If t>u :

D(t,u) = *Prob* (having been repaired at time t-u *and* not having been repaired from t-u to t)
= *Prob* (having age 0 at time t-u) *Prob* (not being repaired from t-u to t)
= D(t-u,0) R(u)
= J(t-u) R(u)

The probability J(t) of having a repair at time t is :

$$J(t) = \int_0^t \textit{Prob}\text{ (age u at time t) } \textit{Prob}\text{ (immediate repair, age being u) du}$$
$$= \int_0^t D(t,u)\, \lambda(u)\, du,$$

where $\lambda(u) = I(u)/R(u) = -R'(u)/R(u)$ is the failure rate at age u.

$$= \int_0^t \lambda(u)\, R(u)\, D(t-u,0)\, du$$
$$= \int_0^t I(u)\, D(t-u,0)\, du$$
$$= \int_0^0 I(u)\, \delta_{t-u}(0)\, du + \int_{>0}^t I(u)\, J(t-u)\, du$$
$$= I(t) + \int_0^t I(u)\, J(t-u)\, du$$

assuming that J(0)=0.

$$= I(t) + I{*}J(t)$$

Quod erat demonstrandum.

The remaining task is to compute J knowing I, using the equation J=I+I*J. It would be natural to consider the Laplace function $\mathcal{L}$, which would transform convolutions into simple products.

$$J = \mathcal{L}^{-1}\left[\mathcal{L}(I) / (1 - \mathcal{L}(I))\right]$$

But is not that obvious to write down the Laplace transform of a Weibull law. We used then finite differences on time to compute J : J(t) can be calculated knowing only I and J(u<t). The results were stored in tables for further simulations. The calculations were perfomed for each value of β, but for only one value of η. This is allowed by the fact that η does not change in time.

We first computed J for one value of η, eg $\eta=1$.
Let $s(t)=t/\eta$, $I_0(x)=-W'(x)$ and $J_0 = \sum_k I_0^{*k}$.

Then, for any other value of η, $R(t) = W(s(t))$, hence $I(t) = s'(t)\, I_0(s(t)) = I_0(t/\eta)/\eta$. I derives from I_0 by a simple change in time scale.
We shall then prove that J derives from J_0 by the same change in time scale, ie. $J(t)= s'(t)\, J_0(s(t)) = J_0(t/\eta)/\eta$.

Let us prove first that $I^{*k}(t)=1/\eta\, I_0^{*k}(t/\eta)$. This is true for k=1. If it is true for any value n, then :

$$I^{*n+1}(t) = \int_0^t I^{*n}(t-u)\, I(u)\, du$$
$$= \int_0^t I_0^{*n}(s(t-u))\, I_0(e(u))\, du$$
$$= \int_0^t I_0^{*n}(s(t)-s(u)))\, I_0(e(u))\, du$$

s(t-u) = s(t) – s(u), traffic being constant

$$= \int_0^t I_0^{*n}(t/\eta-u/\eta)\, I_0(u/\eta)\, \eta^{-2}\, du$$

Let $v=u/\eta$. Then $du=\eta dv$ and

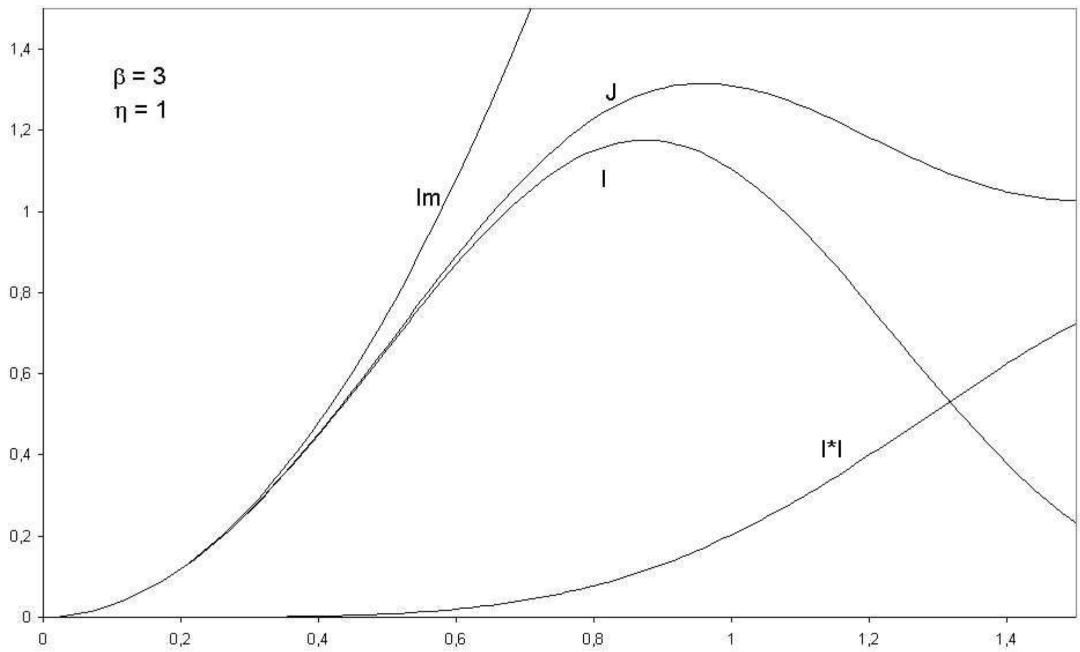
$$I^{*n+1}(t) = \int_0^{t/\eta} I_0^{*n}(t/\eta - v) \quad I_0(v)\, \eta^{-2}\, \eta \; dv$$
$$= 1/\eta \int_0^{t/\eta} I_0^{*n}(t/\eta - v)\, I_0(v)\, dv$$
$$= 1/\eta \quad I_0^{*n} * I_0(t/\eta)$$
$$= 1/\eta \quad I_0^{*n+1}(t/\eta)$$

quod erat demonstrandum. Then :

$$J(t) = \Sigma_k\, I^{*k}(t)$$
$$= \Sigma_k\, 1/\eta\, I_0^{*k}(t/\eta)$$
$$= 1/\eta\, J_0(t/\eta)$$

which completes our proof and shows that only one function J_0 has to be calculated for each value of β, the others being derived from J_0 by a simple change in time scale.

The following figure 1 shows I, J and I_m for $\beta=3$ and $\eta=1$. It appears that I_m is a very short term approximate, and that the « first repair function » I(t) can last a little longer as an approximate of J.



Traffic growth

The reliability function for sleepers depends on the traffic. It would be probably be the same for rails if we could take into account speed and axle load parameters. Now, what would the reliability function be in case of traffic changes ?

The reliability function has been set in the form $R(t) = W(t/\eta(A))$, where A includes all traffic information, and $\eta(A)$ is an average lifetime in a specific traffic situation. Now we have to consider the case where A is a certain function A(t) of time, past or future.

The reliability function R(t) in that case must verify some conditions.

First, it should be independent of time scale : one year at traffic A and one year at traffic A is equal to 2 years at traffic A. This means that R(t) must be associative according to time periods.

Then, it should be independent of time direction : one year at traffic A and one year at traffic B is equal to one year at traffic B and one year at traffic A. This means that R(t) must be commutative according to time periods. Last, if the traffic is constant, R(t) should be the same as before.
All this means that any infinitesimal time period should have the same impact on the global reliability function as if the traffic was constant, whatever the changes in traffic.

If the traffic is constant, then :

$$R(t) = W[t/\eta(A)] = W[\int_0^t du / \eta(A)]$$

We can generalize this last expression with :

$$R(t) = W[\int_0^t du / \eta(A(u))]$$

It can be noticed that this sum is associative, commutative, and gives the same results as before with constant traffic. It is thus a good candidate.

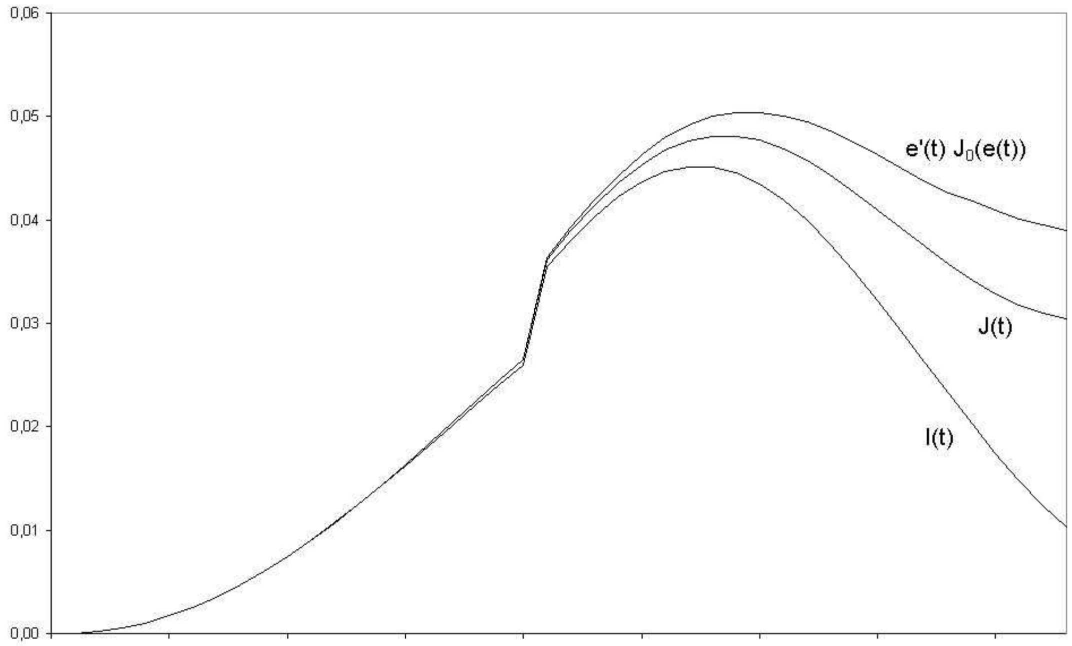
> *As an example, let us consider the case when a track has half the time a traffic which should give it a 20 years lifetime, and the other half time a traffic for 30 years lifetime. Its reliability function would be : R(t) = W (t/2*20 + t/2*30) = W (t/24) corresponding to a lifetime of 24 years. This does not go against common sense.*

Let $s(t) = \int_0^t du / \eta(A(u))$, thus R(t) = W(s(t)) and $s'(t)=1/\eta(A(t))$. The repair flow function I(t) is then :

$$\begin{aligned} I(t) &= -R'(t) \\ &= - s'(t)\, W'(s(t)) \\ &= - W'(s(t)) / \eta(A(u)) \end{aligned}$$

A problem is that the calculation of J does now depend on the traffic history A(u), and can not anymore computed only once for each value of β.

In case of constant traffic $s(t)=t/\eta$, we proved above that $J(t) = 1/\eta\, J_0(t/\eta) = s'(t)\, J_0(s(t))$. But that proof used the property s(x-y)=s(x)-s(y) with is not valid anymore. The following figure shows I(t), J(t) and $s'(t)\, J_0(s(t))$ for $\beta=3$ when the traffic doubles at a certain moment. J_0 was calculated with constant traffic.



Large sections

Now that we know the probability J(t) for a single component, eg ONE sleeper, to be repaired at time t, we have to consider what will happen on longer parts of tracks, eg 10 Km with 16,000 sleepers. What is the probability $B_{n,p}(t)$ of having p components out of n repaired at time t ?
The answer can be given using a simple binomial distribution :

$$B_{n,p}(t) = C_n^p \; J^p(t) \; (1-J(t))^{n-p}$$

It is well known that if n is great, and p stays close to J(t)n, then this binomial law will approach a Poisson distribution $P_{n,p}(t) = \alpha^p e^{-\alpha} / p\,!$, where $\alpha = n\,J(t)$.

The mean value of $P_{n,p}(t)$ is α and its standard deviation is $\sqrt{\alpha}$. As an example, if we are dealing with 100 repairs, it is with a standard deviation of 10. This shows that, when dealing with a subsequent length of track, J(t) is a good estimate of the proportion of components to be repaired per time and length units.

Computer optimization

The aim of the software was to find when and what to renew. Of course, it should have the capability to simulate the consequence of decisions in economical terms. But the difficulty was the number of alternatives.
For one homogeneous section of track, with 3 components that could be renewed or not each year for a duration of a hundred years (minimal value to yield consistent results), the number of alternatives was $3^{100} = \infty$. But this number must be multiplied if considering common strategies for neighbouring different sections.

The first topic considered was the issue of processing neighbouring sections together or separately. A cost model for renewal according to the length of trackwork was derived from statistics of past renewals. It showed that the price is quite higher for very short sections, but did not vary much above 5 km. A simulation was then performed on the whole network, with two assumptions on costs : all costs equal to the minimal value, or modelled costs. The results were only 5% apart, due to the fact that, although there were indeed many very short sections, they did not account for an important length of the network, hence of the cost of renewal. This enlightened the importance of having consistent policies, and allowed us to overlook the problem. Sections were then processed separately, the fate of very small sections being left to the goodwill of planners.

The issue of renewing components together or separately could not be overlooked so easily. The difference between complete renewal and fully separate operations amounts to one third of the total. Besides, the operation staff does not like to multiply the frequency of trackwork, and is quite sensitive on the subject. And the lifetime of components are quite differents : they can range on the very same high traffic track from 25 years for the ballast to 65 years for concrete sleepers.

To reduce the number of alternatives, two ideas arose.

The first one was to establish a catalog of situations, and process them all once and for all. This did not work well at first. Combinations of rail, sleeper, track types and ages can number up to tens of thousands, which is not quite less than the number of track sections. This is why our results can not be stated in a simple one page procedure.

The other idea, which worked better, was to eliminate all improbable alternatives, eg. renewing the rail each year. This led to the elaboration of rules for possibly acceptable decisions to be simulated. These rules were called « strategies », and written down as parameterized programs. The software would try these decision programs, with many candidate values taken by parameters, and keep the set of parameters yielding the best economics. It quickly appeared that 3 parameters was the most that could be used in less than a week calculation for the whole network. Different strategies were tested, and compared in terms of economical results.

The best strategy that we finally came up with was the following, with 3 parameters E, A, M. It uses the above described function $e(t) = \int_0^t du / \eta(A(u))$.

function RenewalDecision (E, A, M) is
    for each track component never renewed :
        decision is yes in any of the following cases
            - *e(t)>E*

- *e(t)>A and another component is renewed (but for rail and sleepers whose common renewal does not yield many savings)*
- *its renewal becomes technically necessary (eg. ballast renewal implied by wooden sleeper renewal on high traffic lines, ballast layers being too thin for concrete sleepers)*

for each track component already renewed :
decision is made on the same pattern,
using values M.E and M.A instead of E and A.

This strategy did provide the best solution in more cases than others, but not always. All strategies having yielded at least once a better solution than others have been added to the package, and now amount to 10.

This package was tested in many different cases, and in very few cases only could experts come with a slightly better solution. Its application on the whole network took 5 days on a PentiumIII© computer.

Enhancements were searched in order to reduce this duration. Five days may not look much if the software is to be used once every five years. But its results are eventually updated every year, and many simulations are performed to evaluate sensitivity to general parameters such as interest rates, traffic growth, …

The process does now take « only » 16 hours, due to two major improvements.

The first one was to use part of the first idea about a catalog. The software searches for similar sections. If one section looks a lot like one already processed, then its set of parameters are copied. Only if it does not is the above package applied.

The second improvement was to stage the examination of parameters. On the first version, each parameter was tried for each section with 10 to 20 different values, which led to 1000 to 8000 simulations. The second version tries only 6 values for each parameter, which leads to only 200 preliminary simulations, and investigates only areas showing local minima, which are usually few. The investigation is a new 3D-grid with 6 values per axis. The results proved to be the same, although no assumptions could me made about the convexity of cost functions.

Operational results

The main results yielded today by the software are the following :

- track maintenance budget for the next 5 to 150 years and sensitivity to interest rates, traffic growth, …
- track renewal policies, in particular whether or not to separate the renewal of track components,
- in some cases, spot renewal decisions,
- evaluation of marginal track cost of traffic increase,
- evaluation of marginal cost of track enhancement.

These results helped getting a better idea of the amount of money, hardware and manpower needed in the next years. It also helped forecasting the network future state, in terms of mean age and safety.