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OPTIMAL INFRASTRUCTURE MANAGEMENT DECISIONS UNDER UNCERTAINTY

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Abstract—The planning of maintenance and rehabilitation activities for transportation facilities uses information on facility condition from two sources: measurement and forecasting. Both of these sources are characterized by the presence of significant uncertainties, which have important life-cycle cost implications. State-of-the-art decision-making models ignore the uncertainty either in one or both sources of information. This paper presents a methodology (the Latent Markov Decision Process) that explicitly recognizes the presence of random measurement errors in the measurement of facility condition. The methodology can also be used to quantify the "value of more precise information," which allows an agency to evaluate measurement technologies of different precisions and costs. A parametric study, which demonstrates such an evaluation in the case of highway pavements, is performed.

1. INTRODUCTION

The infrastructure management process refers to the set of decisions made by a transportation agency concerning the allocation of funds among a network of facilities and over time to maximize its performance. The term performance is used interchangeably with the term "condition" in the literature and in this paper.

The basic maintenance and rehabilitation (M&R) decision that an agency has to make is: "in every time period, what M&R activity should be performed on each facility in the network?" This M&R decision requires several data items, such as the available budget, the cost and effectiveness of different activities, the current and projected levels of usage, etc., but the most important item is the condition of the facilities in the network.

There are two forms of information on infrastructure condition: information on current condition, which is provided by facility inspection, and information on future condition, which is provided by the forecast of a performance model. Performance models are mathematical relations having as a dependent variable the condition of the facility, and as independent variables the facility's age, traffic, environmental variables, historical M&R activities, etc.

The relationships between the measurement and forecasting of facility condition and M&R decision making are shown in Fig. 1 (Ben-Akiva, Humplick, Madanat, & Ramaswamy, 1991, 1993). The facility condition information collected using different inspection technologies is used in two ways. First, it is one of the items used in the estimation of infrastructure performance models. Second, as mentioned above, it is one of the inputs in selecting maintenance and rehabilitation strategies for the current time period. Infrastructure performance models are used in selecting activities for the current period and for planning future M&R activities. The decision-making block selects, in addition to M&R activities, inspection strategies on which basis future data are collected. This effect is represented by the feed-back loop of Fig. 1.

Both forms of condition information are characterized by a large degree of uncertainty. Inspection output has a number of errors from a variety of sources: technological limitations, data processing errors, errors due to the nature of the infrastructure surface inspected, and errors due to environmental effects. These sources of errors interact and produce measurement biases and random errors. If the magnitudes of the biases are known, then the measurements can be corrected for their presence by suitable subtraction and multiplication. The random errors, on the other hand, can only be described in terms of the parameters of their statistical distributions, if known, and cannot be corrected for (Humplick, 1992). Model forecasts are also characterized by a high degree of uncertainty due to measurement errors in the variables used in the model, inherent randomness, and the inability to model the true process of deterioration perfectly. This uncertainty can be quantified by the standard error of the forecast of the performance model.

The motivation for the research presented in this paper is the presence of uncertainty, in both infrastructure condition measurement and prediction, which has important implications for the management process. These implications are discussed separately below.

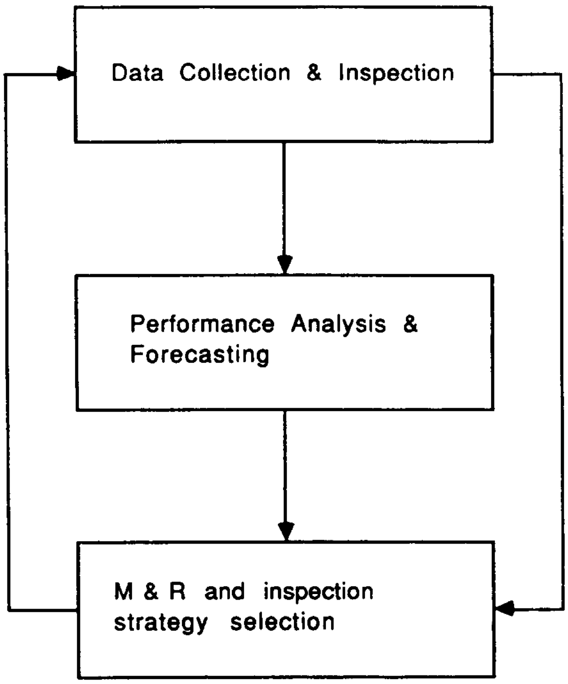

Fig. 1. The infrastructure management process.

Uncertainty in facility condition prediction has been addressed by representing the deterioration process by Markov transition probabilities, and by using Markov Decision Processes (MDP) as a methodology for M&R decision making (Carnahan, Davis, Shahin, Keane, & Wu, 1987; Feighan, Shahin, Sinha, & White, 1988). In such a model, the facility condition at any time is measured by a discrete state, and the deterioration process is represented by discrete transition probabilities of the form:

$$p(x_{t+1} = j \mid x_t = i, a_t); \quad 1 \leq i,j \leq n; \quad t = 0,1, \ldots T - 1$$

where:

x_t = condition state of the facility at the beginning of year t,
a_t = M&R activity performed during year t,
n = number of possible states the facility can be in, and
T = number of years in the planning horizon.

Because the probabilistic distributions of the future condition states can be obtained by using the transition probabilities, the decision maker can, at the beginning of the planning horizon, evaluate alternative M&R policies for the entire horizon. The underlying assumption in this methodology is that an inspection is performed at the beginning of every year, and that inspections *reveal the true condition state of the facility, with no error*. As a result, after an inspection, the decision maker can apply the activity prescribed by the optimal policy for that condition state of the facility.

Uncertainty in the outcome of condition measurement, on the other hand, has not been addressed in the existing M&R decision models. This uncertainty affects M&R decisions because a measurement error can lead to the selection of a "wrong" activity if the prescribed M&R activity for the true condition and that prescribed for the measured condition are different. This "wrong" selection translates into an increase in the total lifecycle costs of an infrastructure facility if the "correct" M&R activity is the one that achieves minimum cost. This increase in lifecycle costs becomes more pronounced when measurements are repeated throughout the lifecycle of that facility, such as in Markov Decision Processes.

In this paper, a methodology for M&R activity selection, which accounts for the presence of both forecasting and measurement uncertainty, is presented. This methodology, the *Latent Markov Decision Process* (LMDP), is an extension of the traditional MDP methodology, but differs from it in one major aspect: It does not assume the measurement of facility condition to be necessarily error-free. Instead, it assumes that the decision maker observes "outputs" from the measurement that are only probabilistically related to the true condition of the facility (Eckles, 1968; Smallwood and Sondik, 1973).

2. THE LATENT MARKOV DECISION PROCESS

The difficulty with introducing measurement uncertainty into the MDP problem is that it violates the central assumption of knowledge of the condition state after an inspection. What the decision maker observes at the beginning of t is now a measured state, which is only probabilistically related to the true state of the system. This function can be mathematically stated as:

$$q(\hat{x}_t = k \mid x_t = j); \quad 1 \leq j,k \leq n \quad t = 0,1, \ldots, T$$

where:

$\hat{x}_t$ = measured condition state of the facility at start of t,
x_t = true condition state of the facility at start of t,
j,k = values belonging to the set of the discrete condition states, and
q = a known probability mass function.

To address the problem created by the violation of the assumption of perfect inspection, state augmentation (Bertsekas, 1987) is used. State augmentation consists of redefining the state of the system at any point in time to include all the information available to the decision maker that is relevant to future decisions.

When the condition state of the facility is measured with uncertainty, the information available to the decision maker at the beginning of t includes the entire history of measured states up to t and the decisions made up to $t - 1$. Moreover, since the measured state at t is only probabilistically related to the true state at t, knowledge of the measured state is not sufficient for decision making. As a result, all the previous measured states and decisions in the history can be relevant to future decisions, and have to be included in the augmented state. Denoting the new state by I_t, we have:

$$I_t = \{I_0, a_0, \hat{x}_1, a_1, \ldots, \hat{x}_{t-1}, a_{t-1}, \hat{x}_t\}; \quad t = 1,2, \ldots, T$$
$$I_0 = \{\hat{x}_{-\tau}, a_{-\tau}, \ldots, \hat{x}_{-1}, a_{-1}, \hat{x}_0\}$$

where:

τ = number of years between first inspection of facility and start of planning horizon.

It follows that:

$$I_t = \{I_{t-1}, a_{t-1}, \hat{x}_t\} \quad t = 1, \ldots, T$$

from which we can write:

$$P(I_t \mid I_0, a_0, \hat{x}_1, I_1, \ldots, I_{t-2}, a_{t-2}, \hat{x}_{t-1}, I_{t-1}, a_{t-1}) = P(I_t \mid I_{t-1}, a_{t-1}) \quad t = 1, \ldots, T. \quad (1)$$

Assuming I_0 to be known, the transition probabilities $P(I_t \mid I_{t-1}, a_{t-1})$ define the evolution of the state of information, and this evolution is Markovian, by virtue of eqn (1).

We can thus write a dynamic programming formulation over the space of the information states (for notational simplicity, this will be done using the generic cost function $g(x_t, a_t)$). To do that, the cost function has to be rewritten in terms of the new variables. The cost per stage as a function of the new state, I_t, and of the activity a_t, is:

$$\bar{g}(I_t,a_t) = \underset{x_t}{E}\{g(x_t,a_t) \mid I_t\} \tag{2}$$

where:

$\underset{x_t}{E}\{g \mid I_t\}$ is the conditional expectation of g over x_t conditional on I_t.

The dynamic programming formulation is given by:

$$J_T(I_T) = \underset{x_T}{E}\{g(x_T) \mid I_T\}$$

and

$$J_t(I_t) = \min_{a_t}\left(\underset{x_t}{E}\{g(x_t,a_t) \mid I_t\} + \alpha \sum_{\forall L_{t+1}} P(I_{t+1} = L_{t+1} \mid I_t,a_t)J_{t+1}(L_{t+1})\right) \quad t = 0, \ldots, T-1 \tag{3}$$

where:

$$\begin{aligned} \alpha &= \text{annual discount amount factor,} \\ P(I_{t+1} = L_{t+1} \mid I_t,a_t) &= \text{transition probabilities for the state of information,} \\ L_{t+1} &= \{I_t,a_t,\hat{x}_{t+1}\}, \text{ and} \\ \hat{x}_{t+1} &= \text{measured condition state at } t+1. \end{aligned}$$

In order to solve program (3), the relationship between I_t and the probabilistic distribution of x_t has to be established. In other words, we need to know $p_t(x_t \mid I_t)$, $\forall x_t$, $\forall I_t$, $\forall t$ or, in vector form, $P_t \mid I_t$, $\forall I_t$, $\forall t$ where $P_t \mid I_t$ is an n-dimensional vector (the information vector) with elements $p_t(x_t \mid I_t)$.

If we assume $P_0 \mid I_0$ to be known, then $P_t \mid I_t$ can be calculated recursively for all t, starting from $t = 1$, using Bayes' law, the known measurement probabilities and the known transition probabilities. Given $I_t = \{I_{t-1},a_{t-1},\hat{x}_t\}$, each element of $P_t \mid I_t$ is given by:

$$\begin{aligned} p_t(x_t = j \mid I_t) &= \frac{\text{prob}(x_t + j,I_{t-1},a_{t-1},\hat{x}_t)}{\text{prob}(I_{t-1},a_{t-1},\hat{x}_t)} \\ &= \frac{\text{prob}(x_t = j,\hat{x}_t \mid I_{t-1},a_{t-1})}{\text{prob}(\hat{x}_t \mid I_{t-1},a_{t-1})} \\ &= \frac{q(\hat{x}_t \mid x_t = j)\sum_i p(x_t = j \mid x_{t-1} = i,a_{t-1})p_{t-1}(x_{t-1} = i \mid I_{t-1})}{\sum_j q(\hat{x}_t \mid x_t = j)\sum_i p(x_t = j \mid x_{t-1} = i,a_{t-1})p_{t-1}(x_{t-1} = i \mid I_{t-1})}, \quad j = 1, \ldots, n. \end{aligned} \tag{4}$$

Using the elements $p_t(x_t = i \mid I_t)$ calculated above, (3) can be rewritten as:

$$J_T(I_T) = \sum_{i=1}^{n} p_T(x_T = i \mid I_T)g(x_T); \quad \forall I_T$$

and

$$J_t(I_T) = \min_{a_t}\left(\sum_{i=1}^{n} p_t(x_t = i \mid I_t)g(x_t,a_t) + \alpha\sum_{k=1}^{n} P(\hat{x}_{t+1} = k \mid I_t,a_t)J_{t+1}(I_t,a_t,\hat{x}_{t+1} = k)\right) \quad \forall I_t \quad t = 0, \ldots, T-1. \tag{5}$$

The expression $P(\hat{x}_{t+1} = k \mid I_t,a_t)$ can be decomposed into known quantities:

$$P(\hat{x}_{t+1} = k \mid I_t,a_t) = \sum_{j=1}^{n} q(\hat{x}_{t+1} = k \mid x_{t+1} = j)\sum_{i=1}^{n} p(x_{t+1} = j \mid x_t = i,a_t)p_t(x_t = i \mid I_t).$$

Substituting in (5), we obtain:

$$J_T(I_T) = \sum_{i=1}^{n} p_T(x_T = i \mid I_T)g(x_T) \quad \forall I_T$$

and

$$J_t(I_t) = \min_{a_t}\left(\sum_{i=1}^{n} p_t(x_t = i \mid I_t)g(x_t,a_t) + \alpha\sum_{i=1}^{n} p_t(x_t = i \mid I_t)\sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i,a_t)\sum_{k=1}^{n} q(\hat{x}_{t+1} = k \mid x_{t+1} = j)J_{t+1}(I_t,a_t,\hat{x}_{t+1} = k)\right)$$
$$\forall I_t, \quad t = 0, \ldots, T-1. \tag{6}$$

The model defined by this formulation will be referred to as the Latent Markov Decision Process with annual inspections, because it assumes that the state of the facility is latent, and because it assumes that a measurement of facility condition $\hat{x}_t$ is available at the start of every year, or every time period, t.

The similarity between the LMDP and the classical MDP can best be explained in terms of the underlying decision trees. Figure 2 depicts a classical MDP tree. At the beginning of time period t, the true state x_t is observed. Based on this knowledge, the decision maker selects an activity a_t. Given the *facility* state x_t and the selected activity a_t, the facility moves to one of the states $x_{t+1} = j$ with probability $p(x_{t+1} = j \mid x_t,a_t)$. The same process is then repeated in time period $t + 1$, and so on.

In Fig. 3, an LMDP tree is shown. We start in period t, when the decision maker has available the state of the information I_t. Based on this information, an activity a_t is selected. Given the *information* state I_t and a_t, the system moves to one of the states $I_{t+1} = K$, with probability $P(I_{t+1} = K \mid I_t,a_t)$. The same process is then repeated in time period $t + 1$, and so on.

3. THE DYNAMIC PROGRAMMING SOLUTION

Once the true condition state probabilities are calculated from $t = 1$ to $t = T$, using (4), we could solve program (6) recursively from $t = T$ to $t = 0$, and for all states of the information

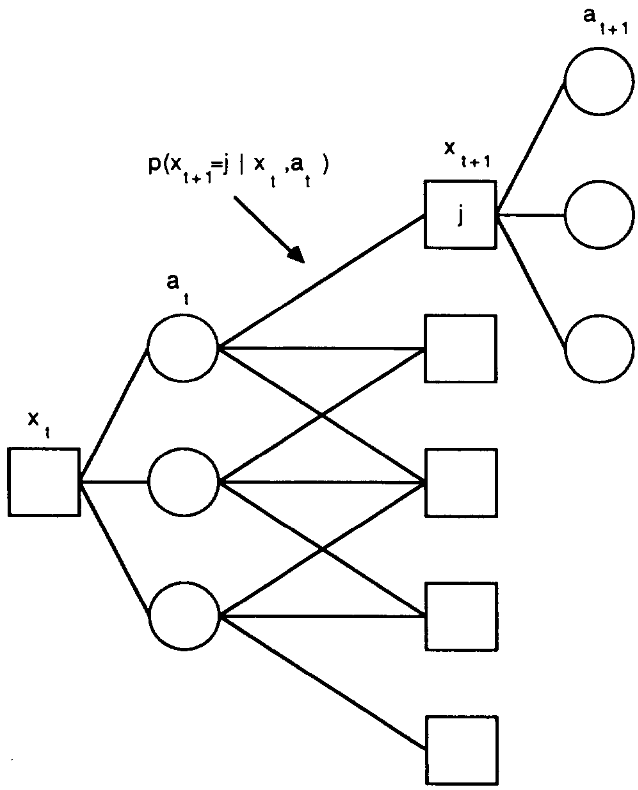

Fig. 2. Decision tree for the Markov decision process.

I_t at each year t, to obtain the minimum expected costs $J_0(I_0)$ and the optimum policy $\pi^* = \{\mu_0^*(I_0), \mu_1^*(I_1), \ldots, \mu_{T-1}^*(I_{T-1})\}$. This, however, would be computationally very expensive. The reason for this is that the number of states I_t for which eqn (6) has to be solved at each year grows exponentially with t. To verify this, note that if the number of possible condition states is n and the number of permissible activities is m, and if I_0 is given, then there are $(n*m)^t$ possible values of I_t at t, which creates a problem of prohibitive size for any reasonable planning horizon T. The problem is that even when two states I_t have similar "information contents" as far as future decisions are concerned (that is, they produce similar distributions of the true state $p_t(x_t = i \mid I_t)$, $i = 1, \ldots, n$), they still are considered as separate states by the dynamic programming algorithm.

It is thus of interest to replace I_t with a quantity of smaller dimension, but having the same information content. Such a quantity is referred to as a "sufficient statistic" for I_t. Since, as observed earlier, I_t affects decisions only through $p_t(x_t = i \mid I_t)$, $i = 1, \ldots, n$, an ideal sufficient statistic is the information vector $P_t \mid I_t$.

The advantage of this sufficient statistic is that it allows for direct comparison among states at a given t. Two states of the information that consist of very different histories I_t may have similar $P_t \mid I_t$ vectors. Whereas it is impossible to compare directly the vectors I_t, it is possible to compare the information vectors $P_t \mid I_t$ by pairwise comparison of corresponding elements. When two states are found to have equal, or almost equal, values of $P_t \mid I_t$, they can be combined into a single state, which reduces the number of times eqn (6) has to be applied. The subject of combination of different states is covered in Madanat (1991) in more detail.

Cost structure of the problem

So far, the cost per stage of the problem has been assumed to be captured by the function $g(x_t, a_t)$. It is necessary, in order to formulate the dynamic programming for the M&R decision problem, to specify the components of this cost function. These components are:

The expected cost of performing the M&R activity. The cost of an activity depends on the type of activity and on the extent of this activity. The extent of an activity is a function of the

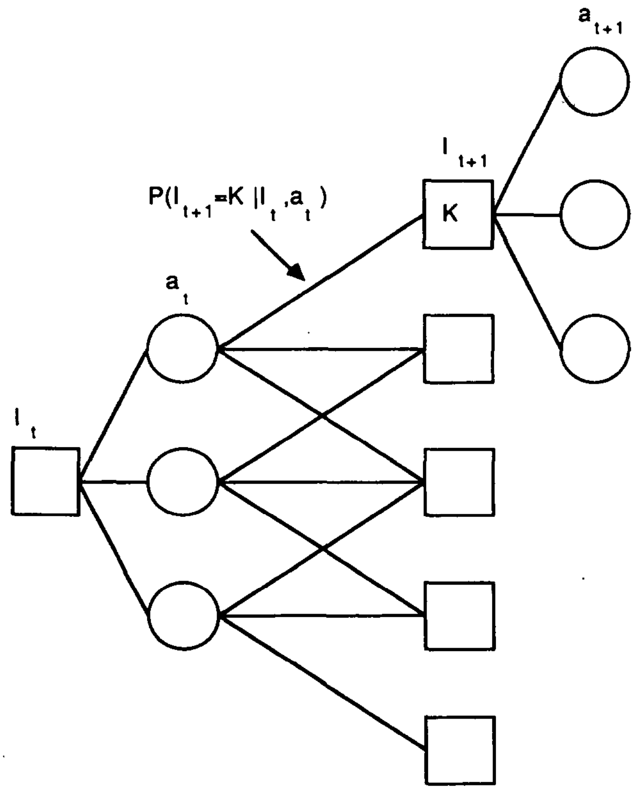

Fig. 3. Decision tree for the LMDP with annual inspections.

true state of the facility, x_t. Since x_t is not known, we use the expected cost of an activity, taken over the distribution of x_t, denoted by:

$$\sum_{i=1}^{n} p_t(x_t = i \mid I_t)^* ca(a_t \mid x_t = i)$$

where:

$ca(a_t \mid x_t = i)$ = cost of performing activity a_t, when the true condition state of the facility is i.

The inspection cost. This is incurred at the beginning of every time period t, and is constant: $cm(r_t)$ where:

$cm(r_t)$ = cost of inspecting the facility using measurement technology r_t (the technology is assumed fixed for the time being).

Expected user costs. User costs are a function of the condition state of the facility; since the condition state is not known, its distribution is used. Because user costs models for infrastructure facilities are not readily available, they can be replaced by a minimum allowable condition state, which acts as a constraint in the cost-minimization algorithm. This constraint is introduced by specifying a penalty to take a value of zero if the condition state is above the minimum allowable state and a value of infinity if it falls below that minimum. The mathematical expression for the expected penalty is:

$$\alpha \sum_{i=1}^{n} p_t(x_t = i \mid I_t) \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) cu(x_{t+1} = j)$$

where:

$cu(x_{t+1} = j)$ = penalty incurred for year t if the facility is in state j at the end of the year.

The dynamic programming formulation of the M&R decision problem, over the state space of the information vectors $P_t \mid I_t$ is presented below:

$$J_T(P_T \mid I_T) = 0, \quad \forall P_T \mid I_T$$

and

$$\begin{aligned} J_t(P_T \mid I_t) = \min_{a_t} \Bigg\{ \sum_{i=1}^{n} p_t(x_t = i \mid I_t) \Bigg[& ca(a_t \mid x_t = i) + \alpha \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) cu(x_{t+1} \\ & = j) + \alpha cm(r_{t+1}) + \alpha \sum_{j=1}^{n} p(x_{t+1} = j \mid x_t = i, a_t) \sum_{k=1}^{n} q(\hat{x}_{t+1} = k \mid x_{t+1} \\ & = j, r_{t+1}) J_{t+1}(P_{t+1} \mid I_{t+1}) \Bigg] \Bigg\} \quad \forall P_t \mid I_t, \quad t = 0,1, \ldots, T-1. \end{aligned} \tag{7}$$

where:

$I_{t+1} = \{I_t, a_t, r_{t+1}, \hat{x}_{t+1} = k\}$
r_{t+1} = inspection technology used by agency, assumed given.

Note that the cost of the inspection performed at the beginning of period $t + 1$ has been included in the cost-to-go expression, J_t, for time period t instead of that for $t + 1$, and that the measurement probabilities have been made conditional on the inspection technology r_{t+1}.

Finally, we assume that $P_0 \mid I_0$ is given, in order to calculate $P_t \mid I_t$, for all t, using (4). This initial vector, the prior distribution of the true states, represents the prior belief of the decision maker regarding the state of the facility. This belief may have been formed from an inspection, just prior to the beginning of the planning horizon.

Solving program (7) for a specified planning horizon T will yield the minimum expected cost $J_0(P_0 \mid I_0)$ and the optimal sequence of policies $\pi^* = \{\mu_0^*(P_0 \mid I_0), \mu_1^*(P_1 \mid I_1), \ldots, \mu_{T-1}^*(P_{T-1} \mid I_{T-1})\}$, where $\mu_t^*(P_t \mid I_t)$, the optimal policy for period t, specifies the optimal M&R activity for each "possible" information vector at time t, $P_t \mid I_t$. A "possible" information vector is one that can be reached with non-zero probability at time t, given $P_0 \mid I_0$, and given the optimal policies up to t: $\mu_0^*(P_0 \mid I_0), \mu_1^*(P_1 \mid I_1), \ldots, \mu_{t-1}^*(P_{t-1} \mid I_{t-1})$.

4. PARAMETRIC STUDY: THE EFFECT OF MEASUREMENT UNCERTAINTY

In this section, a parametric study is performed to investigate the effect of measurement uncertainty on the minimum expected cost obtained by the LMDP. This study also demonstrates a useful capability of the Latent Markov Decision Process: the quantification of the benefits of using more precise measurement technologies. This can be done by comparing the minimum expected lifecycle cost obtained, for a given infrastructure facility and planning horizon, by using measurement technologies of different precisions.

Table 1. Routine maintenance transition matrix

$x(t+1)$	7	6	5	4	3	2	1	0
$x(t)$ 7	0.85	0.15						
6		0.73	0.27					
5			0.62	0.38				
4				0.52	0.48			
3					0.43	0.57		
2						0.35	0.65	
1							0.29	0.71
0								1

Data

This empirical study used data from the field of highway pavements, because of the extensive previous research in that application area, which has made available a large number of suitable models. The condition scale of a pavement section was described by the PCI (Pavement Condition Index, Shahin and Kohn, 1981), which has values in the range 0 to 100. This range was divided into eight states, that is: $i = 0, 1, \ldots, 7$, as in Carnahan *et al.* (1987). The transition probabilities and associated unit costs were adapted, with few modifications, from Carnahan *et al.* (1987), for the following three activities: routine maintenance, two-inch overlay, and reconstruction. The three transition matrices are shown in Tables 1 to 3, and the associated M&R costs are shown in Table 4. The minimum allowable state for the facility was set to $i = 3$ (i.e. the penalty has a value of infinity for states 0, 1, and 2, and a value of 0 elsewhere).

The planning horizon (T) for the study was set to 10 years and the interest rate to 5%, which corresponds to a discount amount factor (α) of 0.9524. The initial information vector, $P_0 \mid I_0$, was set to: $p_0(x_0 = 7 \mid I_0) = 1.0$, $p_0(x_0 = i \mid I_0) = 0.0, \quad i = 0, \ldots, 6$.

Five hypothetical cases of measurement precision were analyzed in the parametric study. The measurement errors were assumed to be normally distributed, with zero mean and standard deviations of 0.0, 2.5, 5.0, 7.5, and 10.0 PCI units, respectively. These distributions were then transformed into discrete measurement probabilities by using basic theorems of probability.

The five hypothetical measurement technologies analyzed in the study were assumed to have equal unit costs. This assumption was made to isolate the effects of measurement precision on minimum expected cost. A comparison of existing technologies conducted for the FHWA (Hudson, Elkins, Uddin, and Reilly, 1987) provided unit costs for different technologies. The average of these unit costs was used as the unit cost of measurement in this study. The discrete measurement probabilities corresponding to each measurement precision are shown in Table 5, together with the unit cost of measurement.

Results

The results of the study are summarized in Fig. 4. It shows the variation in minimum expected costs, $J_0(P_0 \mid I_0)$ as a function of the standard deviation of measurement, for the LMDP algorithm. The points on this curve were obtained using program (7).

The main observation that can be made is that the minimum for $J_0(P_0 \mid I_0)$ lies at the extreme left, that is, when the standard deviation of the measurement technology is the smallest. This trend shows that minimum expected lifecycle cost increases with an increase in measurement uncertainty. This is an intuitively appealing result, which can be explained by considering

Table 2. Overlay transition matrix

$x(t+1)$	7	6	5	4	3	2	1	0
$x(t)$ 7	0.85	0.15						
6	0.73	0.27						
5	0.62	0.38						
4	0.52	0.48						
3	0.43	0.57						
2	0.35	0.65						
1	0.29	0.71						
0		1						

Table 3. Reconstruction transition matrix

$x(t+1)$	7	6	5	4	3	2	1	0
$x(t)$ 7	0.85	0.15						
6	0.85	0.15						
5	0.85	0.15						
4	0.85	0.15						
3	0.85	0.15						
2	0.85	0.15						
1	0.85	0.15						
0	0.85	0.15						

the shape of the cost-to-go function of the LMDP for the case where there is no measurement uncertainty.

The cost-to-go function for that problem (which can be written as $J_t(x_t)$ because x_t is observed) is generally convex in x_t. This is shown in Fig. 5 where the cost-to-go function, for different years in the planning horizon, is plotted. It can be seen that $J_t(x_t)$ is convex over the range of x_t where the facility is allowed to be, that is, for the states above $i = 2$. The result of this convexity is that, even when no measurement bias exists, an increase in uncertainty about the condition state of the facility leads to an increase in minimum expected costs. In other words, we expect that costs will be monotonically non-increasing with the precision of the measurement technology. This is indeed what the results of the parametric study show.

The "value of 1 more unit of measurement precision" can be calculated from Fig. 4. It is given by the difference, on the y axis, corresponding to one unit of precision, on the x axis. This value corresponds to the savings in life-cycle costs, which are accrued by increasing the precision of measurement by one unit of PCI, in dollars per square yards. This can be used as a criterion in evaluating the feasibility of investing in new measurement technologies.

5. CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this paper, a methodology for optimizing M&R decisions in infrastructure management was developed. This methodology, the Latent Markov Decision Process, is sufficiently general to be applied to different types of infrastructure facilities, but it extends the state-of-the-art specifically in the area of pavement management. Before making this methodology operational within a pavement management system, there is one issue that needs to be addressed: the incorporation of network level constraints. This issue is discussed below.

A major limitation of facility-level M&R decision-making models is that they do not account for network-level considerations, such as budget constraints. This limitation, however, can be overcome if the LMDP model is extended to the network-level problem through the use of "randomized policies."

The type of policies produced by the LMDP (and by the ordinary MDP) are non-randomized policies. This is because, given a state of the system I_t, the model specifies a single M&R activity. This specification is given by the policy $\mu_t^*(I_t)$.

A randomized policy does not specify a single optimal activity for each state of the system. Instead, it specifies optimal **probabilities** for different activities for each state of the system. For example, an optimal randomized policy, for M&R only, would take the form:

Table 4. Costs of the M&R alternatives (in dollars per square yard)

State	Routine Maintenance	Two-inch Overlay	Reconstruction
7	0.04	3.81	25.97
6	0.15	3.91	25.97
5	0.31	4.11	25.97
4	0.65	6.64	25.97
3	0.83	9.06	25.97
2	1.4	10.69	25.97
1	2	12.31	25.97
0	6.9	21.81	25.97

Table 5. Precisions and costs of measurement

SD (q) (PCI units)	$q(\hat{x} = j - 2 \mid x = j)$	$q(\hat{x} = j - 1 \mid x = j)$	$q(\hat{x} = j \mid x = j)$	$q(\hat{x} = j + 1 \mid x = j)$	$q(\hat{x} = j + 2 \mid x = j)$
0	0	0	1	0	0
2.5	0	0.08	0.84	0.08	0
5	0	0.16	0.68	0.16	0
7.5	0.01	0.22	0.54	0.22	0.01
10	0.04	0.24	0.44	0.24	0.04

Unit cost of measurement = \$0.065/sq. yd.

$$\mu_t^*(I_t) = \{p_t(a_t = 1 \mid I_t), \ldots, p_t(a_t = m \mid I_t)\}$$

where:

$p_t(a_t = k \mid I_t)$ is the optimal probability of performing activity k at time t for system state I_t, $k = 1, \ldots, m$, and m = number of allowable M&R activities.

At the facility level, the concept of the probabilities of different activities is ambiguous. If we are dealing with a number of facilities in the same information state, on the other hand, these probabilities can be interpreted as fractions of activities. In other words, the optimal policies would specify the optimal fractions of activities to be applied to facilities in state I_t. The choice of the specific activity to be applied to each facility in this state is left to the local engineer. This procedure recognizes that there exist other considerations in the choice of M&R activities that are not captured by the model; for example, materials and labor availability, traffic disruption, and others. Due to these considerations, it is necessary to allow for some flexibility in the model, which can be exploited by the engineer in charge of these facilities.

By using randomized policies, we can apply the LMDP model to several facilities at once. It is also possible to include in the formulation network-level constraints, such as budget constraints, a feature that allows us to use the new model for the solution of the network-level problem. This is achieved by formulating the problem as a Linear Program, where the objective is to minimize expected lifecycle cost, subject to minimum performance constraints, and where the decision variables are the fractions of activities to be applied to facilities in each state of the

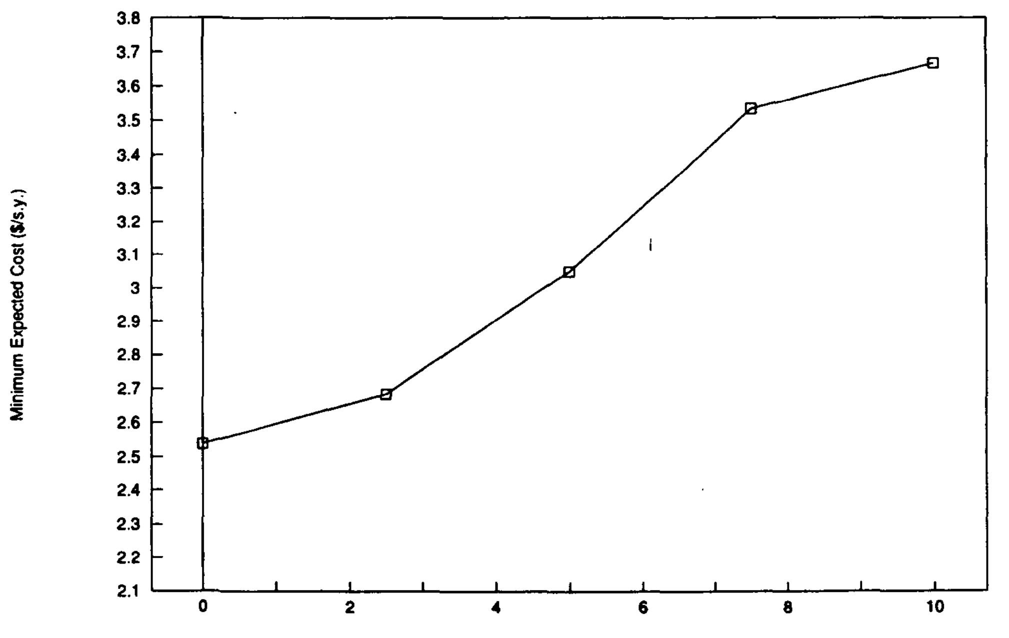

Fig. 4. Effect of measurement uncertainty on minimum expected cost.

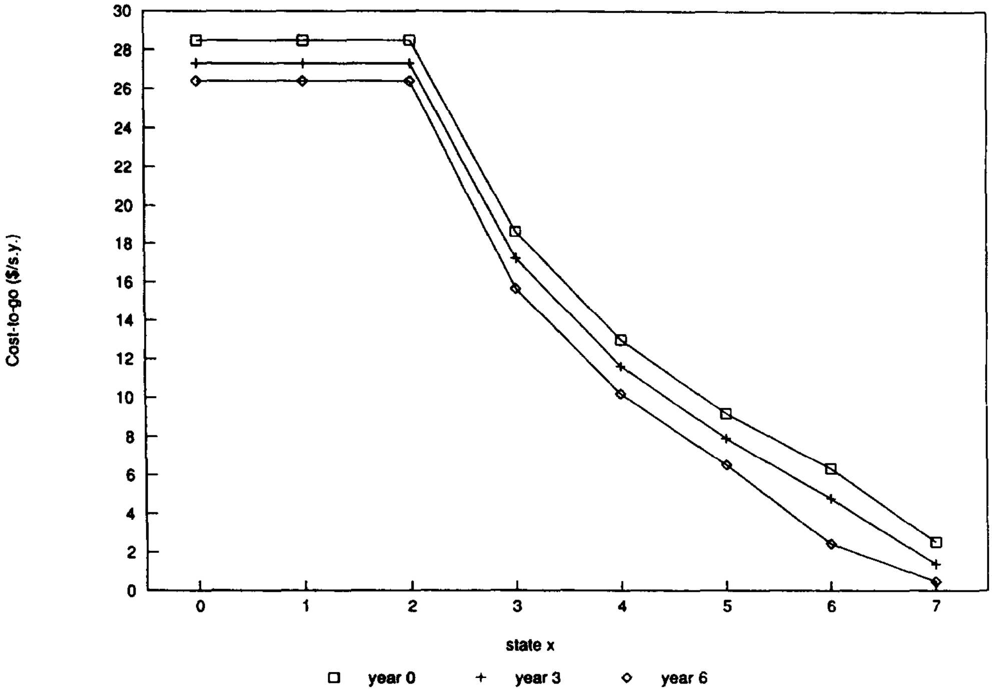

Fig. 5. Cost-to-go when SD (q) = 0 under annual inspections.

information. The concept of randomized policies has been used extensively in the context of classical MDP models; see Golabi, Kulkarni, and Way (1982).

Another issue not discussed in this paper is the incorporation of inspection policies in the infrastructure management process, by extending the LMDP formulation to include inspection decisions. This topic is investigated in depth in Madanat and Ben-Akiva (in press).

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