ESTIMATION OF INFRASTRUCTURE DISTRESS INITIATION AND PROGRESSION MODELS

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ABSTRACT: Infrastructure distress models predict the initiation and progression of distress on a facility over time as a function of age, design characteristics, environmental factors, and so on. Examples of facility distress included cracking, potholing, and rutting. Facility condition survey data sets typically include a large number of structural zeros indicating absence of distress at the time of observation. Most distress progression models in the literature are simple regression models that are estimated using the sample of observations for which distress has been initiated. These models are statistically erroneous because they suffer from selectivity bias due to the nonrandom nature of the estimation sample used. In this paper, we apply two econometric methods to estimate joint discrete-continuous models of infrastructure distress initiation and progression while correcting for selectivity bias. These methods are Heckman's procedure and the full information maximum likelihood method. An empirical case study demonstrates these methods for the case of highway-pavement-cracking models. It is shown that selectivity bias can be a very serious problem in such models.

INTRODUCTION

Modeling of facility deterioration is a critical component of the infrastructure management process at both the project and network levels. Deterioration models provide predictions of infrastructure condition over time, which is a necessary input for planning maintenance and rehabilitation activities (Ben-Akiva et al. 1993).

The literature contains a vast body of work on deterioration modeling of facilities such as pavements and bridges. These models primarily relate indicators of infrastructure condition to explanatory variables such as traffic loads, age, and environmental factors. The basic measures of infrastructure condition are the distress indicators, which can be classified as indicators of structural or functional performance. Structural performance is the ability of the infrastructure facility to carry the design loads, while functional performance relates to its serviceability and safety. These distresses could be caused by load, moisture, temperature, construction defects, or a combination of these. In some infrastructure distress data sets, such as those for cracking, spalling, and potholing, there may be a large number of structural zeros, which implies the absence of distress at the time of observation. Due to this reason, infrastructure distress models usually include separate initiation and progression phases. Moreover, deterioration models usually account for the interaction between different distress types: for example, cracking influences potholing and patching, and they in turn influence rut-depth progression.

The most important previous studies aimed at developing models of infrastructure distress progression and initiation are summarized in the following. Our review emphasizes studies in the area of highway-pavement deterioration.

The ARE study (Butler et al. 1985) developed models of pavement distress and serviceability as a function of explanatory variables. The distress types modeled in the ARE study are cracking, raveling, potholes, rut depth, and roughness. Two different sets of models were developed for the initiation and progression phases of the deterioration process. For each time period and distress type, these models predict the change in the extent of distress. The EAROMAR model system (Markow and Brademeyer 1981) predicts pavement performance, as well as maintenance and rehabilitation costs. The EAROMAR model differs from the ARE model in that deterioration is predicted by modeling the change in the subsurface material properties as a result of traffic loads and precipitation, rather than simply specifying the condition as a function of a vector of explanatory variables. The Queiroz-Geipot models (Queiroz 1981) have separate regression equations that predict cracking initiation and the rate of crack progression. The cracking-initiation model used the number of equivalent single axles to initiation as the dependent variable and the modified structural number as the explanatory variable, whereas cracking progression was specified to be a function of both structural and age parameters of the pavement. The RITM2 team (Hodges et al. 1975; Peirsly and Robinson 1982) also developed separate models for cracking initiation and progression. The cracking-initiation model predicts the amount of cumulative equivalent single axles applied during the period before cracking initiation for a given modified structural number. The cracking progression model predicts the incremental change in the area of cracking and patching as a function of the modified structural number and the incremental cumulative traffic loading since the most recent resurfacing. The highway design and maintenance (HDM) models developed by the World Bank (Paterson 1987) predict the initiation and progression of various pavement distresses, namely cracking, rutting, potholing, raveling, and roughness. Each distress model includes a number of explanatory variables such as age, traffic, design parameters, environmental factors, and other distresses.

The models discussed make predictions based on the expected value of condition. Unlike these models, the Arizona Department of Transportation pavement-deterioration models (Golabi et al. 1982) make probabilistic predictions about future condition. This model system assumes that the propagation of distress over time follows a Markov process. Therefore, the probability of transition from a given state to a more deteriorated state depends only on the present-state variables and is independent of the previous states. The components that describe the state of a pavement section at any point are present roughness, present cracking, change in cracking during the previous year, and the index to the first crack, which is a measure of pavement strength. Stochastic models for pavement performance were also developed by Moavenzadeh and Brademeyer (1977). Here, a set of models was developed

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Note. Associate Editor: Timothy L. Jacobs. Discussion open until February 1, 1996. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on November 3, 1994. This paper is part of the *Journal of Infrastructure Systems*, Vol. 1, No. 3, September, 1995. ©ASCE, ISSN 1076-0342/95/0003-0146–0150/$2.00 + $.25 per page. Paper No. 9542.

to account for the interactions that exist among the material properties, environment effects, traffic, and maintenance aspects of the pavement system. One of the models analyzes pavement distress and performance. Inputs to the model include response functions, environmental factors, such as temperature history, and traffic-load parameters such as rate, duration, and magnitudes of the loadings.

Most deterioration models reviewed consist of separate equations for distress initiation and progression. In such a representation the progression models could be subject to selection bias. Cracking initiation for example was represented in the HDM study by a duration model, where the dependent variable is the probability distribution of the time to cracking. Cracking progression was developed as a regression model, where the estimation sample consists only of facilities that have cracked. Such an estimation procedure is subject to selection bias that is introduced by nonrandom sampling of observations in the progression mode. This nonrandom sampling leads to data sets that are said to be incidentally truncated. Effectively, the data subset used in the estimation of the progression model is likely to differ significantly from the entire data set in its coverage. As such, these progression models are not representative of the entire pavement population from which the sample was drawn.

In the present paper, we apply a structured econometric method for developing deterioration models of distress initiation and progression. We use a joint discrete continuous model consisting of a discrete-choice model for distress initiation and a regression model for distress progression. We note that the estimation sample for the progression model is biased, as it contains a disproportionately large fraction of facilities with high deterioration rates, because they are more likely to have already started cracking (they have lower initiation times) than those facilities with low deterioration rates. This selectivity bias may be corrected by using Heckman's procedure (Greene 1993) or by maximum likelihood estimation to obtain consistent coefficient estimates. Furthermore, we explicitly treat the models for the endogeneity bias that may be introduced by including the values of some distresses as explanatory variables in the progression model of other distresses.

We will first briefly describe the methodology for model estimation with incidentally truncated samples. We then summarize the data set used in this study and present some empirical results from the application of this method. We conclude by discussing the importance of these modeling techniques in the field of infrastructure management.

MODELING USING INCIDENTALLY TRUNCATED SAMPLES

In this section, we present the mathematical formulation and estimation procedures for incidentally truncated regression. In incidental truncation, sample selection depends on a host of factors, of which only a few are known. The sample selection process is modeled by a discrete-choice model, and the observations that are selected are then used to develop a continuous model. In this study, the initiation of distress forms the discrete model and the progression of distress forms the continuous model. In the discrete model, the dependent variable is a zero/one indicator. If distress initiation has not yet occurred, the indicator variable takes the value 0; if it has occurred, the indicator variable takes the value 1. Those observations for which the indicator variable takes the value 1 are included in the sample for estimating the continuous models. This modeling system of interrelated processes suffers from selectivity bias in the estimation of the continuous equation. Selectivity bias occurs because the error terms of the discrete and continuous models are correlated due to common unobserved effects. Intuitively, selectivity bias occurs in this study because the distress progression model is estimated from observations that are drawn disproportionately from facilities with high deterioration rates.

The model system is formulated as follows.

The selection (initiation) model is

$$Z_i^* = \mathbf{u}'\mathbf{w}_i + u_i, i = 1, \ldots, N;$$

$$Z_i = 1 \text{ if } Z_i^* > 0; Z_i = 0 \text{ if } Z_i^* < 0 \quad (1)$$

where Z_i^* = a latent variable representing the distress initiation propensity for facility i; Z_i = the indicator variable; $\mathbf{w}_i$ = a vector of independent variables; $\mathbf{u}$ = a vector of coefficients to be estimated; and u_i = a random error term, which is to be assumed normally distributed with zero mean and unit variance.

The selection model therefore takes the probit form (Ben-Akiva and Lerman 1985) as follows:

$$\text{Prob}(Z_i = 1) = \Phi(\mathbf{u}'\mathbf{w}_i); \quad \text{Prob}(Z_i = 0) = 1 - \Phi(\mathbf{u}'\mathbf{w}_i) \quad (2)$$

where Φ ( ) = the standard normal cumulative distribution function.

The regression (progression) model is

$$Y_i = \boldsymbol{\beta}'\mathbf{X}_i + \varepsilon_i, \text{ if } Z_i = 1 \quad (3)$$

where $\boldsymbol{\beta}$ = a vector of coefficients; $\mathbf{X}_i$ = a vector of independent variables; Y_i = the dependent variable representing the extent of distress; and ε_i = a normal random error term with mean zero and variance σ_ε^2. Eq. (3) is observed only if $Z_i = 1$. We assume that u_i and ε_i are random error terms with correlation coefficient ρ. We may thus combine the preceding models to obtain a model that applies to the observations in the progression model data set:

$$E[Y_i|Y_i \text{ is observed}] = E[Y_i|Z_i^* > 0] = E[Y_i|u_i > \mathbf{u}'\mathbf{w}_i] = \boldsymbol{\beta}'\mathbf{X}_i + E[\varepsilon_i|u_i > \mathbf{u}'\mathbf{w}_i] \quad (4)$$

The last term on the right-hand side of (4) is given by (Greene 1993)

$$\beta_\lambda[\phi(\mathbf{u}'\mathbf{w}_i/\sigma_u)/\Phi(\mathbf{u}'\mathbf{w}_i/\sigma_u)] \quad (5)$$

where ϕ = standard normal probability density function; $\beta_\lambda = \rho\sigma_\varepsilon$ = a coefficient to be estimated; and σ_u = standard deviation of the error term in (1), and is normalized to 1. Eq. (4) can be written as

$$E[Y_i/Z_i^* > 0] = \boldsymbol{\beta}'\mathbf{X}_i + \beta_\lambda\lambda_i(\alpha_u) \quad (6)$$

where

$$\lambda_i(\alpha_u) = \phi(\mathbf{u}'\mathbf{w}_i)/\Phi(\mathbf{u}'\mathbf{w}_i) \quad (7)$$

The parameters of this model system may be estimated by Heckman's procedure, which is a sequential estimation technique. This method estimates the parameters of the discrete probabilities and those of the continuous model sequentially, using a two-step procedure as given here:

1. Estimate the probit equation by maximum likelihood to obtain the estimates of $\mathbf{u}$ denoted by $\hat{\mathbf{u}}$. For each observation in the selected sample we compute the following:

$$\hat{\lambda}_i = \phi(\hat{\mathbf{u}}'\mathbf{w}_i)/\Phi(\hat{\mathbf{u}}'\mathbf{w}_i) \quad (8)$$

2. Estimate $\boldsymbol{\beta}$ and $\beta_\lambda = \rho\sigma_\varepsilon$, by least square regression of (6) on the incidentally truncated sample as follows:

$$Y_i = \boldsymbol{\beta}'\mathbf{X}_i + \beta_\lambda\lambda_i(\alpha_u) + \gamma_i \quad (9)$$

where, γ_i = a random error term.

The second term in the right-hand side ($\beta_\lambda \lambda_i$) of (9) is the correction term (Train 1986) for selectivity bias in this regression equation. If this term is not included in this model, as is often the case in the literature, the ordinary least squares (OLS) estimation results in biased and inconsistent estimates of the vector $\boldsymbol{\beta}$. This is a typical case of bias due to missing relevant explanatory variables. It can be shown (Greene 1993) that, if $\rho > 0$, this results in an upward bias in the estimated value of $\boldsymbol{\beta}$. Clearly, this bias increases with the absolute magnitude of β_λ, which is an estimate of the size of shared unobserved effects among the initiation and progression models.

Though Heckman's procedure gives unbiased parameter estimates, this sequential estimation process produces biased standard errors of the coefficient estimates in the regression model and is statistically inefficient. However, the advantage of Heckman's procedure is that it is computationally easier than the full information maximum likelihood (FIML) procedure.

For the parameters of the selection model to be both unbiased and efficient, the parameters should be estimated by FIML. Here, we estimate the parameters of both equations (discrete and continuous) simultaneously using all the observations in the data set. Moreover, with this method, the estimated standard errors of the coefficient estimates in the regression models are unbiased.

EMPIRICAL CASE STUDY

The data set was taken from the World Bank's road deterioration studies undertaken in Brazil from 1975 to 1982, under the project entitled, "Research and Interrelationships between the Costs of Highway Construction and Maintenance and Utilization."

The study comprised a total of 116 test sections observed over time, which produced 763 observations. The data set includes pavement condition, cumulative traffic, environmental factors, and pavement strength at given dates. The variables used in our study and a brief description of these variables are given in the following:

- *crx*: area of crack index as a percent of total area
- *snc*: structural number of pavement
- *def*: Benkelmann beam surface deflection under 80-kN dual single axle load
- *h*: thickness of surfacing
- *sqhs*: square of *h*
- *ye*41*cc*: annual number of equivalent 80-kN equivalent single axle load (ESAL) per lane per year
- *yesn*: (*ye*41*cc*/*snc*) number of wheel passes per unit strength of pavement

In developing our pavement-cracking model, we corrected for an econometric problem that had not been accounted for properly in the pavement-deterioration literature and that is common in multiple-equation model systems: the problem of endogeneity in the explanatory variables.

Endogeneity occurs in a model system when the disturbance terms are correlated across equations. To illustrate this bias, we use the same models as those used by Paterson (1987) in the World Bank study. The progression of cracking is modeled as a function of pavement strength (*snc*) and an interactive term consisting of the number of wheel passes per unit strength of the pavement (*yesn*) as shown here

$$\log(crx) = \beta_0 + \beta_1 snc + \beta_2 yesn + u \qquad (10)$$

The pavement strength can be modeled as a function of the deflection and the number of wheel passes as

$$snc = \alpha_0 def^{\alpha_1} ye41cc^{\alpha_2} \varepsilon \qquad (11)$$

which can be linearized as follows:

$$\log(snc) = \alpha_0^* + \alpha_1 \log(def) + \alpha_2 \log(ye41cc) + \varepsilon^* \qquad (12)$$

This equation is a representation of a pavement-design procedure. For a given projected traffic, designing a pavement for a low deflection yields a stronger pavement. Similarly, a higher projected traffic requires designing a pavement with higher strength for a fixed deflection. Therefore, we expect the coefficient of α_1 to be negative and the coefficient of α_2 to be positive.

Notice that in (10), *snc* is correlated with the error term u. This is due to the correlation between ε^* and u resulting from the presence of shared unobserved effects such as environmental and subgrade characteristics. Therefore, estimation of the parameters of (10) by OLS regression results in biased and inconsistent estimates. The Hausman specification test (Pindyck and Rubinfeld 1991) can be used to determine whether or not endogeneity does indeed exist. The test involves estimating first the regression model for *snc* and finding its residuals, which is calculated as the difference between *snc* and its predicted value obtained from (12). The cracking progression model is then estimated with the residuals being added as an additional independent variable, as can be seen in

$$\log(crx) = \beta_0 + \beta_1 snc + \beta_2 yesn + \beta_3 resid + \nu \qquad (13)$$

Under the null hypothesis, if there is no endogeneity present, the coefficient for *resid* (β_3) should equal 0. A simple two-sided t-test can be used to test this hypothesis. The results of the preceding regressions are shown in Table 1.

From the results obtained in Table 1, we may conclude that endogeneity does indeed exist, since the coefficient of the variable *resid* does not equal zero and the t-statistic is significant at the 95% level. Endogeneity can be corrected for by the method of instrumental variables (IV) (Pindyck and Rubinfeld 1991). Basically, the IV method consists of substituting the predicted value of the endogenous explanatory variable obtained in (11) in place of its measured value in (10). This method was used to correct for endogeneity in the present paper. In our case, predicted values of *snc*, (*snchat*), were used in the following cracking models as an instrumental variable to correct for endogeneity. The cracking models are estimated using both simple OLS and the joint discrete-continuous modeling method for comparison.

Simple OLS Regression

From basic knowledge of pavement behavior, we expect that the deterioration of a pavement with higher strength is slower, and that a higher number of wheel passes per unit

TABLE 1. Hausman's Test for Endogeneity

Variables (1)	Coefficient (2)	t-statistic (3)
	(*a*) Eq. 12[a]	
Constant	1.22	167.418
log(*def*)	−0.204	−27.795
log(*ye*41*cc*)	0.036	2.626
	(*b*) Eq. 13[b]	
Constant	7.046	22.743
snc	−1.558	−19.458
resid	1.051	8.612
yesn	2.144	3.828

[a]Dependent variable: log(*snc*); observations: 763; R-squared: 0.53.
[b]Dependent variable: log(*crx*); observations: 763; R-squared: 0.35.

TABLE 2. OLS Estimation

Variables (1)	Coefficient (2)	t-statistic (3)
Constant	6.899	21.623
snchat	−1.543	−18.716
yesn	2.556	4.455

Note: Dependent variable: log(*crx*); observations: 253; R-squared: 0.316.

TABLE 3. Discrete Model (Probit Estimation)

Variables (1)	Coefficient (2)	t-statistic (3)
Constant	11.925	7.895
h	−0.41	−3.758
sqhs	0.906E-04	4.022
snchat	−2.198	−13.869
yesn	1.871	2.181

Note: Dependent variable: *cr*; observations: 763; ρ^2: 0.376.

TABLE 4. Discrete Model (Probit Estimation) without Correction for Endogeneity

Variables (1)	Coefficient (2)	t-statistic (3)
Constant	6.45	5.396
h	−0.02369	−2.521
sqhs	0.50535E-04	2.631
snc	−1.156	−13.982
yesn	0.3871	0.549

Note: Dependent variable: *cr*; observations: 763; ρ^2: 0.287.

TABLE 5. Continuous Model Estimation

Variables (1)	Coefficient (2)	t-statistic (3)
Constant	6.41	3.184
snchat	−0.787	−1.155
yesn	1.645	1.449
λ	−0.781	−1.802

Note: Dependent variable: *crx*; observations: 253; ρ^2: 0.202.

strength increases pavement deterioration. The estimated cracking progression model after correcting for endogeneity is as follows (see also Table 2):

$$\log(crx) = 6.899 - 1.543 snchat + 2.556 yesn \quad (14)$$

Note that we used *snchat* as an instrumental variable for *snc*, as discussed earlier, to correct for endogeneity bias. From Table 2, it can be seen that the t-statistics for the variables *snchat* and *yesn* are significant at the 99% level. The coefficients also have the appropriate signs, as expected, which justifies our a priori hypothesis.

The estimated values of the coefficients obtained by ordinary least squares are expected to be biased, as mentioned before, due to the incidentally truncated sample used for model estimation.

Joint Discrete-Continuous Model

We applied Heckman's method to estimate the joint discrete-continuous model system of pavement cracking initiation and progression. As described earlier, this method consists of a sequential estimation of the two models.

The initiation mechanism is represented by a probit model, as shown in Table 3. The dependent variable is an indicator variable *cr*; if there is no crack, then *cr* takes the value 0, and if a crack is observed then *cr* takes the value 1. In the sample, there were 501 observations showing no crack and 253 observations showing cracks. An appropriate specification is

$$\text{Prob}(cr = 1) = \Phi[\beta_0 + \beta_1 h + \beta_2 h^2 + \beta_3 snchat + \beta_4 yesn] \quad (15)$$

Intuitively, we expect that an increase in pavement thickness and pavement strength decreases the propensity to crack, and that an increase in traffic increases the probability of pavement-crack initiation. We also include the square of pavement thickness in the preceding to account for the nonlinear effect of thickness on cracking propensity. The estimated model is (see Table 3)

$$\text{Prob}(cr = 1) = \Phi[11.925 - 0.041h + 0.0000906h^2 - 2.198 snchat + 1.871 yesn] \quad (16)$$

The t-statistics show that pavement strength, surface thickness, and traffic loading are significant explanatory variables of cracking initiation. Furthermore, the coefficients of pavement strength and surface thickness have a negative sign, whereas the coefficient of traffic is positive, which confirms our a priori hypotheses. To show exactly how endogeneity can affect such a model, the probit (initiation) model was estimated again, this time using *snc* instead of *snchat*. The results are shown in Table 4.

Results from Table 4, clearly show that the model without the correction for endogeneity underestimated the coefficients of the independent variables by almost a factor of 2, which can have serious consequences when using such a model for prediction.

To correct for sample selectivity bias, we include the correction term derived earlier ($\beta_\lambda \lambda$) in the cracking progression model, which we estimate by OLS. The estimated regression model with the correction term is as follows (see Table 5)

$$E[crx|cr = 1] = 6.41 - 0.787 snchat + 1.645 yesn - 0.781\lambda \quad (17)$$

From the results, it can be seen that the t-statistics of all the variables are significant. Note that the coefficient estimates in this table are indeed different from those in Table 2, which is a sign that the magnitude of selectivity bias in this model is substantial. This is confirmed by the t-statistic of β_λ, which indicates that the size of the correlation between the error terms of the two models is significantly different from zero.

The parameter estimates obtained by using the maximum likelihood estimation are shown in Table 6. The first set of estimation coefficients in Table 6 represents the probit model.

TABLE 6. Maximum Likelihood Estimates

Variables (1)	Coefficient (2)	t-statistic (3)
(*a*) Initiation		
Constant	11.925	7.783
h	−0.041	−3.425
sqhs	0.906E-04	3.654
snchat	−2.198	−16.981
yesn	1.871	1.613
(*b*) Progression		
Constant	6.405	3.698
snchat	−0.787	−1.387
yesn	1.645	1.512
σ_ε	1.332	9.397
ρ	−0.586	−2.224

Note: Dependent variable: *crx*; observations: 763; ρ^2: 0.532.

The estimated values of the coefficients of the regression model are shown in the lower half of Table 6. The correlation coefficient between the error terms of the two models is ρ and σ_ε represents the standard deviation of the error term in the regression model. The value of β_λ may be obtained as a project of σ_ε and ρ, and its value is the same as that obtained by the sequential estimation procedure.

The estimates obtained by FIML shown in Table 6 may be compared to the estimates obtained by Heckman's procedure in Tables 3 and 5. The estimated coefficients of the two models are equal, as was discussed, though the t-statistics are different. This is because the estimated standard errors in Heckman's procedure are biased.

CONCLUSIONS

A structured econometric approach for modeling the initiation and progression of infrastructure distress was presented. Using OLS regression for modeling distress progression leads to biased and inconsistent estimates, as the sample contains only facilities for which distress has been initiated, thereby introducing selectivity bias in the estimation. The correct approach consists of developing separate but interrelated models for distress initiation and progression, which recognizes and corrects for the presence of selectivity bias by using either a sequential or a simultaneous estimation procedure. The paper also highlighted the potential endogeneity bias introduced when the measured extent of one distress is included as an explanatory variable in the initiation or progression models of other distresses, and presented an approach for correcting this bias.

The importance of accurate predictions of facility deterioration for effective infrastructure management was quantified in the literature (Madanat 1993). It was shown, through parametric analyses, that improvements in the accuracy of condition predictions lead to substantial reductions in the expected life-cycle costs of infrastructure facilities. The proper estimation of distress models, as illustrated in this paper, contributes to such improvements. The size of the biases due to endogeneity and selectivity were found to be rather substantial, which renders the uncorrected models highly inaccurate. Researchers in the field of infrastructure management need to be aware of the potential for such model-estimation problems and of the existence of proper correction techniques in these cases.

ACKNOWLEDGMENTS

We are grateful to W. D. O. Paterson of the World Bank for providing us with the HDM data set that was used for developing our models. We have benefited from thoughtful comments provided by three anonymous reviewers.

APPENDIX. REFERENCES

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