# **Balancing Connectivity of Deteriorating Bridge Networks** and Long-Term Maintenance Cost through Optimization

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**Abstract:** Due to aggressive environmental stressors and increasing traffic loads, highway bridges are undergoing significant deterioration in both condition and safety. Timely and adequate maintenance interventions are therefore crucial to ensure the functionality of existing bridges in a network. Under budget constraints, it is important to prioritize maintenance needs to bridges that are most significant to the functionality of the entire network. In this paper, the network-level bridge maintenance planning problem is posed as a combinatorial optimization and is automated by a genetic algorithm (GA) to select and allocate maintenance interventions of different types among networked bridges as well as over a specified time horizon. Two conflicting objective functions are considered simultaneously: (1) The overall performance of a bridge network expressed by the time-dependent reliability of connectivity between the origin and the destination locations and (2) the present value of total maintenance cost over the specified time horizon. A variety of maintenance types, which differ in unit costs as well as in effects on bridge performance in terms of improvement in structural reliability levels, are used in the optimization. An event tree analysis is carried out to obtain a closed-form expression for the network connectivity reliability. As an illustration example, the GA-based procedure is applied to deteriorating deck slabs of an existing 13-bridge network located in Colorado. It is shown that the proposed maintenance planning procedure has the capability of both prioritizing scarce maintenance needs to deteriorating bridges that are most crucial to the network performance and cost-effectively distributing maintenance interventions over the time horizon.

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#### Introduction

Satisfactory performance of constructed civil infrastructure systems such as lifeline highway transportation systems is of vital importance to the continuing economic growth and social development of a modern society. In particular, highway bridges represent the most crucial as well as the most vulnerable components of the transportation network. Bridges, if being posted, can severely affect the traffic flow capacity of the relevant travel paths. Bridge failure due to local or global collapse severs existing paths connecting the origin and destination locations of interest. In addition, it may take long time for bridge repair or replacement work. More importantly, bridge malfunction due to structural deficiency can significantly hinder emergency responses to natural hazards (e.g., earthquake, hurricane, flood) and man-made disasters (e.g., terrorism action, vehicle/vessel collision). At the same time, aggressive environmental conditions (e.g., alkali-silica reaction, chloride contamination, sulfate attack) and ever-increasing

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traffic loading effects contribute to progressive deterioration of bridge structures over their lifetime. If no timely or adequate maintenance/rehabilitation activities were carried out, bridge safety and condition levels would be severely lowered as time elapses, which may lead to unacceptable structural/functional performance or even catastrophic collapse.

Tremendous efforts have been devoted by civil engineering researchers and practitioners to developing methodologies and techniques for long-term health/condition assessment and maintenance management of deteriorating bridge structures. Because financial resources for maintenance activities are typically inadequate to maintain the safety of all networked bridges at the same time, cost-effective budget allocation schemes become necessary in order to meet bridge managers' specified requirements for the optimum balance between the bridge performance and whole-life maintenance costing. Bridge management systems (BMSs) now in use by state departments of transportation include Pontis (Thompson et al. 1998) and BRIDGIT (Hawk and Small 1998). These software packages are based on visual inspection-related discrete condition states and a Markov deterioration model with stationary transition probabilities; effects of uncertainty associated with the deterioration process as well as with maintenance interventions are not fully considered (Frangopol et al. 2001). In order to solve this problem, frameworks of reliability-based lifecycle maintenance management using optimization were proposed by Thoft-Christensen (1995), Frangopol et al. (1999), and Frangopol and Das (1999). Mathematical deterioration models are derived from available realistic historical inventory data; inspection results can be incorporated using Bayesian techniques (Enright and Frangopol 1999; Estes et al. 2004) for updated condition assessment.

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Most of the previous research focused on project-level bridge maintenance management that deals with individual deteriorating bridges or a group of similar bridges. The inter-relationship of bridges in a common network, however, is not considered. Financial resources are allocated along the time axis such that bridge performance is maintained at or above a predefined level over a specified time horizon. The maintenance planning problem is usually formulated as an optimization problem with the long-term cost being the most commonly used objective function, which consists of inspection cost, maintenance cost, and possibly user costs (Frangopol et al. 1997). Recently, multiobjective optimization techniques have been used for bridge maintenance planning where multiple conflicting objectives (e.g., cost versus performance) are treated simultaneously (Liu et al. 1997; Miyamoto et al. 2000; Liu and Frangopol 2004, 2005). This formulation can lead to a group of alternative maintenance solutions that represent an optimized tradeoff among conflicting objectives under consideration. Bridge managers are then able to select a compromise maintenance solution according to preferred balance among these objectives.

Although the vital role of highway bridges in the transportation network has long been recognized, maintenance and rehabilitation prioritization solutions resulting from a network viewpoint have not received adequate attention. Indeed, the ultimate goal of bridge maintenance management should be improvement of the performance of the entire bridge network rather than merely of individual bridges in the network. Therefore, network-level maintenance planning methodologies represent a significant advancement. Augusti et al. (1994) studied allocation of retrofitting efforts for seismic protection of a ten-bridge highway network using dynamic programming; the goal was to maximize the network connectivity reliability at prescribed seismic intensities and under budget constraints. Later, Augusti et al. (1998) extended this procedure to prioritize maintenance interventions for the highway bridge network considering life-cycle maintenance cost. Basöz and Kiremidjian (1995) proposed a method to prioritize seismic retrofitting of high-risk networked bridges for emergency preparedness. Adey et al. (2003) developed bridge management procedures using a supply and demand system approach.

In this paper, the long-term maintenance planning for deteriorating highway bridges in a transportation network is investigated. First, the concept of bridge network reliability in terms of origindestination connectivity is introduced as a function of timedependent system reliability (or equivalently failure probability) of each individual bridge in the network. This concept is then illustrated using an existing 13-bridge network by an event tree analysis. Second, four maintenance types with different unit costs and effectiveness are described. Third, a genetic algorithm (GA)based numerical procedure is proposed to automatically allocate maintenance needs among networked bridges as well as along the time axis with simultaneous consideration of both the present value of the total maintenance cost and the network connectivity reliability. Formats of the resulting solutions are lists of optimally prioritized maintenance interventions in an annualized fashion. Fourth, this procedure is applied to the example bridge network for the long-term network-level maintenance planning. Finally, important findings are summarized and future research needs are pointed out.

# **Bridge Network and Reliability Analysis**

# An Existing Bridge Network

The example bridge network used in this study is located in the northwest metropolitan area of Denver, Colorado (Akgül and Frangopol 2004). As shown in Fig. 1, this network consists of 13 highway bridges of different structural characteristics, including prestressed girder bridges, steel rolled I-beam bridges, and combined welded steel plate/reinforced concrete girder bridges. These bridges were built between year 1951 and 1995, indicating an average age of 31.4 years based on the reference year of 2004.

# Bridge System Reliability

There exist various indicators that describe performance of a highway bridge system. For example, visual inspection-based condition state and field live-load capacity ratings have been traditionally used to measure the bridge's physical condition and load-carrying capacity (AASHTO 1994). It aids prioritization of maintenance needs by identifying bridges with the lowest rating level. In this study, the system reliability index is used to quantify the structural safety level of a bridge. Compared to the subjective bridge condition rating practice, the structural reliability measure is a more objective means to assess structural capacity of an existing bridge, provided accurate probabilistic structural and load models are readily available. More importantly, the bridge system reliability can be conveniently related to the overall performance measure for the bridge network as explained later.

The entire bridge system consists of superstructure (e.g., deck slab, girders) and substructure (e.g., pier cap and column, footing) systems, which may be idealized as a series-parallel system for evaluating the system reliability (Estes and Frangopol 1999). Akgül and Frangopol (2004) defined performance limit state functions for structural component reliability analysis using standard code formulations from AASHTO specifications (AASHTO 1996); the relevant parameters are either treated as random variables following prescribed probability distributions or are assumed deterministic values. By introducing probabilistic structural deterioration and live load models, the lifetime system reliability profiles for different bridge types can be predicted. For example, deterioration of concrete deck slabs is caused by the deicing chemicals related corrosion, whose propagation in concrete may be predicted by Fick's law of diffusion. Effects of corrosion are gradual reduction of reinforcement area. The live load model is the same as the one used for calibrating AASHTO LRFD bridge design specifications (AASHTO 1998). Monte Carlo simulation is performed to account for effects of parameter randomness on bridge performance prediction. At any time instant, the system reliability of an individual bridge is calculated based on the structural deterioration and live load intensity (in terms of average daily truck traffic) at that time instant.

## Bridge Network Connectivity Reliability

A highway transportation network, which consists of individual bridges and roads that interconnect these bridges to form the network, aims to provide redundant paths for travelers and cargoes going from the origin to destination locations. There are various approaches to describing the performance of a transportation network. For example, the network reliability can be used to measure the degree of the satisfactory network performance. Two metrics

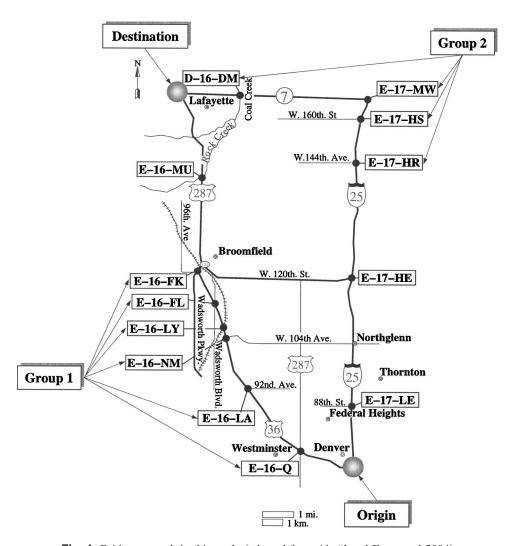


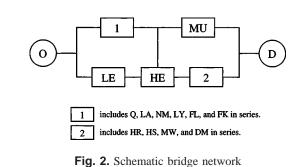
Fig. 1. Bridge network in this study (adapted from Akgül and Frangopol 2004)

exist in this category (Bell and Iida 1997): the connectivity reliability, which states the probability that traffic can reach the destination from the origin, and the travel time reliability, which is the probability that traffic can reach the destination within a given period of time. It is obvious that the travel time reliability takes into account both traffic demand and capacity estimation of respective roads that link bridges, which affects the amount of origin-destination travel time. Chen et al. (2002) introduced the traffic capacity reliability as an alternative measure that evaluates the probability that the traffic capacity can accommodate a given traffic demand within a specified period of time. These network reliability metrics have mainly been considered in the literature for a deteriorating transportation network where the road pavement surface quality is considered (Nicholson and Du 1997). In this paper, it is assumed that highway bridges are the only vulnerable elements of a transportation network, i.e., roads that link any two bridges never fail. The lifetime network performance is evaluated in terms of the origin-destination connectivity reliability.

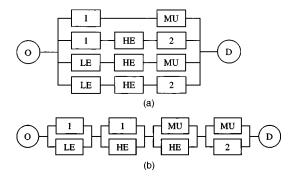
Six bridges (i.e., Q, LA, NM, LY, FL, and FK) in series are lumped as Group 1 and four bridges (i.e., HR, HS, MW, and DM) in series are lumped as Group 2 (see Fig. 1). The original example bridge network is then schematically shown in Fig. 2, where "O" and "D" represent the origin and destination sites, respectively. In this study, possible correlations among failures of networked

bridges due to, for example, similar harsh environment, live loads, and structural types, are neglected. It is then assumed that the bridge failure events are independent. The topology of this bridge network makes it impossible to convert this network into a simple series-parallel system for which a network reliability expression is readily available (e.g., Augusti et al. 1994). Moreover, the traditional minimum path set and minimum cut set approaches in network analysis are not applicable either because, as shown in Fig. 3, some bridges may appear in more than one path (or set) and thus introduce dependence among paths (or sets).

In this study, the network connectivity reliability is calculated systematically using an event tree analysis (Ang and Tang 1975).



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**Fig. 3.** Representation of schematic bridge network by: (a) Minimum path sets in parallel; and (b) minimum cut sets in series

Fig. 4 sketches all 2<sup>5</sup>=32 different branches of functioning and failed individual bridges and bridge groups. The corresponding status of origin–destination connectivity is also identified for each branch. The occurrence probability of each branch is then obtained by simply multiplying all bridge state (i.e., functioning or failure) probabilities for that particular branch. At any time instant, the network disconnectivity probability is obtained by adding occurrence probabilities of all disconnected branches:

$$P_{\rm dc} = \sum_{i=1}^{N_{\rm dc}} \left( \prod_{j=1}^{N_b} P_{ij} \right)$$
 (1)

where  $N_{\rm dc}$ =number of all disconnected branches, which is equal to 19 for the bridge network considered (see Fig. 4);  $N_b$ =total number of individual bridges and bridge groups in the network, which is equal to 5 for the bridge network considered, i.e., Bridges LE, HE, and MU in addition to Bridge Groups 1 and 2 (see Fig. 2);  $P_{ij}$ =state probability associated with bridge (or group) j in branch i.

After some algebraic simplification, the final expression of network disconnectivity probability is as follows:

$$\begin{split} P_{\rm dc} &= P_{f,\rm MU} (1 - P_{f,1}) \big[ 1 - (1 - P_{f,\rm HE}) (1 - P_{f,2}) \big] \\ &+ P_{f,1} \big[ 1 - (1 - P_{f,\rm LE}) (1 - P_{f,\rm HE}) (1 - P_{f,2} P_{f,\rm MU}) \big] \end{split} \tag{2}$$

where  $P_{f, \text{LE}}$ ,  $P_{f, \text{HE}}$ , and  $P_{f, \text{MU}}$ =failure probabilities of bridges LE, HE, and MU, respectively. For Groups 1 and 2, each of which is a series system, failure of any bridge in a group amounts to failure of that bridge group. Thus, the failure probabilities take the following forms, respectively,

$$P_{f,1} = 1 - (1 - P_{f,Q})(1 - P_{f,LA})(1 - P_{f,NM})(1 - P_{f,LY})$$

$$\times (1 - P_{f,FL})(1 - P_{f,FK})$$
(3)

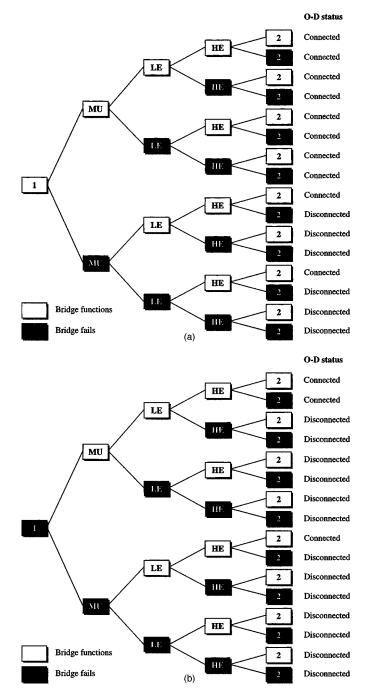
$$P_{f,2} = 1 - (1 - P_{f,HR})(1 - P_{f,HS})(1 - P_{f,MW})(1 - P_{f,DM})$$
 (4)

The nominal network connectivity reliability index is therefore

$$R_{\rm nw} = -\Phi^{-1}(P_{\rm dc}) \tag{5}$$

where  $\Phi^{-1}(\cdot)$ =inverse of the cumulative probability function for the standard normal distribution.

Note that the network disconnectivity probability can also be equivalently obtained as the complement of the summed occurrence probabilities of all connected branches:



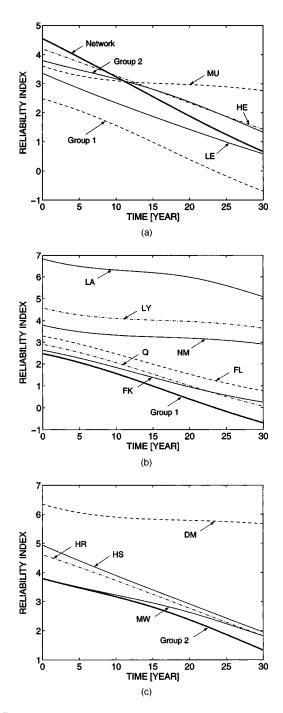
**Fig. 4.** Event tree analysis for bridge network connectivity: (a) Group 1 functions; and (b) Group 1 fails

$$P_{dc} = 1 - \sum_{i=1}^{N_c} \left( \prod_{j=1}^{N_b} P_{ij} \right)$$
 (6)

where  $N_c$ =number of all connected branches, which is equal to 13 for the bridge network considered (see Fig. 4).

# Reliability Profiles of Bridge Network Connectivity and of Individual Bridges under No Maintenance

The structural flexure failure of bridge slabs is considered in this study. The time-dependent connectivity reliability profile of the bridge network under no maintenance intervention is computed using Eqs. (2)–(5). The time horizon for maintenance planning is



**Fig. 5.** Time-dependent reliability index under no maintenance for: (a) Network connectivity; (b) Group 1; and (c) Group 2

taken as 30 years starting from the current year, at which it is assumed, without loss of generality, that all bridges in the network recover their initial reliability levels, respectively. The reliability profiles for deck slabs of all 13 bridges under no maintenance are obtained based on the work of Akgül (2002).

Fig. 5(a) plots the reliability profiles of the network connectivity and of each networked bridge or group of bridges, respectively. It is observed that the network connectivity reliability decreases gradually as time elapses due to deck slab deterioration of each bridge in the network. These bridges, however, contribute with varied degrees to the network reliability due to their different geographical locations in the network and different deterioration severities over the time horizon. It is observed in Fig. 5(a) that

deterioration profiles of Bridges LE, HE, and Groups 1 and 2 more or less keep pace with that of the network. In contrast, the reliability of Bridge MU deteriorates very slowly over the time. Groups 1 and 2 are of low reliability levels largely because they consist of multiple individual bridges in series. Fig. 5(b) illustrates contributions of constituent bridges to the reliability profile of Group 1. A series configuration of independent bridges causes the reliability level of Group 1 to be always lower than that of all its constituent bridges. The deterioration of Group 1 is primarily caused by the vulnerable Bridges FK, Q, and FL. On the other hand, Bridges LA, LY, and NM are associated with both higher reliability levels and slower deterioration over time. Similar trends are observed in Fig. 5(c) for the reliability profiles of Group 2, from which one can clearly observe that Bridges HR and HS experience the most severe deterioration during the time period of 30 years. Although the reliability level of Bridge MW is consistently the lowest among all bridges in Group 2, its deterioration rate is smaller than that of Bridges HR and HS. Bridge DM remains at a very high reliability level over the entire time horizon.

Also observed in Fig. 5, under no maintenance, reliability indices of the individual bridges drop to as low as 0.06 [Bridge Q in Group 1, see Fig. 5(b)] at the end of 30 years, which corresponds to a failure probability of 0.476. The network connectivity reliability index gradually deteriorates from 4.550 to 0.668 over the time horizon [see Fig. 5(a)], i.e., the network disconnectivity probability increases from  $2.68 \times 10^{-6}$  to 0.252. Dependent upon acceptable reliability levels set forth by bridge managers, timely and appropriate maintenance interventions are needed to satisfactorily assure the performance (e.g., in terms of reliability index) of the entire network and/or individual bridges at or above respective minimum acceptable levels over the specified time horizon.

# Sensitivity of Network Disconnectivity to Bridge Failure Probabilities under No Maintenance

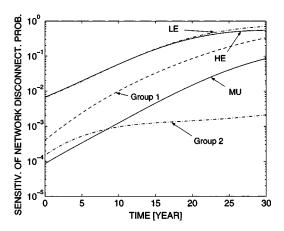
It may be of interest to investigate the sensitivity of the network disconnectivity probability,  $P_{\rm dc}$ , to the deck slab failure probability of each individual bridge and bridge group in the network. This provides information on the relative significance of each constituent bridge or group to the overall network performance. The closed-form expression of the network disconnectivity probability in Eq. (2) offers direct derivation of the sensitivity of  $P_{\rm dc}$  with respect to the deck slab failure probability of each individual bridge or group:

$$\frac{\partial P_{dc}}{\partial P_{f,1}} = 1 - P_{f,MU} + (1 - P_{f,HE}) [P_{f,MU} (1 - P_{f,2}) - (1 - P_{f,LE}) (1 - P_{f,2} P_{f,MU})]$$
(7)

$$\frac{\partial P_{dc}}{\partial P_{f,2}} = P_{f,MU} (1 - P_{f,HE}) (1 - P_{f,1} P_{f,LE})$$
 (8)

$$\frac{\partial P_{dc}}{\partial P_{f,LE}} = P_{f,1} (1 - P_{f,HE}) (1 - P_{f,2} P_{f,MU}) \tag{9}$$

$$\frac{\partial P_{\text{dc}}}{\partial P_{f,\text{HE}}} = P_{f,\text{MU}} (1 - P_{f,1}) (1 - P_{f,2}) + P_{f,1} (1 - P_{f,\text{LE}}) (1 - P_{f,2} P_{f,\text{MU}})$$
(10)



**Fig. 6.** Sensitivity of network disconnectivity probability to bridge failure probabilities

$$\begin{split} \frac{\partial P_{\text{dc}}}{\partial P_{f,\text{MU}}} &= 1 - P_{f,1} - (1 - P_{f,\text{HE}}) [(1 - P_{f,1})(1 - P_{f,2}) \\ &- P_{f,1} P_{f,2} (1 - P_{f,\text{LE}})] \end{split} \tag{11}$$

The presence of multiple bridge failure probabilities existing in Eqs. (7)–(11)) signifies the fact that the importance of a bridge or group to the functionality of the entire bridge network depends on its specific geographical location in the presence of other bridges in the network. Fig. 6 plots the sensitivity profiles over the scheduled maintenance period of 30 years. It is observed that the order of bridges, in terms of sensitivity of the network disconnectivity probability to the change of each bridge failure probability from the highest to the lowest, is LE, HE, Group 1, MU, and Group 2. The higher the sensitivity is, the more the increase in the failure probability (i.e., decrease in reliability index) of a bridge or group will affect the network connectivity.

It should be noted that a bridge to which the network has a higher sensitivity may not necessarily be prioritized for maintenance unless at the same time its reliability level deteriorates significantly. That is, prioritization of maintenance interventions to bridges in the network depends not only upon the sensitivity of the network functionality to the performance of each individual bridge as discussed above, but also upon the deterioration status of the individual bridge at a given time instant. These two aspects must be considered jointly so that one can determine the most critical bridges for maintenance in order to maximize the overall network performance. As another important issue, this sensitivitybased approach does not directly provide an answer to when and how the selected bridges should be maintained in order to simultaneously consider both the network performance and total maintenance cost from a long-term point of view. This problem becomes more complex when some of the bridge reliability profiles have been changed upon application of maintenance interventions. Repeated sensitivity analyses are thus required and the maintenance prioritization becomes cumbersome and intractable. Further, this approach can by no means guarantee cost-effective prioritization solution for the present network-level bridge management over a specified time horizon.

# **Different Maintenance Types**

# Maintenance Types, Effects, and Unit Costs

Four different maintenance types are considered in this study for enhancing performance of the deteriorating bridge deck slabs: Resin injection, slab thickness increasing, steel plate attaching, and total replacement. Rigorously, effects of maintenance interventions on reliability profiles require detailed reliability analysis of bridge deck slabs upon application of different maintenance interventions. In this study, in order to perform maintenance prioritization with reasonable computational expenses, maintenance effects as well as the associated unit costs are provided in Table 1, respectively, based on Furuta et al. (2004) and Kong and Frangopol (2004).

Resin injection is the least costly maintenance type among the four alternatives. It injects epoxy resin into voids and seals cracks in concrete, which repairs the aging deck slabs by reducing the corrosion of reinforcement due to exposure to the open air. In this study, the deterioration rate in reliability is assumed 0.03/year with an effective time interval of 15 years. As a comparison, the other three maintenance types instantly improve the bridge reliability level by various amounts upon application. Increasing slab thickness and attaching steel plate increase the bridge deck slab reliability indices by a maximum of 0.7 and 2.0, respectively. As a tradeoff, unit costs associated with these two maintenance types are \$300/m² and \$600/m², respectively. The most effective yet the costliest maintenance type is the replacement of the entire aging bridge slabs, which causes the structural system to restore its initial reliability level with a unit cost of \$900/m².

Theoretically, the cost model could be made more sophisticated by considering not only the reliability improvement but also the structural reliability level right prior to maintenance application (Kong and Frangopol 2004). Note that the parameter values presented in Table 1 may be highly dependent on bridge types, level of interaction, and loading schemes on bridges. In addition, there exist inevitable uncertainties associated with these parameters due to the very complex nature of the deterioration processes with and without maintenance. Therefore, it would be more desirable to treat them as random variables with specified probability distributions if more information can be gathered; Monte Carlo simulation is then needed to evaluate bridge performance profiles in a probabilistic manner (Liu and Frangopol 2004, 2005).

Table 1. Four Different Maintenance Types

Maintenance type	Reduction of deterioration rate (year <sup>-1</sup> )	Improvement of bridge deck slab reliability index	Unit cost (\$/m²)
Resin injection	0.03 (with a duration of 15 years)	0	200
Slab thickness increasing	0	0.7	300
Steel plate attaching	0	2.0	600
Replacement	0	Recover the initial reliability level	900

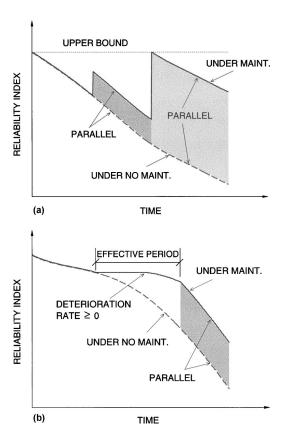


Fig. 7. Evaluation of reliability profiles of bridges under maintenance

#### Computational Considerations

In order to compute the reliability profile of an individual bridge under a sequence of maintenance actions over a specified time horizon, some assumptions are made in this study.

- Upon application of slab thickness increasing, steel plate attaching, or replacement, the reliability index is immediately increased by an appropriate amount without exceeding the respective maximum value provided in Table 1. The resulting reliability index value, however, cannot exceed that at year zero, which is assumed as the upper bound reliability level. Then the reliability profile goes conservatively parallel to the original reliability profile representing deterioration under no maintenance. Fig. 7(a) illustrates these considerations.
- The net deterioration rate as a combined effect of the original structure and maintenance (here resin injection only) can only take a nonnegative value. That is, resin injection can only reduce or at most postpone but not reverse the reliability deterioration. As result, the lowest possible deterioration rate at any time instant is zero, which is illustrated in Fig. 7(b).
- In order to prevent excessive application of maintenance interventions, which is very likely to occur especially when multiple conflicting objectives are considered, it is assumed that, for any bridge, application times of any two maintenance interventions should be at least four years apart.
- Effects of resin injection can be superposed with those of other types of maintenance except for the replacement option, for which the resulting entirely new deck slab voids effects of any resin injection that has been applied before replacement is carried out.

• The duration needed to perform any maintenance intervention is assumed short (say, less than 0.3 year) and, consequently, ignored in computation.

The present value of the total maintenance cost,  $C_{PV}$ , over the specified planning period is the sum of costs from all maintenance interventions that are discounted to the present time, respectively,

$$C_{PV} = \sum_{i=1}^{N_b} \sum_{j=1}^{N_i} \frac{C_{i,j}}{(1+\nu)^{t_{ij}}}$$
 (12)

where  $N_b$ =number of bridges in the network, which is 13 for the bridge network considered;  $N_i$ =total number of maintenance interventions for the ith bridge over the time horizon;  $C_{i,j}$ =unit cost of the jth maintenance intervention for the ith bridge;  $t_{ij}$ =application time of the jth maintenance intervention for the ith bridge; and  $\nu$ =monetary discount rate, which is assumed 6% in this study.

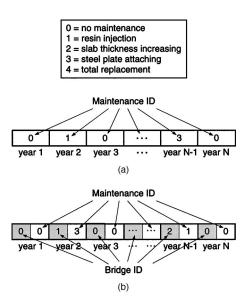
# Network-Level Maintenance Prioritization by Genetic Algorithm

#### **Objective Functions**

This study aims to prioritize limited financial resources in order to cost-effectively maintain the safety of deteriorating bridges in a transportation network. Compared to the traditional project-level BMSs that only deal with maintenance allocation along the time axis, the present network-level maintenance planning procedure distributes maintenance needs not only over the specified time horizon but also within the network. Accordingly, two objective functions are considered in formulating the present maintenance planning problem: the lowest value of the network connectivity reliability over the time horizon and the present value of total maintenance costs. These two objective functions represent the "benefit" and "cost" aspects, respectively, of the present maintenance prioritization. Because of their conflicting nature, however, no unique solution is available that can optimize them simultaneously. Instead, the bi-objective optimization problem leads to a group of maintenance solutions that establish a tradeoff trend between these objectives in the Pareto optimal sense. That is, for any of these optimized solutions, one objective cannot be further improved without sacrificing the other. In contrast to a traditional single maintenance solution that is the result of minimizing the total maintenance cost under performance constrains, the large set of maintenance solutions from the present bi-objective optimization provides the bridge manager with much freedom to determine a final solution with a preferred balance between maximization of the network connectivity performance and minimization of the total maintenance cost over the specified time horizon.

# **Problem Statement**

As noted previously, in order to obtain the largest benefit under budget constraints, a network-level bridge management system needs to prioritize maintenance interventions not only over a specified time horizon but also among different bridges in the network. Bridges in a network, however, are not of equal importance to the overall network performance. Naturally, maintenance interventions should be allocated to bridges whose failures are more detrimental to the network functionality. At the same time, maintenance activities should also be intelligently distributed along the time horizon to obtain the maximum economy in the



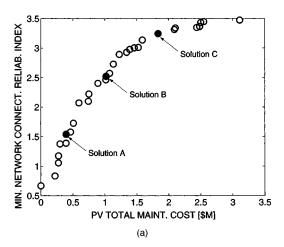
**Fig. 8.** Illustration of the encoded maintenance sequence in genetic algorithm for (a) a single bridge and (b) a bridge group

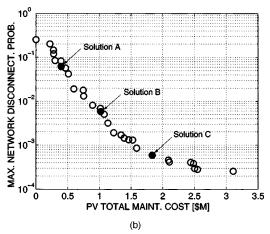
presence of monetary discounting over time. In compliance with realistic bidding practice (Das 1999), a list of prioritized annualized maintenance interventions to be scheduled at discrete years over the time horizon, is desirable. For each individual bridge (LE, HE, and MU), at most one maintenance activity could possibly be carried out at any year (taken as the mid point of one year interval). For a bridge group, it is decided that only one bridge could be maintained at any given year.

## Genetic Algorithms

The resulting bi-objective optimization for the annual combinatorial maintenance planning problem can be effectively solved by GA due to its flexibility and robustness (e.g., Miyamoto et al. 2000; Liu and Frangopol 2004, 2005). In accordance with the assumption made in the above paragraph, a generic maintenance planning solution, i.e., a sequence of maintenance activities distributed among bridges in the network as well as over the specified time horizon, is encoded in the present GA as a real-valued string that contains concatenated segments for each bridge or bridge groups. As schematically shown in Fig. 8(a), for each of bridges LE, HE, and MU, the encoded segmental sub-string has a length equal to the time horizon in years, i.e., 30 bits for one bridge. The four maintenance types considered in this study are sequentially numbered from "1" to "4," respectively. In addition, an identifier of "0" indicates there is no maintenance activity during a particular year. As schematically shown in Fig. 8(b), the encoded sub-string for one group has  $30 \times 2 = 60$  bits. That is, there are two bits for a single year, one bit indicating the bridge in the group that needs to be maintained during that year and the other bit indicating the type of applied maintenance. In total, a complete maintenance solution for the present network is encoded in GA as a string containing 210 bits [(90 bits for Bridges LE, HK, and MU (i.e., 30 bits for each individual bridge) and 120 bits for Groups 1 and 2 (i.e., 60 bits for each bridge group)].

The initial GA population is composed of 1,000 trial maintenance planning solutions. For each solution, five maintenance interventions not necessarily of different types are applied, with a uniform distribution, to randomly selected five bridges in the network at randomly selected discrete years. Each of the subsequent





**Fig. 9.** Tradeoff between the present value (PV) of the total maintenance cost and network performance in terms of (a) the minimum connectivity reliability index and, equivalently, (b) the maximum disconnectivity probability for 30 optimized solutions at the 50th generation

GA generations contains 200 offspring solutions plus the non-dominated elite solutions from its parent generation. Fitness evaluation of each candidate solution is determined using the non-dominated sorting technique together with a crowding distance niching scheme (Deb 2001), which can simultaneously and separately handle multiple conflicting objectives using the Pareto optimality condition. A constrained binary tournament selection operation (Deb 2001) is applied to form the mating pool. A uniform crossover with a probability of 50% and a uniform mutation with a probability of 5% are used for the encoded string representations of maintenance planning solutions.

# **Analysis of Optimized Maintenance Solutions**

The proposed GA-based bi-objective optimization procedure is now used to prioritize maintenance needs to deteriorating bridges in the present example network. A time increment of 0.1 year is used to predict the time-dependent reliability profiles of both individual bridges and bridge groups under possible maintenance interventions. A total of 30 optimized maintenance solutions are produced at the 50th generation. As shown in Fig. 9(a), these different solutions represent a wide spread between the conflicting network connectivity reliability and the present value of the total

Table 2. Maintenance Prioritization by the Representative Solution A

Year	Bridge LE	Bridge HE	Bridge MU	Bridge group 1	Bridge group 2
(1)	(2)	(3)	(4)	(5)	(6)
1					
2					
3					
4					
5			·		
6					
7					
8					
10					
11					
12				<u> </u>	
13					
14					
15					
16					
17					
18					
19	Slab thickness				
	increasing				
20					
21					
22					
23					
24					
25				Bridge Q: Steel plate attaching	
26		Slab thickness			
		increasing			
27					Bridge HS:
					Slab thickness increasing
28					
29		<del></del> _	Slab thickness		
			increasing		
30					
Present value of the total maintenance cost					US\$0.396 million
Minimum value of network connectivity reliability index				1.539	

maintenance cost objectives. Equivalently, the tradeoff between the maximum network disconnectivity probability and the present value of the total maintenance cost is plotted in Fig. 9(b). The leftmost point in Fig. 9 corresponds to the situation where no maintenance activities are carried out over the time horizon, resulting in the minimum network reliability index of 0.668 or equivalently the maximum network disconnectivity probability of 0.252. These optimized maintenance solutions lead to different degrees of improvement of the network performance that necessitate different amounts of financial resources in order to reach such network connectivity reliability levels.

Facing this group of many alternative maintenance solutions, bridge managers can select one that compromises the two competing objectives most desirably. If sufficient budgets are available, the network performance in terms of connectivity reliability can be maintained to a reasonably high level. On the other hand,

if budgets are scarce, bridge managers have to make rational decisions on maintaining bridges that are most crucial to the network functionality. As a result, the network connectivity reliability can be maximized under budget constraints. As an illustration, three representative maintenance solutions labeled as A, B, and C are selected and are identified in Fig. 9. Tables 2–4 provide listing of maintenance interventions that are distributed to different bridges and at different years over the time horizon. The corresponding network connectivity reliability profiles and the present values of the cumulative maintenance cost for these representative maintenance solutions are plotted in Fig. 10, respectively.

Solution A increases the value of network connectivity reliability index objective from the original 0.668 under no maintenance to 1.539, which is associated with a maximum network disconnectivity probability of 0.0619 over the time horizon.

Table 3. Maintenance Prioritization by the Representative Solution B

Year	Bridge LE	Bridge HE	Bridge MU	Bridge group 1	Bridge group 2
(1)	(2)	(3)	(4)	(5)	(6)
1					Bridge HR: Resin injection
2					
3					
4	Resin injection				
5					
6					
7					
8					
9					
10					
11					
12					
13	Slab thickness				
14	increasing				
15	increasing	Slab thickness	<u> </u>		
1.5		increasing			
16		8			Bridge HR:
					Resin injection
17					
18					
19	Slab thickness increasing				
20					****
21					
22	_				
23					
24				Bridge FK: Slab thickness increasing	
25					
26		Slab thickness increasing			
27	Resin injection				Bridge HS: Slab thickness increasing
28					
29					
30					
		sent value of the to			US\$1.020 million
Minimum value of network connectivity reliability index				2.522	

A total of five maintenance interventions are applied (Table 2). Specifically, Bridges LE, HE, and MU are subject to slab thickness increasing at year 19, 26, and 29, respectively. In addition, the intervention of steel plate attaching is carried out for Bridge Q of Group 1 at year 25 whereas slab thickness increasing is scheduled at year 27 for Bridge HS of Group 2. Note that use of a monetary discount rate of 0.06 discourages the maintenance applications at earlier years in order to reduce the total maintenance cost down to an limited amount of \$0.396 million.

If more funding is available, the bridge network can be further maintained at a higher performance level over the time horizon. Solution B may be viewed as the best solution (up to the present 50th GA generation) that can most effectively increase the mini-

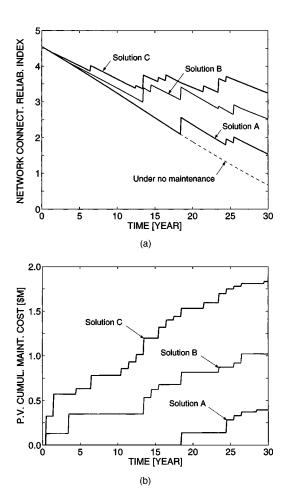
mum network reliability index to 2.522 when a total maintenance budget of \$1.020 million is provided. Compared to Solution A, five additional maintenance interventions have to be scheduled. It can be seen in Table 3 that Bridges LE and HE receive four and two maintenance interventions, respectively, due to their significant deterioration over the time [see Fig. 5(a)] as well as their vital importance to the network performance (see Fig. 6). In contrast, the very slow deterioration experienced by Bridge MU calls for no maintenance at all. For the two bridge groups, Bridge FK in Group 1 and Bridges HR and HS in Group 2 are selected for maintenance because they are among the bridges with least reliability levels and subject to the fastest deterioration in their respective bridge groups [see Figs. 5(b and c)].

Table 4. Maintenance Prioritization by the Representative Solution C

1	Bridge LE (2)	Bridge HE (3)	Bridge MU (4)	Bridge group 1 (5)	Bridge group 2 (6)
2 Resin inje 3 4 5 6 7 8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27	(2)	(3)	(1)	Bridge Q:	Bridge HS:
3 4 5 6 7 8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27				Resin injection	Resin injection
4 5 6 7 7 8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27	sin injection				
5 6 7 8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
6 7 8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
7  8  9  10  11  12  13  14 Slab thick increasi 15  16  17  18  19 Resin inje 20 21 22  23 24 Slab thick increasi 25  26  27			Resin injection		
8 9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27				Bridge FK:	
9 10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27				Slab thickness increasing	
10 11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
11 12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
12 13 14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27		Resin injection			
13  14 Slab thick increasi  15 16  17  18  19 Resin inje 20 21 22  23 24 Slab thick increasi 25  26  27		Resin injection		Bridge FK:	
14 Slab thick increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27				Resin injection	
increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27				Bridge FL:	·
increasi 15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27			[ [	Slab thickness increasing	
15 16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27	ab thickness				
16 17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27	ncreasing				
17 18 19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27					
18  19 Resin inje 20 21 22  23 24 Slab thick increasi 25  26  27			;	Bridge Q:	
18  19 Resin inje 20 21 22  23 24 Slab thick increasi 25  26  27		Slab thickness		Slab thickness increasing	
19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27		increasing			
19 Resin inje 20 21 22 23 24 Slab thick increasi 25 26 27		mercasing		Bridge FL:	<del></del>
20 21 22 23 24 Slab thick increasi 25 26				Resin injection	
20 21 22 23 24 Slab thick increasi 25 26	sin injection				••••
22 23 24 Slab thick increasi 25 26 27	<u>*</u>				·····
23 24 Slab thick increasi 25 26 27					
24 Slab thick increasi 25 26 27				Bridge FK:	
24 Slab thick increasi 25 26 27				Slab thickness increasing	
increasi 25 26 27	1 .1 . 1				
26 27	nb thickness ncreasing				
27		Slab thickness increasing			
			:		Bridge HR: Resin injection
28				Bridge FK:	Resin injection
/X I				Resin injection	
29	<del> </del>				
30		Resin injection			
_ 50	D+2	<del></del>	tal maintenance of	net	US\$1.835 million
Present value of the total maintenance cost  Minimum value of network connectivity reliability index				3.243	

By further increasing the maintenance expenses to \$1.835 million, Solution C can enhance the network performance to the minimum connectivity reliability index of 3.243 over the time horizon. Fig. 11 plots reliability profiles of the network connectivity and of individual bridges when subject to representative maintenance Solution C. As shown in Table 4, two resin injections and two slab thickness increasing are applied to Bridge LE as well as to bridge HE. These maintenance applications spread

more or less evenly over the time horizon of 30 years. Bridge MU, due to its high reliability level and relative immunity to deterioration, is allocated with only one resin injection at year 5. In the presence of an increased budget, more bridges in Group 1 are selected for maintenance. As observed in Fig. 5(b), the order of bridges with reliability levels from the lowest to the highest is FK, Q, FL, NM, LY, and LA, which is also an approximate bridge order in terms of deterioration rates from the largest to the



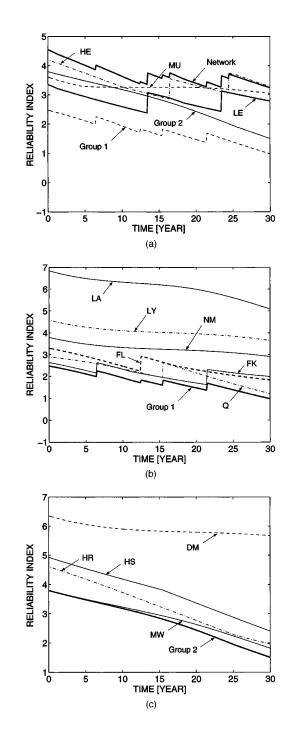
**Fig. 10.** Three representative maintenance solutions: (a) Profiles of the network connectivity reliability index; and (b) profiles of present value of the cumulative maintenance cost over the time horizon

smallest. Accordingly, bridges FK, Q, and FL receive four, two, and two maintenance interventions, respectively. Similarly, Bridges HR and HS are the only two bridges in Group 2 that are subject to maintenance. The reason that no maintenance is allocated to Bridge MW by Solution C is that the rate of deterioration of Bridge MW is not as serious as that of Bridge HR or HS, as evidenced in Fig. 5(c).

It is noted that the expensive maintenance type of steel plate attaching is rarely used and the costliest replacement of the total bridge deck slabs is not selected by these representative maintenance solutions. The predominant application of resin injection and slab thickness increasing emphasizes the importance of preventive maintenance as opposed to expensive essential maintenance interventions in cost-effectively improving the long-term performance of bridge deck slabs. On the other hand, more accurate data are urgently needed in supporting the above observations regarding optimized solutions as well as in making more rational decisions on selecting the most efficient maintenance sequence for the long-term performance of individual bridges as well as the entire bridge network.

# **Conclusions and Future Study**

Highway transportation systems play a pivotal role in the sustainable economic growth and social development of a modern



**Fig. 11.** Time-dependent reliability index under representative maintenance Solution C for: (a) Network connectivity; (b) Group 1; and (c) Group 2

society. Due to aggressive environmental stressors and increasing traffic loads, highway bridges undergo significant deterioration in both condition and safety performance. Timely and adequate maintenance interventions are therefore crucial to ensure the satisfactory performance of an existing bridges network over a specified time horizon. Under budget constraints, it is important to prioritize maintenance needs to bridges that are most significant to the functionality of the network, in addition to prudently scheduling these maintenance interventions over the time horizon in order to achieve the overall cost effectiveness. In this paper, a network-level bridge maintenance management procedure was

proposed. The overall performance of a bridge network was assessed in terms of the origin-destination connectivity reliability, which was evaluated by an event tree analysis, utilizing structural reliability index profiles of all deteriorating bridges in the network. Each maintenance solution was treated as a sequence of maintenance activities that are scheduled at discrete years and are applied to selected bridges. The conflicting objectives of maximizing the network connectivity reliability and minimizing the present value of total maintenance cost were considered in the posed combinatorial bi-objective optimization problem that was solved by GA. The following conclusions are drawn.

- The proposed GA-based procedure is able to locate a set of different maintenance solutions that represent the optimized tradeoff between the two conflicting objectives. In particular, under given budgets, one can find the maintenance solution that best improves the network performance (in terms of connectivity); alternatively, if a threshold is specified for the network performance, a maintenance solution that requires the least total maintenance cost can be found.
- The present GA-based procedure can automatically prioritize
  maintenance needs among networked bridges and over the
  time horizon. It can effectively identify bridges that are more
  important to the functionality of the network and then prioritize limited resources to these bridges at selected years.
- Use of preventive maintenance interventions, in general, leads to the most economical solution in maintaining a bridge network. In contrast, use of essential maintenance interventions, due to their higher costs, is often discouraged unless much higher reliability levels are demanded.

The study presented in this paper can be extended by the following research efforts.

- Deterioration of bridge deck slabs is considered in this paper as an illustration to demonstrate the efficiency of the proposed procedure. Other bridge components (e.g., girders, piers) may also be considered in bridge maintenance management programs. This requires reliable datasets that accurately reveal the interaction between maintenance interventions and bridge system performance.
- The overall performance of a deteriorating bridge network can also be assessed by calculating the present value of the total expected cost over the time horizon. This cost consists of direct agency costs due to various maintenance expenses and bridge failure costs as well as the indirect user costs due to loss of service. These costs can also be treated as objective functions for optimization.

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