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Optimal worksites on highway networks subject to constraints

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Abstract

Advances in the management of civil infrastructure are making it possible to determine the optimal management strategy for specific structural elements and, with the help of agency rules, the optimal management strategy for entire structures. The optimal management strategies on the element and structure levels however cannot often be followed exactly because there are constraints on the network level. For example, it is not desirable to return to the same stretch of highway in successive years and disrupt traffic flow and it is not desirable to have traffic interrupted on the road network for long distances.

In this paper an algorithm is proposed that can be used to determine the optimal worksite on a highway network. The algorithm allows consideration of agency and user costs that are related to both the individual interventions and to the worksite as a whole. The term worksite is defined herein as a group of structures on which interventions are performed. The algorithm is explained via an illustrative example. The optimal worksite on the example highway network is determined for different budgets and a maximum permitted worksite length of 15 km.

1. Introduction

State-of-the-art management systems, such as Pontis (Cambridge Systematics 2001) and KUBA (Ludescher et al. 1998) help infrastructure managers determine optimal management strategies for their bridges. These systems first determine the optimal interventions and optimal times of intervention for individual elements such as bridge decks and piers, knowing that these elements are subjected to deterioration, that a given number of interventions can improve their condition state, and the cost of each of these interventions.

Once the optimal element level strategies are determined they are combined, using agency rules, to give management strategies for individual structures. These agency rules ensure that general agency policies are adhered to when structure level strategies are determined. For example, in Pontis there are *major rehabilitation rules* that are used to recognize the practical considerations of performing rehabilitation work when following the optimal strategy. More specifically this means that when a rehabilitation project is performed on a structure it is necessary to address all of the intervention needs on the bridge and that it is not possible to leave one of the elements in a less than desirable condition state simply because the optimal element level strategy is to replace it in the following year.

Once the structure level strategy is determined, all of the best structure level interventions are ranked based on their calculated benefit-cost ratio (Farid et al. 1994), and the decision of which interventions to perform is left to the infrastructure manager. Existing management systems currently have no provision to take into consideration where the structures are located on the network or the alteration of direct and indirect costs that occurs when structures are grouped together into worksites. It is exactly these network level considerations, however, which often prevent optimal structure level management strategies from being carried out (Swiss Federal Roads Authority 1994).

This paper gives an illustrative example of an algorithm (Rafi et al. 2004) developed to address the combination of interventions to form an optimal worksite on a highway network. The algorithm takes into consideration the agency and user costs associated with each individual intervention as well as those that are attributable directly to the worksite. The algorithm also takes into consideration network connectivity. The structuring of the solution procedure is given step by step.

2. Example network

The example network is the highway network in the southern portion of the canton of Vaud, Switzerland (Figure 1). The network is modelled assuming that traffic flows into the network at node 1 and out through nodes 11, 26, 31 and 38. A complete model of the network would require modelling of traffic beginning at nodes 11, 26, 31 and 38. This is, however, not necessary for illustration of the algorithm. The network is divided into 37 sections, herein referred to as *road sections* and 38 nodes. The 37 road sections are grouped into 7 links (Table 1). The nodes represent either, the start or end points of each possible intervention - traffic configuration, road intersections, or network limits. An intervention-traffic configuration represents the specific combination of the type of intervention to be performed on a structure and the traffic configuration, or perturbation, required during the intervention, e.g. closing one lane. A link is defined herein as a part of the network comprised of all road sections between either intersections in the network, intersections and network limits, or between network limits, that consists of a single path. To avoid any potential political problems entirely fictitious numbers are used. As only fictitious numbers are used the nodes in each link were selected to be equally spaced along each link to simplify illustration.

The optimal worksite is determined assuming a 15 km maximum worksite length constraint and a variable budget, ranging from an amount that can be considered unlimited, i.e. all possible interventions can be performed, to an amount small enough so that no interventions can be performed. The budget in each case is reduced in intervals of 4.3 units from the initial "unlimited" budget of 25.4 units to 3.9 units.

It is assumed that there are three intervention options for each road section and only one possible traffic configuration per intervention. Herein, for simplicity, each intervention-traffic configuration combination is referred to as simply an intervention option. The three intervention options for each road section are shown as arcs on the directed graph used to model the network (Figure 2) and are indicated as follows:

1) high benefit intervention options - with a 2 after the physical node number, e.g. for node 2, the vertex on the directed graph is 02,2;
2) low benefit intervention options - with a 1 after the physical node number, e.g. for node 2, the vertex on the directional graph is 02,1; and
3) a do nothing intervention option - with a 0 after the physical node number, e.g. for node 2, the vertex on the directed graph is 02,0.

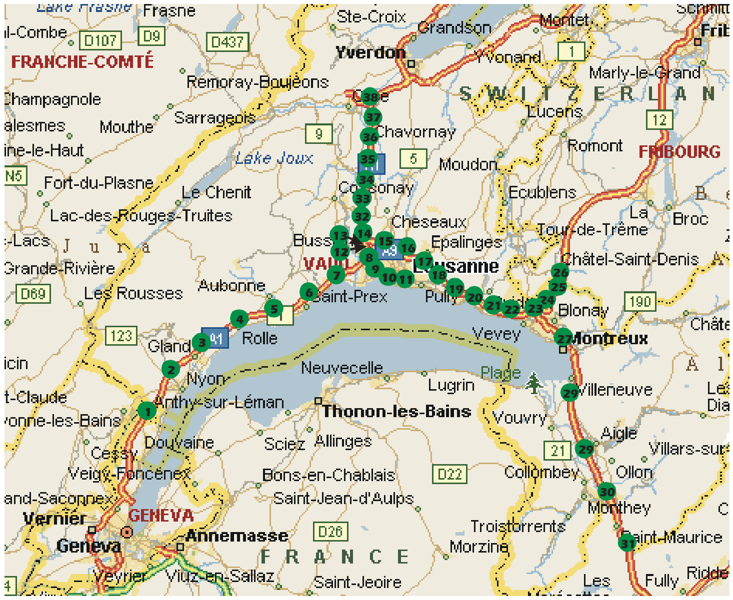

Figure 1. Physical nodes on southern portion of highway network in the canton of Vaud, Switzerland

To avoid confusion, the nodes in Figure 1 that represent a physical location on the network are referred to as *nodes* and the nodes on the directed graph used to model the network (Figure 2) are referred to as *vertices*. The intervention costs are comprised of agency costs and user costs. The agency costs include the costs of traffic management and are dependent on the type and the length of the worksite.

$$V = \tilde{V} + v \cdot d_{ij} \qquad (1)$$

where $\tilde{V}$ is the fixed agency costs of a change in traffic configuration both now and in the future; and v is the cost per unit length of traffic configuration change or the cost per unit length of maintaining the traffic configuration both now and in the future; and d_{ij} is the length between nodes i and j.

Table 1. Characteristics of road network

Link	Start node	End node	Length (km)	Number of nodes	Road sections where interventions possible
1	1	8	49	8	2-3, 4-5, 6-7
2	8	11	6	4	9-10
3	8	14	3	4	12-13
4	14	23	27	10	15-16, 17-18, 19-20, 21-22
5	23	26	6	4	24-25
6	23	31	35	6	27-28, 29-30
7	14	38	21	8	32-33, 34-35, 36-37

Figure 2. Network model (directed graph) of example network

User costs are the costs to the users of the highway network while an intervention is being performed, including time delay costs, vehicle operating costs, accident costs and environmental costs.

$$U = \tilde{U} + u \cdot d_{ij} \tag{2}$$

where $\tilde{U}$ represents the user costs, such as the increased accident rate, related to the changing of traffic configurations, and u represents the user cost per unit length of traffic configuration change or the cost per unit length maintaining the traffic configuration both now and in the future.

Table 2. Fixed and variable costs associated with intervention costs

	Fixed costs	Variable costs	Distance over which cost incurred
Change in traffic configuration	$\tilde{V} + \tilde{U}$ (units)	$v + u$ (units/km)	*d (km)*
(i,0) to (j,2)	*4*	*0.2*	*1*
(i,0) to (j,1)	*2*	*0.15*	*1*
(i,2) to (j,0)	*4*	*0.2*	*1*
(i,2) to (j,1)	*6*	*0.35*	*1*
(i,1) to (j,2)	*6*	*0.35*	*1*
(i,1) to (j,0)	*2*	*0.15*	*1*
(i,2) to (j,2)		*0.2*	d_{ij}
(i,1) to (j,1)		*0.15*	d_{ij}

i is the road section start node and *j* is the road section end node.

The benefits are assumed to be 50 units for high benefit intervention options and 25 units for low benefit intervention options. The values used to calculate the intervention costs, namely agency costs (V) user costs (U), are shown in Table 2. The costs, C, where $C = V + U$, and the net agency benefits, $\overline{N}$, attributed to each intervention option can be found in [Hajdin et al. 2005].

3. Objective function and constraints

In this example, the problem is to determine the worksite, from the point of view of traffic originating at node 1 (Figure 1) on the southern portion of the highway network of the canton of Vaud that provides the greatest utility for budgets ranging, from high enough so that all intervention options are possible, to low enough so that no intervention options are possible, for the case where the maximum worksite length constraint is 15 km. The objective function is:

$$Maximize \quad Z = \sum_{([i,k],[j,l])\in A} y_{[i,k],[j,l]} \cdot \left(\overline{N}_{[i,k],[j,l]} - C_{[i,k],[j,l]}\right) \tag{3}$$

$$where: \; C_{[i,k],[j,l]} = V^{i}_{[i,k],[j,l]} + V^{l}_{[i,k],[j,l]} + U^{i}_{[i,k],[j,l]} + U^{l}_{[i,k],[j,l]} \tag{4}$$

where $\overline{N}_{[i,k],[j,l]}$ is net agency benefit, i.e. the difference between the most expensive intervention option and the optimal intervention option; $V^{i}_{[i,k],[j,l]}$ is initial agency costs; $V^{l}_{[i,k],[j,l]}$ is subsequent agency costs, $U^{i}_{[i,k],[j,l]}$ is initial user costs; and $U^{l}_{[i,k],[j,l]}$ is subsequent user costs, and $y_{[i,k],[j,l]}$ is a binary variable that takes the value of 1 if arc $[i,k],[j,l]$ belongs to the path that maximizes Z and 0 if it does not, and $[i,k]$ and $[j,l]$ indicate vertices on the network model.

Subject to the following continuity constraints:

$$1 + \sum_{([j,l]):([j,l],[a])\in A} y_{[j,l],[a]} = \sum_{([j,l]):([a],[j,l])\in A} y_{[a],[j,l]} \tag{5}$$

$$\sum_{([j,l]):([j,l],[e])\in A} y_{[j,l],[e]} = 1 + \sum_{([j,l]):([e],[j,l])\in A} y_{[e],[j,l]} \tag{6}$$

$$\sum_{([j,l]):([j,l],[i,k])\in A} y_{[j,l],[i,k]} = \sum_{([j,l]):([i,k],[j,l])\in A} y_{[i,k],[j,l]} \quad for\ all \begin{array}{l}[i,k]\in (N \cap N_s)\wedge \\ [i,k]\neq a \wedge [i,k]\neq e\end{array} \tag{7}$$

$$y_{[s],[j,l]} - \sum_{([j-1,k]):([j-1,k],[j,l])\in A} y_{[j-1,k],[j,l]} = 0 \;\; for\ all\ [j,l]\in N_v \tag{8}$$

$$y_{[j,l],[e]} - \sum_{([j-1,k]):([j-1,k],[j,l])\in A} y_{[j-1,k],[j,l]} = 0 \;\; for\ all\ [j,l]\in N_z \tag{9}$$

where a and e are the start and end nodes, respectively, s and e are the additional source and end nodes introduced as each bifurcation and unification, respectively, N the set of all nodes, N_s is the set of intersection nodes, N_v is the set of bifurcation nodes (where one arc becomes multiple arcs); N_z is the set of unification nodes (where multiple arcs become one arc); and A is the set of arcs in the network.

the following maximum worksite length constraint:

$$\sum_{([c,l]):([b,0],[c,l])\in A\wedge l\neq 0} y_{[b,0],[c,l]} - \sum_{l} y_{[f,l],[g,0]} \leq 0 \tag{10}$$

$$\textit{for all } s \in N \textit{ and } d_{[c,l],[d,l]} \leq \Lambda < d_{[c,l],[g,0]}$$

where [*b,l*] is the last node traveled before the work site, [*c,l*] is the first node of the worksite [*f,l*] is the last node of the worksite, [*g,l*] is the first node after the worksite has ended, and Λ is the maximum worksite length.

and the following budget constraint:

$$\sum_{([i,k],[j,l])\in A} y_{[i,k],[j,l]} \cdot C^{i}_{[i,k],[j,l]} \leq R \tag{11}$$

where R is the maximum resources to be allocated.

4. Solution Steps

In this section the steps used to set-up and solve the problem are given. All calculations were done using Excel (Walkenbach 2001) with the "What's Best!" solver add-in (Lindo Systems 2003).

Step 1: Write the continuity constraint table. The continuity constraint table is used to ensure that only one arc per road section (between two physical nodes) is selected. The continuity constraint matrix has the nodes of the network model labeled on one axis and the arcs of the network model on the other axis. A +1 in the cell representing a node and an arc indicates that the node is the incoming node for the arc. A -1 indicates that the node is the outgoing node for the arc. All source and end nodes are included in the continuity constraint matrix.

A sample of the continuity constraint matrix for the example network is given in Table 3 with the vertices labeled on the vertical axis and the arcs on the horizontal axis. Both the connectivity and the continuity of the example network can be read from Table 3. For example, it can be seen in Table 3 that 1 unit flows into the network through vertex S,0 and out of the arc S,0-01,0 through vertex 01,0 because there is a +1 at the intersection of vertex S,0 and arc S,0-01,0 and a -1 at the intersection of vertex 01,0 and arc S,0-01,0. It can also be seen that the unit that flows out of vertex 01,0 can flow into either arc 01,0-02,2 and 01,0-02,0 and 01,0-02,1 because there is a +1 in the cells for vertex 01,0 and arcs 01,0-02,2 and 01,0-02,0 and 01,0-02,1.

Step 2: Write the worksite length constraint table. The worksite length constraint table is used to indicate where worksites start and end. It is necessary in order to ensure that the worksites do not exceed the maximum allowable worksite length. The worksite length constraint table has all possible worksites labeled on one axis and each arc of the network model (the intervention options) on the other axis. A +1 is placed at the intersection of a worksite and an arc if the arc represents the first or last intervention in the worksite, where a change in traffic configuration is required.

A sample of the worksite length constraint table used for the example network is given in Table 4. The start and end nodes of each possible worksite as well as whether or not the change in traffic configuration is required, for the example network can be read from Table 4. For example, it can be seen that worksite 04-05, 09-10 requires a traffic configuration change at the beginning of the worksite for the high and low benefit options and no traffic configuration change for the do nothing option because there is a 1, 0 and 1 placed at the intersection of worksite 04-05, 09-10 and the arcs 04,2-05,2 and 04,0-05,0 and 04,1-05,1, respectively. It can also be seen that worksite 04-05, 09-10 requires a traffic configuration change at the end of the worksite for the high and low benefit options and no traffic configuration change for the do nothing option because there is a 1, 0 and 1 placed at the intersection of worksite 04-05, 09-10 and arcs and the arcs 09,2-10,2 and 09,0-10,0 and 09,1-10,1, respectively.

Table 3. Sample of continuity constraint table for example network

	Arcs															
Nodes	S,0-01,0	01,0-02,2	01,0-02,0	01,0-02,1	. . .	S,1-08,2	S,1-08,0	S,1-08,1	. . .	37,2-38,0	37,0-38,0	37,1-38,0	38,0-E,0	**Sumproduct**	Constraint	RH
S,0	1													**1**	=	1
01,0	-1	1	1	1										**0**	=	0
02,2		-1												**0**	=	0
02,0			-1											**0**	=	0
02,1				-1										**0**	=	0
.																
.															=	
.																
S,1						1	1	1						**1**	=	1
08,2						-1								**0**	=	0
08,0							-1							**0**	=	0
08,1								-1						**0**	=	0
.																
.															=	
.																
37,2										1				**0**	=	0
37,0											1			**0**	=	0
37,1												1		**0**	=	0
38,0										-1	-1	-1	1	**0**	=	0
E,0													-1	**-4**	=	-4

Step 3: Write the costs and benefits for each arc in the direct graph and set up the binary variables and objective function. These are calculated using the arc lengths, which can be calculated from the values presented in Table 1 or taken directly from Figure 2, and the fixed and variable costs given in Table 2. A sample of the costs, benefits, objective function and the binary variable for the example network is given in Table 5. For example, it can be seen that arc 04,2-05,2, which signifies a high benefit intervention between physical nodes 4 and 5 (Figure 1), will cost 1.4 units and yield 50 units in benefit, meaning that its part of the objective function is 48.6 units.

Step 4: Write the formulas that link the binary variables and the continuity constraints. These formulas are the sum of the products of the continuity constraints for each vertex – arc combination and the binary variables that signify whether or not the arc is selected. This ensures that the quantity of flow entering a vertex also leaves the vertex. For any valid solution, i.e. that satisfies all of the constraints, these will equal 0, except for source nodes which will be +1 and end nodes which will depend on the number of branches that must be rejoined before exiting the directed graph. A sample of the results for the example network when a 25 km maximum worksite length is imposed and an available budget of 8.2 units are indicated in the shaded column of Table

3. The end node has 4 units of flow leaving the network because a total of 4 units were introduced into the network, one at the beginning and one at each of the three intersections (Figure 2).

Table 4. Sample of worksite length constraint matrix for example network

	Arcs																	
Worksites	S,0-01,0	…	04,2-05,2	04,0-05,0	04,1-05,1	…	09,2-10,2	09,0-10,0	09,1-10,1	…	12,2-13,2	12,0-13,0	12,1-13,1	…	38,0-0E,0	**Sumproduct**	Constraints	RH
.																		
.																	<=	
.																		
04-05, 09-10			1	0	1		1	0	1							**0**	<=	1
04-05, 12-13			1	0	1						1	0	1			**0**	<=	1
.																		
.																	<=	
.																		
34-35, 36-37																**2**	<=	2

Step 5: Fill in right hand values for the continuity constraints. This sets the entry and exit points for flow units in the directed graph. A positive integer indicates a source vertex where unit of flow is introduced into the network and a negative integer indicates an end vertex where unit of flow leaves the network. A sample of the right hand values for the continuity constraints for the example network can be seen in the far right hand column, marked RH, in Table 3.

Step 6: Fill in right hand values for maximum worksite length constraints. These are used during optimization to verify if a worksite is longer than the maximum allowed distance. A 1 is used if the maximum allowed distance is exceeded and a 2 if the maximum allowed distance is not exceeded. A sample of the right hand values for the example network is shown in the far right hand column, marked RH, of Table 4. From Table 4 it can be seen that the worksite 04-05, 09-10 is greater than the maximum allowed distance and that worksite 34-35, 36-37 is less than the maximum allowed distance, because the right hand side values are 1 and 2, respectively.

Step 7: Optimization. The optimization of the network utility is done by changing the binary variables y until the worksite is found that has the highest network utility and satisfies all of the continuity, worksite length and budget constraints. A sample of the y values determined for the example network when there is a 25 km worksite length constraint and a 8.2 unit budget constraint is shown in the shaded row of Table 5. It can be seen that the intervention option 04,2-05,2 is not included in the optimal solution because the binary variable is 0. In this example, the linear integer problem was solved in excel using the "global solver" in the "*What's Best!*" add-in [Lindo Systems, 2003]. This global solver uses a branch and bound technique to exhaustively search for an optimal solution. A constraint must be added here to ensure that multiple interventions on the same link are not chosen.

Table 5. Costs, benefits, objective function and binary variables

	Arc										
	S,0-01,0	. . .	04,2-05,2	04,0-05,0	04,1-05,1	. . .	09,2-10,2	09,0-10,0	09,1-10,1	. . .	38,0-E,0
Costs	0		1.4	0	1.05		0.4	0	0.3		0
Benefits	0		50	0	25		50	0	25		0
Objective function	0		48.6	0	24.0		49.6	0	24.7		0
y (binary)	**1**		**0**	**1**	**0**		**0**	**1**	**0**		**1**

5. Results

Table 6 shows the changes in the interventions that comprise the optimal worksite, and the reduction in net benefit, as the budget is reduced in intervals of 4.3 units. The available budget and net benefit are given in the left hand columns. The road sections on which the interventions are performed are the column headers. The dark shaded cells indicate the high benefit intervention options and the light shaded cells indicate the low benefit intervention options. For example, it can be seen in Table 6 that for an allowed budget of 25.4 units that the high benefit intervention option on road section 12-13 is part of the optimal worksite.

When there is no budget constraint, and a worksite length constraint of 15 km, the optimal worksite is comprised of the high benefit interventions, namely 9-10, 12-13, 15-16 and 32-33. The net benefit of this worksite, and therefore the maximum possible, is 175.8 units.

<u>Step 7: Optimization.</u> The optimization of the network utility is done by changing the binary variables *y* until the worksite is found that has the highest network utility and satisfies all of the continuity, worksite length and budget constraints. A sample of the *y* values determined for the example network when there is a 25 km worksite length constraint and a 8.2 unit budget constraint is shown in the shaded row of Table 5. It can be seen that the intervention option 04,2-05,2 is not included in the optimal solution because the binary variable is 0. In this example, the linear integer problem was solved in excel using the "global solver" in the "*What's Best!*" add-in [Lindo Systems, 2003]. This global solver uses a branch and bound technique to exhaustively search for an optimal solution. A constraint must be added here to ensure that multiple interventions on the same link are not chosen.

Table 6 Interventions on network for varying budget constraints

Available budget	Net benefit	Road sections on which interventions are performed														
		2:3	4:5	6:7	9:10	12:13	15:16	17:18	19:20	21:22	24:25	27:28	29:30	32:33	34:35	36:37
25.4	175.8															
21.1	155.0															
16.8	138.6															
12.5	138.6															
8.2	68.5															
3.9	0															

6. Discussion of Results

By using the algorithm presented in this paper it is possible to determine the optimal worksite on a highway network when distance and budget constraints are imposed. Some of the general observations when analyzing the results presented in the previous section are that the optimal worksite includes an intersection, and that a small reduction in budget can change the road sections to be included in the optimal worksite.

The optimal worksite is centered on an intersection due to the increased number of possible intervention options without exceeding the distance constraints. This is illustrated by the following small example. If three interventions are to be performed on road sections that are each 7 km long and are in series, and there is a distance constraint of 15 km, it is not possible to do all three interventions (21 km > 15 km). If the three interventions are connected at an intersection they will all be possible (14 km < 15 km) (Figure 3).

Small reductions in budget can result in a change in the road sections on which to perform interventions in the optimal worksite because the highest benefit worksite is always determined regardless of the location on the network. For example, when the maximum worksite length constraint is 15 km and the budget is reduced from 21.1. to 16.8 units, the high benefit interventions on road section 12-13 and 15-16 are removed from the optimal worksite and the high benefit interventions on road section 34-35 and 36-37 are added (Table 6). In some cases such a reduction in budget could result in an increase in the number of interventions although not seen in this example. This would occur due to a greater increase in the number of low benefit interventions than decrease in the number of high benefit interventions. There would, of course, still be a decrease in the number of high benefit interventions.

(a) (b)

Figure 3. Illustration of intervention configurations

Conclusions

The presented algorithm is an efficient algorithm to be used to determine optimal worksites on a highway network. Although it is only a first step, it is envisaged that such a method, once further developed, and implemented into infrastructure management systems, will allow drastic improvement in the determination of optimal management strategies for civil infrastructure at the network level.

The next steps are to investigate 1) real world interventions, 2) different numbers of intervention options, 3) multiple worksites, 4) optimal single worksites in time, and 5) optimal multiple worksites in time.

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