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TREND CURVE OPTIMAL CONTROL MODEL FOR HIGHWAY PAVEMENT MAINTENANCE: CASE STUDY AND EVALUATION

KOJI TSUNOKAWA
The Overseas Economic Cooperation Fund (Japan), Tokyo, Japan

and

JOSEPH L. SCHOFER
Department of Civil Engineering, Northwestern University, Evanston, IL 60201, U. S. A.

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Abstract—This paper presents a control theoretic dynamic model for optimizing the timing and intensity of major highway pavement maintenance actions. The solution procedure is demonstrated through a simple case study, and the worthiness as a problem-solving tool is discussed through a systematic evaluation of the solutions.

1. INTRODUCTION

This paper presents a control theoretic dynamic model for optimizing the timing and intensity of major highway pavement maintenance actions (e.g. overlays).[1]

Highway pavement maintenance has traditionally been based on standards that define minimum allowable serviceability levels and a set of maintenance technologies to be applied when deterioration reaches the threshold levels. Recently, however, the importance of life-cycle cost analysis has been widely recognized in the highway maintenance area (see for example Haas and Hudson, 1978; Iwamatsu, *et al.*, 1992), and methodologies for establishing economically sound maintenance strategies have been called for. Optimal control theory is a natural approach for analyzing life-cycle cost minimization problems and has been used in the study of optimal machinery maintenance strategies (Thompson, 1968; Kamien and Schwartz, 1971). In the highway maintenance area, Buttler and Shortreed (1978) and Friesz and Fernandez (1979) first applied optimal control models for routine maintenance in which maintenance applications are represented as piecewise, continuous investments over time. Optimization of major maintenance actions is formulated by Fernandez (1979) as an optimal control model with discrete jumps in the state variable that represent discontinuous changes in pavement serviceability resulting from applications of major maintenance actions. Optimal control problems with discrete jumps in the state variable, however, are generally quite cumbersome, and the applicability of this model as a practical problem-solving tool is limited.

The proposed model for major highway maintenance actions employs a smooth trend curve for approximating a sawtooth-like pavement trajectory curve to alleviate this mathematical difficulty. Through this approximation, optimization of major maintenance actions is formulated as a standard optimal control model without discrete jumps in the state variable, which may be solved for a wide range of specific problems. In what follows, such a model is formulated by employing four basic relationships of pavement maintenance, i.e. the relationships of road deterioration, maintenance effect, agency

[1]Time staging of maintenance actions discussed in this paper is different from that of capacity investment in the literature (see Fernandez and Freisz (1981) for literature review), in that not only the timing but also the extent of each investment is considered as a decision variable, and not only the direct impact on user cost but also indirect effect through infrastructure deterioration is considered in the analysis of the trade-off between investment and user cost in the present paper.

costs and user costs. The procedure for solving this model is then demonstrated using a simple case study, and finally, worthiness of this approach is examined through a systematic evaluation of the solutions.

2. MODEL FORMULATION

2.1. Sawtooth curve model

The serviceability of a given highway pavement, s (defined here as a measure of "roughness," or more accurately *un*serviceability), follows a sawtooth-like trajectory curve over time as pavement deteriorates and receives maintenance (Fig. 1). We assume that the traffic is exogenous and fixed over time in this paper. This assumption may readily be relaxed to exogenous, time-dependent traffic.[2] The case of endogenous traffic is discussed elsewhere (Tsunokawa, 1986). If climatic aggression is constant over time, the rate of deterioration may be assumed as solely dependent on the current serviceability level and written as follows:

$$ds/dt = F(s(t)). \tag{1}$$

The amount of serviceability recovery after a maintenance application, Δs, may be assumed as a function of the maintenance intensity, w (e.g. overlay thickness), and serviceability level immediately before the application and written as:

$$\Delta s_n = G(w_n, s(t_n^-)), \tag{2}$$

where the subscript n represents the nth maintenance application, and the superscript $^-$ represents the moment immediately before the application. If relationship (1) holds in each interval (t_{n-1}, t_n), $n = 1, 2, \ldots$, and relationship (2) at each maintenance application t_n, $n = 1, 2, \ldots$, the sawtooth-like serviceability trajectory curve is uniquely determined for a given pavement, provided that a maintenance strategy (t_n, w_n), $n = 1, 2, \ldots$, is given, and the initial serviceability is given as:

$$s(0) = s_0. \tag{3}$$

Costs for the agency and the user are assumed to be functions of serviceability and

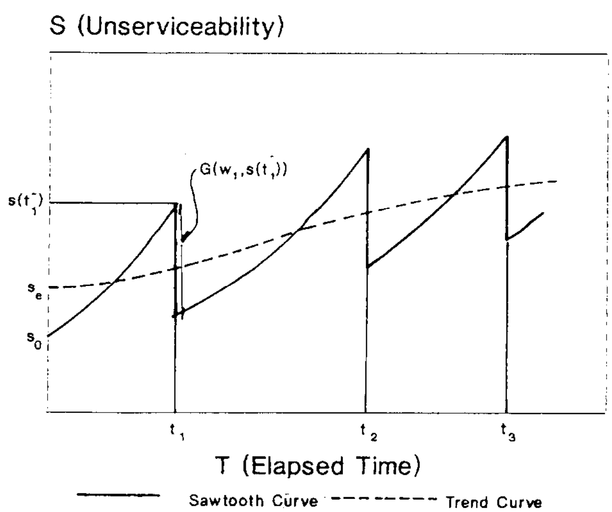

Fig. 1. Sawtooth curve and trend curve of pavement serviceability.

[2]In this case, the deterioration function and the user cost function are modified to $F(t,s(t))$ and $C(t,s(t))$, respectively, to denote the dependence of these functions on time. These modifications do not necessitate any alteration in the first order optimality conditions. However, the solution procedure set forth in this paper needs to be modified appropriately to take account of the nonautonomousness of the relationships.

maintenance intensity, respectively, and may be written as $C(s)$ and $M(w)$. Using these functions, net present value of the total life-cycle costs for the agency and the user attributable to a given pavement, J, can be written as follows:

$$J = \sum_{n=1}^{\infty} \left\{ \int_{t_{n-1}}^{t_n} C(s(t))\, e^{-rt} dt + M(w_n)\, e^{-rtn} \right\}, \tag{4}$$

where r = social discount rate, and $t_0 = 0$. An infinite time horizon is chosen to avoid employing an arbitrary assumption about the residual value at the end of a service life. The problem of finding the optimal maintenance strategy for a given pavement segment may then be defined as the one of finding the optimal values of (t_n, w_n), $n = 1, 2, \ldots$, that minimize expression (4) while the serviceability trajectory, $s(t)$, is determined by relationships (1) and (2) and initial condition (3).

Fernandez (1979) considered a similar model for optimizing major highway maintenance actions. However, optimal control models with discrete jumps in the state variable, such as the above, are generally quite cumbersome (Bryson and Ho, 1975), and efficient solution procedures have not yet been developed. The proposed method employs an approximation for the aforementioned sawtooth-like serviceability trajectory curve by means of a smooth trend curve to circumvent this mathematical difficulty.

2.2. Trend curve model

To develop a tractable model that may be used for solving the above problem, we consider that, associated with each maintenance strategy, (t_n, w_n), $n = 1, 2, \ldots$, there exist two functions of time, maintenance frequency, $h(t)$, and maintenance intensity, $w(t)$, which are as defined below. If maintenance is performed in every n years, the frequency is $1/n$. We assume such frequency varies over time and is given by $h(t)$. Given this function, each maintenance application time, t_n, is determined from the following recursive relationships:

$$\int_{t_{n-1}}^{t_n} h(t)\, dt = 1 \qquad (n = 1,2, \ldots). \tag{5}$$

The maintenance intensity function, $w(t)$, gives the intensity of a maintenance action to be applied at time t. Thus, given this function, the intensity of each maintenance action, w_n, is simply determined as:

$$w_n = w(t_n). \tag{6}$$

Using these functions, the trend of a sawtooth curve determined by relationships (1), (2) and initial condition (3) is represented by a smooth curve (trend serviceability curve hereafter) represented by (Tsunokawa, 1986; forthcoming):

$$ds/dt = F(s(t)) - h(t)\, K(w(t),s(t)) \tag{7}$$

$$s(0) = s_e, \tag{8}$$

where $K(\,.\,)$ is defined as

$$K(w,s) = 0.5\{G(w,s) + \tilde{G}(w,s)\} \tag{9}$$

$$\tilde{G}(w,s) = G(w,s + \tilde{G}(w,s)), \tag{10}$$

and s_e = effective initial serviceability. This is the initial value of the trend curve, $s(0)$, determined from the condition that the trend curve passes through the midpoint of the vertical line segment of the sawtooth curve at time t_1. Mathematically, it is related to true initial roughness, s_0, through the following relationship (see Fig. 1):

$$s_0 + \int_0^{t_1} F(s)\,dt - 0.5\,G(w_1, s(t_1^-)) = s_e + \int_0^{t_1} \{F(s) - h\,K(w,s)\}\,dt, \tag{11}$$

where the variable s on the left side is the serviceability represented by a sawtooth curve, while that on the right side is the serviceability represented by the corresponding trend curve. Tsunokawa (1986; forthcoming) obtained an expression for the deviation between a smooth curve connecting the midpoints of vertical line segments of a sawtooth curve (average serviceability curve) and the trend curve represented by relationship (7) with (8), and showed that it vanishes as h tends to both zero and infinity.

Using $h(t)$ and $w(t)$, objective function (4) may also be approximated as follows:

$$J = \int_0^{\infty} \{C(s(t)) + h(t)\,M(w(t))\}\,e^{-rt}dt. \tag{12}$$

It should be noted that the variable s in this expression represents the serviceability along a trend curve. Thus, the first term of expression (12) represents an approximation for the user costs associated with a sawtooth curve via those associated with the corresponding trend curve. The second term is an approximation for the discrete summation of individual maintenance costs by way of an integral of the continuous stream of expected costs. The validity of these approximations is discussed in Tsunokawa (1986; forthcoming) for a simple case where both $C(s)$ and $M(w)$ are linear functions.

In this formulation, both objective function (12) and constraint (7) are linear in $h(t)$. This implies that the problem cannot be solved unless $h(t)$ is bounded within an appropriate range. Fortunately, however, $h(t)$ appears to have some natural bounds. If maintenance intervals are too long, i.e. $h(t)$ is too small, pavement would probably deteriorate excessively between maintenance actions. Conversely, if maintenance is applied too often, i.e. $h(t)$ too large, the optimal maintenance intensity would become so small that it becomes technically infeasible. Thus we assume that:

$$h_1 \le h(t) \le h_2 \;\forall\; t, \tag{13}$$

where h_1, h_2 = minimum and maximum feasible maintenance frequencies, respectively, in number of maintenance actions per unit period of time. We assume that both h_1 and h_2 are exogenously given, positive finite numbers.

Minimization of expression (12) under constraints (7), (8) and (13) is a standard optimal control problem (see Kamien and Schwartz, 1981, for example) that may be solved for a variety of specific problems. Once the solution to this problem is obtained, an approximate solution for the optimal maintenance strategy, (t_n, w_n), $n = 1, 2, \ldots,$ that minimizes expression (4) may be obtained by using relationships (5) and (6). Convexity of $F(\,.\,)$, $C(\,.\,)$, $M(\,.\,)$ and concavity of $G(\,.\,)$ have been shown to be sufficient for this minimization problem (Tsunokawa, forthcoming).

3. SOLUTION PROCEDURES

In what follows, the solution procedure outlined above is discussed in more detail through a simple case study. A complete description of the procedure for more general cases is found in Tsunokawa (1986; forthcoming).

3.1. Problem definition

Based on the results of field studies in Brazil and other countries, a set of road deterioration relationships for different distress modes were built at the World Bank (Watanatada, *et al.*, 1988). When combined, these relationships imply that serviceability trajectory, $s(t)$, follows approximately an exponential function of time within its reasonable range. In this case study, the following relationship will be assumed for road deterioration that is consistent with this finding:

$$F(s) = f_1 s. \tag{14}$$

The value of f_1 is assumed to be 0.05 with serviceability, s, measured in QI roughness units and time, t, in years. With this deterioration rate, a good pavement of 25 QI will deteriorate to a poor condition of 100 QI in 28 years. In practical applications, these parameters can be selected to reflect actual or expected experience.

Overlay will be considered as the only maintenance action in this case study. Roughness reduction due to an overlay is an increasing function of both overlay thickness and the pavement roughness before overlay. The aforementioned study finds that the effect of current roughness on roughness reduction is mostly linear, while the effect of overlay thickness is nonlinear with the diminishing contribution of thickness. The following simplified relationship is used in this case study for the overlay impact function:

$$G(w,s) = g_1 SQRT(w) + g_2 s + g_3, \tag{15}$$

where $G(\,.\,)$ gives the roughness reduction in QI unit with overlay thickness, w, measured in mm. The assumed values of the parameters appearing in this relationship are: $g_1 = 5.0$, $g_2 = 0.78$ and $g_3 = -66.0$. This implies that the roughness of a poor pavement of 100 QI is reduced to 53 QI after a 50 mm overlay. Using this relationship with definitions (9) and (10), $K(\,.\,)$ is derived as follows:

$$K(w,s) = k_1 SQRT(w) + k_2 s + k_3, \tag{16}$$

where $k_1 = 13.86$, $k_2 = 2.163$ and $k_3 = -183.0$.

The objective of this case study is to find the optimal timing and the thickness of overlays that minimize the net present value of life-cycle social costs for a given highway segment, the deterioration and improvement of which are described by relationships (14) and (15) (with the assumed parameter values).

User cost and agency cost functions in this case study are assumed linear of the following forms:

$$C(s) = c_1 s + c_2 \tag{17}$$

$$M(w) = m_1 w + m_2. \tag{18}$$

Assuming that the rate of increase in user costs per one QI unit is 0.00025 \$/veh/km/QI and the average annual traffic volume 4.0 million vehicles/year for both directions, the value of c_1 becomes 1000. It is not necessary to specify the value of c_2. The variable and fixed costs of an overlay, m_1 and m_2, will be assumed 3000 \$/mm/km and 150,000 \$/km, respectively.

Using these relationships, the current value Hamiltonian for the present problem is written as follows:

$$H = (c_1 s + c_2) + h(m_1 w + m_2) + z\{f_1 s - h(k_1 SQRT(w) + k_2 s + k_3)\}, \tag{19}$$

where z is the adjoint variable. The solution calls for the minimization of Hamiltonian (19) under the constraint that h satisfies (13). The values of h_1 and h_2 are assumed 0.1 and 1.0, respectively, meaning that overlay must be applied no less than or equal to once in ten years but no more than once a year. The social discount rate, r, is assumed to be 7%.

Note that Hamiltonian (19) is nonlinear with respect to one decision variable. A case where Hamiltonian is linear in both decision variables is discussed in Tsunokawa (1988).

3.2. Solving the optimal control problem

The aforementioned sufficient conditions are satisfied in this problem, since $C(s)$, $M(w)$ and $F(s)$ are linear and $G(w,s)$ is concave in s and w. Therefore, the optimal solution that minimizes the objective function can be obtained by solving first order conditions. The first order necessary conditions for this optimal control problem are written as follows:

$$ds/dt = H_z = (f_1 - k_2h)s - k_1h\ SQRT(w) - k_3h \tag{20}$$

$$dz/dt = rz - H_s = (r - f_1 + k_2h)z - c_1 \tag{21}$$

$$H_w = h\{m_1 - k_1z/2/SQRT(w)\} = 0 \tag{22}$$

$$H_h = m_1w + m_2 - z\{k_1SQRT(w) + k_2s + k_3\} \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \Leftrightarrow h = \left\{\begin{matrix} h_1 \\ h_s \\ h_2 \end{matrix}\right\}, \tag{23}$$

where h_s is the singular solution to be discussed below.

Conditions (20)–(23) are essentially a pair of first order differential equations, since w and h may be eliminated through relationships (22) and (23). Such a differential equation system generally requires two boundary conditions. One of them is given by initial condition (7). In an infinite time horizon problem, another boundary condition is typically replaced by the equilibrium assumption that the solution approaches a steady state where both time derivatives of s and z become zero—i.e. s and z converge to certain constant values. At a steady state, the control variables, h and w, also remain constant over time, defining the optimal steady state strategy. Thus, under this assumption, the solution to this problem specifies the optimal steady state strategy and the optimal transitional strategy for bringing the current road condition to that of the optimal steady state.

Relationship $H_h = 0$ along with (22) ($h \neq 0$) specifies a curve on $s - z$ plane, because w may be eliminated from these relationships to establish a relationship only between s and z. The s-z plane is divided by this curve into three subspaces, i.e. $H_h > 0$, $H_h < 0$ and $H_h = 0$ (the curve itself). Figure 2 illustrates the s-z plane with the locus of $H_h = 0$. Condition (23) states that the value of h is h_1, h_2 and h_s in these subspaces, respectively.

A singular control is that portion of an optimal control that does not explicitly depend on exogenously specified boundary values of the control variable. Thus, singular control, h_s, is determined from the condition that $H_h = 0$ holds for a finite period of time (Bryson and Ho, 1975). It can be shown that this may be obtained by solving the system of equations: $H_w = 0$, $H_h = 0$, $ds/dt = 0$ and $dz/dt = 0$. In this case, it should be noted that a singular control, if these exists any, defines a steady state, because it satisfies conditions, $ds/dt = 0$ and $dz/dt = 0$. In the present problem with the assumed parameter values, however, no singular control exists within the feasible range of h (13).

With h taking these constant values in subspaces, $H_h > 0$ and $H_h < 0$, conditions (20)–(22) represent a family of curves. Among these curves, we wish to identify the

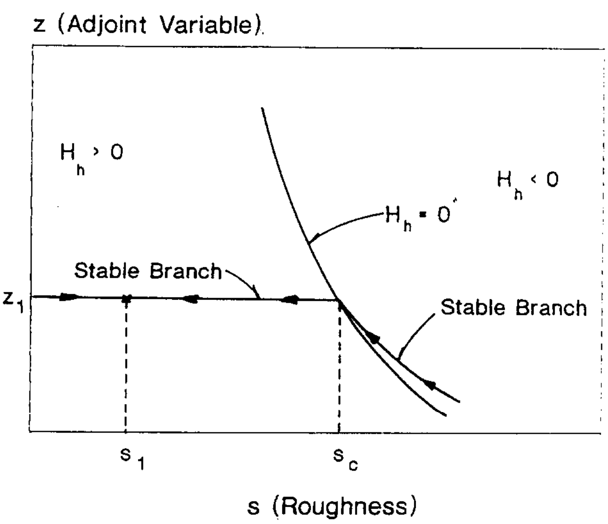

Fig. 2. Subdivision of s-z plane and stable branches.

curve that satisfies the aforementioned boundary conditions. First, the steady points corresponding to two values of h, h_1, and h_2, are found to be (4232, 28.5) and (458.1, 79.7), respectively, by solving the system of equations: $ds/dt = 0$ and $dz/dt = 0$ along with (22). Between these points, the former is the only feasible steady point in this problem satisfying condition (23). A necessary and sufficient condition for this steady point to represent an equilibrium point is

$$(f_1 - k_2h_1) < 0, \tag{24}$$

which is satisfied with the assumed parameter values. Otherwise, there is no equilibrium and thus no solution for this problem. This is a saddlepoint, as is generally the case with the discounted value autonomous problem (Kamien and Schwartz, 1971).

The remaining task is to find a curve (a stable branch) that leads to the equilibrium point found above and also satisfies the initial condition, (7). Because condition (24) implies that the parameter of z in relationship (21), $(r - f_1 + k_2h)$, is positive in subspace $H_h > 0$ (thus a saddlepoint), it follows that the stable branch in this subspace must be a horizontal line represented by:

$$z = z_1, \tag{25}$$

where $z_1 = c_1/(r - f_1 + k_2h_1) = 4232$. Of course this value of z coincides with the equilibrium z value. Along this stable branch, relationship (22) implies that w also takes a constant value, or $w = w_1 = 95.6$. Thus differential eqn (20) may be solved in a simple, closed form as:

$$s = (s_e - s_1) \exp (f_1 - k_2h_1)t + s_1, \tag{26}$$

where s_1 is the equilibrium roughness value, or $s_1 = 28.5$.

The stable branch (25) intersects with the locus of $H_h = 0$ at (69.7, 4232). The roughness value of this point of intersection is called critical roughness. In subspace $H_h < 0$, the stable branch is determined by numerically integrating the pair of differential equations starting from this point. Results of this numerical integration are shown in Fig. 3. The stable branches thus obtained for subspaces $H_h > 0$ and $H_h < 0$ are also illustrated in Fig. 2.

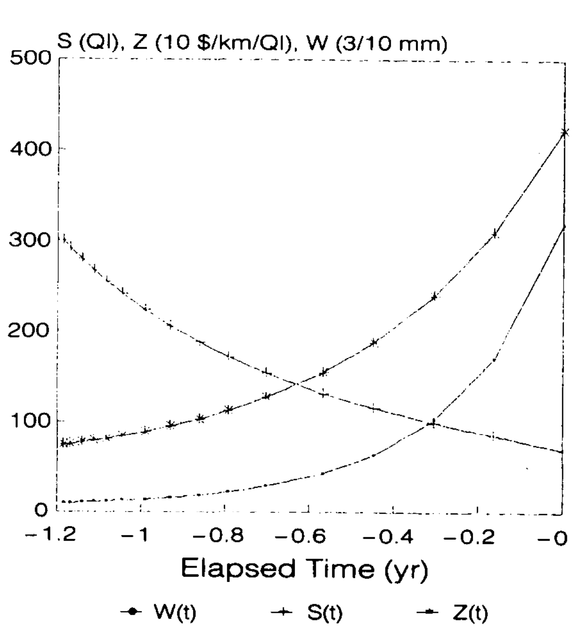

Fig. 3. Optimal trajectories of $S(t)$, $Z(t)$ and $W(t)$ in subspace $H_h < 0$.

For any given value of s_e, time trajectories of optimal maintenance strategy, $h(t)$, $w(t)$ and pavement roughness, $s(t)$, may be constructed using the results obtained above. Since exact correspondence between true initial roughness s_0 and effective initial roughness s_e is not yet known (to be discussed below), a trial value is assumed for s_e here. If the assumed value of s_e is not larger than the critical roughness, only the stable branch in subspace $H_h > 0$ is followed. Thus, $h(t)$ and $w(t)$ only take constant values, h_1 and w_1, respectively, and $s(t)$ is given by (26). If the assumed value of s_e is larger than the critical roughness, the stable branch in subspace $H_h < 0$ is first followed. Thus, until roughness is reduced to the critical roughness, $h(t)$ takes the value, h_2. The time it takes to reduce roughness from the assumed s_e value to the critical roughness is found by using Fig. 3 (or the results of the aforementioned numerical integration). The optimal value of $w(t)$ and the resulting value of $s(t)$ during this period are also found through this figure (or the numerical results). After roughness is reduced to the critical value, time trajectories are constructed in a similar manner to that used above. Namely, the optimal maintenance strategy for this period is (h_1, w_1), and the roughness follows a trajectory curve similar to (26). Figure 4 illustrates trajectory diagrams for selected values of s_e.

3.3. Determining the optimal maintenance strategies

Given time trajectories of instrumental decision variables, $h(t)$ and $w(t)$, the corresponding values of true decision variables, t_n and w_n, $n = 1, 2, \ldots$, can be obtained using definitional relationships (5) and (6). Figure 4 also shows the values of t_1, t_2, w_1, w_2 thus determined. Furthermore, given an overlay strategy, (t_n, w_n), $n = 1, 2, \ldots$, and an initial roughness, s_0, the sawtooth-like roughness trajectory curve can be constructed by sequentially using relationships (1) and (2).

It should be noted here that an effective initial roughness, s_e, and a true initial roughness, s_0, are related through relationship (11). This relationship, however, cannot be used to solve for s_e given s_0. This is because the values of t_1 and w_1 must be known to solve for s_e by using (11), but as is shown above, these values can only be found given a particular value of s_e. Therefore, to find the optimal maintenance strategy for a given initial roughness, s_0, an iterative procedure is required. This is an ordinary, straightforward one-dimensional search. Figure 5 shows the $s_e - s_0$ relationship of this case study. This figure shows that the s_e values appearing in Fig. 4 correspond to s_0 values of 20, 60

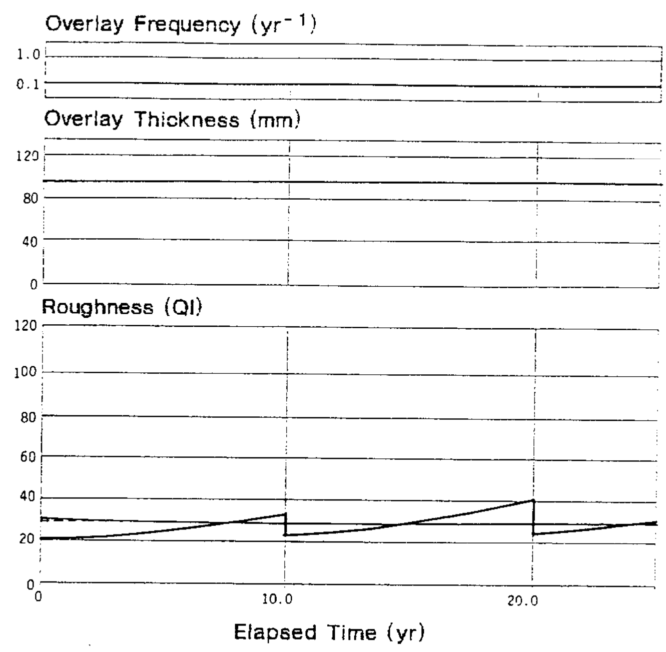

Fig. 4a. Optimal overlay strategy and roughness trajectory ($s_e = 29.3$, $s_0 = 20.0$).

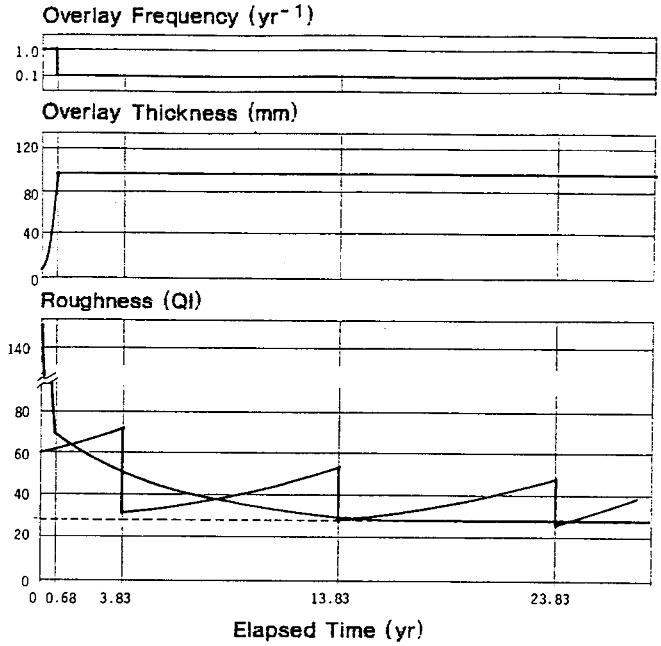

Fig. 4b. Optimal overlay strategy and roughness trajectory (s_e = 150.3, s_0 = 60.0).

and 120. The sawtooth curves shown in Fig. 4 are the roughness trajectories determined by these s_0 values and the maintenance strategies obtained above.

Once roughness trajectories are determined, objective function (4) can easily be evaluated. Table 1 summarizes the optimal maintenance strategies for a selected values of s_0 found through this method and the associated values of the objective function (first-order search is discussed below).

As discussed above, the optimal maintenance strategy in $h(t)$ and $w(t)$ found by solving the trend curve optimal control problem consists of two parts: optimal steady

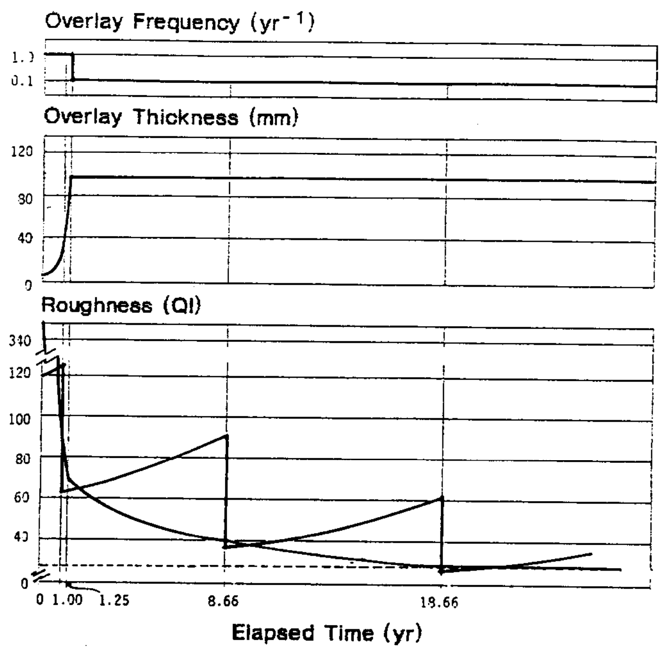

Fig. 4c. Optimal overlay strategy and roughness trajectory (s_e = 337.0, s_0 = 120.0).

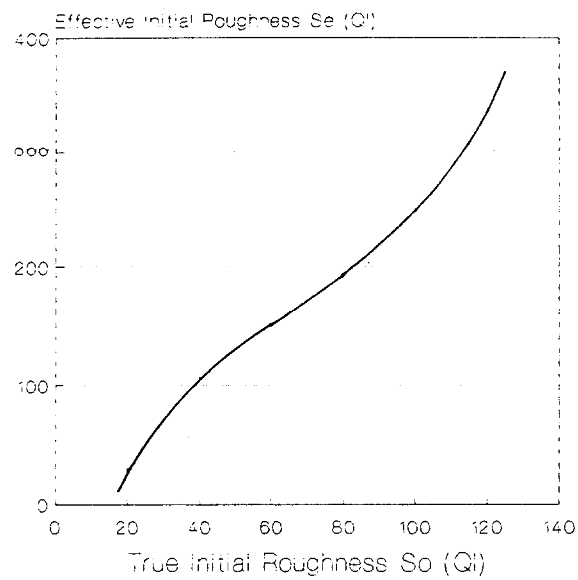

Fig. 5. Correspondence between true and effective initial roughness.

state strategy and optimal transitional strategy. This is illustrated in Fig. 4. It is interesting to note that the corresponding solutions for t_n and w_n also display such characteristics. Table 1 indicates that, for all initial roughness values, the steady state optimal strategy is to apply 96 mm overlay in every 10 years. Depending on the initial roughness, however, the optimal transitional strategies are to apply one or two overlays that are thinner than or as thick as those of the steady state strategy at shorter intervals (except for the case where the initial roughness is 20 QI, in which no transitional overlay is needed).

4. EVALUATION OF THE TREND CURVE METHOD

The optimum maintenance frequency, $h(t)$, and intensity, $w(t)$, obtained through the above procedure are the solution to the optimal control problem of minimizing expression (12) subject to constraints (7), (8) and (13). This problem, however, is an approximate

Table 1. Optimal overlay strategies by trend curve method and comparison with first order search optima

	Trend Curve Method						First Order Search			
	Optimal Overlays					Object Fun. J	Optimal Overlays		Object Fun. J	Comparison
	1st OL		2nd OL		....					
Initial Roughness s_0	t_1	w_1	t_2	w_2	....	(a)	dt	w	(b)	(a/b)
(QI)	(yr)	(mm)	(yr)	(mm)		(10**5 $/km)	(yr)	(mm)	(10**5 $/km)	
20.0	10.0	96	*		*	8.39	10.0	85	8.38	1.001
40.0	6.8	96	*		*	11.13	10.0	85	10.59	1.051
60.0	3.8	96	*		*	13.07	10.0	85	12.80	1.022
80.0	2.0	96	*		*	14.25	8.7	67	14.95	0.953
100.0	1.0	76	10.5	96	*	15.00	7.0	45	16.73	0.897
120.0	1.0	36	8.7	96	*	15.84	5.8	33	18.21	0.870

J = Net present value of the life-cycle costs (i.e., objective function); dt = Overlay interval of a first order search optimum; * = Overlay of the same thickness as that of the previous one to be applied ten years after the previous one.

formulation to the true optimization problem of minimizing expression (4) subject to constraints (1), (2) and (3) [and implicitly, 13]. Thus, the "optimal" solutions for (t_n, w_n), $n = 1, 2, \ldots$, obtained above are necessarily approximate. A natural question to be asked, then, is how good are these approximate, trend curve, solutions? It should be noted here that because the true solutions are not known, the goodness of trend curve solutions cannot be evaluated by comparison against them. Also, it is not feasible to compare the value of expression (4) evaluated at a trend curve solution with those evaluated at all possible combinations of t_n and w_n, $n = 1, 2, \ldots$, because there are an infinite number of such combinations. However, a systematic assessment of the proposed method is still possible by comparing the values of the objective function evaluated at trend curve solutions with those evaluated at a limited number of (t_n, w_n) combinations. In what follows, two such (t_n, w_n) combinations are considered.

4.1. Comparison with the first order search optima

One simple combination of (t_n, w_n), $n = 1, 2, \ldots$, would be that of a constant w_n and equal-interval t_n, such as $(t_n, w_n) = (5, 20), (10, 20), (15, 20), \ldots$ This is equivalent to assuming that $h(t)$ and $w(t)$ are constant over time. In this example, they are 0.2 and 20, respectively. The search for the best among such maintenance strategies will be called first order search. Figure 6a–6c illustrate the results of such first order search for selected values of s_0, where $h(t)$ is varied within the range of (13), and $w(t)$ between 30 and 150 mm (which is deemed to be a practical range for the overlay thickness). When initial roughness is 40 QI, Fig. 6a shows that, regardless of the overlay thickness, the value of the objective function monotonically decreases as overlay interval increases. Figure 6c, on the other hand, shows that, when the initial roughness is 120 QI, there are internal optimal overlay intervals with smaller overlay thicknesses. Although less conspicuous, Fig. 6b shows that, when initial roughness is 80 QI, the situation is similar to that of Fig. 6c.

More detailed results of the first order search analysis are summarized in Table 1, along with the results of the trend curve analysis. These show that when initial roughness is smaller than 80 QI, the objective function is minimized with the largest feasible overlay interval (i.e. 10 years). The optimal overlay thickness is the same (85 mm) for all these cases. When initial roughness is larger than or equal to 80 QI, however, more frequent

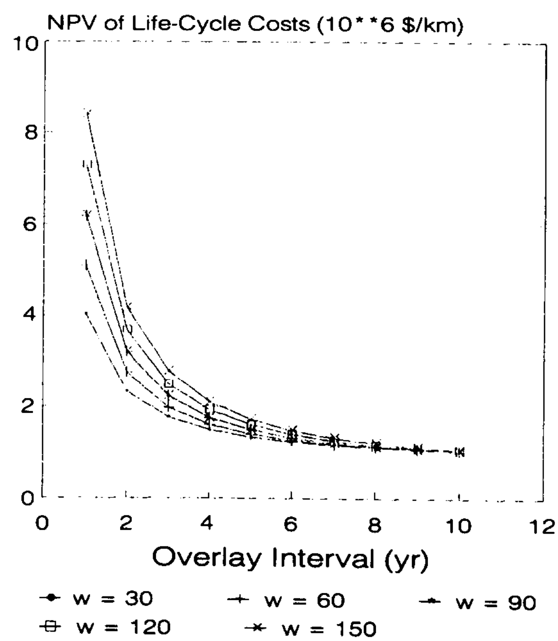

Fig. 6a. First order search (s_0 = 40.0).

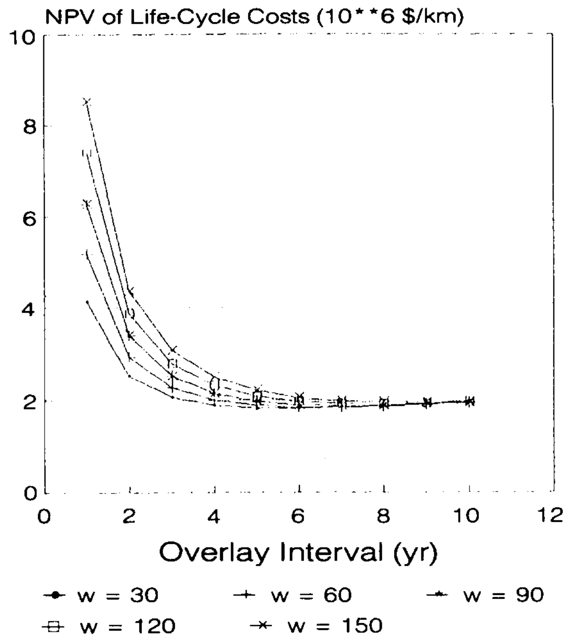

Fig. 6b. First order search (s_0 = 80.0).

but thinner overlays are optimal with the increased frequency and the decreased thickness as a function of the pavement roughness.

The values of the objective function associated with the first order search optima are computed and compared with those associated with the trend curve solutions. These results are listed in the last two columns of Table 1, which may be summarized as follows:

1. When the initial roughness is 20 QI, the values of the objective function associated with the trend curve solution and the first order search solution are virtually the same.

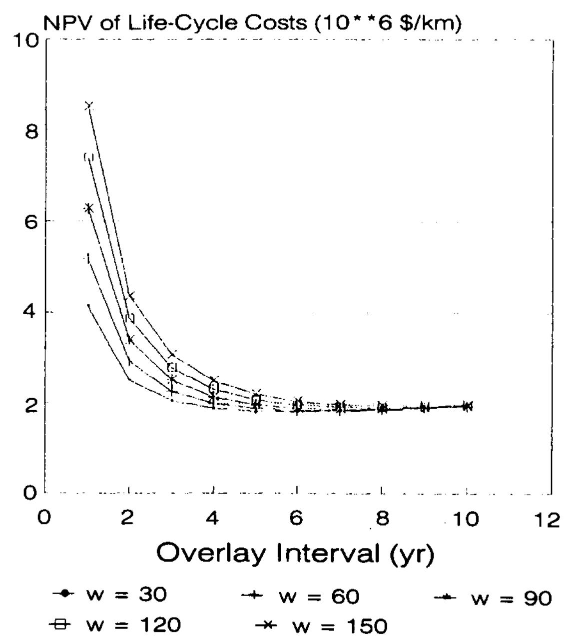

Fig. 6c. First order search (s_0 = 120.0).

2. In cases where the initial roughness values are 40 and 60 QI, the first order search solutions make the objective function slightly smaller than the trend curve solutions.
3. Where the initial roughness is greater than 60 QI, the trend curve solutions make the objective function smaller than the first order search solutions. The difference is larger for larger roughness values, and it is as large as 13% when initial roughness is 120 QI.

4.2. Comparison with the secondary search optima

Trend curve solutions consist of "optimal" strategies both in transition and in a steady state. To see whether the first order search solutions obtained above may further be improved, and if so, how the trend curve solutions compare with the improved solutions, transitional strategies are considered along with the first order search solutions in a similar manner to that implied by the trend curve solutions. That is, in this section, the restriction that every overlay be applied in a same thickness and at a same interval is relaxed to provide a good transition. To limit the space to be searched, however, only the first overlay is considered here for transition. Thus, according to this scheme, while the second and subsequent overlays are applied according to the optimal first order search schedule, t_1 and w_1 are changed systematically within the same ranges as above (i.e. (13) and $30 < w_1 < 150$) to see whether the first order search solutions can be further improved. In this case, the value of t_1 is varied within the range of [1, 10] to be consistent with condition (13). This procedure will be called secondary search, and the results are illustrated in Figs. 7a–7c for selected values of s_0.

Figure 7a shows that, when initial roughness is 40 QI, the objective function decreases monotonically as t_1 increases with all overlay thicknesses. This implies that no improvements over first order search can be made through this secondary search, because the first order optimum of t_1 is 10 (see Table 1). As shown in Fig. 7c, however, when initial roughness is 120 QI, the objective function monotonically increases as t_1 increases with all overlay thicknesses, implying that the first overlay should be applied as early as possible. Figure 7b shows that, when initial roughness is 80 QI, the situation is in between the other two: there exists an internal optimum value of t_1 for each overlay thickness that minimizes the objective function.

Table 2 summarizes the results of the secondary search analysis in more detail for the same initial roughness values used in Table 1. It shows more clearly the tendency for

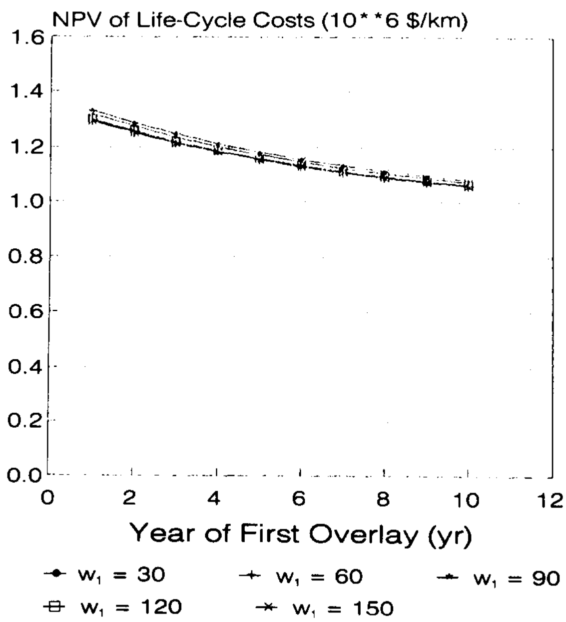

Fig. 7a. Secondary search (s_0 = 40.0).
F. O. S. Optimum: dt = 10.0, w = 84.8.

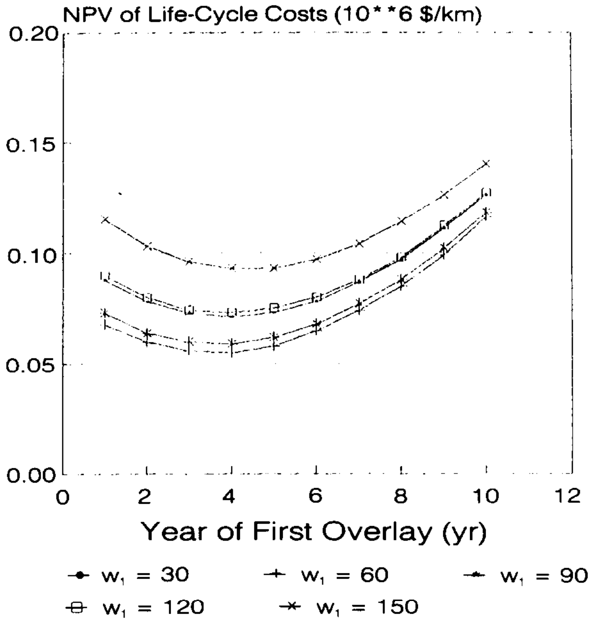

Fig. 7b. Secondary search (s_0 = 80.0).
F. O. S. Optimum: dt = 8.77, w = 67.6.

t_1 to become smaller as the initial roughness becomes larger. It also lists the optimal values of overlay thickness found through this search. It is interesting to note that the optimal values of w_1 found through this search (almost) perfectly coincide with the optimal values of w_n found through the first order search for all initial roughness values.

Table 2 also lists the values of the objective function associated with the secondary search optima and compares them with those associated with the trend curve solutions. Because no improvements can be made in the case where the initial roughness is 20 QI or

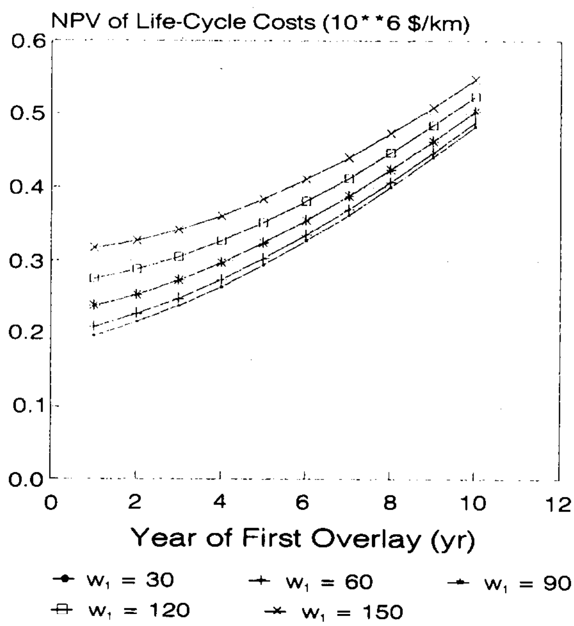

Fig. 7c. Secondary search (s_0 = 120.0).
F. O. S. Optimum: dt = 5.84, w = 32.6.

Table 2. Optimal overlay strategies by secondary search and comparison with trend curve optima

	Secondary Search						Trend Curve Method	
	Optimal Overlay							
	1st OL		2nd OL		.....	NPV of Life-Cycle Costs (obj. function)		Comparison
Initial Roughness s_0	t_1	w_1	t_2	w_2	....	(a)	(b)	(a/b)
(QI)	(yr)	(mm)	(yr)	(mm)		(10**5 $/km)	(10**5 $/km)	
20.0	10.0	85	*		*	8.38	8.39	1.001
40.0	10.0	85	*		*	10.59	11.13	1.051
60.0	8.4	85	18.4	85	*	12.77	13.07	1.024
80.0	3.5	67	12.3	67	*	14.53	14.25	0.980
100.0	1.0	45	8.0	45	*	15.91	15.00	0.943
120.0	1.0	33	6.8	33	*	16.98	15.84	0.933

* = Overlay of the same thickness as that of the previous one to be applied at the same interval as that between the previous one and the one before (or time zero if the previous one is the first overlay).

40 QI (i.e. the solutions are the same with the two searches), ratios between the values of the objective function are the same as those shown in Table 1. For the cases where the initial roughness is larger than 40 QI, secondary search does reduce the values of the objective function, although the extent of the improvement is not very large. Thus, the trend curve solutions are still better than the secondary search solutions in the cases where the initial roughness is larger than 60 QI. When the initial roughness is 120 QI, the trend curve solution is still better by as much as 7%.

5. CONCLUSIONS

A new, tractable solution procedure for optimizing the timing and intensity of major pavement maintenance actions was demonstrated using a simple case study in which the Hamiltonian was nonlinear in maintenance intensity. To evaluate the worthiness of the methodology, a systematic comparison was conducted between the solutions obtained through this method and those through alternative search procedures. The proposed method was found to provide solutions that were as good as or better than those of the first order search for a wide range of initial roughness values. The situation is not very different with the secondary search. In a few cases where initial roughness is small, better solutions were found through search procedures, but the difference was minimal. The proposed method is an approximate solution procedure based on the approximate representations of the basic relationships, but this study suggests that it gives good solutions.

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