BAYESIAN DECISION ANALYSIS
FOR PAVEMENT MANAGEMENT

by

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ABSTRACT

Ideally, pavement management is a process of sequential decisions on a network of pavement sections. The network is subjected to uncertainties arising from material variability, random traffic, and fluctuating environmental inputs. The pavement manager optimizes the whole system subject to resource constraints, and avoids suboptimization of sections. The optimization process accounts for the dynamics of the pavement system. In addition to objective data the manager seeks information from a number of experts, and considers selected social-political factors and also potential implementation difficulties.

Nine advanced schemes that have been developed for various pavement administrations are compared to the ideal. Although the schemes employ methods capable of handling the pavement system's complexities in isolation, not one can account for all complexities simultaneously.

Bayesian decision analysis with recent extensions is useful for attacking the problem at hand. The method prescribes that when a decision maker is faced with a choice in an uncertain situation, he should pick the alternative with the maximum expected utility.

To illustrate the potential of Bayesian decision analysis for pavement management, the author develops a Markov decision model for the operation of one pavement section. Consequences in each stage are evaluated by multi-attribute utility. The states are built of multiple pavement variables, such as strength, texture, roughness, etc. Group opinion and network optimization are recommended for future research, and decision analysis suggested as a promising way to attack these more complex problems.

This thesis emphasizes the utility part of decision analysis, while it modifies an existing approach to handle the probability part. A procedure is developed for Bayesian updating of Markov transition matrices.

where the prior distributions are of the beta class, and are based on surveys of pavement condition and on engineering judgement.

Preferences of six engineers are elicited and tested in a simulated decision situation. Multiattribute utility theory is a reasonable approximation of the elicited value judgements and provides an expedient analytical tool. The model is programmed in PL1 and an example problem is analysed by a computer.

Conclusions discuss the pavement maintenance problem from the decision analytical perspective. A revision is recommended of the widespread additive evaluation models from the standpoint of principles for rational choice. Those areas of decision theory which may be of interest to the pavement engineer, and to the civil engineer in general, are suggested for further study and monitoring.

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CHAPTER 1

INTRODUCTION

Roads and streets play a vital role in a nation's transportation system, and pavements are an important component of these networks. Management of pavements requires a set of practices that can help in achieving the best value possible for the available resources. A research pioneered by Canadian engineers has materialized in a set of such practices (1). The procedure makes an efficient use of existing knowledge and allows the replacing of component parts as better methods become available.

This thesis investigates whether the state of the art can be advanced by applying new methods that have recently emerged for decision analysis of complex systems. The study is carried out in three steps. First, the existing pavement management procedures are examined and compared with an idealized method to identify gaps that may require bridging. Second, a methodology that is capable of providing the bridge is reviewed. Third, it is applied to the optimization of maintenance of one pavement section.

The presentation follows the same steps. Chapter 2 outlines the pavement system and the existing management methods. The complexities inherent in the system are compared to the methods' capabilities in Chapter 3. The chapter concludes that more realistic models are required for decision making. Chapter 4 introduces the proposed methodology which is presented in more detail in the subsequent chapters. Chapter 8 presents an application of the theory to a hypothetical example. The thesis closes with conclusions and recommendations for future research.

CHAPTER 2

DIFFICULTIES OF PAVEMENT MANAGEMENT

Many road and street pavements that were built after the Second World War reach the end of the designed service life and now require upgrading. Newer facilities deteriorate faster under increasing traffic than initially predicted. Combined with a shortage of public funds, these facts pose a challenge for pavement management. Building roads will no doubt continue but greater efforts must be diverted to looking after the already built system.

The objectives that presently guide pavement management have to account for a growing public concern about the safety, the environmental, and the natural resource effects of transportation. The experience which accumulated from the design of new facilities can certainly be utilized to solve the problem that now faces pavement engineers but the decision situation is new and a fresh approach may be able to provide solutions that better respond to the present circumstances.

This chapter presents preliminaries of the search for a better mangement approach. The objectives of the pavement system are reviewed as they help to diagnose the problems in the existing management procedures. A sample of the most advanced schemes for pavement management is studied from a decision analysis point of view, and symptoms of the problem are discussed.

2.1. Pavement System and Objectives

Problem areas in management practices can be diagnosed through an investigation of objectives that guide the agency's decisions. This method helps to quickly reach a conclusion that overviews a problem. To define pavement management objectives, let's isolate a "pavement system" - a whole which comprises all factor that should concern a pavement manager. The pavement system becomes a subsystem when placed in a hierarchy under broader

systems. If it is to fit into the hierarchy, its objectives must be compatible with those of the higher-level systems.

SYSTEMS HIERARCHY	SYSTEMS OBJECTIVES
THE WORLD	Maximize well-being of humans (present and future generations) and well-being of nature.
↓ NATIONAL COMMUNITY	Maximize people's well being.
↓ PUBLIC FACILITY SYSTEMS	Economic efficiency, Employment, Regional redistribution of income, Quality of life, National unity Other?
↓ ROADS	Subject to resource, social and political constraints, optimize: Access, Capital, operation and maintenance costs, Quality of service Other?
↓ ROAD PAVEMENTS	Subject to resource, social and political constraints, optimize: Safety, comfort, expediency, economy for the user, Capital and maintenance costs of the administration, Quality of life effects on nonusers, Job creation in unemployment periods, Other?

FIGURE 1: PAVEMENT SUBSYSTEM AND ITS OBJECTIVES RELATIVE TO BROADER SYSTEMS

Figure 1 illustrates this idea. The pavement system falls into the class of public facility systems (2). These systems are supposed to meet a variety of public demands, economic and noneconomic. The economic objectives encompass economic efficiency, maximum employment and regional redistribution of income (2). The noneconomic objectives include quality of water, air, land and bioresources, aesthetics, public health, community preservation, resource conservation (4), as well as national unity and security, and people's prestige needs (2). Other objectives may emerge in the future as the public attitudes and the economic reality change.

Most of the objectives are accounted for in the regional planning but still there remains a multiplicity of values for consideration by the pavement manager. The following list is compiled from (1) and (3):

1. Economic efficiency for the agency.
2. Adequate load-carrying capacity.
3. Limited physical deterioration due to traffic and environment.
4. Economy, safety, serviceability for the user.
5. Good aesthetics.
6. Limited noise and air pollution during construction.
7. Access to other major transport facilities, other communities, etc.
8. Limited disruption of adjoining land use.
9. Use of local materials and labour.
10. Conservation of aggregates.
11. Prevent adverse changes in traffic volumes as they affect local economy.
12. Response to public attitudes and complaints.

Several points are evident from the list. It is dynamic as witnessed by recent addition of the energy conservation objective. Different administrations will put different emphasis on various objectives, according to the local reality. There is contextual overlap between some objectives. It may be very difficult to find a common evaluator for all objectives. Many objectives are conflicting and may require difficult trade-offs.

2.2. The Process of Making Pavement Decisions

Many agencies have established a set of procedures, here called "pavement management scheme", for aiding decisions. The schemes evolve from local experience and, consequently, similar problems are dealt with by different methods. The schemes are developed to various ends: individual projects or networks; new designs or upgrading of existing facilities; highway pavement or streets. Regardless the purpose of a scheme, options

always exist and an optimal decision is sought. This decision process consists of four phases:

1. Generate alternatives,
2. Predict pavement condition for each alternative,
3. Evaluate consequences of having a pavement in a condition,
4. Select an alternative which offers the "best" consequence.

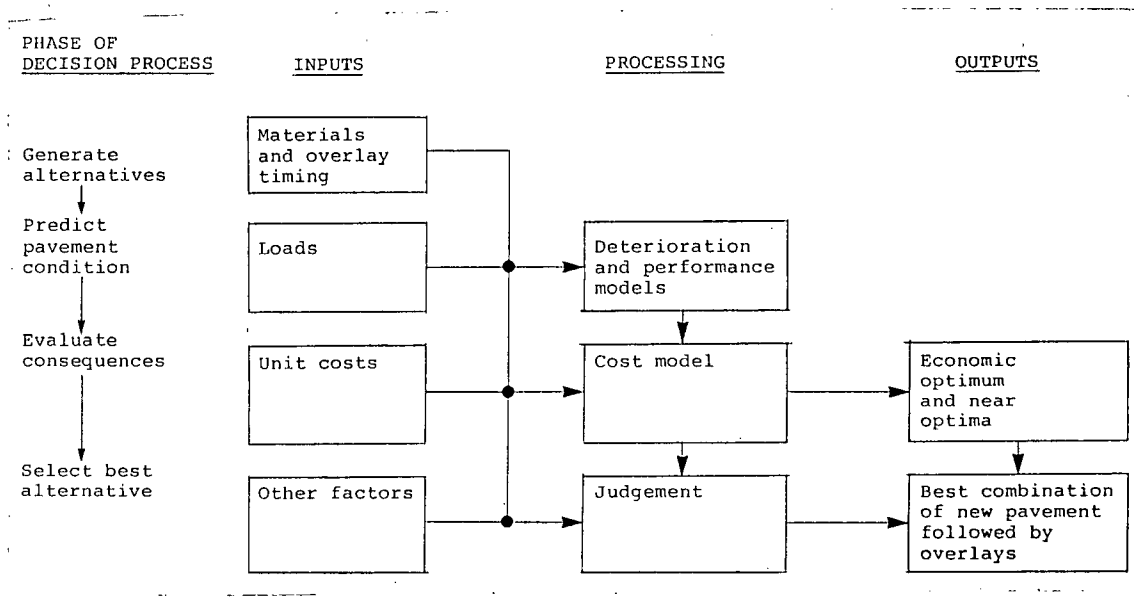


FIGURE 2: SELECTING OPTIMUM ALTERNATIVE FOR NEW PAVEMENT AND SUBSEQUENT OVERLAYS

As an example take the choice of the best alternative among several proposals of a new pavement and subsequent overlays (Figure 2). Pavement condition is predicted by the deterioration model from the materials, climate and load inputs. The condition variables are transformed in the performance model into a measure that is related to user costs. The present worth of construction, maintenance and salvage are also calculated. The total present worth of agency and user costs provide a basis for selection of the optimum. The optimum and the near-optima are evaluated by judgement to account for criteria other than economic, and the best alternative is identified.

2.3. Existing Pavement Management Schemes

Table 1 summarizes nine pavement management schemes according to the four phases of decision-making. It is evident that the four phase pattern applies to all schemes irrespective of the decision situation.

Phase 1 - Generation of Alternatives:

This phase varies with the purpose of a scheme. For A, B and C (the alternatives are a number of possible thickness designs combined with subsequent overlays. Schemes D and E consider maintenance options for one pavement section over time, while F, G, H and I do it for a network. The more complex formulations (D, E, F, G) have the alternatives constructed by the optimization model such as linear programming.

Phase 2 - Prediction of Pavement Condition:

A pavement deteriorates under traffic and climatic loads and its condition is described by unevenness, rutting, cracking, wear, etc. These variables are calculated from regression equations (5), are judged from periodic surveys (10, 13, 14), are predicted by Markov models (9, 11, 12), or are obtained by means of complex mechanistic formulae (7, 8). Significant progress has been made in the development of all approaches but the empirical methods are still favoured by the practising engineer. Simplicity, speed, readily available input data are the main advantages of these methods. They can also accommodate engineering judgement and theoretical results in addition to experimental data (9, 11, 12) - a feature not offered by the mechanistic methods. Once the deterioration variables are estimated, other models predict the measure of performance for pavements. These models usually relate pavement deterioration variables to a serviceability index (8, 9, 11, 12) although some management schemes (5, 7) proceed directly to this phase of analysis from the basic design input variables - pavement thickness or deflection, type of subgrade, traffic, age, climatic factors. The formulae are empirical, signifying that performance is difficult to

Purpose	Scheme and Reference	Generation of Alternatives	Prediction of Pavement Condition: Present State	Prediction of Pavement Condition: Future State	Evaluation of Consequences
ONE SECTION	A (5)	Single project construction/ overlay	"AASHO Interim Guide" regression: PSI = f (thickness, traffic, climate, subgrade) Stochastic variation of inputs		Total cost = administration cost + user cost
ONE SECTION	B (6, 7)	as A	Elastic layer analysis and field deflection. AASHO and Brampton Road Tests regression: RCI = f (deflection, traffic, climate, subgrade, age)		as A
ONE SECTION	C (8)	as A	Viscoelastic layer analysis	AASHO Road Test regression: PSI = f (roughness, rutting, cracking). Stochastic variation of inputs	as A
ONE SECTION	D (9)	Maintenance options for one pavement section	Subjective and objective evaluation of pavement	Two-variable state Markov model. Bayesian updating	as A Parametric user costs
ONE SECTION	E (11)	as D	as D	One-variable state Markov Model	as A
NETWORK	F (10)	Maintenance options for network scheduled next year	as D	Monitoring and judgement of any number of deterioration variables	Total potential gains of pavement rating over long period
NETWORK	G (12)	Maintenance options for urban pavement network	Subjective rating	as E Bayesian updating	as A
NETWORK	H (13)	as F	as D	Monitoring and judgement of PSI, deflection, cracking and skid resistance	Final Index = weighted four porformance variables
NETWORK	I (14)	as F	Yearly monitoring and judgement of: 1. Skid resistance 2. Maintenance cost, 3. Roughness, 4. Rutting, 5. Reflectivity, 6. Noise		Multidimensional quality index

Abbreviations: AASHO = American Association of State Highway Officials
PSI = Present Serviceability Index
RCI = Riding Comfort Index

Note: "Maintenance" stands for routine maintenance, surface treatments, overlaying and rebuilding with or without recycling.

TABLE 1: SUMMARY OF NINE PAVEMENT MANAGEMENT SCHEMES

Scheme	Selection of Optimum Alternative			Product	User
	Objective Function	Explicit Constraints	Supplementary Constraints		
A	Minimize present worth of total cost over finite time horizon	Time between overlays, capital for construction, pavement geometry, service	Accounted for by judgement of decision maker at higher level	List of economically optimal designs on project basis	Texas, Florida
B	as A	as A	as A	as A	Ontario
C	as A	as A	as A	as A	Massachusetts
D	as A Infinite time horizon	as A	as A	Optimal policy = f (pavement condition, weight of user costs)	California
E	as A	as A	as A	Optimal policy = f (pavement condition)	Washington
F	Maximize total gains of pavement rating over network and time	Available capital, supplies, equipment, labour, overhead, minimum quality standards	None	Maintenance priorities on network basis	Texas
G	Minimize user costs over network and time	Service + agency budget	as A	as F	City of Waterloo
H	Maximize Final Index	as A	as A	as F	Utah
I	Satisfy or exceed quality standards	Budget, minimum pavement quality	None	as F	Denmark

TABLE 1: (continued)

model mechanistically. One index is used to measure performance in most cases. However, the more comprehensive schemes employ multidimensional indices (9, 10, 13, 14) which can facilitate decision making through "specialization" of cost functions in areas of pavement safety, rideability, structural adequacy, etc.

To produce the history of performance, deterioration variables are generated for each year of the analysis period and substituted into the serviceability regression (5, 7, 8). A simpler approach employs the Markov model to describe the "motion of performance index through time" (9, 11, 12). Still others prefer to estimate performance histories from existing data and engineering judgement (10, 13). One system has no need for predictions, as decisions are made each year after collection of deterioration data in the field (14).

Phase 3 - Evaluation of Alternatives:

Most of the researchers studied measure the desirability of a project in money (5, 6, 8, 9, 11, 12). They minimize the total annual cost which equals the cost to the administration, plus the extra costs incurred by the pavement users as a result of pavement deterioration. The agency costs are calculated assuming unit material and labour prices and constant discount rate for the analysis period of 10 to 40 years. User costs are assumed to be a function of travelling speed, which is assumed to depend largely on the roughness of pavement. Proponents of the non-monetary valuation of alternatives employ a fairly comprehensive list of deterioration indices. The model of Lu (10) can accommodate any number of rating indices of pavement condition. The objective function is a sum of potential long-term gains of pavement rating resulting from one maintenance action. It is maximized in a zero-one linear programming model capable of optimizing an extensive road network.

Phase 4 - Optimization:

The optimization produces a list of the least expensive alternatives for a project, and the authors indicate that a decision maker at a higher level should subjectively re-examine the list with respect to technical, social or political factors that have not been quantified in the core models. Further decision analysis of the feasible designs on the pavement network basis is required to ascertain a true overall network optimization rather than project-by-project suboptimization.

A relatively simple procedure is employed by pragmatically oriented agencies. Deterioration indices for each pavement section are listed in decreasing order on a priority list (13). The Danes (14) have developed the simplest model. For each road section the indices of safety, maintenance economy, riding comfort, rutting, reflectivity and noise are listed as a multidimensional index in the above order of priority and compared to multidimensional quality standards, at which maintenance actions should be carried out. A priority list is produced, maintenance costs calculated and after comparing to the budget, a number of projects from the top of the list slated for next implementation.

2.4. Symptoms of the Problem

The approaches represented in Table 1 have a wide range of procedures, methodologies and mathematical models. The diversity of methods may demonstrate a dissatisfaction with the present state and a need for a better approach. It is not difficult to find symptoms which indicate that a problem does indeed exist. Comparison of hypothetical objectives outlined in Figure 1 to their practical counterparts reveals that economic efficiency and riding comfort are overemphasized at the expense of other important objectives.

The nonuser consequences such as excessive noise, exhaust and vibrations due to rough road surfaces and frequent maintenance operations (15), are neglected. One would hope that these objectives are contained in constraints but the formulation transfer the evaluation task onto an unidentified decision maker who uses subjective methods. In the public interest all objectives should receive due consideration, preferably by a quantitative method if the task is too complex for a human brain to handle.

The management schemes reviewed generally fail to meet even incomplete objectives. Several authors have recognized that safety must be included into the management systems (5, 6, 13, 15) but they fail to specify attributes suitable for measurement. Skid resistance is a popular measure, but reflectivity, width of paved shoulders and drainability also have a definite impact on road safety under dry and wet weather conditions for day and night (16).

The majority of management systems use the concept of serviceability. Although conceived for measuring performance of pavements in the broad area of road users' objectives (17), the original idea has been distorted. The serviceability indices that have emerged in practice do not include user safety and account only for the comfort of users. Numerous schemes erroneously relate accident costs exclusively to the pavement roughness which is the basis for the narrowly defined pavement serviceability. This is clearly a rather inadequate account as theory and experience pinpoint that pavement surface texture and drainability are the decisive factors for traffic safety. For analytical convenience, too many schemes also operate with scalar measures such as the present serviceability index, where multidimensional indices are needed to evaluate pavement performance.

To sum up, in spite of successful developments in measurement and evaluation of objectives other than economic, the management schemes cannot

include the findings because of modeling difficulties, and they stick by the concepts developed for now invalid decision situations.

The following chapter defines the characteristics of an idealized decision aid for pavement management. The ideal is compared to the sample schemes of Table 1. These steps lead to the formulation of the thesis problem in terms of the requirements for a better method.

CHAPTER 3

PROBLEM DEFINITION

3.1. Characteristics of the Pavement System

Theoretically, managing pavements is a process of making sequential decisions on a multiobjective network of sections subjected to variable material properties, uncertain traffic and fluctuating environmental inputs. Ideally, the pavement manager optimizes the whole network and avoids sub-optimization of sections. In addition to objective data, the manager seeks information from a number of experts and considers selected social-political factors and also potential implementation difficulties. Figure 3 summarizes characteristics of the pavement system.

Network Optimization:

Optimizing complex systems must often be approached by decomposition into simpler subsystems to facilitate analysis. The network of highways or streets is broken down into sections. When the information about individual sections is synthetized to arrive at a solution optimal for the network, it must describe interactions between sections or the system will be misrepresented and suboptimization may result.

Multiple Objectives:

The objectives are multiple, conflicting and change in time. They can be adequately described only by incommensurable attributes and some are intangible. It is often difficult for the pavement manager to identify the people who gain or lose in a particular situation, to derive the relevant objectives and to decide how they should be weighted. A common measure must be devised for all attributes so that trade-offs and evaluation of outcomes can be made. The evaluation method must produce a simple index, for even one pavement section requires a large number of alternatives to be considered for design

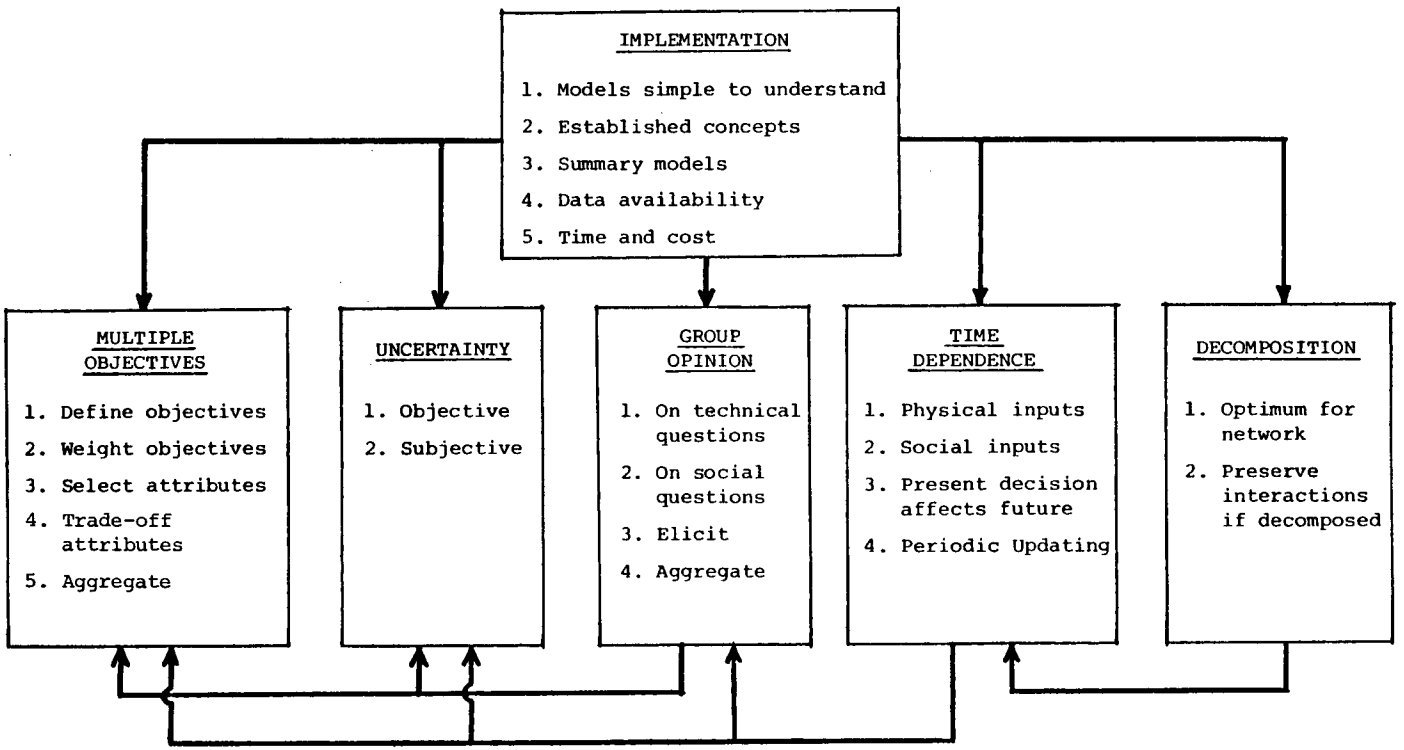


FIGURE 3: COMPLEXITIES IN THE PAVEMENT SYSTEM (BLOCK HEADINGS), INTERACTIONS (ARROWS) AND PROPERTIES DESIRED FROM AN IDEAL MANAGEMENT METHODOLOGY (CONTENTS OF BLOCKS).

and maintenance.

Uncertainty:

Pavement decisions are made in an atmosphere of uncertainty. That is, the outcome resulting from any action is seldom known with certainty due to random inputs into the pavement system. Traffic and climatic loads are unpredictable. Material properties change with construction technique, place and time. Mechanistic structural models cannot provide accurate information and condition surveys are subject to errors. Future costs, discount rates and budgets may at best be guessed. Objectives are dynamic and make the assessment of future desirability of actions extremely difficult.

Philosophic complications may arise in modeling because there are two types of uncertainties. One is associated with random inputs that occur in the system repeatedly and can be assessed by statistical methods. The other cannot be measured "objectively" and is expressed by the degree of belief an analyst has about its occurrence.

Group Opinion:

The physical pavement system requires technical expertise to operate and exists in a complex social-political environment. It follows that the decision process should ideally involve a number of technical experts as well as representatives of social groups that may be affected by the decision. The experts have different opinions about the probabilities of outcomes whereas the social groups have different preferences for consequences of actions.

In each case, the individual judgements must be elicited by the analyst and combined into a representative function. In situations requiring interpersonal comparisons, conflicts often occur. Experts stick by their opinion often solely for personal reasons. Social groups exert pressure on each other in order to achieve their own objectives.

Time Dependence:

The issue of sequential dependence of actions is typical of renewal situations and has been recognized in pavement maintenance. An action is executed periodically on a pavement section and affects the sequel. When decision analysis extends over longer periods, the time-dependent factors must be predicted. Prediction is a straightforward exercise for processes that are measurable and behave in a steady-state fashion. Traffic loads, available resources, material prices, however, may follow random cycles and socio-political components of the pavement system usually change unpredictably. The best known method of accounting for time variability in public systems is the periodic updating of management schemes.

Implementation:

The pavement management frameworks should be updated periodically not only to adapt to the changes in physical and socio-political factors but also to accommodate new findings in modeling and operation of the system. Changes in an organization lead to implementation problems. Departments are reorganized and restaffed, novel equipment is acquired, new procedures drawn. The employees must adapt to these changes but it takes time, money and effort before the managing organization can operate the pavement system effectively. Even the "best" methodology for pavement management will fall short of expectations if it does not account for the physical and behavioural constraints imposed on the organization.

3.2. Capabilities of the Existing Schemes

There is a large gap between the practical management schemes represented in Table 1 and the ideal method outlined in Figure 3.

Multiple Objectives:

The largest disparity between the models reviewed and that proposed is

in the area of system objectives. Most schemes consider riding comfort and structural condition because performance measures for these objectives are well established and convertible to money values. Other objectives are dealt with marginally because they cannot be treated by monetary cost models. When evaluation models do account for the nonmonetary factors, the sequential dependence of construction and maintenance activities is ignored as well as uncertainties (10,13,14).

No systematic procedure is reported which could identify the set of objectives important from the points of view of social groups affected by the decisions. The studied schemes leave an impression that a set of objectives is assumed uncritically as the point of departure. Analysts spend most time on developing the physical representation of the system.

Network Optimization:

With two exceptions (10, 12) the management schemes first optimize individual pavement sections and then arrange them in a priority list. Criteria for placing projects on the list include decision maker's judgement on intangibles in addition to dollar value and rating of measurable attributes. It is hard to believe, however, that the manager is able to process mentally a large number of intangible factors for interrelated projects. Moreover, the approach of suboptimizing some sections to match budget is less rational than optimizing the network, subject to budget constraint.

Uncertainty:

Uncertainty is accounted for by random input variables for which the distributions are assumed to be known (5,8). This approach is not realistic since sample sizes are usually too small to converge to the sizes of populations whose distributions are to be estimated. Neither is a deterministic approach (7,10,13,14) practical because variables with extremely narrow margins of uncertainty are rare, mechanistic models inaccurate and sampling

subject to error.

Bayesian formulation of the system inputs reflects the fact that the information about the system is imperfect, and it also allows for a gradual improvement of knowledge as new objective information and judgement are acquired. Many management methods make use of this concept (9,11,12) but only apply Bayesian inference principles to the performance model. Costs and benefits, however, may be even less predictable than pavement performance.

Group Opinion:

The advantages of combining objective information with subjective judgements are recognized by some management schemes but none suggests how to agglomerate the differing individual opinions. The failure occurs not only for uncertain technical inputs but also in preferences for outcomes and social interests. The desirable method elicits information in an atmosphere free of interpersonal pressures and objectively aggregates individual opinions.

Implementation:

Pavement management recognizes the implementation and adaptation problems. Computer software makes allowances for model and data updating. Some frameworks have been reorganized and computationally improved two or three times within the last decade (5,8,13). The objectives have not been revised, and many schemes tend to oversimplify while others remain extremely complex. The choice of too simple a model cannot be defended in times when many methods are available. On the other hand, some schemes (5, 8) employ complex mechanistic formulae to predict performance variables usually from laboratory data. These models are too idealized for a system that more appropriately requires simulation rather than modeling.

Time Dependence:

Sequential analysis or time dependence has been widely accepted in

pavement management (5,6,8,9,11,12) but is confined to actions expressable in monetary terms. The monetary values are then converted to a present worth. When other consequences are included the models fail to reflect the fact that an action is taken periodically and affects the sequel (10,13,14).

An algorithm is needed that can evaluate multiattribute outcomes occurring sequentially. A crucial property of the algorithm is to relate probable values generated at different dates to some common measure.

3.3. The Problem

The studied management schemes employ models that cannot handle complexities posed by the pavement system. The greatest difficulty arises because analysts cannot combine multiattribute evaluation, uncertainty, sequential dependence of actions and network optimization in one method.

Table 2 suggests that inclusion of a set of complexities excludes the complementary set from the model and methods may be grouped accordingly. Methods A,C,D,E account for uncertainties and sequential dependence of actions but can neither consider all objectives nor optimize the network. Method B can be added to this group if the performance model is made probabilistic as in (5). These methods may be modified for network optimization using a technique employed in (12) but multiple objectives still cannot be handled. Methods H and I account for all objectives but uncertainties, time dependence and network optimization are ignored. Method F is superior to H and I with respect to network optimization but rates low in the overall evaluation.

Analysts simplify the pavement system for analytical and practical convenience. Modeling conceals the most relevant aspects and hardly rational operation of the system may result. Simplifications must be a compromise between the implementation advantages and the loss of realism that may affect

Property of Ideal Method		Pavement Management Scheme from Table 1				
		A,C, D,E	B	G	H,I	F
All objectives included		No	No	No	Yes	Yes
Optimizes	a project	Yes	Yes	Yes	Yes	Yes
	network	No	No	Yes	No	Yes
Accounts for uncertainty of	physical state	Yes	No	Yes	No	No
	costs, benefits	No	No	No	No	No
Time dependence of actions		Yes	Yes	Yes	No	No
Aggregates group opinion		No	No	No	No	No

TABLE 2: CLASSIFICATION OF EXISTING MODELS FOR PAVEMENT MANAGEMENT ACCORDING TO THE COMPLEXITIES THAT CAN BE HANDLED.

the effectiveness of resource allocation to pavement maintenance.

The problem requires a search for a method that can unify all the complexities of pavement management - a task broad in scope. The scope of this thesis, however, is confined to the choice of optimum alternatives for the maintenance and rehabilitation of one pavement section when pavement condition data are available and preferences of pavement manager known.

This type of choice occurs at the project level of pavement management. However, individual projects must be compared at a higher level in order to satisfy the objectives of the whole pavement network. Network optimization and group opinion are not addressed in this thesis, because its objective is to investigate the applicability of decision analysis to the simpler case of only one section and one decision maker. It is hoped that understanding of the problem for one pavement section will help attack the more complex problem by decision analytical methods.

CHAPTER 4

PREVIEW OF THE PROPOSED APPROACH

Managing pavements may be viewed as an optimal control process. Choosing actions for the optimal control of a system may be accomplished by the Bayesian decision methodology. The basic model is explained in engineering terms in (18). Although this basic model supplies a convenient measure of the decision maker's preferences and prescribes a criterion for choosing the optimal action in uncertain decision situations, it must be extended to handle the complexities of the pavement system.

The following chapters describe the synthesis of some recent extensions in decision theory, and the Markov model of pavement behaviour, into a new optimization approach to pavement maintenance. To demonstrate the feasibility of the proposed model, it is programmed for the computer and used to analyze a hypothetical problem numerically.

4.1. Structure of Objectives and Attributes

To evaluate the alternative actions the manager must first know what objectives he would like to accomplish. A numerical evaluator, called an attribute, must be identified for each objective in order to measure the extent to which an objective is achieved.

The analyst must interact with the decision maker to produce a comprehensive list of objectives and attributes. A number of systematic methods have been proposed to this end. The Delphi method (20) may be used to elicit objectives and their interactions from social groups that will be affected by a decision. The structure of objectives may be identified from this information by a method called by Sage (19) "an

interpretive structural modeling".

For a simpler structure, literature surveys and insight into the system suffice to produce all relevant objectives. Experts are also useful in generating objectives but care must be exercised to eliminate bias.

It is important to include only those objectives that will have a significant bearing on the final selection, because evaluations become too expensive and time consuming as their scope increases. One practical technique that can help the analyst screen potential objectives, without consuming a great deal of effort, has been described and tested (21).

Each attribute must clearly convey the extent that the associated objective is achieved. An attribute to be useful must satisfy two requirements. First, for each alternative it must be possible to obtain a probability distribution over the possible levels of attribute. Second, the decision maker must be able to express his utility function for all values that the attribute can assume. The resources available for data gathering and processing, the time and the political constraints significantly affect the choice of attributes. The final set of attributes must quantify the consequences of actions and enable the decision maker to compare alternatives.

There is no universally applicable set of objectives; it depends on the socio-economic environment of the pavement system and can vary between countries and between regions in the same country. If the objectives and attributes are not adequate then the chosen actions may be nonoptimal and have some unwanted effects that were not accounted for in the evaluation.

4.2. Multiattribute Consequences

Utility theory quantifies a decision maker's judgement of the desirability of actions in probabilistic choice situations. A utility function associates a numerical index with each possible consequence. The indices

reflect the preferential ranking of the consequences.

Utility theory was originally developed to evaluate one-attribute consequences but has been recently extended to multiattribute outcomes (22). The theory resolves the difficulties traditionally encountered in the evaluation of intangible and incomparable attributes. The utility function synthesizes pieces of information relevant for optimal choice: the complete list of objectives, the decision maker's attitudes toward risk and the trade-offs between objectives.

Numerous applications of the multiattribute utility model have demonstrated that it is appropriate for many realistic problems and can be operationally verified in practice. Keeny and Raiffa (22) report on applications to air pollution control, emergency operations, nuclear power plant siting, selection of computer systems, consulting company policy, airport planning. Later applications include salmon management (23), water resource development (24), state energy policy (25). The number of attributes considered ranges from five to twelve.

4.3. Markov Decision Process with Expected Utility Criterion

A pavement may be viewed as a dynamic system that undergoes probabilistic deterioration. At every point in time it is in an observable condition that can be described by a number of variables jointly defining the state of the system. As the pavement deteriorates due to traffic and environmental forces, it makes transitions to states of worse condition. The pavement manager can, through a periodic choice of action, upgrade the pavement. Although he cannot totally control which transition will occur afterwards, he can affect its probability.

When viewed in a temporal setting, the system undergoes state transitions at equal intervals, termed stages. At each stage an action is taken

and a multiattribute consequence results. Compared to the basic Bayesian decision model there is a sequence of actions — called policy — and associated outcomes rather than a single action and consequence. In addition to the multiattribute utility function for every stage, the decision maker must define a temporal utility function to evaluate every sequence of outcomes within the planning horizon.

The management problem is to choose an optimal policy that maximizes the expected utility of the future stream of outcomes. The solution may be approached in two ways. The first approach neglects the temporal preferences of the decision maker. All policies are represented in a multistage decision tree. The consequences in a stream associated with a policy are added up at the tip of the decision path. Every path in the decision tree is then evaluated and the one with maximum expected utility identifies the optimal policy.

This approach does not account for the decision maker's attitudes toward risks that arise in sequential decision problems. It is not the sum of individual consequences but their timing, magnitude and probability that matter in a dynamic decision problem. More important, the solution is not yet computationally feasible. A small problem involving only 6 stages, 5 states and 2 actions requires the evaluation of more than one million decision paths.

The alternative approach employs a Markov decision model which is computationally efficient if applied with dynamic programming (26). The model is used extensively for the analysis of sequential decisions. Many authors have found it well suited for the optimal control of dynamic systems that undergo deterioration (27,28,29). Smith (30) incorporated the model into a pavement management framework. Gerchak and Waters (31) proposed Klein's formulation (28) for the economic analysis of road maintenance.

The application proposed in this thesis differs from previous work. The consequences — 'rewards' in Markov process terminology — cover all relevant objectives rather than just those expressable in money. They are evaluated by the utility function which can include the decision maker's position toward risks posed by the interactions between probabilities of states, magnitudes of consequences and timing of decisions.

The dynamics of pavement deterioration is assumed to be Markovian. The sequence of successive states of the system forms a discrete chain with transition probabilities. The Markov property of the chain implies that the system has no memory in the sense that once it is in a state, the past events which led it to enter the state will not influence the states it enters in the future.

Holbrook (32) has shown that the Markov assumption is applicable to the deterioration of concrete pavement joints. Three independent studies have demonstrated that Markovian deterioration is reasonable for flexible pavements — the theory, engineering judgement and experience agree (30,33,34).

Dynamic programming can solve the Markov decision problem having the expected temporal utility criterion, provided the objective function has a particular form. Objective functions decompose as required by the dynamic programming technique if they are separable (35). The separable utility functions must be screened to eliminate those forms which do not meet behavioural assumptions describing the decision maker's attitude towards risk in sequential choice problems (36).

It seems natural that decision makers responsible for the operation of public facilities be temporally risk averse. Oksman (37) has derived general results for two appealing forms of separable, risk averse utility functions that can be used in a Markov decision model. One of these functions is employed in the example of Chapter 8.

CHAPTER 5

THE BASIC MODEL AND LIMITATIONS

5.1. The Basic Bayesian Decision Model

Decision analytical methods are either descriptive or normative. Descriptive methods are used in psychology to study human behaviour in complex choice situations. Normative methods provide rules which when followed by the decision maker remove inconsistencies necessarily present in human decisions. According to Raiffa (38), Bayesian decision analysis is conditionally normative; if the decision maker wishes to follow certain logical principles of rational choice, then the procedure provides a means for assuring that his decisions are consistent with these principles.

The word Bayesian has two implications. Bayes suggested in the 18th century that subjective probability judgements should be combined with objective probabilities calculated from the relative frequencies. These principles are most relevant for engineering decisions (18), and have been applied to pavements (30). Bayes' philosophy has been extended to the evaluation phase of decisions. The decision maker must eventually use preferences, subjective by definition, in order to identify the best action for a socio-technical system.

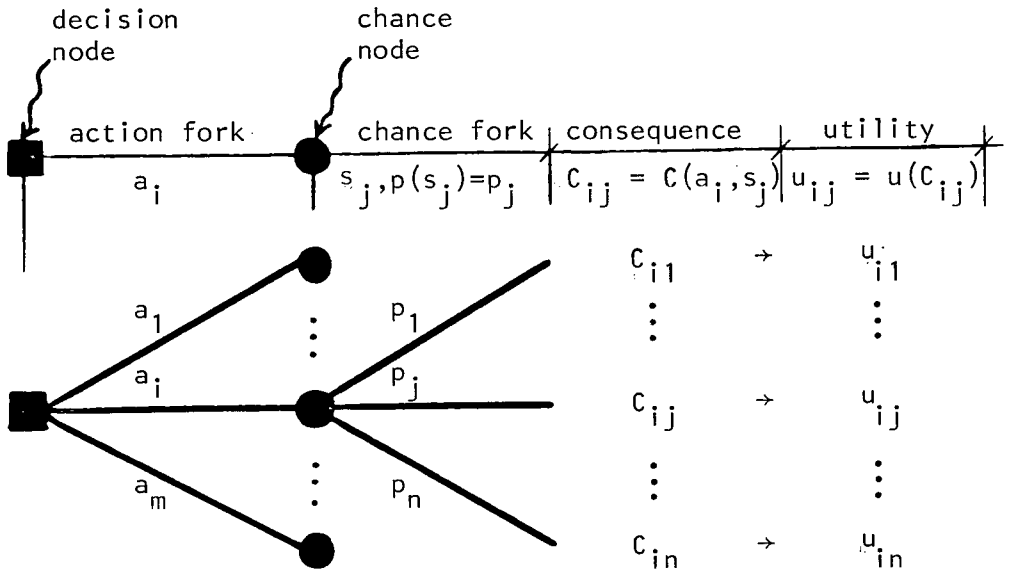
In practice, however, major emphasis is on the objective facts that cannot possibly capture all aspects of decisions. It is not uncommon that the decision maker is supplied with results of time-consuming and costly investigations and has only a limited time to add the subjective elements, often without the aid of any rational tools. For example, the user safety and environmental factors are excluded from analytically advanced pavement management schemes. The imbalance may be convenient for those who conceal value judgements. In the operation of engineering facilities, however, the

justification often seems out of place. Bayesian decision analysis has met acceptance in a broad range of applications. The success can undoubtedly be attributed to: the relative fidelity with which the real world is represented by Bayesian decision models, flexible data requirements and conceptual simplicity of the approach.

A decision model is represented in Figure 4. It consists of a set of possible actions, $a_1,\ldots,a_n$ and a set of possible states, $s_1,\ldots,s_m$ of the system under analysis. The states are uncertain and described by probability distributions. Probabilities can be estimated by engineering judgement, if necessary, and combined with objective estimates through Bayes' theorem. When action a_i is taken and true state s_j occurs, a consequence C_{ij} results. The desirability of C_{ij} for the decision maker is expressed by a numerical measure u_{ij} called utility. The system of action forks followed by the chance fork forms a structure called the decision tree.

When the states are uncertain the theory prescribes the choice of an action with the highest expected utility. The approach is straightforward. The major task of the decision analysis is to specify probabilities and utilities. Probability theory allows the manager to make maximum use of information available, while utility theory guarantees that the choice will relfect the decision maker's true preference.

The basic model is general and may be interpreted according to the analytical needs. No restriction is placed on the dimensionality of a consequence which can be described by a vector of attributes in multi-objective decision problems. Action a_i can denote a sequence of decisions over a period. State s_j becomes then a sequence of states that can occur with joint probability p_j.



Expected utility of action a_i:

$$E(u(a_i)) = \Sigma_j p_j u(C_{ij})$$

Best action:

$$a^* \leftrightarrow u(a^*) = \max_i E(u(a_i))$$

FIGURE 4: BASIC MODEL OF BAYESIAN DECISION ANALYSIS

5.2. Unidimensional Utility Theory

Before discussing utility theory, the notion of a lottery must be introduced. A lottery is a risky option which results in r prizes $C_1, C_2, \ldots, C_r$ with probabilities $p_1, p_2, \ldots, p_r$, respectively, where $\Sigma_{k=1}^{r} p_k = 1$. The general lottery is denoted $L = (p_1C_1, p_2C_2, \ldots, p_rC_r)$ and shown in Figure 5A. By convention the prizes are indexed from the most preferred $(C_1 = C^*)$ to the least preferred $(C_r = C^o)$. The branches emanating from a chance node of action a_i in Figure 4 may be thought of as a lottery facing the decision maker when a_i is chosen. The prizes of this lottery, C_{ij}, obtain with probabilities p_j.

The unidimensional utility theory presupposes that the lottery prizes can be described by a single attribute, so that no trade-offs are involved within a prize. The theory is founded on six assumptions (39).

Assumption 1:

An individual can order any two outcomes according to a preference or indifference relation. This relation is also transitive, so that if $C_1 \gtrsim C_2$ (which is read C_1 is indifferent or preferred to C_2) and $C_2 \gtrsim C_3$, then $C_1 \gtrsim C_3$.

Assumption 2:

When different chance mechanisms lead to two equal outcomes, the decision maker will be indifferent between the two. It means that he finds no utility in gambling.

A compound lottery may be reduced by the rules of probability and the decision maker will find both lotteries equally attractive, for example, L_1 and L_2 in Figure 5B.

Assumption 3:

Each prize C_i of a lottery is indifferent to so called basic lottery ℓ_i involving only the most preferred and least preferred prizes, C_1 and C_r,

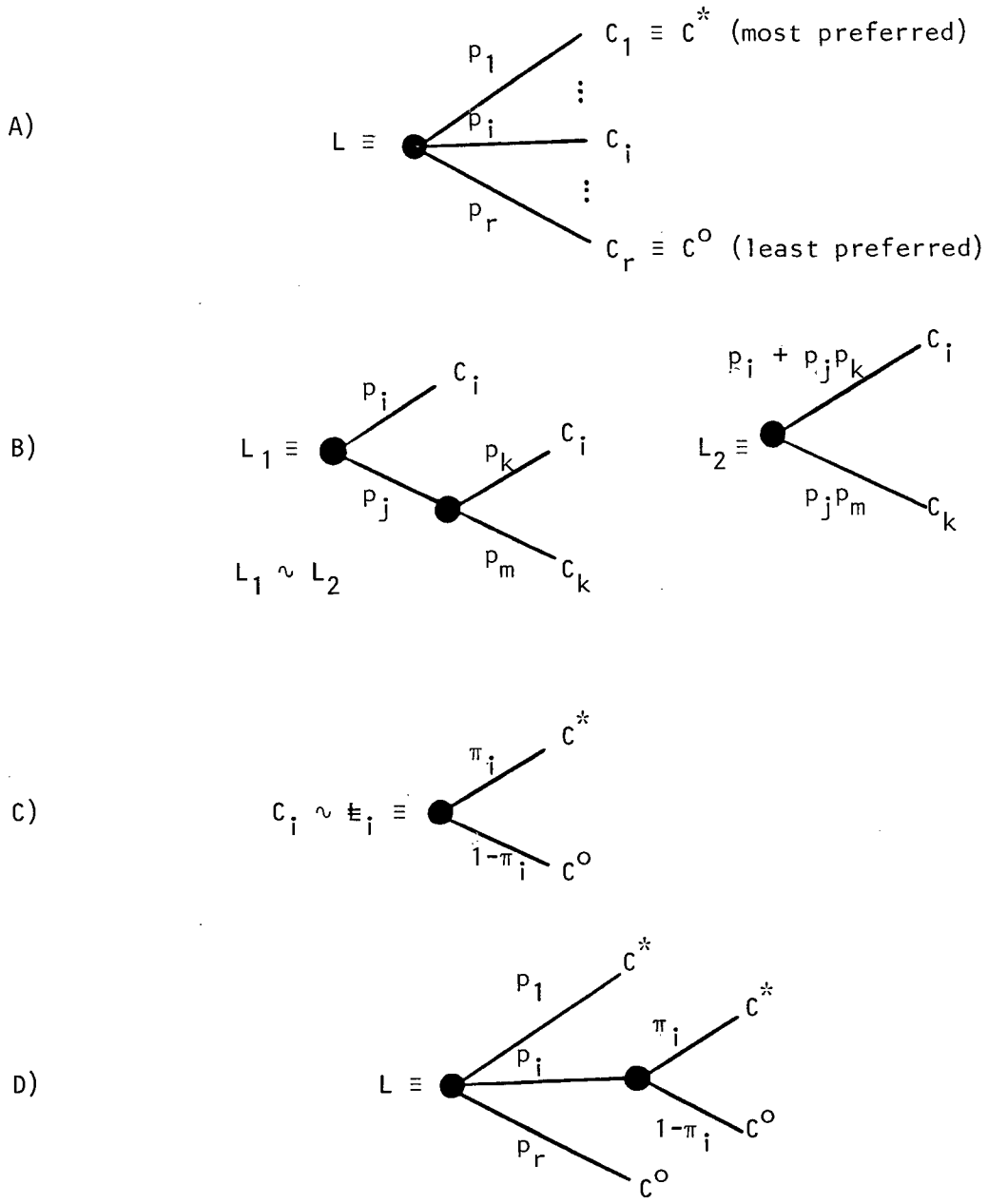


FIGURE 5: CALCULUS OF LOTTERIES

A. General lottery L with r possible prizes
B. Averaging out by Assumption 2
C. Basic lottery of Assumption 3
D. Substitution principle of Assumption 4

with probabilities π_i and $(1 - \pi_i)$, respectively (Figure 5C).

Assumption 4:

In any lottery, the prize C_i may be replaced by its equivalent basic lottery $\mathbb{E}_i$ constructed according to Assumption 3 without altering the preference for that lottery. Substitution of $\mathbb{E}_i$ for C_i creates a compound lottery (Figure 5D), which may be reduced to a simple lottery as described in Assumption 2.

Assumption 5:

The preference and indifference relations among lotteries are transitive. After C_i in L are replaced by $\mathbb{E}_i$ for all i, the lottery is expressed in terms of only C_1 and C_r. Applying Assumption 2 reduces such lottery to a simple two-outcome lottery and we get by Assumption 5 that L is indifferent (indicated by $\sim$) to the two-outcome lottery:

$$\begin{aligned} L = (p_1C_1,\ldots,p_iC_i,\ldots,p_rC_r) &\sim (p_1C_1,\ldots,p_i\mathbb{E}_i,\ldots,p_rC_r) \\ &\sim (p_1C_1,\ldots,p_i(\pi_iC^*,(1-\pi_i)C^o),\ldots,p_rC_r) \\ &\sim (\pi C^*,(1-\pi)C^o), \end{aligned}$$

where $\pi = p_1\pi_1 + p_2\pi_2 + \ldots + p_r\pi_r$.

For convenience, the two-outcome lottery is denoted (C^*,π,C^o).

Assumption 6:

For any two lotteries $L = (C^*,\pi,C^o)$ and $L' = (C^*,\pi',C^o)$, $L \gtrsim L'$ if and only if $\pi \geq \pi'$. We can determine which of two lotteries is preferred by applying Assumptions 1 through 5 and comparing the resultant π and π'.

The task of comparing two actions, possibly described by very complex branches in the decision tree, is thus reduced to comparing two numbers. If action a_1 results in $L = (C^*,\pi,C^o)$ and action a_2 in $L' = (C^*,\pi',C^o)$, then a_1 is preferred to a_2, if and only if π is greater than π'. It is exactly the property required from the utility function, hence π is such a function. Noting that outcomes C_1 and C_r are equivalent to

$(C^*, 1.0, C^o)$ and $(C^*, 0.0, C^o)$, respectively, it follows that $u(C^*) = 1.0$ and $u(C^o) = 0.0$.

For the intermediate values, C_i, utility is elicited from the decision maker. He is asked what probability π_i would make the basic lottery (Figure 5C) indifferent to C_i. This subjective probability is equal to $u(C_i)$ and C_i is called the <u>certainty equivalent</u> of the lottery. An alternative method might be easier for some decision makers. Instead of fixing C_i and judging π_i, he estimates the certainty equivalent of a lottery with fixed probabilities (Figure 6 top).

Since the expected utility of this lottery is $0.5\ u(C^*) + 0.5\ u(C^o) = 0.5 \cdot 1 + 0.5 \cdot 0 = 0.5$, the estimated certainty equivalent has utility 0.5 and is denoted $C_{.50}$. Next, we elicit the certainty equivalents for two other lotteries (Figure 6).

Since the expected utilities of these lotteries equal 0.75 and 0.25 respectively, and correspond to the certainty equivalents $C_{.75}$ and $C_{.25}$, we have five points to fair the utility function through (Figure 6 bottom).

Once the utility function is defined on all outcomes, the expected utility of the whole lottery resulting from action a_i may be calculated

$$u(a_i) = \Sigma_j p_j \pi_j = \Sigma_j p_j u(C_{ij}).$$

By Assumption 6 the best action, a^*, maximizes the expected utility,

$$u(a^*) = \max_i u(a_i).$$

Because utility is an indicator of preference rather than an absolute measure, it is unique up to a positive linear transformation of the form

$$u'(a_i) = a + bu(a_i),\ b > 0.$$

The two functions u and u' are <u>strategically equivalent</u> and will rank consequences in identical order. An analogy in physics is temperature measurement by different scales. Various temperatures can be ordered from

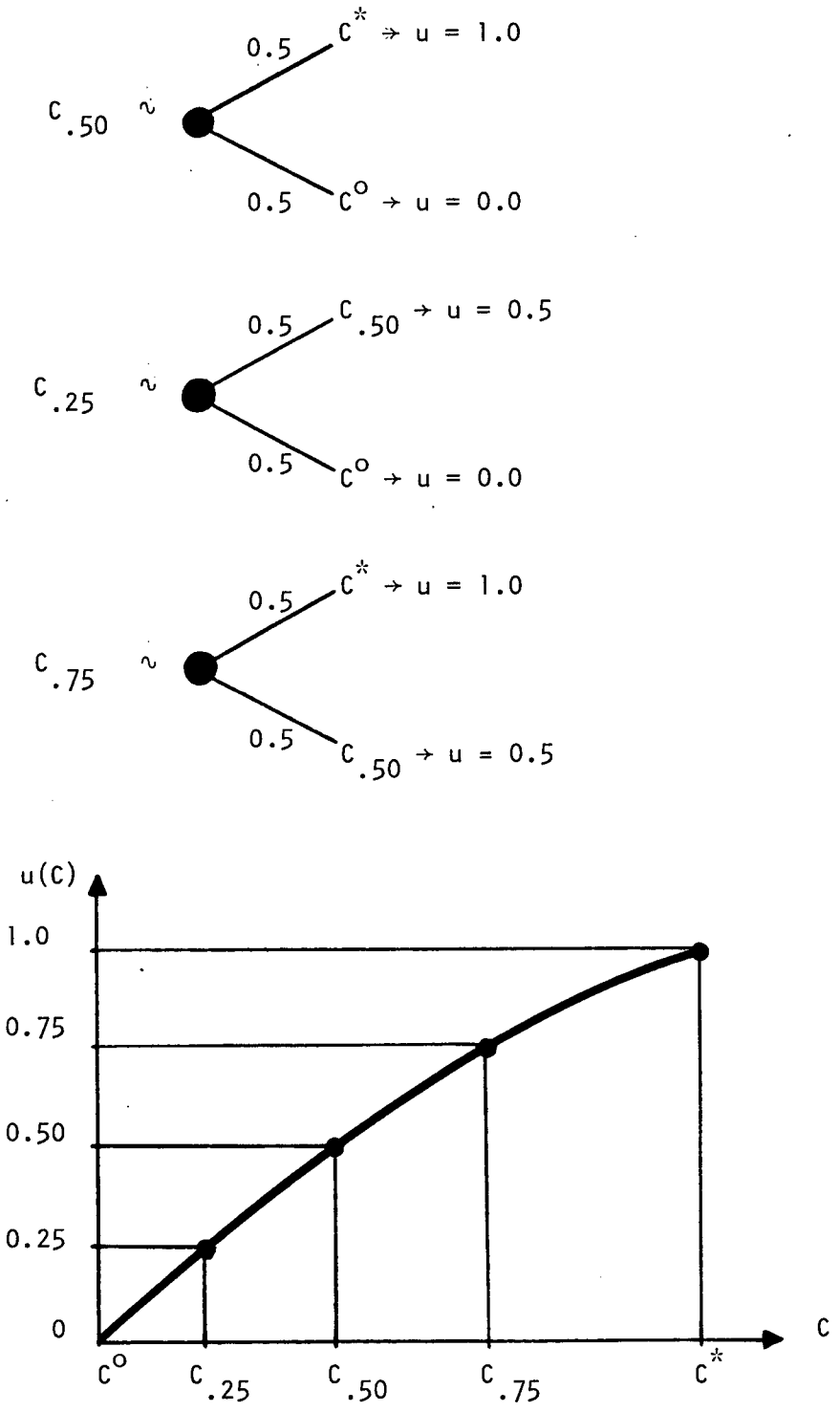


FIGURE 6. FIVE-POINT ASSESSMENT OF UTILITY FUNCTION

highest to lowest identically by the Celsius, Fahrenheit and Kelvin scales.

5.3. Risk Attitudes

Decision maker's risk attitudes have important implications on the functional form of the utility function. Consider first the monotonically increasing functions. When the certainty equivalent of any lottery is equal to the expected outcome of the lottery, the decision maker is risk neutral. When the certainty equivalent is less than the expected outcome, he is said to be risk averse because he will settle for less than the expected outcome of the lottery in order to avoid risk taking.

The difference between the expected outcome and the certainty equivalent is called the risk premium, since this is what he is willing to pay in order to avoid risk. If the decision maker's certainty equivalent exceeds the expected outcome, he is risk prone (risk seeking). The risk premium is negative for a risk seeking person, that is, the person must be paid to give up gambling. Equivalently, such an individual is willing to pay in order to seek risk.

For monotonically decreasing functions, the risk premium is defined as the certainty equivalent minus the expected outcome so that the sign of risk premium is the same as for increasing functions.

An alternative definition of risk attitudes does not depend on whether the utility function is monotonically increasing or decreasing. Risk averse individuals prefer the certainty equivalent to any lottery that has an expected prize equal to the sure option. Risk prone persons prefer the lottery over the certainty equivalent and risk neutral individuals are indifferent between the two options. This definition provides means for a quick check of the shape of utility function.

Risk aversion implies concave utility functions. Consider a risk

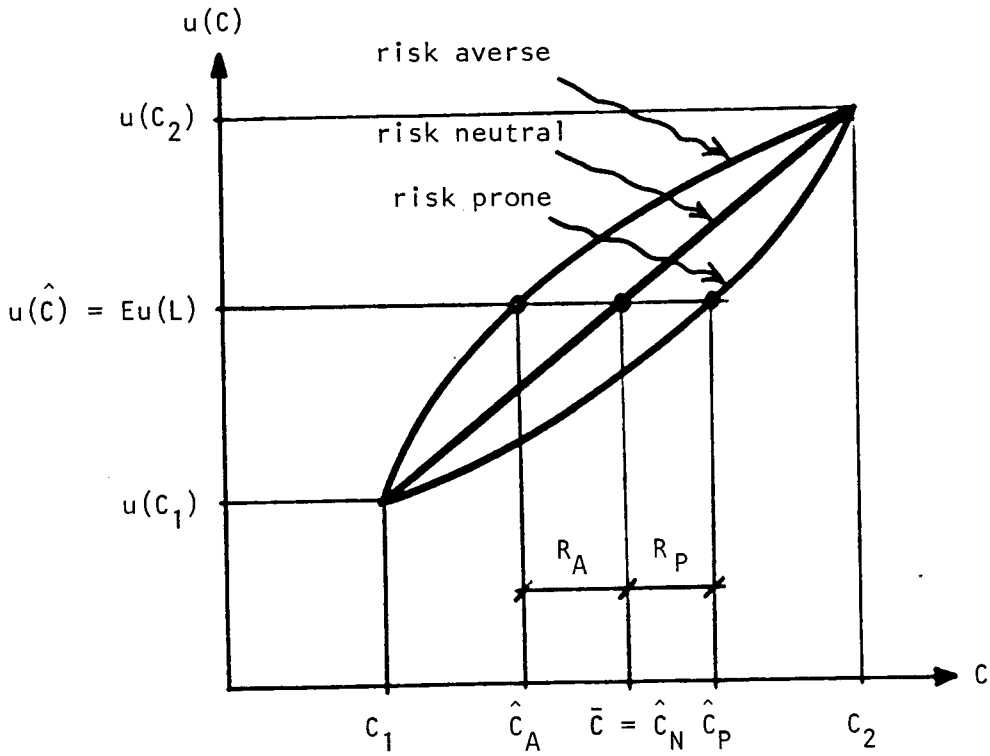
neutral piece of a utility function in Figure 7. The certainty equivalent $\hat{C}$ coincides with the expected outcome, $\bar{C} = 0.5C_2 + 0.5C_1$, of the lottery shown on the left and the risk premium is zero. When the risk premium increases by moving the certainty equivalent to position $\hat{C}_A$, the function becomes concave, since the utility of $\hat{C}_A$ is the same as that of $\hat{C}_N$, and the same as that of the lottery. By moving the certainty equivalent to $\hat{C}_P$, one can demonstrate that risk proneness implies convex utility functions.

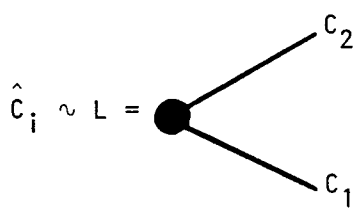
The fact that a particular risk attitude determines the shape of a utility function has been exploited to devise an analytical measure of risk behaviour. The local risk aversion at C is defined for monotonically increasing function by

$$r(C) = -u''(C)/u'(C) \tag{5.1}$$

where u'' and u' are the second and the first derivatives of the utility function with respect to C. The shape of the function is measured by u'', which is negative, zero or positive when the utility function is concave, straight line or convex, respectively. Division by u' ensures that strategically equivalent functions have the same risk properties. For monotonically decreasing functions the positive of u'' rather than the negative is substituted into the formula.

When r(C) is positive (negative, zero) for all C, then u(C) is concave (convex, straight line) everywhere and the decision maker is risk averse (prone, neutral). When r(C) is an increasing (constant, decreasing) function of C, then the risk attitude is additionally qualified. For example, a positive r decreasing in C indicates a decreasingly risk averse utility function. The risk aversion function is thus a convenient analytical tool for selecting a utility function that conforms to the observed risk behaviour of the decision maker.





$\hat{C}_i$ = certainty equivalent of L

i = averse (A), neutral (N), prone (P)

$\bar{C}$ = expected consequence = $0.5C_1 + 0.5C_2$

R_i = risk premium = $\bar{C} - \hat{C}_i$

$R_A > 0$

$R_N = 0$

$R_P < 0$

FIGURE 7: FORMS OF A MONOTONICALLY INCREASING UTILITY FUNCTION AND DECISION MAKER'S RISK ATTITUDES

5.4. Limitations of the Basic Model

The opponents of Bayesian methodology have scrutinized the assumptions of utility theory. Behavioural experiments demonstrated that human subjects do not always exhibit transitive preferences (Assumption 1). Since utility theory is normative, the decision maker may remove his inconsistencies if he wishes to satisfy principles of rational choice. Subjects usually improve when intransitivity is pointed out to them (38).

The writer hypothesizes that other assumptions cannot be defended by using the normative versus descriptive rationale. The procedure of assigning utility to an outcome (Assumption 3) is supposed to capture the decision maker's attitude toward the risks that he faces in his decisions. However, the gambling situation of the basic lottery (Figure 5C) simulates a simple risk situation involving two fixed greatly divergent outcomes and cannot possibly capture preferences for the actual situation being analyzed. When utility is substituted for outcomes (Assumption 4) and a decision branch is reduced to a simple lottery (Assumption 5), Assumption 2 cannot hold. Details about the actual risks are lost and the simple lottery cannot be regarded equivalent to the actual decision problem.

The hypothesis is supported by the apparent inconsistency of behaviour in the Allais paradox discussed by Raiffa (38). The subjects are inconsistent with the axioms of utility theory because they view the total choice situation with interrelations between risks rather than decompose the analysis into independent lotteries.

Raiffa approaches resolution of the Allais paradox from the normative standpoint. He demands from subjects that Assumption 2 holds, which is humanly impossible. As an alternative, he suggests including the utility from gambling into the consequences. While the last trick will work for simple decision trees, it will not be useful, in the present writer's

opinion, in the analysis of complex problems. In extensive decision trees it is difficult, if not impossible, to realize the scale of gambling involved and to assess the risk of every individual path.

Similar objections are made by Dreyfus and Dreyfus (40). They conclude that decision analysis is appropriate for some problems. There are, for a risk-averse person, disutilities associated with choosing to risk a risk, actually taking a risk and feeling guilty after loosing a gamble if it could have been avoided. The analysis is applicable where each possible decision-outcome path can be individually valued without considerations and ethical judgements of this kind. Also amenable are those complex problems where the expertise which is based on intuition about total situations represented by a large branch of the decision tree cannot be obtained.

The authors (40) further suggest that there may be a connection between the application of decision analysis as the model of rationality and the tendency towards a progressively impersonal approach to the complex decisions now facing our society. Their conclusion seems pessimistic considering that the tool has not yet been used extensively. Moreover, it is not certain whether alternative methods offer advantages over decision analysis. It is doubtful that they can handle the complexities of the emerging problems. Decision analysis appears the best tool available and will remain so until a better approach is developed.

The preceding argument may by itself defend the applicability of Bayesian decision model to the analysis of the pavement system, but a further two follow from the characteristic features of this system. In one-of-a-time decision problems — for example, whether to build a dam or not — the disutility of the worst outcome, even if it were to occur with very small probability, affects the decision maker's perception of the risks of a lottery and upsets Assumption 2. Because pavement decisions are

repeatable both over the network and time, maximizing expected utility appears to be a valid decision criterion for the operation of pavements. The consequences reoccur and the decision maker may tolerate the less desirable outcomes as long as the accumulated effect is not worse than that of an infrequent but more desirable consequence.

The third argument follows from the condition regarded by Dreyfus and Dreyfus (40) necessary for application of decision analysis. It is unlikely that the pavement manager can develop expertise based on intuition about the sequential operation of the multiobjective system and, by the lack of contenders, Bayesian decision analysis is a valid tool for this analysis.

The unidimensional utility model cannot be applied in the present form to the pavement problem. When the outcomes are represented by attribute vectors rather than single numbers, it is no longer possible to define utility function because humans cannot order complex outcomes as required by Assumption 1. Moreover, maintenance decisions form a sequence of actions taken periodically and the decision maker is interested in finding the "best" sequence for a given analysis period. Because the time dimension is added to the decision tree, the utility function must encode preferences for the probabilistic outcomes distributed over the period. That sequence of actions is optimal which maximizes the expected utility defined on the sequence of outcomes.

Further pragmatic difficulties with the concept of utility arise when the decision affects several interest groups and the decision maker must transform their preferences into his own. This problem is discussed by Benjamin and Cornell (18) in the context of professional decisions by civil engineers. In public facility systems, such as the pavement system, some mechanisms must be provided, which will ascertain representation of utility functions of all interest groups in the decision making process.

These complications have been resolved by theoreticians. The extensions require additional assumptions about the system and it is mandatory to check them before the extended model is applied. The chapters that follow explore those approaches to multiattribute consequences and to sequential problems which are both compatible with the Bayesian decision scheme. Group opinion can be accommodated by suitable models and is not considered.

CHAPTER 6

MULTIATTRIBUTE UTILITY THEORY AND CONTENDERS

6.1.Multiple Objective Decision Methods

Of multiple objectives, uncertainty, and time dependence in decision analytical problems the first complexity has historically absorbed most attention by theoreticians probably because it is the simplest. Interaction between the first two complexities can be effectively handled only by the multiattribute utility theory, while some specializations of this theory can also attack problems having all three complexities and interactions. For an analyst who wishes to explicitly preserve these complexities in the structure of a decision analytical model, multiattribute utility applied in the Bayesian decision analysis has no contenders. The other models are worth mentioning, though, for it is often possible to structure a problem so that it is amenable to the simpler methods.

Decision analytical metholodogy may be classified as normative or descriptive. Normative analysis explains what rules a decision maker should follow in order to improve his decisions. Normative methods, such as the Bayesian decision theory, help the decision maker remove inconsistencies from the decision making process. Descriptive analysis explains how actually decisions are made by individuals who are not aided by the normative methods. Descriptive methods are of interest mainly to the psychologists who attempt to predict human behaviour in complex choice situations.

Research in evaluation of multiobjective choices has been growing exponentially in the last twenty years and a large number of methods resulted. Many authors have attempted to overview the field. MacCrimmon's (41) overview seems most comprehensive, although the multiattribute utility theory is underrepresented. Only some of what he labels "Weighting Methods" are

compatible with the Bayesian decision analysis. "Sequential Elimination Methods" such as elimination by aspects, dominance and comparison to standards are often used for filtering admissible alternatives in the Bayesian analysis. "Mathematical Programming Methods" that require decision maker's interaction with the program offer features deemed interesting for the normative assessment of complex utilities.

Descriptive methods in utility theory infer preferences of decision makers from past decision situations or by direct questioning. These methods are based on multiple regression techniques and relate intuitive judgements of multiattribute alternatives to values of the attributes.

These methods require that the decision maker evaluate complex situations repetitively. Regression models to be meaningful should be based on a large number of observations. Psychologists have found that human ability to perform repetitive mental tasks without error is limited and judgement of situations involving more than five or six variables at a time is subject to random errors. For a reasonable number of attributes, however, descriptive utility techniques could be used as an intermediate step in the assessment of normative utilities.

Interactive multiobjective programming methods are also interesting for the normative evaluation of utilities. These methods do not require explicit knowledge of the decision maker's utility functions, but uses it on an interactive basis by asking certain questions of him. The fact that these methods actively involve the decision maker may facilitate their implementation and acceptance of the solutions.

The process "teaches" the decision maker to recognize what he considers as good solutions and important objectives. To teach the decision maker, a number of calculation and decision-making cycles are required. During the decision-making phase of each cycle, the decision maker examines

the results of the calculation phase and develops new insights and information about his objectives. This information in turn is used in the calculation phase of the next cycle, thereby providing a guide for the search of the best compromise.

The interactive methods have these behavioural assumptions in common:

1. An individual cannot express his preferences analytically. Rather, they will implicitly guide his search for the "best" solution.
2. The decision maker first concentrates on what he feels are the more significant aspects of the alternatives and then proceeds to the less important.
3. Exploration of the feasible alternatives is a learning process and the information so generated feeds back and changes the decision maker's preferences.

If these assumptions hold, the assessment of multiattribute utilities will possibly benefit from adopting the feedback feature of the interactive multiobjective programming.

6.2. Approaches to Multiattribute Utility

Let $X \equiv X_1 * X_2 * \ldots * X_p$ be an outcome space, where X_i is the ith attribute. A specific outcome is designated by x or $(x_1, x_2, \ldots, x_p)$, where x_i designates a specific amount of X_i for $i = 1,2,\ldots,p$. The symbol $x^* \equiv (x_1^*, x_2^*, \ldots, x_p^*)$ designates the most desirable outcome and $x^o \equiv (x_1^o, x_2^o, \ldots, x_p^o)$ designates the least desirable.

We are interested in assessing the utility function over X, denoted $u(x_1, x_2, \ldots, x_p)$ or $u(x)$. We might have the following attribute vector in a pavement decision problem resulting from action a_i and state s_j:

$$x(i,j) = (\$\ \text{cost}(i,j),\ \text{comfort score of users } (i,j), \ldots,$$
$$\text{person-days of maintenance employment } (i,j))$$
$$= (x_1(i,j),\ x_2(i,j), \ldots, x_p(i,j))$$

Expectation of multiattribute utility taken with respect to probabilities of states

$$E(u(x(i,j))) = \Sigma_j p_j u(x(i,j)) \tag{6.1}$$

provides an appropriate criterion for choosing between alternative actions. Action a_i is at least as desirable as a_k if and only if

$$E(u(x(i,j))) \geq E(u(x(k,j))) \tag{6.2}$$

The utility function is assumed to be monotonically increasing in each X_i and bounded. Utilities of the extreme outcomes are conventionally set

$$u(x^o) = 0 \text{ and } u(x^*) = 1$$

but any positive linear transformation

$$u'(x) = a + b\ u(x),\ b > 0 \tag{6.3}$$

preserves the properties of utility function.

A number of approaches have been developed to calculate multiattribute utilities (42). If the set of possible outcomes is small and attributes only a few, it may be reasonable to assign a utility to each of these directly. The procedure is similar to that used in elicitation of unidimensional utility (Figure 5C). For each x a probability π is assessed such that the outcome is indifferent to the basic lottery yielding either x^* with probability π or x^o with probability $(1 - \pi)$. The direct approach cannot be applied to large problems.

Indirect approaches reduce the dimension of the attribute vector through transformation. One method first trades all attributes off into one and then ascribes a utility function to this single attribute. Application of this procedure is limited since in real-world systems not all attributes are convertible into one.

Another method transforms riskless additive utility functions into risky utility functions. To calculate the riskless function, scores are assigned to each attribute and summed up for each outcome. This type of utility function is used in transportation for evaluation of alternative plans by the additive rating scale procedures. To convert the riskless utility function to a form suitable for use in probabilistic choices, several outcomes are selected which cover the full range of the scores and which are easy to compare with the two extreme outcomes x^* and x^o. Utilities are next assigned to these selected outcomes by the direct method and plotted against the rating scores for the selected outcomes. The utility function is thus defined on a single attribute "rating score".

The main drawback of this method is that it does require the direct assessments of utility of several multiattribute outcomes. When the number of attributes is large, these judgements place a very heavy load on the decision maker. He must simultaneously consider two probabilities and all of the attributes of three outcomes: x, x^* and x^o. Sometimes it may be possible to reduce the dimension of x by trading off some of the attributes. Implementation may be relatively easy because the additive rating scale technique is well established in transportation planning.

6.3. The Decomposition Approach

Other indirect approaches break down the multiattribute assessment procedure into parts. We would like to find a scalar valued function of a simple form such that

$$u(x) = u(x_1, x_2, \ldots, x_p) = f(u_1(x_1), u_2(x_2), \ldots, u_p(x_p))$$

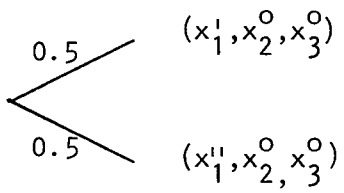
where u_i is a utility function of attribute x_i, for $i = 1,2,\ldots,p$. If this decomposition is possible the assessment of the p-dimensional utility function then resolves into an assessment of p unidimensional utility

functions. The theoreticians have focussed on the identification of assumptions which permit one to find such a simple form of the multi-attribute utility function.

The assumptions concern the concepts of utility independence and preferential independence (22). Their role in multiattribute utility theory is similar to that of statistic independence in multivariate probability theory. The more independence conditions exist among attributes, the simpler the function f and consequently the easier the assessment.

Attribute vector Y, where $Y \subseteq X$, is <u>utility independent</u> of its complement $\bar{Y}$ (shortly: Y is utility independent) if the conditional preference order for lotteries involving only changes in the levels of attributes in Y does not depend on the levels at which the attributes in $\bar{Y}$ are held fixed.

Let $X = (X_1, X_2, X_3)$ and $Y = (X_1)$ and consider the following lottery:



where attribute X_1 has level x_1' in one prize and x_1'' in the other, but the remaining attributes are held at the lowest levels x_2^o and x_3^o in both prizes. Suppose the decision maker feels that his certainty equivalent for the above lottery is $\hat{x}_1$. If the levels of X_2 and X_3 are next changed and the decision maker still assigns $\hat{x}_1$ to the new lottery, his preference order for the lotteries does not depend on the levels of X_2 and X_3. If this is valid for any pair of levels of X_1, the conditional utility function over X_1, given that $\bar{X}_1$ is fixed at any value, will be a positive linear transformation of the conditional utility function over X_1, given that $\bar{X}_1$ is fixed at any other value.

Since utility functions are unique up to a positive linear

transformation, utility independence implies that

$$u(y,\bar{y}) = f(\bar{y}) + g(\bar{y})u(y,\bar{y}'), \text{ for all } y \text{ and } \bar{y},$$

where f and g correspond to a and b in (6.3) and $\bar{y}'$ is arbitrarily chosen specific amount of $\bar{Y}$. Functions f and g depend, in general, on the specific value of $\bar{y}'$ but not on the variable y. If the conventional scaling is chosen and $\bar{y}'$ set equal to the worst value $\bar{y}^o$, then

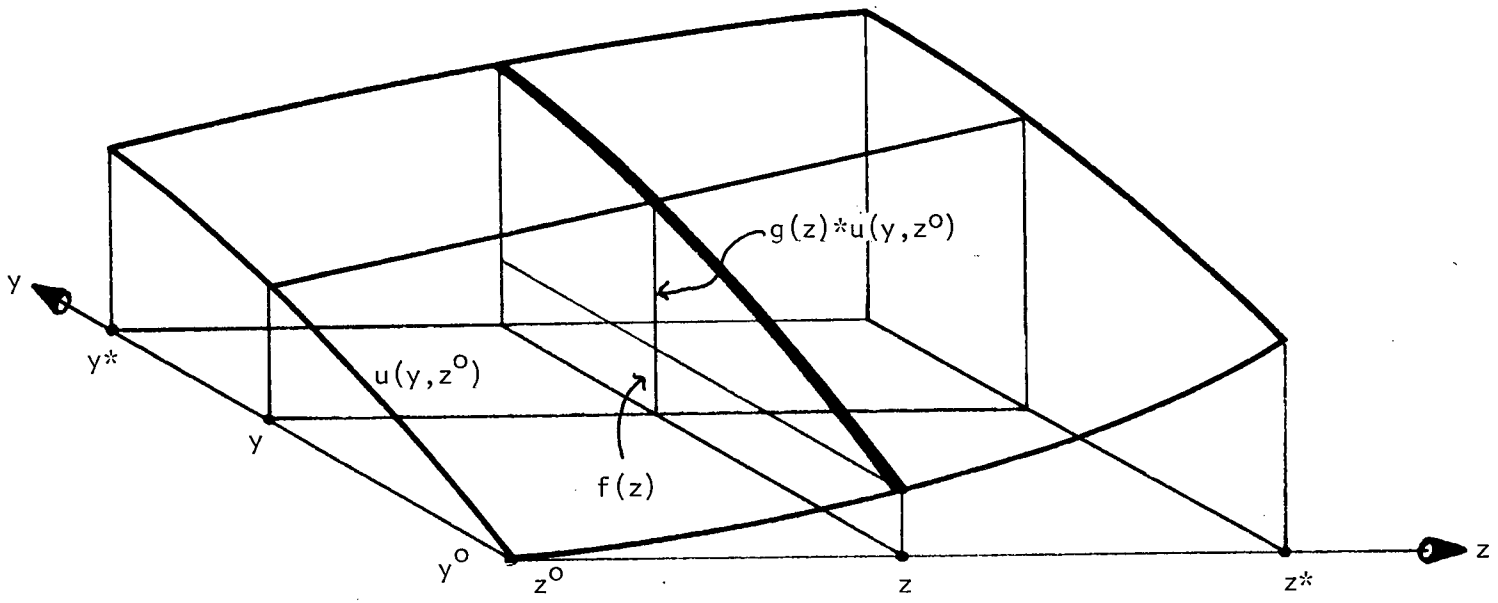
$$f(\bar{y}) = u(y^o,\bar{y})$$

and $$u(y,\bar{y}) = u(y^o,\bar{y}) + g(\bar{y})\ u(y,\bar{y}^o) \tag{6.4}$$

Utility independence is interpreted in Figure 8 for two attributes, Y and $\bar{Y} \equiv Z$. The thickest line represents equation (6.4) and the curvilinear surface is obtained by calculating (6.4) for all $\bar{y}$. If other cuts are made through the surface perpendicular to z axis, the intersections will all have the same general shape as the thick line. Their elevation above the yz plane will vary with z but they will rank y identically for all z. Figure 8 demonstrates that utility independence is not reflexive. When Y is utility independent of Z the reverse is <u>not</u> automatically true. The utility curve over Z for y fixed at y^o has a risk prone shape. This shape changes to risk averse at y^*. Functions $u(y,\cdot)$ have thus different general shape depending on y and are not strategically equivalent in this case.

To make the verification of utility independence assumptions practical, the set Y should contain only one attribute. However, when X_i are utility independent of $\bar{X}_i$, $i = 1,2,\ldots,p$, the functional form of f is rather complex and $2^p - 2$ constants must be assessed. Even for a problem with five attributes the number of assessments becomes prohibitively high.

The simplest form of the multiattribute utility function with only p constants to be assessed results when the attributes are <u>mutually utility</u> independent, that is when every subset of $(X_1,X_2,\ldots,X_p)$ is utility independent of its complement. The condition cannot be used, though, since



Y is utility independent of Z

Z is <u>not</u> utility independent of Y

FIGURE 8: GEOMETRIC INTERPRETATION OF UTILITY INDEPENDENCE FOR TWO ATTRIBUTES, WHERE $z \equiv \bar{y}$ OF EQUATION (6.4).

it is not possible for humans to consistently judge lotteries having variable levels in more than about two attributes.

Fortunately, several weaker conditions imply mutual utility independence and drastically reduce the complexity of required verifications (Theorem 6.2 in (22)). One condition exploits preferential independence as well as utility independence and has been found particularly useful in applications.

Preferential independence is defined in a manner similar to utility independence, except that it concerns preferences for deterministic outcomes rather than lotteries. Attribute vector Y is <u>preferentially independent</u> of $\bar{Y}$ (shortly: Y is preferentially independent) if the preference order of outcomes involving only changes in the levels in Y does not depend on the levels at which attributes in $\bar{Y}$ are held fixed. The property is reflexive.

For example, if (X_1,X_2) is preferentially independent of X_3, then if the outcome (x_1',x_2',x_3') is preferred to (x_1'',x_2'',x_3) for one value of X_3 it must be preferred for all possible x_3. This is equivalent to assuming that trade-offs under certainty between various amounts of X_1 and X_2 do not depend on X_3. The preferential independence assumption implies that the indifference curves over $X_1 \times X_2$ are the same regardless of the value of X_3.

6.4. The Multiattribute Utility Model

A suitable combination of preferential and utility independence conditions that are equivalent to the mutual utility independence are exploited in the most important result in the decomposition approach.

If the number of attributes $p \geq 3$ and, for some X_i, (X_i,X_j) is preferentially independent for all $j \neq i$, and X_i is utility independent, then either

$$u(x) = \Sigma_{i=1}^{p} k_i \, u_i(x_i), \text{ if } \Sigma_i k_i = 1 \tag{6.5a}$$

or

$$1 + k\, u(x) = \Pi_{i=1}^{p}(1 + kk_i\, u_i(x_i)), \text{ if } \Sigma_i k_i \neq 1 \qquad (6.5b)$$

where u and u_i are utility functions scaled from 0 to 1; $k_i = u(x_i^*, \bar{x}_i^o)$ are scaling constants with $0 \leq k_i < 1$; $0 < k < \infty$ if $\Sigma\, k_i < 1$; and $-1 < k < 0$ if $\Sigma\, k_i > 1$. Keeney and Raiffa (22) describe in detail the assessment and the consistency checks of the scaling constants k_i and k.

There is an interesting interpretation of the interaction factor k in (6.5b). Consider two lotteries with two-attribute prizes,

where $y' > y''$, $z' > z''$ and preferences are increasing in both Y and Z. The marginal probability distributions of y and z are identical in both lotteries but their joint distributions are not. L_1 offers a 50-50 chance at either the better (y',z') or the worse prize (y'',z''). L_2 is less dramatic; one cannot win the better prize but cannot loose as much as in L_1, either.

To prefer L_2 over L_1 implies that it is important to do well in terms of at least one attribute. Thus Y and Z can be thought of as substitutes for each other. On the other hand, if a person prefers L_1 it means that he or she needs to do well in both attributes. The full worth of having one attribute at high level cannot otherwise be exploited and Y and Z complement each other. Finally, if no interaction of preference exists between Y and Z, both lotteries are judged equivalent.

The appropriate form of (6.5) can be determined prior to the assessment of k_i. Once the independence conditions of (6.5) are verified, it is sufficient to probe the preference for the above lotteries. If the preference is consistent for all $y' > y''$, $z' > z''$ then $k \neq 0$ and (6.5b) is

appropriate. If $L_1 \sim L_2$ for all $y' > y''$, $z' \gtrsim z''$, then Y and Z are additive independent, $k = 0$, and (6.5a) is the valid model. This can be shown by calculating expected utilities using the multilinear utility function

$$u(y,z) = u(y,z_o) + u(y_o,z) + k\, u(y,z_o)\, u(y_o,z) \tag{6.6}$$

This form generalizes (6.5) for $p = 2$ when Y and Z are mutually utility independent. The proofs can be found in (22) for $p = 2$ and $p \geq 3$.

Form (6.6) yields the following expected utilities:

$$u(L_1) = kA\, u(y_o,z') + kB\, u(y_o, z'')$$

and $$u(L_2) = kA\, u(y_o,z'') + kB\, u(y_o, z')$$

where $A = u(y',z_o)$ and $B = u(y'', z_o)$. The terms which occur for both L_1 and L_2 are omitted by using a positive linear transformation (6.3). It follows from the monotonicity condition that $A > B$. By substituting $u(y_o,z') = v + d$ and $u(y_o,z'') = v - d$, $v \geq 0$, $d > 0$, and using a positive linear transformation again, the following obtains

$$u(L_1) = kd(A-B)$$

$$u(L_2) = kd(B-A)$$

When $L_1 > L_2$ then $u(L_1) > u(L_2)$ and this implies $k > 0$. $L_1 \lesssim L_2$ implies $k < 0$, whereas $L_1 \sim L_2$ only if $k = 0$.

CHAPTER 7

MARKOV DECISION PROCESS FORMULATION

7.1. The Sequential Decision Problem

Consider an engineer responsible for managing the maintenance of a pavement section. At every point in time the pavement is in a particular known condition described by a number of variables which define the state of the system. The system makes state transitions as the pavement deteriorates due to traffic and environmental forces or when the condition is upgraded by maintenance. Although the engineer cannot totally control which transition will occur, he can, by his choice of action affect the probability of any particular transition. The various decisions entail rewards consisting of costs and benefits and the manager has a preference structure for every sequence of rewards. His objective, at any point in time, is to choose his actions so as to maximize the expected utility of the future stream of rewards. An important feature of the process is that decisions are made periodically, but each decision influences all of the following rewards.

This type of problem can be displayed graphically by a multiperiod decision tree built from the basic tree of Figure 4. Starting in a particular state the pavement manager has several options. Once he exercises an option, the pavement enters a new state with a known probability and the process repeats. The decision maker's preferences for different states of pavement condition are solely determined by future transition rewards he can achieve from starting in these states.

7.2. Markov Decision Process

The pavement decision problem can be structured by the Markov decision model (26). The model employs a state space, an action space, a

law of deterioration, and rewards. We consider a system that can be observed periodically and its condition classified as one of the countable states $i = 1,2,\ldots,N$. State 1 is the initial state of the system and represents the process before any deterioration takes place. State N is the terminal state after which no further deterioration can take place. No ordering is implied in the labelling of the intermediate states. After determining the state of the system, one of a number of actions must be taken. For each state i in the state space there is a finite set A_i of possible actions available when the system is in state i. When the system is in state i and we take action $k \in A_i$, two things happen. First, the system moves to a new state j with probability $p_{ij}(k)$ which is determined by the law of deterioration. Second, conditional on the event that the new state is j, we obtain a reward $c_{ij}(k)$.

When viewed in a temporal setting, the system undergoes state transitions at equal intervals termed stages. At each stage an action is taken and a reward results. When the time span of interest consists of n stages, we adapt the convention of indexing transition times so that t is the number of stages remaining to the end of the analysis period, i.e. $t = n$ is the first stage's beginning and $t = 0$ is the terminal time. We define the history of an n-stage process, h^n, as the sequence of states and actions which occur between $t = n$ and $t = 0$

$$h^n = (X_n, Y_n;\ X_{n-1}, Y_{n-1}; \ldots;\ X_1, Y_1;\ X_0)$$

where X_t denotes the state occupied at time t and Y_t is the action taken at time t. The rewards are implied by h^n since $X_t = i$, $Y_t = k$ and $X_{t-1} = j$ specify $c_{ij}(k)$. The reverse assertion need not be true for if different actions lead to identical rewards, we cannot reconstruct h^n.

The deterioration law of the system is assumed to be Markovian. Formally, if the present state and action are i and k, respectively, and j

is the following state the Markov property implies that,

$$P(X_{t-1} = j \mid (X_n, Y_n; \ldots; X_{t+1}, Y_{t+1}; X_t = i, Y_t = k)$$
$$= P(X_{t-1} = j \mid X_t = i, Y_t = k) = p_{ij}(k), \qquad (7.1)$$

that is, the transition probability from i to j at stage t does not depend on the history prior to t. The applicability of Markov assumption to pavement deterioration is discussed in Chapter 4.3.

The notation for transition probabilities implies that they depend on the states occupied, i,j, and the action taken k, but are independent of the time index. It means that the same law of deterioration governs the process regardless of the time point in the analysis period. A process with this property is called stationary or hamogeneous. Contrary to the Markov assumption, the stationarity assumption is not critical and the decision model may be modified to account for time variability of the deterioration law, the state space, the action space, and the rewards. These elements are assumed stationary in this thesis in order to simplify the presentation.

For each possible starting state, i, the decision maker can choose his action from the set A_i. A vector f which assigns a particular action $k \in A_i$ to each possible state is called a <u>decision rule</u>. One decision rule might be

f_1 = (DO NOTHING if state 1, DO NOTHING if state 2, REPAIR if state 3, ..., OVERLAY if state N).

Another decision rule f_2 might differ from f_1 by having DO NOTHING for states 1, 2, 3, and other actions same as in f_1. Any collection of decision rules is a <u>decision set</u>. Thus (f_1, f_2) is a decision set and so is each decision rule taken separately. However, in order to assure that the analysis considers all possible alternatives as required by the principles of systems analysis, the decision set must be the Cartesian product of the individual action sets, $A_1 * A_2 * \ldots * A_n$. Such a decision set contains all

possible decision rules and is called complete. Since the decision maker must exercise a sequence of decisions over time, we define a policy. An n-period policy π_n is a sequence of n decision rules,

$$\pi_n = (f_n, f_{n-1}, \ldots, f_1). \tag{7.2}$$

If all decision rules are the same, the policy is called stationary and is denoted (f^n).

Suppose that at time n the system is in state i and the decision maker wants to pursue the n-period policy $\pi_n = (f_n, \ldots, f_1)$. Since the deterioration law is probabilistic, he cannot predict the resultant sequence of events with certainty. The future history is a random variable $\tilde{h}^n(i, \pi_n)$ and has a probability mass function $m(\tilde{h}^n(\cdot))$. For a particular history h_k^n given the starting conditions, the probability $m(\tilde{h}^n(\cdot) = h_k^n) = m_k$ and $\Sigma_k m_k = 1$. Given a history, m_k can be calculated as the product of one-stage transition probabilities since the transitions between states are independent events.

7.3. Optimal Policies and Dynamic Programming

Associated with each history is a sequence $(c_n, c_{n-1}, \ldots, c_o)$ of one-stage outcomes. Here, c_t is a vector reward at stage t. The decision maker has a preference structure defined on all sequences of rewards and encoded in the utility function $U(c_n, \ldots, c_o)$. Since the histories are random variables conditional on the starting state and the policy used, the associated reward streams are sequences of random rewards. The utility of rewards associated with history $\tilde{h}^n(i, \pi_n)$ is thus

$$\tilde{U}(i, \pi_n) = U(\tilde{c}_n(X_n, X_{n-1}, f_n), \tilde{c}_{n-1}(X_{n-1}, X_{n-2}, f_{n-1}), \ldots, \tilde{c}_1(X_1, X_o, f_1), \tilde{c}_o(X_o))$$

where $\sim$ denotes random variable, X_t is state in which stage t begins, $c_o(X_o)$ is reward associated with terminating in state X_o and the starting conditions

are $X_n = i$ and $f_n = k$.

The decision problem is to choose an optimal sequence of actions with which to manage the system over the analysis period to maximize expected utility. If we start in state i and pursue policy π_n, the expected utility of the streams of rewards possible in the future is

$$v_i(\pi_n) = E_{\tilde{h}^n(i,\pi_n)}\tilde{U}(i,\pi_n)$$

If one tried to find the optimal policy in a problem involving N states, K actions and T stages, one would have to evaluate about $N(K*N)^T$ paths in the decision tree corresponding to all possible histories. A realistic problem may have N = 10, K = 4 and T = 10 and will require about 40^{10} calculations. This is a prohibitive number even for a high speed computer. Clearly, a more powerful technique is required for computational efficiency.

The structure of Markov decision problems is suitable for optimization by the dynamic programming technique (26) which is based on the principle of optimality. The principle asserts that regardless of the present state of the system and the present decision, the remaining decisions in the sequence must constitute an optimal policy with regard to the state resulting from the present decision. For our purpose, "decision" should be interpreted as "decision rule".

If $\pi_n^* = (f_n^*, f_{n-1}^*, \ldots, f_1^*)$ is an optimal n-stage policy, then $\pi_{n-1}^* = (f_{n-1}^*, \ldots, f_1^*)$ must be an optimal (n-1) stage policy and $\pi_n^* = (f_n^*, \pi_{n-1}^*)$. By using the principle of optimality recursively, one can start from the last stage and by backward induction identify the optimal sequence of decision rules. Analysis of the n-stage problem is thus simplified to n problems of finding an optimal decision rule, f_t^*, for one stage at a time by the following criterion

$$\underline{v}(f_t^*, \pi_{t-1}^*) \geq \underline{v}(f_t, \pi_{t-1}^*), \text{ for all } f_t. \tag{7.3}$$

Here, $\underline{v}(\cdot)$ denotes a vector with components $v_i(\cdot)$, i = 1, ..., N, that is a

vector of expected utilities for all starting states. It is termed the return function. The left hand side vector in the above criterion is said to dominate the other, that is

$$v_i(f_t^*,\pi_{t-1}^*) \geq v_i(f_t,\pi_{t-1}^*) \text{ for all } i,$$

$$v_i(f_t^*,\pi_{t-1}^*) > v_i(f_t,\pi_{t-1}^*) \text{ for some } i.$$

Dominance is guaranteed when the decision set is complete. The dominating policy (f_t^*,π_{t-1}^*) is the optimal t-period policy. It is not unique.

Dynamic programming makes the analysis computationally feasible but puts restrictions on the form of the objective function. Sequential decision problems with expected utility in the objective function are difficult to solve, but recently mathematical formulations have emerged for this class of Markov decision processes (37). In general, objective functions decompose as required for the application of dynamic programming if they are separable (35). A function is separable if

$$g(\tilde{c}_n(X_n,X_{n-1},f_n),\ldots,\tilde{c}_1(X_1,X_o,f_1)) = g_1(\tilde{c}_n(\cdot);\ g_2(\tilde{c}_{n-1}(\cdot),\ldots,\tilde{c}_1(\cdot))).$$

If, in addition, g_1 and g_2 are real valued functions, and g_1 is a monotonically nondecreasing function of g_2 for every $\tilde{c}_n(\cdot)$, then

$$\max_{f_n,\ldots,f_1} g(\tilde{c}_n(\cdot),\ldots,\tilde{c}_1(\cdot))$$

$$= \max_{f_n} g_1(\tilde{c}_n(\cdot);\ \max_{f_{n-1},\ldots,f_1} g_2(\tilde{c}_{n-1}(\cdot),\ldots,\tilde{c}_1(\cdot)). \tag{7.4}$$

The maximization with respect to $(f_{n-1},\ldots,f_1)$ moves inside the expected utility of the nth stage and backward induction is now possible. To show this, define

$$\underline{v}\ (\pi_{n-1}^*) = \max_{f_{n-1},\ldots f_1} g_2(\tilde{c}_{n-1}(\cdot),\ldots,\tilde{c}_1(\cdot))$$

and we have

$$\underline{v}(\pi_n^*) = \max_{f_n,\ldots,f_1} g(\tilde{c}_n(\cdot),\ldots,\tilde{c}_1(\cdot))$$

$$= \max_{f_n} g_1(\tilde{c}_n(X_n,X_{n-1},f_n);\ \underline{v}(\pi_{n-1}^*)). \tag{7.5}$$

Given the optimal return function $\underline{v}(\pi_{t-1}^*)$ for stage t-1, we can find the optimal decision rule for stage t. If we start in stage 1 and find $\underline{v}(f_1^*)$, we can next identify the optimal 2nd stage decision rule f_2^* and the optimal return function $\underline{v}(f_2^*,f_1^*) = \underline{v}(\pi_2^*)$. Given these, we can find f_3^* and $\underline{v}(f_3^*,\pi_2^*) = \underline{v}(\pi_3^*)$. The sequence of optimal decision rules forms an optimal policy. The information about all stages t-1, t-2, ..., 1 following stage t is summarized by the optimal return function $\underline{v}(\pi_{t-1}^*)$. By virtue of the principle of optimality, it is never necessary to analyse the effect of a current decision rule on those made in the remaining stages. For this reason it is possible to optimize over one stage at a time.

7.4. Separable, Risk-Averse Temporal Utility Functions

Choosing the optimal policy π_n^* by the criterion of maximum expected utility can be stated as

$$\underline{v}(\pi_n^*) = \max_{f_n,\ldots,f_1} E_{\tilde{h}^n(i,\pi_n)} U(\tilde{c}_n(\cdot),\ldots,\tilde{c}_1(\cdot),\tilde{c}_o(X_o))$$

where $\tilde{c}_t(\cdot) = \tilde{c}(X_t,X_{t-1},f_t)$ and $\tilde{c}_o(X_o)$ is the terminal reward. In order to use dynamic programming, the maximization with respect to $(f_{n-1},\ldots,f_1)$ must move inside the nth stage expected utility and the expectation is calculated over one stage rather than the n-stage history:

$$\underline{v}(\pi_n^*) = \max_{f_n} E_{\tilde{c}_n(\cdot)} U(\tilde{c}_n(\cdot),\ \underline{v}(\pi_{n-1}^*)). \tag{7.6}$$

The optimal return function for stage 1 depends on the terminal reward $\tilde{c}_o(X_o)$,

$$\underline{v}(f_1^*) = \max_{f_1} E_{\tilde{c}_1(\cdot)} U(\tilde{c}_1(\cdot), \tilde{c}_o(X_o)). \tag{7.7}$$

From this, the optimal policy may be determined by proceeding one stage at a time, until stage n is reached, provided the return function conforms to the conditions of separability and monotonicity.

The separable utility functions must be screened to eliminate those forms which do not meet behavioural assumptions describing the decision maker's attitude toward risk in multistage gambles.

When rewards c_i in an n-period lottery are thought of as attributes x_i in the sense of Chapter 6.3, it is possible to use a model like (6.5) in a temporal setting,

$$U(c_n,\ldots,c_o) = \Sigma_{t=1}^{n} K_t u_t(c_t), \text{ if } \Sigma_t K_t = 1 \tag{7.8a}$$

or $$1 + KU(c_n,\ldots,c_o) = \Pi_{t=1}^{n}(1 + KK_t u_t(c_t)), \text{ if } \Sigma_t K_t \neq 1, \tag{7.8b}$$

where U is the multiperiod utility function; u_t are the utility functions for single periods; K_t are the scaling constants for single periods; and K is the temporal interaction factor.

Meyer shows in Chapter 9 of Keeney and Raiffa's book (22) that the mutual utility independence conditions which are necessary for (6.5) to hold in a temporal situation are implied by two behavioural features. First, it is acceptable for the decision maker to make his decision about the future (in each period) without regard to his past stream of rewards. Second, the terminal reward does not affect the decisions made prior to the terminal time.

Testing the additive independence assumption will determine which of the models (7.8a) and (7.8b) is appropriate, and what sign the interaction factor K assumes. The risk behaviour in a two-stage problem may be tested using the following lotteries, where c' > c:

$$L_1 = \begin{cases} (c,c) \\ (c',c') \end{cases}$$

$$L_2 = \begin{cases} (c,c') \\ (c',c) \end{cases}$$

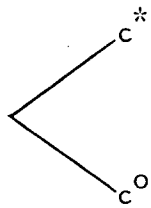
Lottery L_1 gives the decision maker a 50-50 chance of receiving either the smaller rewards, c, in both stages, or the greater reward, c', in both stages. In L_2, the decision maker receives either the smaller reward followed by the greater reward in the second stage, or, with equal probability, he receives a stream of rewards in the reversed order.

The decision maker is temporally risk averse for two-period reward streams if he does not prefer L_1 to L_2 ($L_2 \gtrsim L_1$) for all c and c'. He is risk neutral if and only if $L_1 \sim L_2$ for all c and c'. Finally, he is risk seeking if $L_1 \gtrsim L_2$ for all c and c'. The first lottery may be regarded as receiving "all the best or all the worst" with a 50-50 chance. The second lottery's consequences assure that the decision maker will receive one of the more desirable rewards and one of the less. Risk aversion is attributed to preferring some good and some bad to all or nothing. The concept of temporal risk attitudes may be extended to multistage lotteries.

It seems natural that decision makers responsible for the operation of public systems be temporally risk averse. Assuming the contrary would imply that the decision maker is indifferent or prefers the n-period lottery

$$L_3 = \begin{cases} (c^*,c^*,\ldots,c^*) \\ (c^o,c^o,\ldots,c^o) \end{cases}$$

to the sequence $(\hat{c},\hat{c},\ldots\hat{c})$ where c^* and c^o are the best and the worst rewards, respectively, and $\hat{c}$ is the certainty equivalent for the one-period lottery.

$L_4 =$ 

Lottery L_3 gives him a 50% chance of the worst rewards for n years. Preferring this lottery to the sequence of certainty equivalents could possibly lead to decisions with catastrophic outcomes. Indifference would imply that he behaves as if he were only facing a 50% chance of the worst reward for one stage.

One can conclude for most of the public decision situations that the additive model (7.8a) is not valid and K in (7.8b) has to be negative. Form (7.8b) is known as the negative multiplicative (36,43).

Imposing a condition of stationarity on the decision maker simplifies (7.8b),

$$1 + KU(c_n,\ldots,c_o) = \Pi_{t=1}^{n}(1 + bu(c_t)) \tag{7.9}$$

where $b = KK_t$; $K_t = \text{const}$, $t = 1,\ldots,n$.

Stationarity implies that if faced at time t with a decision problem which affects future reward streams, the decision maker will make a decision which is independent of the absolute time.

When K is negative, note that

$$W(c) = W(c_n,\ldots,c_o) \equiv -(1 + KU(c_n,\ldots,c_o))$$

and $\quad w(c_t) \equiv -(1 + bu(c_t)),\ b < 0$

are utility functions over C and C_t, respectively, so

$$-W(c) = \Pi_{t=1}^{n}(-w(c_t)) = \Pi_{t=1}^{n}(1 + bu(c_t)). \tag{7.10}$$

The risk aversion function can be calculated for the so called level streams of rewards, that is when $c_t = e$ for all t. Then (7.10) becomes

$$W(c) = -(1 + bu(e))^n \tag{7.11}$$

for which the risk aversion function (5.1) is

$$-W''/W' = -u''(e)/u'(e) - (n-1)bu'(e)/(1 + bu(e)).$$

For n = 1, the risk aversion is that of U as expected. As the number of periods increases, the risk aversion increases linearly within n.

Calculation of the risk aversion function for the additive model (7.8a) under conditions of stationarity and level streams shows that the n-period risk aversion is the same as the single period risk aversion:

$$U(c) = nau(e), \; a = K_t \text{ for all } t$$

and $$-U''(c)/U'(c) = -u''(e)/u'(e).$$

CHAPTER 8

APPLICATION

8.1. Summary of Method

Figure 9 outlines the proposed method. The manager decides which objectives and attributes are relevant (Chapter 4.1). The analyst elicits the manager's preferences for multiattribute consequences and selects an appropriate utility model for one-stage consequences (Chapter 6.3). The planning horizon is chosen and the analyst decides which temporal utility function best represents the manager's risk attitudes (Chapter 7.4). The function transforms the sequence of one-stage utilities into a criterion index in the dynamic programming algorithm.

The available actions, the associated consequences and the law of deterioration are input into the optimization algorithm. A one-stage utility function transforms multiattribute consequences into single values to be used in evaluation. The law of deterioration is represented by Markov transition matrices. Using these, the dynamic programming algorithm calculates the expected temporal utility and identifies an optimal policy (Chapter 7.3).

To illustrate the method it is applied to a hypothetical example which is processed by a computer program. This test application emphasizes the utility part of decision analysis. The utilities are elicited from a group of engineers, while the structure of objectives and the Markov transition matrices are assumed. The elicitation exercise tests the behaviour of subjects in a simulated pavement decision situation and provides a basis for selecting an appropriate preference model. Temporal preferences are tested superficially as the theory is still in the development stage.

Computational runs on several versions of the problem probe whether the model behaves reasonably for simply varied inputs. These tests investigate the effect of variation in the elicited scaling constants. The effect

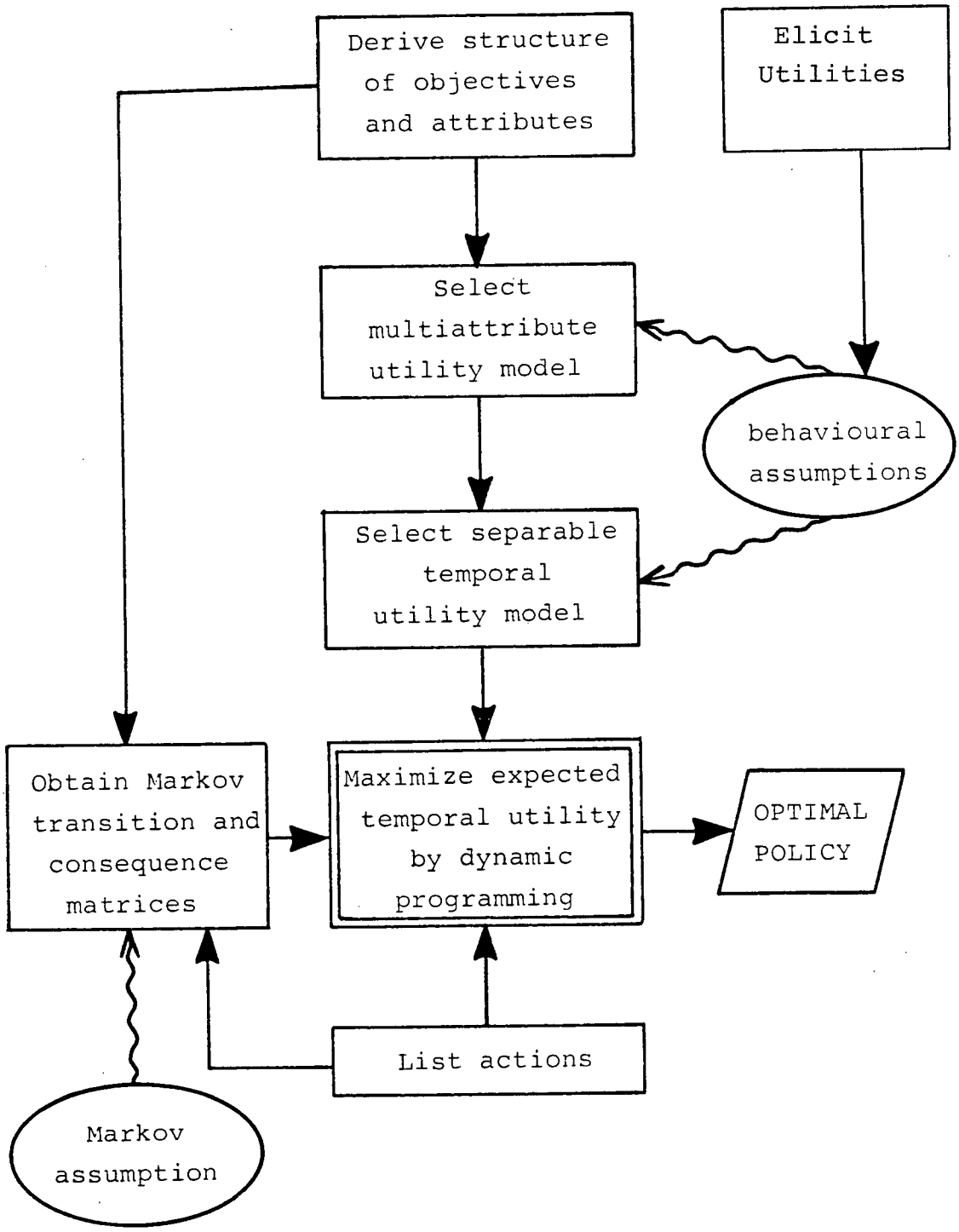


FIGURE 9. SUMMARY OF PROPOSED METHOD

on the optimal policy of Markov transition probabilities and of the temporal utility are also tested.

The number of states is set at a minimum and the formulas for calculating consequences are simplified to facilitate judgement of the model's behaviour. For this reason, the program may not be adopted for immediate application by a highway department. However, these simplifications can be easily rectified and the model implemented, provided the user subscribes to the underlying assumptions.

8.2. Hypothetical Example Problem

A Department of Highways (DOH) requires a plan for the upkeep of a 20 km long section of a four-lane suburban highway. The objectives for managing the pavement section reflect DOH's role of a public servant (Figure 10). The road user objectives of safety, economy, and travel time concern the average daily traffic (ADT) of 60,000 vehicles/day that use the section at an operating speed of 100 km/hr. The public objectives include DOH's cost of maintaining the section, access to adjoining communities provided by the section, job creation in the region, and conservation of gravel. The nonusers are 2000 households in a traffic noise zone along the section. DOH desires to minimize the agency cost for the section, because the unused funds can be utilized on other links in the regional network. The region experiences unemployment and a shortage of construction aggregates. To ameliorate the situation, DOH will maximize employment and minimize the quantity of gravel required for highway maintenance.

DOH has the following actions available:

1. do nothing,
2. routine maintenance (fill cracks and potholes),
3. seal coat and chips (waterstop with rock chips rolled for good tire friction),

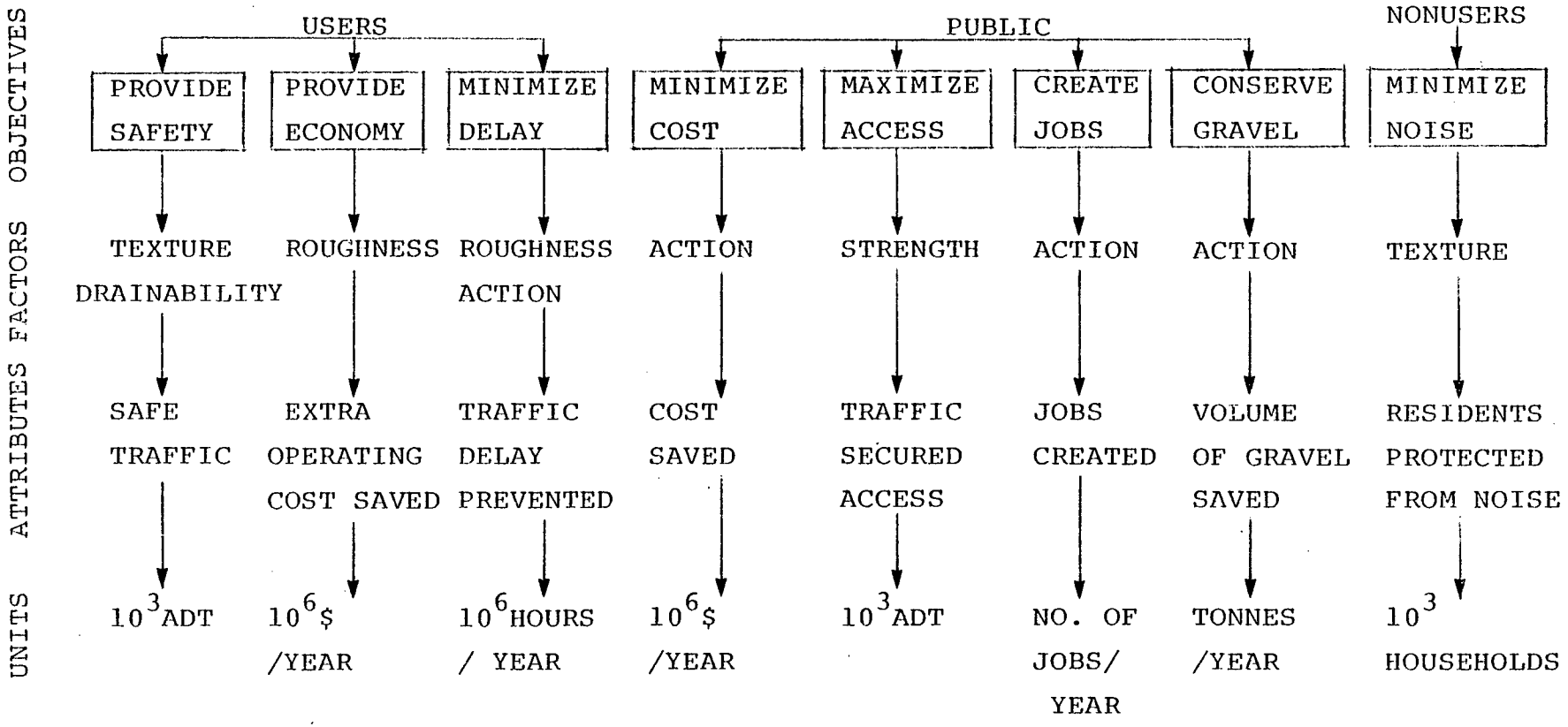


FIGURE 10: STRUCTURE OF OBJECTIVES, CONTROLLABLE FACTORS AND ATTRIBUTES

4. overlay (layer of asphalt concrete laid on existing pavement).

One of these actions will be implemented every year and DOH wishes to determine that sequence of actions which will satisfy the objectives in a best way possible within a planning horizon. DOH recognizes the probabilistic nature of pavement behaviour and will represent the laws of pavement deterioration by Markov transition matrices.

Figure 10 outlines the objectives, the related factors controllable by the pavement manager, and the measures of effectiveness (attributes) for all objectives. The attributes are either adopted from highway transportation planning or assumed ad hoc. They are further explained in Chapter 8.3.

Safety can be measured by the number of accidents that occur due to inadequate pavements. Minimizing the number of accidents can be accomplished by controlling the cuasative pavement characteristics: drainability, texture and colour. Assume that light colour and harsh microtexture are always provided as a matter of good engineering practice. Then safety is a function of drainability measured by depth of ruts and macrotexture measured by depth of pavement surface texture.

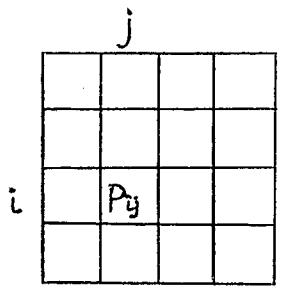
8.3. Probabilities and Updating

The state of the pavement system can be described by a number of variables. These variables are either under the pavement manager's control (pavement condition variables) or outside his control (traffic, number of residents).

The condition of pavements can be predicted from their past performance by the Markov transition model. A state of the pavement is jointly defined by a number of pavement variables such as roughness, texture, rutting and strength. Each of these follows its own law of deterioration given by a Markov transition matrix which depends on the action applied.

This law may be conditional on the values of other variables and also may change with time, but these variations are not included here for the sake of simplicity of presentation.

Markov transition matrices for pavement condition variables can be easily obtained from pavement condition surveys. Consider construction of the matrix for a pavement variable under action k:



In order to estimate $p_{ij}(k)$ take a sample of size $n_i(k)$ on those pavements which were at state i one stage ago and were subjected to action k since that time. Count the number of pavements in the sample which are in state j at present, $r_{ij}(k)$. The best estimate of $p_{ij}(k)$ is simply the ratio of $r_{ij}(k)$ and $n_i(k)$.

Pavement networks are usually surveyed periodically and data so obtained provide a basis for gradual updating of Markov transition matrices by using Bayes' formula. Sampling for r_{ij} may be regarded Bernoulli trials and then the sample-likelihood function is given by the binomial formula. It is computationally convenient to represent the prior information about p_{ij} by a beta distribution, f', which is compatible with, or a conjugate of, the binomial sample likelihood function, L.

Smith (30) has suggested the use of a normal conjugate but the method presented here appears to be more natural for at least two reasons. First, while the Gaussian approximation of the prior distribution may be good around the mean, it cannot be good in the tails because p_{ij} is a random variable from a restricted interval of numbers (0,1) rather than from an infinite interval $(-\infty,\infty)$. Second, there is no reason to assume that f'', L

and f' are bell-shaped and symmetric as implied by the normal distribution.

The prior beta distribution is given by

$$f'(p_{ij}) = B'p_{ij}\exp(r_{ij}'-1)(1-p_{ij})\exp(n'_i-r_{ij}'-1) \tag{8.1}$$

where B' is a normalizing constant, and the primed symbols refer to a sample collected at previous survey. The mean and variance are:

$$\bar{p}_{ij} = r_{ij}'/n_i' \tag{8.2}$$

$$\sigma^2 = \bar{p}_{ij}(1-\bar{p}_{ij})/(n_i'+1). \tag{8.3}$$

With the present availability of electronic calculators and computers, the class of beta distributions does not seem to be less tractable than the normal distribution. The class is very flexible for use in describing empirical data on probabilities. The distributions can assume a wide variety of symmetric and asymmetric shapes, including the bell, the rectangle, the triangle and the J-shape.

When the likelihood function is binomial and f' beta, then f'' is also a beta distribution with parameters

$$r_{ij}'' = r_{ij} + r_{ij}'$$

$$n_i'' = n_i + n_i'$$

that is

$$f''(p_{ij}) = B''\, p_{ij}\exp(r_{ij}''-1)(1-p_{ij})\exp(n_{ij}''- r_{ij}''-1)$$

The simplicity of an updating procedure is obvious. Pavement behaviour is summarized at any time by r_{ij}'' and n_i''. After the next condition survey is carried out, the present posterior information becomes prior and is updated to a new posterior by Bayes' theorem using the new sample. Once the prior has been updated by the new sample, it may be discarded, for its information contents is now stored in the new posterior. At any time t, the mean and variance for a Markov transition probability are

$$\bar{p}_{ij}(t) = \sum_{t=1}^{t} r_{ij}(t)/ \sum_{t-1}^{t} n_i(t)$$

$$\sigma^2 = \bar{p}_{ij}(t)(1-\bar{p}_{ij}(t))/(1 + \sum_{t=1}^{t} n_{ij}(t))$$

where $r_{ij}(t)$ and $n_i(t)$ are sample results obtained by condition survey at the beginning of stage t. This updating procedure secures the most current information available for the pavement manager and minimizes record keeping.

A consequence of Bayes' theorem is of particular interest from an information value standpoint. In those cases where the prior source of information or the other one is relatively weak, the shape of the posterior distribution is very nearly that of the dominant distribution. When alternative distributions are available, for example one based on a condition survey and one based on predictive mechanistic formulae, then the analyst can choose that one which is more "peaked". Moreover, whatever prior probabilities are adopted at time t=1, they will be swamped by the data incoming at subsequent condition surveys. They can thus be chosen uniformly distributed if no better estimate is available at t=1.

It is possible to set up Markov transition matrices based on engineering judgement, without having the data from pavement condition surveys available. Engineering judgement represents an experience that is often superior to the results of predictions based on limited condition surveys and analytical models.

In order to tap this experience and to structure it to a form suitable for decision analysis one can proceed two ways. One, ask the engineer for an estimate of r_{ij}' and n_i' and then use these in (8.1). Two, ask for estimates of $\bar{p}_{ij}$ and σ^2 in (8.2) and (8.3), solve for r_{ij}' and n_i' and use these in (8.1).

Because other work (30,33,34) have demonstrated a good command of the probabilistic part of decision analysis for pavements, this thesis puts less emphasis on probabilities relative to utilities. Although a practical application will operate with about three to five states to describe each pavement condition variable, the program uses only two states - acceptable and unacceptable, and all Markov transition matrices are of the order of two.

This simplification offers advantages as well as disadvantages. The programming is kept reasonably simple and program outputs can be evaluated by intuition. Due to an interaction between probabilities and utilities in the decision model the results may be distorted, however, even if the utility part is accurate. This fact should be remembered when results of the present program are interpreted.

8.4. Consequences

Consequences are vectors which consist of all attributes and accrue in each stage. The numerical values of attributes depend on the state at stage's beginning, on the action chosen, and on the state at stage's end. It is assumed in the presented model that actions are applied at stage's beginning and the starting state changes instantly into the end state. A consequence thus depends on the action and on the end state only. This assumption may distort results for short planning horizons. If this is a valid concern, then consequences can be made dependent on the starting state. The refinement is omitted in this example.

To calculate a consequence given end state, the state variables must enter respective attribute functions. The state is described by traffic, number of residents and by pavement condition variables. A particular attribute, however, depends on a subset of these variables, as explained below. The worst values for all attributes are chosen at zero and the best values are always greater than zero. This ascertains increasing utility functions.

The traffic and number of residents are assumed constant in the example. If necessary, a time function can calculate future values of these variables, while probability distributions can express uncertainty about these values.

Traffic safety is measured by the proportion of traffic that is prevented from hazards arising from inadequate pavement properties. The best level of safety is achieved when all traffic is safe, the worst when all traffic is exposed to hazards. The attribute for safety is a function of traffic, pavement drainability and texture. For the simple case of either acceptable or unacceptable pavement variables,

$$X_1 = \text{Safe Traffic} = F(D)*F(T)*\text{Traffic}$$

where D and T denote drainability (depth of ruts) and texture (depth of macrotexture), respectively; $F(\cdot) = 1$ if pavement variable acceptable and $F(\cdot) = 0$ otherwise.

Road user economy is measured by the dollar savings in vehicle operating costs which are achieved relative to a rough pavement condition. Highest savings obtain on smooth pavements and zero savings - on the roughest pavements,

$$X_2 = \text{Operating Cost Savings} =$$
$$= F(R)*\text{Length}*\text{Traffic}*\text{Cost}$$

where $F(R) = 1$ if pavement roughness acceptable, $F(R) = 0$ otherwise; Length = total length of lanes on the road section; Cost = vehicle operating cost per one vehicle kilometre travelled on rough pavement (59).

User time delay is an attribute that depends on pavement roughness and action taken. Rough surfaces cause the road users to slow down and so do maintenance operations on the road. There exist accurate data which make it possible to calculate both types of delays, but a simpler formula is used here. To make the utility function increasing, the savings in travel time relative to the worst condition are chosen for the attribute.

$$X_3 = \text{Travel Time Savings} =$$
$$= \text{Delay} -(F(R)*\text{Length}*\text{Cycles} +$$
$$+ E(A)/365)*\text{Time}*\text{Traffic}$$

where Delay = maximum delay possible; F(R) = 1 if roughness unacceptable, F(R) = 0 otherwise; Length = total length of lanes on the road section; Cycles = number of vehicle slowdown cycles per km; Time = average excess time per slowdown cycle; E(A) = number of days that a maintenance operation will cause traffic slowdowns. Both Cycles and Time are typical values compiled from (59), while E(A) is assumed for each action.

Agency cost is equal to the savings in expenditures on pavement maintenance relative to the most expensive maintenance action.

The access objective reflects the fact that one of the most important tasks of pavement engineers is the protection of the facility from structural deterioration. The seriousness of this consequence increases with the importance which given road section has in a transportation network. Let the attribute be,

X_5 = Access Provided = F(S)*Traffic

where F(S) = 1 if pavement strength measured f. ex. by the Benkelman beam is acceptable, and F(S) = 0 otherwise.

The number of jobs that are necessary for carrying out a maintenance action is a straightforward attribute for the employment objective. "Overlaying" is most laborious and "do nothing" does not create any employment.

Gravel saved is measured in tonnes relative to the action which consumes most of the material. "Do nothing" and "routine maintenance" can provide largest savings in gravel.

The effect of traffic noise on nonusers is also measured by a simple function. The noise is generated by tires on deeply textured pavements. When pavement texture is shallow, the noise level does not affect any of the residents in the noise zone along the road section. When pavement texture is deep, all residents in the zone are affected:

X_8 = Noise Prevented = F(T)*Residents

where F(T) = 0 if pavement texture is deep enough to cause unacceptable noise levels and F(T) = 1 otherwise; Residents = number of households in the noise zone.

8.5. Elicitation of Preferences

Five cooperative persons were asked to imagine themselves in the role of a pavement manager. They were motivated by the possibility of exploring their own preferences in a decision analytical game. All of these people are engineers and it is likely that their value judgements are similar to those of a pavement manager.

Each subject was introduced to the decision problem individually. He or she was asked to act as an impartial decision maker who is not subjected to any institutional, personal or political pressures. The person first became familiar with the decision scenario and with the objectives as in Chapters 8.2 and 8.4. If the subject was showing signs of misunderstanding, additional explanations were provided.

The elicitation of preferences was carried out in two phases. The first phase assessed preferences of the five subjects qualitatively in order to identify possible interactions between objectives. The second phase elicited utility functions from one subject for use in the decision model.

Qualitative Assessment

The elicitation was aided by a series of forms on which the subjects marked their answers. The preferential independence was tested using a square graph (Appendix A) on which the ordinate represents values of one attribute, X_i, and the abscissa represents another attribute, X_k. The origin represents an outcome with both X_i and X_k at the worst levels and the top right corner represents an outcome with both attributes at the best

levels. The square thus contains all possible pairs (X_i, X_k). The values at which the complementary attributes are held fixed are presented in an accompanying table.

Trade-offs shall be investigated in the whole domain (X_i, X_k). For practical reasons, however, only three points were chosen: the upper right corner, the centre of the square, and a point close to the lower left corner. The subject is shown the square and the table, and is asked which of the two attributes would he or she rather drop from the current value to the worst value, given other attributes have levels as in the table. If the answer is X_i, the subject must estimate how much of X_i should be given up in exchange for keeping X_k at the current value. This answer is marked on the square.

Each of the three (X_i, X_k) pairs was tested with the following four cases of the complementary attributes, in order to investigage whether trade-offs change when the levels of attributes change:

Case 1 = all complementary attributes at the best levels,

Case 2 = all complementary attributes are best, except "Jobs", "Gravel" and "Noise" which are at the worst levels,

Case 3 = all complementary attributes at the best levels, except the road-user ones, which are at the worst levels,

Case 4 = all complementary attributes at the worst levels.

These four cases are contained on one sheet (see Appendix A). For n = 8 attributes, n - 1 = 7 sheets are required to test trade-offs. Thus, 3 x 4 x 7 = 84 questions (3 questions per square, 4 squares per sheet, 7 sheets) would have to be answered by each subject. This number appears to be high, but answers follow relatively fast after a "warming-up" period of about five questions. Moreover, numerical estimates are not required for a qualitative study and statements by the subjects about trade-offs relative to previously answered cases were regarded as sufficient answers.

The five subjects were divided into two groups. The first group (subjects A,D,E) first made trade-offs on all of the top squares of randomly ordered sheets, then on all the second squares of rearranged sheets, then on the third squares, and finally on the fourth squares. This scheme ascertains that the subject keeps in mind the complementary attribute values until a new case of complementary attributes is considered. The order of sheets was random and different for each case. The second group (subjects B,C) answered questions sheet by sheet, but the order of squares on each sheet was kept random. This second scheme requires that the subject constantly change the mental picture of the complementary attribute levels as the questions move randomly between the four cases. The process is more exhausting and time-consuming than the first scheme.

The forms for testing utility independence also contain four cases of complementary attributes, but have four lotteries instead of four trade-off squares (see Appendix A). The subject is asked to estimate the certainty equivalent of a lottery involving one attribute, conditional on the remaining attributes being held fixed. A total of 3 x 4 = 12 certainty equivalents (3 points for one conditional utility curve as Figure 6, 4 cases to condition upon) would have to be given by each subject. However, it was judged that asking for $C_{0.50}$ and either $C_{0.25}$ or $C_{0.75}$ is sufficient in a qualitative study. Consequently, two certainty equivalents were elicited from each subject for one attribute.

Results of the qualitative assessment are presented in Appendix C and are summarized in Table 3. The preferential and utility independence assumptions appear to be a good approximation for three of the five subjects, and may be a workable approximation for the remaining two subjects.

Subject	Variation in Trade-Offs	Variation in Certainty Equivalents
A	Invariant for 7 pairs	Invariant for 2 attributes
B	Invariant for 7 pairs	Invariant for 2 attributes
C	Invariant for 3 pairs. Changed for 3 pairs by less than 10%. Dropped 1 attribute.	Invariant for 1 attribute. Changed for 1 attribute.
D	Invariant for 6 pairs. Changed for 1 pair by less than 10%.	Invariant for 2 attributes
E	Invariant for 7 pairs	Invariant for 2 attributes

TABLE 3: VARIATION OF TRADEOFFS AND CERTAINTY EQUIVALENTS WITH LEVELS OF COMPLEMENTARY ATTRIBUTES

Quantitative Assessment

Preferences were elicited from subject E two days after the first phase. Utility functions over single attributes were assessed by the certainty equivalent method as in Figure 6. Each utility curve was smoothed by hand through the five points, and the HALF value was estimated from the curve. The HALF values were used as an input into the computer program to approximate utility functions by the exponential form based on a theory developed in Appendix B. Although these forms do not reproduce the hand-smoothed curves exactly the discrepancies are comparable to the uncertainties inherent in smoothing. After the utility curves were established the subject was asked to rank order the attributes. They were all set at their least desirable levels equal zero, and the subject had to decide which one he would most like to have at its best level. The scaling factor associated with this attribute is the largest. The attribute he would next prefer to have alone at its most desirable level has the second largest scaling factor, and so on. The following ranking was obtained.

$$k_5 > k_1 > k_2 > k_3 > k_4 > k_7 > k_8 > k_6. \tag{8.6}$$

Three inequalities were also elicited for consistency checks,

$$k_6 + k_7 + k_8 < k_2 \tag{8.7a}$$

$$k_2 + k_3 < k_1 \tag{8.7b}$$

$$k_1 + k_2 + k_3 > k_5 \tag{8.7c}$$

To enumerate the scaling constants, trade-offs were considered on pairs of attributes. The subject first chooses the attribute, X_k, which he would rather push from zero to the best value, x_k^* and the subject has to find an indifferent lower value, x_k. The indifference pair implies an equation of the form,

$$k_1 = k_k \, U_k(x_k)$$

Seven indifference pairs provided seven equations of the above form.

These equations are not independent, and an additional one is required in order to find a unique solution for the vector of scaling constants. The equation was sought by asking the subject this question: Here is a lottery yielding all attributes at their best levels with probability p or all attributes equal zero with probability 1-p. Estimate p so that the lottery becomes indifferent to a consequence with SAFETY at its best level and all other attributes equal zero.

The answer to this question came with much more difficulty compared to trade-off questions. The subject eventually estimated a band for p rather than a single value. Similar question was repeated as a consistency check with USER ECONOMY as the single attribute at the best level. The answer came again in a banded form, and two additional trade-off questions were posed. Thus a set of twelve equations resulted, including four redundant equations for consistency checks.

To solve the set, it was first necessary to fix the two imprecise scaling factors that were given in a banded form:

$$k_1 = 0.60 \pm 0.10$$

$$k_4 = 0.20 \pm 0.05$$

The low, the medium, and the high value of k_1 was substituted into the set and thus three solution vectors were obtained. The same procedure was repeated with k_4.

The exercise revealed two problems. First, the obtained values for k_4, k_6, k_7 and k_8 disagreed with (8.6). Second, $k_4 \geq 0.18$ caused k_5 to exceed unity. The first problem was found to be implanted in inconsistent trade-offs involving X_4, X_6, X_7 and X_8. These were reexamined with the subject, and the trade-offs were corrected. The second problem simply indicated that the subject has specified his band too wide and all solutions containing $k_4 \geq 0.18$ must be neglected as inconsistent.

The set of equations was now solved for the three values of k_1, and for $k_4 = 0.15$ and $k_4 = 0.17$. The solutions complied to (8.6) and (8.7) but only those obtained by substituting $k_1 = 0.60$ and $k_4 = 0.15$ were in a reasonable agreement. In fact, the agreement was to less than 1% absolute error. This error was corrected by the subject through a fine-tuning of two trade-offs. The following set of consistent equations resulted:

$$k_2 = k_1 u_1 (30 \times 10^3 \text{ ADT}) = 0.60 k_1$$

$$k_8 = k_1 u_1 (8 \times 10^3 \text{ ADT}) = 0.16 k_1$$

$$k_3 = k_2 u_2 (2.4 \times 10^6 \text{ \$/year}) = 0.67 k_2$$

$$k_4 = k_2 u_2 (1.5 \times 10^6 \text{ \$/year}) = 0.42 k_2$$

$$k_6 = k_2 u_2 (0.7 \times 10^6 \text{ \$/year}) = 0.20 k_2$$

$$k_4 = k_5 u_5 (6 \times 10^3 \text{ ADT}) = 0.18 k_5$$

$$k_6 = k_7 u_7 (16.0 \text{ t/year}) = 0.58 k_7$$

$$k_6 = k_8 u_8 (1500 \text{ households}) = 0.75 k_8$$

$$k_8 = k_4 u_4 (0.9 \times 10^6 \text{ \$/year}) = 0.63 k_4$$

$$k_1 = 0.60$$

$$k_4 = 0.15.$$

The following solution obtains:

$$k_1 = 0.600$$

$$k_2 = 0.360$$

$$k_3 = 0.240$$

$$k_4 = 0.151$$

$$k_5 = 0.839$$

$$k_6 = 0.072$$

$$k_7 = 0.124$$

$$k_8 = 0.096.$$

The sum of the scaling constants is greater than one. Thus, the multi-attribute utility model is multiplicative, and the interaction factor k must be negative. It is computed by subprogram KEENEY (Appendix D).

Time Dependence:

The choice of planning horizon must be a compromise between analytical convenience and realism. For very large time horizons it is possible to regard the Markov decision process as if it were to continue indefinitely. The terminal returns are so distant that they have only a limited effect on the decision maker's behaviour. The optimal policy may become stationary and will constitute a rule for the "best" operation of the pavement system in the long run.

The large horizon approach is clearly unrealistic. The uncertainties about distant future become too large to serve a useful purpose. Technological innovation will affect traffic and pavement materials which will cause the pavement deterioration laws and the set of available actions to change. The financial picture and the social-political reality even for a future as close as ten years is unpredictable.

These disadvantages are diminished for short planning horizons. Stationarity may reasonably be assumed for all the model's elements: probabilities, utilities and actions. The effect of terminal returns may, however, affect the results of optimization more than for a longer horizon.

A short time horizon of five years is assumed for this example. The Markov transition probabilities are stationary within this period and so are the uniattribute utility functions and the scaling constants in the multiattribute utility model.

Subject E was tested qualitatively for his temporal preferences by a procedure that is outlined in Chapter 7.4. He indicated temporal risk aversion, but quantification of constants for the applicable model (7.8b) was not attempted because no elicitation procedure is yet available. Instead , the temporal scaling constants were all set equal 0.3, which implies temporal risk aversion.

8.6. Behaviour of the Model

The model as outlined in Figure 9 is programmed for computer. The program, named BELLMAN, is described in Appendix D and listed in Appendix E. The program was tested on a problem that has been previously solved manually. Results of this test were identical to those obtained by manual computation.

The example problem is based part on assumed data and part on real data. Assumed are Markov transition matrices, resource requirements for actions, and data for calculation of consequences. Real data are parameters of preferences elicited from one of the subjects (Chapter 8.5).

The program prints out (Appendix F) the inputs, and also the computed joint stochastic matrices, the exponential uniattribute utility curves, the matrices of consequences, the type of multiattribute and temporal utility model, and the optimum decisions. The last decision rule printed out is the stationary rule, and it is of greatest interest for the long-term operation of a pavement section. For each joint state the stationary rule specifies an action which after implementation will result in consequences that are optimal relative to the decision maker's preferences. Joint states are explained in Table 4 in terms of states of texture, drainability, roughness and strength. Joint state 1, for example, has all these pavement variables at acceptable levels, whereas in joint state 16 all variables are at unacceptable levels.

The steady-state decision rule for the original data is shown in Table 5 under Run 1. Subsequent runs test the response of the model to input variations. Run 2 has the scaling constant for AGENCY COST increased to 0.30 and for ACCESS decreased to 0.60. The weight of SAFETY thus increases relative to ACCESS. The effect is that SEAL and ROUTINE supplant OVERLAY in states 3, 5 and 6. One can expect this change, for OVERLAY is the most expensive action. The supplanting actions are nearly as effective through pavement texture on the SAFETY objective, which receives relatively

JOINT STATE	STATE OF PAVEMENT VARIABLE 1 = acceptable ; 2 = not acceptable			
	TEXTURE	DRAINABILITY	ROUGHNESS	STRENGTH
1	1	1	1	1
2	2	1	1	1
3	1	2	1	1
4	2	2	1	1
5	1	1	2	1
6	2	1	2	1
7	1	2	2	1
8	2	2	2	1
9	1	1	1	2
10	2	1	1	2
11	1	2	1	2
12	2	2	1	2
13	1	1	2	2
14	2	1	2	2
15	1	2	2	2
16	2	2	2	2

TABLE 4: EXPLANATION OF JOINT STATES

JOINT STATE	STATIONARY DECISION RULES				
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5
1	SEAL	SEAL	SEAL	SEAL	OVERLAY
2	SEAL	SEAL	SEAL	SEAL	OVERLAY
3	OVERLAY	ROUTINE	OVERLAY	ROUTINE	OVERLAY
4	OVERLAY	OVERLAY	OVERLAY	ROUTINE	OVERLAY
5	OVERLAY	ROUTINE	OVERLAY	ROUTINE	OVERLAY
6	OVERLAY	SEAL	OVERLAY	ROUTINE	OVERLAY
7	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY
8	OVERLAY	OVERLAY	OVERLAY	ROUTINE	OVERLAY
9	OVERLAY	OVERLAY	OVERLAY	SEAL	OVERLAY
10	OVERLAY	OVERLAY	OVERLAY	ROUTINE	OVERLAY
11	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY
12	OVERLAY	OVERLAY	OVERLAY	ROUTINE	OVERLAY
13	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY
14	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY
15	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY
16	OVERLAY	OVERLAY	OVERLAY	OVERLAY	OVERLAY

TABLE 5: RESULTS OF FIVE COMPUTATIONAL RUNS

high weight in Run 2, after the main competitor, ACCESS, carries smaller weight.

Run 3 tests whether the policy changes when the scaling constants are normalized to sum up to one. This implies an additive multiattribute utility model. With given data, there is no change in the optimal policy. The result should not be generalized, however, since the output is a result of interactions between the utility, the probability and the other inputs. It is conceivable that with different data and more accurate model there would be a change in the optimal policy. Run 4 has weights for all attributes equal. The effect on the decision rule is substantial, but OVERLAY is still chosen for the most deteriorated states.

Run 5 demonstrates the effect of Markov transition probabilities on the policy. OVERLAY is made more effective on pavement texture relative to Run 1. The change is from 0.50 to 0.90 for p_{11} and p_{21} and consequently SEAL is eliminated from the decision rule. The original set of inputs is not suitable for testing the temporal utility scaling, because the steady-state rule obtains in the first stage iteration. A different data set was made up which produces a three-stage optimal policy. A change in the temporal scaling factors from 0.3 to 0.9 slightly affects the optimal policy, but the stationary decision rule remains unchanged. However, for a scaling constant 0.5 for the most distant stage in the future and linearly decreasing to 0.1 for the present, the policy becomes more protective. ROUTINE is supplanted by OVERLAY - an intuitively correct result for a situation with the future more "important" than the present.

The computer runs have also shown that the model is economically feasible. With five stages, four actions, eight attributes, four pavement variables, and two states for each variable, it takes five seconds of CPU time to compile the program. Once compiled the program require 0.7 CPU seconds to execute a problem. The execution time is roughly proportional to

the number of joint states N,

$$N = l^p$$

where p = number of pavement variables, l = number of states for each pavement variable. Suppose that a problem has p = 4, l = 4. Compared to the present problem (p = 4, l = 2), the CPU will need 16 times longer time to execute, i.e. about 10 seconds. A problem with p = 5 and l = 3 requires approximately the same execution time. At present rates it amounts to about two dollars for CPU time.

CHAPTER 9

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

This chapter starts with conclusions regarding the application of decision analysis to the maintenance of one section. It is followed by a discussion of the pavement maintenance problem. Because the complexities inherent in the pavement problem are widespread in engineering decisions, Chapter 9.3 discusses the applicability of decision analysis to other areas of civil engineering. Chapter 9.4 suggests study areas which may be of interest not only to pavement management but also to civil engineering in general.

9.1. Conclusions

The proposed decision model applied to the optimization of maintenance for one pavement section sastisfies the characteristics required from an ideal method (Figure 3). The only exception is network optimization which was excluded from the scope of this thesis as it requires more research.

Multiple objectives are handled by the multiattribute utility approach. Uncertainty of pavement behaviour is expressed in the form of Markov transition matrices that are updateable and can accommodate engineering judgement. Risks arising from these uncertainties are encoded in the utility functions for single attributes. Group opinion was not included into this thesis but can be routinely accounted for by any model that has probabilities and utilities for inputs. Time dependence is modeled by the temporal utility applied in the Markov decision scheme.

Although the skeleton of the model is conceptually simple, the multiattribute and the temporal utility components create more complexity. Potential implementation obstacles due to complexity are not insurmountable

and occur or do not occur depending on the attitude towards innovation and improvement in the user's organization.

The model's elements have an "overview" effect, and avoid going into too great detail that often is unnecessary. The Markov model of pavement behaviour summarizes the uncertain effects of traffic, climate and materials, and facilitates the inclusion of the time dimension. The multiattribute utility model aggregates incommensurable consequences into one number, and the temporal utility model makes it possible to reduce the number of dimensions by further aggregation.

Data required by the model are commonly available. Markov transition matrices become a standard item of information as the approach is gaining recognition in many departments of transportation and municipalities (9,11,12). Markov transition matrices can be easily established from pavement condition surveys, which now are a common practice.

The utility data derived from the manager's preferences is also obtainable, and the cost and time required are disproportionately low compared to a pavement condition survey. It took about three hours in this research to familiarize an engineer with the objectives and attributes, to check the behavioural assumptions by a qualitative elicitation procedure, and to elicit the trade-offs and the utility curves for eight attributes. Add one hour for fine tuning of answers after consistency checks. Even if a real situation will require ten times as much time for elicitation and revision of preferences after feedback from program runs, one week of work does not seem too excessive for information that can be reused for many pavement sections and for a number of years.

The model behaves well, at least as judged from responses to input changes that can be predicted by intuition. It is recognized, however, that only an extensive testing on real-world data can determine the usefulness of the model. The number of levels for pavement variables must increase and the

formulas for quantification of consequences must improve before useful results can be expected from the model.

The computer runs demonstrate that the model is computationally and economically feasible for the number of variables that more likely will occur in a real application. There is certainly room for improving program efficiency by a professional programmer.

A possible adoption of the model to suit local needs will be enhanced by the flexibility in setting up the structure of objectives, in deciding which pavement variables are important, and in selecting formulas for calculation of consequences. Markov transition matrices also reflect local conditions. They are less expensive to produce than regression equations, and are more accurate models of pavement behaviour for management purposes than are systems of mechanistic formulas.

Without doubt, introducing a new model and then providing data to operate it will cost time and money. However, the user can decide how many variables need to be included and what compromise should be made between available resources and the quality of model's output.

9.2. The Pavement Maintenance Problem

Approaches currently used for optimizing pavement maintenance cannot account even for the most important complexities posed by the pavement system. This research uncovers that decision analysis under uncertainty with recent extensions can effectively approach all the complexities in an unified fashion, and produce a rational answer.

Through the inquiry into the structure of pavement manager's objectives, decision analysis provides a framework for the much needed definition of the problem's scope. The process reveals divergent social interests, identifies uncertain variables of the pavement system, and highlights the importance of including the time dimension into analysis. Once all relevant

factors are sorted out, the pavement manager knows what information must be collected and processed in order to use the decision model. Unlike many management methods studied, decision analysis correctly places the data gathering phase after the problem definition, shifts the emphasis from roughness to other important pavement variables, and signifies those objectives that cannot be translated into money.

Although the additive models prevail in the existing pavement management schemes, their validity is questionable from a rigorous point of view. Objectives of the pavement system interact in a manner resembling a substitution, if not a complementary, effect between attributes. The time span of the pavement problem is sufficiently long to make the temporal risk factors signficant. In short, a rational pavement manager has a preference structure which, according to the multiattribute utility theory, excludes the additive models from consideration.

A Markov decision process formulation is proposed in this thesis, and the expected multiattribute utility appears in the objective function. Introduction of this new criterion does not create any special computational or operational problems in present formulation. The model behaves well, and a real-world problem of optimizing the maintenance of one pavement section will take only a marginal amount of computer time.

If interpersonal comparisons is the issue in pavement management, one will investigate what scheme is most suitable for aggregating various opinions. First of all one may investigate whether the pavement manager may be considered a benevolent dictator of choice - a case most expedient to analyse. Other situations can also be handled as mentioned in Chapter 9.2.

The real problem of maintenance may be a yearly budget which is insufficient to upgrade all sections to the satisfactory state of repair. When all the money is spent on a few sections in a given year, future maintenance of the neglected sections may prove more costly than if the budget

was distributed over the network more uniformly. If so - and research is needed into this problem - then an optimization model must be built to account for network interactions. The criterion may be to minimize adverse effects of underinvestment in maintenance, or equivalent - to maximize the effectiveness of maintenance.

Decision analysis will still be relevant for the network problem. The details of one section behaviour may need to be compressed into a more concise model, and states will represent different variables. Sections will probably be categorized according to their importance. Decision maker's preferences will refer to the whole network region, rather than to the microcosmos of one pavement section. Alternatives may become fractions of the budget assigned to each subnetwork, and be translated into particular activities to match subregions' maintenance practice and resources. Decision analysis will allow one to investigate what sacrifices can be imposed on the users and nonusers in order to fulfill the public objectives like cost minimization and protecting the highway investment. The theoreticians may want to look into the possibilities of developing a multiattribute utility model for spatial systems such as networks.

It seems that extension of the model to network considerations will suppress the resolution on the time dimension. If the maintenance budget can be predicted only a few years in advance, then it makes sense to have an approximate simulation of the network's long-time behaviour and a more accurate model to decide yearly on the distribution of maintenance over the network. The model of Lu and Lytton (10) illustrates this philosophy.

Until a Bayesian decision model is developed for network optimization, the user may want to follow the presently popular approach of section-by-section optimization. The individual optima obtained by a model such as the one proposed in this thesis can be matched yearly with available resources by using a resource allocation model.

9.3. Decision Analysis in Civil Engineering

Civil engineers design, build and operate socio-technical systems. Although the emphasis is on the technical side, the engineer must consider the social, economic and sometimes the political aspects of a problem as well. This fact gives rise to multiple objectives and the implications of group opinion must inevitably follow. The engineering systems are built to serve for a length of time comparable to the span of human life. The time dimension cannot be neglected, but it introduces uncertainties about the future and makes it necessary to consider risks. Other types of uncertainty and risk enter many problems because civil engineering works are built of variable materials, are subject to random loads and environmental effects, and often crucial information required by the engineer is vague.

Decision analysis appears extremely relevant for civil engineering practice. Numerous applications in a wide selection of engineering areas have already demonstrated that this method is appropriate for most realistic and complex problems, and can be verified in practice. Intangibles such as safety, environmental impact, aesthetics and political feasibility are included beside objectives expressible in money. Uncertainties are effectively handled by probabilities of factors that are under the decision maker's control or outside his control.

Time dependencies are accounted for by dividing the analysis horizon into a suitable number of periods. Appropriate weights are assigned to periods, given their relative importance and the certainty with which the variables are known for each period. Temporal risk sensitivity is incorporated into the model, as well as the attitudes associated with trade-offs between objectives. The interests of many groups can be included in the evaluation of alternatives, the conflicts illuminated and resolved.

Decision analysis can remove many potential implementation difficulties. The method can be universally applied to problems infiltrated by any

subset of the above complexities. It represents the real problem relatively faithfully, is conceptually simple, and has fairly flexible requirements for data. Decision analysis breaks the complex decision task into parts that can be independently assessed by various experts, and then combined with due regard to interactions. The component parts reflect the decision making process and are laid out in a clear scheme. With only a reasonable effort the user can understand the model and its function.

If necessary, the method can rigorously use information based on engineering judgement in addition to the objective data. The updating of information can also be routinely handled. Decision analysis can accommodate component models which best suit the practical requirements of the problem at hand. These flexibilities constitute a most important asset, since engineering decisions often have to be made with only limited data available.

Decision analysis is in fact a major part of the scheme for the scientific method of inquiry. Once a problem is identified, decision analysis provides a structured methodology for the formulation of the problem, the modeling of the identified system, and the generation and evaluation of alternatives. It provides an aid which can assist the decision maker in difficult problems of choice. He can also employ it as a learning tool. The impacts of different alternatives may be assessed analytically, and this information used as a feedback to improve the next decision. Because the model has a meaningful structure, the effect of a change in inputs can be evaluated by sensitivity analyses. These will help the user to gain further insight into the problem.

Through supplying the model, the analyst does not usurp the decision making capacity. The modeling process requires a strong interaction with the user in all phases. With the analyst's help, the decision maker is forced to think hard about the objectives, the trade-offs, and the values he

attaches to different consequences of decisions. The answers provide a function which is unique for a decision maker, in that it reflects his - not the analyst's - relative desirability of each consequence.

Some managers may object to the time required for this elicitation process, but no substitution exists for decision maker's judgement. Until an alternative method is invented, decision analysis is superior for those people who want to make, or who help others to make, rational choices in complex situations.

9.4. Promising Study Areas in Decision Analysis

Many researchers point out that the inaccessibility of decision makers is a main obstacle in decision analyses of major projects. It may be worthwhile to investigate how far the analysis can be carried out without an accurate knowledge of the decision maker's preference structure. Simplifications will most likely be discovered through sensitivity analyses, once a decision model is set up for a particular problem. In any case, aids such as interactive computer programs that can quickly teach a decision maker about his preferences, may substantially reduce the problem of unavailable preference data.

Nevertheless, the popular belief that senior managers do not have time to have their preferences quantified must be investigated. The time necessary for elicitation is reasonably short and it appears that rather than reflect a time problem, the belief may have a different reason; the managers may fear to lose part of their decision making authority if a utility model is applied. It should be clear at this point that the fear is not justified. Decision analysis should be seen by the managers as a powerful tool that can help them to arrive at a better decision more efficiently, with full use of their value judgements. This argument is particularly valid for complex

problems where few if any managers are able to process all variables rationally without resorting to analytical aids.

The preceding objection can be softened by educating the managers on the strengths and on the nature of decision analysis. Some managers object to analysis because they do not want to disclose the preferences for political reasons. There is not much one can do to abate such a stance without upsetting the distribution of political power. Public servants, however, may be persuaded by the increasing demands from the public for better accountability of the governments they elect.

In the light of new findings in normative theory of choice, it is mandatory to revise the additive evaluation models which practically dominate the analytical fields of transportation, urban and regional planning, environmental impact studies, and many other civil engineering and related endeavours. This research has found an evidence of interactions between decision criteria, for which additive models cannot account.

There appears to be a fruitful ground for researching new approaches to optimization. Many practitioners question the usefulness of predicting the "best" choice, because the present rate of change may make the calculations obsolete before an alternative is implemented. The philosophy is to take actions which allow the decision maker the greatest flexibility to adapt to the future, rather than search for the absolutely best solutions given present information. Multiple criteria evaluation models may be adapted to suit this requirement by introduction of an additional objective which will measure actions' flexibility.

Developments in temporal utility theory should be studied, for one may expect findings useful for problems where time is a factor. Interesting formulations emerge than can realistically handle sequential analyses (22). By relaxing the temporal utility assumptions, the so-called semi-separable form is obtained. It generalizes the additive and the multipli-

cative forms at the expense of a more complex structure. One may want to investigate whether the added complexity has a significant impact on the results. The analyst will find the semi-separable function fairly useful, however, for problems where time can be divided into an immediate future for which all factors are reasonably well known, and a distant future - the vague and ill-perceived years beyond the horizon. Other utility structures are being developed for streams of consequences with an uncertain horizon, and for situations where past experience does affect preferences for the future. Both cases may be relevant for sequential analyses in civil engineering.

With respect to group decisions, some philosophical questions about aggregating individuals' probabilities and utilities remain unanswered, but the engineer will find many workable approaches - both axiomatic and intuitive (22,38,56,58). The decision model need not be reshaped to accept inputs representative of a group, because these are transformed into functions compatible with the model.

9.5. Final Comment

The writer feels that the thesis accomplishes the objective stated in Chapter 3. The pavement maintenance problem has been structured for one section with due regard to all relevant complexities. A new optimization model has been synthetized from powerful developments in decision analysis and pavement theory. The model accounts for all complexities of the pavement system except network optimization, which can, hopefully, be attacked by a decision analytical approach in future research.

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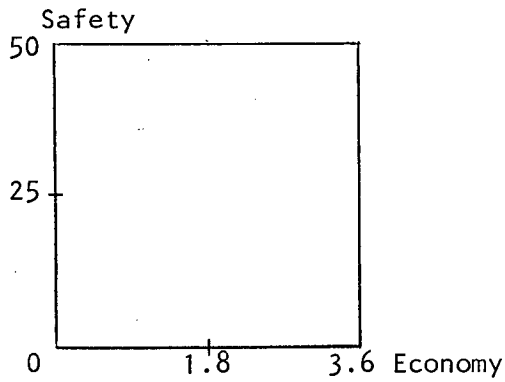
APPENDIX A

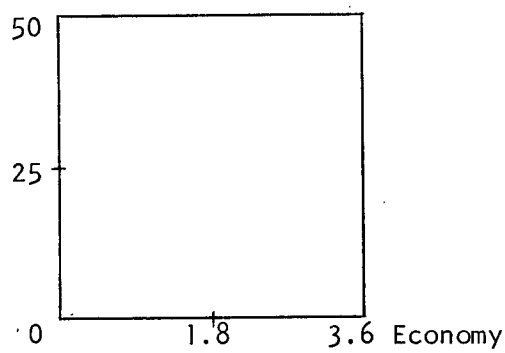
FORMS FOR TESTS ON PREFERENCES

These forms are used to examine qualitatively if a subject's preferences change when levels of the complementary attributes vary. The procedure is described in Chapter 8.5. The first form is for testing preferential independence for "user safety" and "economy". Similar forms were prepared for the remaining pairs of attributes but are not reproduced here. The second form is for testing utility independence for "user safety". Similar forms for the other attributes are not reproduced.

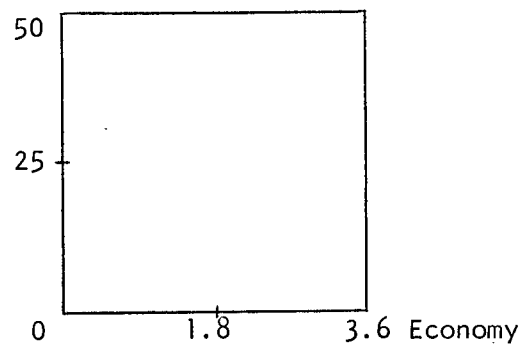
TRADE-OFFS BETWEEN SAFETY (10^3ADT) AND ECONOMY (10^6\$/yr)



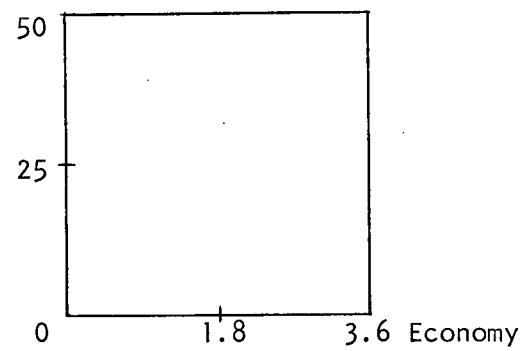
Time	0.7 mln hrs/yr
Cost	1.6 mln \$ /yr
Access	50,000 ADT
Jobs	240 jobs/yr
Gravel	32 tonnes/yr
Noise	2,000 households



Time	0.7 mln hrs/yr
Cost	1.6 mln \$ /yr
Access	50,000 ADT
Jobs	0
Gravel	0
Noise	0

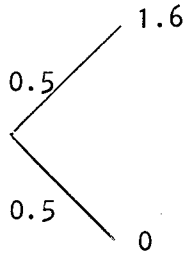


Time	0
Cost	1.6 mln \$/yr
Access	50,000 ADT
Jobs	240 jobs/yr
Gravel	32 tonnes/yr
Noise	2,000 households

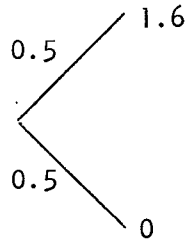


Time	0
Cost	0
Access	0
Jobs	0
Gravel	0
Noise	0

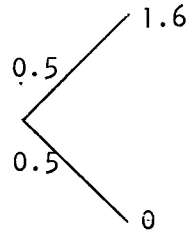
LOTTERIES FOR AGENCY COST (10^6 \$/yr)



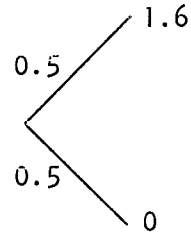
Safety	50,000 ADT
Economy	3.6 mln \$/yr
Time	0.7 mln hrs/yr
Access	50,000 ADT
Jobs	240 jobs/yr
Gravel	32 tonnes/yr
Noise	2,000 households



Safety	50,000 ADT
Economy	3.6 mln \$/yr
Time	0.7 mln hrs/yr
Access	50,000 ADT
Jobs	0
Gravel	0
Noise	0



Safety	0
Economy	0
Time	0
Access	50,000 ADT
Jobs	240 jobs/yr
Gravel	32 tonnes/yr
Noise	2,000 households



Safety	0
Economy	0
Time	0
Access	0
Jobs	0
Gravel	0
Noise	0

APPENDIX B

DERIVATION OF CONSTANTS FOR EXPONENTIAL UTILITY CURVES

Applications of multiattribute utility theory (Chapter 4.2) have demonstrated that utility curves with constant risk properties can approximate decision maker's behaviour accurately enough for a first cut analysis and sometimes sufficiently even for a more detailed analysis. The curves of Chapter 8 are constructed using this approach.

It can be shown graphically that an increasing exponential function that is bounded between 0 and 1 has this form

$$u(x) = a(1 - \exp(cx));\ x \geq 0$$

where $a < 0$, $c > 0$ for a risk prone behaviour and $a > 0$, $c <$ for a risk averse behaviour. Two points of this curve are defined by the required scaling.

$$u(WORST) = 0$$
$$u(BEST) = 1$$

where WORST is the minimum (least desirable) and BEST is the most desirable x. The WORST value equals 0 for all attributes in the example problem. A third point will be specified by the decision maker. The simplest candidate for this point is the x (labelled HALF) whose utility equals 0.5,

$$u(HALF) = 0.5$$

When HALF/BEST < 0.5 then the utility function is concave and the decision maker risk averse. When HALF/BEST > 0.5 then the function is convex (risk prone). When HALF/BEST = 0.5 the function is straight line (risk neutral).

<u>Risk prone case</u>

$$u(x) = a(\exp(cx) - 1));\quad 0 \leq x \leq BEST,\ a > 0,\ c > 0 \tag{B.1}$$

$$cx = \ln a - \ln(a + u(x))$$

I. $u(HALF) = 0.5$

$$cHALF = \ln a - \ln(a + 0.5) \tag{B.2}$$

II. $u(BEST) = 1$

$$cBEST = \ln a - \ln(a + 1) \tag{B.3}$$

Dividing (B.2) by (B.3) yields

$$HALF/BEST = (\ln a - \ln(a + 0.5))/(\ln a - \ln(a + 1)),\ a > 0 \tag{B.4}$$

Constant a can be iterated from this equation and then c can be obtained from (B.3).

<u>Risk averse case</u>

$$u(x) = -a(\exp(-cx) - 1),\ 0 \leq x \leq BEST,\ a > 0,\ c > 0 \tag{B.5}$$

$$cx = \ln a - \ln(a - u(x))$$

I. $u(HALF) = 0.5$

$$cHALF = \ln a - \ln(a - 0.5) \tag{B.6}$$

II. $u(BEST) = 1$

$$cBEST = \ln a - \ln(a - 1) \tag{B.7}$$

Dividing (B.6) by (B.7) yields

$$HALF/BEST = (\ln a - \ln(a - 0.5))/(\ln a - \ln(a - 1)),\ a > 1 \tag{B.8}$$

Iteration by computer will yield constant a, and c can be calculated from (B.7)

Figure 11 is a sketch of HALF/BEST as given by (B.4) and (B.8). A suitable transformation of the risk averse case can make the function identical to the risk prone case. To show this, notice that if the lower curve is shifted to the origin it will form a symmetrical image of the upper curve provided both have the same shape.

$$\begin{aligned}
&1 - HALF/BEST = \\
&\quad = 1 - (\ln(a + 1) - \ln(a + 1 - 0.5))/(\ln(a + 1) - \ln(a + 1 - 1) \\
&\quad = 1 - (\ln(a + 1) - \ln(a + 0.5))/(\ln(a + 1) - \ln a) \\
&\quad = (\ln(a + 1) - \ln a - \ln(a + 1) + \ln(a + 0.5))/(\ln(a + 1) - \ln a) \\
&\quad = (\ln(a + 0.5) - \ln a)/(\ln(a + 1) - \ln a),\ a > 0
\end{aligned}$$

and the right hand side of this equation is indeed identical to the risk prone case (B.4). Iteration for both cases can then be handled by one procedure, provided the following transformations are made in the risk averse case:

HALF transformed = BEST - HALF

and

a = 1 + a calculated.

The program for iterating the constants from HALF/BEST ratio is described in Appendix D.

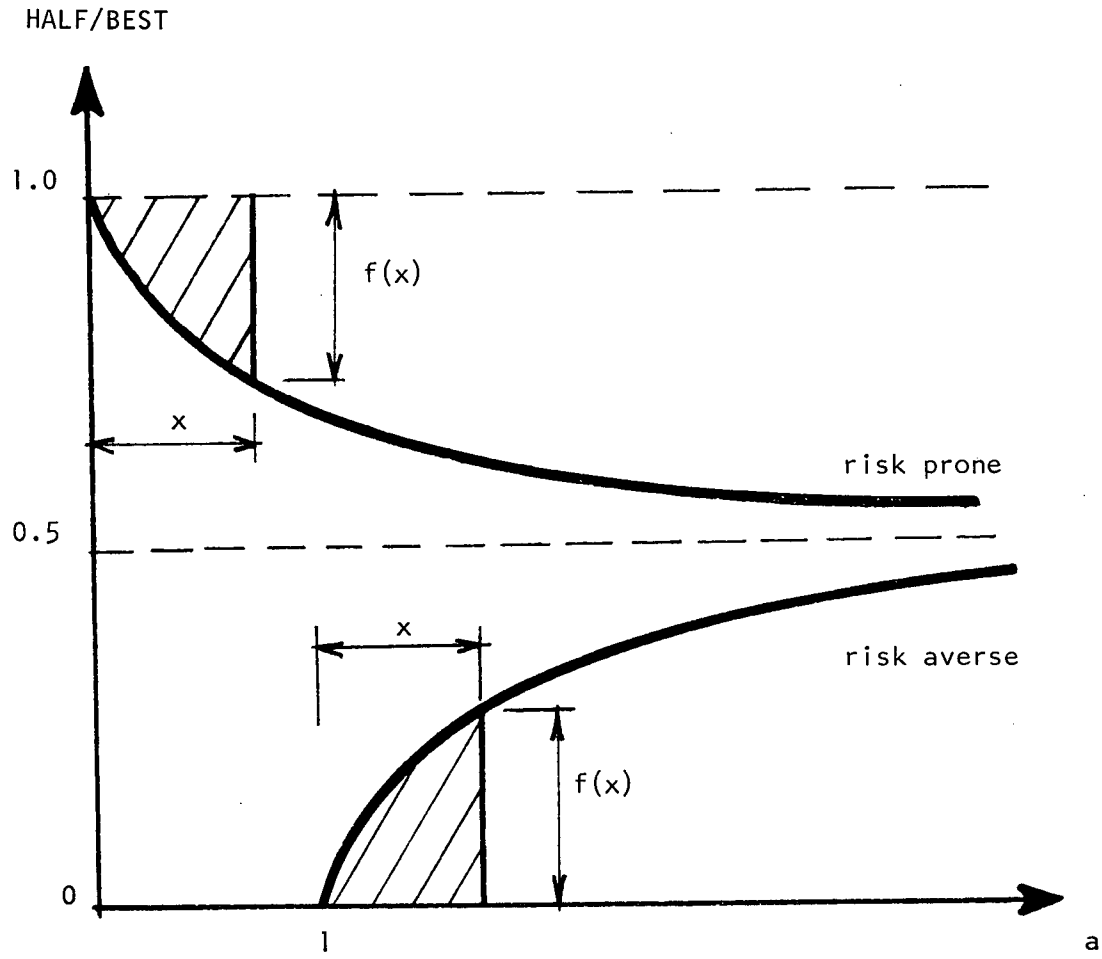


FIGURE 11. SKETCHES OF THE HALF/BEST FUNCTIONS

APPENDIX C

RESULTS OF QUALITATIVE ASSESSMENT

The tests were carried out on subjects A, B, C, D and E using forms shown in Appendix A and according to a procedure described in Chapter 8.5.

Figures 12 to 18 correspond to one trade-off pair of attributes each, and Figures 19 to 23 correspond to certainty equivalents for one attribute each. Cases 1 to 4 of complementary attributes are on the abscissa. The unit on the ordinate is percent of the BEST (most desirable) value of the "more important" attribute. Percentages, as scaled from the respond sheets with 5 percent precision, rather than absolute units are used to make comparisons between trade-off pairs possible. Note that there are differences of opinion as to which attribute is "more important" (Figures 14, 15, 17, 18).

Letters on the graphs denote subjects and lines connect their answers scaled from questionnaire sheets. Mean and standard deviation are given for those subjects who stated that their trade-offs or risk attitudes did not change with Cases 1 to 4. One can expect some variance in answers from one subject even if the person verbally states that changing the cases from 1 to 4 does not affect the trade-offs. It is interesting to note that subjects A and D exhibited a standard deviation of up to 13 percent of the maximum attribute value, but maintained that their preferences did not change.

Figures 12 to 18 show trends for trade-offs made in the upper right hand corner of the trade-off square. In the middle region of the trade-off domain, the pattern was very similar, except for the magnitude of the trade-off. The standard deviations never exceeded 8 percent and subjects C and D exhibited a change in trade-offs at cases 3 and 4 for one attribute each. In the lower left corner of the square all subjects had difficulty to make

accurate trade-offs due to small quantities involved. Subjects B, D, E stated that their trade-offs would not be affected by the variation of cases 1 to 4. Subjects A and C indicated that their trade-offs would be similar in direction and proportional in magnitude to trade-offs in the upper right corner.

The graphs suggest that for subjects A, B and E the preferential independence assumptions hold well. Subject C violated preferential independence assumptions for three out of seven trade-off pairs, while D violated one assumption. The violations occur at cases 3 and 4 of the complementary attributes, but are relatively small and within 5 to 10 percent of the maximum attribute value relative to cases 1 and 2.

The utility independence assumption is fulfilled by subjects A, B, D and E. Subject C was utility independent for one of the two attributes tested, but stated a change in the certainty equivalent with case 4.

In conclusion, the preferential and utility independence assumptions appear to be a good approximation for three of the five subjects, and may be a workable approximation for the remaining two subjects.

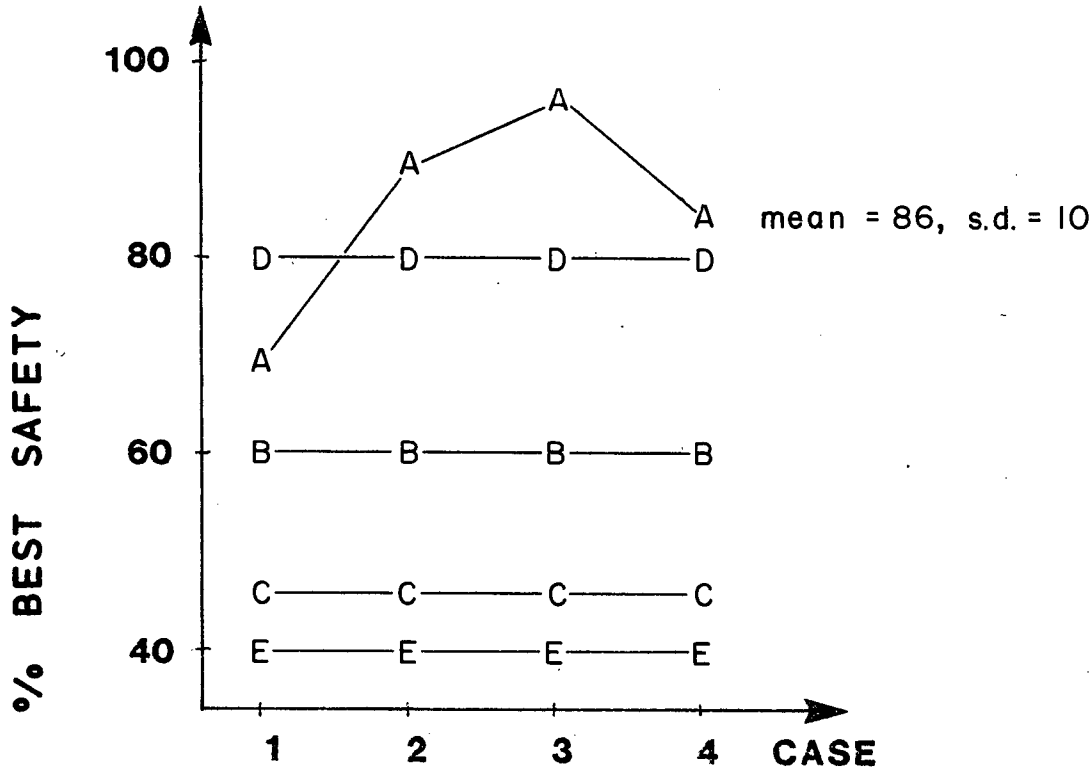


FIGURE 12: TRADE-OFF USER SAFETY AND ECONOMY

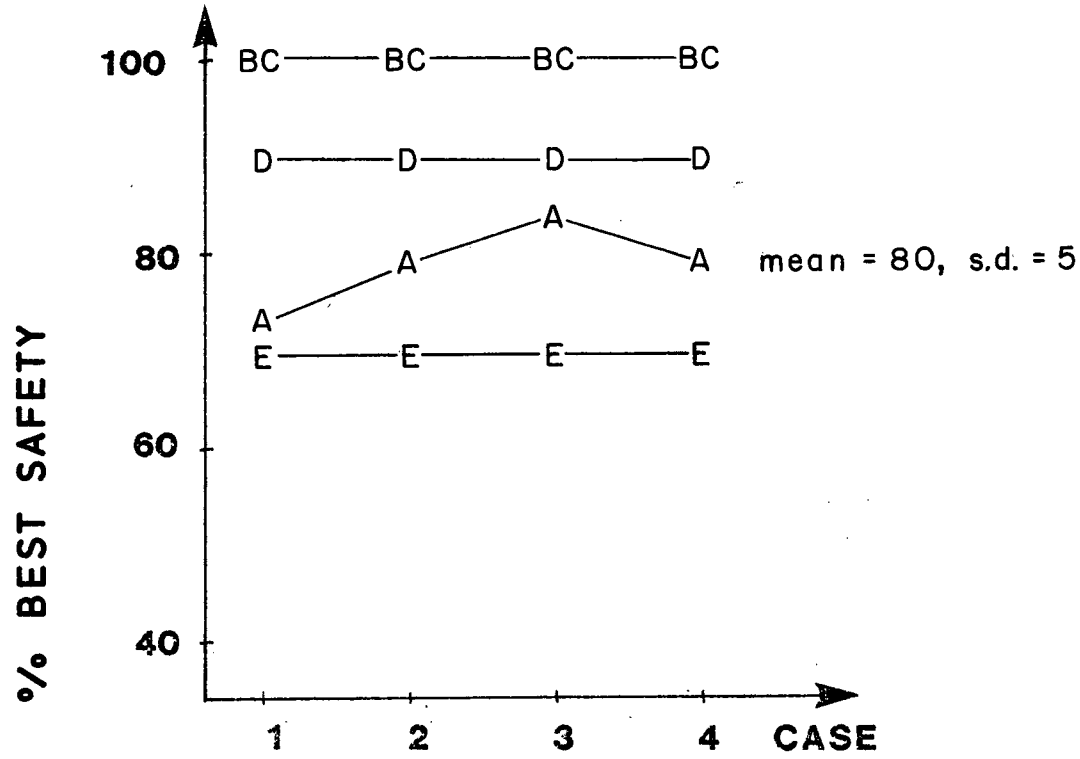
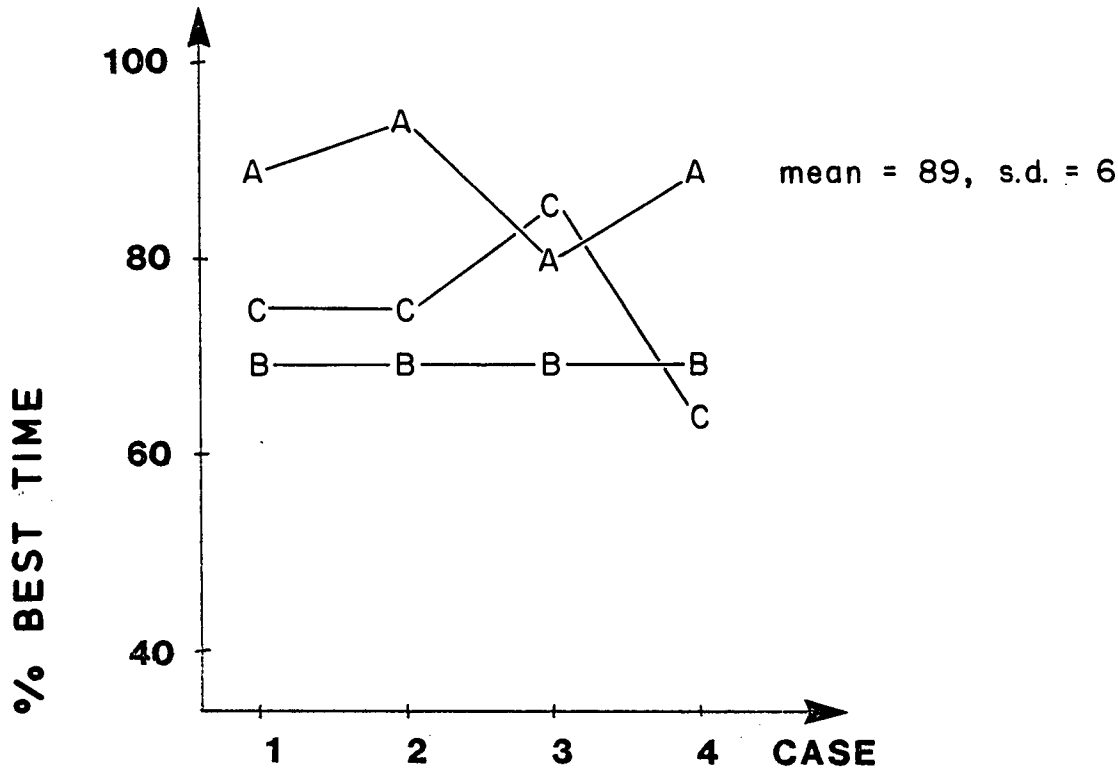


FIGURE 13: TRADE-OFF USER SAFETY AND NOISE



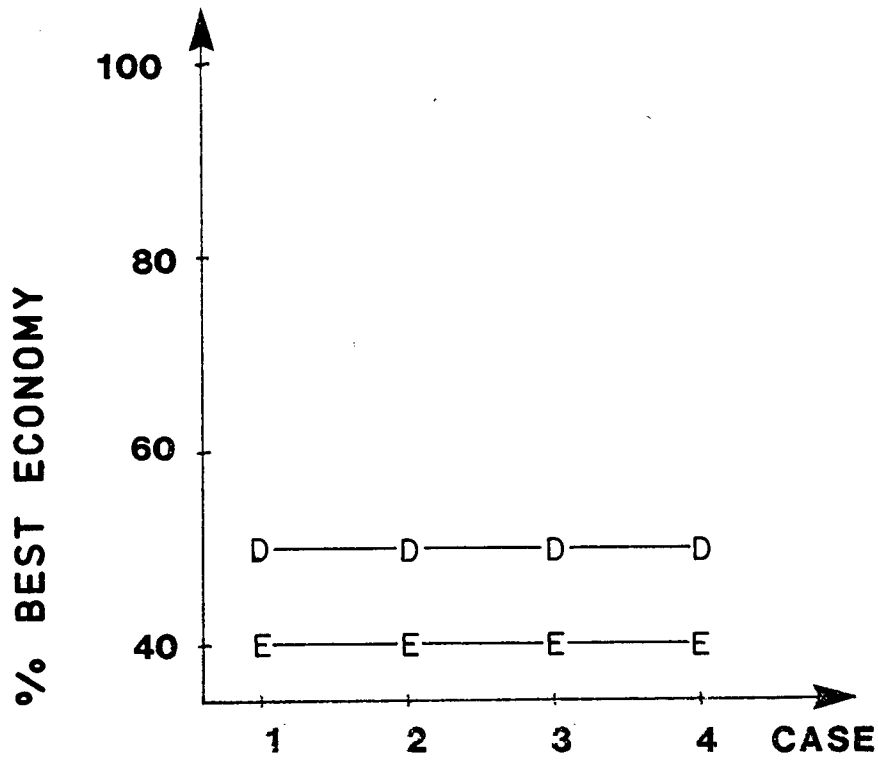
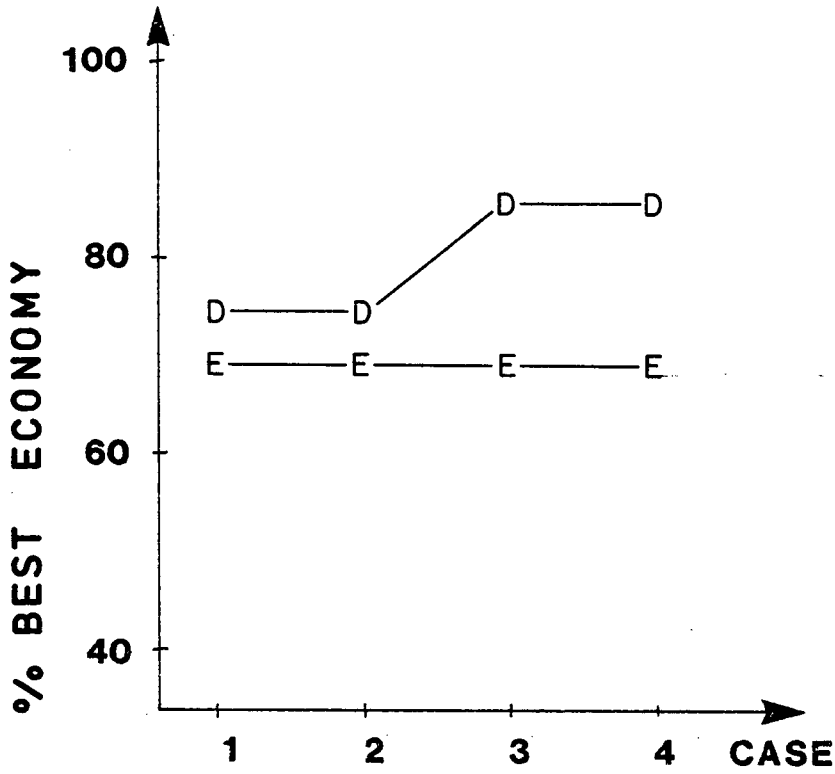
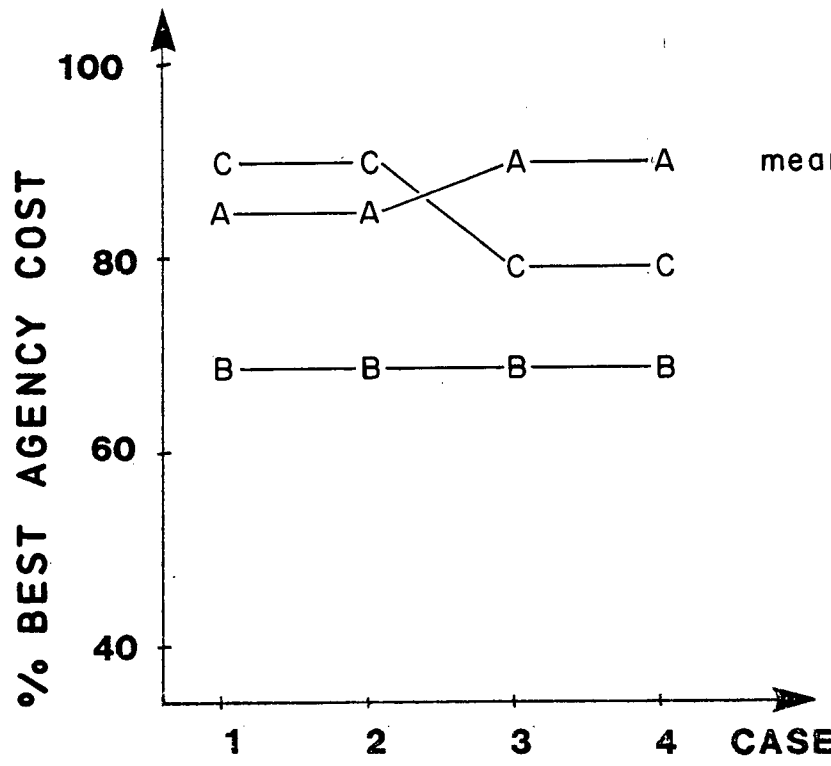


FIGURE 14: TRADE-OFF USER ECONOMY AND TIME





mean = 88, s.d. = 3

FIGURE 15: TRADE-OFF USER ECONOMY AND AGENCY COST

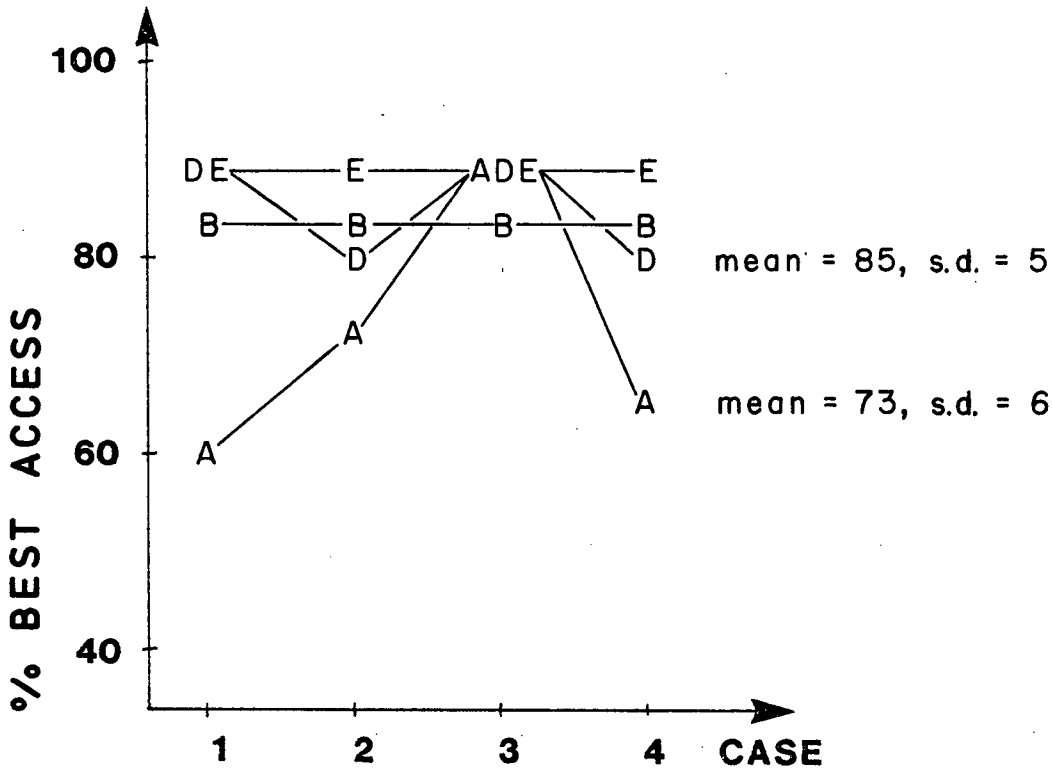
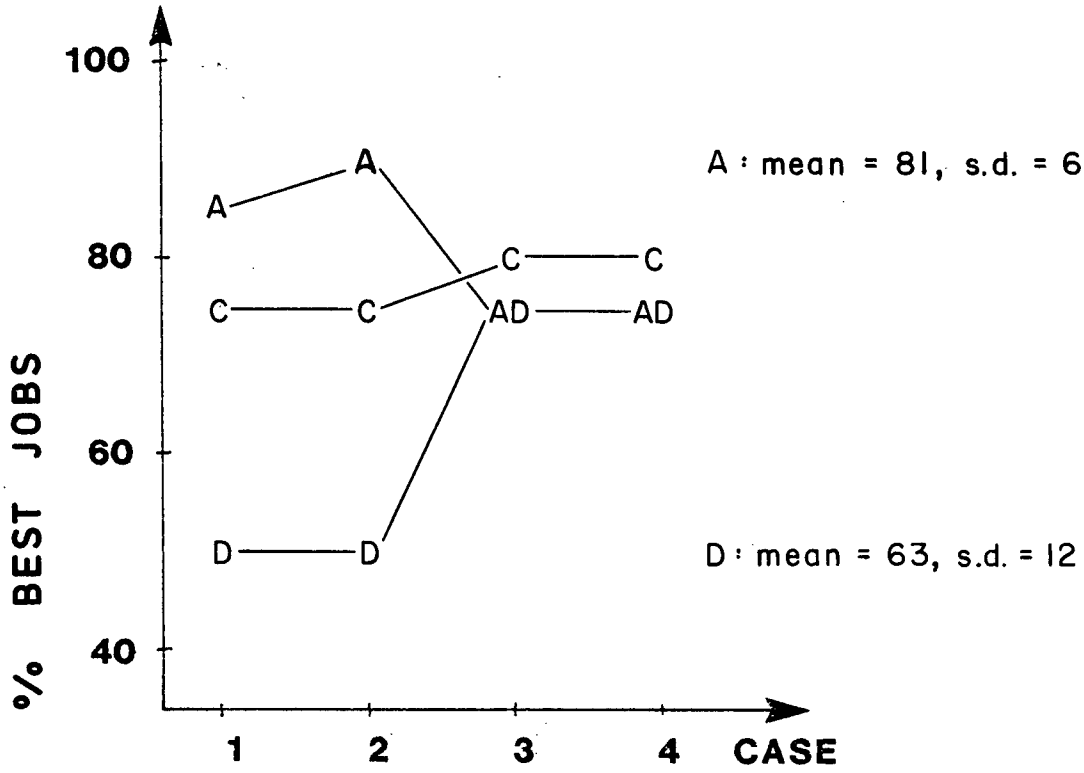


FIGURE 16: TRADE-OFF AGENCY COST AND ACCESS



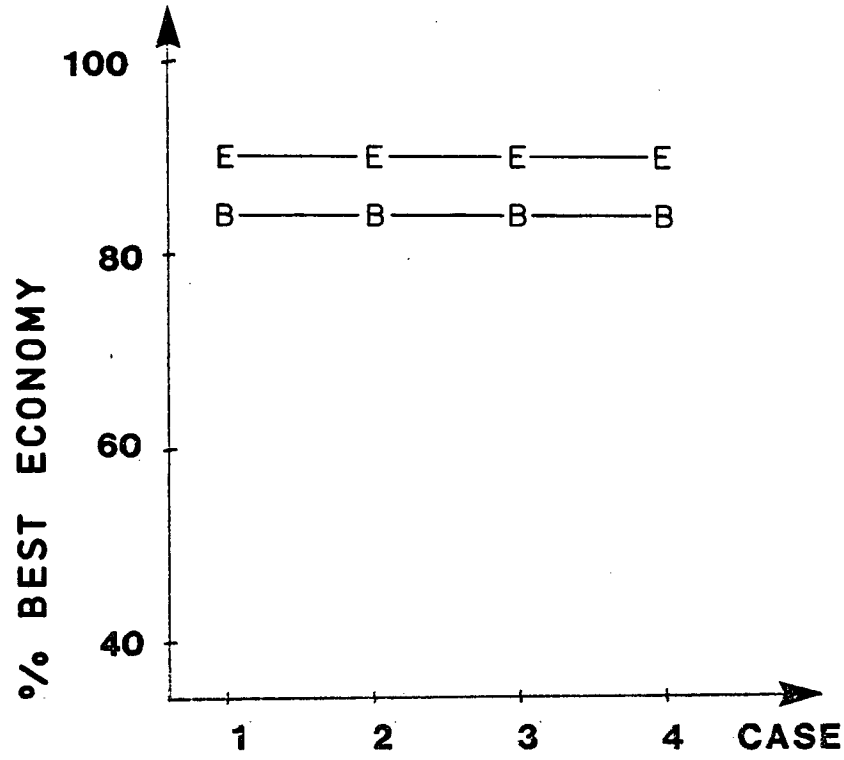
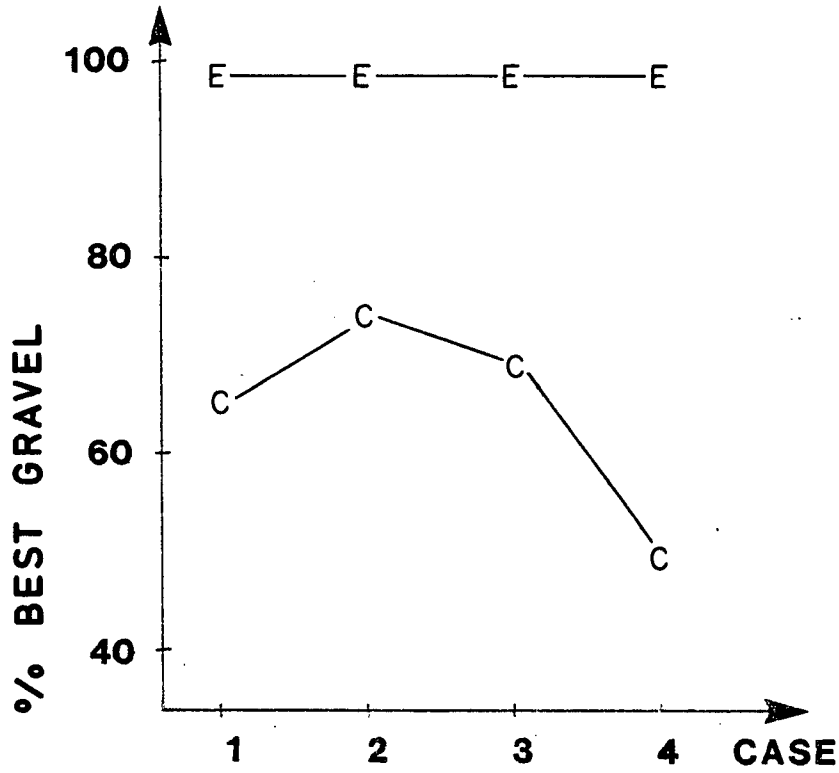


FIGURE 17: TRADE-OFF USER ECONOMY AND JOBS



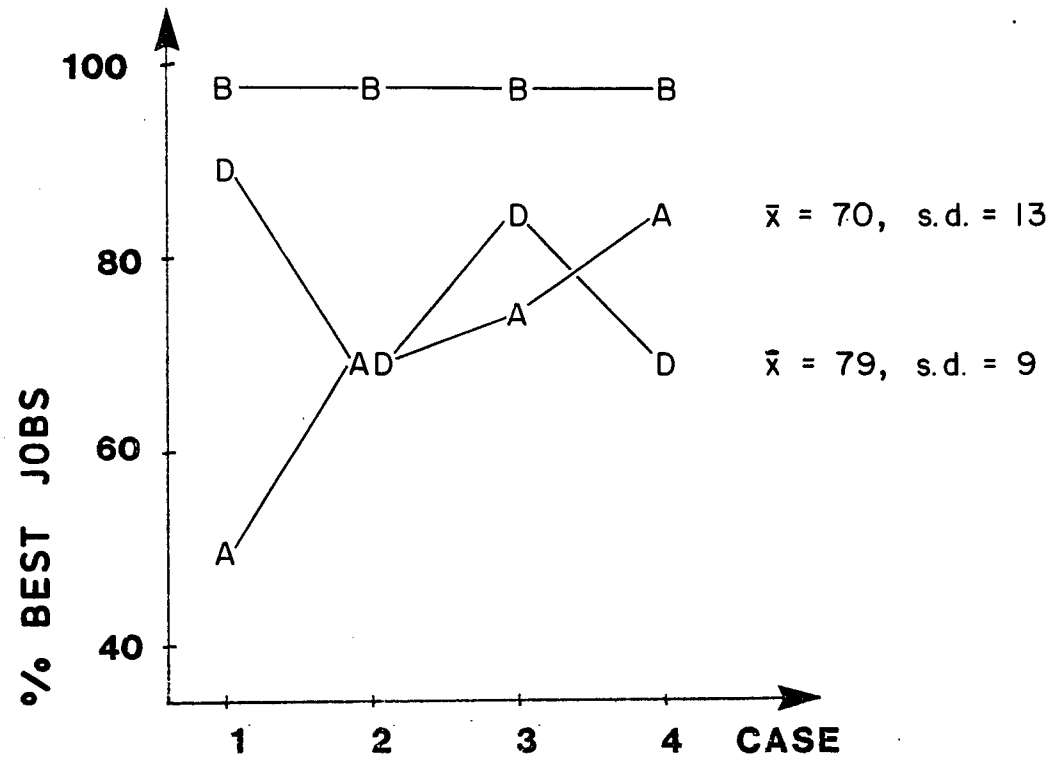
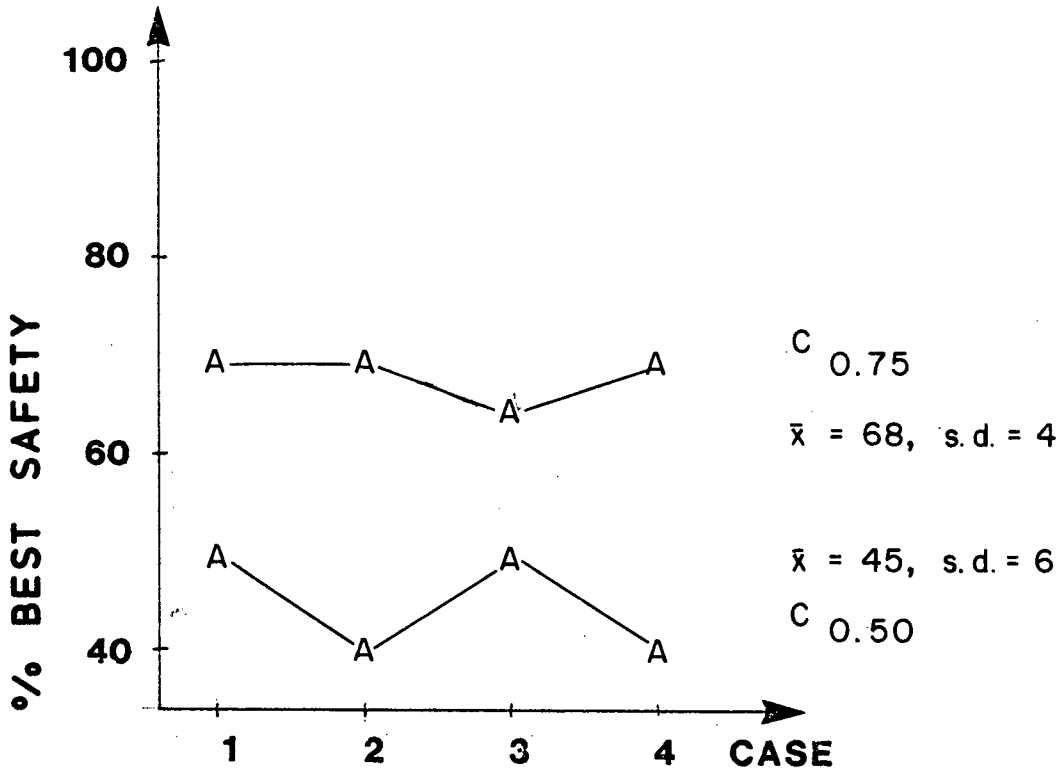


FIGURE 18: TRADE-OFF JOBS AND GRAVEL



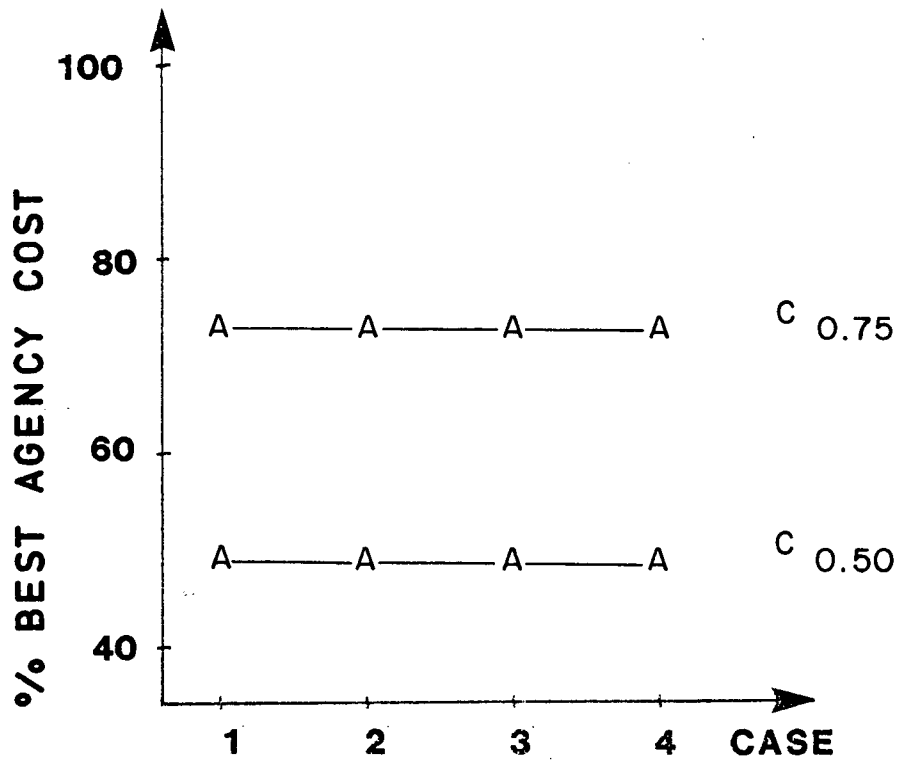
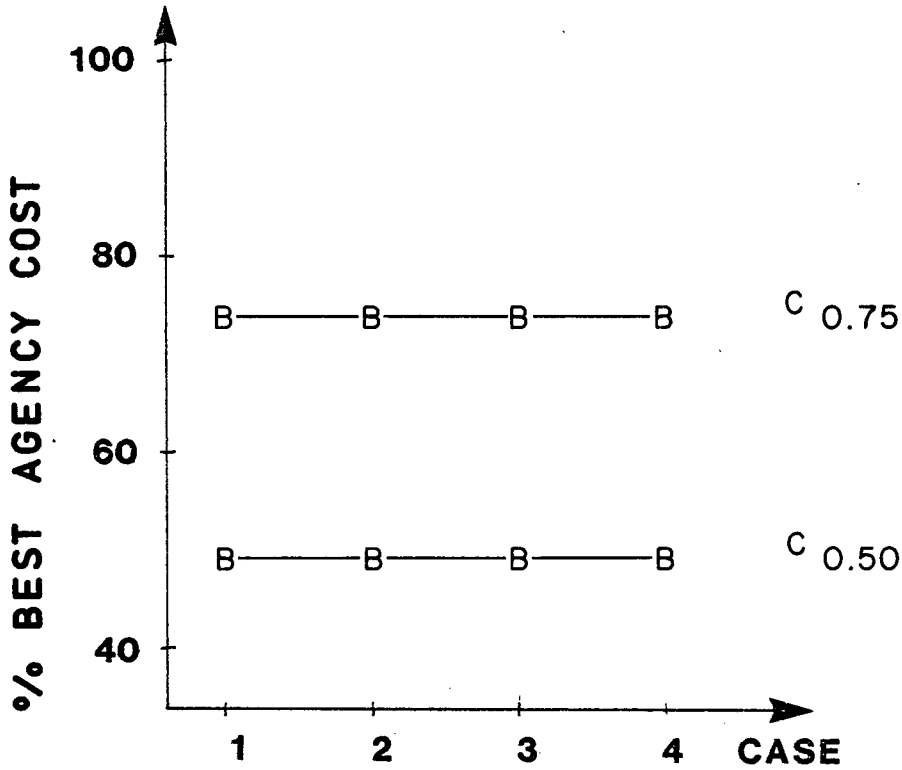


FIGURE 19: CERTAINTY EQUIVALENTS FOR SUBJECT A



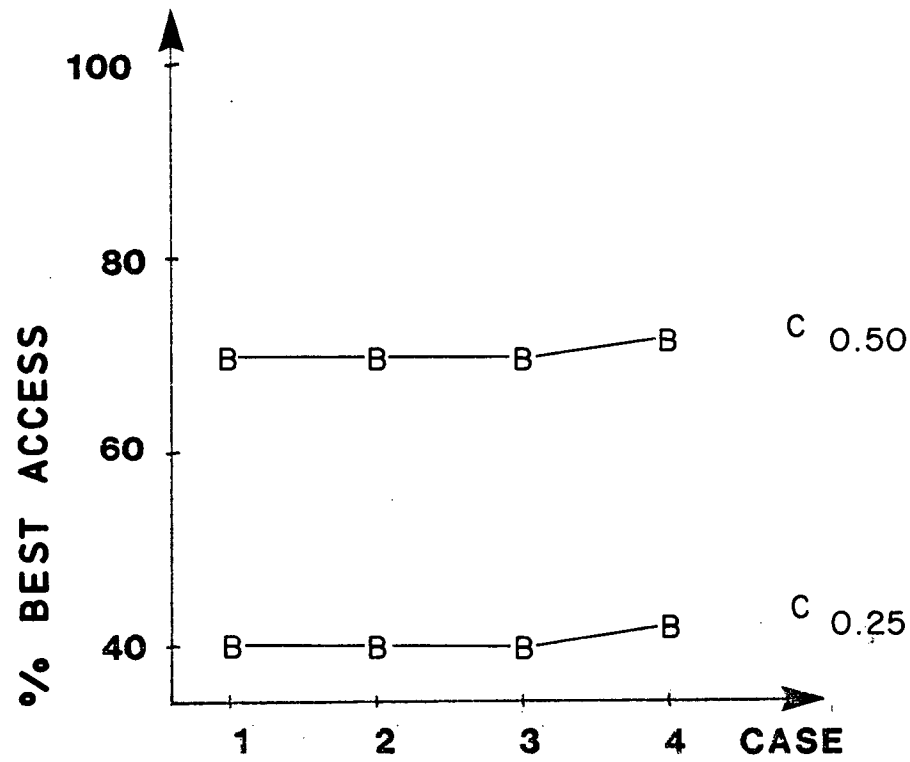
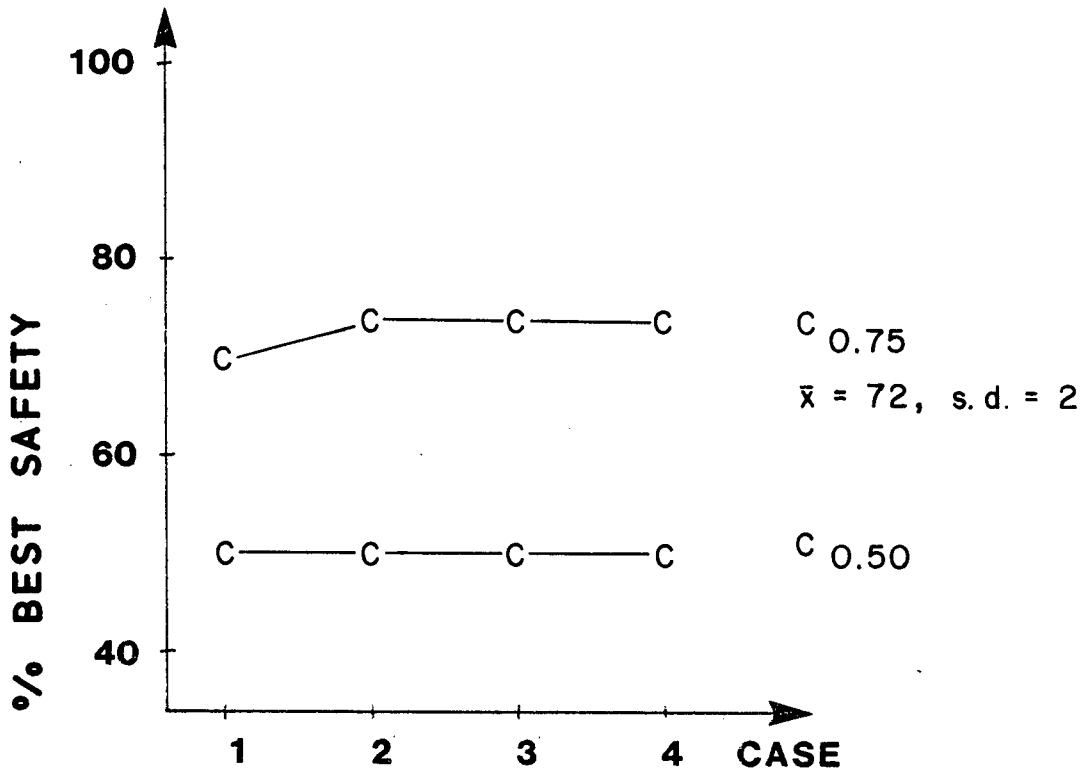


FIGURE 20: CERTAINTY EQUIVALENTS FOR SUBJECT B



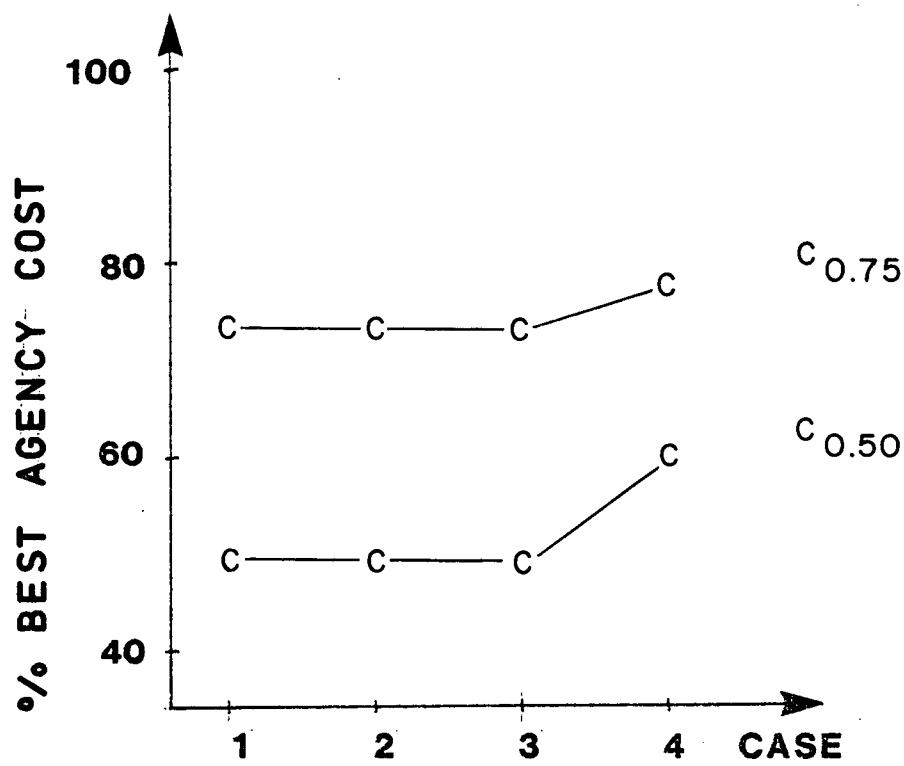
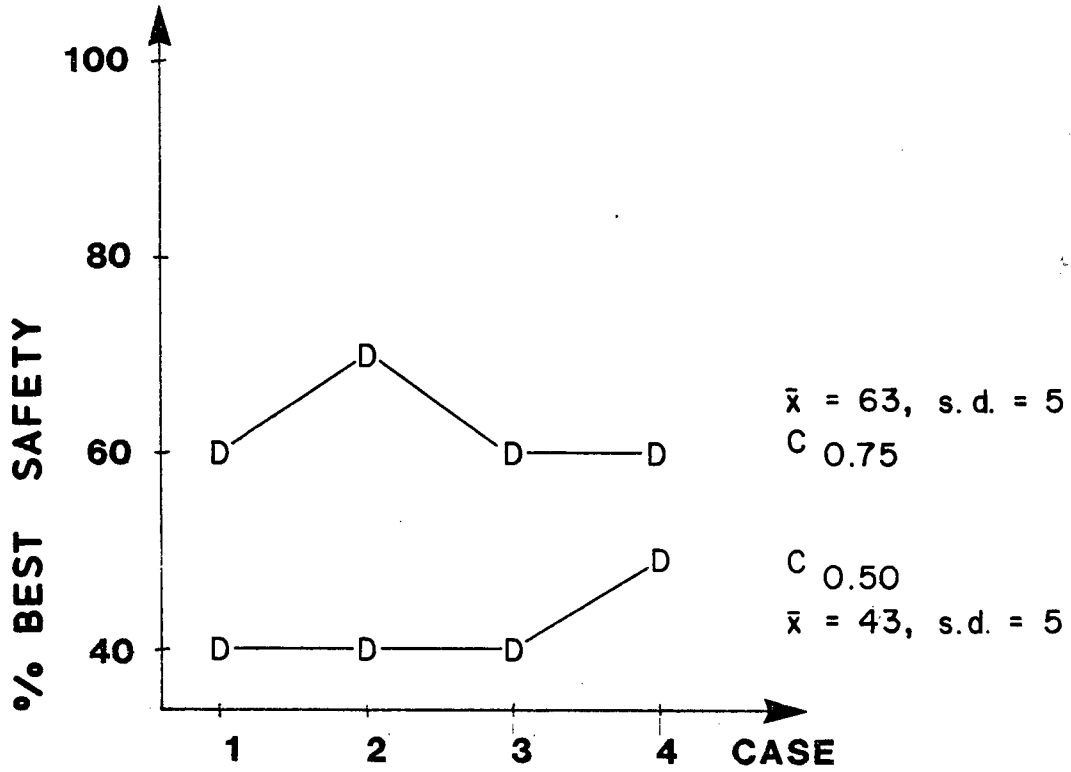


FIGURE 21: CERTAINTY EQUIVALENTS FOR SUBJECT C



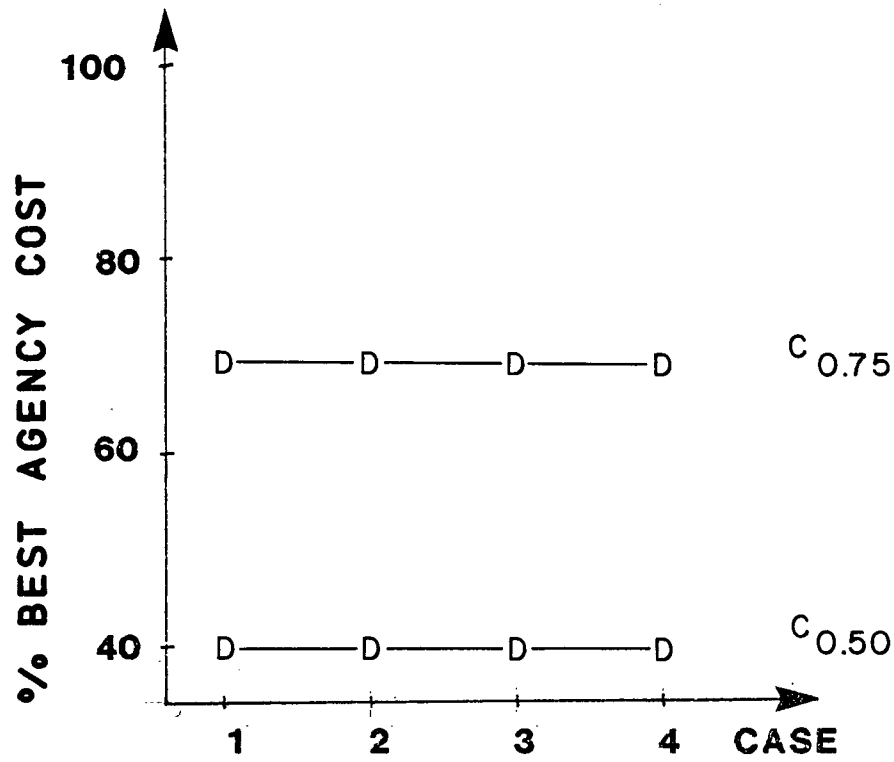
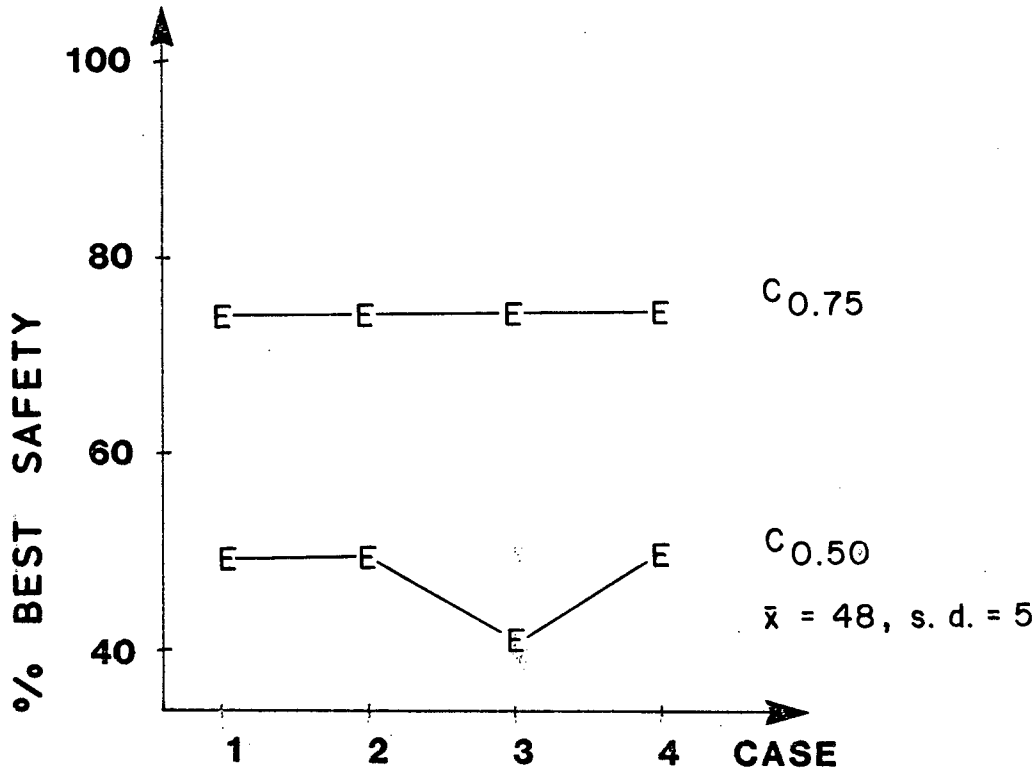


FIGURE 22: CERTAINTY EQUIVALENTS FOR SUBJECT D



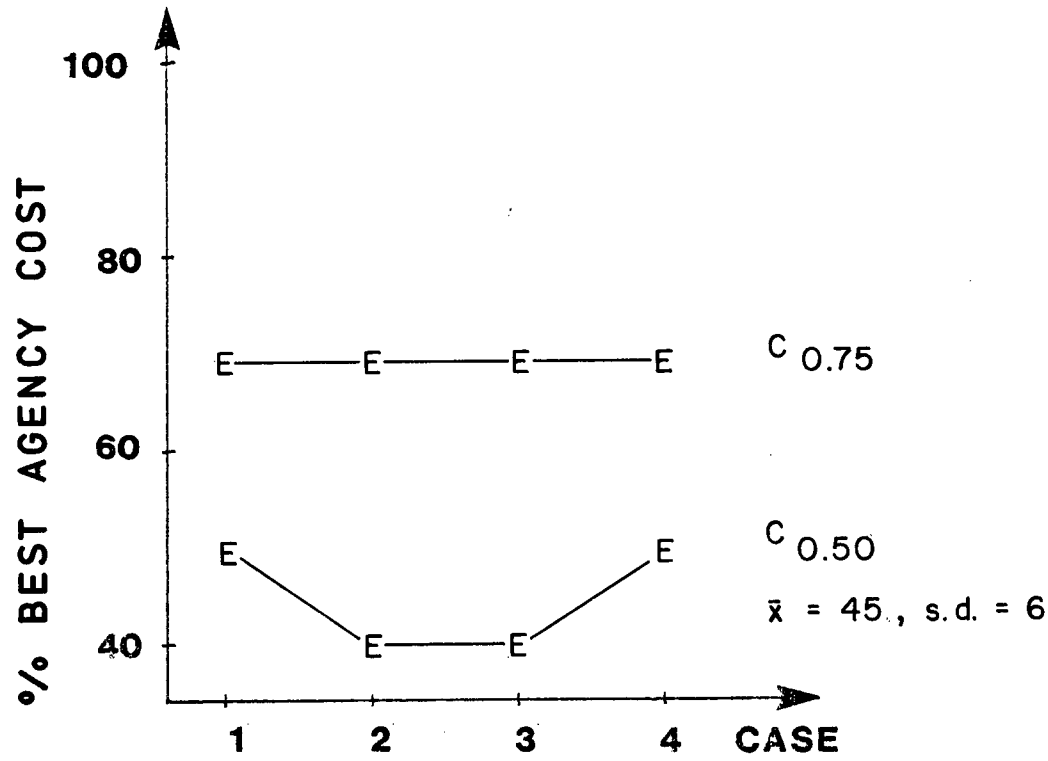


FIGURE 23: CERTAINTY EQUIVALENTS FOR SUBJECT E

APPENDIX D

COMPUTER PROGRAM 'BELLMAN'

The program, called BELLMAN, is written in PL1 which offers advantages over FORTRAN in handling matrices, data conversion, input and output formats, language efficiency and program structuring. The main program is divided into modules corresponding to different parts of the decision analysis. The structure is shown in Figure 24. Appendix E contains listing of the program.

Input files:

Raw data are stored in files for easy overview and changes at a conversational terminal. The contents of these files are read in at the beginning of BELLMAN and preprocessed for use in the subprograms that follow. Note that each line in an input file must end with a comma.

File MISCELL contains miscellaneous data and program operating variables. File ACTIONS lists names for available actions, their costs and other resource requirements. File PAVARS lists names of pavement variables and elements of Markov transition matrices for each action. File ATTRIBS lists names of attributes, units of measurement, the HALF/BEST ratios and the scaling constants for the multiattribute model (6.5). File TEMPOS contains the temporal scaling constants, KT(T), and the terminal value of temporal utility, TVSTAR, for the model (7.8).

Subprogram MARKOV:

MARKOV computes the joint stochastic matrix Q for each action as outlined in Figure 25. The subprogram takes the Markov transition matrix of first pavement variable P(1) and scalar multiplies it by the transition matrix of the second pavement variable P(2). The result is then scalar multiplied by the third transition matrix, and so on until the last pavement

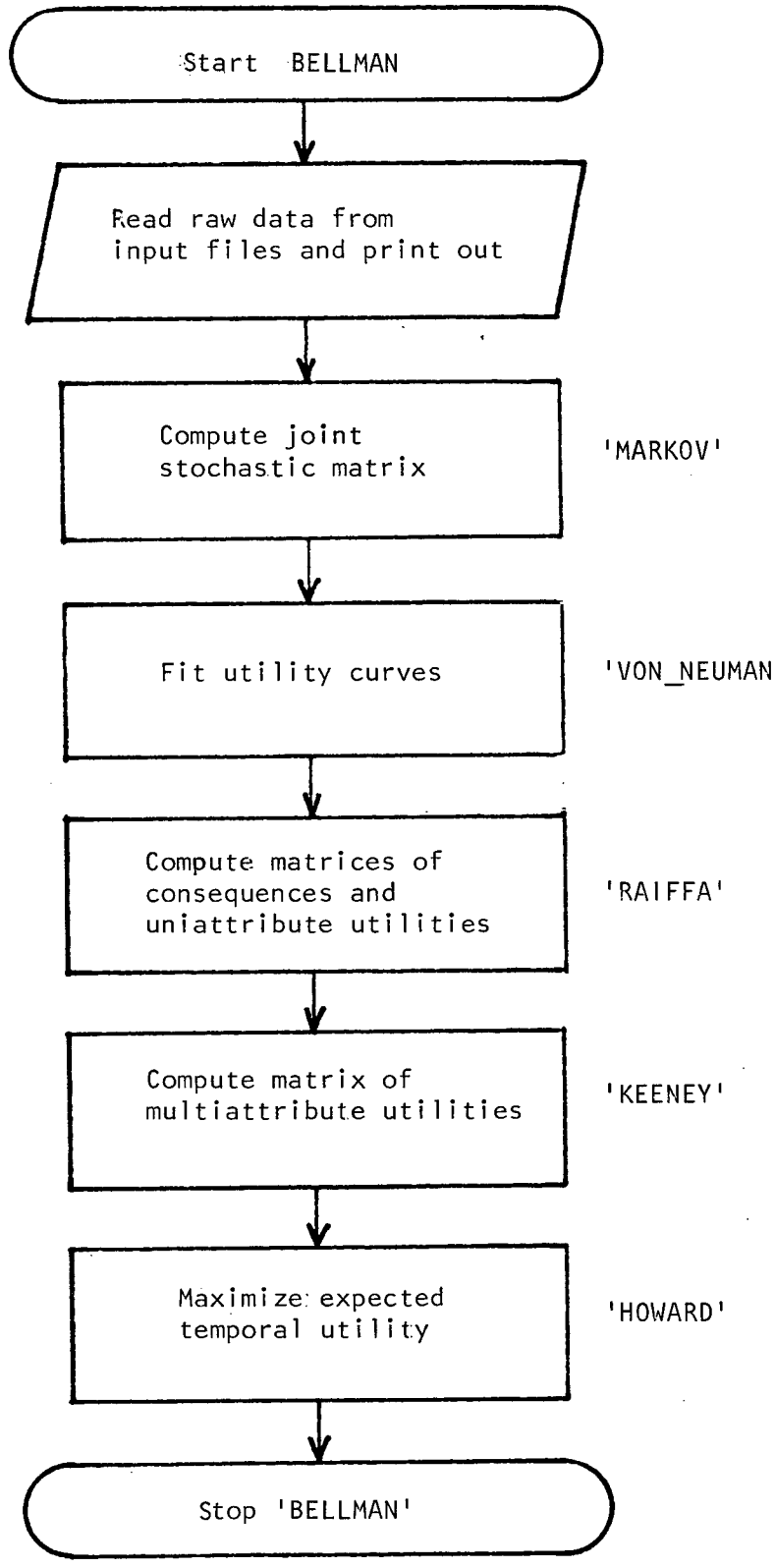


FIGURE 24: MAIN BLOCKS OF THE PROGRAM 'BELLMAN'

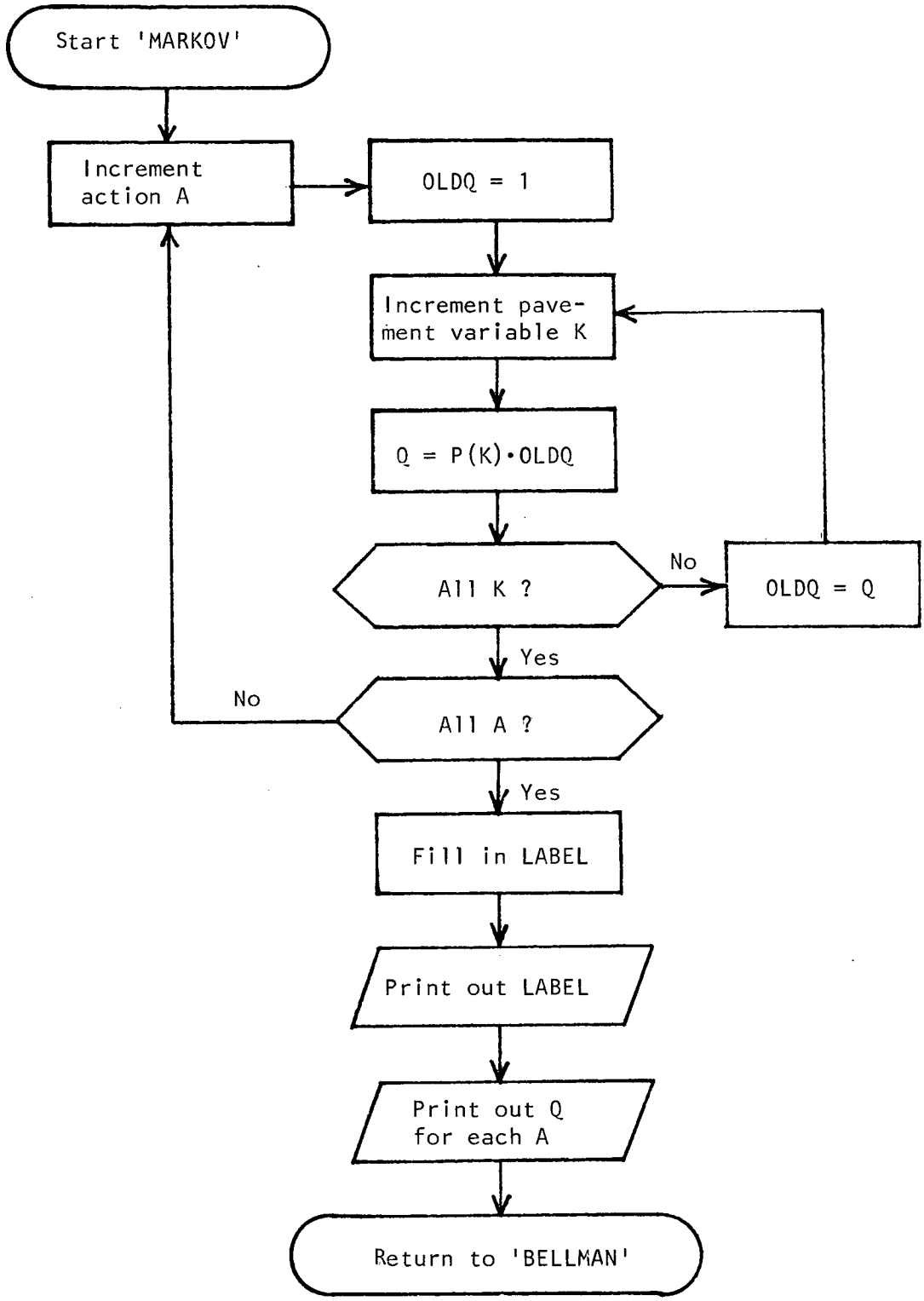


FIGURE 25. SUBPROGRAM 'MARKOV'

variable, N,

$$Q = P(N)*(P(N-1)* \cdots *(P(2)*(P(1))) \cdots). \qquad (D.1)$$

A subset of the loops for this equation is then repeated in order to store the state numbers of pavement variables that enter a joint state number. The numbers are stored in LABEL and make it possible to refer back to individual pavement variables for the calculation of consequences. The explanation of joint state numbers, and the joint transition matrices are printed out. The latter is checked by the program and an error message produced if a matrix is not stochastic.

Subprogram VON NEUMANN:

This subprogram approximates the uniattribute utility functions by exponential curves. It is based on the theory in Appendix B.

The program (Figure 26) starts from a test on the risk attitude. For neutral cases, the procedure computes the slope of straight-line utility curve. For the nonneutral cases HALF/BEST ratio is tested and constant a of (8.1) iterated. If the case is risk averse, it is transford to risk prone.

Iteration starts at A = 0 with an increment DELTA = 0.1. The functions EXA is the right hand side of (B.4) whereas EX is the other side. EPS is the value on which a check is made whether the iterated A is larger than the true A. As soon as this is true, the program checks if the difference between both sides of (B.4) is small enough to terminate the iteration. If the difference is too large, A is reduced by one increment and then increased by a new increment DELTA/10. This continues until EPS is sufficiently small. The current A is then the correct value of constant a in Formula (B.1) if the case is risk prone. For risk averse case, A is transformed by adding 1 and constant c calculated.

Subprogram RAIFFA:

Once the joint states are defined in terms of states of individual pavement variables and uniattribute utility curves fitted for all attributes,

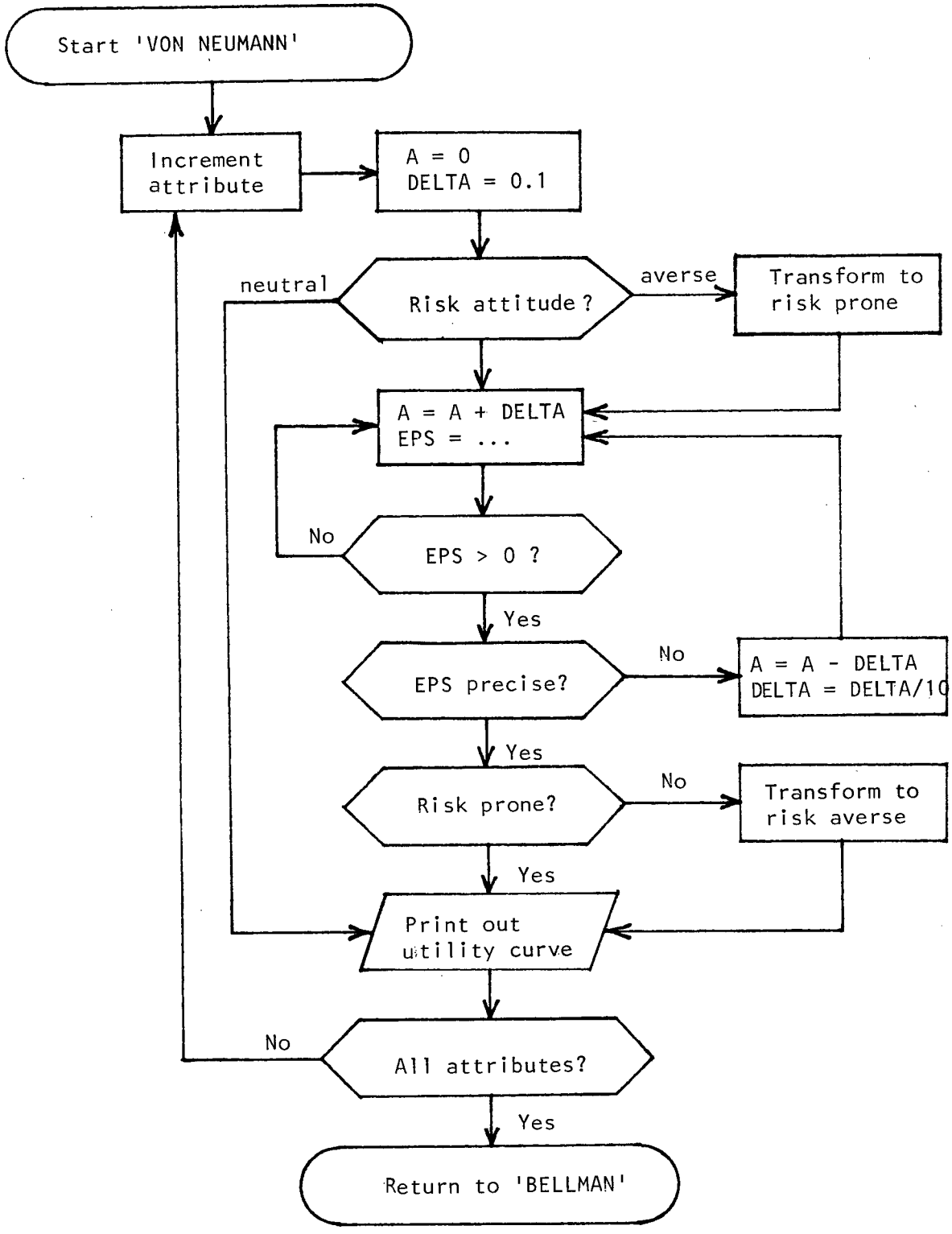


FIGURE 26. SUBPROGRAM 'VON NEUMANN'

RAIFFA computes the matrix of consequences CONS and corresponding matrix of uniattribute utilities U. The latter is checked for proper sign and scaling. The formulas for computing attributes X must be changed if the structure of objectives changes or the number of states of an individual pavement variable exceeds 2. See Figure 27 for block diagram.

Subprogram KEENEY:

This subprogram (Figure 28) aggregates a vector of uniattribute utilities into the multiattribute utility scalar for all actions and all joint states at a stage's end.

It first tests the additive independence condition by checking the sum of scaling factors. Complementary cases (Chapter 6.4) are rejected by the program and terminate execution. Supplementary cases cause the program to iterate the interaction factor KAY according to the theory in Section 6.6.5 of reference (22). The procedure for iterating KAY is, except for the testing for risk attitude, similar to that used for iterating A in VON NEUMANN. After KAY is obtained, KEENEY computes the matrix of multi-attribute utilities, MUTIL, for all actions by the multiplicative model. The additive model is used for problems in which the scaling factors sum up to one.

Subprogram HOWARD:

Figure 29 outlines this program. HOWARD starts with testing whether the temporal utility model is additive or multiplicative. Multiplicative cases cause HOWARD to iterate the temporal interaction factor KAYT. The testing and interation are very similar to those in KEENEY, except for notation. The dynamic programming loops contain recursive equations for both the additive and the multiplicative model of temporal utility.

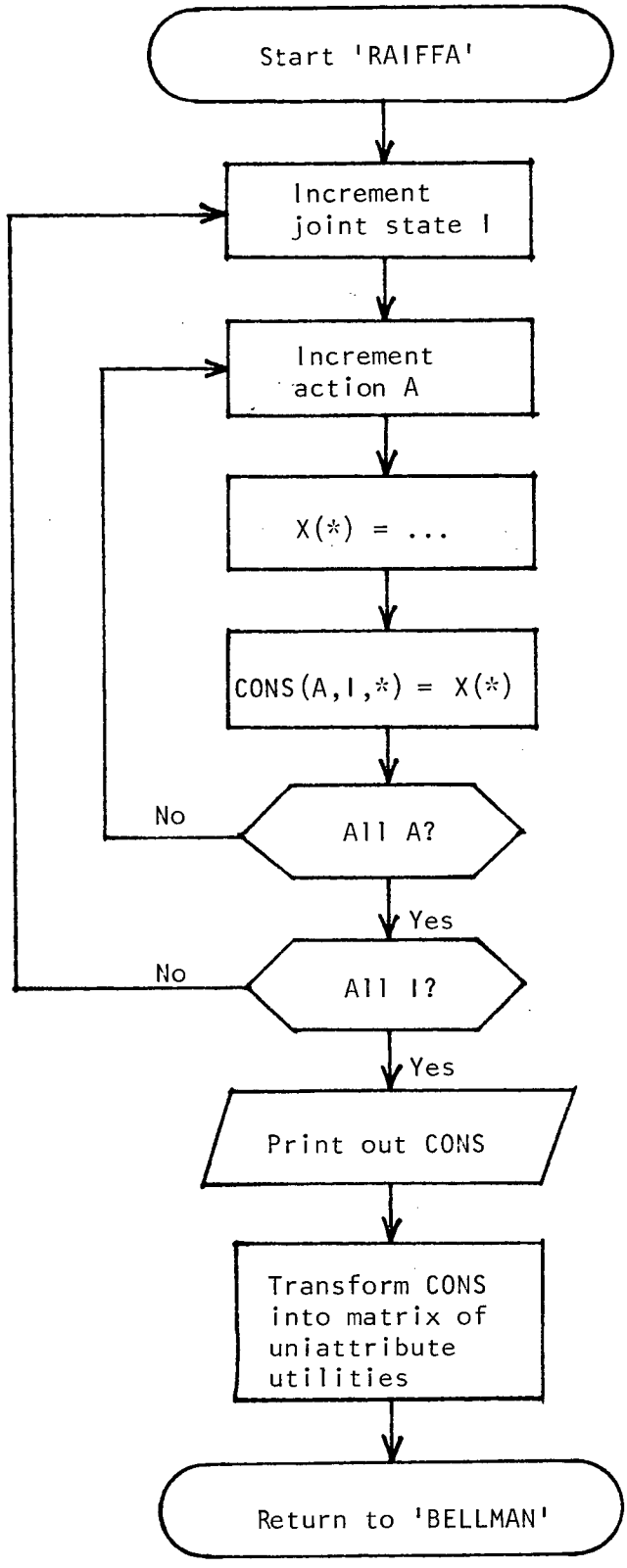


FIGURE 27: SUBPROGRAM 'RAIFFA'

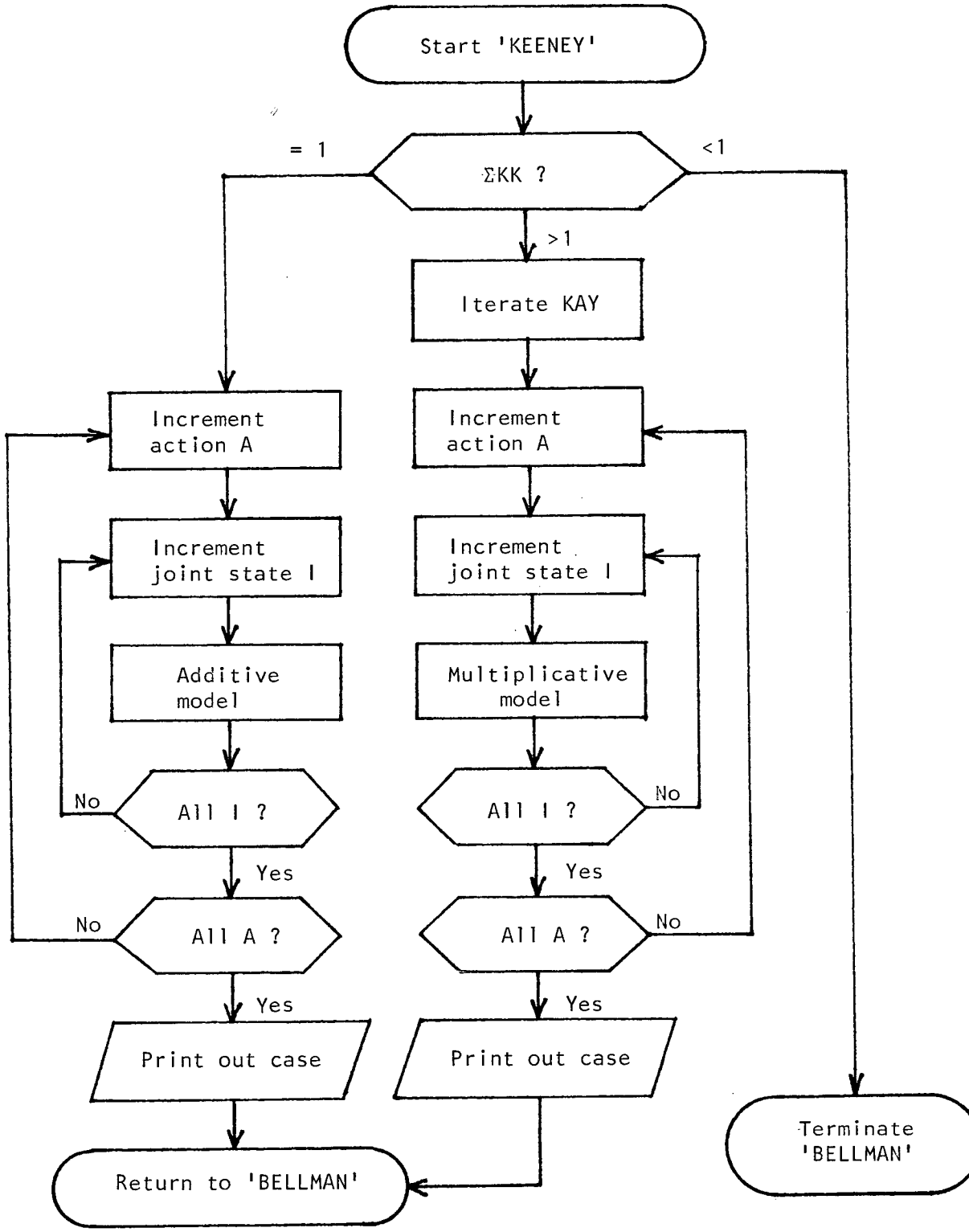


FIGURE 28: SUBPROGRAM 'KEENEY'

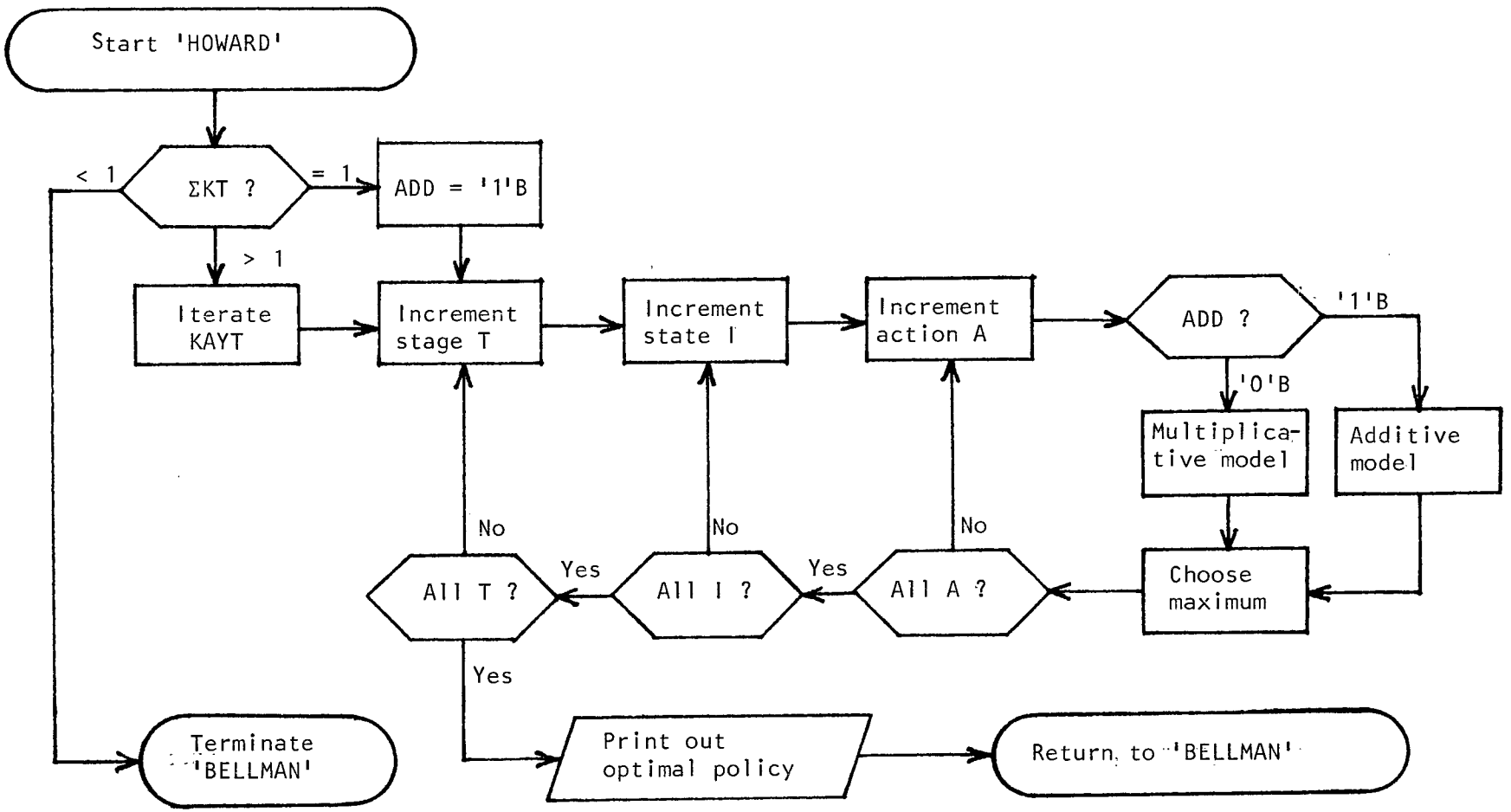


FIGURE 29: SUBPROGRAM 'HOWARD'

HOWARD prints out the optimal decision rules for all stages. If the policy consists of fewer decision rules than there are stages, the last rule printed out is the steady-state decision rule.

APPENDIX E

COMPUTER PROGRAM LISTING

```
 BELLMAN: PROC OPTIONS(MAIN) REORDER;
-DCL (MISCELL,ACTIONS,ATTRIBS,PAVARS,TEMPOS) FILE STREAM;
ODCL EPSILON INIT(1E-6); /* PRECISION OF COMPUTATIONS */
ODCL PREC INIT(1E-2); /* PRECISION OF EMPIRICAL ESTIMATES */
ODCL ( A,ABEST,I,J,K,L,M,N,T) FIXED BIN(15,0);
ODCL (VBEST,VHOLD) FLOAT;
ODCL NAME(2) CHAR(20) VARYING;
ODCL (ENDACT,ENDATT,ENDPAV,ENDMIS,ENDTEM) BIT(1) INIT('0'B);
OON ENDFILE(MISCELL) ENDMIS='1'B;
 ON ENDFILE(ACTIONS) ENDACT='1'B;
 ON ENDFILE(ATTRIBS) ENDATT='1'B;
 ON ENDFILE(PAVARS)  ENDPAV='1'B;
 ON ENDFILE(TEMPOS) ENDTEM='1'B;
OOPEN FILE(ACTIONS) INPUT,
      FILE(ATTRIBS) INPUT,
      FILE(PAVARS) INPUT,
      FILE(MISCELL) INPUT,
      FILE(TEMPOS) INPUT;
-/**** READ IN AND PRINT OUT MISCELLANEOUS DATA:
       IDENTIFICATION OF CURRENT RUN,
       AVERAGE DAILY TRAFFIC(1000 VEHICLES/DAY),
       NUMBER OF RESIDENTS(1000 HOUSEHOLDS),
       LENGTH OF PAVEMENT SECTION(KM),
       NUMBER OF LANES,
       EXTRA VEHICLE OPERATING COST($/VEHICLE KM),
       DELAY PER VEHICLE SLOWDOWN-CYCLE(HOURS),
       NUMBER OF SLOWDOWN CYCLES PER KM(1/KM),
       NUMBER OF ACTIONS,
       NUMBER OF ATTRIBUTES,
       NUMBER OF PAVEMENT VARIABLES
       NUMBER OF STAGES
       TEMPORAL SCALING CONSTANTS
 ****/
ODCL RUNNAME CHAR(60) VARYING;
ODCL (TRAFFIC,RESIDENTS,LENGTH,LANES,
     EVOCOST,DELAY,CYCLES) FLOAT;
ODCL (NACT,NATT,NPAV,NSTG) FIXED BIN(15,0);
OGET FILE(MISCELL) DATA
    (RUNNAME,TRAFFIC,RESIDENTS,LENGTH,LANES,EVOCOST,DELAY,CYCLES,
      NACT,NATT,NPAV,NSTG);
 PUT PAGE EDIT(RUNNAME,
                 'TRAFFIC=',TRAFFIC,'  *1000 VEHICLES/DAY',
                 'RESIDENTS=',RESIDENTS,'  *1000 HOUSEHOLDS',
                 'LENGTH=',LENGTH,'  KM',
                 'LANES=',LANES,'  ',
              'EXTRA VEHICLE OPERATING COST=',EVOCOST,'  DOLLARS/VEHICLE KM',
                 'DELAY=',DELAY,'  HOURS',
                 'CYCLES=',CYCLES,'  /KM',
                 'NUMBER OF ACTIONS=',NACT,
                 'NUMBER OF ATTRIBUTES=',NATT,
                 'NUMBER OF PAVEMENT VARIABLES=',NPAV,
                 'NUMBER OF STAGES=',NSTG)
            (A,SKIP(2),(4)(A,F(4),A,SKIP(2)),(3)(A,F(6,3),A,SKIP(2)),
                 (4)(A,F(3),SKIP(2)));
ONJOINT=2**NPAV; /* NUMBER OF JOINT STATES */
-BEGIN; /* VARIABLES DECLARED IN THE FOLLOWING ARE IN ONE
            BLOCK WITH THE SUBPROGRAMS AND NEED NOT BE CTL */
ODCL X(NATT) FLOAT;
ODCL (COST(NACT),JOBS(NACT),GRAVEL(NACT),TIME(NACT)) FLOAT;
ODCL P(NPAV,NACT,2,2) FLOAT;
```

```
61          ODCL Q(NACT,NJOINT,NJOINT) FLOAT;
62          ODCL CONS(NACT,NJOINT,NATT) FLOAT;
63          ODCL LABEL(NJOINT,NPAV) FIXED BIN(15,0);
64          ODCL C(NATT,3) FLOAT;
65          ODCL U(NACT,NJOINT,NATT) FLOAT;
66          ODCL MUTIL(NACT,NJOINT) FLOAT;
67          ODCL (ATTNAME(NATT),UNIT(NATT)) CHAR(20) VARYING;
68          ODCL (ACTNAME(NACT),PAVNAME(NPAV)) CHAR(20) VARYING;
69          -/**** READ IN AND PRINT OUT ACTION'S NAME AND RESOURCES
70                 REQUIRED PER LANE*KM: COST($), JOBS(PERSON*DAYS),
71                 GRAVEL(TONS), TIME(DAYS/CREW)
72           ****/
73          ODCL (BEST(NATT),HALF(NATT),KK(NATT)) FLOAT;
74          ODCL TEMP(2*NACT) FLOAT;
75          OPUT LINE(31) EDIT
76           ('ACTIONS WITH RESOURCES REQUIRED PER LANE*KM')(A);
77           PUT SKIP(2) EDIT
78           ('ACTION','COST','JOBS','GRAVEL','TIME')
79           (A,COL(11),A,COL(22),A,COL(37),A,COL(45),A);
80           PUT SKIP EDIT('(DOLLARS)','(PERSON-DAYS)','(TONS)','(DAYS)')
81                        (COL(11),(4)(A,X(2)));
82          ODO I=1 TO NACT;
83           GET FILE(ACTIONS) LIST(A,NAME(1),(TEMP(K) DO K=1 TO 4));
84               ACTNAME(A)=NAME(1);
85               COST(A)=TEMP(1);
86               JOBS(A)=TEMP(2);
87               GRAVEL(A)=TEMP(3);
88               TIME(A)=TEMP(4);
89               PUT SKIP(2) EDIT
90               (ACTNAME(A),COST(A),JOBS(A),GRAVEL(A),TIME(A))
91               (A(10),COL(11),F(7),COL(23),F(4,1),COL(38),F(4),
92               COL(45),F(4,1));
93               END;
94          -/**** CALCULATE 'BEST'-VALUE FOR ALL ATTRIBUTES
95           ****/
96          ODCL(MAXCOST,MAXJOBS,MAXGRAVEL,MAXTIME) FLOAT INIT(0);
97          ODO I=1 TO NACT;
98               MAXCOST=MAX(MAXCOST,COST(I));
99               MAXJOBS=MAX(MAXJOBS,JOBS(I));
100              MAXGRAVEL=MAX(MAXGRAVEL,GRAVEL(I));
101              MAXTIME=MAX(MAXTIME,TIME(I));
102              END;
103         OBEST(1),BEST(5)=TRAFFIC;
104          BEST(3)=(CYCLES*364+MAXTIME*LANES)*LENGTH*DELAY*TRAFFIC;
105          BEST(4)=MAXCOST*LENGTH*LANES/1000;
106          BEST(6)=MAXJOBS*LENGTH*LANES;
107          BEST(2)=LENGTH*TRAFFIC*EVOCOST*364;
108          BEST(7)=MAXGRAVEL*LENGTH*LANES/1000;
109          BEST(8)=RESIDENTS;
110         -/**** READ IN AND PRINT OUT PAVEMENT VARIABLE'S NAME AND
111                TRANSITION PROBABILITIES FOR ALL ACTIONS
112          ****/
113         ODO I=1 TO NPAV;
114              PUT LINE(30);
115              GET FILE(PAVARS) LIST(K,NAME(1),TEMP);
116              PAVNAME(K)=NAME(1);
117         O    DO A=1 TO NACT;
118                  P(K,A,1,1)=TEMP(2*A-1);
119                  P(K,A,2,1)=TEMP(2*A);
120                  P(K,A,*,2)=1-P(K,A,*,1);
```

```
121                 PUT SKIP(2) EDIT
122                  ('MARKOV TRANSITION MATRIX FOR ',PAVNAME(K),
123                          'UNDER ACTION=',ACTNAME(A))((2)A,SKIP,(2)A);
124                 PUT SKIP EDIT('1','2')(COL(21),A,X(5),A);
125                 PUT EDIT
126                 ('1',P(K,A,1,1),P(K,A,1,2),'2',P(K,A,2,1),P(K,A,2,2))
127                        (SKIP,COL(18),A,(2)(F(6,2)));
128                 END;
129             END;
130         -/**** READ IN ATTRIBUTE'S NAME, UNIT, HALF/BEST RATIO AND
131                 SCALING CONSTANT
132          ****/
133         ODO  I=1 TO NATT;
134             GET FILE(ATTRIBS) LIST(L,NAME,TEMP(1),TEMP(2));
135             ATTNAME(L)=NAME(1);
136             UNIT(L)=NAME(2);
137             HALF(L)=TEMP(1);
138             KK(L)=TEMP(2);
139             END;
140         -/**** READ IN TEMPORAL SCALING FACTORS
141                 AND TERMINAL VALUE OF TEMPORAL UTILITY
142          ****/
143         ODCL TVSTAR FLOAT;
144         ODCL KT(NSTG) FLOAT;
145         OGET FILE(TEMPOS) LIST((T,KT(T) DO T=1 TO NSTG),TVSTAR);
146         OCALL MARKOV;
147         OCALL VON_NEUMANN;
148         OCALL RAIFFA;
149         OCALL KEENEY;
150         OCALL HOWARD;
151         1/* SCALAR MULTIPLY INDIVIDUAL MARKOV TRANSITION MATRICES
152             TO PRODUCE THE JOINT STOCHASTIC MATRIX Q(A,II,JJ)
153             A=ACTION
154             II='FROM' JOINT STATE
155             JJ='TO' JOINT STATE                                  */
156         OMARKOV: PROC;
157         ODCL(A,I1,I2,II,J1,J2,JJ,N) FIXED BIN(15,0);
158          DCL OLDQ(NACT,NJOINT,NJOINT) FLOAT;
159          DCL CARRY(NPAV) FIXED BIN(15,0);
160         ODO A=1 TO NACT; /* GO THROUGH ACTIONS */
161             Q(A,*,*)=1;
162             DO K=1 TO NPAV; /* GO THROUGH PAVEMENT VARIABLES */
163                N=2**(K-1);
164                OLDQ(A,*,*)=Q(A,*,*);
165                DO I1=1 TO 2;     /* ROWS I1, COLUMNS J1  */
166                   DO J1=1 TO 2; /* OF 2X2 TRANSITION MATRIX */
167                      DO I2=1 TO N;       /* ROWS I2, COLUMNS J2 */
168                         II=(I1-1)*N+I2; /* OF JOINT STOCHASTIC MATRIX */
169                         DO J2=1 TO N;    /* COMPUTED SO FAR     */
170                            JJ=(J1-1)*N+J2;
171                            Q(A,II,JJ)=P(K,A,I1,J1)*OLDQ(A,I2,J2);
172                            END; /* J2 */
173                         END; /* I2 */
174                      END; /* J1 */
175                   END; /* I1 */
176                END; /* K */
177             END; /* A */
178         O/* REPEAT K,I1,I2 LOOPS TO FILL IN 'LABEL' MATRIX */
179         ODO K=1 TO NPAV;
180             N=2**(K-1);
```

```
181             DO I1=1 TO 2;
182                DO I2=1 TO N;
183                   II=(I1-1)*N+I2;
184                   LABEL(II,K)=I1;
185                   END; /* I2 */
186                END; /* I1 */
187             /* FILL IN THE REST OF 'LABEL' MATRIX */
188             DO L=1 TO NJOINT/(2*N) - 1; /* NO. OF GROUPS */
189                DO M=1 TO 2*N; /* NO. IN EACH GROUP */
190                   LABEL(L*(2*N)+M,K)=LABEL(M,K);
191                   END; /* M */
192                END; /* L */
193             END; /* K */
194          O/* PRINT OUT EXPLANATION OF JOINT STATE NUMBERS
195             IN TERMS OF STATES PAVEMENT VARIABLES ARE IN */
196          OPUT SKIP(6) EDIT('LABELS FOR JOINT STATES')(A);
197           PUT SKIP(2) EDIT
198           ('STATE OF PAVEMENT VARIABLE NUMBER:')(COL(17),A);
199           PUT SKIP EDIT((K DO K=1 TO NPAV))(COL(7),(NPAV)(X(9),F(2)));
200           PUT SKIP EDIT('JOINT STATE',(PAVNAME(K) DO K=1 TO NPAV))
201                         (A,(NPAV)(X(2),A));
202           PUT EDIT((J,LABEL(J,*) DO J=1 TO NJOINT))
203                   (SKIP,COL(8),F(2),(NPAV)(X(8),F(2)));
204          O/* PRINT OUT JOINT STOCHASTIC MATRIX FOR EACH ACTION */
205          ODO A=1 TO NACT;
206             PUT LINE(31) EDIT
207             ('100*JOINT STOCHASTIC MATRIX FOR ACTION=',ACTNAME(A))(A,A);
208          OPUT SKIP(2) EDIT((K DO K=1 TO NJOINT))(X(4),(NJOINT)F(3));
209             PUT SKIP;
210             DO I=1 TO NJOINT;
211                IF ABS(SUM(Q(A,I,*))-1)>EPSILON THEN
212                   PUT SKIP EDIT('ROW SUM WRONG IN ROW=',I)(A,F(2));
213                IF ANY(Q(A,I,*)<0) THEN
214                   PUT SKIP EDIT('PROBABILITY<0 IN ROW=',I)(A,F(2));
215                PUT SKIP EDIT(I,100*Q(A,I,*))(F(2),X(2),(NJOINT)F(3));
216                END; /* I */
217             END; /* A */
218          ORETURN;
219           END MARKOV;
220          1/* FIT EXPONENTIAL CURVES TO UTILITY FUNCTIONS GIVEN BY:
221             U(0)=0, U(HALF)=0.5, U(BEST)=1.0                    */
222          OVON_NEUMANN: PROC RETURNS(FLOAT);
223          ODCL (A,B,DELTA,EPS,EX,EXA,U,X) FLOAT;
224           C=0; /* INITIAL ALL COEFFICIENTS FOR EXPONENTIAL CURVES */
225          O/* TEST EACH ATTRIBUTE FOR RISK ATTITUDE */
226          OTAKE:
227               DO I=1 TO NATT;
228               IF KK(I) > 0 THEN DO; /* IGNORE ATTRIBUTES WITH KK=0 */
229                  HALF(I)=HALF(I)*BEST(I);
230                  EX=HALF(I)/BEST(I);
231                  IF ABS(EX-.5)<PREC THEN GO TO NEUTRAL;
232                  A=0; DELTA=0.1; /* INITIAL THE SEARCH VARIABLES */
233                  IF EX-.5<PREC THEN EX=1-EX; /* CONVERT AVERSE TO PRONE */
234                  GO TO PRONE;
235          OSCALE: A=A-DELTA; /* RESCALE  THE SEARCH VARIABLES */
236                  DELTA=DELTA/10;
237          OPRONE: EPS=1;
238                  DO WHILE (EPS>0);
239                     A=A+DELTA;
240                     EXA=LOG(A+.5)-LOG(A);
```

```
                    EXA=EXA/(LOG(A+1)-LOG(A));
                    EPS=(EXA-EX)/EX;
                    END;
                IF EPS<-EPSILON THEN GO TO SCALE;
                B=(LOG(A+1)-LOG(A))/BEST(I);
                IF HALF(I)/BEST(I)-.5 < PREC THEN GO TO AVERSE;
                C(I,1)=-A;
                C(I,2)=B;
                GO TO EXIT;
        OAVERSE: A=A+1;
                C(I,1)=A;
                C(I,2)=-B;
                GO TO EXIT;
        ONEUTRAL:  C(I,3)=1/BEST(I);
        O/* PRINT OUT INFORMATION ABOUT ATTRIBUTES
            AND SKETCH UTILITY CURVES FITTED       */
        OEXIT:
            PUT LINE(31) EDIT(I,ATTNAME(I),'  (',UNIT(I),')')
                            (F(2),X(1),(4)A);
            PUT SKIP EDIT('SCALING CONSTANT KK(',I,')=',KK(I))
                        (A,F(1),A,F(5,2));
            PUT SKIP EDIT('BEST=',BEST(I),'HALF=',HALF(I))
                    ((2)(A,F(6,1),SKIP));
            IF ABS(HALF(I)/BEST(I)-.5)<PREC THEN
            PUT SKIP EDIT('RISK NEUTRAL:  U(X)=',C(I,3),'*X')
                        (A,F(6,3),A);
            ELSE IF (HALF(I)/BEST(I)-.5)<PREC THEN
            PUT SKIP EDIT
            ('RISK AVERSE:  U(X)=',C(I,1),'*(1-EXP(',C(I,2),'*X))')
                        (A,(2)(F(6,3),A));
            ELSE PUT SKIP EDIT
            ('RISK PRONE:  U(X)=',C(I,1),'*(1-EXP(',C(I,2),'*X))')
                        (A,(2)(F(6,3),A));
            PUT EDIT
            ('0','1',(20)' _',' U(X)')(SKIP(2),A,X(37),A,SKIP,A,A);
            U=0;
            DO K=1 TO 20;
               X=K*BEST(I)/20;
               U=C(I,1)*(1-EXP(C(I,2)*X));
               U=U+C(I,3)*X;
               U=40*(U);
               IF U<=1 THEN
               PUT EDIT('*')(COL(1),A);
               ELSE PUT EDIT('!','*')(COL(1),A,COL(U+1),A);
               END;
        O   END; /* IF KK(I)>0 */
        OEND TAKE;
        ORETURN;
         END VON_NEUMANN;
        1/* COMPUTE CONSEQUENCES AND UNIATTRIBUTE UTILITIES */
        ORAIFFA: PROC RETURNS(FLOAT);
        ODCL F(NPAV,2) FLOAT INIT((2*NPAV)0);
         F(*,1)=1.0; /* F(K,J)=1 WHEN PAVEMENT VARIABLE 'K'
                        IS IN ACCEPTABLE STATE (J=1)       */
        O/* COMPUTE A CONSEQUENCE (= VECTOR OF ATTRIBUTES)
            FOR EACH ACTION AND JOINT STATE              */
        ODO I=1 TO NJOINT; /* RUN THROUGH JOINT STATES */
            DO A=1 TO NACT; /* RUN THROUGH ACTIONS */
               X(1)=F(1,LABEL(I,1))*F(2,LABEL(I,2))*TRAFFIC;
               X(3)=(1-F(3,LABEL(I,3)))*CYCLES*364+TIME(A)*LANES;
```

```
300.5            X(3)=BEST(3)-X(3)*LENGTH*DELAY*TRAFFIC;
302              X(4)=BEST(4)-COST(A)*LENGTH*LANES/1000;
303              X(5)=F(4,LABEL(I,4))*TRAFFIC;
304              X(6)=JOBS(A)*LENGTH*LANES;
305              X(2)=F(3,LABEL(I,3))*LENGTH*TRAFFIC*EVOCOST*364;
306              X(7)=BEST(7)-GRAVEL(A)*LENGTH*LANES/1000;
307              X(8)=(1-F(1,LABEL(I,1)))*RESIDENTS;
308       0      CONS(A,I,*)=X; /* A=ACTION, I=JOINT STATE, *=J=ATTRIBUTE */
309       0      END; /* A */
310          END; /* I */
311       0/* PRINT OUT MATRIX OF CONSEQUENCES FOR EACH ACTION */
312       ODO A=1 TO NACT;
313          PUT LINE(25) EDIT
314          ('MATRIX OF CONSEQUENCES FOR ACTION=',ACTNAME(A))(A,A);
315          PUT SKIP(2) EDIT
316          ('ATTRIBUTE',(K DO K=1 TO NATT))(A,COL(13),(NATT)(F(2),X(5)));
317          DO J=1 TO NJOINT;
318             PUT SKIP EDIT('STATE',J,(CONS(A,J,K) DO K=1 TO NATT))
319             (A,F(3),(NATT)(X(1),F(6,2)));
320             END; /* J */
321          END; /* A */
322       0/* TRANSFORM MATRIX OF CONSEQUENCES INTO MATRIX OF
323          UNIATTRIBUTE UTILITIES
324          U(A,I,J): A=ACTION, I=JOINT STATE, J=ATTRIBUTE */
325       ODO A=1 TO NACT;
326          DO I=1 TO NJOINT;
327             DO J=1 TO NATT;
328                U(A,I,J)=C(J,1)*(1-EXP(C(J,2)*CONS(A,I,J)))
329                         +C(J,3)*CONS(A,I,J);
330                IF U(A,I,J)<0|(1-U(A,I,J))<-EPSILON THEN DO;
331                   PUT SKIP(5) EDIT
332                   ('WRONG UNIATTRIBUTE UTILITY=',U(A,I,J),
333                    ' FOR ACTION=',ACTNAME(A),' ATTRIBUTE=',ATTNAME(J),
334                    ' JOINT STATE=',J)(A,F(7,2),(5)A,F(2));
335                   END;
336                END; /* J */
337             END; /* I */
338          END; /* A */
339       ORETURN;
340       OEND RAIFFA;
341       1/* COMPUTE MULTIATTRIBUTE UTILITIES BY APPROPRIATE MODEL */
342       OKEENEY: PROC RETURNS(FLOAT);
343       ODCL (DELTA,EPS,KAY,LEFT,RIGHT) FLOAT;
344       0/* TEST THE ADDITIVE INDEPENDENCE CONDITION */
345       OIF (1-SUM(KK))>PREC THEN DO;
346          PUT PAGE EDIT((10)'*','SUM OF KK < 1')(A,A);
347          GO TO ADIEU; /* TERMINATE EXECUTION */
348          END;
349        ELSE IF ABS(1-SUM(KK))<=PREC THEN GO TO ADDIT;
350       0/* ITERATE THE INTERACTION FACTOR KAY */
351       OKAY=0; /* INITIAL THE SEARCH VARIABLES */
352        DELTA=.1;
353        GO TO ITERATE;
354       OSCALE: KAY=KAY+DELTA; /* RESCALE THE SEARCH VARIABLES */
355               DELTA=DELTA/10;
356       OITERATE:EPS=1;
357               DO WHILE (EPS>EPSILON);
358                  KAY=KAY-DELTA;
359                  LEFT=1+KAY;
360                  RIGHT=1;
```

```
361                 DO I=1 TO NATT;
362                    RIGHT=RIGHT*(1+KAY*KK(I));
363                    END;
364                 EPS=LEFT-RIGHT;
365                 END ;
366        OIF EPS<-EPSILON THEN GO TO SCALE;
367        OMULTIP:/* MULTIPLICATIVE MULTIATTRIBUTE UTILITY MODEL */
368                 DO A=1 TO NACT;
369                    DO I=1 TO NJOINT;
370                    MUTIL(A,I)=PROD(1+KAY*KK*U(A,I,*));
371                    MUTIL(A,I)=(MUTIL(A,I)-1)/KAY;
372                    END; /* I */
373                 END; /* A */
374                 PUT SKIP(5) EDIT('MULTIATTRIBUTE UTILITY IS')(A);
375                 PUT SKIP EDIT('NEGATIVE MULTIPLICATIVE, SUM(KK(I))=',
376                              SUM(KK))(A,F(6,3));
377                 PUT SKIP EDIT('INTERACTION FACTOR KAY=',KAY)(A,F(6,3));
378        O        GO TO EXIT;
379        OADDIT: /*  ADDITIVE MULTIATTRIBUTE UTILITY MODEL */
380                 DO A=1 TO NACT;
381                    DO I=1 TO NJOINT;
382                    MUTIL(A,I)=SUM(KK*U(A,I,*));
383                    END; /* I */
384                 END; /* A */
385                 PUT SKIP(5) EDIT ('MULTIATTRIBUTE UTILITY IS ADDITIVE')(A);
386        OEXIT:
387        ORETURN;
388        OEND KEENEY;
389        1/* TEST FOR THE TEMPORAL UTILITY MODEL
390            AND COMPUTE THE OPTIMUM POLICY   */
391        OHOWARD: PROC RETURNS(FLOAT);
392        ODCL VSTAR(NJOINT,0:NSTG) FLOAT;
393         VSTAR(*,0)=TVSTAR; /* SET THE TERMINAL VALUES */
394         DCL ASTAR(NJOINT,NSTG) FIXED BIN(15,0);
395         DCL (RULE(NJOINT),OLDRULE(NJOINT)) FIXED BIN(15,0);
396         RULE=-1;
397         DCL (DELTA,EPS,KAYT,LEFT,RIGHT) FLOAT;
398         DCL ADD BIT(1) INIT('0'B);
399        OPUT SKIP(5) EDIT
400         ('TERMINAL VALUES OF TEMPORAL UTILITY = ',TVSTAR)(A,F(5,2));
401         PUT SKIP EDIT('TEMPORAL SCALING FACTORS:')(A);
402         DO T=1 TO NSTG;
403            PUT SKIP EDIT('KT(',T,') =',KT(T))(A,F(2),A,F(5,2));
404            END;
405        O/* TEST THE ADDITIVE INDEPENDENCE CONDITION */
406        OIF (1-SUM(KT))>PREC THEN DO;
407            PUT PAGE EDIT((10)'*','SUM OF KT < 1')(A,A);
408            GO TO ADIEU; /* TERMINATE EXECUTION */
409            END;
410         ELSE IF ABS(1-SUM(KT))<=PREC THEN DO;
411            PUT SKIP EDIT('ADDITIVE TEMPORAL UTILITY')(A);
412            VSTAR(*,0)=TVSTAR+1; /* SCALING POSITIVE */
413            ADD='1'B;
414            GO TO DYNAMIC;
415            END;
416        O/* ITERATE THE TEMPORAL INTERACTION FACTOR KAYT */
417        OKAYT=0; /* INITIAL THE SEARCH VARIABLES */
418         DELTA=.1;
419         GO TO ITERATE;
420        OSCALE: KAYT=KAYT+DELTA; /* RESCALE THE SEARCH VARIABLES */
```

```
421                 DELTA=DELTA/10;
422         OITERATE:EPS=1;
423                 DO WHILE (EPS>EPSILON);
424                    KAYT=KAYT-DELTA;
425                    LEFT=1+KAYT;
426                    RIGHT=1;
427                    DO I=1 TO NSTG;
428                       RIGHT=RIGHT*(1+KAYT*KT(I));
429                       END;
430                    EPS=LEFT-RIGHT;
431                    END ;
432         OIF EPS<-EPSILON THEN GO TO SCALE;
433         OPUT SKIP EDIT('TEMPORAL UTILITY IS')(A);
434          PUT SKIP EDIT('NEGATIVE MULTIPLICATIVE, SUM(KT(I))=',
435                                 SUM(KT))(A,F(6,3));
436          PUT SKIP EDIT('TEMPORAL INTERACTION FACTOR KAYT=',KAYT)
437                          (A,F(6,3));
438         O/* DYNAMIC PROGRAMMING ALGORITHM */
439          /* T=STAGE; COUNTS FROM THE FUTURE BACKWARDS
440             I='FROM' STATE
441             *=J='TO' STATE
442             A=ACTION
443          */
444          DYNAMIC:
445         ODO T=1 TO NSTG; /* GO THROUGH STAGES */
446             OLDRULE=RULE;
447             DO I=1 TO NJOINT; /* GO THROUGH JOINT STATES */
448                VBEST=-1;
449                ABEST=1;
450                DO A=1 TO NACT; /* GO THROUGH ACTIONS */
451         O         /* COMPUTE EXPECTATION BY RECURSIVE EQUATION */
452         O         IF ADD THEN DO; /* ADDITIVE TEMPORAL UTILITY */
453                      VHOLD=SUM(Q(A,I,*)*(KT(T)*MUTIL(A,*)+VSTAR(*,T-1)));
454                      END; /* ADDITIVE */
455         O         ELSE DO; /* MULTIPLICATIVE TEMPORAL UTILITY */
456                      VHOLD=SUM(Q(A,I,*)*(1+KAYT*KT(T)*MUTIL(A,*))*
457                            VSTAR(*,T-1));
458                      END; /* MULTIPLICATIVE */
459         O         IF VHOLD > VBEST THEN  DO; /* CHOOSE MAXIMUM UTILITY */
460                      VBEST=VHOLD;
461                      ABEST=A;
462                      END; /* CHOOSE */
463         O         END; /* A */
464                VSTAR(I,T)=VBEST;
465                ASTAR(I,T)=ABEST;
466                RULE(I)=ASTAR(I,T);
467                END; /* I */
468             IF ALL(OLDRULE=RULE) THEN GO TO STATIONARY;
469             END; /* T */
470         OSTATIONARY: THOR=T-1;
471         O/* DISPLAY OPTIMUM DECISION RULES */
472         OPUT SKIP(5) EDIT('O P T I M U M  D E C I S I O N S')(A);
473          PUT SKIP(2) EDIT('STAGE INDEX COUNTS FROM THE PRESENT')(A);
474          PUT SKIP(2) EDIT
475          ('STAGE','STATE','ACTION','UTILITY')((4)(A,X(4)));
476          DO T=THOR TO 1 BY -1;
477             PUT SKIP EDIT((36)'*')(A);
478             DO I=1 TO NJOINT;
479          PUT SKIP EDIT((THOR-T+1),I,ACTNAME(ASTAR(I,T)),VSTAR(I,T))
480                       (X(3),F(2),X(6),F(2),X(4),A(7),X(4),F(6,3));
```

```
481                 END;   /* I */
482              END;   /* T */
483           IF THOR < NSTG THEN PUT SKIP(2) EDIT
484              ('POLICY BECOMES STATIONARY IN STAGE ITERATION = ',THOR)
485              (A,F(2));
486           PUT SKIP(3);
487          ORETURN;
488          OEND HOWARD;
489          OEND; /* END OF READ-IN BLOCK */
490          ORETURN;
491          OADIEU: END BELLMAN;
End of File
```

APPENDIX F

SAMPLE OUTPUT

ILLUSTRATIVE EXAMPLE

TRAFFIC= 50 *1000 VEHICLES/DAY

RESIDENTS= 2 *1000 HOUSEHOLDS

LENGTH= 20 KM

LANES= 4

EXTRA VEHICLE OPERATING COST= 0.010 DOLLARS/VEHICLE KM

DELAY= 0.017 HOURS

CYCLES= 0.100 /KM

NUMBER OF ACTIONS= 4

NUMBER OF ATTRIBUTES= 8

NUMBER OF PAVEMENT VARIABLES= 4

NUMBER OF STAGES= 5

ACTIONS WITH RESOURCES REQUIRED PER LANE*KM

ACTION	COST (DOLLARS)	JOBS (PERSON-DAYS)	GRAVEL (TONS)	TIME (DAYS)
NONE	0	0.0	0	0.0
ROUTINE	100	0.5	0	0.0
SEAL	1000	1.0	50	0.2
OVERLAY	20000	2.5	400	1.0

MARKOV TRANSITION MATRIX FOR TEXTURE
UNDER ACTION=NONE

	1	2
1	0.85	0.15
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR TEXTURE
UNDER ACTION=ROUTINE

	1	2
1	0.90	0.10
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR TEXTURE
UNDER ACTION=SEAL

	1	2
1	1.00	0.00
2	1.00	0.00

MARKOV TRANSITION MATRIX FOR TEXTURE
UNDER ACTION=OVERLAY

	1	2
1	0.90	0.10
2	0.90	0.10

MARKOV TRANSITION MATRIX FOR RUTTING
UNDER ACTION=NONE

	1	2
1	0.80	0.20
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR RUTTING
UNDER ACTION=ROUTINE

	1	2
1	0.80	0.20
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR RUTTING
UNDER ACTION=SEAL

	1	2
1	0.80	0.20
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR RUTTING
UNDER ACTION=OVERLAY

	1	2
1	1.00	0.00
2	1.00	0.00

MARKOV TRANSITION MATRIX FOR ROUGHNESS
UNDER ACTION = NONE

	1	2
1	0.80	0.20
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR ROUGHNESS
UNDER ACTION=ROUTINE

	1	2
1	0.90	0.10
2	0.90	0.10

MARKOV TRANSITION MATRIX FOR ROUGHNESS
UNDER ACTION=SEAL

	1	2
1	0.90	0.10
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR ROUGHNESS
UNDER ACTION=OVERLAY

	1	2
1	1.00	0.00
2	1.00	0.00

MARKOV TRANSITION MATRIX FOR STRENGTH
UNDER ACTION=NONE

	1	2
1	0.95	0.05
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR STRENGTH
UNDER ACTION=ROUTINE

	1	2
1	0.95	0.05
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR STRENGTH
UNDER ACTION=SEAL

	1	2
1	0.95	0.05
2	0.00	1.00

MARKOV TRANSITION MATRIX FOR STRENGTH
UNDER ACTION=OVERLAY

	1	2
1	1.00	0.00
2	1.00	0.00

LABELS FOR JOINT STATES

	STATE OF PAVEMENT VARIABLE NUMBER:			
	1	2	3	4
JOINT STATE	TEXTURE	RUTTING	ROUGHNESS	STRENGTH
1	1	1	1	1
2	2	1	1	1
3	1	2	1	1
4	2	2	1	1
5	1	1	2	1
6	2	1	2	1
7	1	2	2	1
8	2	2	2	1
9	1	1	1	2
10	2	1	1	2
11	1	2	1	2
12	2	2	1	2
13	1	1	2	2
14	2	1	2	2
15	1	2	2	2
16	2	2	2	2

100*JOINT STOCHASTIC MATRIX FOR ACTION=NONE

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	52	9	13	2	13	2	3	1	3	0	1	0	1	0	0	0
2	0	61	0	15	0	15	0	4	0	3	0	1	0	1	0	0
3	0	0	65	11	0	0	16	3	0	0	3	1	0	0	1	0
4	0	0	0	76	0	0	0	19	0	0	0	4	0	0	0	1
5	0	0	0	0	65	11	16	3	0	0	0	0	3	1	1	0
6	0	0	0	0	0	76	0	19	0	0	0	0	0	4	0	1
7	0	0	0	0	0	0	81	14	0	0	0	0	0	0	4	1
8	0	0	0	0	0	0	0	95	0	0	0	0	0	0	0	5
9	0	0	0	0	0	0	0	0	54	10	14	2	14	2	3	1
10	0	0	0	0	0	0	0	0	0	64	0	16	0	16	0	4
11	0	0	0	0	0	0	0	0	0	0	68	12	0	0	17	3
12	0	0	0	0	0	0	0	0	0	0	0	80	0	0	0	20
13	0	0	0	0	0	0	0	0	0	0	0	0	68	12	17	3
14	0	0	0	0	0	0	0	0	0	0	0	0	0	80	0	20
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	85	15
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100

100*JOINT STOCHASTIC MATRIX FOR ACTION=ROUTINE

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	62	7	15	2	7	1	2	0	3	0	1	0	0	0	0	0
2	0	68	0	17	0	8	0	2	0	4	0	1	0	0	0	0
3	0	0	77	9	0	0	9	1	0	0	4	0	0	0	0	0
4	0	0	0	85	0	0	0	9	0	0	0	5	0	0	0	0
5	62	7	15	2	7	1	2	0	3	0	1	0	0	0	0	0
6	0	68	0	17	0	8	0	2	0	4	0	1	0	0	0	0
7	0	0	77	9	0	0	9	1	0	0	4	0	0	0	0	0
8	0	0	0	85	0	0	0	9	0	0	0	5	0	0	0	0
9	0	0	0	0	0	0	0	0	65	7	16	2	7	1	2	0
10	0	0	0	0	0	0	0	0	0	72	0	18	0	8	0	2
11	0	0	0	0	0	0	0	0	0	0	81	9	0	0	9	1
12	0	0	0	0	0	0	0	0	0	0	0	90	0	0	0	10
13	0	0	0	0	0	0	0	0	65	7	16	2	7	1	2	0
14	0	0	0	0	0	0	0	0	0	72	0	18	0	8	0	2
15	0	0	0	0	0	0	0	0	0	0	81	9	0	0	9	1
16	0	0	0	0	0	0	0	0	0	0	0	90	0	0	0	10

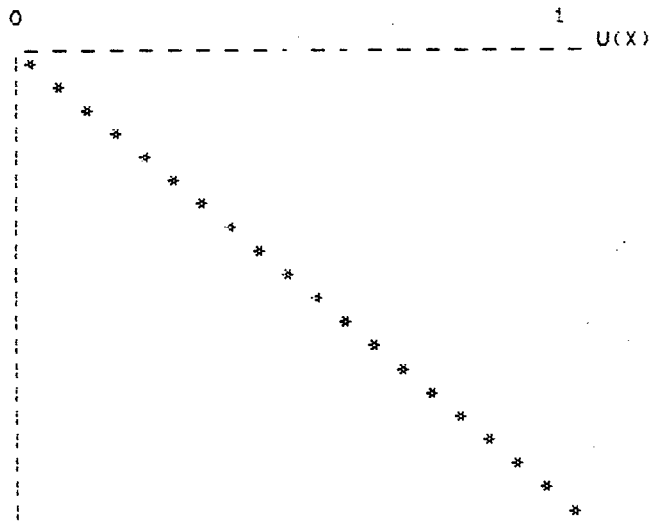
100*JOINT STOCHASTIC MATRIX FOR ACTION=SEAL

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	68	0	17	0	8	0	2	0	4	0	1	0	0	0	0	0
2	68	0	17	0	8	0	2	0	4	0	1	0	0	0	0	0
3	0	0	85	0	0	0	9	0	0	0	5	0	0	0	0	0
4	0	0	85	0	0	0	9	0	0	0	5	0	0	0	0	0
5	0	0	0	0	76	0	19	0	0	0	0	0	4	0	1	0
6	0	0	0	0	76	0	19	0	0	0	0	0	4	0	1	0
7	0	0	0	0	0	0	95	0	0	0	0	0	0	0	5	0
8	0	0	0	0	0	0	95	0	0	0	0	0	0	0	5	0
9	0	0	0	0	0	0	0	0	72	0	18	0	8	0	2	0
10	0	0	0	0	0	0	0	0	72	0	18	0	8	0	2	0
11	0	0	0	0	0	0	0	0	0	0	90	0	0	0	10	0
12	0	0	0	0	0	0	0	0	0	0	90	0	0	0	10	0
13	0	0	0	0	0	0	0	0	0	0	0	0	80	0	20	0
14	0	0	0	0	0	0	0	0	0	0	0	0	80	0	20	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100	0

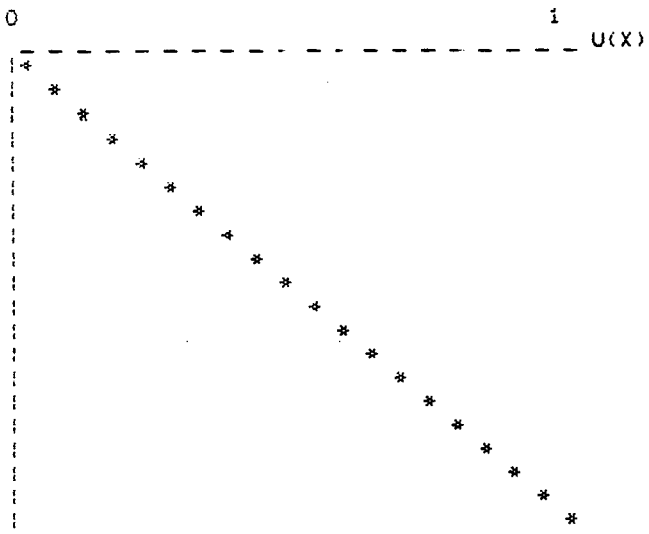
100*JOINT STOCHASTIC MATRIX FOR ACTION=OVERLAY

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	90	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0

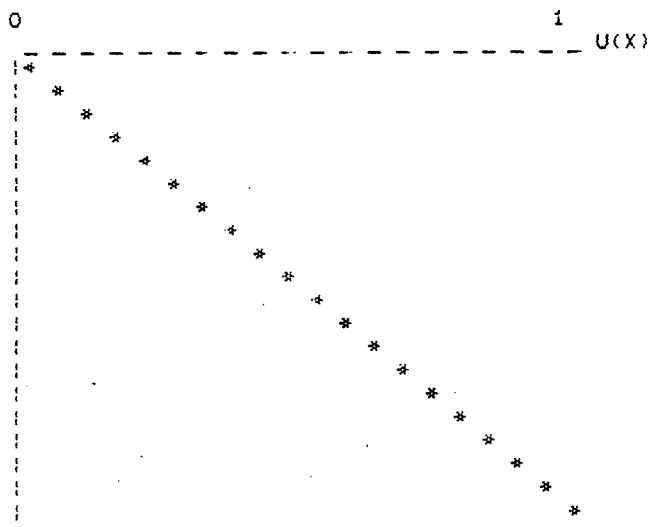
1 SAFETY PROVIDED (1000 VEHICLES/DAY)
SCALING CONSTANT KK(1)= 0.60
BEST= 50.0
HALF= 25.0
RISK NEUTRAL: U(X)= 0.020*X



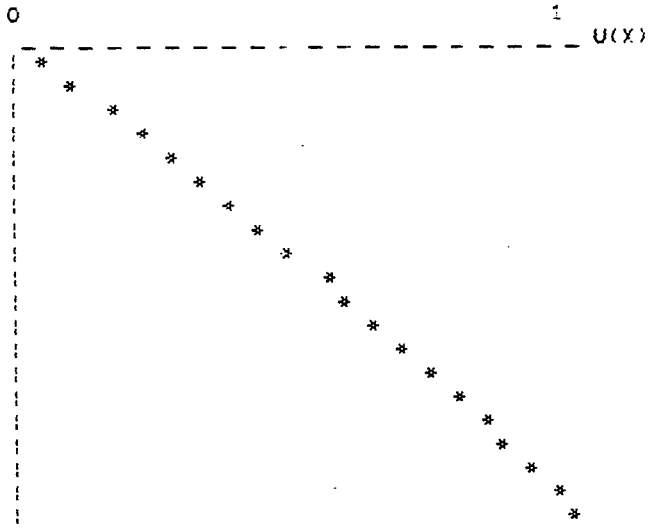
2 USER COST SAVED (1000 DOLLARS/YEAR)
SCALING CONSTANT KK(2)= 0.36
BEST=3640.0
HALF=1820.0
RISK NEUTRAL: U(X)= 0.000*X



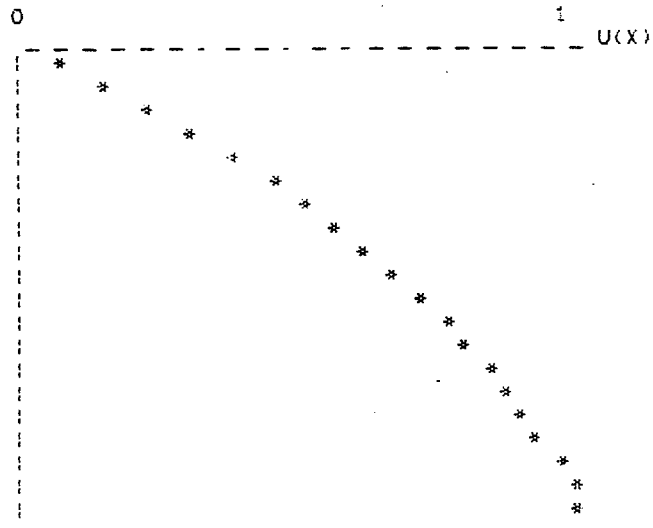
3 DELAY PREVENTED (1000 HOURS/YEAR)
SCALING CONSTANT KK(3)= 0.24
BEST= 673.1
HALF= 336.5
RISK NEUTRAL: U(X)= 0.001*X



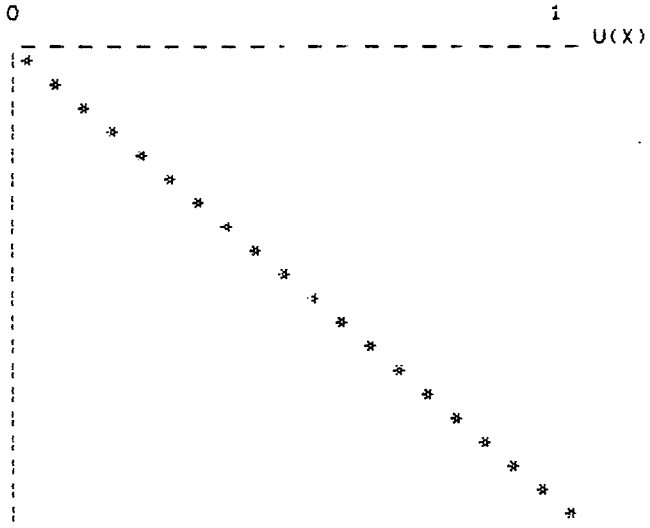
4 AGENCY COST SAVED (1000 DOLLARS/YEAR)
SCALING CONSTANT KK(4)= 0.15
BEST=1600.0
HALF= 720.0
RISK AVERSE: U(X)= 3.017*(1-EXP(-0.000*X))



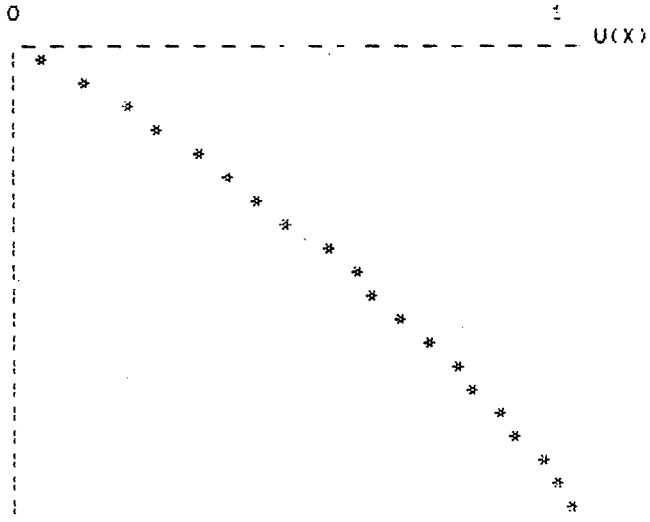
5 ACCESS PROVIDED (1000 VEHICLES/DAY)
SCALING CONSTANT KK(5)= 0.84
BEST= 50.0
HALF= 17.0
RISK AVERSE: U(X)= 1.338*(1-EXP(-0.028*X))



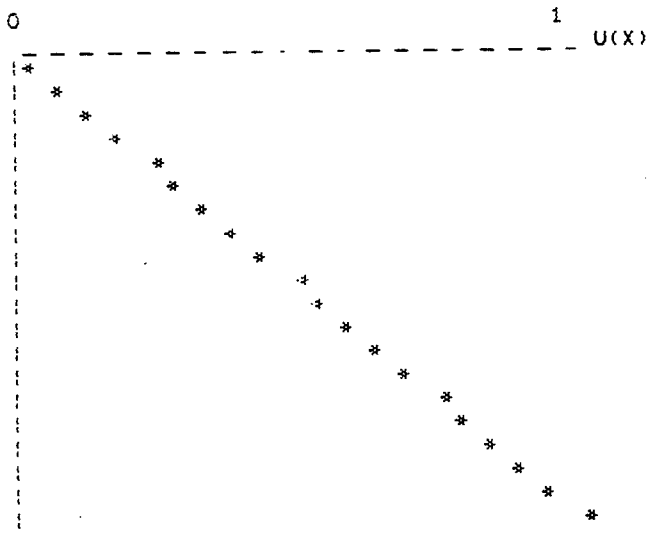
6 JOBS CREATED (PERSON*DAYS/YEAR)
SCALING CONSTANT KK(6)= 0.07
BEST= 200.0
HALF= 100.0
RISK NEUTRAL: U(X)= 0.005*X



7 GRAVEL SAVED (1000 TONS/YEAR)
SCALING CONSTANT KK(7)= 0.12
BEST= 32.0
HALF= 12.8
RISK AVERSE: U(X)= 1.784*(1-EXP(-0.026*X))



8 NOISE PREVENTED (1000 HOUSEHOLDS)
SCALING CONSTANT KK(8)= 0.10
BEST= 2.0
HALF= 1.0
RISK NEUTRAL: U(X)= 0.500*X



MATRIX OF CONSEQUENCES FOR ACTION=NONE

ATTRIBUTE	1	2	3	4	5	6	7	8
STATE 1	50.00	640.00	673.06	600.00	50.00	0.00	32.00	0.00
STATE 2	0.00	640.00	673.06	600.00	50.00	0.00	32.00	2.00
STATE 3	0.00	640.00	673.06	600.00	50.00	0.00	32.00	0.00
STATE 4	0.00	640.00	673.06	600.00	50.00	0.00	32.00	2.00
STATE 5	50.00	0.00	66.64	600.00	50.00	0.00	32.00	0.00
STATE 6	0.00	0.00	66.64	600.00	50.00	0.00	32.00	2.00
STATE 7	0.00	0.00	66.64	600.00	50.00	0.00	32.00	0.00
STATE 8	0.00	0.00	66.64	600.00	50.00	0.00	32.00	2.00
STATE 9	50.00	640.00	673.06	600.00	0.00	0.00	32.00	0.00
STATE 10	0.00	640.00	673.06	600.00	0.00	0.00	32.00	2.00
STATE 11	0.00	640.00	673.06	600.00	0.00	0.00	32.00	0.00
STATE 12	0.00	640.00	673.06	600.00	0.00	0.00	32.00	2.00
STATE 13	50.00	0.00	66.64	600.00	0.00	0.00	32.00	0.00
STATE 14	0.00	0.00	66.64	600.00	0.00	0.00	32.00	2.00
STATE 15	0.00	0.00	66.64	600.00	0.00	0.00	32.00	0.00
STATE 16	0.00	0.00	66.64	600.00	0.00	0.00	32.00	2.00

MATRIX OF CONSEQUENCES FOR ACTION=ROUTINE

ATTRIBUTE	1	2	3	4	5	6	7	8
STATE 1	50.00	640.00	673.06	592.00	50.00	40.00	32.00	0.00
STATE 2	0.00	640.00	673.06	592.00	50.00	40.00	32.00	2.00
STATE 3	0.00	640.00	673.06	592.00	50.00	40.00	32.00	0.00
STATE 4	0.00	640.00	673.06	592.00	50.00	40.00	32.00	2.00
STATE 5	50.00	0.00	66.64	592.00	50.00	40.00	32.00	0.00
STATE 6	0.00	0.00	66.64	592.00	50.00	40.00	32.00	2.00
STATE 7	0.00	0.00	66.64	592.00	50.00	40.00	32.00	0.00
STATE 8	0.00	0.00	66.64	592.00	50.00	40.00	32.00	2.00
STATE 9	50.00	640.00	673.06	592.00	0.00	40.00	32.00	0.00
STATE 10	0.00	640.00	673.06	592.00	0.00	40.00	32.00	2.00
STATE 11	0.00	640.00	673.06	592.00	0.00	40.00	32.00	0.00
STATE 12	0.00	640.00	673.06	592.00	0.00	40.00	32.00	2.00
STATE 13	50.00	0.00	66.64	592.00	0.00	40.00	32.00	0.00
STATE 14	0.00	0.00	66.64	592.00	0.00	40.00	32.00	2.00
STATE 15	0.00	0.00	66.64	592.00	0.00	40.00	32.00	0.00
STATE 16	0.00	0.00	66.64	592.00	0.00	40.00	32.00	2.00

MATRIX OF CONSEQUENCES FOR ACTION=SEAL

ATTRIBUTE	1	2	3	4	5	6	7	8
STATE 1	50.00	640.00	659.74	520.00	50.00	80.00	28.00	0.00
STATE 2	0.00	640.00	659.74	520.00	50.00	80.00	28.00	2.00
STATE 3	0.00	640.00	659.74	520.00	50.00	80.00	28.00	0.00
STATE 4	0.00	640.00	659.74	520.00	50.00	80.00	28.00	2.00
STATE 5	50.00	0.00	53.31	520.00	50.00	80.00	28.00	0.00
STATE 6	0.00	0.00	53.31	520.00	50.00	80.00	28.00	2.00
STATE 7	0.00	0.00	53.31	520.00	50.00	80.00	28.00	0.00
STATE 8	0.00	0.00	53.31	520.00	50.00	80.00	28.00	2.00
STATE 9	50.00	640.00	659.74	520.00	0.00	80.00	28.00	0.00
STATE 10	0.00	640.00	659.74	520.00	0.00	80.00	28.00	2.00
STATE 11	0.00	640.00	659.74	520.00	0.00	80.00	28.00	0.00
STATE 12	0.00	640.00	659.74	520.00	0.00	80.00	28.00	2.00
STATE 13	50.00	0.00	53.31	520.00	0.00	80.00	28.00	0.00
STATE 14	0.00	0.00	53.31	520.00	0.00	80.00	28.00	2.00
STATE 15	0.00	0.00	53.31	520.00	0.00	80.00	28.00	0.00
STATE 16	0.00	0.00	53.31	520.00	0.00	80.00	28.00	2.00

MATRIX OF CONSEQUENCES FOR ACTION=OVERLAY

ATTRIBUTE	1	2	3	4	5	6	7	8
STATE 1	50.00	640.00	606.42	0.00	50.00	200.00	0.00	0.00
STATE 2	0.00	640.00	606.42	0.00	50.00	200.00	0.00	2.00
STATE 3	0.00	640.00	606.42	0.00	50.00	200.00	0.00	0.00
STATE 4	0.00	640.00	606.42	0.00	50.00	200.00	0.00	2.00
STATE 5	50.00	0.00	0.00	0.00	50.00	200.00	0.00	0.00
STATE 6	0.00	0.00	0.00	0.00	50.00	200.00	0.00	2.00
STATE 7	0.00	0.00	0.00	0.00	50.00	200.00	0.00	0.00
STATE 8	0.00	0.00	0.00	0.00	50.00	200.00	0.00	2.00
STATE 9	50.00	640.00	606.42	0.00	0.00	200.00	0.00	0.00
STATE 10	0.00	640.00	606.42	0.00	0.00	200.00	0.00	2.00
STATE 11	0.00	640.00	606.42	0.00	0.00	200.00	0.00	0.00
STATE 12	0.00	640.00	606.42	0.00	0.00	200.00	0.00	2.00
STATE 13	50.00	0.00	0.00	0.00	0.00	200.00	0.00	0.00
STATE 14	0.00	0.00	0.00	0.00	0.00	200.00	0.00	2.00
STATE 15	0.00	0.00	0.00	0.00	0.00	200.00	0.00	0.00
STATE 16	0.00	0.00	0.00	0.00	0.00	200.00	0.00	2.00

MULTIATTRIBUTE UTILITY IS
NEGATIVE MULTIPLICATIVE, SUM(KK(I))= 2.482
INTERACTION FACTOR KAY=-0.977

TERMINAL VALUES OF TEMPORAL UTILITY = -1.00
TEMPORAL SCALING FACTORS:
KT(1) = 0.30
KT(2) = 0.30
KT(3) = 0.30
KT(4) = 0.30

KT(5) = 0.30
TEMPORAL UTILITY IS
NEGATIVE MULTIPLICATIVE, SUM(KT(I))= 1.500
TEMPORAL INTERACTION FACTOR KAYT=-0.691

O P T I M U M D E C I S I O N S

STAGE INDEX COUNTS FROM THE PRESENT

STAGE	STATE	ACTION	UTILITY
1	1	OVERLAY	-0.799
1	2	OVERLAY	-0.799
1	3	OVERLAY	-0.799
1	4	OVERLAY	-0.799
1	5	OVERLAY	-0.799
1	6	OVERLAY	-0.799
1	7	OVERLAY	-0.799
1	8	OVERLAY	-0.799
1	9	OVERLAY	-0.799
1	10	OVERLAY	-0.799
1	11	OVERLAY	-0.799
1	12	OVERLAY	-0.799
1	13	OVERLAY	-0.799
1	14	OVERLAY	-0.799
1	15	OVERLAY	-0.799
1	16	OVERLAY	-0.799

POLICY BECOMES STATIONARY IN STAGE ITERATION = 1