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A bottom-up solution for the multi-facility optimal pavement resurfacing problem

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ABSTRACT

Transportation infrastructure management has been a subject of growing economic importance in recent years due to the magnitude of agency expenditures. Increasingly sophisticated methods have been developed to model pavement deterioration and solve for optimal management strategies. However, it is unclear whether these more complex methods are providing more useful results. This paper presents a simple approach for optimizing the frequency and intensity of resurfacing for multiple highway facilities. It builds upon existing optimization methods for the single-facility, continuous-state, continuous-time problem and corresponding results, which include a threshold structure for optimal solutions. This threshold structure allows for mathematical simplifications and for a straightforward optimization approach to be applied to the multi-facility case. The approach is bottom-up rather than top-down, preserving facility-specific features to develop informative budget allocation results. Application of the approach in a case study indicates that solutions are likely to be robust to deterioration model uncertainty, which is consistent with previous facility-level findings. In addition, the methodology is shown to be robust to the form of the deterioration model.

1. Introduction

In recent decades transportation infrastructure management has been a topic of great importance due to the magnitude of agency expenditures and its influence on user costs (Durango-Cohen, 2007). Presently, despite the enormous spending going towards infrastructure maintenance, this is thought to be far from what is needed to support a safe and cost-effective transportation system. For instance, studies of US roadway infrastructure indicate that poor road conditions cost motorists $67 billion per year in repair and operating costs, 33% of major roads are in poor or mediocre condition, and that the current annual spending level of $70 billion for highway capital improvements is far from the estimated $186 billion needed to substantially improve highways (American Society of Civil Engineers, 2009). In addition, with the environment at the forefront of policy concerns, it has become apparent that pavement management has significant influence on emissions of criteria pollutants and greenhouse gases (Santero and Horvath, 2009; Sathaye et al., 2010).

The significance of pavement management has motivated a plethora of studies related to the optimization of maintenance, rehabilitation and reconstruction (MR&R) activities. Such studies can be classified by their specific modeling features. For the purposes of this paper, we distinguish models by:

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(1) Pavement condition state representation: Finite or Continuous.
(2) The number of facilities: Single or Multiple.
(3) Multi-facility optimization approach: Top-down or Bottom-Up.

As noted by (Durango-Cohen, 2007), MR&R optimization formulations have typically been developed as discrete-time, finite-state, Markov decision processes for multiple facilities, which were originally applied to pavements by (Golabi et al., 1982 and Carnahan, 1988). On the other hand, a minority of the pavement MR&R literature has utilized continuous-state formulations, including (Friesz and Fernandez, 1979; Fernandez and Friesz, 1981 and Markow and Balta, 1985). Although continuous-state formulations have not yet found pervasive use, they have potential advantages over finite-state models and have been used to identify important analytical properties of solutions of optimal pavement MR&R optimization. In this paper we build upon previous studies of continuous-state formulations.

A line of research, involving continuous-state, discrete-time formulations provides a methodology that simultaneously addresses performance prediction, and maintenance and repair decision-making for transportation infrastructure facilities. The underlying resource allocation problem is formulated as a stochastic optimal control problem with linear dynamics and a quadratic cost criterion (Durango-Cohen, 2007), and applicable performance models have been developed (Chu and Durango-Cohen, 2007). (Durango-Cohen, 2007) notes an important advantage of continuous-state MR&R formulations, which stems from the fact that most pavement deterioration models utilize a continuous-state dependent variable. As a result, the discretization of such a variable for the purposes of optimization can lead to flawed results in finite-state models. However, the continuous-state, discrete-time formulations are particularly useful when monitoring technologies are available to collect condition data at high temporal frequencies, which is unlikely to be the case for pavements.

Another line of research involves deterministic, continuous-state, continuous-time formulations for a single pavement facility. Generally, these formulations minimize the sum of user costs as a function of pavement state and agency costs as a function of MR&R intensity, subject to constraints in the form of deterministic deterioration, and MR&R improvement functions. The threshold solution is in the form of an optimal trigger roughness value, at which point a pavement should be resurfaced. Results reveal the robustness of threshold solutions to deterioration model uncertainty for both the infinite (Li and Madanat, 2002) and finite-horizon (Ouyang and Madanat, 2006) cases. The results in these papers provide two other useful conclusions. The first is that the maximum effective resurfacing intensity is optimal, which is similar to the findings in earlier empirical studies (Carnahan, 1988; Darter et al., 1985). This conclusion will be utilized for the formulation in this paper. The second is that the optimal trigger roughness solution enters a steady state soon after the implementation of optimal policies, and thus the solution is largely independent of initial pavement condition. Therefore, the focus of the formulation in the present paper is on the steady-state problem rather than the finite-horizon problem, since the solutions are similar. We note that a trend-curve optimal control method has also been used to solve the continuous-state, continuous-time resurfacing problem (Tsunokawa and Schofer, 1994; Ul-Islam and Tsunokawa, 2006). However, such an approximation has been shown to potentially induce problematic solutions (Li and Madanat, 2002).

In this paper, we extend the concepts from the aforementioned continuous-state, continuous-time, single-facility formulations to solve the multi-facility problem. Multi-facility optimization problems can be categorized as either top-down or bottom-up, of which the former is much more common in the literature (Kuhn and Madanat, 2005). As pointed out by (Durango-Cohen and Sarutipand, 2007), to simplify solution methods, multi-facility problems have typically been solved under the assumption that facilities are homogenous with respect to deterioration and cost parameters. This is a prominent feature of the top-down approach. However the omission of facility-specific features greatly reduces the usefulness of these optimization models, as they provide only coarse information regarding optimal budget allocation for a system composed of heterogeneous facilities (Ferreira et al., 2002; Madanat et al., 2006). Bottom-up pavement management formulations have been developed for both discrete-state (Chan et al., 1994; Ng and Lin, 2009; Ouyang, 2007) and continuous-state (Durango-Cohen, 2007; Ouyang and Madanat, 2004) cases. However, these models typically necessitate complex solution methods, often requiring the number of facilities to be relatively small. In this paper we show that the methods and results for the single-facility, continuous-state, continuous-time problem, provide a basis for a straightforward bottom-up, multi-facility formulation to the pavement resurfacing problem. This approach parallels that of (Robelin and Madanat, 2008) for bridges, in that it exploits the structure of the MR&R optimization problem to develop a tractable bottom-up methodology.

Section 2 presents a bottom-up methodology for the pavement resurfacing problem. Section 3 utilizes the methodology in a case study. This is followed by consideration of the effect of deterioration model uncertainty on optimal solutions and the methodology in Section 4. Section 5 concludes the paper.

2. Methodology

This section presents a methodology for solving the continuous-state, multi-facility, infinite-horizon problem for the case of deterministic deterioration and resurfacing-only policy. The basic formulation will initially be discussed, followed by presentation of a solution method.

2.1. Basic formulation

The basic formulation is given by the objective function in Eq. (1) and constraint in Eq. (2). Eq. (1) is the summation of total infinite horizon costs V_j across all facilities $j = 1,\ldots,J$. V_j is comprised of the maintenance cost for a single resurfacing activity M_j and user costs incurred between resurfacing activities U_j. The infinite horizon costs are represented by a geometric series, where $r < 1$ is the discount rate and τ_j is the time between overlays. B represents the agency budget per time for resurfacing activities, leading to the budget constraint shown in Eq. (2). V_j and M_j/τ_j are derived in Section 2.2 as functions of trigger roughness s_j^-, which will be the decision variable.

$$\min \sum_{j=1}^{J} V_j = \sum_{j=1}^{J} [M_j + U_j]/[1 - e^{-r\tau_j}] \tag{1}$$

$$\text{s.t.} \sum_{j=1}^{J} M_j/\tau_j \leqslant B \tag{2}$$

As previously mentioned, the infinite-horizon formulation is used in this paper, since (Li and Madanat, 2002 and Ouyang and Madanat, 2006) show that optimal solutions are fairly insensitive to initial conditions and that the steady-state solution is reached soon after the implementation of optimal policies. Thus, solving only the infinite-horizon problem simplifies the methodology without hindering its application.

The steady-state budget constraint is appropriate, since we assume that the agency has some flexibility to choose how to allocate an annual budget over the course of the year. In addition, (Li and Madanat, 2002) show that optimal costs are not greatly sensitive to the timing of resurfacing activities. Therefore we can realistically expect that an agency can temporally distribute resurfacings to comply with its budget without greatly affecting total costs.

Our formulation essentially follows the structure of what is commonly termed the ‘resource allocation problem’ (Ibaraki and Katoh, 1988; Patriksson, 2008). This allows us to derive further intuition, as we may interpret the formulation conceptually as the problem of determining optimal budget allocation to maximize some measure of social benefits, which has been the subject of a long history of investigation (Patriksson, 2008). As a result, with the reasonable assumptions for pavement management that corner solutions are suboptimal (i.e. $0 < s_j^- \forall j$) and that the budget constraint is binding, the optimal solution occurs when the marginal social benefit with respect to expenditure is equal for all j. Accordingly, after conversion of the problem to the Lagrangian form in Eq. (3), optimal values for $-dV_j/d(M_j/\tau_j)$ for all j will be equal to the Lagrange multiplier Λ. This insight will also be useful for developing a solution method in Section 2.3, since, as noted by (Li and Madanat, 2002), the derivation of analytical solutions for the pavement resurfacing problem is not typically possible.

$$\min L = \sum_{j=1}^{J} V_j + \Lambda\left[\sum_{j=1}^{J} M_j/\tau_j - B\right] \tag{3}$$

2.2. Cost functions

This section provides derivations for M_j and V_j as functions of trigger roughness s_j^-, for use in the formulation presented in Section 2.1. All formulas are based on realistic, empirical models (Ouyang and Madanat, 2004). Eq. (4) displays the resurfacing cost as a function of the intensity w_j. m_j and n_j are parameters.

$$M_j = m_j w_j + n_j \tag{4}$$

The maximum effective resurfacing intensity w_{maxj} is a linear function of s_j^-, as explained in (Ouyang and Madanat, 2004). w_{maxj} has been shown to be optimal for the infinite-horizon (Li and Madanat, 2002) and finite-horizon problems (Ouyang and Madanat, 2006), and therefore will be substituted for s_j^- in this paper.[1] Eq. (5) gives w_{maxj} as a function of s_j^-. h_j and p_j are parameters.

$$w_{maxj} = h_j s_j^- + p_j \tag{5}$$

Substitution of w_{maxj} into Eq. (4) for w_j results in Eq. (6), which provides M_j as a function of s_j^-.

$$M_j = [m_j[h_j s_j^- + p_j] + n_j] \tag{6}$$

Next, we derive an expression for τ_j. The formula for resurfacing effectiveness G_j, shown in Eq. (7), is the result of an approximation made in (Ouyang and Madanat, 2004). Data indicate that G_j is a linear function of w_j to a certain maximum effectiveness level G_{maxj}, and that G_{maxj} is a linear function of s_j^-. Using this information, we have:

$$G_j(w_j, s_j^-) = s_j^- - s_j^+ = G_{maxj} w_j / w_{maxj} = g_j s_j^- w_j / [h_j s_j^- + p_j] = g_j s_j^- \tag{7}$$

[1] We note that it is possible to construct analytical scenarios in which w_{maxj} is not optimal; however all previous work indicates that w_{maxj} is optimal, since $\partial[\textit{Objective Function}]_j/\partial w_j < 0$. This condition holds for all facilities considered in this paper as well, in which $\partial V/\partial w_j < 0$ for all relevant values of s_j^- and w_j. In addition, $\partial[M_j/\tau_j]/\partial w_j < 0$ for all relevant values of s_j^- and w_j; therefore the introduction of a budget constraint limiting M_j/τ_j does not contradict the optimality of w_{maxj}.

where g_i is a parameter, and where we substituted $w_{\max j}$ for w_j in the last step. Eq. (8) displays the formula for deterioration as a function of condition just after resurfacing s_j^+ and time since resurfacing t. This is taken directly from (Ouyang and Madanat, 2004). $\bar{f}_j$ and b are model parameters.

$$F_j\left(t, s_j^+\right) = s_j = \left[s_j^+ + \bar{f}_j\right]e^{bt} - \bar{f}_j \tag{8}$$

Solving F_j for τ_j, when $s_j = s_j^-$, gives the first step of Eq. (9). Eq. (7) is then used to substitute for s_j^+ in the second step. We note that τ_j could be equivalently used as a decision variable in the formulation, instead of s_j^-, since there is a one-to-one mapping between τ_j and s_j^-.

$$\tau_j = \frac{1}{b}ln\left[\frac{s_j^- + \bar{f}_j}{s_j^+ + \bar{f}_j}\right] = \frac{1}{b}ln\left[\frac{s_j^- + \bar{f}_j}{s_j^-[1-g_j] + \bar{f}_j}\right] \tag{9}$$

The user cost C_j as a function of pavement state s_j, shown in Eq. (10), is again taken from (Ouyang and Madanat, 2004). c_j and d_j are parameters. F_j and G_j are used to substitute for s_j in C_j, as shown in Eq. (10).

$$C_j(s_j) = c_j s_j + d_j = c_j\left[\left[s_j^-[1-g_j] + \bar{f}_j\right]e^{bt} - \bar{f}_j\right] + d_j \tag{10}$$

This is applied to derive the formula for the discounted infinite-horizon user cost in Eq. (11).

$$\begin{aligned} U_j/[1-e^{-r\tau_j}] &= \left[\int_0^{\tau_j} C^j(s_j)e^{-rt}dt\right]/[1-e^{-r\tau_j}] \\ &= \begin{cases} \dfrac{c_j[e^{(b-r)\tau_j}-1]\left[s_j^-[1-g_j]+\bar{f}_j\right]}{[b-r][1-e^{-r\tau_j}]} - \dfrac{[c_j\bar{f}_j - d_j]}{r} & \text{if } b \neq r \\ \dfrac{c_j\tau_j\left[s_j^-[1-g_j]+\bar{f}_j\right]}{[1-e^{-r\tau_j}]} - \dfrac{[c_j\bar{f}_j - d_j]}{r} & \text{if } b = r \end{cases} \end{aligned} \tag{11}$$

2.3. Solution method

In this section we present a simple solution method for the formulation presented in Sections 2.1 and 2.2. Before presenting the solution method, we note that for most plausible cost functions, the feasible region will be convex, since $M_j/[1-e^{-r\tau_j}]$ should be strictly convex. However the objective function $\sum_{j=1}^J V_j$ may not be convex. This is because $U_j/[1-e^{-r\tau_j}]$ is not likely to be convex, since τ_j grows at a decreasing marginal rate with respect to s_j^-. Nevertheless, $\sum_{j=1}^J V_j$ should still be a strictly pseudoconvex[2] function of s_j^-, since it is the sum of a increasing function $U_j/[1-e^{-r\tau_j}]$ and a strictly convex function $M_j/[1-e^{-r\tau_j}]$. These convexity features, of the feasible region and objective function, allow for nonlinear programming techniques to be applied (Bazaraa et al., 2006); these are discussed in Section 4.2. Potentially useful techniques include those focusing on separable programming (Stefanov, 2001), resource allocation problems (Patriksson, 2008), and knapsack problems (Bretthauer and Shetty, 2002). In this paper we use a 'greedy' approach, which is similar to algorithms which are referred to as 'marginal allocation' or 'incremental' (Ibaraki and Katoh, 1988).

The solution method is comprised of three steps, for which an outline is provided in Fig. 1.

(1) First, we specify initial values $s_{j0}^- \forall j$. The subscript k is introduced to denote iteration for the solution method, and $k = 0$ denotes the first iteration. Two possible values for s_{j0}^- provide useful lower and upper bounds to the optimal value of $\sum_{j=1}^J V_j$. A lower bound s_j^{l-} can be estimated by numerically solving for the value of s_j^- that minimizes V_j for each facility independently, using the method of (Li and Madanat, 2002). An upper bound can be estimated be numerically solving for s_j^- that minimizes M_j/τ_j for each facility independently, which we denote by s_j^{u-}. The reason these are useful bounds is shown in Fig. 2, which displays cost functions based on the parameter values for facility 2 in the parametric analyses of (Ouyang and Madanat, 2004). The network-level optimal s_j^- must lie in the interval $\left[s_j^{l-},\ s_j^{u-}\right]$, since outside this interval both V_j and M_j/τ_j can be reduced as a result of a change in s_j^-.

If $B \geqslant \sum_{j=1}^J M_j\left(s_j^{l-}\right)/\tau_j\left(s_j^{l-}\right)$, then values of s_j^{l-} provide the optimal solution. If $B = \sum_{j=1}^J M_j\left(s_j^{u-}\right)/\tau_j\left(s_j^{u-}\right)$, then values of s_j^{u-} provide the optimal solution. If $B < \sum_{j=1}^J M_j\left(s_j^{u-}\right)/\tau_j\left(s_j^{u-}\right)$, then the problem is infeasible if we assume that s_j^- must take on a reasonable value for all facilities. In addition, this holds even if we do not presuppose that the maximum resurfacing effectiveness is optimal, since s_j^{u-} and the corresponding value of w_{maxj} minimize M_j/τ_j.

[2] We apply the definition of (Bazaraa et al., 2006) of strictly pseudoconvex to pertain to $\sum_{j=1}^J V_j$: The function $\sum_{j=1}^J V_j$ is said to be *strictly pseudoconvex* if, for each distinct pair of vectors $\{\boldsymbol{a}^-, \boldsymbol{b}^-\} \in \mathbb{R}_+^J$ satisfying $\nabla f(\boldsymbol{a}^-)^T[\boldsymbol{b}^- - \boldsymbol{a}^-] \geqslant 0$, we have that $f(\boldsymbol{b}^-) > f(\boldsymbol{a}^-)$. $\boldsymbol{a}^-$ and $\boldsymbol{b}^-$ are $J \times 1$ vectors containing values for s_j^-.

(1) Determine lowerbound an upperbound values for $s_j^- \ \forall \ j$. These bounds are used to specify initial trigger roughness values s_{j0}^-.

(2) Iteratively adjust values for s_{jk}^- until the budget constraint is satisfied. The search direction $-\boldsymbol{\lambda}_k/\|\boldsymbol{\lambda}_k\|$ is designed to consider the marginal effects on the objective function versus the budget constraint at each iteration k. The step size α is reduced as iterations are increased to ensure convergence.

(3) Iteratively adjust values for s_{jk}^- until the ratio of marginal changes in V_j to M_j/τ_j, with respect to s_j^-, is equal to the same value Λ^* for all facilities.

Fig. 1. Solution method outline.

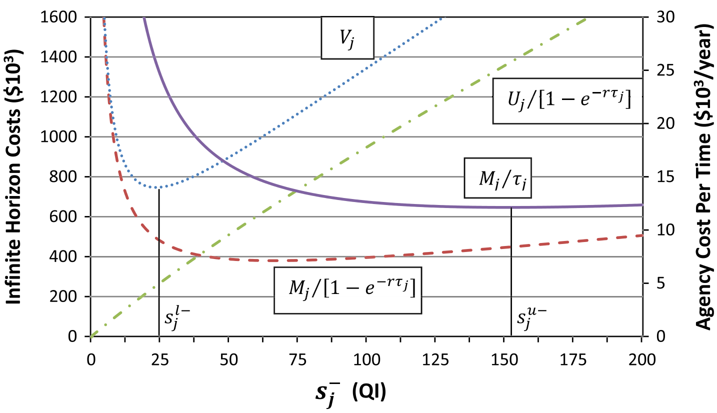

Fig. 2. Costs as a function of trigger roughness for a single-facility.

If $B < \sum_{j=1}^{J} M_j\left(s_j^{l-}\right)/\tau_j\left(s_j^{l-}\right)$ and $B > \sum_{j=1}^{J} M_j\left(s_j^{u-}\right)/\tau_j\left(s_j^{u-}\right)$, in order to pick initial values s_{j0}^-, the value of $\sum_{j=1}^{J} M_j/\tau_j$ can be calculated using values for s_j^{l-} in one case and values for s_j^{u-} in another case. The case which results in $\sum_{j=1}^{J} M_j/\tau_j$ being closer to B will be used to specify initial values s_{j0}^-, as shown in Eq. (12). We select the case which is closer, since it is expected to lead more quickly to the final solution.

$$s_{j0}^- = \begin{cases} s_j^{l-} & \mathit{if} \left|\sum_{j=1}^{J} M_j\left(s_j^{l-}\right)/\tau_j\left(s_j^{l-}\right) - B\right| \leqslant \left|\sum_{j=1}^{J} M_j\left(s_j^{u-}\right)/\tau_j\left(s_j^{u-}\right) - B\right| \\ s_j^{u-} & \text{otherwise} \end{cases} \tag{12}$$

(2) Next, we iteratively adjust s_{jk}^- for each j until the budget constraint is satisfied. k is used to denote iteration. For computational practicality the second condition is converted to $\left|\sum_{j=1}^{J} M_j\left(s_{jk}^-\right)/\tau_j\left(s_{jk}^-\right) - B\right| < \varepsilon$, in which ε is the termination scalar. $\boldsymbol{s}_k^-$ will represent the $J \times 1$ vector containing values for s_{jk}^-. The vector $\boldsymbol{\lambda}_{jk}$, which has dimensions $J \times 1$, contains values for λ_{jk}, as defined in Eq. (13). The derivatives of the cost functions can be found in Appendix A. Values for λ_{jk} are designed to achieve the greatest reduction in the value of V_j relative to the increase in M_j/τ_j when $\boldsymbol{s}_0^- = \boldsymbol{s}^{u-}$, and the greatest reduction in M_j/τ_j relative to increases in the value of V_j when $\boldsymbol{s}_0^- = \boldsymbol{s}^{l-}$

$$\lambda_{jk} = \begin{cases} \frac{d[M_j/\tau_j]}{ds_j^-}\left(s_{jk}^-\right) / \frac{dV_j}{ds_j^-}\left(s_{jk}^-\right) & \text{if } k > 1 \text{ and } \mathbf{s}_0^- = \mathbf{s}^{l-} \\ -\frac{dV_j}{ds_j^-}\left(s_{jk}^-\right) / \frac{d[M_j/\tau_j]}{ds_j^-}\left(s_{jk}^-\right) & \text{if } k > 1 \text{ and } \mathbf{s}_0^- = \mathbf{s}^{u-} \\ \frac{d[M_j/\tau_j]}{ds_j^-}\left(s_{jk}^-\right) & \text{if } k = 0 \text{ and } \mathbf{s}_0^- = \mathbf{s}^{l-} \\ -\frac{dV_j}{ds_j^-}\left(s_{jk}^-\right) & \text{if } k = 0 \text{ and } \mathbf{s}_0^- = \mathbf{s}^{u-} \end{cases} \tag{13}$$

The search direction is $-\boldsymbol{\lambda}_k/\|\boldsymbol{\lambda}_k\|$ and the step size is α. Therefore, at each iteration a new vector $\mathbf{s}_{k+1}^-$ is estimated such that $\mathbf{s}_{k+1}^- = \mathbf{s}_k^- - \alpha \times \boldsymbol{\lambda}_k/\|\boldsymbol{\lambda}_k\|$. α is reduced as k increases to ensure convergence. A simple criterion for reducing α is to change it when the sign of $\sum_{j=1}^{J} M_j\left(s_{jk}^-\right)/\tau_j\left(s_{jk}^-\right) - B$ is different from the sign of $\sum_{j=1}^{J} M_j\left(s_{jk+1}^-\right)/\tau_j\left(s_{jk+1}^-\right) - B$, and reassigning $\mathbf{s}_{k+1}^-$ to equal $\mathbf{s}_k^-$.

(3) Once the budget constraint is satisfied, $\mathbf{s}_k^-$ is adjusted until the second optimality condition is reached. The second condition is that $-\gamma_{jk} = \Lambda^* \forall j = 1,\ldots,J$, where Λ^* represents the optimal value of the Lagrange multiplier in Eq. (3). γ_{jk} is defined as shown in Eq. (14). $\boldsymbol{\gamma}_k$ is a $Jx1$ vector containing values of γ_{jk}.

$$\gamma_{jk} = \frac{dV_j}{ds_j^-}(s_{jk}^-) \Bigg/ \frac{d[M_j/\tau_j]}{ds_j^-}(s_{jk}^-) \tag{14}$$

This condition holds for a convex feasible region for decision variables (s_J^-) and a strictly pseudoconvex objective function (V_J) (Bazaraa et al., 2006). For computational practicality the second condition is converted to $|\gamma_{j_1 k} - \gamma_{j_2 k}| < \varphi \forall \{j_1, j_2\} \in 1,\ldots,J$ and $j_1 \neq j_2$. φ is the termination scalar.

A simple method for adjusting s_{jk}^- to meet the second condition is to split the values for γ_{jk} between those which are less (more negative) than the mean $\left(\sum_{j=1}^{J}\gamma_{jk}/J\right)$ and those which are greater (less negative) than the mean. The set of facilities which have more negative γ_{jk} is denoted by L_k and the set of facilities which have less negative γ_{jk} is denoted by H_k. The search directions are $\rho_{jk} = -w_k \times \gamma_{jk}/\|\boldsymbol{\gamma}_k\| \forall j \in L_k$ and $\rho_{jk} = \gamma_{jk}/\|\boldsymbol{\gamma}_k\| \forall j \in H_k$, since γ_{jk} is a decreasing function of s_{jk}^- for the models and parameter values used in this paper. This ensures that values for s_{jk}^- are generally shifting towards the solution. $\boldsymbol{\rho}_k$ is a $Jx1$ vector containing values for ρ_{jk}. In order to ensure that the budget constraint is maintained, the search direction for $j \in L_k$ is additionally weighted by w_k, as shown in Eq. (15).

$$w_k = \left[\sum_{H_k} \frac{d[M_j/\tau_j]}{ds_j^-}(s_{jk}^-) \times \gamma_{jk}\right] \Bigg/ \left[\sum_{L_k} \frac{d[M_j/\tau_j]}{ds_j^-}(s_{jk}^-) \times \gamma_{jk}\right] \tag{15}$$

At each iteration a new vector $\mathbf{s}_{k+1}^-$ is estimated such that $\mathbf{s}_{k+1}^- = \mathbf{s}_k^- - \alpha \times \boldsymbol{\rho}_k$. α decreases as k increases to ensure convergence. A simple criterion for reducing α is to change it each time the sign of γ_{jk} is not the same as the sign of γ_{jk+1} for all j.

We have aimed to develop a simple and useful solution method, which may not necessarily use the best or most efficient methods for selecting values for s_{j0}^- or the search directions. However, for the case study of Section 3, convergence to the solution occurs quickly for reasonable values of α. Therefore we do not explore convergence properties in detail. We assume that α is initially 10 for both steps 2 and 3 of the solution method, and that α is reduced by one tenth when the criterion in each is step is met. Also, given the straightforward structure of the problem, this solution method should work well even for networks with a large number of facilities, although α must be decreased appropriately.

We note that the solution method is similar to that in (Hochbaum, 1994 and Hochbaum, 1995) for the knapsack problem, in that we utilize α to scale the problem at each iteration. Therefore, a further study of convergence properties would be similar to that found in (Hochbaum, 1995), which presents a polynomial time approximation scheme for the nonlinear knapsack problem.

3. Case study

In this section, based on the methodology presented in Section 2, we provide results for a 3-facility example network which has been used in a previous study (Ouyang and Madanat, 2004). Table 1 presents parameter information for each facility. In accordance with the previous study, $\bar{f}_j^* = \bar{f} \times (1 - e^{-b})$.

Table 1
Parameters for a System of Three Facilities.

	m_j	n_j	h_j	p_j	g_j	b	$\bar{f}_j^*$	c_j	d_j	r
Facility 1	3.0	170	0.55	18.3	0.66	0.0153	2	1.2	0	0.07
Facility 2	2.5	150	0.55	18.3	0.66	0.0153	1.5	1.0	0	0.07
Facility 3	2.5	150	0.55	18.3	0.66	0.0153	1.6	1.1	0	0.07

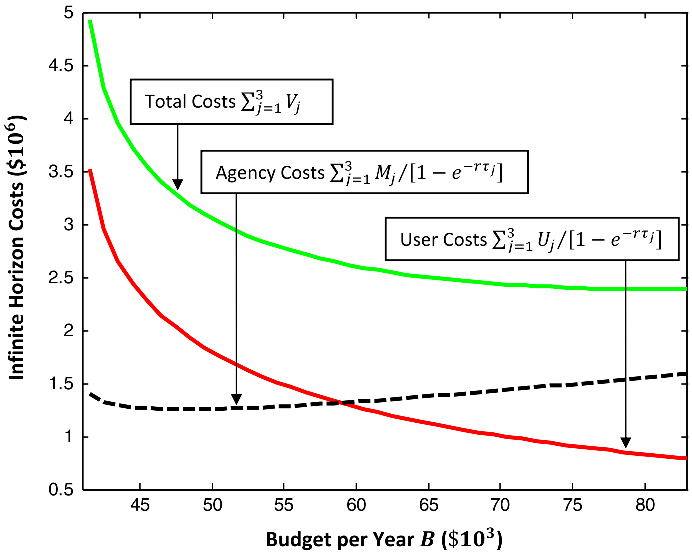

Fig. 3. Effect of budget on network-level optimal costs.

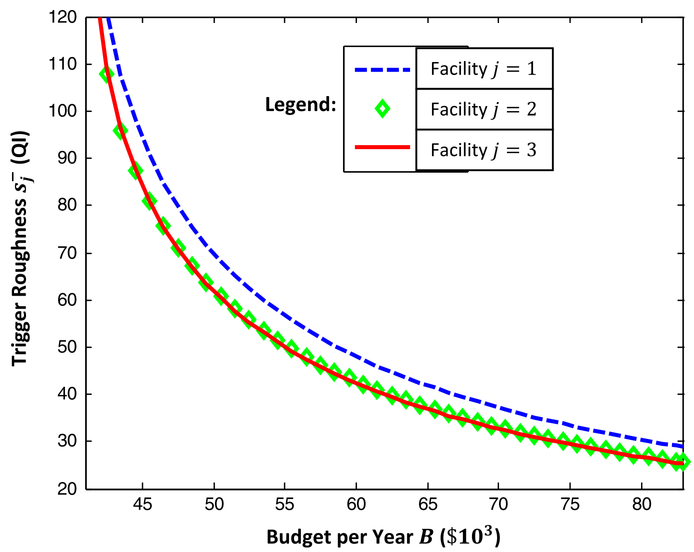

Fig. 4. Effect of budget on trigger roughness values.

Fig. 3 presents the effect of yearly budget B on optimal costs. B must be greater than about \$41,300/year in order for the problem to be feasible. If B is greater than about \$83,300/year, the budget constraint is non-binding. The total infinite horizon cost $\sum_{j=1}^{3} V_j$ is convex, exhibiting significant incremental improvements from increased B, when B is low. On the other hand, when B is high, $\sum_{j=1}^{3} V_j$ does not greatly improve for increased B. Nevertheless, $\sum_{j=1}^{3} V_j$ is a decreasing function of B over the domain displayed in Fig. 3.

Fig. 4 presents the effect of B on optimal trigger roughness values s_j^-. This effect is similar for facilities 2 and 3, since their parameters are similar. The significant improvements in $\sum_{j=1}^{3} V_j$ for incremental increases in B shown in Fig. 3 for low B strongly correlate with the decreases in s_j^-. However, this incremental decrease in s_j^- wanes as B increases.

Figs. 5 and 6 present the effect of a deterioration model parameter, for facility 2, on optimal costs and values for s_j^-. As can be seen, the optimal values for s_j^- change significantly as $\bar{f}_2^*$ is shifted. This result differs from that in (Li and Madanat, 2002), which indicates that optimal trigger roughness is fairly robust to deterioration model uncertainty. The reason for this difference will be further discussed in Section 4.

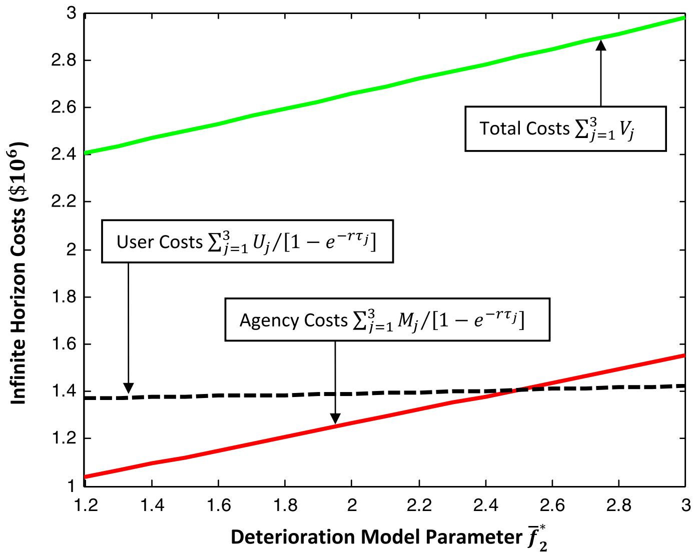

Fig. 5. Effect of deterioration model parameter $\bar{f}_2^*$ on network-level costs for B = $65,000/year.

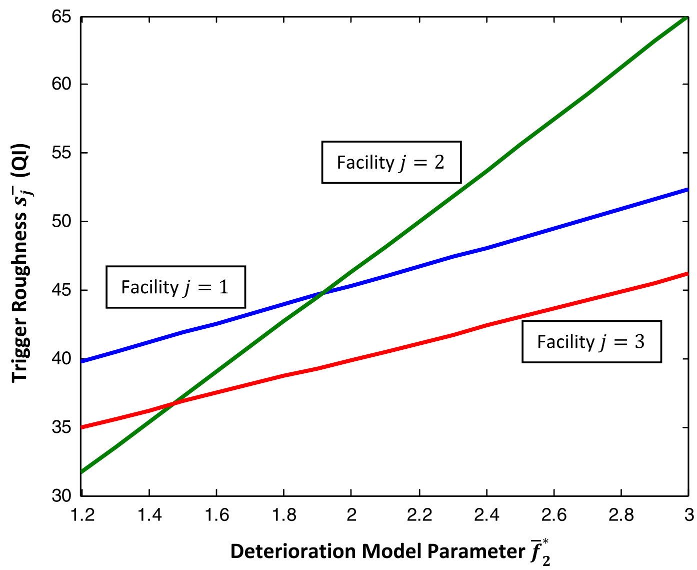

Fig. 6. Effect of deterioration model parameter $\bar{f}_2^*$ on trigger roughness values for B = $65,000/year.

4. Uncertainty in the deterioration process

Pavement deterioration is very difficult to predict accurately. Therefore, in Section 4.1 we consider the robustness of optimal values of s_j^- to deterioration parameter uncertainty. This is followed, in Section 4.2, by consideration for the robustness of the methodology to uncertainty in the form of the deterioration model.

4.1. Parameter uncertainty

In this section we consider the robustness to parameter uncertainty of optimal solutions of s_j^-. A related study to that in this section is (Ng et al., 2010), which aims to estimates the price of uncertainty. (Ng et al., 2010) estimates the additional cost of guaranteeing that optimization models produce solutions that satisfy a minimum acceptable pavement condition at a specified probability. This problem seems more relevant for infrastructure systems like bridges for which the primary goal of MR&R is to maintain facilities above some specified condition at some level of risk. For pavement management, the primary concern should be the influence of parameter uncertainty on the minimum expected costs. The most pertinent question for multi-facility MR&R optimization is whether or not stochastic models provide improved MR&R policies for minimizing costs versus deterministic models. We develop an intuitive, analytical approach to consider this issue.

An agency conducting pavement MR&R chooses to allocate its budget across various facilities and possible actions to minimize costs. Therefore, for deterministic optimization methodologies, the main concern is whether a deterministic model accurately estimates the expected costs associated with each MR&R option. We can approach this concern by considering the total discounted infinite horizon costs as a function of possible actions and uncertain model parameters. In particular, possible actions are represented by values for s_j^- and the most uncertain model parameters are $\bar{f}_j$ and b, since pavement deterioration is stochastic (Madanat et al., 2002; Nakat and Madanat, 2008). Fig. 2 depicts discounted infinite horizon costs as a function of s_j^- for an example facility for which $\bar{f}_j = 98.79$ and $b = 0.0153$. For the same facility, Figs. 7 and 8 depict infinite

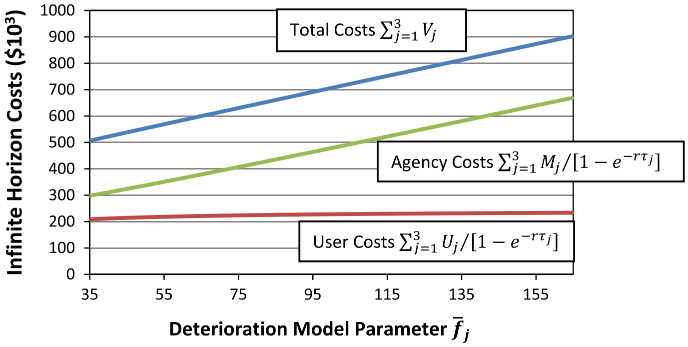

Fig. 7. Costs as a function of a deterioration model parameter $\bar{f}_j$ for trigger roughness $s_j^- = 100\text{QI}$.

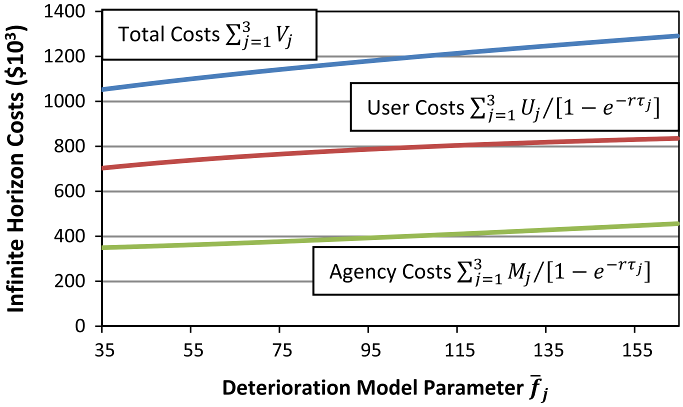

Fig. 8. Costs as a function of a deterioration model parameter $\bar{f}_j$ for trigger roughness $s_j^- = 25\text{QI}$.

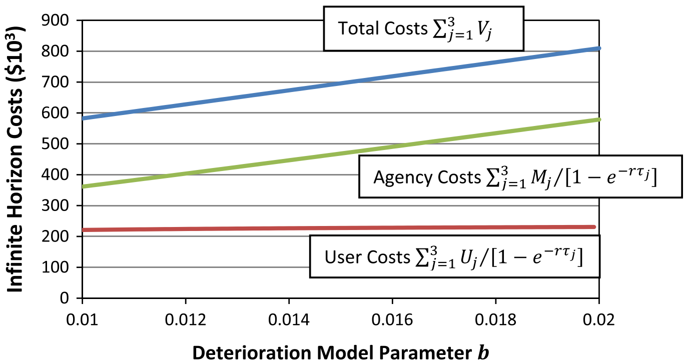

Fig. 9. Costs as a function of a deterioration model parameter b for trigger roughness $s_j^- = 25\text{QI}$.

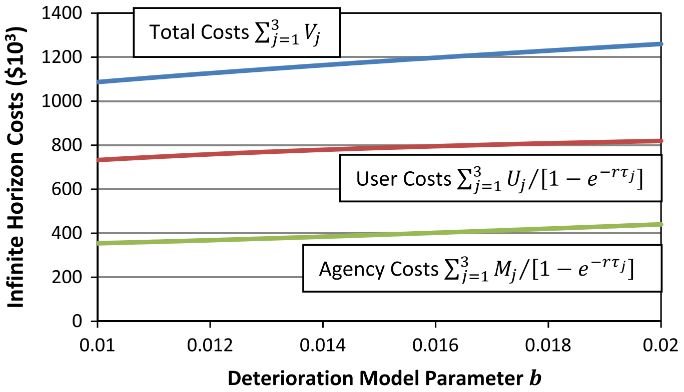

Fig. 10. Costs as a function of a deterioration model parameter b for trigger roughness $s_j^- = 100$QI.

horizon costs as a function of $\bar{f}_j$, and Figs. 9 and 10 do the same as a function of b. As can be seen, total costs $V_j(\bar{f}_j, b)$ is approximately a linear function of $\bar{f}_j$ and b for s_j^- values of 25 QI and 100 QI. This linear relationship holds generally over the range of reasonable values for $\bar{f}_j$ and b. Therefore, if we view $\bar{f}_j$ and b as random variables, then $E[V_j(\bar{f}_j, b)] = V_j(E[\bar{f}_j],\ E[b])$ for a given value of s_j^-.

A similar conclusion regarding the use of a deterministic versus stochastic deterioration model was reached in the single-facility simulation in (Li and Madanat, 2002). However, the conclusion for the multi-facility case is slightly more limiting, as the optimal values of s_j^- are more sensitive to changes in deterioration model parameters, as shown in Fig. 6. This is because the solution for the multi-facility problem depends on budget allocation across facilities, rather than exclusively on cost minimization.

4.2. Uncertainty of the functional form

In this section we consider the robustness of the optimization methodology in Section 2 to the form of the deterioration model $F_j(t, s_j^-)$, which will be presented as a function of t and s_j^-. We show that V_j is a pseudoconvex function of s_j^- and that the feasible region is convex under five conditions. The five conditions are first presented and a description of their implications on convexity features follows. A flow chart of the implications of the conditions on the convexity features is provided in Fig. 11.

We note that for the purposes of this section, $F_j(t, s_j^-)$ is assumed to be a strictly increasing, twice differentiable function of s_j^-, and a twice differentiable function of $F_j^{-1}(s_j, s_j^-) = t$. These assumptions are not essential, but allow for the use of an inverse function $F_j^{-1}(t, s_j^-)$ and differential calculus to develop insights. $F_j(t, s_j^-)$ is also presupposed to have a form which conforms to the reasoning presented in Section 4.1, in that a deterministic model for $F_j(t, s_j^-)$ can be used to accurately estimate $E[V_j]\forall s_j^-$, so that a deterministic optimization approach is useful.

Condition 1 is that the forms of $G_j = g_j s_j^-$, $U_j = \int_0^{\tau_j} [c_j s_j(t) + d_j] e^{-rt} dt$ and $M_j = [m_j[h_j s_j^- + p_j] + n_j]$ are as shown in Section 2.2. The forms of these functions are justified in (Ouyang and Madanat, 2004).

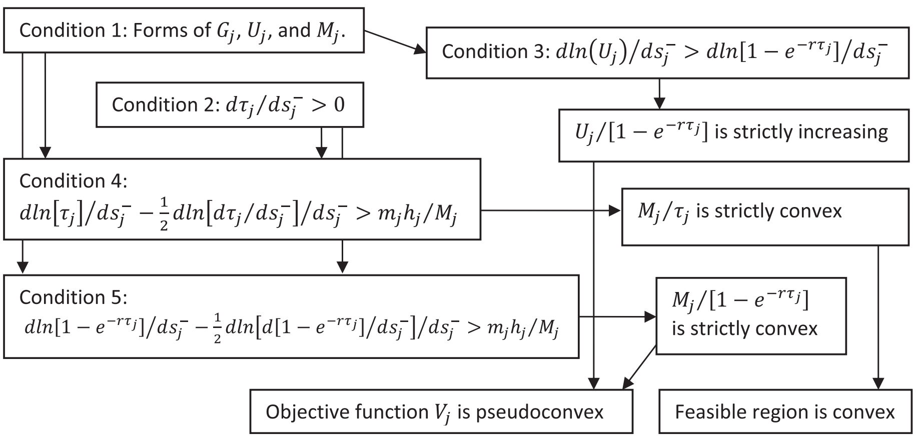

Fig. 11. A summary of the implications of the five conditions on convexity features of the problem.

Condition 2 requires that $d\tau_j/ds_j^- > 0$, which is reasonable, since one can expect that an increase in trigger roughness will allow for resurfacing to take place less frequently. A formula for $d\tau_j/ds_j^-$ is provided in Eq. (17), which is derived from Eq. (16). $F_j^{-1}(s_j, s_j^-) = t$ is the inverse function of $F_j(t, s_j^-) = s_j \cdot z_j^-$ and z_j^+ are introduced to represent the integral limits as functions $z_j^-(s_j^-) = s_j^-$ and $z_j^+(s_j^-) = [1-g] \times s_j^-$.

To develop some intuition for Eq. (17), we consider its terms. The terms in curly brackets can be thought of as the influence of G_j on τ_j, and the integral term can be thought of as the influence of $F_j(t, s_j^-)$ on τ_j, independently of G_j. The term in curly brackets in Eq. (17) is positive for typical functional forms of $F_j(t, s_j^-)$, since G_j increases with respect to s_j^-. In other words, the increase in τ_j, due to an increase in s_j^-, outweighs the decrease in τ_j, due to an increase in s_j^+. This is because $[1-g_j]$ reduces the latter effect, as can be seen in Eq. (17). Therefore, considering the integral term of Eq. (17), the equation indicates that the rate of increase of $\partial s_j/\partial F_j^{-1} = \partial F_j(t, s_j^-)/\partial t$ with respect to s_j^-, on the interval $[0, \tau_j]$, is not typically too large relative to the effect of G_j. This causes the integral term to not take on a very low negative value compared to the positive term in curly brackets, and in turn $d\tau_j/ds_j^-$ is positive and Condition 2 is satisfied. For $F_j(t, s_j^-)$ as shown in Eq. (8), the integral term is equal to 0 and a formula for $d\tau_j/ds_j^-$ is found in Eq. (A.2).

$$\tau_j = \int_{z_j^+}^{z_j^-} \frac{\partial F_j^{-1}}{\partial s_j}(s_j, s_j^-)ds_j \tag{16}$$

$$\begin{aligned}\frac{d\tau_j}{ds_j^-} &= \int_{z_j^+}^{z_j^-} \frac{\partial^2 F_j^{-1}}{\partial s_j^- \partial s_j}(s_j, s_j^-)ds_j + \left\{\frac{ds_j^-}{ds_j^-}\frac{\partial F_j^{-1}}{\partial s_j}(z_j^-, s_j^-) - \frac{ds_j^+}{ds_j^-}\frac{\partial F_j^{-1}}{\partial s_j}(z_j^+, s_j^-)\right\} \\ &= \int_{z_j^+}^{z_j^-} \frac{\partial^2 F_j^{-1}}{\partial s_j^- \partial s_j}(s_j, s_j^-)ds_j + \left\{\frac{\partial F_j^{-1}}{\partial s_j}(z_j^-, s_j^-) - [1-g_j]\frac{\partial F_j^{-1}}{\partial s_j}(z_j^+, s_j^-)\right\}\end{aligned} \tag{17}$$

Condition 3, shown in Eq. (19), is derived by setting $d[U_j/[1-e^{-r\tau_j}]]/ds_j^- > 0$ in Eq. (18), and multiplying by $[1-e^{-r\tau_j}]/U_j$. Condition 3 states that the percentage change in U_j, which is the lefthand side of Eq. (19), must be greater than the percentage change in $[1-e^{-r\tau_j}]$, which is the righthand side of Eq. (19). U_j is in accordance with Condition 1 and the formula for $d\tau_j/ds_j^-$ is in Eq. (17).

$$\frac{d[U_j/[1-e^{-r\tau_j}]}{ds_j^-} = \frac{dU_j/ds_j^-}{1-e^{-r\tau_j}} - \frac{U_j \times d[1-e^{-r\tau_j}]/ds_j^-}{[1-e^{-r\tau_j}]^2} \tag{18}$$

$$\left[\int_0^{\tau_j} c_j \frac{\partial F_j(t, s_j^-)}{\partial s_j^-} e^{-rt}dt + \frac{d\tau_j}{ds_j^-}[c_jF_j(\tau_j, s_j^-) + d_j]e^{-r\tau_j}\right] \Big/ U_j > \frac{re^{-r\tau_j}}{1-e^{-r\tau_j}} \times \frac{d\tau_j}{ds_j^-} \tag{19}$$

To develop intuition about Condition 3, we consider the terms in Eq. (19). The integral term in the numerator on the lefthand side represents the influence of $\partial F_j(t, s_j^-)/\partial s_j^- \forall t \in [0, \tau_j]$ on U_j, which is typically positive. The second term represents the influence of $d\tau_j/ds_j^-$ on U_j, which is also positive, in accordance with Condition 2. Similarly, since $d\tau_j/ds_j^-$ is positive, the term on the righthand side is positive. Therefore, the combined effect of the increase in $F_j(t, s_j^-)$ and the increase in τ_j on the percentage change in U_j, with respect to s_j^- must outweigh the effect on the percentage change in $[1-e^{-r\tau_j}]$ for Condition 3 to be satisfied.

Conditions 4 and 5 are discussed together, since the derivatives of M_j/τ_j and $M_j/[1-e^{-r\tau_j}]$ have similar forms. The derivatives for $M_j/[1-e^{-r\tau_j}]$ just require additional use of the chain rule. Therefore, we use the function T_j to represent either τ_j or $1-e^{-r\tau_j}$, and in turn M_j/τ_j and $M_j/[1-e^{-r\tau_j}]$ are represented by M_j/T_j. Conditions 4 and 5 are shown in Eq. (21), which is derived by setting $d^2[M_j/T_j]/d[s_j^-]^2 > 0$ in Eq. (20), multiplying by $[T_j]^2/[M_j \times dT_j/ds_j^-]$ and using logarithmic derivatives. $[T_j]^2/[M_j \times dT_j/ds_j^-]$ is positive, in accordance with Conditions 1 and 2, so the direction of the inequality is not changed. The terms in Eq. (21) represent the percentage changes in T_j, dT_j/ds_j^- and M_j. M_j is in accordance with Condition 1, $dln[T_j]/ds_j^-$ can be obtained by using Eq. (17), and $dln[dT_j/ds_j^-]/ds_j^-$ can be obtained by using Eq. (22).

$$\frac{d^2[M_j/T_j]}{d[s_j^-]^2} = \frac{2 \times M_j \times [dT_j/ds_j^-]^2}{[T_j]^3} - \frac{2 \times dM_j/ds_j^- \times dT_j/ds_j^-}{[T_j]^2} - \frac{M_j \times d^2T_j/d[s_j^-]^2}{[T_j]^2} \tag{20}$$

$$dln[T_j]/ds_j^- - \frac{1}{2}dln[dT_j/ds_j^-]/ds_j^- > m_jh_j/M_j \tag{21}$$

$$\begin{aligned}\frac{d^2\tau_j}{d[s_j^-]^2} &= \int_{z_j^+}^{z_j^-} \frac{\partial^3 F_j^{-1}}{\partial [s_j^-]^2 \partial s_j}(s_j, s_j^-)ds_j + \left\{\frac{\partial^2 F_j^{-1}}{\partial s_j^- \partial s_j}(z_j^-, s_j^-) - [1-g_j]\frac{\partial^2 F_j^{-1}}{\partial s_j^- \partial s_j}(z_j^+, s_j^-)\right\} \\ &\quad + \left\{\frac{\partial^2 F_j^{-1}}{\partial [s_j]^2}(z_j^-, s_j^-) - [1-g_j]^2\frac{\partial^2 F_j^{-1}}{\partial [s_j]^2}(z_j^+, s_j^-)\right\}\end{aligned} \tag{22}$$

To develop intuition about Conditions 4 and 5 we consider the terms of Eq. (21). $dln[T_j]/ds_j^-$ is positive, according to Condition 2, and represents the influence of T_j being in the denominator of M_j/T_j. $\frac{1}{2}dln[dT_j/ds_j^-]/ds_j^-$ is typically negative, as will be discussed in the next paragraph, and represents the influence of the rate of change of T_j, with respect to s_j^-, on the sign of $d^2[M_j/T_j]/d[s_j^-]^2$. m_jh_j/M_j is positive, and represents the interactive effect of changing T_j and M_j simultaneously with respect to s_j^-. In order for Conditions 4 and 5 to hold, the percentage increase in T_j and decrease in dT_j/ds_j^- must outweigh the associated increase for M_j. This has implications for how τ_j and in turn $F_j(t, s_j^-)$ can change with respect to s_j^-, in accordance with Eqs. (17) and (22).

To develop intuition about Eq. (22) we note that $d^2\tau_j/d[s_j^-]^2$, and in turn $dln[dT_j/ds_j^-]/ds_j^-$ in Eq. (21), are typically negative. This occurs, since the second group of terms in curly brackets is typically negative. $\frac{\partial^2 F_j^{-1}}{\partial[s_j]^2}(z_j^-, s_j^-) = \frac{\partial}{\partial s_j}\frac{\partial t}{\partial F_j}(\tau_j, s_j^-)$ is likely to take on a lesser (more negative) value than $[1-g_j]^2\frac{\partial^2 F_j^{-1}}{\partial[s_j]^2}(z_j^+, s_j^-) = [1-g_j]^2\frac{\partial}{\partial s_j}\frac{\partial t}{\partial F_j}(0, s_j^-)$. This is because of the effect of $[1-g_j]^2$, and since $\frac{\partial^2 F_j}{\partial t^2}(\tau_j, s_j^-)$ is likely to be greater than $\frac{\partial^2 F_j}{\partial t^2}(0, s_j^-)$. For the first group of terms in curly brackets, the change in $\frac{\partial F_j}{\partial t}(\tau_j, s_j^-)$ with respect to s_j^- is generally greater than the change in $\frac{\partial F_j}{\partial t}(0, s_j^-)$ with respect to s_j^-. Considering the integral term, $\frac{\partial^2}{\partial[s_j^-]^2}\frac{\partial F_j}{\partial t}(t, s_j^-)$ should not take on very large negative values. As a result, the first group of terms in curly brackets and the integral term do not become large positive values that outweigh the negative value of the second group of terms in curly brackets. Subsequently, $d^2\tau_j/d[s_j^-]^2$, and in turn $dln[dT_j/ds_j^-]/ds_j^-$, are negative. We note that $dln[dT_j/ds_j^-]/ds_j^-$ could also take on a relatively small positive value, but this of course depends on the values for the other terms in Eq. (21). For $F_j(t, s_j^-)$, as shown in Eq. (8), the integral and the first group of terms in curly brackets, of Eq. (22), are equal to 0 and the second group of terms in curly brackets can be shown to be negative for typical parameter values.

Finally, based on the five conditions, we describe the convexity features of the problem presented in Section 2.1, in accordance with the summary of implications in Fig. 11. M_j/τ_j is strictly convex as a result of Condition 4. In turn, since M_j/τ_j is strictly convex, the feasible region is convex. In addition, V_j is the sum of a strictly increasing function $U_j/[1-e^{-r\tau_j}]$, as a result of Condition 3, and a strictly convex function $M_j/1-e^{-r\tau_j}$, as a result of Condition 5. This implies that V_j is pseudoconvex, as depicted for the facility in Fig. 2. These convexity features can be shown to hold for the deterioration model in Eq. (8), and since the features have been shown to hold under fairly reasonable conditions, problems with different deterioration models can also be expected to be solvable through the methodology in Section 2 or other nonlinear programming methods.

Although the conditions are likely to hold for a variety of deterioration models, the ease with which they can be verified varies. Condition 1 is straightforward, since it specifies forms for M_j, U_j and G_j. The remaining conditions are complicated by the need to consider the restrictions on the change in the deterioration function with respect to trigger roughness, over the interval $t \in [0, \tau_j]$. Nevertheless, Conditions 2 and 3 can be interpreted in a fairly straightforward manner, since $d\tau_j/ds_j^-$ is comprised of two features, having to do with G_j and $\partial F_j(t, s_j^-)/\partial s_j^-$, and Conditions 3–5 are comprised of the percentage changes of various aspects of the functions in the formulation. In addition, Conditions 2–5 are not greatly restrictive, since they do not impose any restrictions on the form on the deterioration model per se. These conditions impose restrictions on the way in which the deterioration function changes as a result of changes in trigger roughness. Therefore, a variety of potential deterioration models can be used.

5. Conclusion

This paper presents a simple methodology for solving the multi-facility pavement resurfacing problem. The formulation is developed based on the structure of optimal solutions for single-facility problems, as revealed by previous research, and is in the form of the resource allocation problem. The formulation utilizes a bottom-up rather than top-down approach, which preserves facility-specific characteristics. This important feature is not present in state-of-the-art multi-facility pavement management optimization approaches. In addition, the models applied in this paper in the formulation are empirical and realistic.

The methodology is also shown to be robust to parameter uncertainty for an empirical, realistic deterioration model, indicating that optimal solutions may be developed with deterministic rather than probabilistic deterioration models. Given the current state of funding for pavement management, condition data collection and deterioration prediction are not likely to improve greatly in the near future, making such robustness an important modeling featuring. To the authors' knowledge, this issue, regarding deterministic vs. probabilistic modeling, has not been previously addressed for the multi-facility problem in the pavement management literature. In addition, under reasonable conditions, the methodology is shown to be robust to the form of the deterioration model. The only restrictions are on how the model changes with respect to the policy variable, trigger roughness.

The solution method efficiently produces optimal solutions and is shown to work well in a case study with three facilities. This method should be applicable for systems with much larger numbers of facilities as well. For systems with large numbers of facilities, efficient nonlinear programming algorithms, such as those used in knapsack or resource allocation problems can be applied, given the simple structure of the formulation.

It should be noted that the approach in this paper does not account for potentially important complexities, such as facility interdependencies (Durango-Cohen and Sarutipand, 2007), and demand responsiveness (Durango-Cohen and Sarutipand, 2009). In such cases, the use of a continuous action space and steady-state conditions may not be useful and a discrete space approach may be necessary. The approach in this paper is better suited to broad planning of pavement management systems, which can be used to inform policy making for more detailed scheduling and localized maintenance decisions.

Nevertheless, the approach presented in this paper provides an attractive alternative to state-of-the-art methodologies for solving the multi-facility pavement resurfacing problem. Future work can be directed towards incorporating additional considerations into the formulation, such as traffic growth or multiple maintenance types. Results can then be compared between optimization methodologies to determine which are likely to be the most useful in various contexts.

Acknowledgements

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Appendix A. Cost function derivatives

In this appendix, we derive formulas for the derivatives used for the methods of Section 2.3.

In order to derive an analytical expression for $\frac{d[M_j/\tau_j]}{ds_j^-}$, we can use the total derivative as shown in Eq. (A.1). Formulas for $\frac{\partial[M_j/\tau_j]}{\partial\tau_j}$ and $\frac{\partial[M_j/\tau_j]}{\partial s_j^-}$ are relatively straightforward. The formula for $\frac{d\tau_j}{ds_j^-}$, shown in Eq. (A.2), results from taking the derivative of Eq. (9) with respect to s_j^-. The derivatives are then combined to yield the analytical expression in Eq. (A.1).

$$\frac{d[M_j/\tau_j]}{ds_j^-} = \frac{\partial[M_j/\tau_j]}{\partial\tau_j}\frac{d\tau_j}{ds_j^-} + \frac{\partial[M_j/\tau_j]}{\partial s_j^-} = \left[\frac{m_j[h_j s_j^- + p_j] + n_j}{-\tau_j^2 b}\right]\left[\frac{1}{s_j^- + \bar{f}_j} - \frac{1}{s_j^- + \bar{f}_j/[1-g_j]}\right] + \frac{m_j h_j}{\tau_j} \tag{A.1}$$

$$\frac{d\tau_j}{ds_j^-} = \frac{d[ln\{[s_j^- + \bar{f}_j]/[s_j^+ + \bar{f}_j]\}]}{b\cdot ds_j^-} = \frac{1}{b}\left[\frac{1}{s_j^- + \bar{f}_j} - \frac{1}{s_j^- + \bar{f}_j/[1-g_j]}\right] \tag{A.2}$$

Next we focus on deriving an expression for $\frac{dV_j}{ds_j^-}$. Accordingly expressions for $\frac{d[M_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ and $\frac{d[U_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ will be derived. $\frac{d[M_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ can be derived similarly to $\frac{d[M_j/\tau_j]}{ds_j^-}$, resulting in Eq. (A.3).

$$\begin{aligned}\frac{d[M_j/[1-e^{-r\tau_j}]]}{ds_j^-} &= \frac{\partial[M_j/[1-e^{-r\tau_j}]]}{\partial\tau_j}\frac{d\tau_j}{ds_j^-} + \frac{\partial[M_j/[1-e^{-r\tau_j}]]}{\partial s_j^-}\\ &= re^{-r\tau_j}\left[\frac{m_j[h_j s_j^- + p_j] + n_j}{-b[1-e^{-r\tau_j}]^2}\right]\left[\frac{1}{s_j^- + \bar{f}_j} - \frac{1}{s_j^- + \bar{f}_j/[1-g_j]}\right] + \frac{m_j h_j}{1-e^{-r\tau_j}}\end{aligned} \tag{A.3}$$

$\frac{d[U_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ can be solved for using the total derivative shown in Eq. (A.4). $\frac{\partial[U_j/[1-e^{-r\tau_j}]]}{\partial\tau_j}$ is shown in Eq. (A.5), $\frac{d\tau_j}{ds_j^-}$ is taken from Eq. (A.2) and $\frac{d[U_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ is straightforward. The combination of these derivatives results in the expression for $\frac{d[U_j/[1-e^{-r\tau_j}]]}{ds_j^-}$ shown in Eq. (A.4).

$$\begin{aligned}\frac{d[U_j/[1-e^{-r\tau_j}]]}{ds_j^-} &= \frac{\partial[U_j/[1-e^{-r\tau_j}]]}{\partial\tau_j}\frac{d\tau_j}{ds_j^-} + \frac{\partial[U_j/[1-e^{-r\tau_j}]]}{\partial s_j^-}\\ &= \frac{c_j}{b}[s_j^-[1-g_j] + \bar{f}_j]\left[\frac{e^{(b-r)\tau_j}}{1-e^{-r\tau_j}} + \frac{re^{-r\tau_j}[1-e^{(b-r)\tau_j}]}{[b-r][1-e^{-r\tau_j}]^2}\right]\left[\frac{1}{s_j^- + \bar{f}_j} - \frac{1}{s_j^- + \bar{f}_j/[1-g_j]}\right]\\ &\quad + \frac{c_j[e^{(b-r)\tau_j} - 1][1-g_j]}{[b-r][1-e^{-r\tau_j}]}\end{aligned} \tag{A.4}$$

$$\frac{\partial[U_j/[1-e^{-r\tau_j}]}{\partial\tau_j} = c_j[s_j^-[1-g_j] + \bar{f}_j]\left[\frac{e^{(b-r)\tau_j}}{[1-e^{-r\tau_j}]} + \frac{re^{-r\tau_j}[1-e^{(b-r)\tau_j}]}{[b-r][1-e^{-r\tau_j}]^2}\right] \tag{A.5}$$

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