Evaluation of Expected Life-Cycle Maintenance Cost of Deteriorating Structures

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Abstract: In a world where financial resources do not keep pace with the growing demand for maintenance of deteriorating structures, it is imperative that those responsible for maintenance decisions make the best possible use of limited financial resources. Decision makers have to evaluate the expected life-cycle maintenance cost of deteriorating structures and use benefit/cost techniques for finding the optimal resource allocation. This paper proposes a methodology for the evaluation of expected life-cycle maintenance cost of deteriorating structures by considering uncertainties associated with the application of cyclic maintenance actions. The methodology can be used to determine the expected number of maintenance interventions on a deteriorating structure, or a group of deteriorating structures, during a specified time horizon and the associated expected maintenance costs. The method is suitable for application to both new and existing civil infrastructures under various maintenance strategies. The ultimate objective is to evaluate the costs of alternative maintenance strategies and determine the optimum maintenance regime over a specified time horizon. In its present format, the first line of application of the method is for highway bridges. However, the method can be used for any structure, or group of structures, requiring maintenance in the foreseeable future. The proposed method can be programmed and incorporated into an existing software package for life-cycle costing of civil infrastructures. An existing reinforced concrete bridge stock is analyzed to illustrate the proposed methodology and to reveal the cost-effectiveness of preventive maintenance interventions.

DOI: 10.1061/(ASCE)0733-9445(2003)129:5(682)

CE Database subject headings: Costs; Deterioration; Life cycle; Rehabilitation; Structural analysis; Maintenance.

Introduction

Most structural systems need appropriate management during their lifetime in order to maintain their operational performance status. Decision makers must decide when and how to repair, rehabilitate, replace, and/or shut-down the deteriorating facilities. Effective costs evaluation methods are needed to assess reasonable expenditures for managing deteriorating structures during their service lives and to help managers to make the most appropriate decisions (FHWA 2000). Usually, the construction cost, the inspection cost, the maintenance cost, the user cost, and the failure cost are essential for the life-cycle cost analysis of deteriorating structures (Chang and Shinozuka 1996; Ang and De Leon 1997; Frangopol et al. 1997, 2000; Ang et al. 1998a,b; Das 1998, 2000; Maunsell Ltd. and Transport Research Laboratory 1998, 1999; Enright and Frangopol 1999; Frangopol 1999; Frangopol and Das 1999; Wallbank et al. 1999).

The times of application of inspection and maintenance and the duration of these actions (also called interventions) depend on various parameters and random variables. Therefore, the application times and durations of various interventions have to be represented by probability distributions. In general, the probability distributions of the times of application of interventions are provided based on the relative time scale (i.e., time is measured from a relative reference point, such as the time of application of previous maintenance). These relative times should be converted to the absolute time (e.g., age of the structure) in order to be used directly during decision making processes such as prioritization of maintenance needs and cost allocation. If the sequence of maintenance actions is prescribed, then the probability of a maintenance intervention at a relative point in time can be converted to the corresponding probability at an absolute point in time by using conditional joint distribution functions.

In this paper, a method to evaluate the expected probability of maintenance at a certain time (age) of a deteriorating structure and the expected life-cycle maintenance cost due to application of a series of subsequent interventions is proposed. During their service life, structural systems can experience various types of inspections and/or maintenance actions at different times. The associated costs of these actions can only be predicted by using conditional joint distribution functions. Since multiple integral steps are required, the solution process is usually computationally inefficient. To increase computational efficiency, the prescribed probability distributions of the times of various maintenance interventions are converted to probability mass functions. The classical event tree model (Ang and Tang 1984) was modified to consider not only available event paths but also lengths (i.e., durations) of these paths. In this manner, multiple integrals are replaced by summations. This is very effective for the evaluation of the expected annual probability of maintenance when cyclic interventions are applied and the expected annual costs associated

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Note. Associate Editor: Jamshid Mohammadi. Discussion open until October 1, 2003. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on December 19, 2001; approved on August 6, 2002. This paper is part of the ***Journal of Structural Engineering***, Vol. 129, No. 5, May 1, 2003. ©ASCE, ISSN 0733-9445/2003/5-682–691/$18.00.

with these interventions have to be evaluated. The method is suitable for application to both new and existing civil infrastructures under various maintenance strategies. The ultimate objective is to evaluate the costs of alternative maintenance strategies and determine the optimum maintenance regime over a specified time horizon. In its present format, the first line of application of the method is for highway bridges. However, the method can be used for any structure, or group of structures, requiring maintenance in the foreseeable future.

The new features included in Fortran-90 make it possible to use the modified event tree approach with more flexibility. For instance, by using the recursive capability, the number of interventions applied during a given time interval has not to be fixed. The proposed method is independent of the type of structure (e.g., bridges, offshore platforms) to which it is applied. In this paper, bridge structures are selected as the main applications.

Bridge management systems have focused renewed attention on cost data estimation and management, especially for the purpose of network-level analysis (Shirole 1994; Thompson 1994; Itoh et al. 1997; Das 1998; Hawk and Small 1998; Lauridsen et al. 1998; Small and Cooper 1998; Söderqvist and Veijola 1998; Thompson et al. 1998; Kong et al. 2000; Roberts and Shepard 2000; Small et al. 2000; Frangopol et al. 2001). The developed method can also be used for a group of similar deteriorating structural systems.

In general, the financial resources necessary for maintenance of civil infrastructures are made available in successive amounts at different times. The time sequence of expenditures can be defined a priori, or be the subject of an optimization process. This time-dependent effect of expenditures can be represented by the discount rate. According to Encyclopedia Britannica Online (1999), the discount rate, also called rediscount rate, or bank rate is the ''interest rate charged by a central bank for loans of reserve funds to commercial banks and other financial intermediaries. This charge originally was an actual discount (an interest charge held out from the amount loaned), but the rate is now a true interest charge, even though the term discount rate is still used.'' The discount rate serves as an important indicator of the condition of credit in an economy. According to Tilly (1997), ''The present costs represent the sum notionally to be set aside to meet all eventual costs after allowing for accumulation of interest on the sums intended to meet future commitments.'' . . . ''The aim was to funnel resources into projects of recognized importance at the expense, but not the abandonment, of projects of lesser importance.'' The discount rate has significant implications in the design and maintenance of civil infrastructures. The uncertainty in the discount rate could have significant effects on the costs of different maintenance strategies. However, in general, the discount rate is established by government agencies and this deterministic value has to be used when computing expected life-cycle cost. The cost evaluation method developed in this paper can accommodate the discount rate along with different probability distributions of associated maintenance interventions. The method of evaluating the probability distributions of associated maintenance interventions is described in Frangopol et al. (2001). An existing reinforced concrete bridge stock is analyzed to illustrate the proposed methodology and to reveal the cost-effectiveness of preventive maintenance interventions.

Probability of Maintenance

Let us define a set of random variables T_i, $i=1,\ldots,n$, where T_i represents the duration between two subsequent essential maintenance (also called rehabilitation) interventions $i-1$ and i. These random variables are represented using the relative timescale (i.e., the time interval between two subsequent interventions). There may be also random variables represented using the absolute time scale. In this case, time is measured from a given origin (i.e., a specified fixed point in time).

The relationship between variables based on the absolute and relative timescales is as follows:

$$T_i^* = \sum_{j=1}^{i} T_j \tag{1}$$

where T_i^* = application time of intervention i based on the absolute timescale and T_j = application time of intervention j based on the relative time scale. For convenience, let us define the random variable R_n instead of T_n^*. This random variable represents the application time of the nth intervention based on the absolute timescale. The following relation between R_n and the random variables T_i exists:

$$R_n = T_1 + T_2 + \cdots + T_n = \sum_{i=1}^{n} T_i \tag{2}$$

The distribution of R_n can be found if all the distributions of the random variables T_i are known. In general, the time of maintenance application is random and its probability distribution can be described by a continuous random variable with a specified probability-density function (PDF). Since multiple maintenance interventions are applied during the lifetime of a structure, multiple PDFs have to be considered. The computational effort resulted from using continuous random variables and their PDFs associated with numerical integration or Monte Carlo simulation is considerable and usually time consuming. In this study, a simpler and efficient method is developed based on replacing PDFs of maintenance application time by the corresponding probability mass functions (PMFs) of the respective discrete variables. It should be noted that the accuracy of the approach would be reduced by this approximation. However, it has been found that the difference between the two approaches is minimal when the accuracy of approximation of the generic continuous variables is high. This high accuracy is preserved in this study.

The occurrence of the intervention i at time T_i may be represented on a discrete time axis by introducing units such as years or months. Any continuous probability-density function (PDF) can be transformed into a probability mass function (PMF). Let us consider a simple case by assuming that a deteriorating system is rehabilitated twice at T_1^* and T_2^*. The application time of the second rehabilitation T_2^* can be described using two variables based on the relative timescale as follows:

$$T_2^* = R_2 = T_1 + T_2 \tag{3}$$

The probability mass function of R_2 is defined as

$$P[R_2(t_2^*)] = P_{T_1,T_2}[t_1,t_2] = P(T_1=t_1 \ \text{ and } \ T_2=t_2) \tag{4}$$

Let us introduce a new variable t_L^* representing a point in time based on the absolute timescale limiting the number of possible rehabilitation paths. This variable is used to select possible combinations of application times that satisfy the constraint $t_1+\cdots+t_n=t_L^*$. Using the new variable, the probability that the second rehabilitation occurs at time t_L^*, including all possible rehabilitation paths, can be computed as

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$$P[R_2(t_L^*)]=P(R_2=t_L^*)=\sum_{\text{if } t_1+t_2=t_1^*} P_{T_1,T_2}[t_1,t_2]$$

$$=\sum_{\text{all } t_1,t_2} P_{T_1,T_2}[t_1,t_2|(t_1+t_2=t_L^*)] \tag{5}$$

where $P[R_2(t_L^*)]=$probability that the second intervention occurs exactly at time t_L^* and $\Sigma_{\text{if } t_1t_2=t_L^*}=$summation over all cases that satisfy the condition $t_1+t_2=t_L^*$. Eq. (5) shows that the joint PMF is needed in this calculation. For the general case of n interventions, a similar result is obtained

$$P[R_n(t_L^*)]=\sum_{\text{if } t_1+\cdots+t_n=t_L^*} P_{T_1,T_2,\ldots,T_i,\ldots,T_n}[t_1,t_2,\ldots,t_i,\ldots,t_n]$$

$$=\sum_{\text{all } t_1,\ldots,t_n} P_{T_1,T_2,\ldots,T_i,\ldots,T_n}$$

$$\times\left[t_1,t_2,\ldots,t_i,\ldots,t_n\middle|\sum_{i=1}^{n} t_i=t_L^*\right] \tag{6}$$

In this case, the joint PMF based on random variables $T_1,T_2,\ldots,T_i,\ldots,T_n$ is needed.

The top graph in Fig. 1 shows the PDFs associated with different intervention cycles at the point in time t_L^*. Note that PDFs can also be represented by PMFs using the formulation based on discrete random variables. Strictly speaking, the PDFs have to be broken in a number of intervals of equal length t_u and the probabilities of random variables falling in each interval t_u have to be calculated. Let us imagine a plane normal to the time axis at $t=t_L^*$. If we assign a unit time period (say, one year) to the plane t_u the unit normal cross-section to the time axis is represented as indicated in the top graph in Fig. 1. The probability associated with each rehabilitation cycle at t_L^*, implicitly associated with the unit time period, can be easily computed (see bottom graph of Fig. 1). Summation of all probabilities associated with this point in time gives the superposed probability of (any) rehabilitation at time t_L^*. This sum is called the superposed probability of rehabilitation (SPR) at t_L^*. For instance, if $P_{R_2}(t_L^*)=0.2$ and $P_{R_m}(t_L^*)=0.1$, and all other probabilities are zero at t_L^*, then the SPR at t_L^* is 0.3. Considering all discrete intervention cycles, the SPR at a given point in time $t=t_L^*$ is

$$\sum_{\text{all } i} P[R_i(t=t_L^*)]=\sum_{\text{all } i} P[R_i(t_L^*)] \tag{7}$$

To evaluate how much the SPR change from time zero to t_L^*, the cumulative SPR can be evaluated as

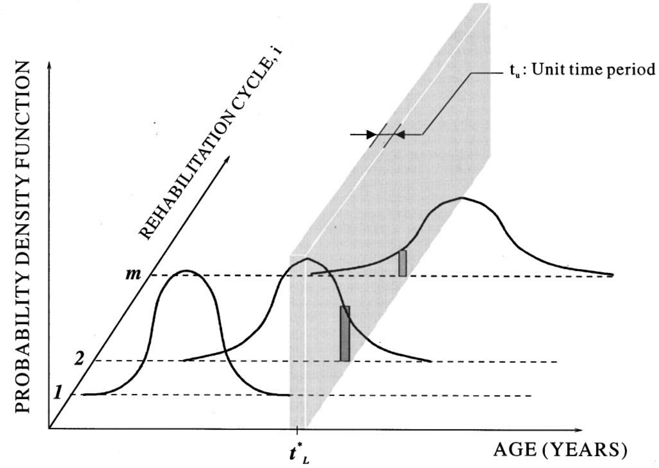
$$\sum_{\text{all } i} P[R_i(t\leq t_L^*)]=\sum_{\text{all } t\leq t_L^*}\left[\sum_{\text{all } i} P[R_i(t)]\right] \tag{8}$$

Eq. (8) represents the expected number of rehabilitations during the time interval $(0,t_L^*]$.

Cost of Maintenance

The starting year of service life of a new structure is assumed as the base year of discounting. The cost of the ith rehabilitation occurring at time t can be calculated by taking into account the discount rate υ as follows (Tilly 1997; Vassie 1997):

$$C_{r_i}(t)=\frac{C_{r_i}}{(1+\upsilon)^t} \tag{9}$$



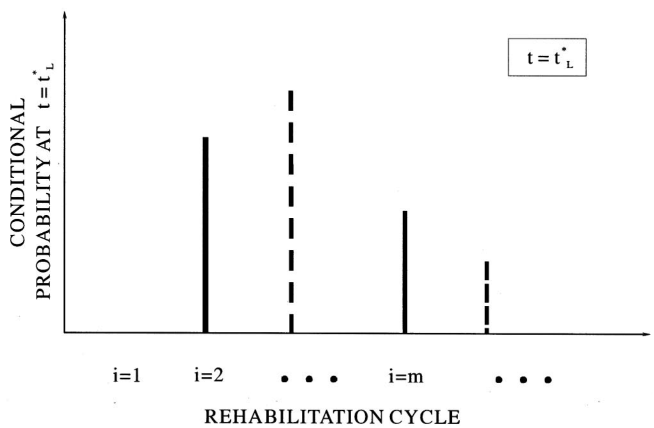


Fig. 1. Probability of any rehabilitation cycle

where $C_{r_i}=$undiscounted cost of the ith rehabilitation. Let us assume the case where n rehabilitations occur at times $t_1^*,t_2^*,\ldots,t_n^*$ based on the absolute timescale. Therefore, $t_i^*=\Sigma_{m=1}^{i} t_m$ where t_m is based on the relative time scale. Consequently, $t_n^*=t_1+t_2+\cdots+t_n=t_L^*$ and the total rehabilitation cost associated with this case is

$$C_{r,n}^T(t_L^*)=C_r^T(t_1^*,\ldots,t_n^*)=C_{r_1}(t_1^*)+C_{r_2}(t_2^*)+\cdots+C_{r_n}(t_n^*)$$

$$=\frac{C_{r_1}}{(1+\upsilon)^{t_1^*}}+\frac{C_{r_2}}{(1+\upsilon)^{t_2^*}}+\cdots+\frac{C_{r_n}}{(1+\upsilon)^{t_n^*}} \tag{10}$$

The expected rehabilitation cost at time t_L^* can be obtained using the probability of rehabilitation as follows:

$$E[C_{r,n}^T]=P_{\mathbf{T},n}[\mathbf{t}]\times C_{r,n}^T(t_L^*)$$

$$=P_{T_1,\ldots,T_n}[t_1,\ldots,t_n]\times C_{r,n}^T(t_1^*,\ldots,t_n^*) \tag{11}$$

with the constraint $t_1+t_2+\cdots+t_n=t_L^*$. Note that $P_{T_1,\ldots,T_n}[t_1,\ldots,t_n]$ is the joint PMF. The cost (11) is associated with the scenario when interventions 1,2, . . . ,n occur at times $t_1^*,t_2^*,\ldots,t_n^*$, respectively.

The total expected rehabilitation cost at time t_L^* associated with all rehabilitation cycles is

$$E[C_r(t_L^*)] = \sum_{\text{all } i} E[C_{r_i}^T] = \sum_{\text{all } i} \{P_{\mathbf{T},i}[\mathbf{t}] \times C_{r,i}^T(t_L^*)\} \quad (12)$$

and the total expected cumulative cost of rehabilitation during the time interval $(0, t_L^*]$ is

$$E[C_{r,\text{cumul}}(t \le t_L^*)] = \sum_{\text{all } t \le t_L^*} \left[\sum_{\text{all } i} \{P_{\mathbf{T},i}[\mathbf{t}] \times C_{r,i}^T(\mathbf{t}_L^*)\} \right] \quad (13)$$

As previously mentioned, the joint PMF of each rehabilitation cycle is needed in computation. In general, however, it is difficult to obtain the joint PMF associated with all rehabilitation cycles.

The correlations between various interventions may be obtained if the time-dependent behavior of a deteriorating system is known. If a number of structures have to be maintained, however, the problem becomes very complex and, in general, it is not possible to find the joint PMF of rehabilitation of groups of deteriorating structures. Moreover, because of other factors which should be considered during computation (e.g., a limited budget, abnormal loading) rehabilitation strategies are seldom designed based on time-dependent system reliability analysis. In this paper, it is assumed that there is no correlation between different intervention cycles. Using this assumption, the expected probability of rehabilitation at time t_L^* can be obtained from the product of all probability mass functions of rehabilitations at this time.

Computational Procedure

A *FORTRAN-90* computer program was developed to compute the SPR and associated expected cost. The formulations derived in the previous section are embedded in this program. A recursive subroutine is used to manage multiple intervention cycles which are usually not decided at the beginning of the computation. The number of intervention cycles is controlled by the given PDFs of application times and the service life of the structure. To calculate the joint probability, two options are available. The first uses direct numerical integration and the second is based on the modified event tree algorithm. The event tree approach adopted to evaluate all possible rehabilitation scenarios is applicable only to problems that consist of discrete probability density functions (i.e., PMFs). The method reduces the computational time significantly and this is essential for solving practical problems associated with large stocks of deteriorating structures (say, hundreds or thousands) such as highway bridges.

The remainder of this paper explains how to obtain the annual and cumulative SPRs for a deteriorating structure and a group of deteriorating structures. Examples are provided to explain the computational procedure. The procedure for calculating expected cost is also described.

Probability Distribution of Rehabilitation Time

In general, each maintenance cycle is represented by its own probability distribution (Maunsell Ltd. and Transport Research Laboratory 1998). To explain the procedures for (1) evaluating the SPR after applying several subsequent rehabilitation cycles and (2) calculating the present value of expected associated cost, an example with three rehabilitation cycles is selected. It is assumed that the distributions of the applications of the first, second, and third maintenance cycles are those indicated in Fig. 2, where $P_{1,A}$ = probability of the first rehabilitation with respect to the absolute timescale. Note that the subscripts A and R are used to indicate that the rehabilitation is based on the absolute and

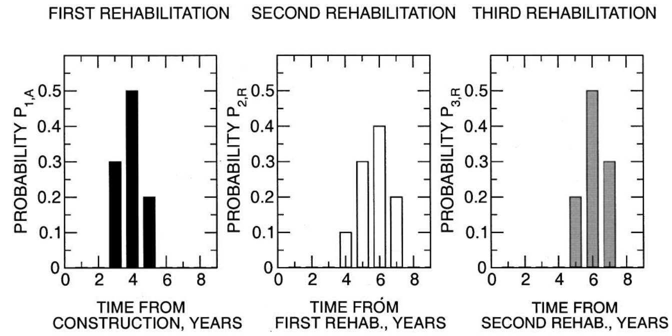


Fig. 2. Distributions of rehabilitation times of three subsequent cycles (relative timescale)

relative time, respectively. The origin of the relative time scale is the time of occurrence of the previous rehabilitation. For example, $P_{2,R}$ indicates that the probability of the second rehabilitation is defined with respect to the time of occurrence of the first rehabilitation.

Fig. 3 shows an event tree with all possible paths including the first and second rehabilitation cycles where $T_{i,A}$, $i=1,2$, is the occurrence (in absolute time scale) of the ith rehabilitation cycle. Similarly, another event tree can be obtained based on all possible paths associated with the three rehabilitation interventions in Fig. 2. In this case, Table 1 indicates the process associated with the computation of the probability of joint rehabilitation for each of the 36 possible paths. The probability that a rehabilitation will occur at a specific time t, $P_{R,t}$, can be obtained from the event tree by selecting all paths ending at time t and by adding all probabilities associated with these paths. For example, in Fig. 3 there are two possible paths ending at $t_{2,A}=8$ years (i.e., second and fifth paths from top) and their probabilities are 0.09 and 0.05, respectively. Thus the probability of rehabilitation at $t_{2,A}=8$ years is $P_{R,8} = P_{1,3}P_{2,5} + P_{1,4}P_{2,4} = 0.30 \times 0.30 + 0.50 \times 0.10 = 0.14$.

If all paths are rearranged according to the same ending time (based on the absolute scale) and probabilities of all paths are added, the probability of the second rehabilitation, including the effect of the first rehabilitation, is obtained. For the third rehabilitation cycle, the same procedure can be applied. Fig. 4 shows the associated results.

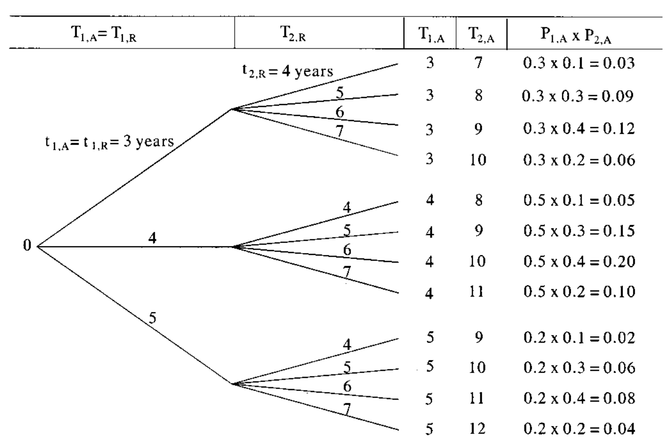


Fig. 3. Event tree after applying second rehabilitation

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Table 1. Computation of Probability of Joint (First, Second, and Third) Rehabilitation

First rehabilitation time (years, absolute time) $T_{1,A}$	Second rehabilitation time (years, relative time) $T_{2,R}$	Third rehabilitation time (years, relative time) $T_{3,R}$	Second rehabilitation time (years, absolute time) $T_{2,A}$	Third rehabilitation time (years, absolute time) $T_{3,A}$	Probability of joint rehabilitation $P_{1,A}\times P_{2,R}\times P_{3,R}$
3	4	5	7	12	$0.3\times0.1\times0.2=0.006$
3	4	6	7	13	$0.3\times0.1\times0.5=0.015$
3	4	7	7	14	$0.3\times0.1\times0.3=0.009$
3	5	5	8	13	$0.3\times0.3\times0.2=0.018$
3	5	6	8	14	$0.3\times0.3\times0.5=0.045$
3	5	7	8	15	$0.3\times0.3\times0.3=0.027$
3	6	5	9	14	$0.3\times0.4\times0.2=0.024$
3	6	6	9	15	$0.3\times0.4\times0.5=0.060$
3	6	7	9	16	$0.3\times0.4\times0.3=0.036$
3	7	5	10	15	$0.3\times0.2\times0.2=0.012$
3	7	6	10	16	$0.3\times0.2\times0.5=0.030$
3	7	7	10	17	$0.3\times0.2\times0.3=0.018$
4	4	5	8	13	$0.5\times0.1\times0.2=0.010$
4	4	6	8	14	$0.5\times0.1\times0.5=0.025$
4	4	7	8	15	$0.5\times0.1\times0.3=0.015$
4	5	5	9	14	$0.5\times0.3\times0.2=0.030$
4	5	6	9	15	$0.5\times0.3\times0.5=0.075$
4	5	7	9	16	$0.5\times0.3\times0.3=0.045$
4	6	5	10	15	$0.5\times0.4\times0.2=0.040$
4	6	6	10	16	$0.5\times0.4\times0.5=0.100$
4	6	7	10	17	$0.5\times0.4\times0.3=0.060$
4	7	5	11	16	$0.5\times0.2\times0.2=0.020$
4	7	6	11	17	$0.5\times0.2\times0.5=0.050$
4	7	7	11	18	$0.5\times0.2\times0.3=0.030$
5	4	5	9	14	$0.2\times0.1\times0.2=0.004$
5	4	6	9	15	$0.2\times0.1\times0.5=0.010$
5	4	7	9	16	$0.2\times0.1\times0.3=0.006$
5	5	5	10	15	$0.2\times0.3\times0.2=0.012$
5	5	6	10	16	$0.2\times0.3\times0.5=0.030$
5	5	7	10	17	$0.2\times0.3\times0.3=0.018$
5	6	5	11	16	$0.2\times0.4\times0.2=0.016$
5	6	6	11	17	$0.2\times0.4\times0.5=0.040$
5	6	7	11	18	$0.2\times0.4\times0.3=0.024$
5	7	5	12	17	$0.2\times0.2\times0.2=0.008$
5	7	6	12	18	$0.2\times0.2\times0.5=0.020$
5	7	7	12	19	$0.2\times0.2\times0.3=0.012$

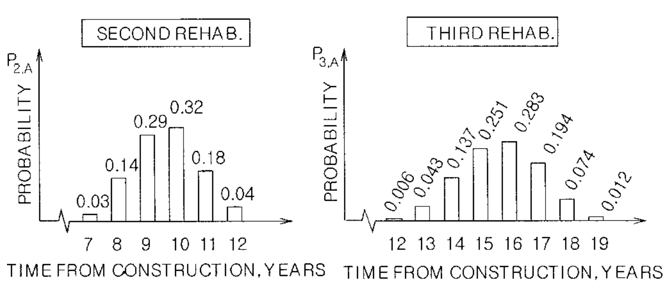

Fig. 4. Probability of second and third rehabilitation (absolute time scale)

Annual Probability of Rehabilitation and Expected Number of Rehabilitations

As previously indicated, the probabilities of the first, second, and third rehabilitations are obtained based on the absolute timescale. The superposed probability associated with the three rehabilitations at a point in time is obtained by adding the individual probabilities at this point in time. The result is expressed as the annual superposed probability of rehabilitation $\Sigma_{i=1}^{n} P_{i,A}(t)$ where $n=3$ is the number of rehabilitation cycles applied. The expected number of rehabilitations during the time interval $(0,t]$, $E[N_{R,t}]$, can be obtained from the annual superposed probability of rehabilitation as follows:

$$E[N_{R,t}]=\sum_{\text{all } t\leq t_L^*}\sum_{i=1}^{n} P_{i,A}(t) \tag{14}$$

For the example presented in Fig. 2, Fig. 5 shows the procedure

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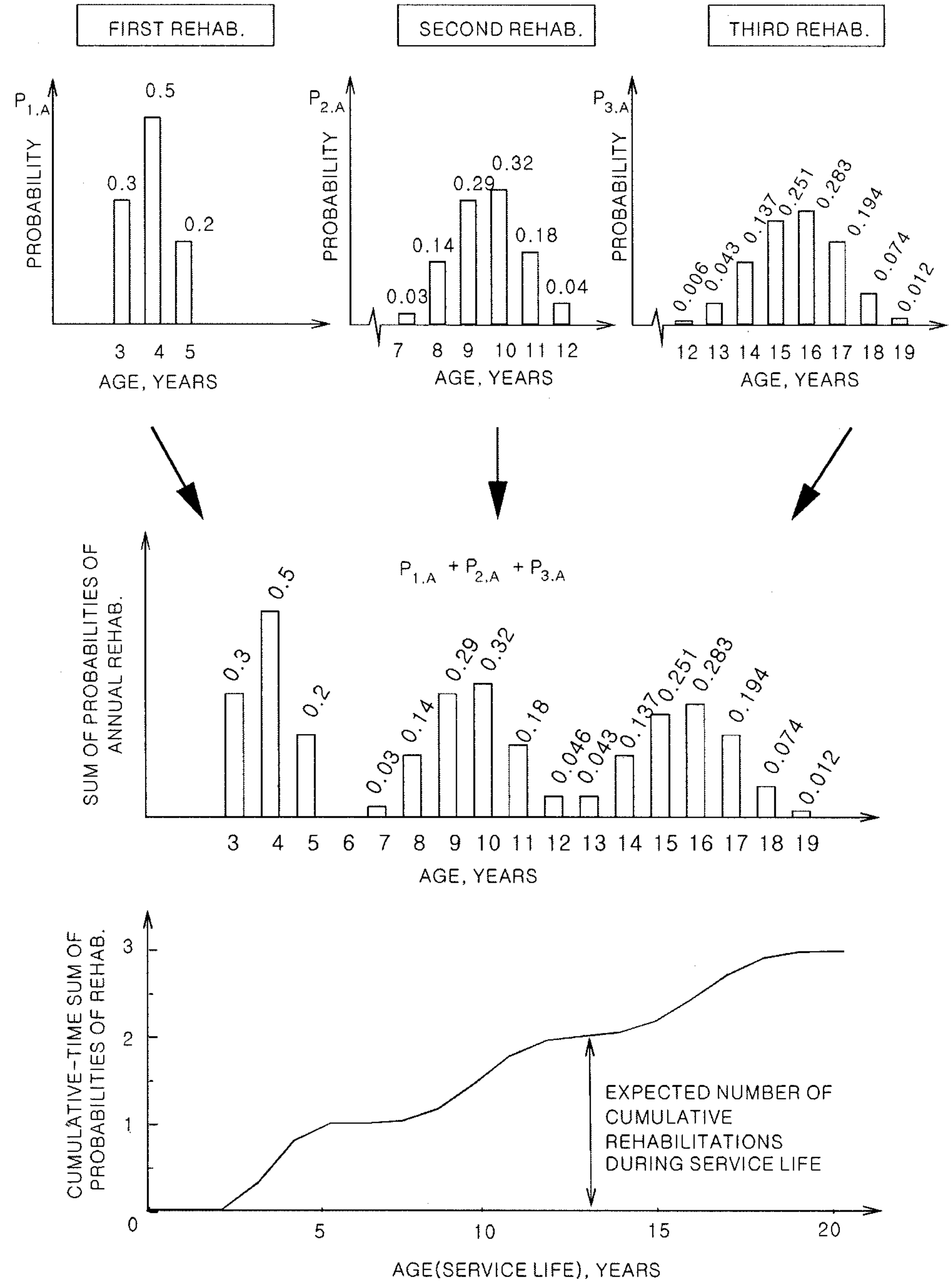

Fig. 5. Procedure for evaluating sum of probabilities of annual rehabilitation and expected number of cumulative rehabilitations

of evaluating the annual and cumulative superposed probabilities of rehabilitation associated with multiple maintenance actions.

Expected Rehabilitation Cost

To evaluate the expected rehabilitation costs, information on all paths in the event tree is needed rather than the superposed annual or cumulative probability of rehabilitation. Fig. 6 shows the probabilities of all possible paths in the event tree when three rehabilitation cycles are applied.

The general formulation for the cost of rehabilitation was explained previously. The present value of the expected cost associated with path j in the event tree can be obtained as

$$E[C_{r_i,p_j}(t)]=\frac{C_{r_i}}{(1+v)^{t_{i,j}}}p_{i,j} \tag{15}$$

where $p_j=j$th path in the event tree, $v=$discount rate, C_{r_i} $=$undiscounted cost of the ith rehabilitation, and $t_{i,j}=$time (in absolute scale) of occurrence of the jth path in the event tree after applying the ith rehabilitation. For example, if $C_{r_3}=100$ and $v=0.05$, then the present value of the expected rehabilitation cost of the first path shown in Fig. 6 is

$$E[C_{r_3,p_1}(t)]=\frac{100}{(1.05)^{12}}\times 0.006=0.3341 \tag{16}$$

This process should be iterated through all paths in the event tree. After this, as mentioned previously, the results associated with each path are rearranged, selected, and superposed according to the same age (i.e., based on the absolute time scale) to obtain the present value of the expected rehabilitation cost at a certain age associated with the third cycle

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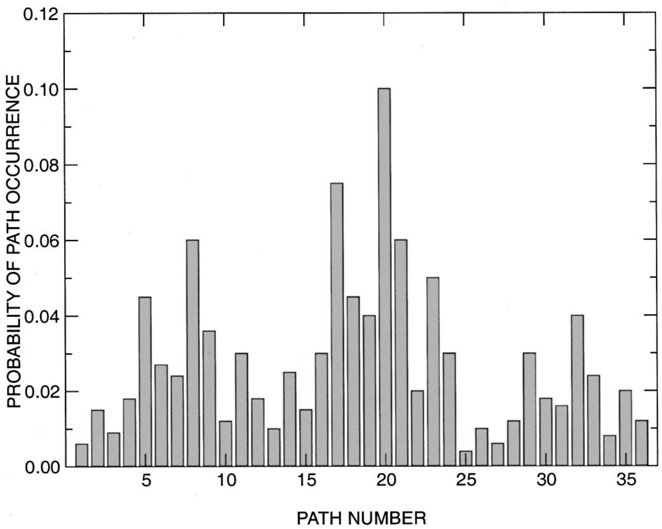

Fig. 6. Probability of paths in event tree

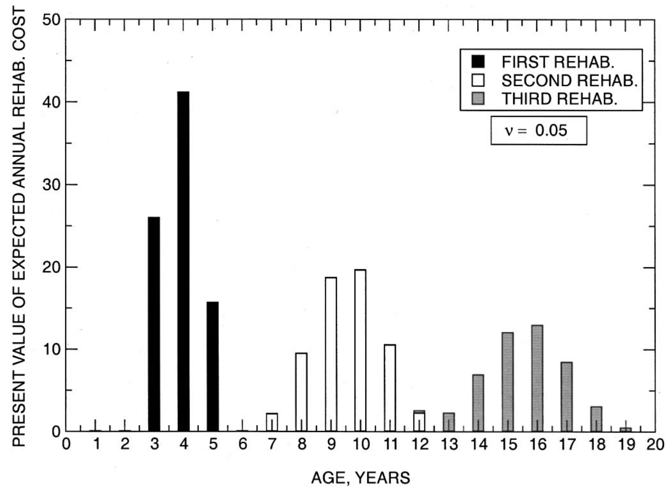

Fig. 7. Present value of expected annual rehabilitation cost

$$E[C_{r_3}(t)]=\sum_{i=1}^{m} E[C_{r_3,p_i}|_{\text{if } t_{r_3,p_i}=t}] \tag{17}$$

where m = number of paths ending at age t. Similarly, considering $C_{r_1}=C_{r_2}=100$, the present value of the expected rehabilitation cost over time associated with the first and second cycle can be obtained.

Table 2 shows annual rehabilitation cost over time associated with three essential maintenance (i.e., rehabilitation) cycles. In this example, the different cycles do not overlap except at age 12. However, if the initial probabilities of rehabilitations are distributed over a wider time range, then the number of overlapped annual rehabilitation costs will increase. It is interesting to note that the same unit cost is used for each rehabilitation cycle, but the sum of expected annual rehabilitation costs of each cycle is less than 100 because of the discount rate. The effect of discount rate increases with time.

Fig. 7 shows the present value of the expected annual rehabilitation cost (superposing the cost results associated with the first, second, and third rehabilitations) according to the procedures explained previously, and Fig. 8 shows the present value of expected cumulative rehabilitation cost associated with three rehabilitations.

Table 2. Present Value of Expected Annual Rehabilitation Cost for Each Rehabilitation Cycle

Age (year)	First rehabilitation cost	Second rehabilitation cost	Third rehabilitation cost
1	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000
3	25.9151	0.0000	0.0000
4	41.1351	0.0000	0.0000
5	15.6705	0.0000	0.0000
6	0.0000	0.0000	0.0000
7	0.0000	2.1320	0.0000
8	0.0000	9.4758	0.0000
9	0.0000	18.6937	0.0000
10	0.0000	19.6452	0.0000
11	0.0000	10.5242	0.0000
12	0.0000	2.2273	0.3341
13	0.0000	0.0000	2.2804
14	0.0000	0.0000	6.9194
15	0.0000	0.0000	12.0735
16	0.0000	0.0000	12.9646
17	0.0000	0.0000	8.4642
18	0.0000	0.0000	3.0749
19	0.0000	0.0000	0.4749
	$\Sigma=82.72$	$\Sigma=62.70$	$\Sigma=46.59$

Group of Deteriorating Structures

The proposed method is equally applicable to a group of deteriorating structures (say, highway bridges) built at different time periods. The computational procedure for evaluating the probability of maintenance application for a group of deteriorating structures is identical to that for an individual structure. However, the cost evaluation should include the different money-discounting

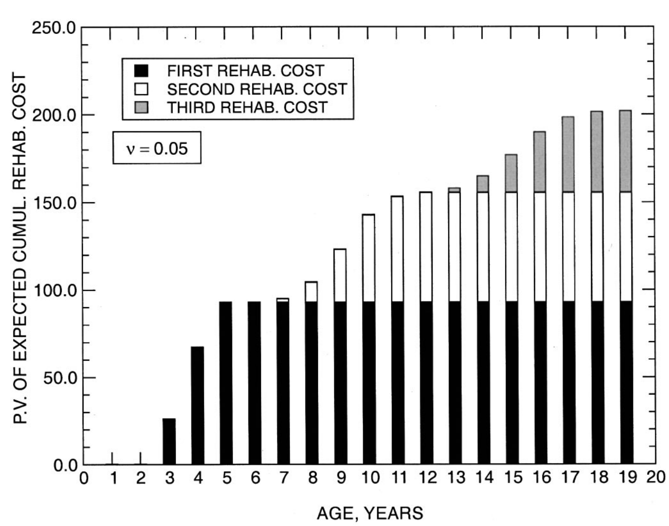

Fig. 8. Present value of expected cumulative rehabilitation cost

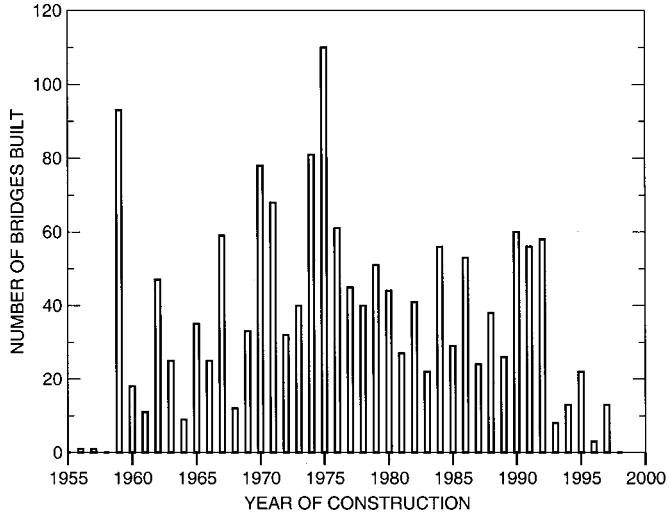

Fig. 9. Stock of 1,570 reinforced concrete bridges by year of construction

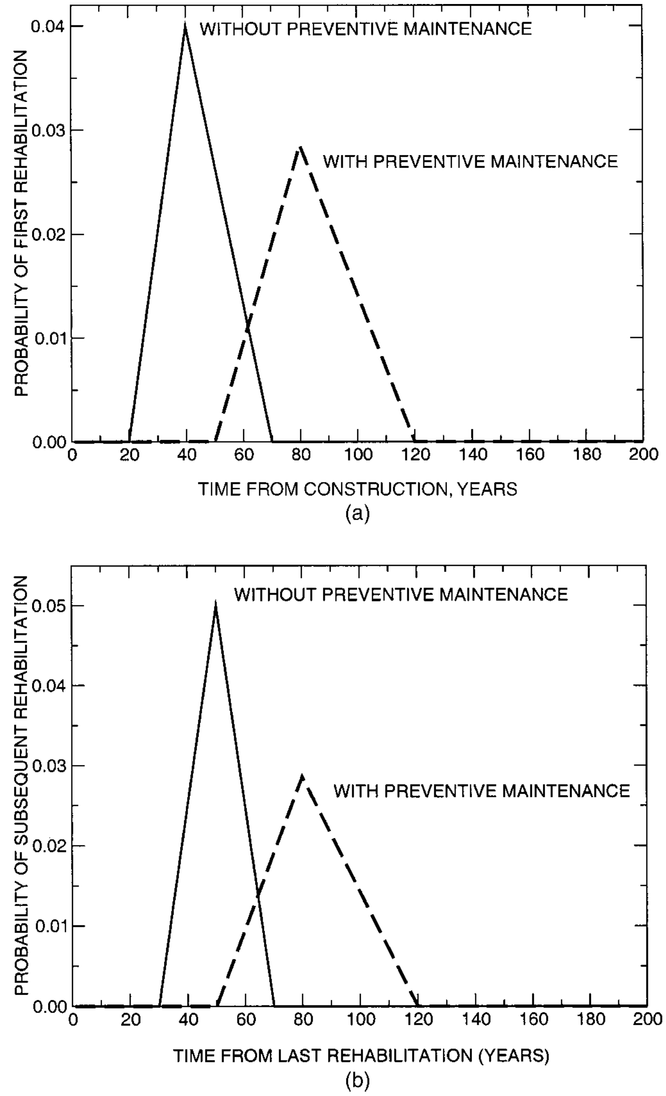

Fig. 10. Reinforced concrete bridges; PDFs of rehabilitations with and without preventive maintenance; (a) first rehabilitation and (b) subsequent rehabilitation (Maunsell Ltd. and Transport Research Laboratory 1998)

effect according to the age of each individual structure in the group. In addition, the number of structures belonging to the same age group should be considered. To demonstrate the applicability of the method to a group of structures, a large stock of 1,570 reinforced concrete bridges built in the United Kingdom before 1999 is considered. The time of construction of all individual bridges in this bridge stock is indicated in Fig. 9 (Frangopol et al. 1999). The five bridge age groups considered are as follows: (1) 207 bridges built before 1964; (2) 463 bridges built from 1965 to 1974; (3) 497 bridges built from 1975 to 1984; (4) 365 bridges built from 1985 to 1994; and (5) 38 bridges built from 1995 to 1998. The unit maintenance cost is indicated in Table 3 considering different types of interventions (Maunsell Ltd. and Transport Research Laboratory 1999). PDFs for the time of intervention with and without preventive maintenance were developed by Maunsell Ltd. and Transport Research Laboratory (1998). These distributions are indicated in Figs. 10(a and b). Based on these distributions, Eq. (7) is used to compute the annual probability of rehabilitation shown in Fig. 11. Using the unit cost values indicated in Table 3 and Eqs. (10) and (14), leads to the present values of the expected cumulative unit cost indicated in Fig. 12. In this figure, logistic and Weibull PDFs (defined in Maunsell Ltd. and Transport Research Laboratory 1999) for the times of intervention with and without preventive maintenance, respectively, are also considered in order to compare the sensitivity of the expected cumulative unit cost to the type of PDF. Note that the results in Figs. 11 and 12 are generated based on an individual bridge. Finally, by iteratively applying Eq. (13) to all different bridge age groups, the expected maintenance cost for the whole stock can be evaluated. Figs. 13(a and b) show for each of the five age groups and for all bridge stock considered the present value (i.e., year 2000) of the expected cumulative rehabilitation cost with and without preventive maintenance, respectively. The benefit of preventive maintenance is clearly demonstrated by comparing the expected costs in Figs. 13(a and b). Furthermore, this benefit is considerably increased if user cost is taken into account.

Conclusions

An approach for computing the probability of maintenance application over a given time horizon and the expected life-cycle maintenance cost of deteriorating structures was presented. The

Table 3. Estimated Unit Maintenance Cost for Reinforced Concrete Bridges (Maunsell Ltd. and Transport Research Laboratory 1999)

Cost type	Without preventive maintenance (£/m^2 deck area)	With preventive maintenance (£/m^2 deck area)
Preventive maintenance cost	0	69
Essential maintenance cost	837	358
User cost for a preventive maintenance	0	157
User cost for an essential maintenance	6,408	660

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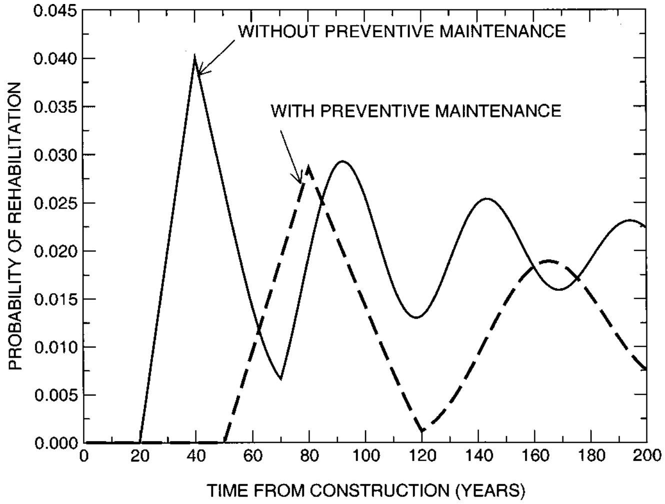

Fig. 11. Reinforced concrete bridges; probability of rehabilitation with and without preventive maintenance

approach is based on a modified event tree analysis. The proposed method is computationally more efficient and simpler than the conventional methods such as numerical integration or Monte Carlo simulation.

The uncertainties in the maintenance actions and the discount rate were taken into account. The overall maintenance costs of different maintenance scenarios were compared for a single bridge. As a result, it was illustrated that the scenario associated with preventive maintenance is more economical than the one associated with essential maintenance beyond a certain time horizon. The proposed method is equally applicable to groups of deteriorating structures. Application to an existing stock of deteriorating reinforced concrete bridges was presented to illustrate the methodology. Preventive maintenance interventions were shown to reduce significantly the expected total cost.

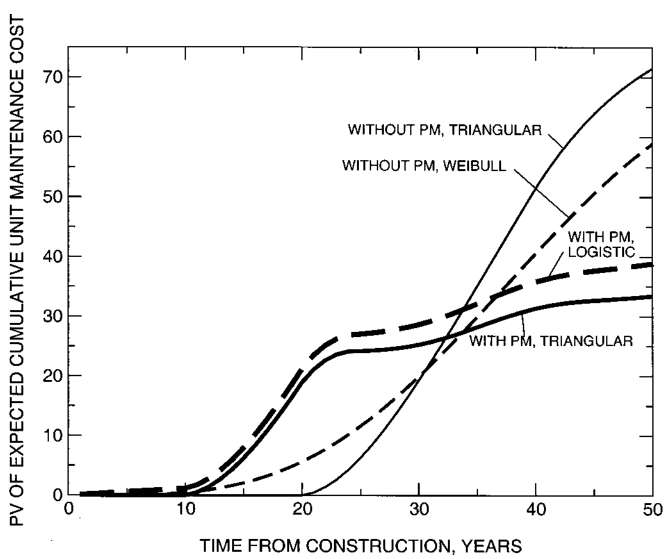

Fig. 12. Reinforced concrete bridges with and without preventive maintenance; present value of expected cumulative unit maintenance cost associated with different probability distributions; discount rate = 6%

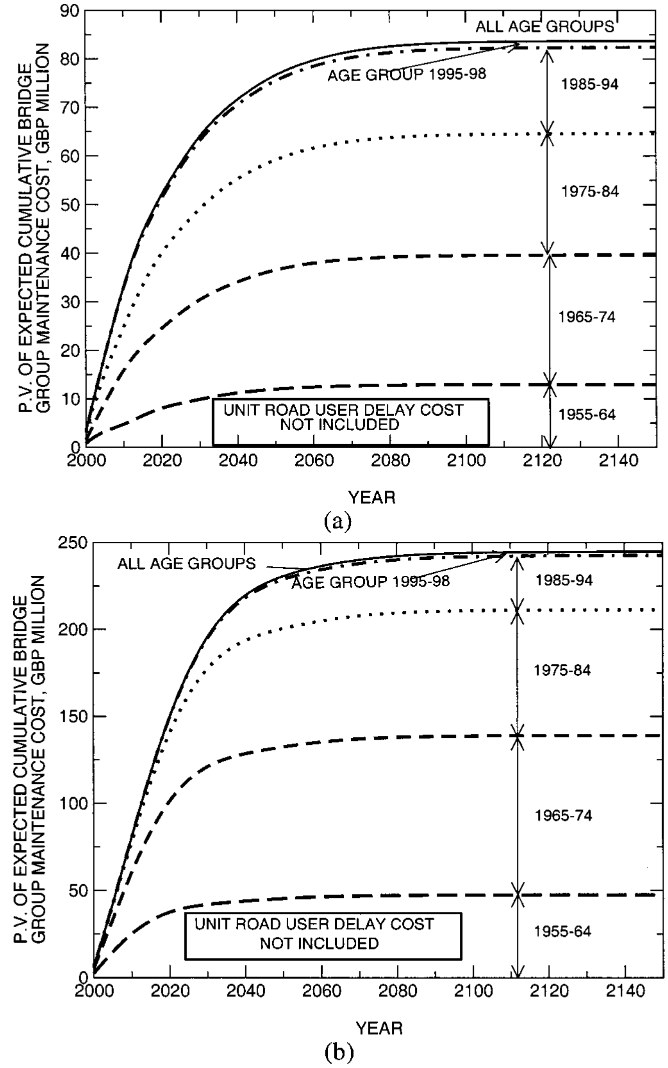

Fig. 13. Reinforced concrete bridges; present values of cumulative bridge group maintenance costs, discount rate = 6%; (a) with preventive maintenance and (b) without preventive maintenance

The study presented here provides engineers in charge of management of deteriorating structures with evidence of the benefits of using the expected maintenance cost approach. A database has to be created that can serve for the calibration, verification, and implementation of the proposed methodology in management of deteriorating civil infrastructures. The resulting developments are likely to produce enhanced cost-based maintenance provisions for deteriorating civil infrastructures.

The proposed method focuses on evaluating the probability of maintenance over a certain time span and its associated expected maintenance cost. However, lifetime performance of a structure or a group of structures as affected by deterioration and maintenance interventions has to be considered separately. In practice, decision-makers often face the problem of choosing between lower cost and higher performance options. For this purpose, a structural lifetime performance assessment method based on reliability is necessary (Kong et al. 2000; Frangopol et al. 2001). The cost evaluation method proposed in this study together with a sound reliability-based lifetime performance assessment method can be used effectively as a general management system to select the optimum maintenance solution in terms of both expected cost and life-cycle performance.

In conclusion, it is emphasized that although mathematical tools to evaluate the expected life-cycle maintenance cost of deteriorating structures are adequate at this time, future effort is needed to consistently deal with insufficient data and uncertainty associated with life-cycle analysis of civil infrastructures. Decisions based on the expected value of life-cycle maintenance cost and possibly its variance are practical in optimizing our resources by selecting optimal options of preventive maintenance, essential maintenance, or both, and even recommending more intensive data gathering information.

Acknowledgments

The partial financial support of the U.S. National Science Foundation through Grant Nos. CMS-9506435 and CMS-9912525, of NATO through Grant No. CRG. 960085, and of the U.K. Highways Agency is gratefully acknowledged. The writers would like to thank Mr. Masaru Miyake for performing the computations associated with Figs. 11–13. The opinions and calculations presented in this paper, which is a revised and extended version of Frangopol and Kong (2001), are those of the writers and do not necessarily reflect the views of the sponsoring organizations.

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