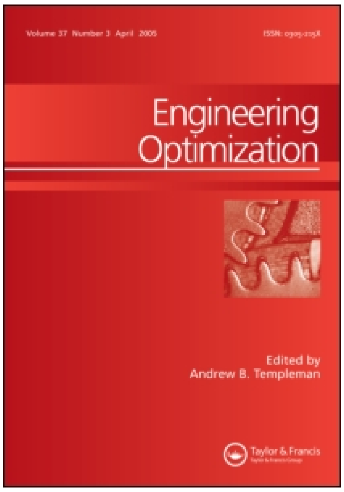

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Optimization procedures for simultaneous road rehabilitation and bridge replacement decisions in highway networks

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OPTIMIZATION PROCEDURES FOR SIMULTANEOUS ROAD REHABILITATION AND BRIDGE REPLACEMENT DECISIONS IN HIGHWAY NETWORKS

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A road-bridge rehabilitation model is formulated as a mixed non-linear programming problem with linear constraints. The purpose of the model is to minimize user travel costs under limited availability of funds. Two constraints are related to budget availability for road rehabilitation and bridge replacement (or rehabilitation). For a given set of replaced bridges, the problem is reduced to a continuous non-linear programming problem that can be further decomposed into a traffic assignment problem (TAP) and a road rehabilitation budget allocation problem (RBAP). The solution to the non-linear problem is found by iteratively solving the TAP and the RBAP. Since the TAP has a non-convex objective function, its solution is only guaranteed to be a local optimum. Several local optima are obtained at each branch of the search tree to estimate a lower confidence limit on the user cost of a global solution.

Keywords: Pavement management; Bridge management; Highway management; Network optimization

1 INTRODUCTION

During the past fifteen years, state highway departments in the U.S. and Canada have substantially increased their efforts aimed at developing pavement management systems (PMS) to ensure that their limited resources are used in the most cost-effective manner possible. In recent past years an interest in bridge management systems (BMS) has been expressed by state highway agencies, as a result of the significant number of structures in need of rehabilitation, reconstruction, or replacement. At present, preliminary research is being focused on the design of highway management systems (HMS) that would integrate a PMS and a BMS into a single system.

The central premise of this article is that a true cost-effective methodology for the best use of limited budgets in transportation planning must integrate and simultaneously investigate both roads and bridges. This paper presents an original operations research approach to develop an optimization methodology that can be used as a building block towards a highway

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management system (HMS), since it allows the combined analysis of both bridges and road sections in a transportation network. Specifically, the proposed modeling approach can be used to determine how much should be invested in the rehabilitation of each road section, and which bridges should be replaced or rehabilitated, in order to minimize total user travel costs without violating budgetary constraints.

Previous analytical approaches for identifying undersized bridges to be replaced in a transportation network are due, among others, to Garcia-Diaz and Liebman [4, 5], Seo and Garcia-Diaz [20], Jiang and Sinha [9], and Carmichael [1]. Models for road rehabilitation are more common. Usually, these models indicate the level of funds needed on each road section in a transportation system. Examples of these models are those due to Christofides and Brooker [3], Phillips and Lytton [17], Prastacos and Romanos [18], and Ridley [19]. Other work in pavement management includes the work of Carnahan *et al.* [2], Grivas *et al.* [7], Li *et al.* [11], and the work of Mbwana and Turnquist [13]. All these contributions addressed either the bridge replacement problem or the road rehabilitation problem, but not both.

In addition to this introductory section (Sec. 1), this article contains four more sections. Section 2 presents the formulation of the mathematical models and a solution methodology. Section 3 briefly describes an available procedure for finding confidence intervals for the global optimal solution of the proposed model. Section 4 shows a numerical example to illustrate the use of the optimization methodology. Finally, Section 5 summarizes the fundamental aspects of the article.

2 MATHEMATICAL MODELS AND SOLUTION METHODOLOGY

The purpose of this section is to formulate a mathematical model and develop a methodology for the selection of cost-effective bridge replacement and road rehabilitation policies in a transportation network. The road-bridge rehabilitation investment problem (RBRIP) to be discussed in this article will be formulated using the following notation:

Network Structure

$\mathbf{A}$ Set of road links in the road network
$\mathbf{N}$ Set of nodes in the network
$\mathbf{A}_b$ Subset of $\mathbf{A}$ with road links having undersized bridges
$\mathbf{V}_i$ Set of road links terminating at node i
$\mathbf{W}_i$ Set of road links originating at node i

Input Data

h_i Net flow (trips originating or demanded) at node i
r_k Replacement or rehabilitation cost for bridge on road link k in $\mathbf{A}_b$
d_j Length of road link j
U_j Upper bound for the maximum investment on road link j
B_1 Available budget for road rehabilitation
B_2 Available budget for bridge replacement

Decision Variables

f_j Traffic flow (number of trips) on link j
x_j Amount of money to be invested in link j
Y_j Binary variable which indicates whether the undersized bridge on road link j is replaced (or rehabilitated) or not

This section consists of three subsections. Subsection 2.1 has the formulation of the RBRIP model. Subsection 2.2 presents a solution methodology for a given set of bridges to be replaced. Subsection 2.3 documents a procedure for identifying the optimal set of bridges to be replaced.

2.1 Model Formulation

It will be assumed that the user travel cost is a linear function of the funds invested in the rehabilitation of a road link. In symbols, the travel cost per mile is given by a decreasing linear function $a - bI_j$, where a and b are known constants and I_j is the amount invested in road link j. Assuming that the current value of the per-mile travel cost is equal to $a - bm_j$, the new cost after an additional investment x_j would be $a - b(m_j + x_j)$, since in this case $I_j = m_j + x_j$. Although two specific values a_j and b_j can be defined for each road section j, the values a and b will be used for all road sections to simplify the notation used in the formulation of the models.

The road bridge rehabilitation investment problem (RBRIP) can be formulated as a nonlinear programming model, as indicated below:

RBRIP

$$\text{Minimize} \quad \sum_j d_j[a - b(m_j + x_j)]f_j \tag{1}$$

$$\text{Subject to} \quad \sum_{j \in \mathbf{W}_i} f_j - \sum_{j \in \mathbf{V}_i} f_j = h_i, \quad i \in \mathbf{N} \tag{2}$$

$$f_k \le MY_k, \quad k \in \mathbf{A}_b \tag{3}$$

$$\sum_j d_j x_j \le B_1, \quad j \in \mathbf{A} \tag{4}$$

$$\sum_k r_k Y_k \le B_2, \quad k \in \mathbf{A}_b \tag{5}$$

$$x_j \le U_j, \quad j \in \mathbf{A} \tag{6}$$

$$Y_k = 1, 0, \quad \text{for } k \in \mathbf{A}_b \tag{7}$$

$$x_j, f_j \ge 0, \quad j \in \mathbf{A} \tag{8}$$

The objective function (1) indicates the total user cost for a specified planning period (typically, one year). Constraint (2) is the flow conservation or demand/supply restriction, which establishes that the number of trips out of node i minus the number of trips into it is equal to a specified number h_i (zero, positive, or negative). Constraint (3) ensures that no flow is allowed through a link having an undersized bridge. In this constraint, M represents an arbitrarily large number (an upper bound on the number of trips on any link). In addition, constraints (4) and (5) are the budget restrictions for the planning period being considered. Constraint (6) establishes an upper bound on the investment on road segment j. Constraint (7) indicates whether an undersized bridge k is replaced ($Y_k = 1$) or not ($Y_k = 0$). Finally, constraint (8) represents the non-negativity conditions for flow and road investment variables.

2.2 Optimal Road Investment Policy Under a Specified Set of Replaced Bridges

The solution technique used to generate a *local optimum* is described below. Constraints (5) and (7) of the RBRIP can be eliminated (relaxed) by assuming a set of bridges to be replaced. This bridge replacement decision, denoted by the ordered set $\{Y_k | k \in \mathbf{A}_b\}$, will be referred to as a *bridge configuration*. Under a specified bridge configuration, the RBRIP reduces itself to a road rehabilitation investment problem (RRIP) formulated as follows:

RRIP

$$\text{Minimize} \quad \sum_j d_j[a - b(m_j + x_j)]f_j \tag{9}$$

$$\text{Subject to} \quad \sum_{j \in \mathbf{W}_i} f_j - \sum_{j \in \mathbf{V}_i} f_j = h_i, \quad i \in \mathbf{N} \tag{10}$$

$$f_k \le MY_k, \quad k \in \mathbf{A}_b \tag{11}$$

$$\sum_j d_j x_j \le B_1, \quad j \in \mathbf{A} \tag{12}$$

$$x_j \le U_j, \quad j \in \mathbf{A} \tag{13}$$

$$x_j \ge 0, \quad f_j \ge 0, \quad j \in \mathbf{A} \tag{14}$$

Given a set of bridges to be replaced or rehabilitated, the solution to the RRIP is a road rehabilitation policy that would result in minimal user travel cost for the specified planning horizon. It is noted that the RRIP minimizes a non-convex objective function subject to linear constraints. Furthermore, this problem can be reduced to that of determining the least-cost paths (routes) for all users if specified values are assumed for the road investment variables x_j. This reduction will be referred to as the traffic assignment problem (TAP). The TAP is formulated below, where $c_j = a - b(m_j + x_j)$ is the cost per mile on link j for a fixed investment level x_j. The TAP is a linear programming problem that can be readily solved by the Simplex method.

TAP

$$\text{Minimize} \quad \sum_j d_j c_j f_j \tag{15}$$

$$\text{Subject to} \quad \sum_{j \in \mathbf{W}_i} f_j - \sum_{j \in \mathbf{V}_i} f_j = h_i, \quad i \in \mathbf{N} \tag{16}$$

$$f_k \le MY_k, \quad k \in \mathbf{A}_b \tag{17}$$

Alternatively, the RRIP reduces to an allocation problem (budget allocation among road sections) when the flow variables f_j are set equal to specified values. When this is done, the reduced problem will be referred to as the rehabilitation budget allocation problem (RBAP). The RBAP formulation is shown below:

RBAP

$$\text{Minimize} \quad \sum_j d_j[a - b(m_j + x_j)]f_j \tag{18}$$

$$\text{Subject to} \quad \sum_j d_j x_j \le B_1, \quad j \in \mathbf{A} \tag{19}$$

$$x_j \le U_j, \quad j \in \mathbf{A} \tag{20}$$

$$x_j \ge 0, \quad j \in \mathbf{A} \tag{21}$$

where f_j is the fixed traffic flow on road segment j. The RBAP is a continuous knapsack problem that can be solved by setting $x_j = U_j$ for links in decreasing order of bf_j values until the road rehabilitation budget B_1 is exhausted. Since b is constant, the optimal solution to the RBAP is achieved by allocating the highest allowable levels of investment to links with highest traffic flow first.

The optimal solution to the RRIP can be obtained by iteratively solving the TAP and the RBAP as shown below:

Step 1 Set $i = 0$ and z_i equal to a large positive number. Obtain a feasible road rehabilitation policy proceeding as follows:

(1) Randomly choose a road link j not yet chosen for rehabilitation and having no undersized bridges.
(2) Set $x_j = U_j$ to rehabilitate road link j up to the highest road condition allowed and compute the user cost per mile as $c_j = a - b(m_j + U_j)$. Compute the budget requirement $d_j x_j$. If the available budget is less than the requirement, adjust the investment level, x_j, and the associated cost c_j by allocating the entire available budget on link j.
(3) Reduce the available budget by the amount allocated to road link j. If the budget is exhausted or all road links have been chosen, continue to Step 2. Otherwise go to (1).

Step 2 Increase i by 1 and solve the TAP model to find the flow f_j on each road link j of the road-rehabilitated network. Let z_i be the user cost for this rehabilitation policy.

(1) With the c_j values corresponding to the current road rehabilitation policy indicated by the x_j values, perform the Simplex algorithm (two-phase method).
(2) Let f_j be the optimal traffic assignment on road j and z_{i+1} be the optimal value of the total travel cost corresponding to the current f_j and x_j values.

Step 3 Solve the RBAP model to find the new values of x_j.

(1) Restore the original c_j values, the user cost per mile of each road link j before rehabilitation, set all $x_j = 0$.
(2) Arrange the current non-zero f_j values in decreasing order. For each f_j set $x_j = U_j$ and compute the user cost per mile $c_j = a - b(m_j + U_j)$ as well as the budget requirement $d_j x_j$ until the remaining available budget is less than what is required to upgrade a road link to its maximal condition. At this point, adjust the investment level, x_j, and the associated cost c_j by allocating the entire available budget on link j. Each remaining road link j continues to have $x_j = 0$.
(3) Determine the new link flows on the road-rehabilitated network. Let z_{i+1} be the user cost for this road rehabilitation policy.

Step 4 If $z_i - z_{i+1} < \varepsilon$, where ε is an arbitrarily small positive number, stop. If not, go to Step 2.

The following two propositions (given without a mathematical proof) state that a better solution is obtained at each iteration between the TAP and the RBAP. Since the RRIP has a bounded solution, the procedure detailed above terminates in a finite number of iterations.

PROPOSITION 1 *Let z_i^* be the optimal cost from the TAP. Let z_{i+1}^* be the optimal solution to the RBAP given the current flows f_j obtained from the most recent solution to TAP. At any iteration i, $z_{i+1}^* \leq z_i^*$.*

PROPOSITION 2 *Conversely, let z_i^* be the optimal cost from the solution to the RBAP. Let z_{i+1}^* be the optimal solution to the RBAP given the current c_j values obtained from the most recent solution to RBAP. At any iteration i, $z_{i+1}^* \leq z_i^*$.*

2.3 Optimal Bridge Replacement Search

The solution procedure developed for the RRIP assumes a given bridge configuration $\mathbf{Q} = \{Y_k | k \in \mathbf{A}_b\}$. A solution to the RBRIP would be the same as that obtained for the RRIP under the most cost-effective bridge configuration. This configuration can be determined through a tree search by means of a procedure based on Glover's implicit enumeration [6].

At any stage of the search, the set of undersized bridges can be partitioned into a subset of bridges for which a decision has been made and a complementary subset of those for which a decision has not been made. Recall that $\mathbf{A}_b$ is the subset of road links having undersized bridges. Once a decision is made concerning the rehabilitation or replacement of the bridges in $\mathbf{A}_b$ it is possible to identify the following sets: $\mathbf{J}^+ = \{k | k \in \mathbf{A}_b, Y_k = 1\}$, $\mathbf{J}^- = \{-k | k \in \mathbf{A}_b, Y_k = 0\}$, and $\mathbf{J}^c = \mathbf{A}_b - \mathbf{J}^+ - \mathbf{J}^-$. The set $\mathbf{J} = \mathbf{J}^+ + \mathbf{J}^-$ is here referred to as a *partial solution*. Additionally, the decision variables in the set $\{Y_k | k \in \mathbf{J}^c\}$ are referred to as *free variables*.

As an illustration, if there are eight undersized bridges, $\mathbf{A}_b = \{1, 2, \ldots, 8\}$. If a decision is made to either replace or rehabilitate bridges 4 and 7 and not to do anything to bridge 1, then $\mathbf{J}^+ = \{4, 7\}$ and $\mathbf{J}^- = \{-1\}$. The corresponding partial solution is given by $\mathbf{J} = \{4, 7, -1\}$. Also, $\mathbf{J}^c = \{2, 3, 5, 6, 8\}$ and the set of free variables is $\{Y_2 Y_3 Y_5 Y_6 Y_8\}$. The search monitors the lowest feasible user cost throughout the process. Note that assigning a binary value to Y_k is equivalent to removing bridge k from $\mathbf{J}^c$ and storing it in either $\mathbf{J}^+$ or $\mathbf{J}^-$; likewise, freeing a rehabilitation variable Y_k is equivalent to moving bridge k from either $\mathbf{J}^+$ or $\mathbf{J}^-$ to $\mathbf{J}^c$.

A *completion* of a partial solution is an assignment of binary values to the free variables. A lower bound on user cost for any partial solution can be obtained by solving the RRIP with a completion where all the free variables are set equal to 1, as justified by the following proposition:

PROPOSITION 3 *Let $\mathbf{S}_1$ and $\mathbf{S}_2$ be two sets of bridges to be rehabilitated. Let z_1 be the optimal user cost corresponding to $\mathbf{S}_1$ and z_2 the optimal user cost corresponding to $\mathbf{S}_2$. If $\mathbf{S}_2 \subset \mathbf{S}_1$, then $z_1 \leq z_2$.*

If the lower bound thus obtained is greater than the current *best* feasible user cost, $z_{\min}$ (*i.e.* the current lowest user cost that satisfies the bridge rehabilitation budget constraint), continuing the tree search by branching from that point is unnecessary. In this case, the partial solution is *fathomed*. Let $\mathbf{S} = \{Y_k | Y_k = 1 \text{ if } f_k > 0, Y_k = 0 \text{ if } f_k = 0, k \in \mathbf{A}_b\}$ be a lowest-cost bridge configuration associated with this solution. If the lower bound on user cost is less than or equal to $z_{\min}$ and $\mathbf{S}$ meets the bridge budget constraint (5), then the corresponding solution becomes the current best feasible solution and the partial solution is always fathomed.

When a partial solution cannot be fathomed, the RRIP is solved for a bridge configuration that just satisfies the bridge replacement budget. An ostensibly efficient way of specifying such completion is to set $Y_k = 1$ for bridges in increasing order of Δ_k, $k \in \mathbf{J}^c$, where

$$\Delta_k = \begin{cases} \dfrac{r_k}{f_k} & f_k > 0 \\ r_k & f_k = 0 \end{cases} \tag{22}$$

until the bridge replacement budget is exhausted, and $Y_k = 0$ for the remaining bridges $k \in \mathbf{J}^c$. If the corresponding optimal user cost, z, is less than $z_{\min}$, z becomes the new $z_{\min}$ and the new solution becomes the current best feasible solution. This step is referred to as a *forward move*. A forward move produces a new partial solution, one without free variables.

The next step in the search is a *backward move*, which consists of removing from $\mathbf{J}^+$ the last assigned bridge, placing it in $\mathbf{J}^-$, and freeing the Y_k variables corresponding to assignments made afterward. A backward move provides a partial solution for the next iteration of

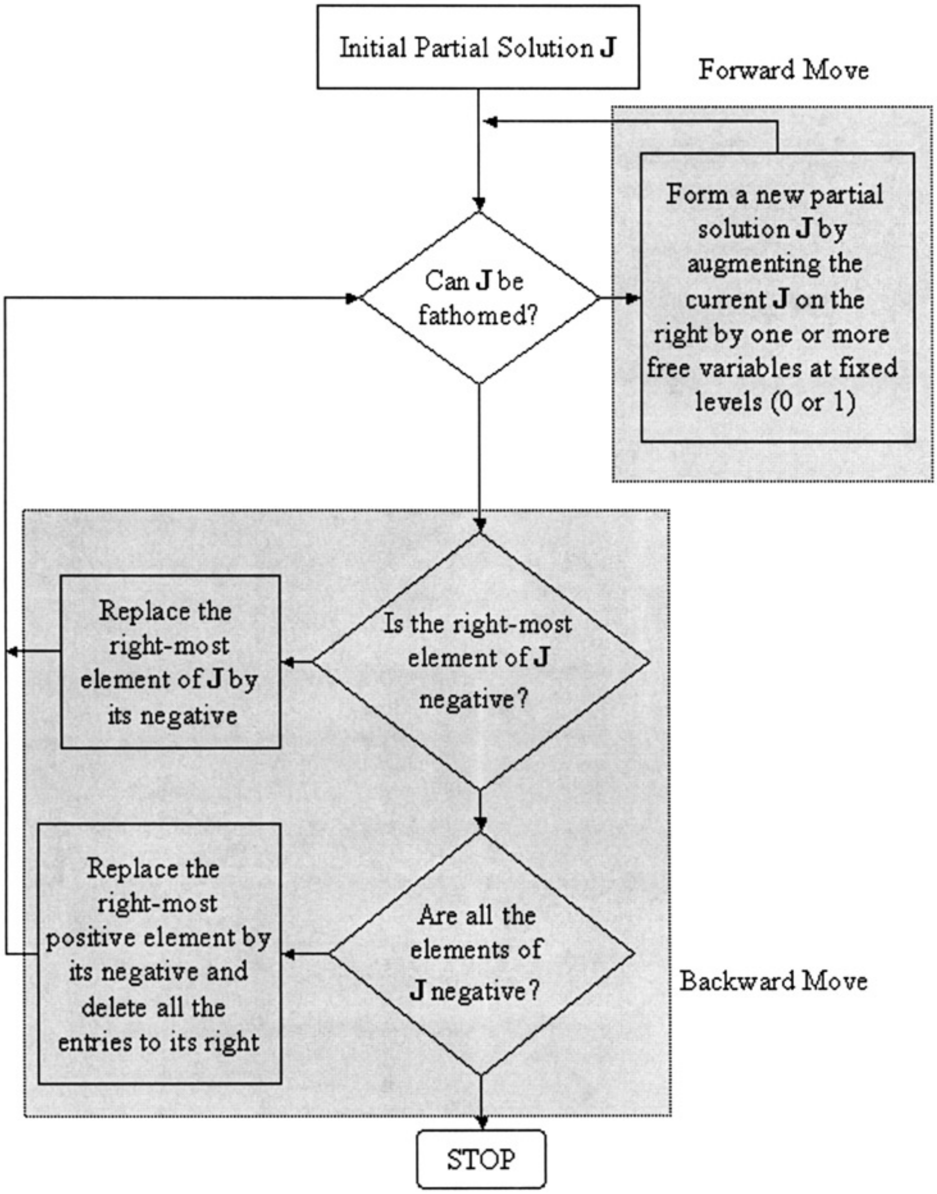

FIGURE 1 Modified Glover's enumeration scheme.

the search. Note that a backward move attempt when $\mathbf{J}^+$ is empty cannot succeed and therefore ends the search. A simplified overview of the modified search procedure is shown in Figure 1.

The RBRIP is solved by iteratively applying the tree search technique previously outlined. The search is started with an arbitrarily high initial value for $z_{\min}$ and an initial partial solution where all Y_k are free. A summary of the steps to be taken is given below:

Step 1 Set the initial lower bound on the minimum user cost m equal to an arbitrary large number. Set $Y_k = 0$ for all k.

Step 2 Solve the RBRIP. Let $\mathbf{S}$ be the set of bridges over which there is positive traffic flow. Find $\Delta_k = r_k/f_k$ for each of these bridges and rank them according the their values. For all other bridges, set $\Delta_k = r$.

Step 3 If $\mathbf{S}$ is feasible go to Step 6. Otherwise, construct a feasible bridge configuration, using first the bridges with smaller Δ_k values. Let z be the minimum user cost for $\mathbf{S}$.

Step 4 If $z > m$ backtrack and go to Step 2. Otherwise, set $m = z$, record the total user cost, traffic assignment, road rehabilitation policy, replaced bridges, and continue to Step 5.

Step 5 If the enumeration is completed, stop. Otherwise, backtrack and go to Step 4.

Step 6 If $\mathbf{S}$ is the first bridge configuration, stop. Otherwise, go to Step 4.

3 CONFIDENCE INTERVAL FOR THE GLOBAL OPTIMUM

The solution to the relaxed RBRIP is guaranteed to be a *local* optimum, instead of a *global* optimum, since the objective function of this problem is not convex. Conceptually, the optimization problem under consideration can be viewed as that of minimizing a non-convex function $g(\boldsymbol{x})$ with respect to a vector $\boldsymbol{x}$ restricted to a feasible region $\mathbf{R}$:

$$G^* = \min_{x \in \mathbf{R}} g(\boldsymbol{x}) \tag{23}$$

Since it is unrealistic to obtain all local minimum solutions in a region $\mathbf{R}$, an acceptable alternative procedure is to identify a confidence interval for the global optimum on the basis of a random sample of n local minima. Let these n solutions be denoted by $G_1, \ldots, G_n$. Furthermore, let the ordered values of this independent sample of G values be denoted by $G(1), \ldots, G(n)$, where $G(1) > \cdots > G(n)$. A $100(1-\alpha)\%$ lower confidence bound on the global optimal solution G^* is defined as the statistic $\hat{G}^*$ such that $P\{G^* > \hat{G}^*\} = 1 - \alpha$.

The work by Monroe [14] is an excellent reference concerning the area of confidence intervals for global optimal solutions. Sielken and Monroe [21] have compared several different procedures to determine a *lower confidence limit* for the global minimum of a mathematical programming problem. Another procedure to compute a lower confidence limit on the largest value of a random variable has been proposed by Van der Watt [22]. This procedure uses the $(n-k)$ smallest order statistic $G(n-k)$ and the smallest order statistic $G(n)$ to determine a lower confidence interval for G^* using a level of significance α:

$$\hat{G}^* = G(n) - [\{1 - (1-\alpha)^{1/k}\}^{-1} - 1]^{-1}[G(n-k) - G(n)] \tag{24}$$

As an illustration, the 95% lower confidence limit using $k = 5$ is given by

$$\hat{G}^* = G(n) - 0.8206[G(n-5) - G(n)] \tag{25}$$

The above 95% lower confidence limit will be used in the RBRIP as an estimator of the true lower bound on the value of the objective function at each iteration of the tree search. This justifies the fathoming of some branches in the tree. Such a justification is needed since the solutions obtained at each iteration are local optima. The fathoming of a branch based on a local minimum solution is equivalent to saying that no other solution under that branch can have a lower value than the local minimum. This is not necessarily true. The 95% lower confidence limit allows the quantification of the degree of uncertainty in the fathoming of a branch.

4 AN APPLICATION

The proposed methodology relies heavily upon knowledge of various items of cost data that may be difficult to estimate in practice. In particular, in a typical government transportation agency, the decision makers will need to have available a linear function that maps levels of investment onto user costs. An approach that uses available data on related variables to generate this linear function is described below in general terms:

Step 1 Develop a linear function linking user costs to pavement condition (PSI, for example). The use of this function is justified by Newbery [15] and Paterson [16].

Step 2 Develop a linear relationship linking pavement condition to investment. This is justified by work done, among others, by Mbwana [12].

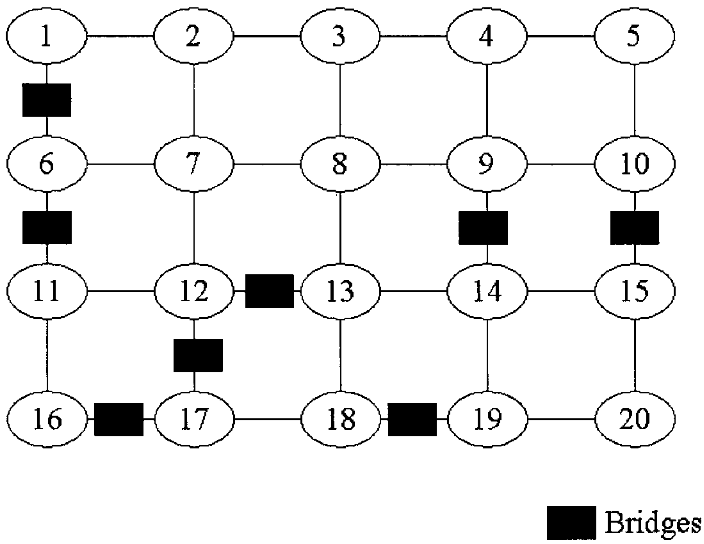

FIGURE 2 Network representation of sample problem.

Step 3 Using the results obtained in Steps 1 and 2, develop a linear relationship that links the user costs to the investment level.

The methodology developed in this article will be now applied to the sample transportation network shown in Figure 2. This example is taken from Ref. [4]. Nodes 1, 3, 6, 8, 9, 12, and 16 are the origin nodes, representing users, such as farm units in a rural agricultural area. It is assumed that the number of round trips made from these nodes is equal to 10. Nodes 17 and 20 are the terminal nodes, representing grain elevators. The arcs represent the roads of the network. Relevant information for Figure 2 is given in Table I. Road and bridge budgets are assumed to be \$300,000 and \$100,000, respectively. Moreover, the investment function for road rehabilitation is given as $y = 1.26 - 0.006846x$ for all roads.

A short description of Table I is provided now. Column 1 enumerates the arcs in the network. Each arc joins node i to node j, with the corresponding labels given in Columns 2 and 3. The length of each road segment, in miles, is given in Column 4. The user cost in dollars per mile is shown in Column 5. This cost is a function of road condition. Average user cost seems to be approximately linear in road condition, according to Newberry [15] and the evidence shown by Paterson [16]. The initial road investment, which reflects current road riding conditions, is given in Column 6. This is estimated by obtaining the user cost from the current condition (Step 1 above), and then using this user cost to estimate the current investment (Step 3 above). The undersized bridges are enumerated in Column 7 and their replacement costs, in thousands of dollars, are shown in Column 8.

TABLE I Network Data.

Arc no.	*From node*	*To node*	*Arc length*	*Travel cost*	*Current investment*	*Bridge no.*	*Bridge cost*
1	1	2	2.5	0.9519	45	0	0.0
2	1	6	1.5	0.7321	80	1	2.5
3	2	3	2.0	0.9119	45	0	0.0
4	2	7	1.5	1.1519	15	0	0.0
5	3	4	1.0	1.1519	15	0	0.0
6	3	8	1.5	0.9519	45	0	0.0
7	4	5	2.0	1.1579	15	0	0.0
8	4	9	1.5	1.1579	15	0	0.0
9	5	10	1.5	0.7123	80	0	0.0
10	6	7	2.5	0.9519	45	0	0.0
11	6	11	2.0	0.7123	80	2	1.5
12	7	8	2.0	1.1579	15	0	0.0
13	7	12	2.0	0.9519	45	0	0.0
14	8	9	1.0	0.9519	45	0	0.0
15	8	13	2.0	1.1579	15	0	0.0
16	9	10	2.0	0.9519	45	0	0.0
17	9	14	2.0	1.1579	15	3	22.5
18	10	15	2.0	0.7123	80	4	31.5
19	11	12	2.5	1.1579	15	0	0.0
20	11	16	1.0	0.7123	80	0	0.0
21	12	13	2.0	0.9519	45	5	22.5
22	12	17	1.0	1.1579	15	6	36.0
23	13	14	1.0	1.1579	15	0	0.0
24	13	18	1.0	0.9519	45	0	0.0
25	14	15	2.0	0.1579	15	0	0.0
26	14	19	1.0	1.1579	15	0	0.0
27	15	20	1.0	0.7123	80	0	0.0
28	16	17	2.5	0.7123	80	7	45.0
29	17	18	2.0	0.7123	80	0	0.0
30	18	19	1.0	0.7123	80	8	31.5
31	19	20	2.0	0.7123	80	0	0.0

Two scenarios will be considered for this problem. The first scenario shows how to find a local optimal solution. In the second scenario 10 local optimal solutions are generated to determine lower bounds on the optimal user cost. The tree search for the first scenario is shown in Figure 3. Initially, the road rehabilitation problem is solved under the assumption that all the undersized bridges can be replaced. Since the total cost of bridge replacement, \$166,500, is greater than the available budget, the solution obtained, which has a user cost $S = \$229.59$, is infeasible. Once the bridges are ranked according to their Δ ratio, their relative order of importance becomes 6, 7, 2, 1, 4; that is, bridge 6 is the most important, bridge 7 is the second most important, and so on.

The next step is to construct a budget feasible bridge configuration with the most important bridges; the procedure sets $Y_6 = 1$, $Y_7 = 1$, $Y_i = 0$ for $i = 1, 2, 3, 5, 5, 8$, since only bridges 6 and 7 can be replaced with the available funds. The solution for this bridge configuration has a user cost equal to $Z = \$258.48$. (Note that a feasible solution has an objective function value denoted by Z, and an infeasible solution a value represented by S).

The backtracking step examines all bridges configurations such that $Y_7 = 0$; at this step all other bridges are considered as candidates for replacement. The objective function value of the solution obtained after backtracking, $S = 258.21$, is used as a lower bound for any bridge configuration not containing bridge 7. Since this lower bound is greater than the current best value solution, this branch is fathomed. The procedure then backtracks to $Y_6 = 0$ and

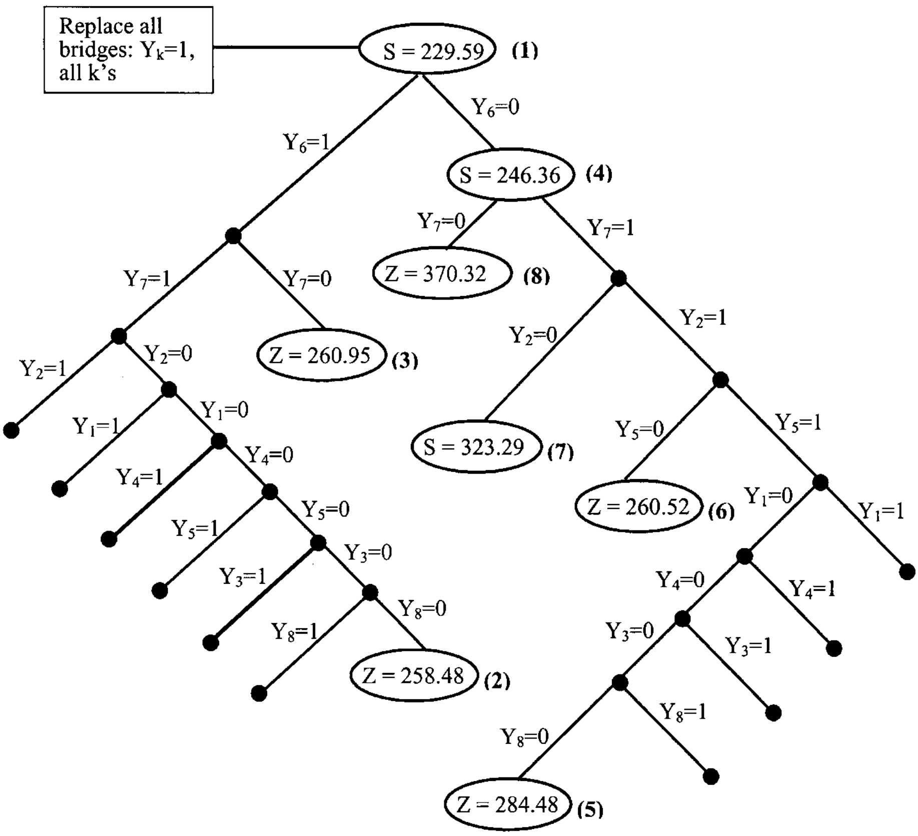

FIGURE 3 Branch and Bound results for first scenario.

$S = 246.36$ is obtained. But this lower bound is less than the current best solution, therefore this branch cannot be fathomed. The forward move produces a new partial solution of $Y_6 = 0$, $Y_7 = 1$, $Y_2 = 1$, $Y_5 = 1$ and $Y_i = 0$ for $i = 1, 4, 3, 8$, which result in the user cost of $Z = 284.48$. In the same way the other branches are fathomed. As a result of this analysis, the current best solution is considered as the optimal solution.

The road sections to be rehabilitated, as well as the bridges to be replaced, according to the optimal solution shown in Figure 3, are indicated below:

Section No.	**Rehabilitation Cost**
22	85,000
24	55,000
15	160,000
Bridge No.	**Replacement Cost**
6	36,000
7	45,000

TABLE II Confidence Interval Results.

	Iter. 1	*Iter. 2*	*Iter. 3*	*Iter. 4*	*Iter. 5*
$G(1)$	237.32	384.09	251.58	364.44	367.11
$G(2)$	229.59	364.44	244.17	364.44	367.11
$G(3)$	222.73	360.75	243.15	346.63	338.61
$G(4)$	222.73	346.63	243.15	338.61	338.61
$G(5)$	222.73	346.63	243.15	338.61	338.61
$G(6)$	217.13	346.63	243.15	338.61	338.61
$G(7)$	217.13	338.61	243.15	338.61	338.61
$G(8)$	217.13	338.61	243.15	338.61	338.61
$G(9)$	217.13	338.61	243.15	338.61	338.61
$G(10)$	217.13	338.61	243.15	338.61	338.61
LCL	209.15	320.65	239.82	328.41	327.35
	Iter. 6	*Iter. 7*	*Iter. 8*	*Iter. 9*	*Iter. 10*
$G(1)$	291.79	244.32	260.52	245.65	299.67
$G(2)$	274.81	244.32	260.52	245.65	299.67
$G(3)$	274.81	244.32	260.52	245.65	299.67
$G(4)$	274.81	238.17	260.52	245.65	299.67
$G(5)$	274.81	237.32	260.52	245.65	299.67
$G(6)$	260.95	237.32	260.52	245.65	299.67
$G(7)$	260.80	237.32	260.52	245.65	299.67
$G(8)$	260.80	221.64	260.52	245.65	299.67
$G(9)$	260.80	221.64	260.52	245.65	299.67
$G(10)$	260.80	221.64	260.52	245.65	299.67
LCL	248.56	212.68	260.52	245.65	299.67
	Iter. 11	*Iter. 12*	*Iter. 13*	*Iter. 14*	*Iter. 15*
$G(1)$	299.67	267.78	258.48	331.87	291.79
$G(2)$	299.67	252.10	258.48	331.87	276.11
$G(3)$	299.67	252.10	258.48	331.87	276.11
$G(4)$	299.67	252.10	258.48	331.87	269.51
$G(5)$	299.67	252.10	258.48	331.87	269.38
$G(6)$	299.67	252.10	258.48	331.87	269.38
$G(7)$	299.67	245.36	258.48	331.87	269.38
$G(8)$	299.67	245.36	258.48	331.87	269.38
$G(9)$	299.67	245.36	252.68	331.87	269.38
$G(10)$	299.67	245.36	252.68	321.03	269.38
LCL	299.67	236.50	241.77	316.75	260.52

The user cost associated with the above optimal solution is equal to $Z = 258.48$. Only one starting road rehabilitation policy is randomly generated at each iteration of this tree search. Since the objective function is not convex, the solution obtained is a local optimum and is not a proper lower bound for the tree search. Therefore, a criterion must be defined to assess the desirability of the solution. This will be achieved by finding a confidence interval for the global optimum at each iteration and using the lower limits as estimators of the true lower bounds.

In the second scenario of the problem being considered, ten different starting road rehabilitation policies were randomly generated at each iteration of the tree search in order to estimate lower-limits on the optimal user cost. The 95% lower confidence limits were obtained using Eq. (25) and are shown in Table II. Table III summarizes the results obtained from the tree search (here not shown) for the second scenario. Column 1 contains the iteration number. Each iteration starts with the results shown in Column 2. Column 3 shows both the binary variables and their values corresponding to the path in the search tree that ends with the results obtained at each iteration. Column 4 has the list of bridges to be replaced. Replacement costs as given in Column 5. Finally, Column 6 shows the user travel cost.

As can be seen in Table III, the best road-bridge rehabilitation policy had a cost of $Z = 246.51$ with the road and bridges to be replaced indicated below:

Section No.	**Rehabilitation Cost**
22	85,000
13	110,000
12	105,000

Bridge No.	**Replacement Cost**
6	36,000
7	45,000

This confidence-limit procedure does not guarantee a feasible solution to the problem, but it provides an indication of how "near" the "near-optimal" solution is to the global optimum. Therefore, in the tree search procedure, the 95% lower confidence limit (LCL values in

TABLE III Results from Branch-and-Bound Procedure.

Iteration no.	*Branching from*	*Values of binary variables*	*Replaced Bridges*	*Bridge cost*	*User cost*
1	–	$Y_k = 1,\ k = 1, \ldots, 8$	4, 7, 2, 1, 6	166,500	$S = 217.13$
2	1	$Y_4 = 1,\ Y_7 = 1,\ Y_2 = 0,\ Y_1 = 1,\ Y_6 = 0,\ Y_3 = 0,\ Y_5 = 0,\ Y_8 = 0$	7	45,000	$Z = 338.71$
3	1	$Y_4 = 1,\ Y_7 = 1,\ Y_2 = 0,\ Y_1 = 0$	4, 6, 7	112,500	$S = 243.15$
4	3	$Y_6 = 0,\ Y_5 = 1,\ Y_3 = 0,\ Y_8 = 0$	7	45,000	$Z = 338.61$
5	3	$Y_4 = 1,\ Y_7 = 1,\ Y_2 = 0,\ Y_1 = 0,\ Y_6 = 0,\ Y_5 = 0$	7	45,000	$Z = 338.61$
6	1	$Y_4 = 1,\ Y_7 = 0$	1, 4, 6	90,000	$Z = 260.80$
7	1	$Y_4 = 0$	1, 2, 6, 7	135,000	$S = 221.64$
8	10	$Y_7 = 1,\ Y_2 = 1,\ Y_1 = 1,\ Y_6 = 0,\ Y_3 = 0,\ Y_5 = 0,\ Y_8 = 0$	1, 2, 7	99,000	$Z = 260.52$
9	10	$Y_4 = 0,\ Y_7 = 1,\ Y_2 = 1,\ Y_1 = 0$	2, 6, 7	112,500	$S = 245.65$
10	12	$Y_6 = 0,\ Y_5 = 1,\ Y_3 = 0,\ Y_8 = 0$	2, 7	76,500	$Z = 299.67$
11	12	$Y_4 = 0,\ Y_7 = 1,\ Y_2 = 1,\ Y_1 = 0,\ Y_6 = 0,\ Y_5 = 0$	2, 7	76,500	$Z = 299.67$
12	10	$Y_4 = 0,\ Y_7 = 1,\ Y_2 = 0$	1, 3, 6, 7	126,000	$S = 245.36$
13	15	$Y_6 = 1,\ Y_1 = 0,\ Y_5 = 0,\ Y_3 = 0,\ Y_8 = 0$	6, 7	81,000	$Z = 246.51$
14	15	$Y_4 = 0,\ Y_7 = 1,\ Y_2 = 0,\ Y_6 = 0$	5, 7	67,500	$Z = 321.03$
15	10	$Y_4 = 0,\ Y_7 = 0$	1, 3, 6	81,000	$Z = 269.38$

Tab. II) is used as an estimator of the true lower bound at a given branch of the tree to decide whether to fathom that branch or not. The budget feasible solution with the minimum user cost defines the road-bridge rehabilitation policy.

The minimum solution obtained in the tree search constitutes a single observation of the behavior of the tree pattern. The sample problem was simulated ten times, and the tree patterns were the same. This supports the hypothesis that the global optimum solution can be estimated with one run of the program, if the estimator of the optimal solution at each branch of the tree is based on a relatively large sample.

5 SUMMARY AND FUTURE WORK

The road-bridge rehabilitation model is formulated as a mixed integer non-linear programming problem with linear constraints. The purpose of the model is to minimize user travel costs under a limited budget and to join pavement management and bridge management systems into one highway management system. For a given set of bridges to replace, the problem is reduced to a continuous non-linear programming problem that can be further decomposed into a traffic assignment problem (TAP) and a road rehabilitation budget allocation problem (RBAP). A local optimum for the problem with a given set of bridges to replace is attained by iteratively solving the TAP and RBAP. Also, the local optimum for this subproblem is used in a branch and bound procedure to produce a solution for the initial problem.

Future work in this area could include the usage of a branch decomposition based algorithm offered by Hicks [8] to obtain an initial feasible upper bound for the search tree and the usage of interior point methods to solve the subproblem used in the search tree. Also, the work of Lee and Grossmann [10] may prove beneficial to solve the initial problem.

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