Optimization for condition-based maintenance with semi-Markov decision process[☆]

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Abstract

The semi-Markov decision model is a powerful tool in analyzing sequential decision processes with random decision epochs. In this paper, we have built the semi-Markov decision process (SMDP) for the maintenance policy optimization of condition-based preventive maintenance problems, and have presented the approach for joint optimization of inspection rate and maintenance policy. Through numerical examples, the improvement of this method is compared with the scheme, which optimizes only over the inspection rate. We also find that under a special case when the deterioration rate at each failure stage is the same, the optimal policy obtained by SMDP algorithm is a dynamic threshold-type scheme with threshold value depending on the inspection rate.
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1. Introduction

Preventive maintenance is defined as the activity undertaken regularly at pre-selected intervals while the device is satisfactorily operating, to reduce or eliminate the accumulated deterioration [5], while repair is the activity to bring the device to a non-failed state after it has experienced a failure. When the cost incurred by a device failure is larger than the cost of preventive maintenance (this cost could be cost of downtime, repair expenses, revenue lost, etc.), it is worthwhile to carry out preventive maintenance.

Generally, there exist two types of preventive maintenance schemes, i.e. condition based and time based preventive maintenance [3]. For condition based preventive maintenance, the action taken after each inspection is dependent on the state of the system. It could be no action, or minimal maintenance to recover the system to the previous stage of degradation, or major maintenance to bring the system to as good as new state. For time based preventive maintenance, the preventive maintenance is carried out at pre-determined time intervals to bring the system to as good as new state [7]. In this paper we focus on the condition based preventive maintenance.

Sim and Endrenyi introduced the multi-stage exponential device failure model in [5], in which the idea of minimal preventive maintenance was introduced. The minimal preventive maintenance is defined as the preventive maintenance activity with limited effort and effect [4]. For deterioration failures modeled as several stages of exponential distributions, minimal maintenance restores the system to the previous deterioration stage. Corresponding to the minimal preventive maintenance, the idea of major maintenance is defined as the maintenance operation by which the device is restored 'as good as new' status.

With condition based preventive maintenance, the maintenance action taken after each inspection is dependent on the state of the system. There could be no action, or minimal maintenance to recover the system to the previous failure stage, or major maintenance to bring the system to *as good as new* state. Hosseini et al. [2] introduced

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the threshold-type policy for the maintenance action. No maintenance action is taken if the device deterioration stage is found to be smaller than the minimal maintenance threshold; minimal maintenance is performed if the device deterioration stage is found to be between the minimal maintenance threshold and the major maintenance threshold; and major maintenance is carried out if the device deterioration stage is larger than the major maintenance threshold. This model was captured by a stochastic Petri net, and its optimal inspection interval to maximize the system availability is presented. Closed-form results for such threshold-type condition-based maintenance problems are reported in [1].

Above works have only considered the optimization of system parameters. We notice that another important factor in determining the overall system availability is the maintenance policy, i.e. the action to be taken at each deterioration stage. Improperly designed policies may considerably hamper the ability of the system to meet certain design objectives, even with carefully chosen parameters. For this reason, in this paper we aim at joint optimization of both system parameters and system maintenance policy.

One possible approach to tackle this problem is by solving the system model and search for the optimal combination exhaustively. However, when the number of failure stages is large, this work is cumbersome. For this reason, we formulate the semi-Markov decision process (SMDP) model [6] for the condition-based maintenance problem, based on the assumption that the system behavior may be captured by Markovian models. Besides deterioration failures, in our model, we have also considered Poisson failures, which are defined as the type of failures under which the system fails abruptly rather than gradually, as with the deterioration failures. For Poisson failures, we assume the repair action is to restore the system to the operable state it was just in before the failure.

The joint optimization of system parameters (the inspection rate in our case) and the maintenance policy is performed by taking the inspection rate as input parameter to the SMDP model. For each individual inspection rate the SMDP model is solved for the optimal policy, based on which the system CTMC model is constructed and solved. From these results, we may thus obtain the best combinations of system inspection rate and system maintenance policy.

This paper is organized as follows: In Section 1 we introduce the notation, and the Markov model and the SPN model for the preventive maintenance problem. In Section 2 we formulate the MDP problem for determining the optimal maintenance policy, and in Section 3 numerical results are presented. Finally, Section 4 concludes the paper.

2. Notation and the SMDP formulation

In this paper, the following notations are used:

k total number of deterioration stages
i running index deterioration stage

g minimal preventive maintenance threshold
b major preventive maintenance threshold
λ_i failure rate at stage i
F_{in} time to inspection trigger interval distribution
F_d distribution of the time to carry out an inspection
F_m preventive minimal maintenance duration distribution
F_M preventive major maintenance duration distribution
F_R failure repair time distribution
F system failure state

Consider the epoch at which the system fails and enter state F as the fictitious decision epoch. The only action at this epoch is to repair the system to as good as new state. Let $i \in I = \{0,1,\ldots,k+1\}$ define a system state where the system is found to be in deterioration stage i with $0 \le i \le k$, or failure state F represented by $i = k+1$, during an inspection.[1] For simplification of representation, in the following of our paper we call state $0 \le i \le k$ inspection state and state $i = k+1$ failure state.

In each state, a decision needs to be made on the action to perform according to the system deterioration stage. The action should include both the parameter(s) to determine the next inspection time θ, and the maintenance action (do nothing, perform minimal maintenance, or perform major maintenance). We represent the action at the nth decision epoch as a two-tuple (A_n, θ) where A_n is the maintenance action to be performed and θ is the mean time to next inspection. Then, for states $i = 0,\ldots,k$ the possible action A_n is

$$A_n = \begin{cases} 0, & \text{no action is taken} \\ 1, & \text{minimal maintenance is performed} \\ 2, & \text{major maintenance is performed} \end{cases} \quad (1)$$

and in state $i = k+1$ the only action is to repair the system to *as good as new* state with θ as the next inspection time distribution parameter.

Let Y_n, $n \in N = \{0,1,\ldots\}$ be the system state at the nth decision epoch. Then, the state transition probability could be derived.

For the transition probability from one inspection state with deterioration stage i to the next inspection state with deterioration stage j we have

$$P(Y_{n+1} = j | Y_n = i, A_n = a, \theta_n = \theta)$$

$$= \begin{cases} \displaystyle\int_0^\infty \tilde{P}_i^j(t)\, \mathrm{d}F_{in}(t,\theta) & a = 0 \\ \displaystyle\int_0^\infty \tilde{P}_{i-1}^j(t)\, \mathrm{d}F_{in}(t,\theta) & a = 1 \\ \displaystyle\int_0^\infty \tilde{P}_0^j(t)\, \mathrm{d}F_{in}(t,\theta) & a = 2 \end{cases} \quad (2)$$

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[1] We assume that the failure state F can be discovered without any inspection.

where $0 \le i \le k+1$, $0 \le j \le k$, and $\tilde{P}_i^j(t)$ is the probability that the system state changes from i to j without any inspection event. Computation of $\tilde{P}_i^j(t)$ is shown in Appendix A.

The transition probability from an inspection/failure state to failure state F is

$$
P(Y_{n+1} = k + 1 | Y_n = i, A_n = a, \theta_n = \theta)
$$

$$
= \int_0^\infty (1 - F_{\text{in}}(t, \theta)) \mathrm{d}(\tilde{P}_i^{k+1}(t)) \tag{3}
$$

where $0 \le i \le k$.

The expected time to the occurrence of next decision epoch, given current state i and action a is

$$
\tau(i,a) = \begin{cases}
\int_0^\infty (1 - \tilde{P}_i^{k+1}(t))(1 - F_{\text{in}}(t))\mathrm{d}t + \int_0^\infty F_{\text{d}}(t)\,\mathrm{d}t & 0 \le i \le k, a = 0 \\
\int_0^\infty (1 - \tilde{P}_{i-1}^{k+1}(t))(1 - F_{\text{in}}(t))\,\mathrm{d}t + \int_0^\infty (F_{\text{d}}(t) + F_{\text{m}}(t))\,\mathrm{d}t & 0 \le i \le k, a = 1 \\
\int_0^\infty (1 - \tilde{P}_0^{k+1}(t))(1 - F_{\text{in}}(t))\,\mathrm{d}t + \int_0^\infty (F_{\text{d}}(t) + F_{\text{M}}(t))\,\mathrm{d}t & 0 \le i \le k, a = 2 \\
\int_0^\infty (1 - \tilde{P}_0^{k+1}(t))(1 - F_{\text{in}}(t))\,\mathrm{d}t + \int_0^\infty F_{\text{R}}(t)\,\mathrm{d}t & i = k + 1
\end{cases} \tag{4}
$$

The system operational cost is assigned as follows

Cost Meaning
c_{m} cost per unit time of downtime due to maintenance
c_{d} cost per unit time of downtime due to inspection
c_{R} cost per unit time of downtime due to repair
c'_{m} cost for each minimal maintenance
c'_{M} cost for each major maintenance
c'_{R} cost for each repair

Then, the cost function in each state x with action a can be written as

$$
c(i,a) = \begin{cases}
c_d \int_0^\infty (1 - F_d(t))\,\mathrm{d}t & i = 0 \\
c_d \int_0^\infty (1 - F_d(t))\,\mathrm{d}t & 1 \le i \le k, a = 0 \\
c_d \int_0^\infty (1 - F_d(t))\,\mathrm{d}t + c_m \int_0^\infty (1 - F_m(t))\mathrm{d}t + c'_m, & 1 \le i \le k, a = 1 \\
c_d \int_0^\infty (1 - F_d(t))\,\mathrm{d}t + c_M \int_0^\infty (1 - F_M(t))\,\mathrm{d}t + c'_M, & 1 \le i \le k, a = 2 \\
c_R \int_0^\infty (1 - F_R(t))\,\mathrm{d}t + c'_R. & i = k + 1
\end{cases} \tag{5}
$$

The value iteration algorithm is summarized as follows:

Step 0. Choose $V_0(x)$ such that $0 \le V_0(x) \le \min_a (c(x,a)/\tau(x,a))$ for all x. Let $n = 1$.
Step 1. Compute the function $V_n(x)$, $x \in I$, from

$$
V_n(x) = \min_{a \in A(x), \theta}, \left[\frac{c(x,a)}{\tau(x,a)} + \frac{\tau}{\tau(x,a)} \sum_{y \in I} P_{xay} V_{n-1}(y) \right.
$$

$$
\left. + \left\{ 1 - \frac{\tau}{\tau(x,a)} \right\} V_{n-1}(x), \right] \quad x \in I, \tag{6}
$$

and find the stationary policy $R(n)$ that minimize the above equation.
Step 2. Compute the bounds $m_n = \min_{x \in I}(V_n(x) - V_{n-1}(x))$ and $M_n = \max_{x \in I}(V_n(x) - V_{n-1}(x))$.
  If $0 \le (M_n - m_n) \le \varepsilon m_n$, where ε is a pre-specified accuracy number, the algorithm is stopped with the policy $R(n)$. Otherwise, go to step 3.
Step 3. $n = n + 1$ and go to step 1.

3. Numerical results

In the numerical evaluation of the MDP scheme, we assume the time to inspection, inspection time, minimal maintenance time, major maintenance time and repair time are all deterministically distributed. Then, we have

$$
F_{\text{in}}(t, \theta) = U(t - \theta), \quad F_{\text{d}}(t) = U(t - t_{\text{d}}),
$$

$$
F_{\text{m}}(t) = U(t - t_{\text{m}}), \quad F_{\text{M}}(t) = U(t - t_{\text{M}})
$$

$$
F_{\text{R}}(t) = U(t - t_{\text{R}})
$$

and the parameters are chosen as follows

$$
k = 10, \quad t_{\text{d}} = 0.5, \quad t_{\text{m}} = 0.5, \quad t_{\text{M}} = 0.5, \quad t_{\text{R}} = 100.
$$

and we assume the deterioration rate λ_i at each stage is the same with a value of 0.03.

The cost for each state is chosen in such a way that the total cost represents the system steady-state availability, i.e. $c_d = 1$, $c_m = 1$ with all other costs equal to zero.

Fig. 1 shows a comparison between the MDP scheme and the scheme with fixed minimal and major maintenance threshold. With MDP scheme, maximal availability of 0.962
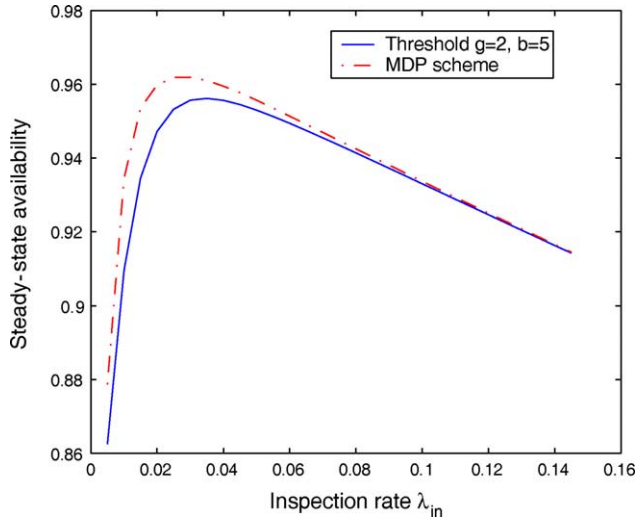
Fig. 1. Comparison between the threshold policy and the MDP policy.

may be achieved by choosing $\lambda_{in} = 0.021$ while with fixed threshold the maximal availability of 0.955 may be achieved with $\lambda_{in} = 0.032$. This figure demonstrates the improvement achievable by the joint optimization approach over the optimization of only the inspection rate.

A deeper look into the optimal maintenance policies reveals that the policy obtained by MDP algorithm is also threshold-type policy, but with different thresholds b and g for different inspection rate λ_{in} For example, the optimal inspection rate $\lambda_{in} \approx 0002$ according to Fig. 1. If we denote the minimal maintenance threshold by g, and denote the major maintenance threshold by b, the optimal maintenance policy corresponding to $\lambda_{in} = 0002$ is $g = 0$ for minimal maintenance threshold and $b = 4$ for major maintenance threshold.

To further explore the relationship between the inspection rate and maintenance policy, we have varied λ_{in} to obtain the corresponding thresholds. Fig. 2 shows the result.
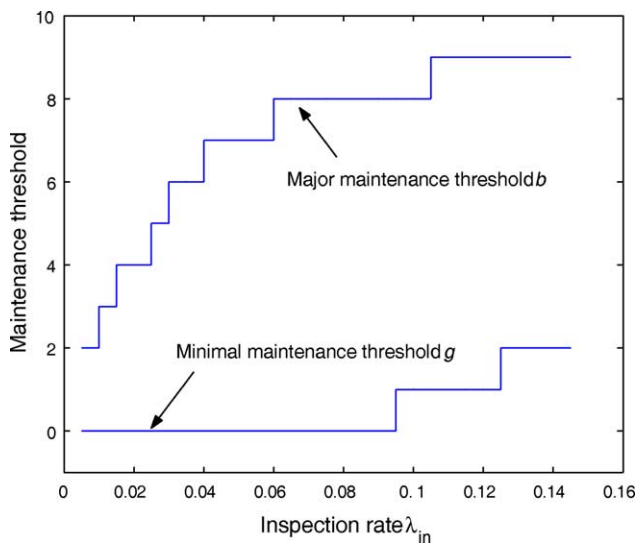

Fig. 2. Thresholds for the optimal maintenance policy.

It may be observed that with more frequent inspections (higher inspection rates), the major maintenance threshold becomes larger, which means more minimal maintenance is performed. Intuitively, this phenomenon may be explained in such way that more frequent inspections lead to more timely warning if the system is near to failure. To reduce system operation cost or increase availability, it is thus better to take risk to perform minimal maintenance even if the system is in its higher deterioration stages.

While the threshold type policies obtained by SMDP approach are easier to understand and employ, it is worthwhile to point out that the optimal policy is not necessarily threshold type in other situations, e.g. when the deterioration rate are not the same or when the objective of optimization is different from the steady-state availability. In these situations, the optimal policy may be more complicated than the threshold-type policies.

4. Conclusion

In this paper, we have presented the application of MDP algorithm in searching for the optimal maintenance policy for condition based maintenance, and we have also presented a joint optimization of inspection rate and its corresponding maintenance policy. Under a special case when the optimization objective is steady-state availability and the deterioration rate at each failure stage is the same, we find that the optimal policy is a threshold-type maintenance policy.

Appendix A. Calculation of $\tilde{P}_i^j(t)$

For state $0 \le i \le k$, $0 \le j \le k+1$, the system balance equation is

$$\frac{d\tilde{P}_i^j(t)}{dt} = 0, \quad j < i \tag{A1}$$

$$\frac{d\tilde{P}_i^i(t)}{dt} = -\lambda_i \tilde{P}_i^i(t), \tag{A2}$$

$$\frac{d\tilde{P}_i^{i+1}(t)}{dt} = -\lambda_{i+1} \tilde{P}_i^{i+1}(t) + \lambda_i \tilde{P}_i^i(t), \tag{A3}$$

$$\vdots \tag{A4}$$

$$\frac{d\tilde{P}_i^j(t)}{dt} = -\lambda_j \tilde{P}_i^j(t) + \lambda_{j-1} \tilde{P}_i^{j-1}(t), \quad i < j \le k \tag{A5}$$

$$\tilde{P}_i^{k+1}(t) = 1 - \sum_{m=i}^{k} \tilde{P}_i^m(t) \tag{A6}$$

and for state $i = k+1$, $0 \leq j \leq k+1$, we have

$$\frac{\mathrm{d}\tilde{P}_{k+1}^0(t)}{\mathrm{d}t} = -\lambda_0 \tilde{P}_{k+1}^0(t), \tag{A7}$$

$$\frac{\mathrm{d}\tilde{P}_{k+1}^j(t)}{\mathrm{d}t} = -\lambda_j \tilde{P}_{k+1}^j(t) + \lambda_{j-1}\tilde{P}_{k+1}^{j-1}(t), \quad 1 \leq j \leq k \tag{A8}$$

$$\tilde{P}_{k+1}^{k+1}(t) = 1 - \sum_{m=0}^{k} \tilde{P}_{k+1}^m(t) \tag{A9}$$

and thus $\tilde{P}_{k+1}^j(t) = \tilde{P}_0^j(t)$, $0 \leq j \leq k+1$..

Performing Laplace transform, for $0 \leq i \leq k$ we have

$$s\tilde{P}_i^i(s) = 1 - \lambda_i \tilde{P}_i^i(s) \tag{A10}$$

$$s\tilde{P}_i^j(s) = -\lambda_j \tilde{P}_i^j(s) + \lambda_{j-1}\tilde{P}_i^{j-1}(s), \quad i < j \leq k \tag{A11}$$

$$\tilde{P}_i^{k+1}(s) = 1 - \sum_{m=i}^{k} \tilde{P}_i^m(s). \tag{A12}$$

and $\tilde{P}_{k+1}^j(s) = \tilde{P}_0^j(s)$, $0 \leq j \leq k+1$.

Therefore, the Laplace transform of $\tilde{P}_i^{i'}(t)$ may be written as

$$\tilde{P}_i^j(s) = \frac{\prod_{m=i}^{j-1} \lambda_m}{\prod_{m=i}^{j}(s+\lambda_m)}, \quad i \leq j \leq k \tag{A13}$$

$$\tilde{P}_i^{k+1}(s) = 1 - \sum_{j=i}^{k} \tilde{P}_i^j(s). \tag{A14}$$

with $\prod_{m=i}^{j} = 1$ for $j < i$ and the time domain results may be obtained by either analytical or numerical inversion of (A13) and (A14).

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