Spatial Variability of Falling Weight Deflectometer Data: A Geostatistical Analysis

by

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A GEOSTATISTICAL ANALYSIS

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Falling Weight Deflectometer (FWD) data and the corresponding pavement stiffness moduli and deflection basin areas vary as functions of both time and space. This paper focuses primarily on extracting information from the spatial variability of FWD deflection data. Spatial variability occurs both horizontally and vertically within a pavement system; it is inherent in the system due to the heterogeneity of the material composing the subgrade, and is further influenced by the construction process and the resulting variations in density, moisture content, and thickness of the subbase, base, and surface layers.

To assess spatial variability of just the subgrade and then the overall pavement structure, FWD tests were conducted on 71 test points on top of each of the layers composing a pavement system, and a statistical analysis was conducted on the data. Test pavements included two mainline (five-year design) and two low-volume test cells at the Minnesota Road Research Project (MN/ROAD).

By comparing differences between pairs of measured deflections at increasing test point separation distances, one can incorporate distance weighting techniques into a statistical form frequently used in the fields of geology and mining. The geostatistical semi-variogram can be applied to pavements and used to model the degree of correlation between data at any two test points. As the distance between test points increases, corresponding data become decreasingly dependent upon each other until, at some appreciable distance, they are independent of each other. From the semi-variogram, one can readily determine the separation distance at which values are independent of each other. Conventional statistical analyses are also used to supplement geostatistical techniques.

Valuable and cost-saving information can be acquired by analyzing baseline FWD data with this technique. The efficiency of future FWD testing can be maximized, and optimum FWD test point spacing can be determined for pavement evaluation and overlay design. Furthermore, the geostatistical techniques discussed are applicable to any problem involving the distribution of a variable in one, two, or three dimensions.

INTRODUCTION

While pavement models and testing methods become increasingly sophisticated, the questions addressing the number and locations of tests remain unanswered. Also, there exists very little emphasis on the development of a strategy for maximizing the efficiency of sampling and testing. Inherent in any in-situ soil deposit are both order and randomness. These two features constitute spatial variability. Geostatistics, a tool commonly used in the disciplines of mining and geology, enables one to quantify the spatial correlation of a site; classical statistics essentially neglects the relative positions of tests or samples.

Intuitively, one recognizes the fact that tests conducted adjacent to each other will yield more similar results than ones taken farther apart. The degree of similarity is measured by a variogram or semi-variogram (one half the variogram). The model is geostatistics' counterpart to classical statistics' normal curve. A semi-variogram is essentially the variance of the difference between any two realizations of, or values for, a random variable distributed in space. Theoretical models have been fit to the experimental semi-variograms for Falling Weight Deflectometer (FWD) deflections obtained from test cells at the Minnesota Road Research Project (MN/ROAD), a large pavement testing facility recently constructed by the Minnesota Department of Transportation (MN/DOT). MN/ROAD includes 40 pavement test cells of varying cross-sections, types, and design lives (MN/DOT 1992).

Fitting variogram models in conjunction with kriging (a geostatistical interpolation procedure that minimizes the variance of the error) and simulations (combining classical statistics' probability density function with the variogram for the purpose of characterizing the extreme values) can be used to locate trends and zones of higher or lower valued test results. The techniques can yield a sampling strategy for maximizing the efficiency of sampling and testing.

FWD tests were conducted on top of the subgrade of four MN/ROAD test cells (Figure 1) in July 1992 by MN/DOT (MN/DOT 1992). Each 500-ft (152-m) pavement test cell included 71 FWD test points in the pattern shown in Figure 2. Additional tests were conducted on top of the base course in two of the original four test cells in August 1992, and again on top of the bituminous concrete surface in August 1993.

FWD tests were complemented by a limited number of dynamic cone penetrometer (DCP) and soil moisture content tests. The DCP and moisture content tests were not part of the spatial variability test plan; rather, test data were taken from the MN/ROAD database. For this reason, test points only occasionally coincided spatially with FWD test points.

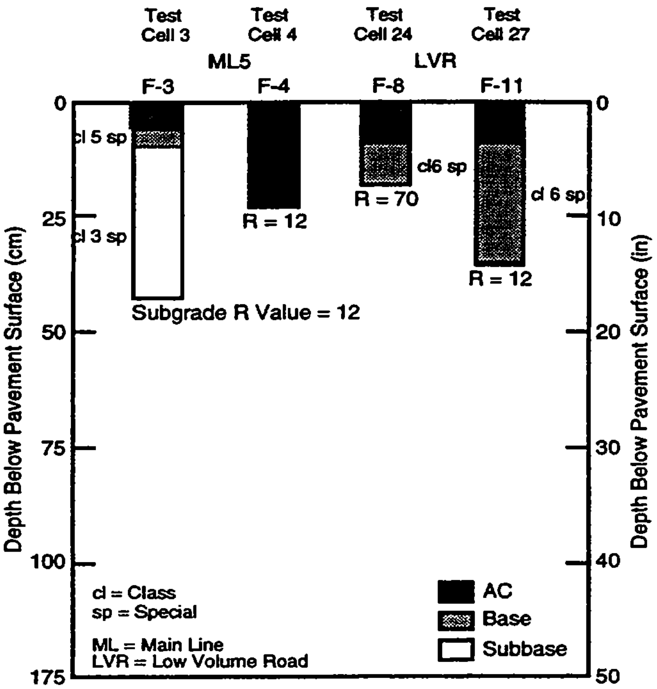

Figure 1. Pavement structure of test sections.

This paper briefly discusses the background of geostatistics, reviews the development of basic geostatistical theory, discusses MN/ROAD test results (primarily test cell ML03 shown in Figure 1) in light of geostatistics, briefly discusses multivariate analyses, and concludes by discussing the potential value of combining classical statistics and geostatistics for pavements engineering.

BACKGROUND: MINING GEOSTATISTICS

Geostatistics is frequently used in the estimation of ore reserves in the mining industry. Soulie, Montes, and Silvestri (1990) state that geostatistics had a "reputation for being complicated and difficult to understand," thus hindering its use outside of mining. The more basic geostatistical estimation techniques can, however, be used wherever (1) there is a continuous measure that is sampled or tested at discrete points in space, and (2) an individual sample value is affected by its position and relationship to neighboring points.

In mining, recommended categories for the classification of in-situ resources may include: 1) measured ore, 2) indicated ore, and 3) inferred ore. The precision of the estimation of, for example, the mean grade is a function of the number of assays, the in situ variability of the grade in the deposit, and the method by which the variability was sampled (Journel and Huijbregts 1978). For this particular mining problem, trends can be detected with variogram analysis, measurements can be estimated at locations where no data have been taken with a geostatistical interpolation procedure termed kriging, and a degree of confidence can be assigned to the kriged estimates. The authors of this paper believe similar techniques can be easily adapted to pavements that both exhibit similar spatial variability and undergo similar discrete sampling and testing. The University of Minnesota has also been studying the application of geostatistics to pavements, but on a larger statewide/geologic scale. They have observed correlations among soil types within relatively large regions of the state (Aboulkheir and Barnes 1992).

SPATIAL VARIABILITY

In classical statistics, trends are typically modeled by linear regressions, and residuals off the trend are assumed to be uncorrelated (Fig. 3). As noted by DeGroot and Baecher (1993), this is incorrect unless there exists an appreciable separation distance between measurements. The value at any one point includes a fixed component along the regression line and a random variable. Each residual value, whether above or below the regression line, is close in value to nearby residual values. At the limit, as the separation distance between residuals approaches zero,

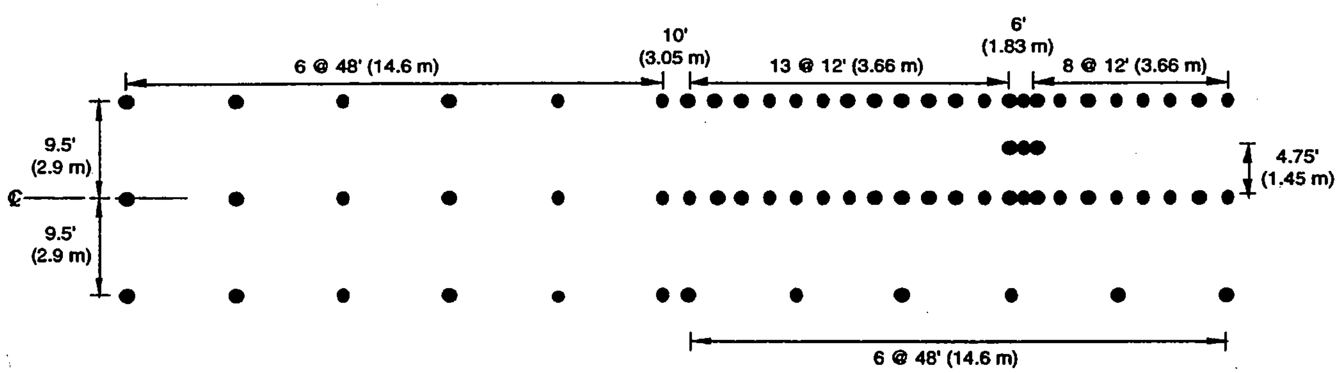

Figure 2. FWD test point grid.

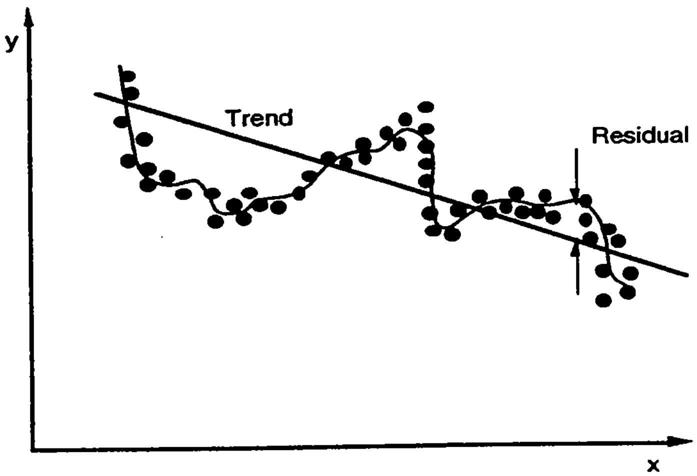

Figure 3. Spatial variability indicated by residual values and trends.

the two respective residuals approach each other. The correlation among residuals indicates that spatial variability is not solely explained by the trend itself. This correlation among residuals constitutes a key element in optimizing estimates at unsampled or untested locations.

VARIOGRAM

The variogram is based upon the assumption that the difference in value between any two positions depends solely upon their separation distance and their relative orientation. If the difference (a measure of similarity) in the parameter of interest between all pairs at the same separation distance were determined, one could construct a histogram through which the statistical distribution could be analyzed. Similar histograms could be built for all possible separation distances and directions. This procedure puts distance weighting techniques into a statistical format. The multiple histograms can then be summarized in a few simple parameters. Since it was assumed that the difference in the desired value between two samples is dependent only upon a separation distance and direction (vector h), the distribution of the differences also depends only upon h. Furthermore, since this is true of the distribution, it is also true of the mean h [m (h)] and variance 2γ (h) (Moore and McCabe 1989). (Conforming to standard geostatistical convention, "*" simply indicates experimental as opposed to theoretical.) Experimental values for $m^*(h)$ and $2\gamma^*(h)$ are:

$$m^*(h) \quad = \quad (1/n)\sum_{i=1}^{n}[z(x_i)-z(x_i+h)] \tag{1}$$

and

$$2g^*(h) \quad = (1/n)\sum_{i=1}^{n}[z(x_i)-z(x_i+h)]^2 \tag{2}$$

where

m^*	=	experimental mean
$2\gamma^*$	=	experimental variance
x_i	=	position of one sample in the pair
$x_i + h$	=	position of the other sample in the pair
n	=	number of experimental pairs of data
z	=	experimental measure (e.g., grade of ore for mining, modulus for pavements).

The first of the above two equations will not be discussed in this paper, as assumptions regarding trends and patterns complicate the issue. The second equation is known as the variogram. More frequently used in geostatistics is the semi-variogram, which is one half the variogram and is indicated by $\gamma^*(h)$. It should be noted that, in geostatistical literature, the terms variogram and semi-variogram are often used synonymously when referring to what has been defined here as the semi-variogram.

Values for the semi-variogram are calculated for as many different values of h as possible. Typically the semi-variogram is graphed with the distance between pairs on the abscissa and the semi-variogram value on the ordinate (Fig. 4).

The semi-variogram essentially summarizes all available structural information into one model. According to Journel, "γ(h) can be regarded as injecting extensive geological experience into the sequence of studies involved in a mining project" (Journel and Huijbregts 1978).

Neglecting system error, two tests performed at exactly the same point exactly the same way yield exactly the same value. Needless to say, the difference is zero, and γ(h) always passes through the origin. As samples/tests are

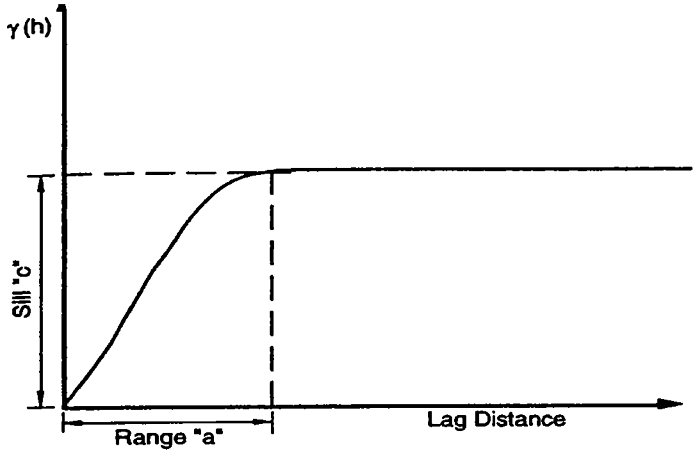

Figure 4. Typical variogram.

When the distance becomes very large, and values become independent of one another, the semi-variogram approaches a constant value "C" termed the sill. This leveling off would be expected, as it is the difference between sets of independent samples, and is a reflection of both the variance and standard deviation. The distance beyond which points are no longer correlated is termed the range and is indicated by "a" in the figure.

A variety of theoretical models can be fit to experimental semi-variograms. Among these models are the spherical model, exponential model, and linear model. The model that best fit our experimental pavements data was the spherical model.

γ(h) = c[(3h/2a)–(h^3/a^3)/2] for h< = a

γ(h) = c for h> = a (Clarke 1979).

The "nugget effect" is sometimes exhibited in the semi-variogram. (Again the origin of the term is from mining (Journel and Huijbregts 1978).) Although close in space, the grades at z(x) and z(x+h) can vary substantially when one contains a nugget and the other doesn't. A distribution of nuggets would be seen on a semi-variogram as a steeply sloped line reaching the sill where "a" equals the nugget dimension. Should the range "a" be smaller than the smallest observation distance, the plotted semi-variogram would remain flat as h decreases and would not curve toward the origin. The microstructure within a macrostructure would tend to yield two ranges, and the variability could be modeled as nested: $\gamma(h) = \gamma_0(h)+\gamma_1(h)$ where $\gamma_0(h)$ corresponds to the small-scale nugget structure, $\gamma_1(h)$, the concentration of nuggets, and γ, the nested variability. In-situ lensing within subgrade material constitutes a nearly analogous pavement phenomenon (vertical spatial variability).

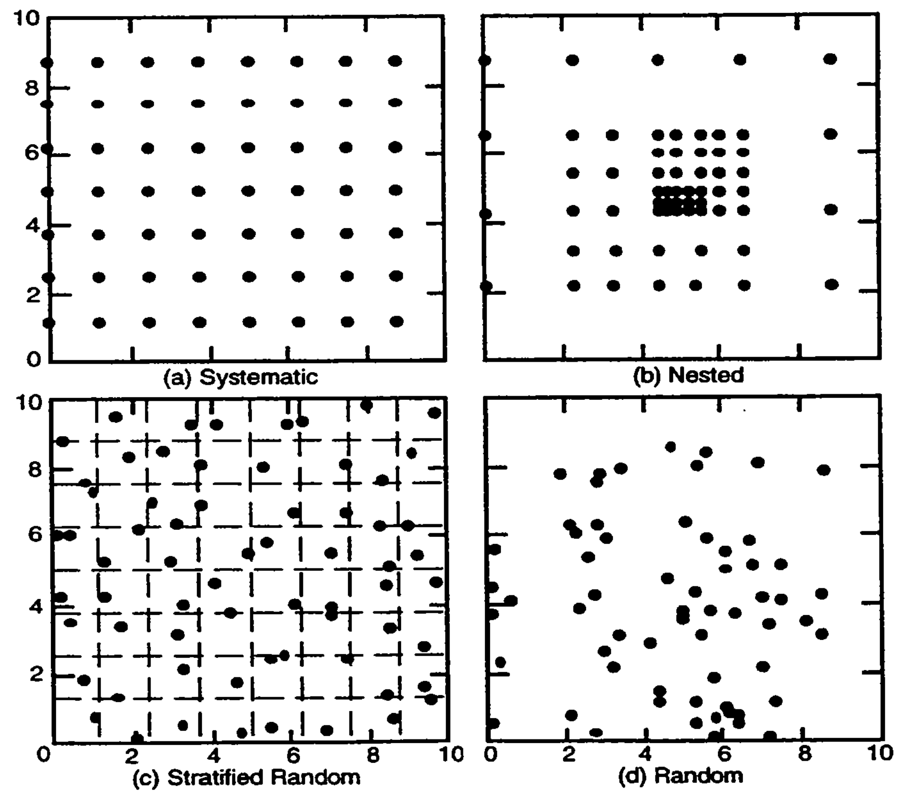

Figure 5. Test patterns (DeGroot and Baecher 1993).

TEST PATTERN / SAMPLING GRID

Since a primary objective of designing any site exploration program (or pavement design and evaluation program) is to gain the most information at the least expense, DeGroot and Baecher (1993) evaluated the four sampling plans shown in Figure 5. Although not shown in the figure, clustered, evaluated prior to DeGroot and Baecher's article, proved to be the least effective for kriging (an interpolation method to be discussed subsequently). The reason for its limited effectiveness is that information was provided at only small and large lag distances. Random sampling was reasonably good, and nested proved to be the best. Nesting was the sampling plan used for the MN/ROAD testing. It should also be noted that a systematic grid that includes a few samples on an even finer grid facilitates the study of nested structures and interpretations of any possible nugget effect (Journel and Huijbregts 1978).

KRIGING

There are a variety of methods that incorporate spatial relationships into estimating values at unsampled locations. Most methods utilize some algorithm by which weights are assigned to the samples based upon their distance from the location of the desired sample. Methods of weighting include inverse-distance, inverse distance squared, or perhaps an arbitrary constant estimating the range of influence minus the separation distance. Each method depends upon the geometric placement of the samples, not on sample values, or rich or poor/high or low zones. Kriging is essentially a stochastic interpolation procedure in which the variance between estimated and known values is minimized. Values at untested points can be estimated taking into account the correlation between adjacent points (Moore and McCabe 1989, Clarke 1979).

NONDESTRUCTIVE TESTING

FWD tests are conducted by applying an impulse load to the respective pavement layer surface though a 17.5-in.-diameter (45-cm) or 12-in-diameter (30-cm) circular plate. Seven sensors are spaced at specified distances from the center of the load and measure velocity. Maximum vertical deflections are determined by integration of the velocity-time history. Each FWD test point experienced a sequence of loads: one or more seating drops at the first (lowest) load level, followed by three additional drops, each at a series of increasing load levels (drop heights). The purpose of the seating drop is to allow for apparatus alignment, etc. While the influence of the seating drops was negligible (coefficients of correlations between deflections including and excluding seating drops = .99+), deflections corresponding to seating drops were not used in the statistical calculations on any raw or processed data. The cross-sectional area (through the center of the plate) bounded by the undeformed pavement surface and the deformed pavement surface is termed the deflection basin area and can be used as an indicator of pavement stiffness (Fig. 6).

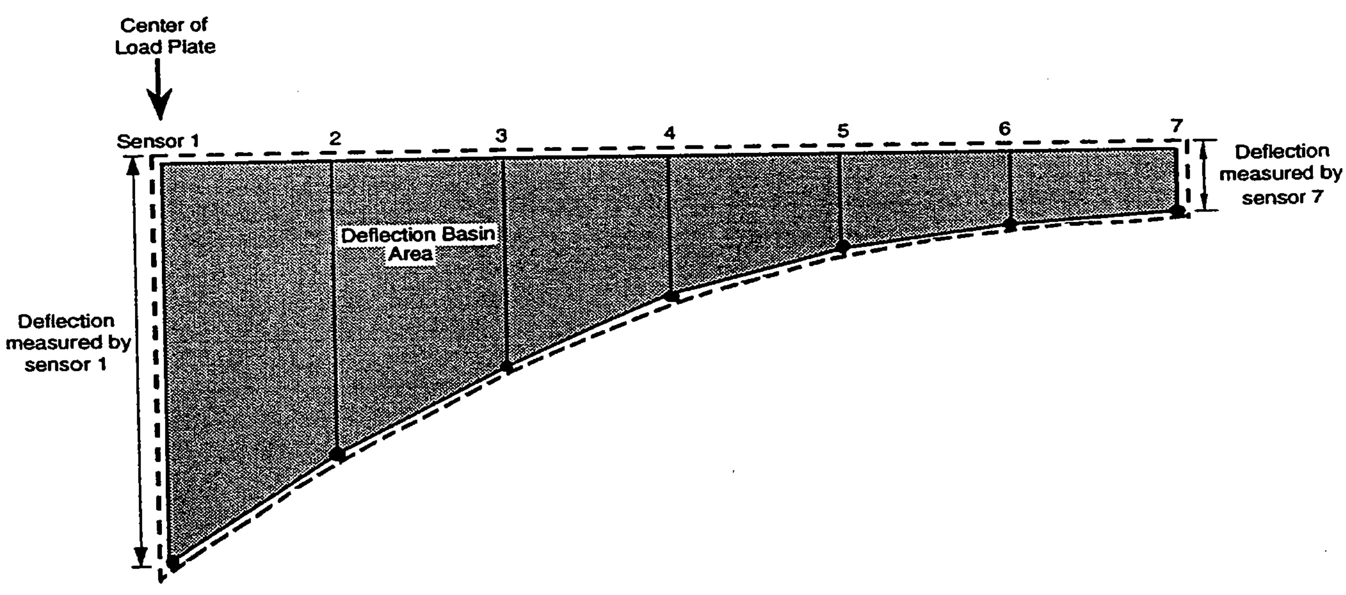

Figure 6. FWD deflection basin area.

Subgrade moduli were approximated from the various sensor deflections by simply assuming one layer theory. The pavement structure was assumed to be elastic, isotropic, and homogeneous. The equation for deflection as a function of the modulus can be expressed as:

$$\text{Defl} = [p(1+\mu)a/E][z/a*A+(1-\mu)H] \quad \text{(Yoder 1975)}$$

This can be reduced to:

$$E = [p(1+\mu)a/\text{Defl}][(1-\mu)H]$$

since the deflections are measured at the pavement surface.

Defl	=	Measured deflection in the center of the load.
μ	=	Poisson's ratio (assumed to be 0.4)
a	=	plate radius.
E	=	Modulus.
z	=	Depth beneath pavement surface.
A	=	Constant (Yoder 1975).
H	=	Constant (Yoder 1975).
p	=	Contact pressure.

DCP DATA

The Dynamic Cone Penetrometer (DCP) was used to determine an index of soil strength. The DCP index was correlated to a CBR strength value by the equation log CBR = 2.46–1.12(log DCP). The correlation was determined by Waterways Experiment Station (WES) based upon a database of field CBR versus DCP index values (Webster, Grau, and Williams 1992), since WES and MN/DOT use the same DCP.

STATISTICAL AND GEOSTATISTICAL ANALYSIS: MN/ROAD

Although the modulus and the deflection basin area are both considered to be good indicators of pavement stiffness, the following discussion deals primarily with analyses of only the deflection data simply because deflections are so quickly and easily determined in the field. Analogous analyses performed with deflection basin areas yielded similar results, although deflection basin area curves were generally smoother or more closely fit the data points than did deflection data. Rigorously, "raw" deflection data was not truly raw data. It was preprocessed inasmuch as average deflections for each load level were divided by average loads. The deflection per pound was referred to as the normalized deflection.

Figure 7a shows the normalized center deflection at all 71 FWD test points on the subgrade of main line test cell ML03 as a scatter plot. It appears that the subgrade toward the lower numbered stations is less stiff than that toward the higher numbered stations; nevertheless, there appears to be a great deal of scatter. The questions one asks are: "How can this spatial variability be quantified?" and "How frequently and where should FWD tests be conducted for pavement design?" (or, in this case, for pavement evaluation).

Figure 7b shows the variation in center deflections [normalized to 9000 LB (40 kn.)] along the centerline of the main line test cell 3 (ML03) (Kestler and Berg 1992). Deflections along the length of the test cell to either side of the centerline are combined with the above figure to yield Figure 7c. Although the deflections generally decrease with increasing station numbers, the variation in deflection offset from the centerline at station 111626.5 should be noted. Slightly more information can be extracted from the centerline normalized deflection histogram in Figure 8a. The distribution is noticeably skewed to the right due to the higher density of test points on one half of the test cell. Although the assumption of a normal distribution is deemed adequate, a beta distribution has been fit to the data and is shown in Figure 8b (Harr 1987).

One half the variance (determined from Equation 2) of the difference between normalized center deflections at increasing separation distances along the subgrade is shown by the variogram in Figure 9a. The experimental semi-variogram is depicted by the discrete points. A spherical model with range 35 and sill 13 was fit to the experimental data. Keeping in mind the earlier discussion, this semi-variogram indicates that there no longer exists any correlation between deflection values at any two points spaced 35 ft (11 m) or greater from each other. Had this been an existing pavement requiring rehabilitation, an FWD overlay testing grid of 50 ft (15 m) would have been inadequate for representative testing. The relatively small range over which the FWD test data was correlated was appreciably less than expected since the subgrade material was assumed to be fairly uniform based on the construction techniques used. Figures 9b and 9c show correlations on top of the base and asphalt concrete pavement to be 10 ft (3 m)

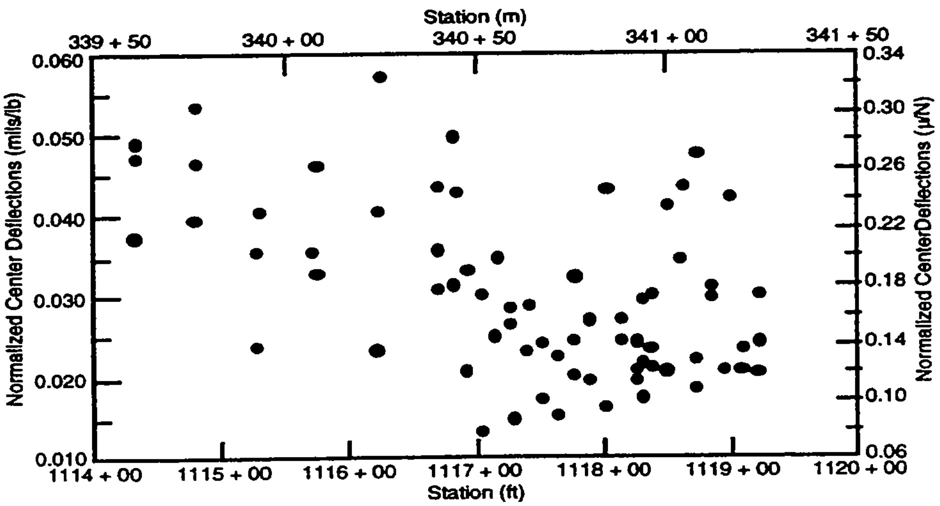

Figure 7a. Scatterplot of normalized center deflections, ML03 subgrade.

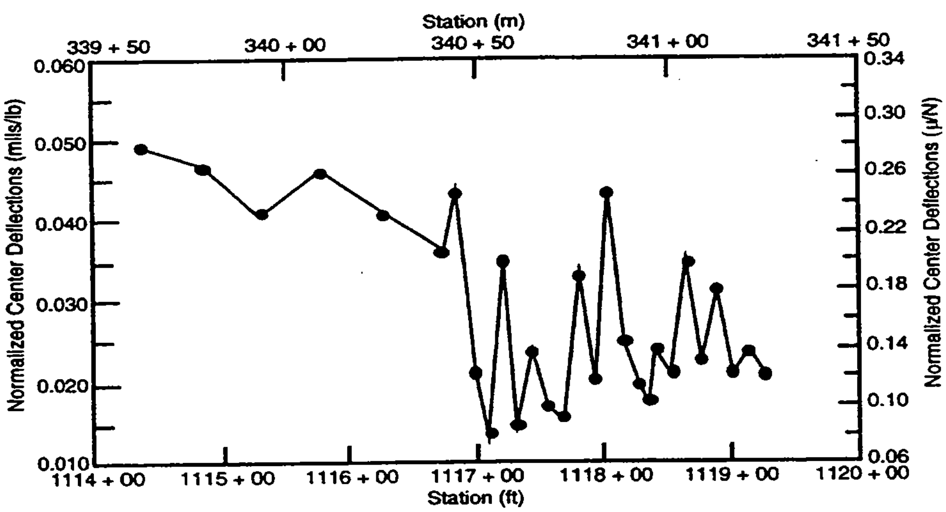

Figure 7b. Normalized FWD center deflections along centerline, ML03 subgrade.

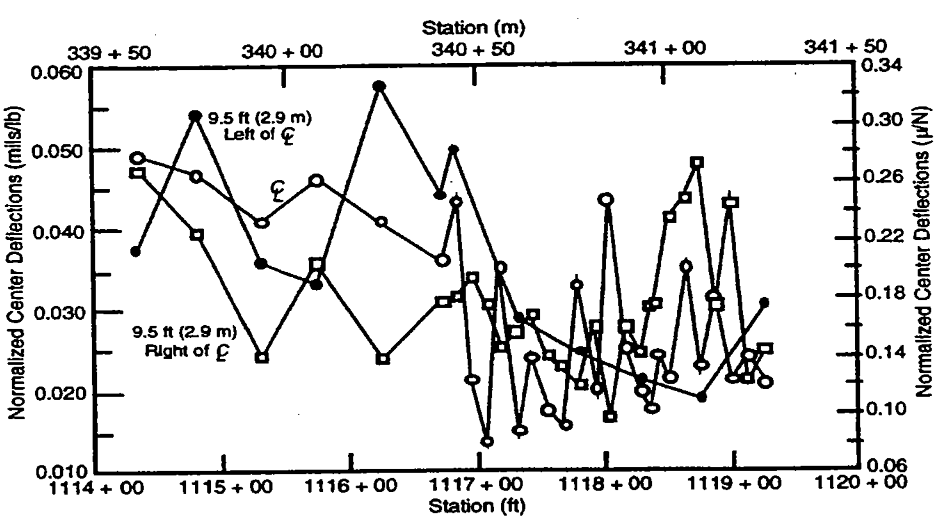

Figure 7c. Normalized FWD center deflections, ML03 subgrade.

and 100 ft (30 m), respectively. Ten ft (3 m) is extremely low; however, it is possible that a better contact is made on bituminous materials than on subgrade or base materials. Additional testing and analysis may confirm or negate this suspicion. In contrast to the short range indicated for the ML03 subgrade deflections, the semi-variogram corresponding to the subgrade in cell 27 of the low-volume road (LV27) exhibited a range of approximately 100 ft (30 m), as is shown in Figure 10. It should be noted that a similar geostatistical analysis (of deflection basin areas) at an airport in Lebanon, New Hampshire, resulted in correlations between areas up to 600 ft± (183 m±). This was the case during spring thaw as well as after recovery. The variation in correlated distances for different project sites is significant. The recommended number of FWD or Dynaflect measurements per highway mile for overlay designs typically range from three to fifty-five (Houston and Perera 1991). Guesswork could be virtually eliminated with a geostatistical analysis of the initial set of baseline data. Elimination of oversampling would reduce unnecessary consumption of valuable time, and elimination of undersampling could cut down on inadequate design and premature failure.

All semi-variograms discussed are unidirectional in the direction of traffic. The variograms in which the direction of the vector h was allowed to range from 0 to 90 degrees showed even smaller correlation distances than unidirectional.

The variances at different separation distances between pairs of both known and estimated pairs of points are shown in Figure 11. Note the zero variances for known values at actual FWD test points and the magnitude of variances of 4-ft (1.2-m) separation distances as opposed to 12-ft (3.7-m) separation distances. These are the variances determined during the process of kriging (S. Yates and M. Yates 1990).

Future testing programs should include a larger number of tests at very small separation distances.

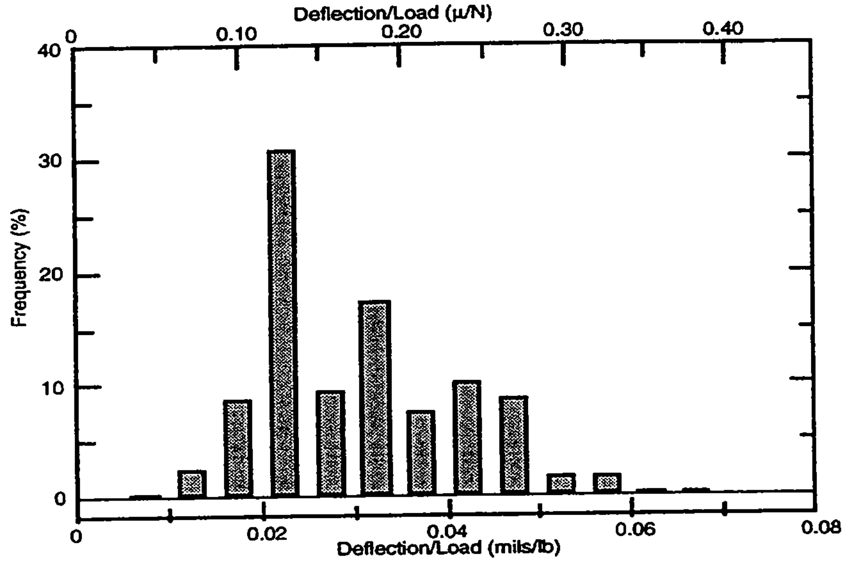

Figure 8a. Histogram of normalized FWD center deflections, ML03 subgrade.

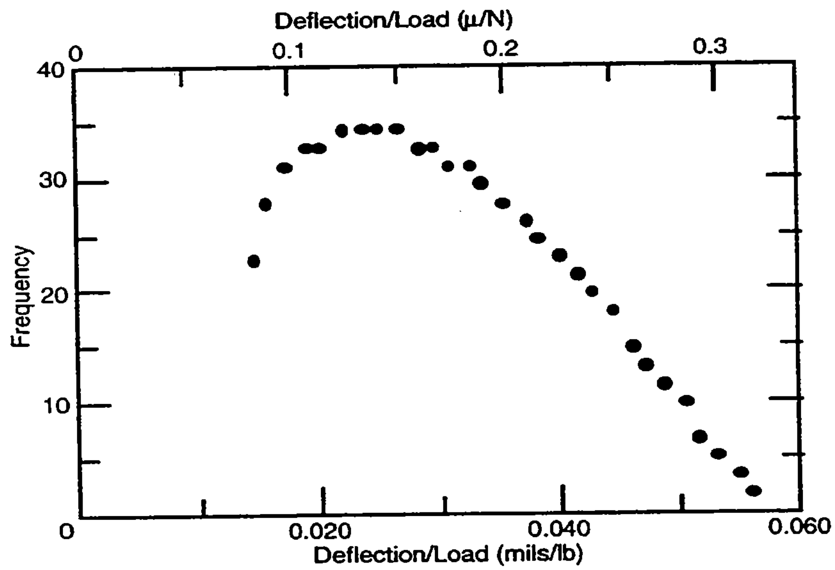

Figure 8b. Beta distribution of normalized FWD center deflections, ML03 subgrade.

Due to an absence of such tests, it could not be determined whether MN/ROAD data exhibited the nugget effect. An additional recommendation for future testing would include conducting repeated tests at selected points. This would enable the separation of system variability from spatial variability, in general, and from the nugget effect, in particular.

Based upon DCP data, the vertical spatial variability was also assessed for each of the test cells. Kriged CBR values are shown for ML03 and LV27 in Figures 12a and 12b, respectively. While ML03 appears to exhibit extreme vertical variability, LV27 appears to be relatively uniform. This is in conformity with the results of the FWD semi-variograms for horizontal spatial variability discussed previously.

MULTIVARIATE ANALYSIS

Multivariate analysis constitutes a simple procedure for gaining stochastic information from any number of random variables (test measurements) (Harr 1991).

The preprocessed and processed data resulting from FWD, DCP, and soil moisture testing are all suspected to serve as either an indicator of, or contributor to, pavement performance. A matrix consisting of both raw and processed data was established. Each row included 1) direct deflection measurements at each of the seven FWD sensors, 2) normalized deflections at each of the sensors, 3) the deflection basin area corresponding to a 9000-LB (40-kN.) load, 4) modulus values based upon the one layer theory as discussed previously (Yoder 1975), 5) backcalculated modulus values obtained by Newcomb and Van Deusen (1994), 6) soil moisture contents (determined in the laboratory from samples) at increasing depths beneath the subgrade surface, 7) weighted averages of CBR values from the subgrade surface to a variety of depths, and 8) weighted averages of CBR values over five-in.-thick layers of subgrade.

Due to the limited number of moisture content tests performed and the non-coinciding locations of all tests, the first analysis includes a larger number of variables at or near fewer test points. The second analysis includes fewer variables, but at 11 test points.

The procedure (Harr 1991) is as follows: Columns are labeled by the values of interest.

E[x] = (1/n) * summation xi, i=1,2,3.... The expected value is determined for each column.

D = Mean-Corrected matrix = the n x m matrix of average loads, average areas, etc. minus the expected value of each. n is the number of test points and m is the number of variables.

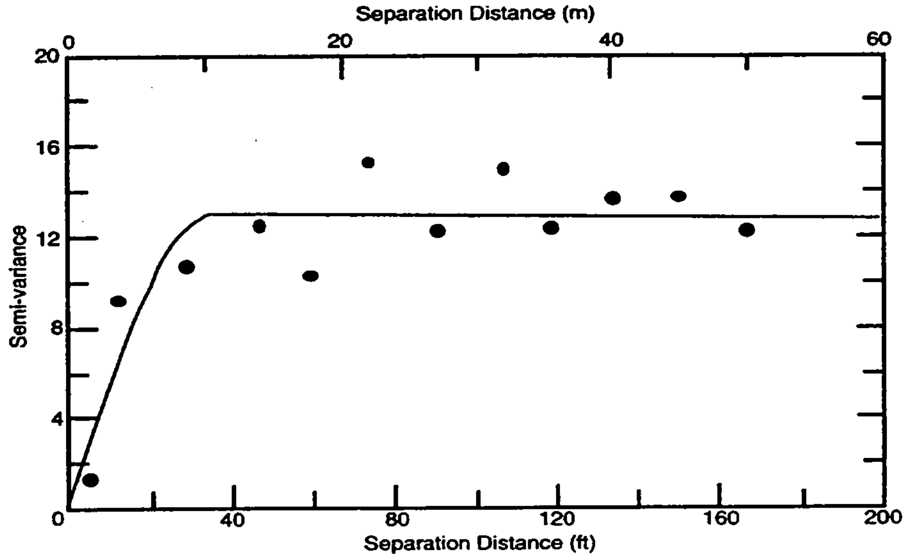

Figure 9a. Variogram of normalized FWD center deflections, ML03 subgrade.

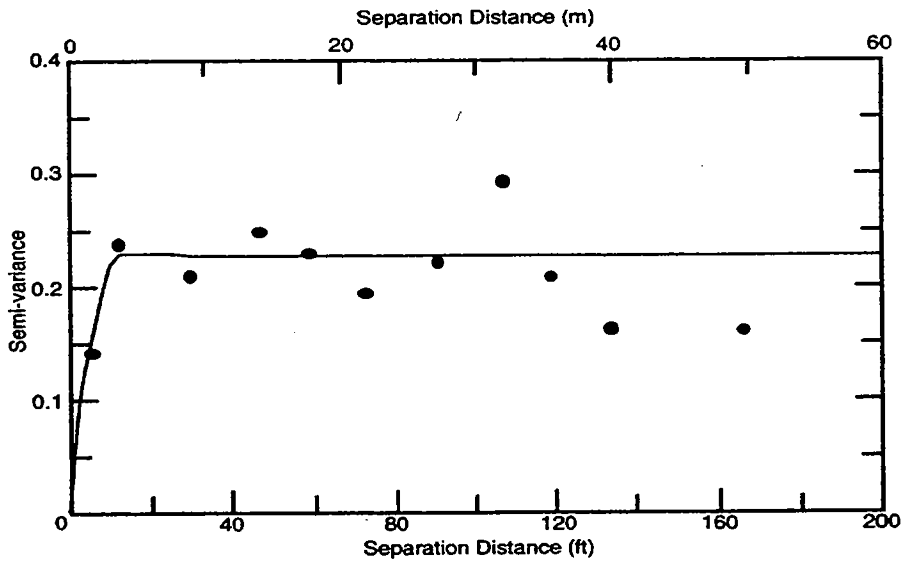

Figure 9b. Variogram of normalized FWD center deflections, ML03 base.

D' = the m x n transpose of the matrix D.

D'D/(n–1) = D' multiplied by D divided by (count – 1) = the covariance matrix. At this stage, the elements on the principal diagonal are the variances corresponding to the initial variables x_i, x_2, and the off-diagonals are the variances.

K = m x m correlation matrix. Since the covariance in the above matrix is defined as $(s_1) \times (s_2) \times (r_{12})$ where s is the standard deviation and r is the correlation coefficient between x_1 and x_2, dividing each element of the covariance matrix by the respective standard deviations will result in this symmetrical correlation matrix.

The matrix algebra simply provides a shortcut method of calculating the coefficients of correlation between every possible pair of parameters considered. The equation for any one correlation coefficient reduces to the standard equation:

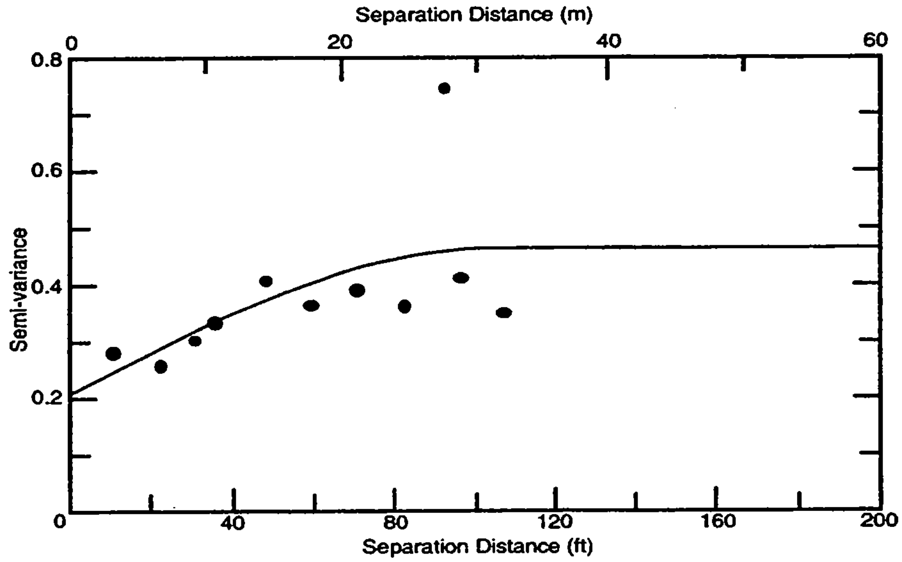

Figure 9c. Variogram of normalized FWD center deflections, ML03 asphalt concrete surface.

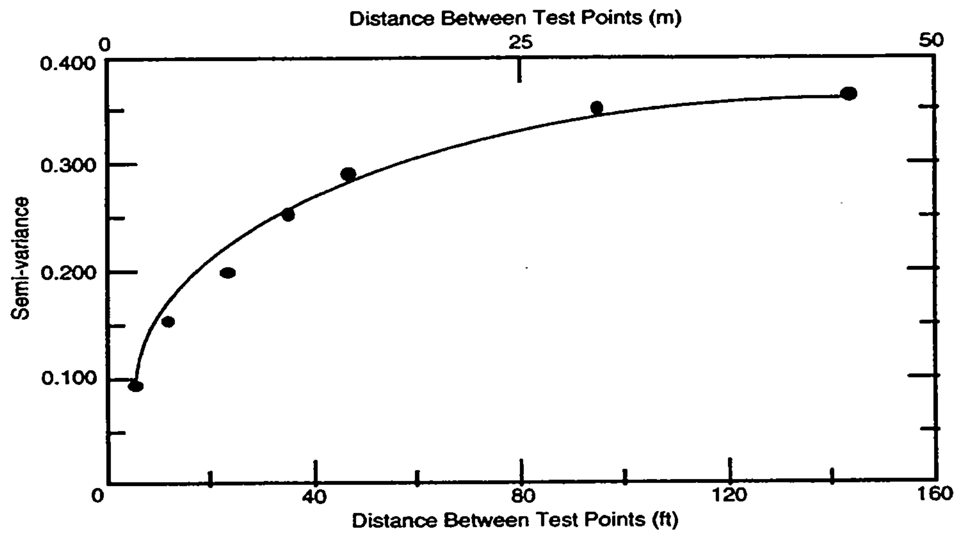

Figure 10. Variogram of normalized FWD center deflections, LV27 subgrade.

$$r = [1/(n-1)]\ [(x-\bar{x})/s_x][(y-\bar{y})/s_y]$$

where $\bar{x},\bar{y}$ = expected value,

n = the number of x or y points, and

s_x, s_y = the standard deviations of x and y respectively.

The matrix manipulation that included test results for soil moisture contents showed the deflection basin area t be most closely correlated to the deflection measured at the second sensor. Although this is clearly dependent upc sensor spacing and the pavement cross-section, this is in conformity with results from several other pavements ol

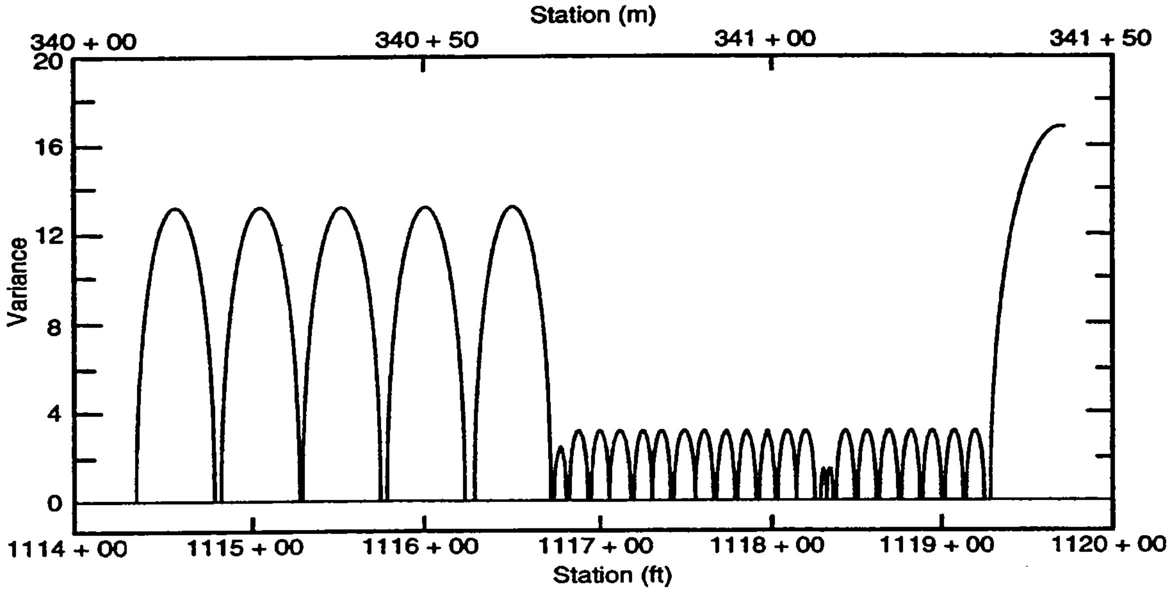

Figure 11. Deflection variances determined during kriging.

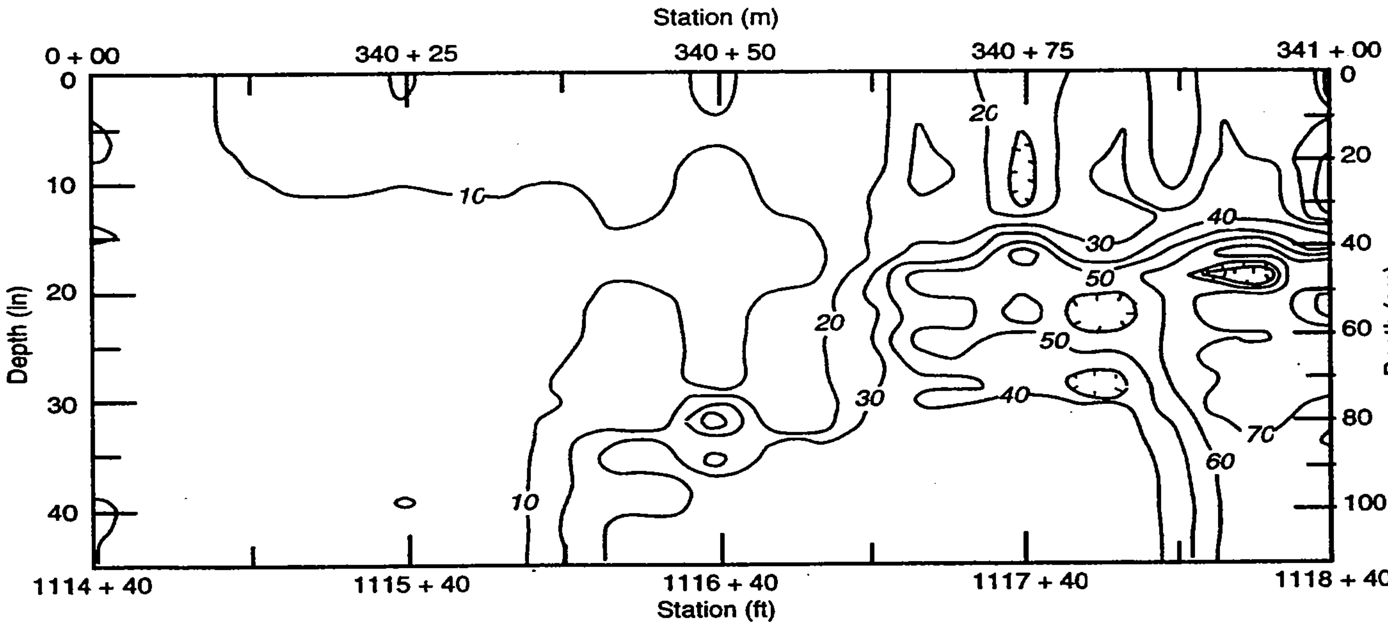

Figure 12a. ML03 subgrade CBR values from DCP tests determined by kriging.

served by the authors. Of particular interest was the coefficient of correlation of –0.99 between the deflection basin area and the weighted average of CBR values to a depth of 20 inches. The CBR correlated reasonably well with most of the deflections as measured.

Although a few coefficients of correlation between water contents and CBR were in the range of –.96 through –.98, correlations between water content and most variables were generally poor. It is believed that correlations would have been improved had the moisture content, DCP, and FWD test points been coincident. A 10-ft (3-m) separation (maximum separation between moisture content and FWD point) is already well into the transition section of the ML03 subgrade variogram for FWD deflections.

The second multivariate analysis, which omitted soil moisture content tests, included more test points but did not yield better results. In this analysis the third, fourth, and fifth sensors exhibited the best correlations with deflection basin areas. Again, coincident tests points for the entire range of tests would probably improve results.

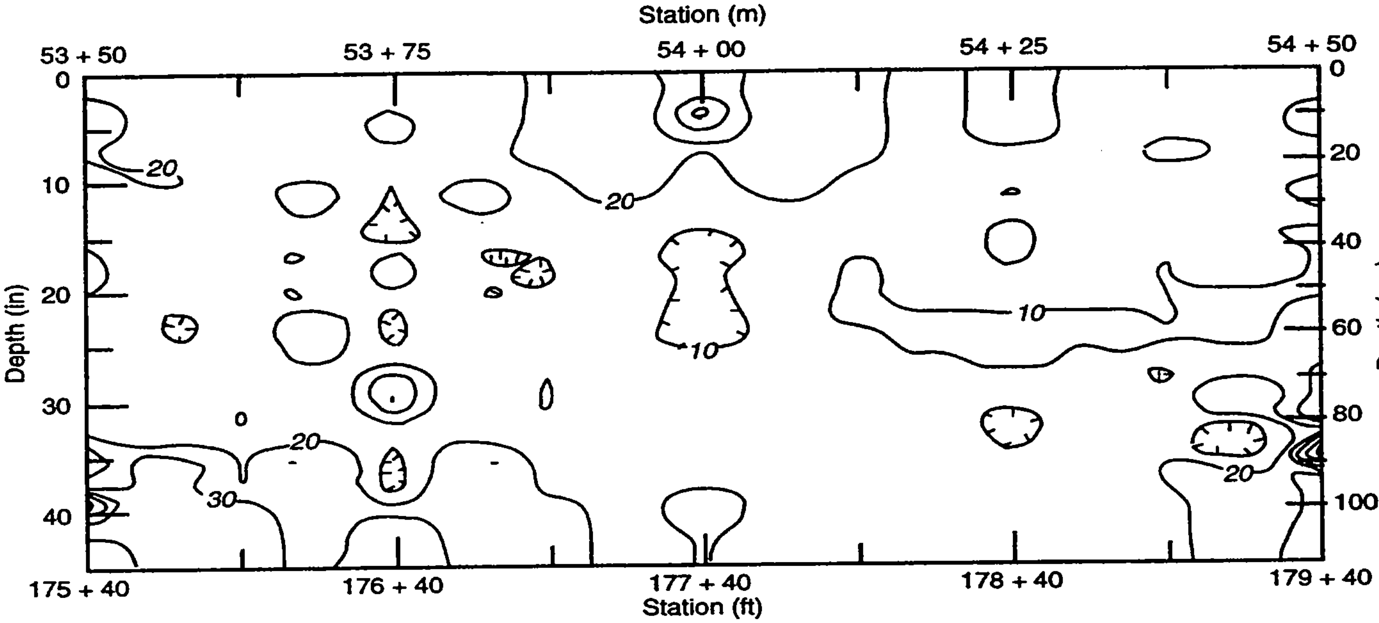

Figure 12b. LV27 subgrade CBR values from DCP tests determined by kriging.

CONCLUSION

Although an extremely powerful tool for analysis, classical statistics tends to neglect the relative positions of samples or test points. Geostatistics, on the other hand, enables one to quantify the spatial variability of a site. Although primarily a mining tool, it has just been shown that the geostatistical semi-variogram model can be readily adapted to pavements, as it incorporates the concept of similarity, which is a function of the separation distance between two points. The model serves as a summary of all available structural information and allows extraction of even more information. Knowledge of the lengths over which any particular indicator value is correlated can translate into substantial cost savings. Sampling strategies can be developed, and optimum FWD test point spacing can be determined from a baseline data variogram for pavement evaluation and overlay design.

Frequently a value midway between two points is assumed to equal the average of the values corresponding to the two points. This, however, is true only if the points are independent of each other. Kriging, a stochastic interpolation procedure, takes into account the correlation between the two points. Furthermore, the prediction error is known.

With multivariate analysis, the correlation structure among any number of variables can be determined relatively quickly. The integration of the resulting correlations with variogram analysis, i.e., a blending of classical statistics with geostatistics, can result in even greater cost savings for private firms, Federal Government, or State Departments of Transportation. A series of simple and economic tests can determine the points about which a variety of tests should be performed and can possibly substitute for complex, costly, and/or time consuming tests necessary for design and evaluation.

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