Life-Cycle Cost Analysis using Deterministic and Stochastic Methods: Conflicting Results

Tarek M. Zayed, S.M.ASCE[1]; Luh-Maan Chang, M.ASCE[2]; and Jon D. Fricker, M.ASCE[3]

Abstract: Life-cycle cost analysis is an essential approach to differentiating alternative rehabilitation strategies for steel bridge paint systems. An economic analysis (EA), which is a deterministic method, and the Markov decision process (MDP), which is a stochastic method, were used to carry out the life-cycle cost analysis. These analyses were applied to data from two state Departments of Transportation. The deterioration curves for steel bridge paint condition rating against age were constructed. Different rehabilitation scenarios were proposed for steel bridge paint. The EA and the MDP were used to analyze and differentiate among the proposed rehabilitation scenarios. The results of the EA were different from those of MDP for the two data sets. MDP favored the "do nothing" scenario until the end of paint life and then a complete repainting. EA indicated that the scenario "do spot repairs at state 3 of the paint life" and repeat that until the end of the bridge life was superior. The results were analyzed to determine the reason for the conflict.

DOI: 10.1061/(ASCE)0887-3828(2002)16:2(63)

CE Database keywords: Stochastic models; Life-cycle cost; Bridges, steel; Painting; Cost analysis.

Introduction

The application of life-cycle cost analysis to the maintenance coating of steel bridges represents a major departure from the practice of basing coating decisions on the lowest initial cost. An economic analysis (EA), which is a deterministic method, and the Markov decision process (MDP) which is a stochastic method, can be used in a variety of situations. A major problem becomes apparent upon a careful examination of the data that feed either model. The cost and performance data are highly variable, and considerable doubt exists about the validity of comparisons among different sources. The reasons for these problems are many, for instance, the cost components are seldom available in adequate detail and format. Typically, bridge protection cost data are in a lump-sum dollar amount that may be transformed into a unit cost per area if the surface area is provided. The Indiana Dept. of Transportation (INDOT) expresses coating costs as a cost per ton of steel. Given the wide range of geometry used in bridge construction, conversion of cost per ton to cost per square foot is difficult.

Another major factor in the variability of the cost data is the presence of hidden costs. For example, two similar bridges with the same surface preparation and coating system that have different accessibility limitations are considered. In one case, access for painting is unrestricted, while in the other case, painting can only be performed during periods of minimal traffic disruption. For the first bridge, scaffolding and containment can be erected and left in place until the job is complete, while the second bridge requires that scaffolding and containment be assembled and disassemble before and after each daily painting period. If all other costs remain identical, these two jobs could easily differ greatly in cost per square foot. Another source of hidden costs results from typical contractor practices that stem from cash-flow problems. Performance data also are badly clouded. The judgment as to when a coating system has failed can be very subjective, particularly if no uniform quantitative standards exist for judging paint failure (Scherer and Glagola 1993).

A performance function or deterioration curve is the relationship between the bridge paint condition rating and the age that reflects the level of service of that paint. The performance functions for steel bridge paint in the INDOT were developed using regression, as a deterministic method, and Markov chains, as a stochastic method. The probabilistic model that was developed with Markov chains was used to reflect the stochastic nature of bridge paint conditions. The Markov model can be used to predict the condition rating of bridge paint at a given age. Based on these performance curves, life-cycle cost analysis was performed using two different methods. These methods were the EA, based on regression, and the MDP, based on Markov chains. The EA used the present value (PV) and the equivalent uniform annual cost (EUAC) as tools for selecting the best rehabilitation scenario. The MDP used the policy improvement technique algorithm (PITA) to select the best rehabilitation scenario (Hillier and Lieberman 1986).

This paper is the third in a series of three dependent papers that complement each other. The first paper discusses the deterioration models for steel bridge paint systems in the INDOT. The second paper discusses the INDOT maintenance plan for steel bridge paint systems based on life-cycle cost analysis considering the deterioration models in the first paper. This paper discusses

[1]Assistant Professor, Construction Engineering and Management Dept., Faculty of Engineering, Zagazig Univ., Egypt; presently, Visiting Assistant Professor, Civil and Environmental Engineering Dept., Univ. of Cincinnati, Cincinnati, OH 45219.

[2]Associate Professor, Construction Division, School of Civil Engineering, Purdue Univ., 1284 Civil Building, West Lafayette, IN 47907.

[3]Professor, Transportation and Infrastructure Systems Engineering Area, School of Civil Engineering, Purdue Univ., 1284 Civil Building, West Lafayette, IN 47907.

Note. Discussion open until October 1, 2002. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on February 26, 2001; approved on October 26, 2001. This paper is part of the ***Journal of Performance of Constructed Facilities***, Vol. 16, No. 2, May 1, 2002. ©ASCE, ISSN 0887-3828/2002/2-63–747474/$8.00+$.50 per page.

the conflict between the deterministic and the stochastic methods as applied to this study.

Study Objectives

A major goal of this study was to develop economic models that can be used to provide a rational framework for the evaluation of alternatives in the paint maintenance of steel bridges. An extensive study of steel bridge maintenance practices was conducted. Based on data and experience from the bridge paint maintenance study, models were formulated and their input data were provided. Because the results of the deterministic and probabilistic methods were quite different, an analysis was conducted to determine the reasons.

Deterioration Models

The deterioration models that the life-cycle cost analysis was based upon were built in the first paper of this series of papers. The results of these models were used to calculate the life-cycle costs of the different paint types using the deterministic (EA) and stochastic (MDP) techniques. Consequently, this paper is a complement to the first paper, representing the deterioration models, which play an essential role in the life-cycle cost calculation.

Stochastic Method (Markov Decision Process)

Managers need to select the optimal rehabilitation action for each paint condition. The action that is selected from several alternatives affects the transition probabilities in the Markov chains model, as well as their initial and subsequent costs. The decision process for selecting the optimal actions for the respective states of paint when considering initial and subsequent costs is referred to as the MDP (Abraham et al. 1998; Wirahadikusumah 1998).

Life-cycle cost analysis for maintaining or rehabilitating the paint on steel bridges can be studied by using the MDP. Life-cycle cost analysis is performed on the basis of the long-term behavior of the paint systems. As the number of planning periods n (years) approaches infinity, there is a limiting probability (called the steady-state probability) that the system will be in state j after a large number of transitions, where this probability is independent of the initial state i. This long-term behavior holds under relatively general conditions and it holds for steel bridge paint rehabilitation problems as well (Abraham et al. 1998; Wirahadikusumah 1998).

Because the long-term behavior of a Markov chain exists for steel bridge paint, the rehabilitation problem can be solved using the assumption that the MDP will be operating indefinitely. The steps of that MDP can be summarized as follows (Hillier and Lieberman 1986; Abraham et al. 1998; Wirahadikusumah 1998):

1. There are five possible states (i=1,2,3,4,5) observed after each transition that are associated with the steel bridge paint condition rating, from state 1 to state 5, with 5 being the worst. The index i is used for the initial state, and the index j is used for the future state. Therefore, state 1 corresponds to condition rating 9, state 2 corresponds to condition rating 8, state 3 corresponds to condition rate 7, etc.;
2. After each observation, a decision (action) k is chosen from a set of K possible decisions (k = 1,2,3,4). Some of the K decisions may not be relevant for some of the states. Table 1 indicates the decisions and their relevant states. For example, decision $k = 1$ (do nothing) is relevant to states 1, 2, 3, or 4 and decision $k = 4$ (complete repainting) is only relevant for state 5;
3. If decision $di=k$ is made in state i, an initial cost is incurred that has an expected value Cik. The costs in Table 2 are the estimated unit costs ($/ton) for steel bridge paint types 1 and 2 that are analyzed based on the INDOT standards. Paint type 1 is lead-based, while type 2 is zinc/vinyl;
4. Decision $di=k$ in state i determines what the transition probabilities will be for the next transition from the initial state i to the future state j. These transition probabilities can be denoted by $P_{ij}(k)$ for i and j=1,2,3,4,5. The parameter k in $P_{ij}(k)$ is used to indicate that the appropriate transition probability depends upon the decision k; and
5. There are several policies that can be used to guide steel bridge paint rehabilitation actions according to paint rating. A policy is a set of decisions (actions) for each paint rating. Table 2 indicates the proposed policies or scenarios and the associated decisions with each rating.

Table 1. Decisions and Relevant States (Indiana Dept. of Transportation Data Set)

Decision k	Action description	Relevant to states
1	Do nothing	1,2,3,4
2	Spot repair	2,3,4
3	Overcoating	3,4
4	Complete repainting	5

In Table 3, policy $Y1$: $(d_1,d_2,d_3,d_4,d_5)=(1,1,1,1,4)$ describes a policy where decision 1 (do nothing) is made in states 1,2,3,4 and decision 4 (complete repainting) is made in state 5. The following transition probability matrix is assumed to be the matrix that corresponds to policy $Y1$. This assumption is made for illustration purposes only:

$$P1=\begin{pmatrix} 0.80 & 0.20 & 0 & 0 & 0 \\ 0 & 0.30 & 0.50 & 0.20 & 0 \\ 0 & 0 & 0.1 & 0.5 & 0.4 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The long-term behavior of $P1$ is reached after seventeen transitions. It is noticed that the five rows have identical entries. This implies that the probability of being in state j after 17 periods appears to be independent of the initial paint condition:

$$(P1)^{17}=\begin{pmatrix} 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \\ 0.362 & 0.226 & 0.191 & 0.143 & 0.078 \end{pmatrix}$$

Table 2. Different Proposed Policies for Painting Rehabilitation (Indiana Dept. of Transportation Data Set)

di	Policy ($Y1$)	Policy ($Y2$)	Policy ($Y3$)	Policy ($Y4$)	Policy ($Y5$)
$d1$	1	1	1	1	1
$d2$	1	1	1	1	2
$d3$	1	2	3	1	1
$d4$	1	1	1	3	3
$d5$	4	4	4	4	4

Table 3. Estimated Unit Cost of Paint Rehabilitation Scenarios (Indiana Dept. of Transportation Data Set)

Decision K	State i	Description	Rehabilitation cost $[C_{ik}$ (\$/ton)] (no disruption costs) at paint types 1	2
1	1,2,3,4	Do nothing	0.0	0.0
2	2	Spot repair at state 2	25	20
	3	Spot repair at state 3	50	40
	4	Spot repair at state 4	90	75
3	3	Overcoating at state 3	110	100
	4	Overcoating at state 4	180	150
4	5	Complete repainting	220	180

Because the long-term behavior of Markov chains exists for each of the five relevant policies, paint rehabilitation problems can be solved using the assumption that the MDP will be operating indefinitely.

6. The objective is to find an optimal policy that minimizes the expected total discounted cost (initial cost and discounted subsequent costs that result from the future processes).

Minimizing Total Expected Discount Cost

Given a distribution $\mathbf{P}\{X_0=i\}$ over the initial states of the system and a policy Y, a system evolves over time according to the sequence of decisions made (actions taken). In particular, when the system is in state i and decision $d_i(Y)=k$ is made, then the probability that the system is in state j at the next observed time period is given by $p_{ij}(k)$. Furthermore, a known expected cost C_{ik} is incurred. The expected total discounted cost of a system starting in state i (at the first observed time period) and evolving for n time periods is denoted by $V_i^n(Y)$ as shown in the following recursive formula (Hillier and Lieberman 1986; Abraham et al. 1998; Wirahadikusumah 1998):

$$V_i^n(Y)=C_{ik}+f\sum_{j=0}^{M} p_{ij}(k)V_j^{n-1}(Y) \quad \text{for } i=0,1,2,\ldots,M \tag{1}$$

where C_{ik}=cost incurred at the first observed time period as a result of the current state i and the decision $d_i(Y)=k$ when operating under policy Y; and $f\Sigma_{j=0}^{M}p_{ij}(k)V_j^{n-1}(Y)$=expected total discounted cost of the process evolving over the remaining $n-1$ time periods. A discount factor $f<1$ is specified, so that the present value of 1 unit of cost m periods in the future is f^m. The discount factor f can be interpreted as equal to $1/(i+1)$, where i is the current interest rate.

This policy can be evaluated using the techniques associated with dynamic programming. It can be shown that, as n approaches infinity, this expression converges to the formula in Eq. (2):

$$V_i(Y)=C_{ik}+f\sum_{j=0}^{M} p_{ij}(k)V_j(Y) \quad \text{for } i=0,1,2,\ldots,M \tag{2}$$

where $V_i(Y)$ can now be interpreted as the expected long-term total discounted cost for a system starting in state i and continuing indefinitely. There are $M+1$ equations and $M+1$ unknowns; hence, $V_i(Y)$ may be obtained by standard methods of solving simultaneous equations.

Policy Improvement Technique Algorithm

The steps for the policy improvement technique algorithm (PITA) are as follows (Hillier and Lieberman 1986; Abraham et al. 1998; Wirahadikusumah 1998):

1. *Value determination:* For an arbitrary chosen policy R_1, use $p_{ij}(k_1)$ and C_{ik1} to solve the set of $M+1$ equations for all $(M+1)$ unknown values of $V_i(Y_1)$, as shown in the following:

$$V_i(Y_1)=C_{ikl}+f\sum_{j=0}^{M} p_{ij}(k_1)V_j(Y_1) \quad \text{for } 1=0,1,2,\ldots,M \tag{3}$$

2. *Policy improvement:* Using the current values of $V_i(Y_1)$, find the alternative policy Y_2 such that, for each state i, $d_i(Y_2)=k_2$ is the decision that makes Eq. (4) a minimum:

$$C_{ik2}+f\sum_{j=0}^{M} p_{ij}(k_2)V_j(Y_1) \quad \text{for } i=0,1,2,\ldots,M \tag{4}$$

Hence, for each state i, the optimization model that finds the appropriate value of k_2 can be represented by the following:

$$\text{Minimize } \left\{ C_{ik2}+f\sum^{M}{}_{ij}\,(k_2)V_j(Y_1) \right\} \quad \text{for } i=0,1,2,\ldots,M \quad k_2=1,2,\ldots,k \quad j=0 \tag{5}$$

3. Then, set $d_i(Y_2)$=minimizing the value of k_2. This procedure defines a new policy Y_2.
4. If Y_2 does not equal Y_1, then return to step 1 by using Y_2 instead of Y_1, and solve for $V_i(Y_2)$, $i=0,1,2,\ldots,M$. Using these values, go to step 2 and find Y_3.
5. Continue in this fashion until one finds two successive Y's to be equal. When they are found, the optimal policy is achieved, and the algorithm terminates.

Application of Markov Decision Process to Indiana Dept. of Transportation Data Set

The MDP procedure was applied to steel bridge paint according to the proposed policies and their costs. One transition probability matrix is estimated from performance data for each four-year period or range of paint age. These four-year ranges are selected to facilitate the calculation of probabilities and the optimization procedure in the Markov chains method. For more information about this range and the selection of four-years, the reader is referred to Zayed and Fricker (1999). This transition probability matrix is changed or enhanced due to different decisions that can be taken at each paint state and according to different policies. Consequently, the PITA algorithm that is explained in the previous section should be repeated in each four-year range or period to decide the optimal rehabilitation policy at each range of paint life. The transition probability matrices were built according to the deterioration curves generated by the Markov chains method. These matrices are indicated in Zayed and Fricker (1999).

Based on the transition probability matrix and the costs associated with a proposed policy according to Table 3, five equations with five unknowns $V_i(Y)$ could be solved to get the values of the estimated costs at each state for the same four-year range of the paint life cycle. These five equations are part of the *policy improvement* step of PITA. The results indicate whether this policy is suitable for this four-year range. If it is not, another policy is applied and checked and the process is repeated. If it is accepted, then this is the optimal policy for that range. This procedure is repeated for each range of the paint life cycle. Consequently, a

Table 4. Application of Markov Decision Process to I1 (0–4 Years Range)

Values of 5 Unknowns		Transition Probability Matrix						
V1	*0.3284*		*0.943*	*0.057*	*0*	*0*	*0*	
V2	*0.9686*		*0*	*0.768*	*0.232*	*0*	*0*	
V3	*1.4326*		*0*	*0*	*0.701*	*0.299*	*0*	
V4	*1.965*		*0*	*0*	*0*	*0.999*	*0.001*	
V5	*220.3*		*1*	*0*	*0*	*0*	*0*	
State 1		**Probability * Values of 5 Unknowns**					***Summation***	***Minimum***
K	***C1k***	***P1*V1***	***P2*V2***	***P3*V3***	***P4*V4***	***P5*V5***	***Value***	***Selection***
1	0	0.309681	0.05521	0	0	0	0.277	**1**
State 2							***Summation***	***Minimum***
K	***C2k***	***P1*V1***	***P2*V2***	***P3*V3***	***P4*V4***	***P5*V5***	***Value***	***Selection***
1	0	0	0.743885	0.332363	0	0	0.817	**1**
2	25	0	0.9686	0	0	0	25.736	
State 3							***Summation***	***Minimum***
K	***C3k***	***P1*V1***	***P2*V2***	***P3*V3***	***P4*V4***	***P5*V5***	***Value***	***Selection***
1	0	0	0	1.004253	0.587535	0	1.209	**1**
2	25	0	0	1.21771	0.29475	0	26.149	
3	110	0	0	1.4326	0	0	111.088	
State 4							***Summation***	***Minimum***
K	***C4k***	***P1*V1***	***P2*V2***	***P3*V3***	***P4*V4***	***P5*V5***	***Value***	***Selection***
1	0	0	0	0	1.963035	0.2203	1.658	**1**
2	90	0	0	0	1.965	0	91.492	
3	180	0	0	0	1.965	0	181.492	
State 5							***Summation***	***Minimum***
K	***C5k***	***P1*V1***	***P2*V2***	***P3*V3***	***P4*V4***	***P5*V5***	***Value***	***Selection***
4	220	0.3284	0	0	0	0	220.249	**4**
The Result								
State I	1	2	3	4	5			
Decision K	*1*	*1*	*1*	*1*	*4*			
Policy Y1	*1*	*1*	*1*	*1*	*4*			

maintenance plan based on the optimal policy can be planned for each bridge paint life cycle.

Based on the estimated costs that are shown in Table 2, the optimal policy was calculated using the MDP for each paint type. This study presents different maintenance policies for two paint types that exist in the INDOT. They are paint type 1 (lead-based) and paint type 2 (zinc/vinyl). The available data set was divided according to four paint categories: interstate paint types 1 and 2 (I1 and I2) and state paint types 1 and 2 (S1 and S2). For I1, I2, S1, and S2, the policy $Y1$ is the optimal one for all the ranges along the life of each paint type. This policy proposes ''do nothing at states 1, 2, 3, and 4 and do complete repainting at state 5.'' An example of MDP application to the first four-year range in I1 is shown in Table 4.

Table 4 describes the steps of the MDP application to I1 for the life range from 0 to 4 years of paint age. The top of this table shows the transition probability matrix that resulted from the Markov chains method application on I1. This transition probability matrix includes the probabilities of the paint being at any state during the four-year range. The corresponding states for this matrix are 1–5. The values of the 5 unknowns $V_i(Y)$ that result from solving the five equations of PITA are shown to the left of the transition probability matrix. Each $V_i(Y)$ value represents the cost value corresponding to its state i, where $V1$ corresponds to state 1, $V2$ corresponds to state 2, and so on. For each state, all the corresponding decisions ki are represented in their specific columns. For each decision within the same state, the model in Eq. (3) should be used to calculate Vi. Therefore, the cost C_{ik1} is estimated and substituted in its column, where $p_{ij}(k_1)V_j(Y_1)$ is calculated by multiplying the values of $V_i(Y)$ by the corresponding probabilities from the transition probability matrix. After multiplying each of the five values of $V_i(Y)$ by the corresponding probability, the resulting products are summed. The decision that corresponds to the minimal value of the equation above is selected. Consequently, the values for the selected decisions ki are 1, 1, 1, 1, and 4, which correspond to the states 1, 2, 3, 4, and 5 as shown in bottom of Table 4, respectively. On the other hand, the selected policy $Y1$ was ''do nothing'' until the end of paint life (state 5). This result recommends that the decision makes use decision 1 until state 5 is reached, and then use decision 4. As a result, the PITA algorithm has confirmed policy $Y1$, because it produces the same decisions as the PITA-selected policy. Therefore, policy $Y1$ is the selected minimal cost policy.

The same technique was applied for all the four-year ranges of I1, I2, S1, and S2. The best policy that resulted from the PITA algorithm was policy $Y1$ in all the four-year ranges for each paint type. Policy $Y1$ proposes ''do nothing'' until the end of paint life, and then completely repaint.

Table 5. Decisions and Relevant States (Michigan Dept. of Transportation Data Set)

Decision k	Action description	Relevant to states
1	Do nothing	1,2,3,4
2	Spot repair	3,4
3	Complete repainting	5

Table 6. Estimated Unit Cost of Three-Coat Paint Rehabilitation (Michigan Dept. of Transportation Data Set)

Decision k	State i	Description	Rehabilitation cost C_r(\$/ft^2), old paint is		Disruption cost (C_d), old paint is		Total cost[a] C_{ik}(\$/ft^2) old paint is	
			Lead	Zinc	Lead	Zinc	Lead	Zinc
1	1,2,3,4	Do nothing	0.0	0.0	0.0	0.0	0.0	0.0
2	3	Spot repair at state 3	1.5	1.0	17.69	17.69	19.20	18.69
	4	Spot repair at state 4	2.5	2.0	26.53	26.53	29.03	28.53
3	5	Complete repainting	4.0	2.8	35.37	35.37	39.37	38.17

[a]Total Cost C_{ik}=rehabilitation cost C_r+disruption cost to travelers due to rehabilitation work C_d.

Application of Markov Decision Process to Michigan Dept. of Transportation Data Set

The steps in applying the MDP to the three-coat paint system, the data set from the Michigan Dept. of Transportation (MDOT), were the same as those of the INDOT data set. Table 5 indicates the proposed policies or scenarios and their decisions at each state for the MDOT data set. The costs in Table 6 are the estimated unit cost (\$/ft^2) for steel bridges that were collected from different departments of transportation (DOTs). In Table 7, policy $Y1:(d_1,d_2,d_3,d_4,d_5)=(1,1,1,1,3)$ describes a policy where decision 1 (do nothing) is made in states 1,2,3,4, and decision 3 (complete repainting) is made in state 5. The objective is to find an optimal policy that minimizes the expected total discounted cost–initial cost and discounts subsequent costs that result from the future processes.

After applying these steps to the three-coat paint system (MDOT dataset), the calculations indicate that policy $Y1$ is the most economical. This policy proposes "do nothing" until the bridge reaches state 5 and then completely repaint the bridge. An example of the MDP application to the three-coat system, in the first 4-year range, is shown in Table 8. For more details on the disruption costs and the application of the MDP to the other four-year ranges of the three-coat system, the reader is referred to Zayed and Fricker (1999).

Deterministic Method (Economic Analysis)

Several researchers have begun to incorporate life-cycle cost analysis or discounted cash flow methods to minimize the cost of bridge maintenance. One study compared the equivalent uniform annual cost of various rehabilitation activities during the service life of a bridge. Others have applied this approach to road pavements in order to reduce their maintenance costs (Tam and Stiemer 1996). In this research, a life-cycle cost analysis using equivalent annual costs was developed to compare the costs of three maintenance strategies: spot repair, overcoating, and complete repainting.

Table 7. Different Proposed Policies for Painting Rehabilitation (Michigan Dept. of Transportation Data Set)

di	Policy ($Y1$)	Policy ($Y2$)	Policy ($Y3$)
$d1$	1	1	1
$d2$	1	1	1
$d3$	1	2	1
$d4$	1	1	2
$d5$	3	3	3

Life-cycle cost analysis is a relatively simple approach for minimizing coating maintenance costs. The general objective of this approach is to determine all the costs associated with the corrosion protection of the structure throughout its remaining service life. The total cost for a combination of a particular maintenance strategy is compared with the total cost of another strategy. The strategy that yields the lowest cost is considered to be the optimal maintenance strategy for the specific structure (Tam and Stiemer 1996).

The basic formula used in life-cycle cost analysis can be obtained from any basic economics textbook (Riggs 1986). The equation of equivalent uniform annual cost (EUAC) is shown in Eq. (6):

$$EUAC=F(A/F,i,N)\cdot L=F\left\{\frac{i\cdot L}{(1+i)^N-1}\right\} \tag{6}$$

where F=future cost; i=interest rate; *EUAC(A)*=equivalent uniform annual cost; L=inflation factor=$(1+i)^N$; and N=maintenance period.

In addition, Tam and Stiemer (1996) stated the following formula:

$$EUAC=P(A/P,i,N)\cdot L=P\left\{\frac{i(1+i)^N\cdot L}{(1+i)^N-1}\right\} \tag{7}$$

where P=present cost and *EUAC(A)*=equivalent uniform annual cost.

The present value of the sequence of costs incurred during the life of the paint, *PV*, can be written (Carnahan and Marsh 1998) as

$$PV=C_0+\sum_{t=1}^{M} C_t[(1+L)(1+i)]^{-t} \tag{8}$$

where C_0=initial cost and C_t=cost incurred in year t.

Economic Analysis Application to Indiana Dept. of Transportation Data Set

In this study, there are five states associated with the five possible conditions of bridge paint ratings. State 1 corresponds to the best condition and state 5 corresponds to the worst condition. Two types of paint systems currently exist in the INDOT: paint type 1 (lead-based paint) and paint type 2 (zinc/vinyl paint). Two different categories of bridges are used in this study: interstate (bridges that pass over or with the interstate roads) and state (bridges that pass over or with state roads). The two available types of paint are used in the two bridge categories. Interstate road bridges, which are painted with paint type 1, are categorized as I1. I2 is the symbol for interstate bridges that are painted using paint type 2. S1 and S2 are the categories of state road bridges using paint

Table 8. Application of Markov Decision Process to Three-Coat System OZEU/Old Paint Zinc (0–4 Years Range)

Values of 5 Unknowns		Transition Probability Matrix						
V1	*0.19*		*0.0001*	*0.9999*	*0*	*0*	*0*	
V2	*0.21*		*0*	*0.856*	*0.144*	*0*	*0*	
V3	*0.372*		*0*	*0*	*0.981*	*0.019*	*0*	
V4	*2.55*		*0*	*0*	*0*	*0.33*	*0.67*	
V5	*2.97*		*1*	*0*	*0*	*0*	*0*	
State 1		Probability * Values of 5 Unknowns					*Summation*	*Minimum*
K	*C1k*	*P1*V1*	*P2*V2*	*P3*V3*	*P4*V4*	*P5*V5*	*Value*	*Selection*
1	0	0.000019	0.209979	0	0	0	0.1396	**1**
State 2							*Summation*	*Minimum*
K	*C2k*	*P1*V1*	*P2*V2*	*P3*V3*	*P4*V4*	*P5*V5*	*Value*	*Selection*
1	0	0	0.17976	0.053568	0	0	0.1551	**1**
State 3							*Summation*	*Minimum*
K	*C3k*	*P1*V1*	*P2*V2*	*P3*V3*	*P4*V4*	*P5*V5*	*Value*	*Selection*
1	0	0	0	0.364932	0.04845	0	0.2748	**1**
2	1	0	0	0.3162	0.3825	0	1.4645	
State 4							*Summation*	*Minimum*
K	*C4k*	*P1*V1*	*P2*V2*	*P3*V3*	*P4*V4*	*P5*V5*	*Value*	*Selection*
1	0	0	0	0	0.8415	1.9899	1.8824	**1**
2	2	0	0	0	2.55	0	3.6953	
State 5							*Summation*	*Minimum*
K	*C5k*	*P1*V1*	*P2*V2*	*P3*V3*	*P4*V4*	*P5*V5*	*Value*	*Selection*
3	2.8	0.19	0	0	0	0	2.9263	**3**
The Result								
State I	1	2	3	4	5			
Decision K	*1*	*1*	*1*	*1*	*3*	**O.K.**		
Policy Y1	*1*	*1*	*1*	*1*	*3*			

types 1 and 2, respectively. The formulas in Eqs. (6)–(8) are used to calculate the EUAC for paint type 1 (lead-based paint) and paint type 2 (zinc/vinyl paint) according to five proposed rehabilitation and maintenance alternatives. The proposed alternatives for rehabilitation and maintenance are as follows:

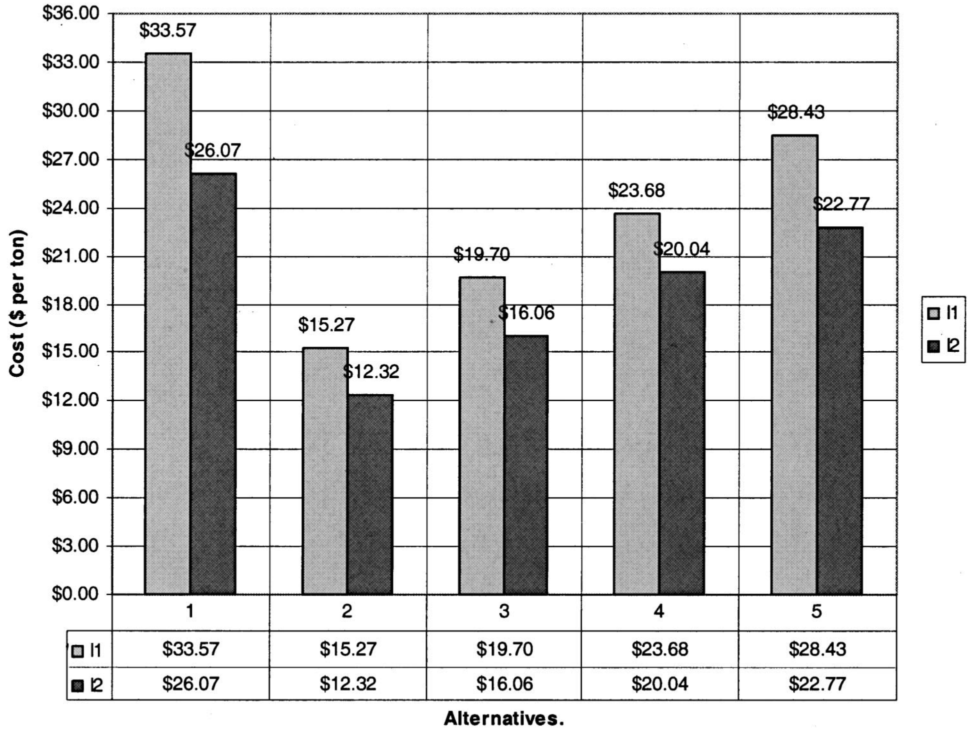
1. Do nothing and do complete repainting after reaching state 5 (approximately 30 years), where the bridge life span is 60 years;
2. Make spot repairs at state 2 and approximately every 10 years until the end of the bridge life of 60 years;
3. Make spot repairs at state 3; repeat every 18 years until the end of the bridge life;
4. Overcoat at state 3; repeat every 18 years until the end of the bridge life; and
5. Overcoat at state 4 after the first 24 years and do spot repairs after 18 years, that is, at the 42nd year, until the end of the bridge life.

The INDOT cost data that has been used in the MDP application is used in EA to unify the input values and guarantee similar inputs to each method. Table 9 shows the above alternatives and their corresponding initial costs, rehabilitation costs, number of years this rehabilitation is done, and the rehabilitation description. The information presented in this table is developed according to INDOT experts. It is noticed that the cost data that fed into EA are the same as those fed into MDP, as shown in Tables 2 and 9. The assigned state for each alternative is mentioned in the same table. For example, alternative 2, described as spot repair every 10 years, has an initial cost of \$220/ton for paint type 1 and \$180/ton for paint type 2. It has rehabilitation cost of \$25/ton for paint type 1 and \$20/ton for paint type 2. This alternative is applied to the bridge paint at state 2. These cost figures do not include traffic disruption costs.

These alternatives are applied to paint systems considering the

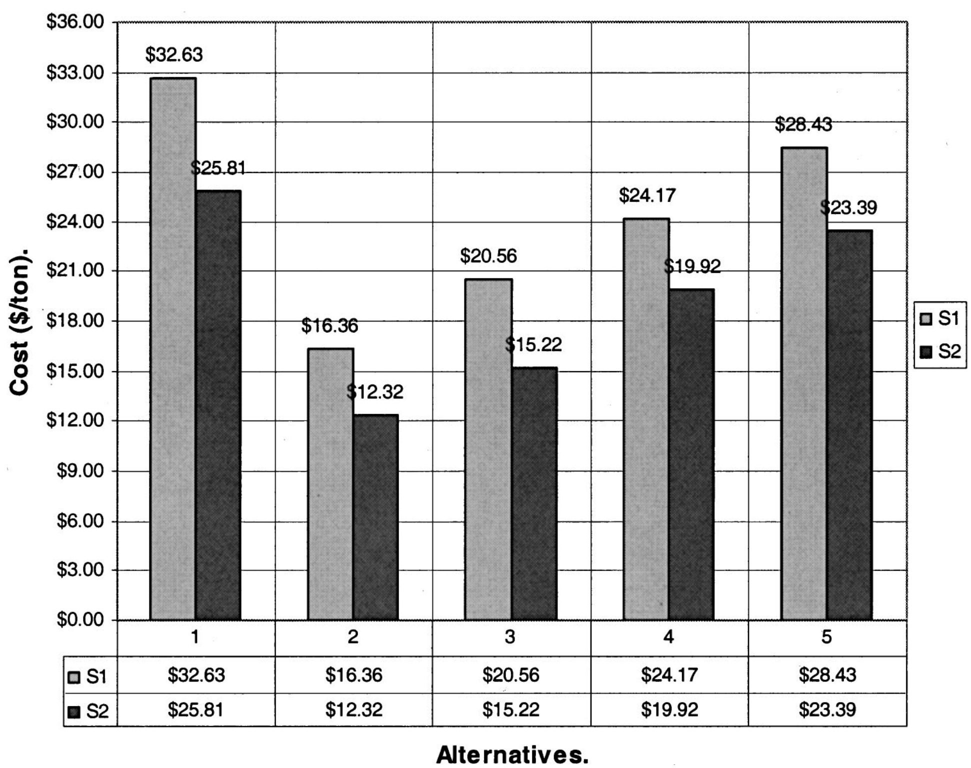
Table 9. Estimated Unit Cost of Paint Rehabilitation (Indiana Dept. of Transportation Data Set)

Description of rehabilitation process	Every *n* years (*n*)	Alternative number	Paint state	Initial cost (\$/ton) at paint types 1	Initial cost (\$/ton) at paint types 2	Rehabilitation cost (\$/ton) at paint types 1	Rehabilitation cost (\$/ton) at paint types 2
Complete repainting	30	1	5	\$220	\$180	\$220	\$180
Spot repair	10	2	2	\$220	\$180	\$25	\$20
Spot repair	18	3	3	\$220	\$180	\$50	\$40
Overcoating	18	4	3	\$220	\$180	\$110	\$100
Overcoating	24	5	4	\$220	\$180	\$180	\$150
Bridge reconstruction	60						



	1	2	3	4	5
I1	$33.57	$15.27	$19.70	$23.68	$28.43
I2	$26.07	$12.32	$16.06	$20.04	$22.77

Fig. 1. Economic analysis comparison for I1 and I2 using equivalent unit annual cost



	1	2	3	4	5
S1	$32.63	$16.36	$20.56	$24.17	$28.43
S2	$25.81	$12.32	$15.22	$19.92	$23.39

Fig. 2. Economic analysis comparison for S1 and S2 using equivalent unit annual cost

Table 10. Estimated Unit Cost of Paint Rehabilitation (Michigan Dept. of Transportation Data Set)

Description of rehabilitation process	Every n years (n)	Alternative number	Paint state	Initial cost ($/ft^2)		Rehabilitation cost C_r($/ft^2), old paint is	
				Lead	Zinc	Lead	Zinc
Complete repainting	25	1	5	$4	$2.8	$4.0	$2.8
Spot repair	15	2	3	$4	$2.8	$2.5	$2.0
Spot repair	20	3	4	$4	$2.8	$1.5	$1.0
Bridge reconstruction	60					$0.0	$0.0

differences in cost among the different types of paint. After calculating EUAC for each alternative considering one paint category, the minimum cost alternative will be the optimal. By analyzing the four categories, I1, I2, S1, and S2, it is apparent that alternative number 2 (make spot repairs at state 2 and repeat them along the life span of the bridge) is optimal. This is true for all paint categories. The cost calculations for these alternatives are indicated in Figs. 1 and 2.

The bar chart in Fig. 1 indicates that the optimal alternative for I1 is number 2, where spot repairs on the paint should be done every 10 years until the end of the bridge life. Its cost is $15.27 per ton. This alternative is less than half the cost of alternative number 1, which does nothing until complete repainting is performed. Similarly, the optimal alternative for I2 is number 2, where it has the EAUC of $12.32 per ton. On the other hand, Fig. 2 shows the same observation where alternative number 2 is optimal for S1 and S2.

In fact, the same procedure was applied to all the other paint systems with the same result. The results favored alternative number 2, with spot repairs made every 10 years until the end of the bridge life. Based on these results, there was a conflict between the results of the PITA algorithm and EA. The PITA favored alternative number 1 (do nothing until the end of paint life) and the EA favored alternative number 2 (spot repairs should be done every 10 years until the end of the bridge life). This conflict will be analyzed later in this paper.

Economic Analysis Application to Three-Coat System (Michigan Dept. of Transportation Data Set)

The economic formulas in Eqs. (6)–(8) are used to calculate the PV and EUAC for the three-coat paint system according to three proposed rehabilitation and maintenance alternatives. Table 10 shows these alternatives and their corresponding information. The three-coat paint system cost data that has been used in the MDP application is used in EA to unify the input values and guarantee similar inputs to each method. It is noted that cost data that fed into EA are similar to those fed into MDP, as shown in Tables 6 and 10. The proposed alternatives for rehabilitation and maintenance are as follows:

1. Do nothing and do complete repainting after reaching state 5 (approximately 25 years). Bridge life span is 60 years.
2. Make spot repairs at state 3 and then approximately every 15 years until the end of the bridge life of 60 years.
3. Make spot repairs at state 4, and repeat every 20 years until the end of the bridge life.

These alternatives were applied to the three-coat system. After calculating the PV and EUAC for each alternative in a paint category, the minimal cost alternative will be the optimal policy. After analysis, it was concluded that alternative number 2 (make spot repairs at state 3 and repeat it over the life span of the bridge) is the optimal policy for all the categories. The calculations of PV and EUAC for the three-coat paint over lead or zinc as the old paint systems are indicated in Table 11.

Table 11 indicates that the optimal alternative for the three-coat system over lead is alternative number 2, where spot repairs are done every 15 years. This alternative present value is a PV of $5.8/ft^2. This is approximately two-thirds the cost of alternative number 1, which does nothing until complete repainting is performed. It also shows that the best policy is alternative number 2, with an EUAC of $0.413/ft^2.

Table 11 indicates that the optimal alternative for the three-coat system over zinc is alternative number 2, where spot repairs are done every 15 years. This alternative present value is $4.0/ft^2. This is approximately two-thirds the cost of alternative number 1, which does nothing until complete repainting is performed. It also shows that the best policy is alternative number 2, with an EUAC of $0.285/ft^2.

It was thought that adding the traffic disruption costs to the model might change the decision. On the contrary, even after adding the traffic disruption cost to the paint cost, the best scenario does not change, as indicated in Table 11. The disruption cost depends upon the delay of traffic caused by each lane closure and the frequency of such closures over the bridge life span. The best scenario is making spot repairs every 15 years and/or when the bridge reaches a paint condition rating of 7. For more details on traffic disruption cost figures and the detailed calculations for the numbers in Table 11, the reader is referred to Zayed and Fricker (1999). Based on these results, there was a conflict between the results of the PITA algorithm and the EA in their ap-

Table 11. Maintenance Scenarios Economic Analysis for Three-Coat System Application to Indiana Dept. of Transportation Bridges Paint

Paint systems	Do nothing		Spot at State 3		Spot at State 4	
	PV ($/ft^2)	EUAC ($/ft^2)	PV ($/ft^2)	EUAC ($/ft^2)	PV ($/ft^2)	EUAC ($/ft^2)
Three-coat on lead (no disruption cost)	$6.50	$0.46	$5.80	$0.41	$5.95	$0.42
Three-coat on zinc (no disruption cost)	$4.55	$0.32	$4.00	$0.28	$4.36	$0.31
Three-coat on lead (with disruption cost)	$63.98	$4.56	$44.71	$3.18	$53.13	$3.78
Three-coat on zinc (with disruption cost)	$62.03	$4.42	$42.91	$3.06	$51.54	$3.67

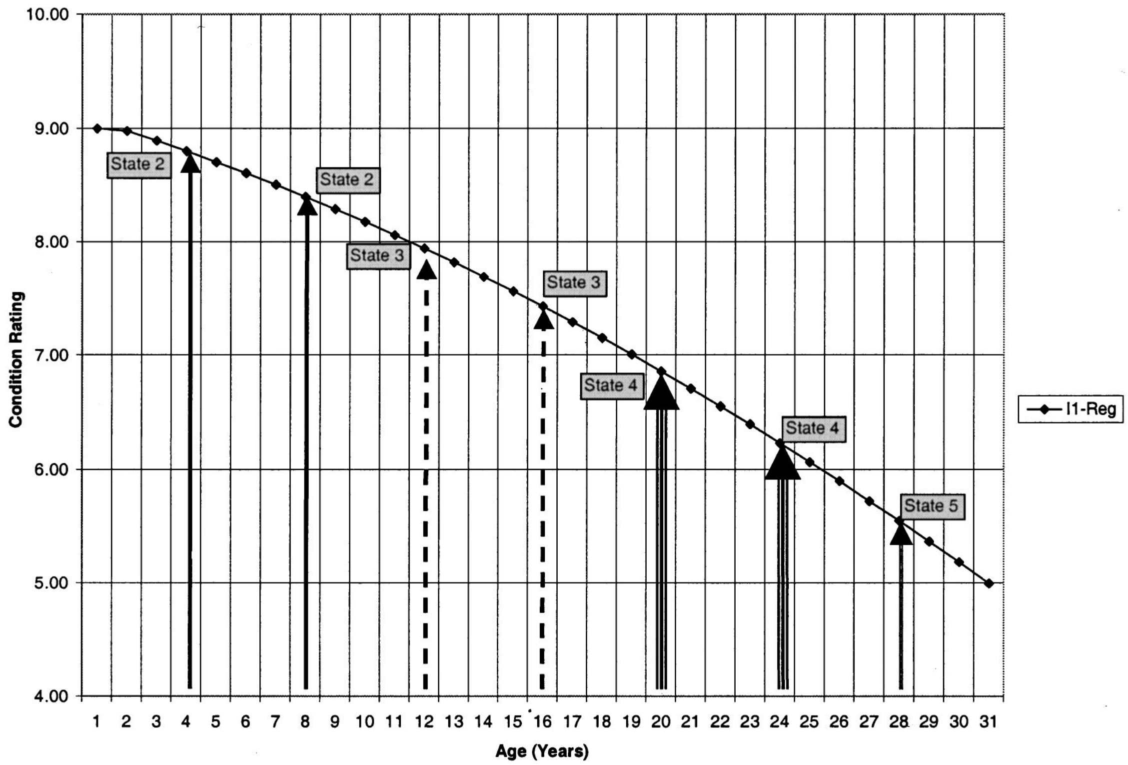


Fig. 3. Developed deterioration curve for different paint (I1) life stages

plication to the three-coat system data set. The stochastic method favored alternative number 1 (do nothing until the end of paint life) and the deterministic method favored alternative number 2 (spot repairs should be done every 15 years until the end of the bridge life). This conflict will be analyzed in the next section.

Discussion of Conflicting Results

There is a difference between the results of the EA and the MDP methods. A choice between the two results has to be made so that the analyst can decide which paint maintenance plan to use. The EA proposes policy number 2, while the MDP proposes policy number 1. The EA uses deterministic values of cost and time, while the MDP uses probabilistic time and deterministic costs. Therefore, the conflict between the results of these two methods needs to be examined further. The nature of each method's results will be explained first. The EA provides a central value (mean) of cost associated with the different payments for a specific scenario. The scenario represents the decisions that should be taken at specific states of the bridge condition ratings. The optimal scenario, from the EA prospective, is the one that has lowest central cost value (mean). On the other hand, the MDP provides only the set of decisions that are optimal, without mentioning their cost values. No central value or standard deviation for the cost of the optimal policy is concluded from the application of the MDP. Consequently, one can not make a direct comparison between a cost total and a policy.

For example, the EA proposes that the optimal policy for the three-coat system over zinc is number 2, where spot repairs are done every 15 years. The policy's present value is $\$4.0/\text{ft}^2$. Hence, the outcome of this method is a cost figure ($\text{PV}=\$4.0/\text{ft}^2$). On the other hand, MDP proposes the optimal policy for the same paint type as policy $Y1$ (1,1,1,1,3). This MDP result does not provide any cost figures with its solution. Consequently, it is not possible to compare the PV of $\$4.0/\text{ft}^2$ (the EA result) with $Y1$ (1,1,1,1,3) (the MDP result) to determine the best method for maintenance plan usage. Consequently, the results of the two methods should have the same units to allow comparison. The writers implemented the comparison methodology illustrated below.

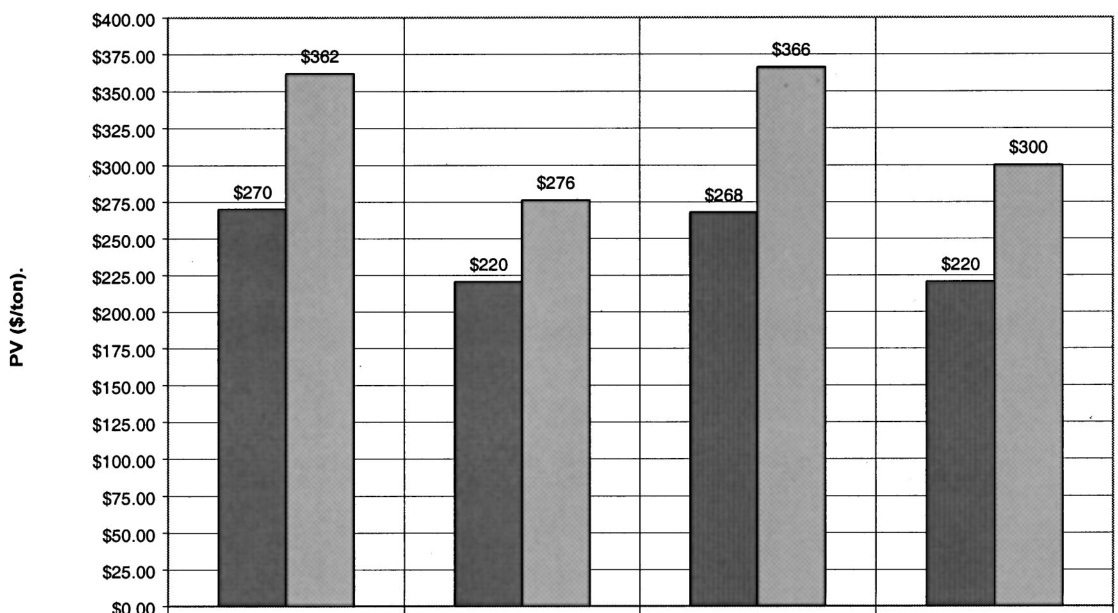
Comparison Methodology

In order to compare the EA and the MDP outcomes, they should have common characteristics or comparison criteria. The MDP method uses the PV methodology to calculate its optimal solution through the discount factor f ($f=1/(1+i)$) (Hillier and Lieberman 1986). The expected total discounted cost of the MDP method is denoted by $V_i(Y)$, which represents the total PV that results from its application to the data set. Hence, the comparison criteria between the EA and the MDP methods are PV and $V_i(Y)$, respectively. But the challenge is that the MDP calculation is divided into four-year ranges of the paint system life. The basic question that should be answered is "How should PV $[V_i(Y)]$ be computed for the MDP method throughout the entire paint system life?" The following methodology summarizes the computation procedure for PV $[V_i(Y)]$ of the MDP method:

1. Determine the paint system's state at the end of the four-year range of its life. This step is done as shown in Fig. 3. It represents the deterioration curve for paint system I1. The indicated arrows represent the end of each four-year range. The corresponding paint's state for each range is shown in the same figure. For example, the first and second ranges are located in state 2, while the third and fourth ranges are located in state 3. If the paint age is 20 or 24 years old, it is located in state 4. Hence, the states are defined throughout the paint type life.

Table 12. Markov Decision Process Results for Different Ranges of Paint System I1 (All Cost Figures are in $/Ton)

State	Age ranges (year) (0–4)	(5–8)	(9–12)	(13–16)	(17–20)	(21–24)	(25–28)	Total
State 1	$V_{1jk}(Y)$	$V_{2jk}(Y)$	$V_{3jk}(Y)$	$V_{4jk}(Y)$	$V_{5jk}(Y)$	$V_{6jk}(Y)$	$V_{7jk}(Y)$	
K								
1	$0.28	$2.44	$4.29	$4.33	$7.28	$11.32	$15.92	
State 2								
K								
1	$0.82	$6.79	$8.50	$8.47	$11.17	$17.90	$20.68	
2	$25.74	$31.09	$32.65	$31.43	$34.18	$41.11	$43.90	
State 3								
K								
1	$1.21	$13.78	$20.77	$19.32	$24.73	$25.79	$25.97	
2	$26.15	$36.17	$46.77	$45.17	$49.15	$49.74	$49.04	
3	$111.09	$118.99	$128.69	$126.70	$131.59	$133.21	$132.93	
State 4								
K								
1	$1.66	$76.90	$43.61	$41.31	$42.98	$37.07	$34.63	
2	$91.49	$113.55	$129.25	$129.88	$128.68	$123.37	$120.33	
3	$181.49	$203.55	$219.25	$219.88	$218.68	$213.37	$210.33	
State 5								
K								
4	$220.25	$222.20	$223.86	$223.90	$227.05	$230.18	$234.04	
Optimal scenario (PV)	$0.82	$6.79	$20.77	$19.32	$42.98	$37.07	$234.04	$361.78



	I1	I2	S1	S2
Spot at State 3-EA	$269.98	$220.48	$267.71	$220.48
Do Nothing-MDP	$361.78	$276.10	$366.20	$299.93

Fig. 4. Economic analysis (EA) versus Markov decision process (MDP) (Indiana Dept. of Transportation data set)

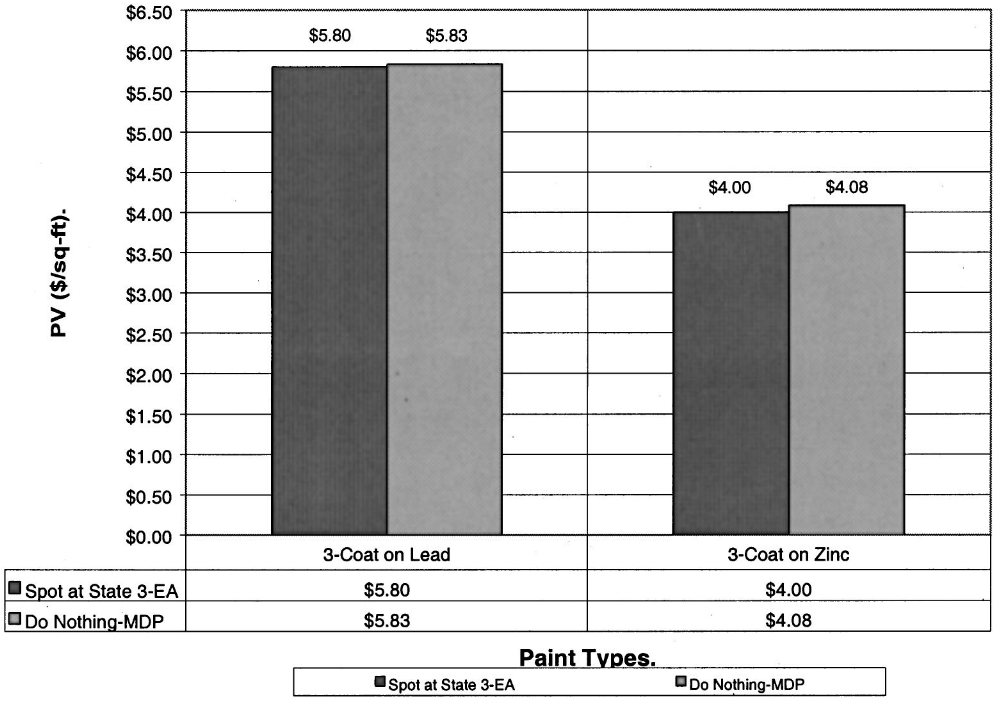


Fig. 5. Economic analysis (EA) versus Markov decision process (MDP) (Michigan data set)

2. The output of the MDP method is PV $[V_i(Y)]$ corresponding to the optimal policy in each range as shown in Tables 4 and 8. The interpretation of the optimal policy as it is described in both tables is that, if the paint system in state 2, then, perform decision 1 (do nothing) as an example. Similarly, if the paint system in state 4, perform decision 1. The MDP results for all the possible four-year ranges in I1 paint age are summarized in Table 12, where each column represents the output of each four-year range.
3. The following formula is used to determine the final MDP method outcome $\{\mathrm{PV}[V_i(Y)]\}$:

$$\text{Total PV } [V_i(Y)] = \sum_{i=1}^{n} \left\{ \prod_{j=1}^{m} \delta_j * \left(\min_{k=1,2,3,4} [V_{ijk}(Y)] \right) \right\} \tag{9}$$

   where PV=total present value resulted from MDP method application; δ_j=constant, equals 1 if state *j* is selected in the range *i*, and 0 otherwise, to match the deterioration model; n=number of four-year ranges available; m=state number (maximum is 5); and k=decision number (maximum is 4, as shown in Table 1).
4. Table 12 shows the application of Eq. (9) to paint system I1. For example, in range number 1 $(n=1)$, the selected state at the end of the four-year range is state number 2 $(m=2)$. Hence, δ_2 equals 1 and all the other constants equal 0. There are two available decisions; hence, $k=2$. The minimum value of $V_{12k}(Y)$ is selected to be the optimal where it is added to the other optimal values from the other ranges. The final outcome is the total PV of this optimal policy. Total PV for Paint I1 is $361.78/ton, which represents the MDP method's PV for optimal policy $(Y1)$.
5. Repeat steps 1–4 for the other paint systems, such as I2, S1, S2, and three-coat.

Once the above 5 steps have been completed, the PV for each optimal policy that results from the MDP method is determined. Therefore, a comparison between the EA and the MDP results can be performed. Fig. 4 shows the PV for scenarios 1 and 2 of the EA and policy number 1 of the MDP methods using the INDOT data set. This figure shows that the PV of the EA selected optimal scenario (spot repair at state 3) is always less than that of the MDP (do nothing) for all the paint systems. Note that although the ''do nothing'' scenario using the EA is not optimal, it has a PV less than that of the same policy using the MDP. Consequently, the writers decided that the EA is better than the MDP for the INDOT data set, because it provides lower PV values for optimal scenarios. Hence, the optimal scenario that results from the EA is selected to be used in planning INDOT maintenance for its paint systems.

Implementing the same procedure for the MDOT data set leads to the results shown in Fig. 5. It shows the PV for ''do nothing'' (the optimal policy results from the MDP) and the ''spot repair at state 3'' (the optimal scenario results from the EA). Contrary to the INDOT data set, the MDOT data set provides very close results for the EA and the MDP. The optimal policy using the EA has a PV of $5.80/ft^2, while the optimal one using the MDP has a PV of $5.83/ft^2. Hence, PV is similar, but the strategies are different.

Conclusions

Using the available data from INDOT and MDOT, existing and new bridge paint systems have been analyzed. Two methods (the EA and the MDP) were applied to INDOT and MDOT data sets. The methods led to different results. A comparison was performed to select the superior method. The EA proved its superiority over MDP using INDOT data. Consequently, the results of the EA were considered for this data set. The results indicated that the best rehabilitation scenario for I1, I2, S1, and S2 is doing spot repairs every 10 years.

Because the costs of the EA and the MDP optimal strategies in the MDOT data set are very close, MDOT could choose either to get similar results. The important finding here is that the clear superiority of the EA-based strategy in the INDOT was not confirmed in the MDOT. The EA method's advantages (simple to use and understand, applied widely) are offset by the MDP's ability to incorporate the inherent stochastic nature of the phenomenon being modeled. Of course, the data needed to construct such a stochastic model must be available.

Acknowledgments

The writers would like to express their gratitude to the INDOT and the MDOT for their generous contribution of time, energy, and knowledge in this study. They would also like to thank the INDOT for its financial support of this research. The writers appreciate the unlimited help from the Ohio, Illinois, Kentucky, and Connecticut DOTs.

References

Abraham, D. M., Wirahadikusumah, R., Short, T. J., and Shahbahrami, S. (1998). ''Optimization modeling for sewer network management.'' *J. Constr. Eng. Manage.* 124(5), 402–410.

Carnahan, J. V., and Marsh, C. (1998). ''Comparative life-cycle cost analysis of underground heat distribution systems.'' *J. Transp. Eng.* 124(6), 594–605.

Hillier, F. S., and Lieberman, G. J. (1986). *Introduction to operations research*, 4th Ed., Holden-Day, Oakland, Calif.

Riggs, J. L. (1986). *Engineering economics,* McGraw-Hill, New York.

Scherer, W. T., and Glagola, D. M. (1993). ''An overview of Markovian models for bridge management systems.'' *Proc., Infrastructure: Planning and Management*, ASCE, New York, 382–386.

Tam, C. K., and Stiemer, S. F. (1996). ''Development of bridge corrosion cost model for coating maintenance.'' *J. Perform. Constr. Facil.* 10(2), 47–56.

Wirahadikusumah, R. (1998). ''Sewer management life cycle cost approach.'' PhD research proposal, Purdue Univ., West Lafayette, Ind.

Zayed, T. M., and Fricker, J. D. (1999). ''Life-cycle cost analysis and maintenance plan.'' *Rep. Prepared for INDOT*, Construction Division, School of Civil Engineering, Purdue Univ., West Lafayette, Ind.