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A probabilistic computational framework for bridge network optimal maintenance scheduling

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ABSTRACT

This paper presents a probabilistic computational framework for the Pareto optimization of the preventive maintenance applications to bridges of a highway transportation network. The bridge characteristics are represented by their uncertain reliability index profiles. The in/out of service states of the bridges are simulated taking into account their correlation structure. Multi-objective Genetic Algorithms have been chosen as numerical tool for the solution of the optimization problem. The design variables of the optimization are the preventive maintenance schedules of all the bridges of the network. The two conflicting objectives are the minimization of the total present maintenance cost and the maximization of the network performance indicator. The final result is the Pareto front of optimal solutions among which the managers should chose, depending on engineering and economical factors. A numerical example illustrates the application of the proposed approach.

1. Introduction

For the economic and cultural development of any nation, the transportation infrastructure plays a role of utmost importance. Moreover, after the occurrence of an extreme event, either natural or man-made, the efficiency of the highway system is critical for a prompt response to the emergency and for the recovery activities. Nevertheless, according to ASCE [1], in the next five years the investment shortfall for civil infrastructure will be $1.176 trillion. In this situation, an optimal allocation of the limited available resources is necessary.

The main actions that can improve the reliability of an existing transportation network are maintenance, monitoring, repair, and replacement. The focus of this paper is the optimal bridge maintenance scheduling at the network level under uncertainty.

In the literature, it is possible to find many studies that deal with the optimal maintenance planning for individual bridges (see the comprehensive review paper [2] and references therein). Some of these studies have been a source of inspiration for the present paper. However, the maintenance management is usually planned by institutions and agencies that are in charge of entire transportation networks or, at least, of several bridges. For this reason, many studies have been focusing on bridge network analysis. For instance, Liu and Frangopol [3,4] developed a procedure for the time-dependent reliability analysis of a bridge network; Shinozuka and his co-workers [5,6] performed a cost–benefit analysis of maintenance in terms of seismic retrofit on transportation networks; Lee et al. [7] proposed a technique for the assessment of the flow capacity of a transportation network after the occurrence of an extreme event. Similar studies have focused on other civil infrastructure lifelines, such as power lines [8–10], and on the interaction between different networks [11–14]. In particular, some papers treat the topic of bridge maintenance optimization at the transportation network level [15–17] and the interest in this topic is strongly increasing lately [18–20].

Under the assumption that accurate profiles of the variation in time (due for instance to corrosion, external stressors, aging, and fatigue) of the individual bridge reliabilities are available, it is possible to assess the effect of the time-dependent reliability of individual bridges on the reliability of the overall network. Unfortunately, this assumption is not realistic in most of the cases. Only the most important bridges of a network are usually thoroughly modeled and sometime monitored, so that their predicted reliability profiles can be considered realistic. For all the other bridges, this information is, in general, unavailable. For this reason, in the present study, the proposed network analysis technique is used together with life-cycle reliability models developed by Frangopol and his co-workers [21–24]. This kind of model, that includes uncertainties, can be assessed knowing some basic characteristics of the individual bridges. Therefore, the reliability profile can be assessed without the need (and the cost) of thorough studies on every bridge. The associated epistemic uncertainty (as well as the intrinsic one) is accounted for by the proposed approach, through simulation.

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Another numerical tool that is part of the framework is the computer program HAZUS-MH MR4 [25]. HAZUS is a free software distributed by the US Federal Emergency Management Agency that is specially meant for loss estimations and therefore is coupled with a very rich Geographic Information System (GIS) that collects data on all the bridges in the US. These data and HAZUS are used to assess the degree of correlation between the states of the bridges in the network by means of a series of seismic fragility analyses [26]. It should be specified that earthquakes scenarios are used solely with the purpose of assessing the correlation [27]. This kind of external load effect has been chosen because it is particularly well addressed by HAZUS, but other extreme events (such as hurricanes) or a combination of different actions could be used. In fact, the assessment of the optimal maintenance scheduling is not based on earthquake actions, but on the generic bridge reliability, as provided by the time-dependent models mentioned previously.

The two pillars of the stochastic model are the time-dependent reliability functions (which provide the marginal distribution of the bridge state) and the HAZUS-based technique [27] that assesses the correlation among the state of different bridges of the same network.

As previously stated, the main goal of the proposed approach is the optimization of the maintenance scheduling under uncertainty. Unfortunately, the main objectives of the optimization, that are the maximization of the network performance and the minimization of the maintenance cost, cannot be computed in closed form, but only numerically. When the closed form expression of the objective function(s) is not available, traditional optimization techniques cannot be employed. In this kind of problem, numerical optimization methods that make use only of discrete values of the objective function(s) and do not require additional information (such as gradients) are required. These optimization procedures are generally known as heuristic methods. Points in the feasible design domain are generated and tested for the satisfaction of objectives through the evaluation of the objective function(s). The most used class of heuristic techniques are Evolutionary Algorithms. They are numerical optimization procedures that find their origin in the Darwinian theory of evolution. Genetic Algorithms [28–30] are a set of specific methodologies that belong to this class. Genetic Algorithms (GAs), or the more general case of Evolutionary Algorithms, have been widely used in many fields of civil engineering, including structural identification [31–34] and maintenance optimization [35,36]. In the present paper, multi-objective GAs [37] are used to find the Pareto front of optimal preventive maintenance schedules at the network level. It should be noted that multi-objective GAs are particularly suitable to this purpose, also because their characteristic to simultaneously evaluate the objective functions at many points of the domain, automatically yields a tentative Pareto front. The decision makers must chose one solution among the various Pareto optimal ones, based on economic and engineering considerations.

Section 2 provides a description of the different types of maintenance interventions that are considered in the proposed framework. Section 3 and its subsections detail the theoretical background and the computational aspects of the procedure. Section 4 presents the numerical application of the technique to a bridge network of 13 bridges, for which maintenance interventions are planned over a period of 75 years. Finally, Section 5 collects the concluding remarks.

2. Preventive, essential and required maintenance

Three different types of maintenance can be considered: (i) preventive, (ii) essential, and (iii) required. Fig. 1 provides a graphical representation of these types of maintenance.

Preventive maintenance (PM), also called "time-based" maintenance, consists of all those interventions that are scheduled *a priori* in order to always keep the bridge at a good service level. Usually, this kind of intervention has the lowest impact on the bridge safety and the lowest cost.

Essential maintenance (EM) is a "performance-based" intervention. This maintenance intervention is applied when an indicator of the bridge performance crosses a predefined threshold. The most used indicator is the bridge reliability index, but several other indicators can be considered. For instance, Okasha and Frangopol [38] investigated also availability and redundancy. The most common case of reliability index threshold crossing is considered henceforth. First of all, a bridge limit state has to be defined, for instance it can be the collapse or just the excessive deformation of the main girders. Event E_1 is defined as the bridge reaches the investigated limit state. For this specific event E_1, it is possible to define the "time to failure" TF_{E_1} as the time between a reference instant $t=0$ and the moment at which E_1 occurs. The reliability at time t is then defined as the probability that E_1 does not occur in the interval $[0,t]$:

$$REL_{E_1}(t) = \mathcal{P}(t < TF_{E_1}) \tag{1}$$

where $\mathcal{P}(\cdot)$ denotes the probability of the event in brackets. There are many available techniques to compute the reliability index. Each of these techniques has advantages and shortcomings (in terms of simplicity and accuracy). A popular way is to assume that the reliability index $\beta_{E_1}(t)$ is obtained from the reliability as follows:

$$\beta_{E_1}(t) = \Phi^{-1}[REL_{E_1}(t)] \tag{2}$$

where Φ^{-1} is the inverse standard Gaussian cumulative distribution function. If a lower threshold $\overline{\beta}$ for the reliability index is fixed, a second event E_2 can be defined as $\beta_{E_1}(t) \le \overline{\beta}$. EM is applied whenever event E_2 occurs.

Required maintenance (RM) is a "failure-based" intervention. For a specific limit state, RM is applied when event E_1 (as previously defined) is imminent, or when it has just occurred. A special case of RM is when event E_1 is assumed to be the collapse of the bridge; in this case, RM is the bridge restoration. However, RM is a more general concept than restoration, since it includes minor interventions if E_1 is a different limit state (e.g. serviceability limit, excessive corrosion, excessive deformation).

Since the occurrence of EM (triggered by event E_2) is based on the definition of the event that yields RM (event E_1), it is evident that the two are strongly interconnected. However, they are not the same. On one hand, E_2 can occur even if E_1 never happened. In fact, the threshold $\overline{\beta}$ is usually high, therefore when $\beta_{E_1}(t)$ downcrosses it, the reliability is still very close to one (i.e. event E_1 is still very unlikely to occur). On the other hand, E_1 can happen at any time, even much earlier than E_2 (e.g. during or just after the bridge construction, since the bridge reliability is never equal to unity, that means that there is always a chance of failure).

When the focus is on an individual bridge, it makes perfect sense to focus on EM (event E_2). In fact, if a single bridge is studied, it means that this bridge is considered important, and that not only the distress caused by the occurrence of event E_1 should be avoided, but even the probability of having a low reliability level.

On the contrary, when an entire transportation network is considered, RM seems more realistic. For most of the bridges, thorough studies on the time-dependent reliability profile $\beta_{E_1}(t)$ could be unavailable. Therefore, if the moment in which $\beta_{E_1}(t)$ downcrosses the threshold $\overline{\beta}$ is unknown, there will certainly be no EM interventions applied at that unknown instant. It appears much more realistic to assume that only RM will be applied when the bridge shows an imminent state of distress or when the distress has just occurred.

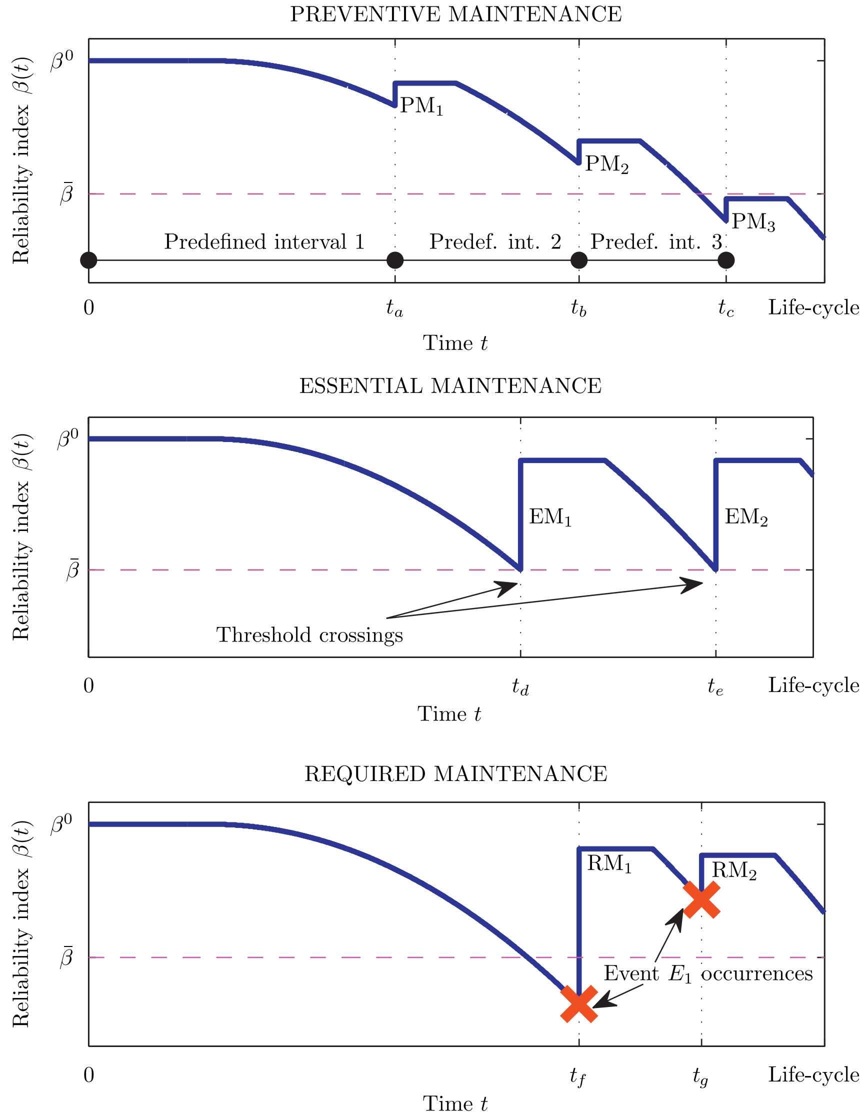

Fig. 1. Preventive maintenance is applied at predefined time intervals. Essential maintenance is applied when the reliability index downcrosses a threshold $\overline{\beta}$ (i.e. event E_2 is the trigger). Required maintenance is applied when event E_1 occurs or is imminent; this can happen for any value of $\beta(t)$, and the probability of occurrence is given by $1-\Phi[\beta(t)]$.

In conclusion, in this study only PM and RM are considered (meaning that PM schedule is optimized and RM is accounted for during the optimization process). However, the case of application of EM on the most important bridges of the network can be easily included into the proposed approach.

3. Proposed approach

3.1. Description

The problem at hand involves a large number of topics. Therefore the solution is necessarily obtained by means of several modules with an intricate set of interactions. Because of the complexity of the algorithm, a first overall description of the approach is useful, for the sake of clarity.

Fig. 2 shows the interactions among the various modules of the proposed methodology. The time-dependent reliability model manages the effects of aging and maintenance interventions. This module interacts mostly with the bridge state simulation module, but it also sends information to the maintenance cost module. The network model performs the traffic analyses and transfers the results to the network performance module. All this information is finally sent to the GAs module.

The core of the procedure is the optimization module. In this module, multi-objective GAs are employed. The design variables are the times of application of the PM actions, while the objectives are the minimization of the maintenance cost and the maximization of the network performance.

Since many of the variables at hand must necessarily be modeled as random, inside the GAs optimization a Monte Carlo simulation (MCS) is performed. The samples considered in the MCS are sets of time-dependent reliability index profiles (one profile for every bridge of the network). At every considered time step, depending on the current reliability of the individual bridges, the in/out of service state of each bridge is simulated (for each bridge and each sample). Depending on the in/out of service bridge states, the network performance indicators are computed and the maintenance cost (that includes RM in case of failures) is assessed. This way, GAs can determine if the PM schedule assumed for that specific "individual" is optimal (i.e. belongs to the Pareto front or not).

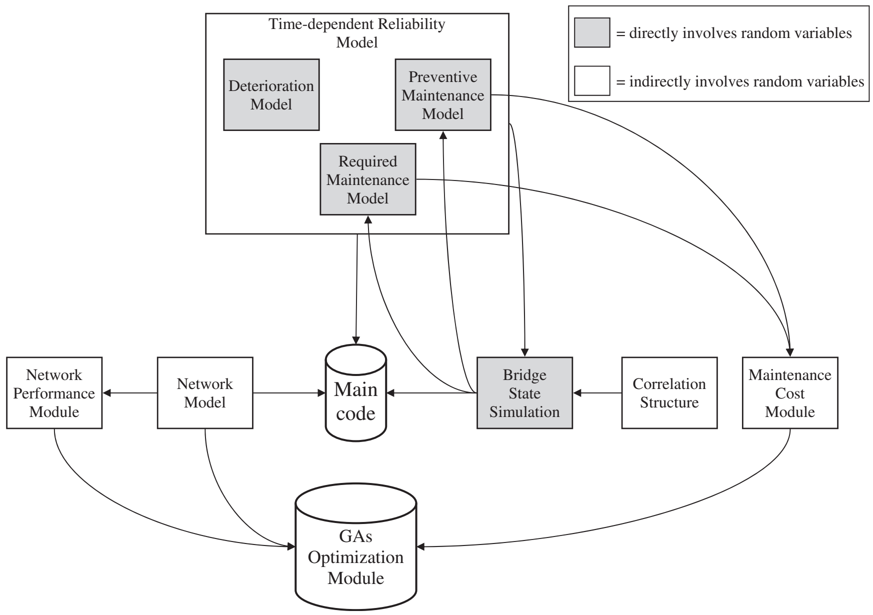

Fig. 2. Flowchart of the interactions among the various modules of the proposed methodology.

Bridges that belong to the same transportation network are most often subject to similar environmental conditions, usually experience similar traffic loads, undergo the same natural extreme events, might be designed with the same codes and built with similar technologies. All these factors imply a certain degree of correlation between the bridges of the same network. It has been shown [27] that this correlation strongly influences the network performance indicators. Therefore, in this study the correlation will be assessed and taken into account for the simulation of the individual bridge states.

A schematic representation of the tasks performed by the computational code implemented for the optimal scheduling of PM is shown in Fig. 3. More details on the procedure are given in the following subsections.

- Assess the correlation (correlation coefficients and correlation distance)
- Select the time-dependent reliability model for every bridge
- Define the probability distributions of the model parameters
- Define the Preventive Maintenance (PM) actions as superposition of basic components and define the probability distributions of the involved parameters
- Simulate the random variables for
  - Reliability index model
  - PM actions model
- Compute samples of $\beta_{bi}(t)$ without maintenance for every bridge b and for every sample i

GAs optimization

- Design variable: PM schedule
- Superimpose PM to $\beta_{bi}(t)$ and obtain $\hat{\beta}_{bi}(t)$
  - Analysis for every time instant $t = T$
  - Given the values of $\hat{\beta}_{bi}(t)$ and given the correlation coefficients, simulate the bridge in/out of service states $s_{bi}(T)$ for every bridge b and every sample i
  - Use the lookup table to retrieve the network performance indicators Total Travel Time $TTT_i(T)$, Total Travel Distance $TTD_i(T)$ and the combined index $\Gamma_i(T)$ for every sample i
  - Apply Required Maintenance (RM) to every failed bridge:
    - Modify $\hat{\beta}_{bi}(t > T)$ applying random restoration
    - Shift the PM schedule for $t > T$ for the restored bridge that failed a $t = T$
    - Update RM cost
  - Go to next time step $t = T + 1$
- Compute the cost of PM actually applied during the life-cycle (considering the shifts)
- Compute mean values of the (present) total maintenance cost (PM + RM) and network performance NP (that are the objectives of the optimization)

Fig. 3. Procedure of the proposed computational technique.

3.2. Assessment of the correlation

The correlation among the in/out of service states of the individual bridges is assessed by means of the technique proposed by Bocchini and Frangopol [27] and briefly recalled herein.

The structural damage level due to a prescribed extreme event scenario (e.g. earthquake or hurricane) is considered a synthetic measure of the structural characteristics of a bridge. Therefore, the computer program HAZUS-MH MR4 [25] is used to perform fragility analyses for the bridges of the network and to compute the expected value of the damage level. The database associated with HAZUS provides information on every bridge in the United States, including pier type, number of spans, structure type, design code, and further details. Depending on these data, every bridge belongs to one out of 28 classes. For every class, there are four predefined fragility curves [39]. Each one of these curves is associated with a certain damage level (e.g. no damage, moderate damage, etc.) and provides the probability of exceedance (vertical axes) of that damage level as functions of the ground spectral acceleration (horizontal axis). Then, these four curves are adapted to better represent some key individual bridge characteristics, such as maximum span length and skewness [40,41]. Given an earthquake scenario e (i.e. earthquake type, magnitude, epicenter location and depth), HAZUS uses an attenuation function to compute the value of the intensity measure (i.e. spectral acceleration) at the locations of all the bridges and then it exploits the fragility curves to assess the probabilities to attain the various damage states for every bridge. Eventually, it is possible to compute the expected value of the damage level as follows:

$$l_{b,e} = 0 \cdot \mathcal{P}(\text{no damage}) + 1 \cdot \mathcal{P}(\text{minor damage}) + 2 \cdot \mathcal{P}(\text{moderate damage}) + 3 \cdot \mathcal{P}(\text{major damage}) + 4 \cdot \mathcal{P}(\text{collapse}) \quad (3)$$

where $l_{b,e}$ is the average damage level of the bridge b due to the extreme event scenario e; $\mathcal{P}(\cdot)$ indicates the probability of bridge b being in a certain damage state, as computed by HAZUS. Eq. (3) also implies that the five states are assumed to be "equally spaced" in the damage level domain (different assumptions could be made, this one is consistent with the description of the damage levels provided by HAZUS), therefore $l_{b,e}$ is a real variable that takes values in the interval [0, 4] where 0 and 4 represent no damage and collapse states, respectively.

The difference between expected values of the damage level of two bridges of the network subject to the same extreme event scenario incorporates information on the distance between them, on their characteristics, and on their structural similarities. If several analyses with different earthquakes are performed, it is possible to assess the correlation coefficient:

$$\rho_{bc} = \frac{\sum_{e=1}^{n_E}[(l_{b,e}-\mu_b)\cdot(l_{c,e}-\mu_c)]}{n_E\cdot\sigma_b\cdot\sigma_c} \quad \forall b,c \tag{4}$$

where ρ_{bc} is the correlation between the states of bridges b and c; the index e runs over the n_E considered scenarios; $l_{b,e}$ and $l_{c,e}$ are the structural damages of bridges b and c in the e-th scenario, as computed by Eq. (3); and μ_b, μ_c and σ_b, σ_c are the mean values and standard deviations of the damage levels for bridges b and c among the various considered scenarios. Note that only the deviation from the average damage level $(l_{b,e}-\mu_b)$ is considered for each bridge. Thus, the fact that a bridge is safer than another does not affect the computation of the correlation.

The discrete results obtained by Eq. (4) for every pair of bridges b and c can be collected in a plot of ρ_{bc} versus ξ_{bc}, where ξ_{bc} is the distance between bridges b and c. Such a series of points can then be interpolated by a continuous analytical function as, for instance, the following:

$$\rho(\xi) = A\exp\left(-\frac{\xi^2}{\lambda^2}\right)+K \tag{5}$$

where λ is the correlation length; and A and K are a scaling factor and a constant shifting parameter that can be estimated by least square method or other techniques.

The main parameter of this correlation model is the correlation length λ. The proposed estimation procedure implicitly takes into account also the structural characteristics of the individual bridges, but the correlation coefficient $\rho(\xi)$ is explicitly a function only of the distance between bridges. More complex models that define ρ as a function also of some primary structural assets (e.g. the main building material) can also be considered.

Finally, it should be noted that the choice of the extreme event scenarios does not have to reflect the expected characteristics of earthquakes in the investigated region. This correlation assessment should be considered as a benchmark test. In benchmark tests, specimens are not tested for realistic loads, that they would have been likely to be subject to if they were used for the construction, but they are tested for extreme loads that lead to their total failure. Similarly, earthquake magnitudes in this analysis should be chosen so that all the bridges are very likely to be in the total collapse state in at least one considered scenario. In fact, this way the whole information given by the fragility curves is exploited. Moreover, the epicenters of the various scenarios should be spread all over the area covered by the network. Bocchini and Frangopol [27] showed that if all the scenarios share the same epicenter the correlation estimation is heavily biased, because it appears much stronger than what it actually is.

3.3. Assessment of bridge life-cycle reliability models

In a few circumstances, an accurate description of the time-dependent reliability of all the bridges of the network might be available [42,43]. More often, only the critical bridges are subject to a thorough study and, in general, the reliability of most of the bridges of the network is uncertain. In these cases, an analytical model of the time-dependent reliability for individual bridges should be employed.

In the literature, different models have been proposed. For instance, the so-called "bilinear" model is very popular [22,24], but also quadratic models have been used and herein a novel exponential model is introduced.

The analytical definitions of such models in the case where no maintenance is applied are:

$$\text{Bilinear} \quad \beta(t) = \begin{cases} \beta^0 & \text{for } 0 \le t \le T^I \\ \beta^0 - R^L(t-T^I) & \text{for } t > T^I \end{cases} \tag{6}$$

$$\text{Quadratic} \quad \beta(t) = \begin{cases} \beta^0 & \text{for } 0 \le t \le T^I \\ \beta^0 - R^Q(t-T^I)^2 & \text{for } t > T^I \end{cases} \tag{7}$$

$$\text{Square root} \quad \beta(t) = \begin{cases} \beta^0 & \text{for } 0 \le t \le T^I \\ \beta^0 - R^S\sqrt{t-T^I} & \text{for } t > T^I \end{cases} \tag{8}$$

$$\text{Exponential} \quad \beta(t) = \begin{cases} \beta^0 & \text{for } 0 \le t \le T^I \\ (\beta^0 - Q)\exp\left[-\left(\dfrac{t-T^I}{S}\right)^2\right] & \\ \quad -R^E(t-T^I)+Q & \text{for } t > T^I \end{cases} \tag{9}$$

where t is time elapsed from a reference instant $t=0$; $\beta(t)$ is the time-dependent reliability index; $\beta^0=\beta(0)$ is the starting value of the reliability index; R^L, R^Q, R^S, and R^E are degradation rate parameters; S is a shape parameter that tunes the position of the inflexion point in the exponential model; Q is a degradation parameter that tunes the slope of the exponential model during the first years, T^I is the initiation time of the deteriorating process. The four models are plotted together in Fig. 4.

The new exponential model in Eq. (9) has the advantage that its shape and versatility usually yield reliability profiles very similar to those that result from analytical studies, such as those presented by Akgül [43]. The drawback is that it requires to estimate five parameters versus the three parameters required by the other models. The parameters of the newly proposed model can be assessed with the same technique reported in [22].

The choice of the model for every bridge depends on its structural characteristics, age, probable loading conditions, and the environment in which the bridge is located. The choice can be done looking at the time-dependent reliability profile of bridges in similar conditions, or by engineering judgment.

In any case, uncertainty is included into the model by means of its parameters (β^0, T^I, R^L, R^Q, R^S, R^E, S, Q) that are considered random variables. The probabilistic distribution of some of the parameters for the cases of bending and shear limit states have been provided by Frangopol et al. [21].

The randomness of the parameters implies that the values of the reliability index are uncertain. Fig. 5 shows the probability density function of $\beta(t)$ at some selected time instants. The standard deviations become larger during the years, since the present uncertainties propagate in time.

In conclusion, for every bridge of a network, either a deterministic model of the time-dependent reliability index (if available) or a model taking into account uncertainties has to be selected. When the uncertain model is chosen, the probability distribution of its random parameters must be assigned, based on engineering judgment. Eventually, for every bridge b with an uncertain $\beta(t)$ profile, a large number n_S of $\beta_{bi}(t)$ samples can be generated by means of well-established simulation techniques.

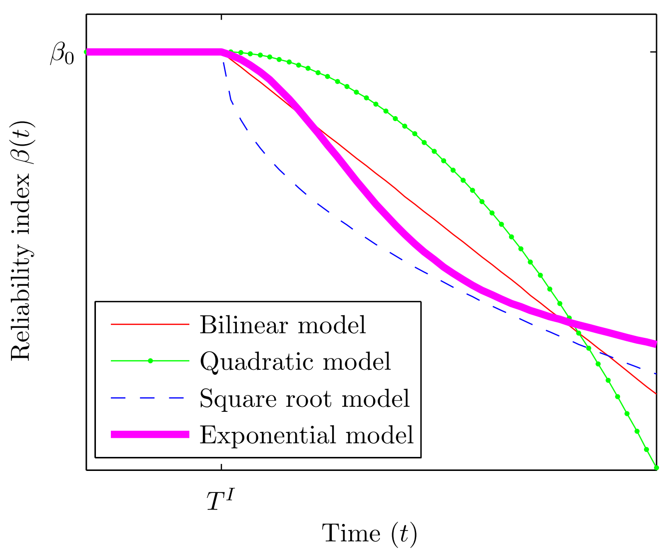

Fig. 4. Four models for the time-dependent reliability of bridges. The initial flat portion is the same for all the models.

3.4. Definition of preventive maintenance actions

The goal of the current procedure is to optimize the schedule of applications of PM interventions on a bridge network. The type of PM intervention is assumed to be the same (in a probabilistic sense) for all the applications and for all the bridges. Therefore, the characteristics of the PM have to be defined *a priori*. The numerical code that has been developed considers four basic components of the PM effect that are represented in Fig. 6. The first one is an improvement in the value of $\beta(t)$ that determines an increase in the reliability index that is constant in time, after the moment of its application. Even if the maintenance operations are assumed to be all the same, their effects are always subject to uncertainty. Therefore, the actual amount of the improvement is modeled as a random variable. The second basic maintenance component is a delay in the deterioration process. Therefore, the value of $\beta(t)$ remains constant for a certain period, that is again modeled as a random variable. The third maintenance component is a linear increase in the reliability index that stabilizes and becomes constant after a while. Both the duration and the slope are random variables. The fourth and last basic maintenance component is an improvement in $\beta(t)$ with a parabolic shape. Also in this case, the duration and the parameter R^Q_{PM} of the parabolic increase $R^Q_{PM}(t-t_{application})^{0.5}$ are random.

The analyst has to define a combination of the four basic components as maintenance intervention. Moreover, the probability density functions of the random parameters of the involved components have to be defined. Therefore, a real maintenance intervention can be described by a minimum of one (if only the first or the second component is used) up to a maximum of six (if all the components are involved) random

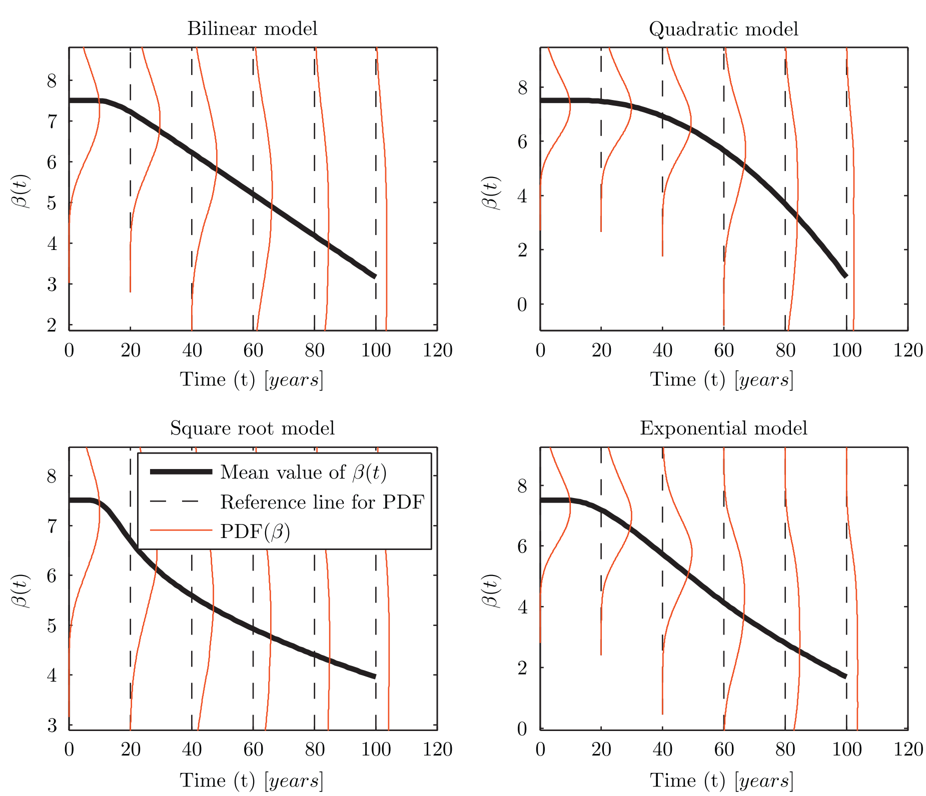

Fig. 5. Mean value and probability density functions of the reliability index with different models and under the assumption of no maintenance interventions. For every model 50 000 samples have been generated. The parameters are distributed to model the case of bending limit state as follows: $\beta^0 \sim \mathcal{L}ogN(7.5,1.2)$; $T^I \sim \mathcal{L}ogN(15,5)$; $R^L \sim \mathcal{U}niform(0.002,0.1)$; $R^Q \sim \mathcal{U}niform(0.000035,0.001765)$; $R^S \sim \mathcal{U}niform(0.0153,0.7543)$; $R^E \sim \mathcal{U}niform(0.002,0.1)$; $S \sim \mathcal{N}ormal(40,3)$; $Q \sim \mathcal{N}ormal(6,1)$.

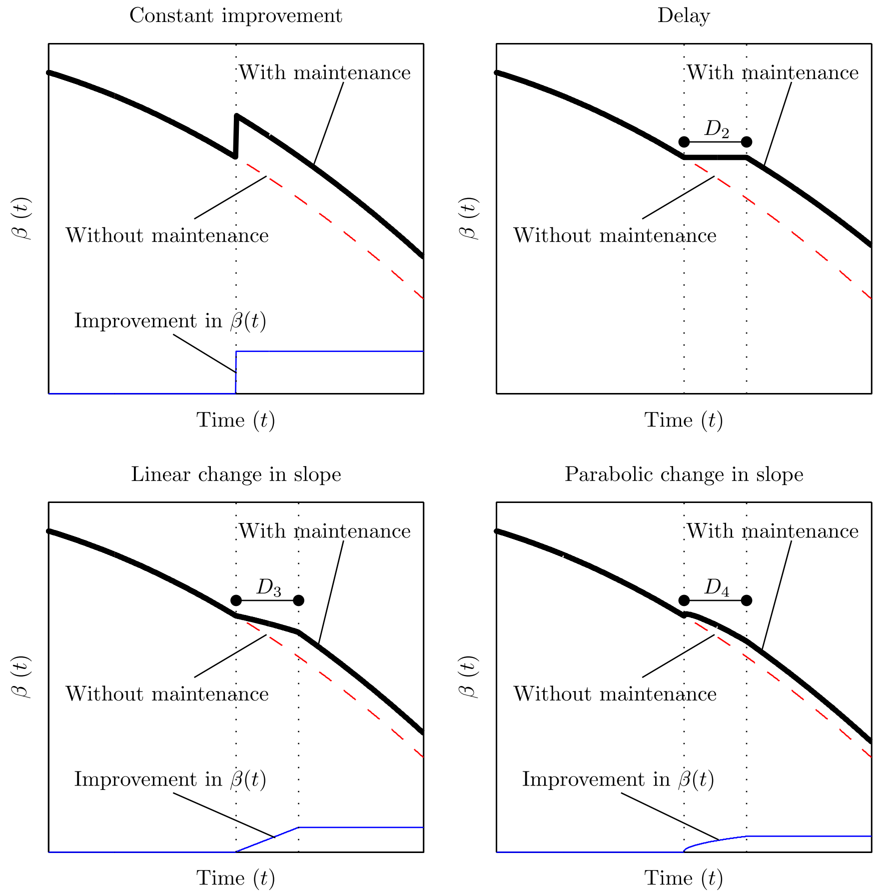

Fig. 6. Four basic components of PM actions. The continuous thin lines represent the actual improvement on the reliability index profile. For the second case, "Delay", this line has not been plotted, because its shape depends on the model chosen for $\beta(t)$, while the effects of the other three maintenance components are independent of the $\beta(t)$ model. Symbols D_2, D_3, and D_4 indicate the duration of the delay for the second component and the duration of the improvement for the third and fourth components, respectively.

parameters. The analyst has to define also a maximum number of PM interventions per bridge life-cycle n_{MPM}. This way, if the number of bridges is n_B and the number of samples to be simulated is n_S, then the total number of random instances for every maintenance parameter that will be generated is $n_{MPM} \cdot n_B \cdot n_S$. It should be noted that not all of these PM instances will be actually used by the procedure, since the optimization algorithm might select for some bridges maintenance schedules that require less interventions than n_{MPM}.

These values are generated at this stage of the analysis (as opposed to inside the GAs optimization) for two reasons. The first one is that this choice is more computationally efficient. The second one is that if they were generated directly by GAs, the fitness values would be strongly affected not only by the maintenance schedule (that is the design variable), but also by the random instances of the maintenance parameters. This would lead to a much slower convergence of the optimization technique.

Finally, for the chosen maintenance type, the cost (*costPM*) must be assessed.

3.5. Optimization process

The design variables of the optimization are the times of application of the PM actions on every individual bridge of the network. The objectives of the optimization are the maximization of the network performance and the minimization of the total maintenance cost (PM and RM). Since the two objectives are clearly conflicting, there will not be just one optimal solution, but a Pareto front. By definition, a solution (i.e. a certain PM schedule) is Pareto optimal if there is no other solution that leads to an improvement in one of the objectives without a worse result for at least one of the others. All the maintenance schedules that lay on the Pareto front are optimal. Therefore, the final choice will be made depending on the available financial resources, the desired performance target, and engineering judgment.

As already mentioned, GAs are a heuristic method that mimics the concepts of the biological theory of evolution. Trial sets of design variables (i.e. possible PM schedules) are called "individuals". A group of individuals is a "population". For every individual of the population, GAs numerically evaluate the objective function(s) and the result(s) is called "fitness" of that specific individual. Depending on their fitness, the individuals of the population are ranked and a new "generation" of individuals is produced. The individuals of this "offspring" can be generated by the combination of several techniques (e.g. "elite", "crossover", "mutation", "migration"). In general, the new generation will preserve the characteristics of the most fitted individuals of the previous population, but it will also include individuals generated more randomly, so that the whole feasible domain is investigated and local minima are avoided. Since they are based on trial solutions, multi-objective GAs automatically compute also the Pareto front. A detailed description of GAs can be found in [28].

The version of multi-objective GAs used in this paper is the one provided by the software Matlab [44], which is a modified version of the NSGA-II algorithm [37,45]. It should be mentioned that except for the GAs library, the entire procedure has been coded by the authors (including, for instance, the multi-objective fitness function used by GAs).

The proposed methodology can deal both with PM actions that can occur at any instant in the continuous time domain and with actions that can take place only at discrete time steps (e.g. six months, one year, five years) with which the lifetime is modeled. The analyst has to choose which domain, continuous or discrete, and, therefore, integer or real solutions. The proposed implementation of multi-objective GAs is meant for real variables. Therefore, if the integer domain is chosen, a "creation function" and a "mutation function" that have been written *ad hoc* for integer variables are exploited, together with the "two point crossover" method.

Consequently, an individual consists in the list of times at which the PM actions are applied to every bridge b. These maintenance interventions (whose random parameters have been previously simulated) are superimposed to the reliability index profiles $\beta_{bi}(t)$ (the subscript b runs over the bridges, i runs over the samples used for MCS). Every sample i then consists of a new reliability index profile $\hat{\beta}_{bi}(t)$ for every bridge b of the network, where the hat indicates that the PM has been superimposed.

At this point, a loop over all the considered time instants starts. For every time instant $t=T$, the bridge in/out of service states are simulated. This is done by means of the multivariate binary random variable $s_{bi}(t)$ defined as follows:

$$s_{bi}(t) = \begin{cases} 0 & \text{if bridge } b \text{ of sample } i \text{ is out of service at time } t \\ 1 & \text{if bridge } b \text{ of sample } i \text{ is in service at time } t \end{cases} \tag{10}$$

At the time instant T, the marginal probabilities of $s_{bi}(T)$ are assumed to be distributed as Bernoulli random variables, with mean value $\nu_{bi}(T)$:

$$s_{bi}(T) \sim \mathcal{B}ernoulli[\nu_{bi}(T)] \tag{11}$$

where

$$\nu_{bi}(T) = E[s_{bi}(T)] = \Phi[\hat{\beta}_{bi}(T)] \tag{12}$$

in which $E[\cdot]$ denotes the expected value operator and Φ is the standard Gaussian cumulative distribution function. The correlation between the variables $s_{bi}(T)$ of different bridges in the same sample i is given by Eq. (5). Knowing the marginal distribution and the correlation structure, variables $s_{bi}(T)$ can be simulated by means of the Dichotomized Gaussian method [46,47] in the modified version presented by Bocchini and Frangopol [48]. The result is a set of in/out of service states that perfectly match the target marginal distribution (i.e. the bridge reliability) and that is compliant, with good approximation, with the target correlation structure.

For every sample i, given the configuration of bridge in/out of service states, the network performance indicators are then computed. This task is accomplished by means of the technique proposed by Bocchini and Frangopol [48] by modifying the highway segment capacities depending on the state of their bridges. Then, it solves the joint traffic distribution and assignment problem. This means that the travels of the network users are distributed among the various nodes of the networks (e.g. cities) according to a so-called "gravitational model" [49] that measures the attraction between two points as inversely proportional to the time required to go from one to the other. Moreover, this procedure imposes the "user equilibrium" [50,51] by satisfying the following two principles [52]: (i) travel times on all routes actually used are equal and they are lower than those which would be experienced by a single vehicle on any unused route; (ii) the total network travel time is a minimum. It should be mentioned that this technique does not assume that the travel origins and destinations are fixed *a priori*, instead they are adaptive to the bridge in/out of service states. This choice complicates further the algorithm, but it is considered necessary to the purposes of a realistic transportation network model. Eventually, the technique computes two performance measures for every sample i that are the total travel time TTT_i and the total travel distance TTD_i. The former is the sum of all the times spent by all the users of the network to reach their destinations (considering the travels that start in the unit time):

$$TTT_i(T) = \sum_{x \in X} \sum_{y \in Y} \int_0^{f_i^{xy}(T)} \tau^{xy}(f)\, \mathrm{d}f \tag{13}$$

where X is the set of nodes of the network; Y is the subset of nodes connected to node x; $f_i^{xy}(T)$ is the traffic flow (number of car-equivalent vehicles per unit time) that transit over the highway segment between nodes x and y, as computed by the previously mentioned technique for sample i; $\tau^{xy}(f)$ is the time required to cover the highway segment x–y with traffic flow f. Similarly, the Total Travel Distance is the distance covered by all the travels occurred in the unit time and is computed as follows:

$$TTD_i(T) = \sum_{x \in X} \sum_{y \in Y} f_i^{xy}(T) \cdot d^{xy} \tag{14}$$

where d^{xy} is the length of highway segment x–y. The overall performance of the network is represented by the indicator $\Gamma_i(T)$:

$$\Gamma_i(T) = \frac{1}{\gamma_T \cdot TTT_i(T) + \gamma_D \cdot TTD_i(T)} \tag{15}$$

where γ_T is a balancing factor (cost) associated with the time spent by the network users and γ_D is a similar factor associated with the distance traveled. More details on the traffic distribution and assignment algorithm can be found in [53,27]. Actually, since the number of possible combinations of the s_b variables (i.e. possible network layouts with bridges in service and out of service) is usually much lower than the number of calls made by the optimization code (i.e. number of MCS samples multiplied by the number of individuals in the GAs population multiplied by the number of generations required to meet convergence), it is convenient to use the lookup table technique [54]. This means that the value of $\Gamma(T)$ is computed for all the possible combinations of s_b before the execution of the optimization algorithm and the results are stored. Then, GAs just retrieve the value of $\Gamma_i(T)$ for the specific bridge configuration of sample i at time T. If also the total number of combinations is too high, the bookkeeping technique might be proficiently employed to reduce the computational cost. In this case, the lookup table is built by GAs. Every time the network performance must be evaluated, the code checks whether $\Gamma(T)$ has already been computed before for the same configuration. If yes, the available result is retrieved; if not, the traffic distribution and assignment routine is called and the result is added to the lookup table. Either one of these two computational tools can dramatically improve the efficiency of the algorithm.

At this point, the algorithm applies RM to the bridges that are out of service. This has three effects: (i) changes the reliability index profile for $t>T$; (ii) changes the PM schedule for $t>T$; and (iii) changes the total maintenance cost. RM is here modeled as retrofit. Therefore, the shape of the reliability index profile is set back to the initial one, including the initiation time during which the degradation does not occur, as shown in Fig. 7. Since it is very unlikely to perfectly restore the initial conditions, the shape of $\hat{\beta}_{bi}(t>T)$ is shifted down by a random quantity $\Delta \cdot [\beta_{bi}^0 - \hat{\beta}_{bi}(T)]$ where Δ could have a triangular distribution:

$$\Delta \sim \mathcal{T}riangular[0,0.5,0.2] \tag{16}$$

Moreover, all the PM interventions scheduled for $t>T$ are postponed by T. This might put some interventions after the end

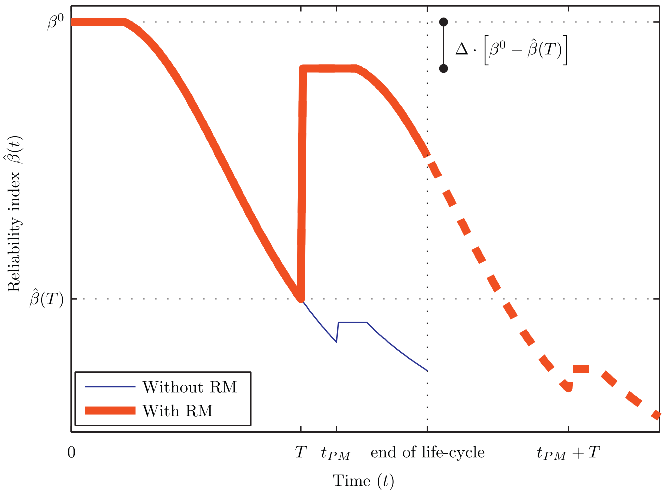

Fig. 7. Reliability index profile with and without the application of RM. The thin line shows the reliability profile for the entire life-cycle, without RM. It can be noted that a PM intervention was scheduled for time t_{PM}, shortly after the time instant T (and before the end of the bridge life-cycle). The bridge reaches a limit state at time T, therefore RM is applied. Due to this restoration, the shape of $\hat{\beta}(t)$ for $t>T$ is the same of the interval [0,life-cycle], but shifted down by the random quantity $\Delta \cdot [\beta^0 - \hat{\beta}(T)]$. Since the application of the PM is postponed after the end of the life-cycle (time $t_{PM}+T$), it actually does not take place.

of the life-cycle (see Fig. 7) and, therefore, they are neglected. Finally, the cost of RM must be updated, including this intervention. The cost of the a-th RM intervention on bridge b of sample i is assessed as a unit value *costRM* multiplied by the improvement in $\hat{\beta}_{bi}$ due to RM:

$$\begin{aligned} costRM_{i,b,a} &= costRM \cdot [\hat{\beta}_{bi}(T+1) - \hat{\beta}_{bi}(T)] \\ &= costRM \cdot (1-\Delta)[\beta^0_{bi} - \hat{\beta}_{bi}(T)] \end{aligned} \tag{17}$$

At this point, the algorithm proceeds to the next time step (T+1) and repeats the same operations.

When all the time steps have been considered, until the end of the investigated time period, the cost of the PM actions actually applied can be computed, neglecting those interventions that have been shifted after the end of the life-cycle.

Then, the mean value among the various samples i of the total maintenance present cost (PM and RM) can be computed as

$$cost = \frac{1}{n_S}\sum_{i=1}^{n_S}\sum_{b=1}^{n_B}\sum_{a}\frac{costPM}{(1+r)^{TPM_{i,b,a}}} + \frac{1}{n_S}\sum_{i=1}^{n_S}\sum_{b=1}^{n_B}\sum_{a}\frac{costRM_{i,b,a}}{(1+r)^{TRM_{i,b,a}}} \tag{18}$$

where index a runs on the various PM/RM interventions applied to one bridge of one sample; *costPM* is the cost of the fixed PM intervention; $costRM_{i,b,a}$ is computed by Eq. (17); $TPM_{i,b,a}$ is the time of the a-th application of PM on bridge b of sample i; $TRM_{i,b,a}$ is the time of the a-th application of RM on bridge b of sample i; and r is the discount rate of money.

The value of the network performance (NP) is computed as

$$NP = \left\{ \min_t \left[\frac{1}{n_S}\sum_{i=1}^{n_S} \Gamma_i(t) \right] - \Gamma^0 \right\} \cdot \frac{100}{\Gamma^{100} - \Gamma^0} \tag{19}$$

where Γ^{100} is the value of $\Gamma_i(t)$ when all the bridges are in service; and Γ^0 is the value of $\Gamma_i(t)$ when none of bridges is in service.

3.6. Problem formulation

All the described tasks provide the fitness of an individual of the GAs optimization procedure. Therefore, the analytical formulation of the problem is

Find:

$$TPM_{b,a} \tag{20}$$

so that

$$NP = \text{maximum} \tag{21}$$

$$cost = \text{minimum} \tag{22}$$

subject to the constraints

$$cost \leq \text{available fundings} \tag{23}$$

$$a \leq n_{MPM} \tag{24}$$

$$TPM_{b,a} < \text{prescribed time horizon} \quad \forall b,a \tag{25}$$

Other constraints could be added to narrow the feasible domain. For instance, Okasha and Frangopol [38] proposed to require to leave at least two years between any maintenance interventions on the same bridge and to avoid any PM intervention in the first two and last two years of the life-cycle. These and other linear constraints could be easily added using the proposed multi-objective GAs implementation.

As already mentioned, the final result is a Pareto optimal set of PM schedules. Each of them will yield a certain total cost and a network performance and the final decision about the best trade-off of the two competing criteria has to be taken by engineering and economic judgment.

4. Numerical example

The sample network shown in Fig. 8 is used for the numerical application. Input data for the network analysis have been computed in [27] and are collected in Tables 1 and 2.

The correlation length is estimated using HAZUS to perform nine fragility analyses for earthquakes with the nine epicenters in

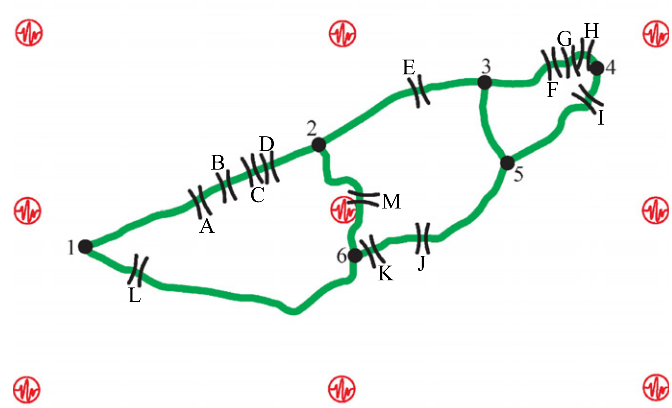

Fig. 8. Layout of the network used for the numerical example. Numbers indicate the six nodes, letters indicate the 13 bridges. The nine epicenters used for the correlation assessment are also indicated by a symbol.

Table 1
Node characteristics.

Node	Trips generated Cars/h	Trips attracted Cars/h
1	10500	10500
2	12000	12000
3	5000	5000
4	8500	8500
5	9500	9500
6	10000	10000

Fig. 8, with magnitude 5.5 and depth 10 km. Eqs. (3) and (4) give a set of discrete points that can be fitted by least square method and the analytical expression in Eq. (5). The result for the correlation length is $\lambda = 15.8$ km.

Different reliability models have been chosen for the various bridges of the networks. For the seven main bridges (letters D, E, G, I, J, L, M in Fig. 8) a deterministic reliability index profile is assumed to be known and described by the exponential model in Eq. (9). Bridges A and B are assumed to have an uncertain bilinear reliability index profile described by Eq. (6), while for bridges C and F the quadratic model in Eq. (7) has been used and for bridges H and K the square root model in Eq. (8). The characteristics of the bridges are collected in Table 3, the probability distributions of the model parameters can be found in Table 4, and the reliability index profiles are plotted in Figs. 9 and 10. The values of the reliability indexes are lower than those presented in Fig. 5 because in this case it is assumed to combine all the major failure modes of the bridges.

Table 2
Highway segment (edge) characteristics for the undamaged state (all bridges in service).

1st node[a]	2nd node[a]	Free flow time (min)	Practical capacity (cars/h)
1	2	10.1	4000
2	3	7.2	4000
3	4	5.2	4000
4	5	8.0	5000
5	6	6.0	5000
6	1	11.3	5000
2	6	7.6	4000
3	5	3.4	4000

[a] All the highway segments are represented by two edges, since all of them are both ways. Therefore, the network model has a total of 16 edges. Taken from [27].

Table 3
Bridge characteristics.

Bridge	x position (km)	y position (km)	Model (see Fig. 5)
A	3.803	4.453	Bilinear
B	5.288	5.526	Bilinear
C	7.313	6.493	Quadratic
D	7.322	6.493	Exponential
E	17.589	11.503	Exponential
F	26.727	13.358	Quadratic
G	28.431	13.565	Exponential
H	28.516	13.268	Square root
I	28.982	12.431	Exponential
J	17.469	2.412	Exponential
K	13.670	0.000	Square root
L	0.000	0.371	Exponential
M	14.084	4.639	Exponential

The PM actions are assumed to be a combination of three of the four available components (see Fig. 6): a small constant increase on the level of $\beta(t)$, a delay in the degradation process and a linear improvement in $\beta(t)$. The probability distributions of the four parameters that tune the involved components are collected in Table 5. The uncertain effect of the selected PM action on $\beta(t)$ is shown in Fig. 11 as superposition of the three components.

At this point, samples of the involved random variables are generated. For the parameters of the reliability index profiles, $n_S = 100$ samples for every bridge are generated. For the parameters of the preventive maintenance components the samples are $n_{MPM} \cdot n_B \cdot n_S = 2 \cdot 13 \cdot 100 = 2600$. The samples of the parameters for the reliability index models are used into Eqs. (6)–(9) to compute the n_S samples $\beta_{bi}(t)$ for every bridge b (see Fig. 10). For the main bridges with deterministic reliability profile, the samples are all the same.

The relative costs of the interventions are assumed as follows: $costPM = 0.55$, $costRM = 1$. The balancing coefficients for the travel time cost and travel distance cost are $\gamma_T = 2 \times 10^{-7}$ and $\gamma_D = 10^{-7}$. One of the factors that can heavily affect the optimal scheduling is the discount rate of money r. In fact, a higher discount rate makes strongly preferable all the expenses (PM and RM) that are delayed in time. In order to investigate the effect of the annual discount rate of money, three values of r have been considered: 5%, 2%, and 0%. The maximum allowable present cost is strongly dependent on the discount rate too. Therefore, three different maximum costs have been assumed: 3, 10.5 and 26 for $r = 5\%$, 2%, and 0%, respectively.

The network life-cycle is set equal to 75 years and the time steps of the analysis are five years. It should be noted that, as already mentioned, the domain of application of the PM actions

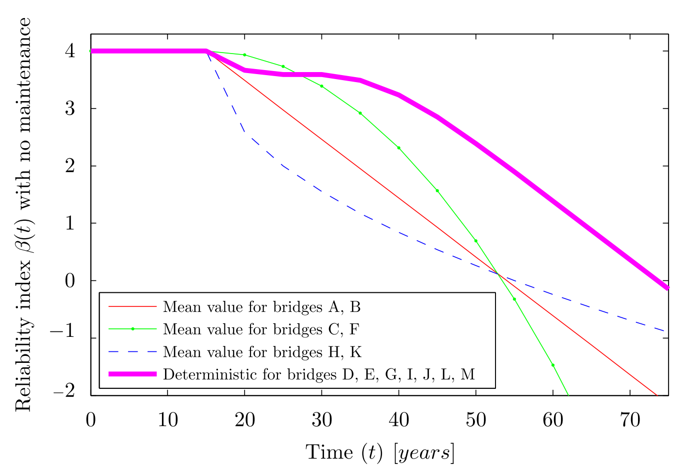

Fig. 9. Reliability index profiles of the 13 bridges considered in the numerical example, under no maintenance actions.

Table 4
Probability distributions of the reliability index profile model parameters.

Parameter	Units	Distribution	Mean	Standard deviation	Minimum	Maximum
β^0	–	Lognormal	4	1.5	0	$+\infty$
T^I	years	Lognormal	15	5	0	$+\infty$
R^L	years^{-1}	Uniform	0.1025	0.56292	0.005	0.2
R^Q	years^{-2}	Uniform	0.002703	0.001482	0.000135	0.00527
R^S	years$^{-0.5}$	Uniform	0.6323	0.34641	0.0323	1.2323
R^E	years^{-1}	Deterministic	0.1025	0	0.1025	0.1025
S	years	Deterministic	16.5	0	16.5	16.5
Q	–	Deterministic	6	0	6	6

For the exponential model, also parameters $\beta^0 = 4$ and $T^I = 15$ have been assumed to be deterministic.

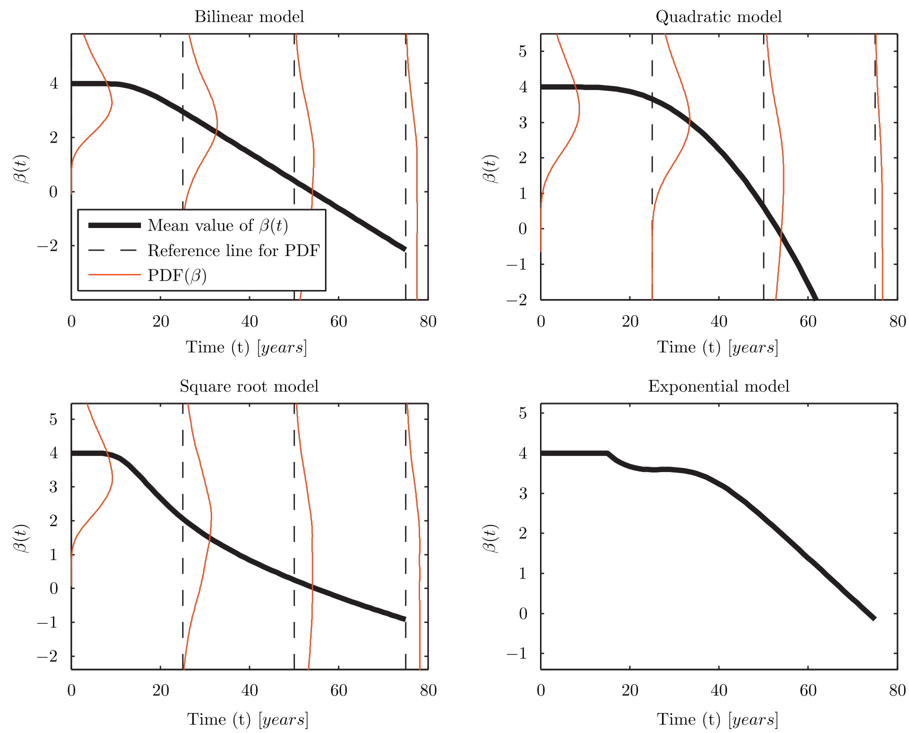

Fig. 10. Probability density functions of the reliability index profiles of the 13 bridges considered in the numerical example in case of no maintenance interventions (neither PM nor RM). The main bridges have a deterministic reliability index model.

Table 5
Probability distributions of the maintenance parameters.

Component	Parameter	Units	Distribution	Mean	Standard deviation	Minimum	Maximum
1	Increase	–	Uniform	0.55	0.02083	0.3	0.8
2	Duration	years	Triangular	7.667	1.0556	5	10
3	Rate	$\text{years}^{-0.5}$	Uniform	0.0015	8.33×10^{-8}	0.001	0.002
3	Duration	years	Uniform	7.5	2.0833	5	10
RM	Δ	–	Triangular	0.233	0.010556	0	0.5

"Component 1" indicates the sudden increase in $\beta(t)$, "Component 2" indicates the delay in the deterioration process, "Component 3" indicates the linear improvement in the deterioration rate; "RM" indicates the RM parameter.

can be either continuous or discrete and it is not related to the time steps used for the analysis. In case it is discrete, the discretization used for the PM interventions can be different from the discretization used for the analysis, and therefore for $\beta(t)$. In fact, even when specific studies on the time-dependent profile of $\beta(t)$ are available, data are usually provided at relatively coarse time steps [43], while the resolution of the optimal PM planning should be no longer than one or two years. Therefore, the algorithm has been implemented so that it is able to handle different discretizations in the time domain. In this numerical example, the PM interventions are supposed to be applied on a yearly base, while the time steps used for the analysis and for $\beta(t)$ are five years long.

With all these inputs, the optimization problem is the following:

Find:

$$TPM_{b,a} \quad \text{with } b=1,2,\ldots,13;\;\; a=1,2 \tag{26}$$

so that

$$NP=\text{maximum} \tag{27}$$

$$cost=\text{minimum} \tag{28}$$

subject to the constraints

$$cost \le \begin{cases} 3 & \text{for } r=5\% \\ 10.5 & \text{for } r=2\% \\ 26 & \text{for } r=0\% \end{cases} \tag{29}$$

$$a \le 2 \tag{30}$$

$$TPM_{b,a} < 75\text{ years} \quad \forall b,a \tag{31}$$

Eq. (26) shows that GAs have to optimize a total of $n_B \cdot n_{MPM} = 13\cdot 2 = 26$ design variables. These variables are the years of application of the PM actions on the various individual bridges. They can also take value 0, meaning that PM on a certain bridge should be applied less than n_{MPM} times. Therefore, the algorithm not only optimizes the times of application of PM but also, automatically, the number of actions that should be applied to each bridge.

The initial population is composed by 100 individuals and after 100 generations GAs provide the Pareto fronts in Fig. 12. Three optimal solutions are highlighted for each discount rate and the relative PM schedules are reported in Table 6 and Fig. 13. For instance, considering the case where r=5%, schedule "S1" yields a lower cost, but it provides a lower level of network performance. Schedule "S3" requires more fundings, but it guarantees a high

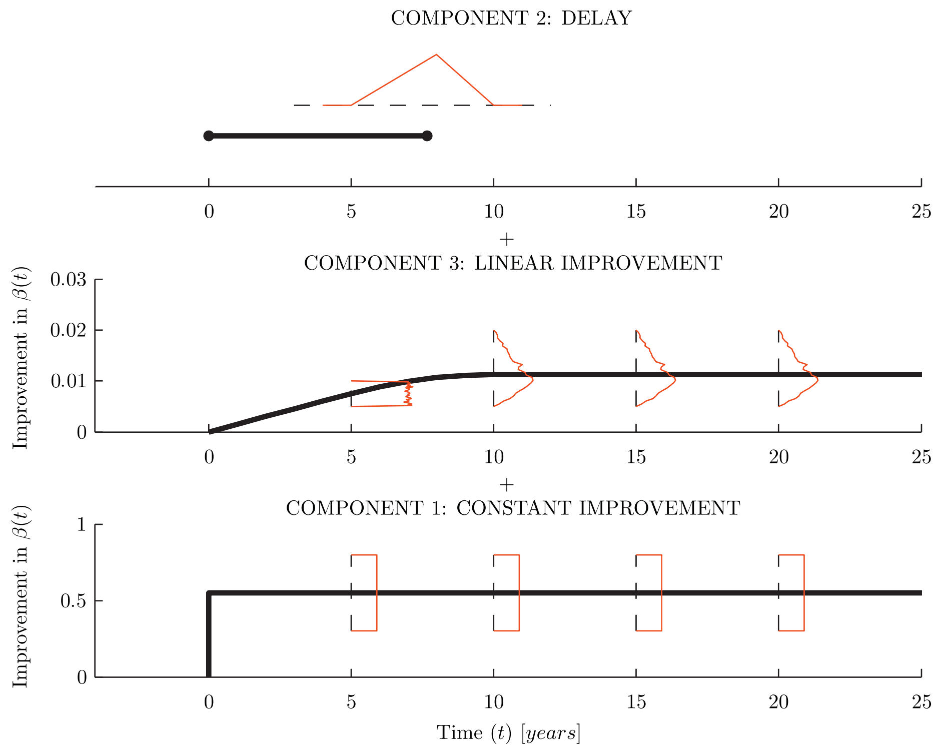

Fig. 11. Applied PM action. The applied PM intervention is the superposition of a delay in the degradation process (main effect), a small constant improvement in $\beta(t)$, and a very small linear improvement in $\beta(t)$. The continuous thin lines represent the probability density functions of the effect of PM. The probabilistic characterizations of the three components are collected in Table 5.

network performance level for the entire life-cycle. Schedule "S2" is a trade-off. Similar considerations can be made for S4–S6 and for S7–S9. Decision makers can chose the schedule depending on the desired performance and the amount of fundings that they can allocate.

Some interesting considerations can be drawn from these results, focusing, for instance, on the case where $r=5\%$.

Fig. 12 shows that the Pareto front for $r=5\%$ presents a rapid increase after the region where solution S2 is located. This means that all the optimal solution on the right of the group to which S2 belongs, such as S3, determine a small increase in the network performance (*NP*), with a large increase in the expected total cost. On the contrary, the optimal solutions on the left, such as S1, yield a limited reduction in the total cost, with a large loss in the network performance, as graphically shown in Fig. 14. Therefore, in this specific example, the overall most convenient solutions are those that belong to the group of S2.

Table 6 shows that it is not convenient to apply the maximum number of PM interventions ($n_{MPM}=2$) on all the bridges. In particular, all optimal schedules S1–S3 plan only one PM action on bridges I, J, K, L, and M. Solutions S2 and S3 require two PM interventions on bridge D around years 46 and 65 or 69, whereas solution S1 plans only one intervention in between (year 56).

Finally, it is worth noticing that all the proposed optimal solutions for $r=5\%$ exclude PM interventions for bridge E. This bridge is considered one of the main bridges, therefore it has the same deterministic $\beta(t)$ model of bridges D, G, I, J, L, and M. Nevertheless, while the algorithm suggests one or two PM interventions for the other main bridges, it computes that interventions on bridge E are anti-economical. This is a proof of the fact that the maintenance optimization at the network level gives very different results from the maintenance optimization at the individual bridge level. In fact, for bridges with the same reliability profile, the algorithm suggests two (bridges G and D for S2 and S3), one (bridges I, J, L, M, and D only for S1) or none PM intervention, depending on (i) the location of the bridge on the network, (ii) the network topology, (iii) the traffic characteristics, (iv) the correlation structure between the bridge states, and other network characteristics. In this particular case, bridge E is the only one on the highway segment 2–3, therefore even if it is out of service, the delay that it determines cannot be combined with the delay determined by other bridges out of service on the same highway segment. Moreover, Table 1 shows that the number of trips generated and attracted by node 3 is very limited. Therefore, most of the traffic flow on segment 2–3 can be easily detoured on the other segments of the network. In conclusion, even if bridge E has the same characteristics of the other main bridges, its condition is less important to the purposes of the network performance optimization, thus GAs suggest to allocate the available resources to other bridges.

The value of the discount rate of money not only heavily affects the total present cost, as shown in Fig. 15, but also the optimal distribution of PM interventions. In fact, Fig. 16 shows the density of the various years of PM application in the Pareto optimal schedules computed for different values of r. When the discount rate is 0% or 2%, the first optimal PM intervention for every bridge is around years 20–25 and the second around years 45–55 (see the mode values of the histograms). On the contrary, when $r=5\%$, very few PM interventions (about 15% of the total) are applied in the first half of the life-cycle. Most of the maintenance actions are applied after year 40 and many of them (more than 20%) are scheduled for the last 15 years of the life-cycle, to avoid very low values of $\beta(t)$.

Another interesting indicator is the number of suggested PM applications. When $r=0\%$, the solutions laying on the Pareto front recommend 1.81 PM interventions per bridge on average (i.e. two interventions per bridge are scheduled for almost all the bridges in all the optimal solutions). This number decreases to 1.49 for

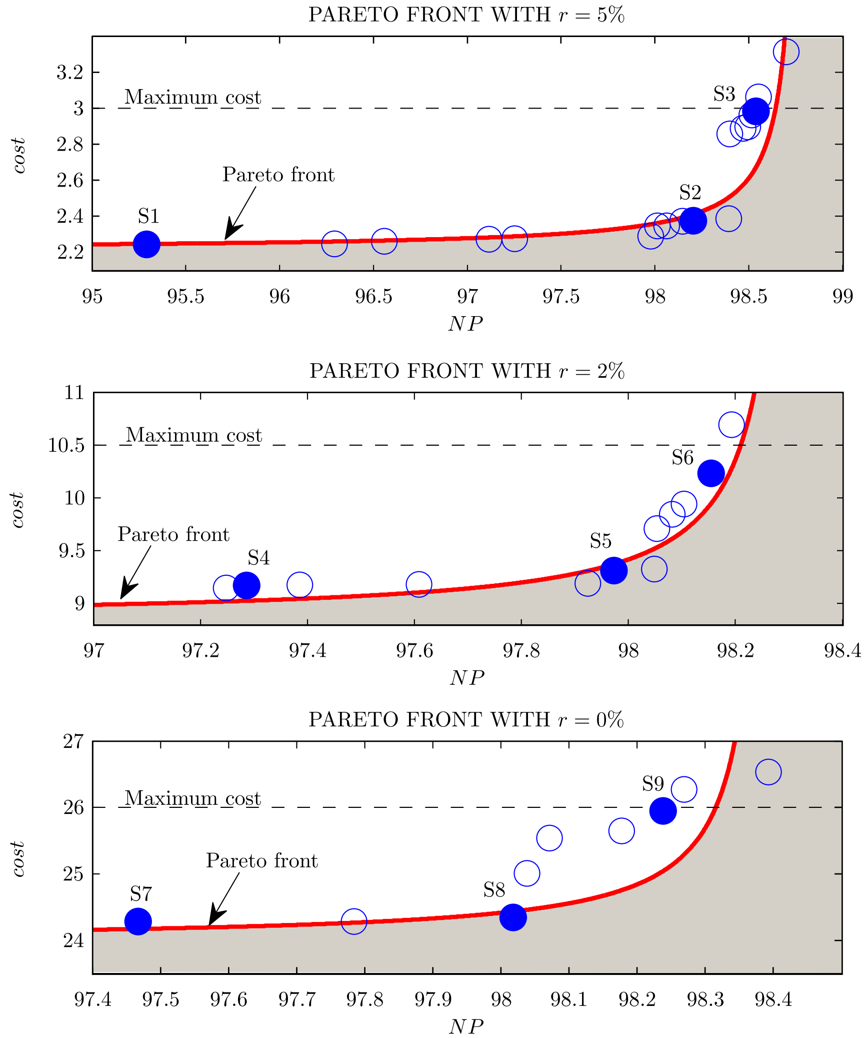

Fig. 12. Pareto fronts. All the dots represent Pareto optimal solutions. Three optimal schedules for each money discount rate are highlighted (black dots). The years of preventive maintenance application of the highlighted solutions are reported in Table 6.

Table 6
Pareto optimal schedules.

Bridge	Years of PM application																	
	Solutions with r=5%						Solutions with r=2%						Solutions with r=0%					
	S1		S2		S3		S4		S5		S6		S7		S8		S9	
A	32	45	40	51	32	38	24	42	24	42	23	50	19	48	19	48	19	45
B	32	48	32	47	32	38	24	47	24	50	23	50	19	48	19	48	19	48
C	43	55	46	56	46	57	24	49	24	57	23	56	24	51	24	51	24	51
D	56	–	46	55	46	57	38	56	38	55	30	51	24	51	24	51	24	44
E	–	–	–	–	–	–	54	–	54	–	54	–	19	48	19	48	19	48
F	48	58	46	57	46	57	45	–	45	–	45	–	53	–	53	–	53	–
G	48	58	48	58	46	57	41	–	41	–	41	59	47	59	47	59	47	–
H	18	38	18	37	18	38	11	44	11	44	11	42	7	33	7	33	7	40
I	65	–	45	–	45	–	48	–	48	–	48	–	22	43	22	43	22	48
J	55	–	69	–	69	–	58	–	58	–	63	–	19	43	19	45	72	–
K	48	–	48	–	75	–	19	21	19	21	61	–	11	38	11	44	11	40
L	48	–	48	–	48	–	59	–	59	–	47	–	24	46	24	46	34	52
M	60	–	60	–	18	–	49	–	49	–	25	50	24	46	24	46	26	47
cost	2.24		2.38		2.98		9.17		9.31		10.2		24.3		24.4		25.9	
NP	95.3		98.2		98.5		97.3		98.0		98.2		97.5		98.0		98.2	

For each solution the years of applications of the interventions are reported. The bottom lines show the results for the two objective functions: total present cost and minimum network performance.

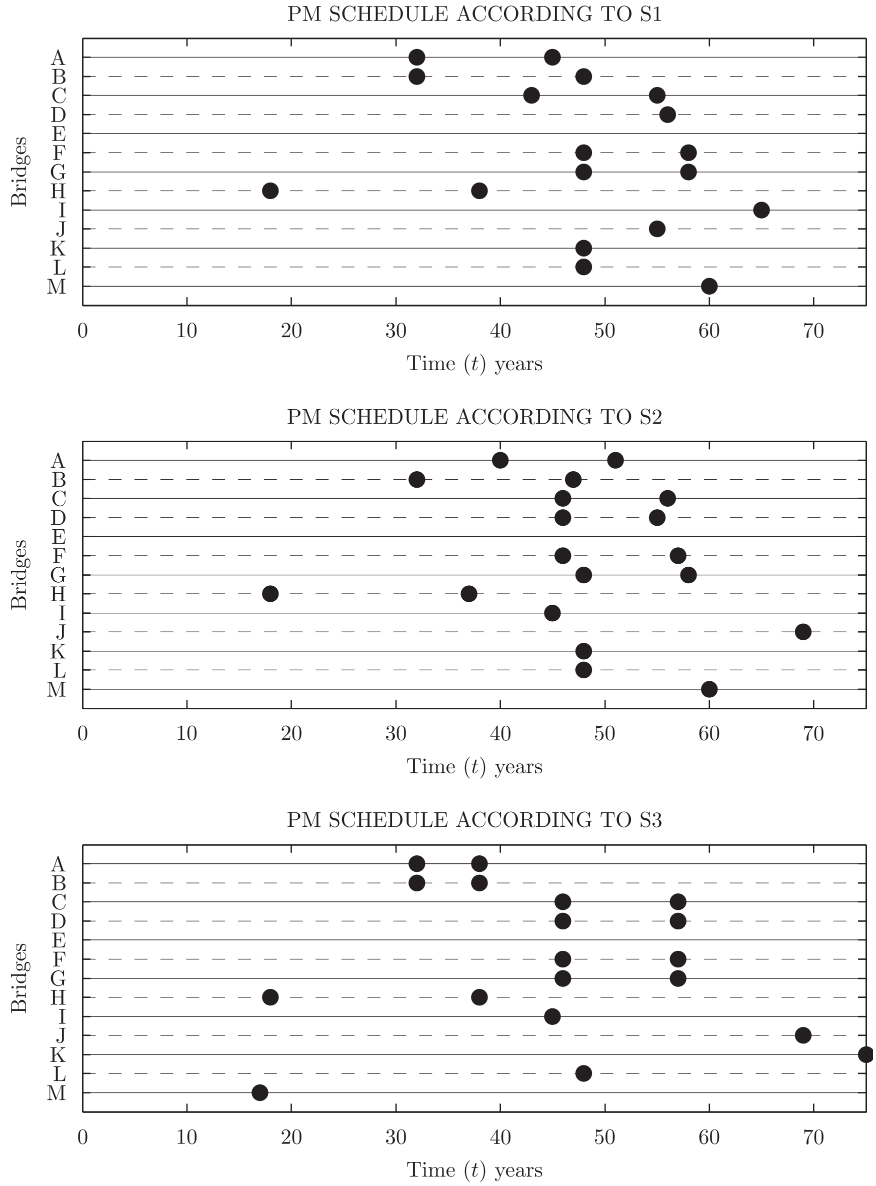

Fig. 13. Three optimal schedules. Graphical representation of the PM interventions according to S1, S2 and S3.

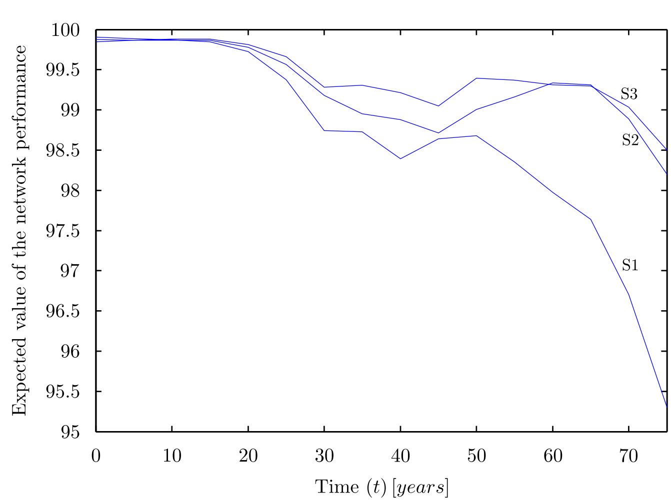

Fig. 14. Comparison of the time-dependent network performance indicator defined as $\{[1/n_S \sum_{i=1}^{n_S} \Gamma_i(t)] - \Gamma^0\} \cdot 100/(\Gamma^{100} - \Gamma^0)$. Note that the value of *NP* used as objective for the GAs optimization is the minimum over time t (for each of the plotted functions). Since the actual performance indicator is uncertain, the expected value is plotted.

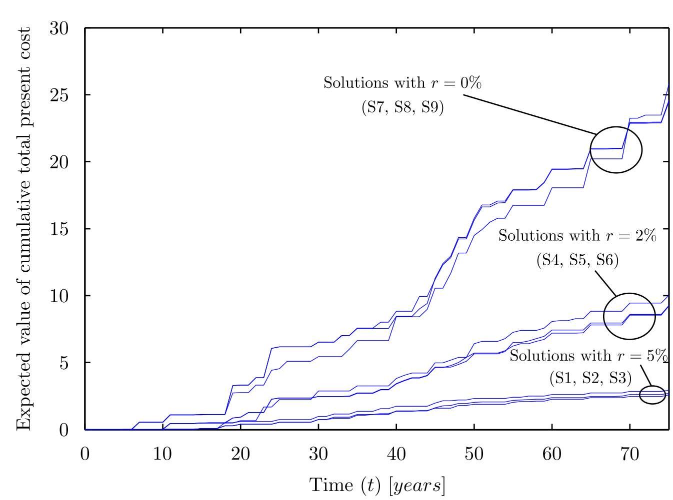

Fig. 15. Comparison of the cumulative cost of the representative optimal solutions (S1–S9). Since the actual cost is uncertain, the expected value is plotted.

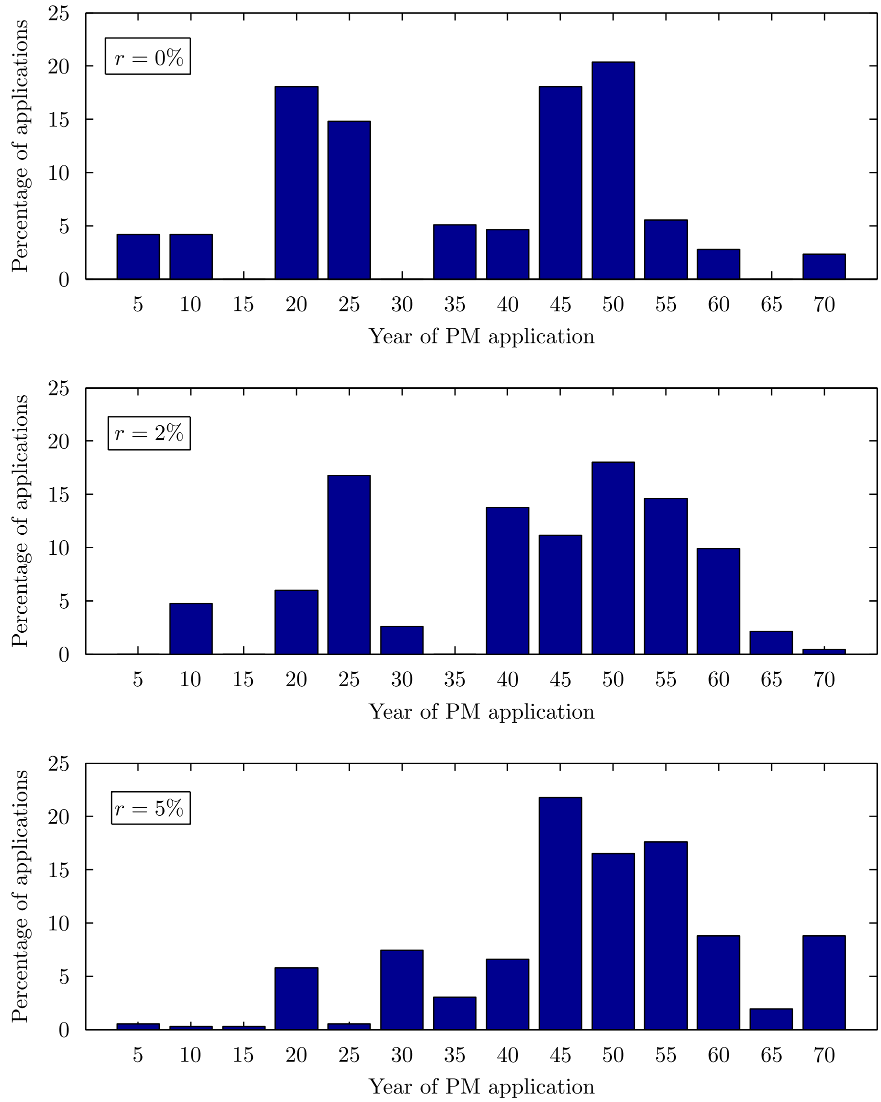

Fig. 16. Influence of the money discount rate r on the optimal scheduling. Higher rates determine optimal solutions with more interventions delayed toward the end of the life-cycle.

r=2% and to 1.46 for r=5%. This means that when the discount rate is higher, it becomes more convenient to apply less PM interventions, possibly only one (or even none) for the bridges that affect less the network performance.

As expected, early and more frequent applications of the PM actions guarantee higher reliability indexes for the individual bridges. Fig. 17 shows that schedule S8 (optimal for r=0%) implies values of $\beta(t)$ around 2.5, and never below 2. Fig. 17 also shows that EM actions have not been considered. There is no threshold $\overline{\beta}$ below which the individual bridges are systematically maintained. As already mentioned, for the bridges that are thoroughly modeled and monitored, and therefore the $\beta(t)$ profile is known with good confidence, it is possible to add also this kind of intervention.

The case with r=5% has been solved on a single-core desktop computer produced in 2006 with 2 GB of RAM and a 3.4 GHz processor. The whole analysis took 323.86 min (similar times have been spent for the other discount rates of money). This CPU-time is considered low for such a study that provides a plan for 75 years of maintenance. Therefore, for real applications, a higher number of samples is suggested, aiming at increasing the accuracy of the simulation.

5. Conclusions

A computational framework for the Pareto optimal PM scheduling at the bridge network level has been presented. An advanced probabilistic model of the bridge reliability and an efficient algorithm for the network traffic flow distribution and assignment are used together with multi-objective GAs. The final result is a Pareto front of optimal solutions that minimize the total present maintenance cost and maximize the network performance.

The main assets of the proposed approach can be summarized as follows.

1. Even if, for some of the network bridges, accurate studies on the reliability index time-dependent profiles are not available, the proposed procedure can still be applied taking advantage of probabilistic models of $\beta(t)$. These models reflect the increase in time of the epistemic uncertainty and have already been calibrated for shear and bending limit states [21]. Moreover, a new exponential model, with the advantage to be very versatile, has been introduced.

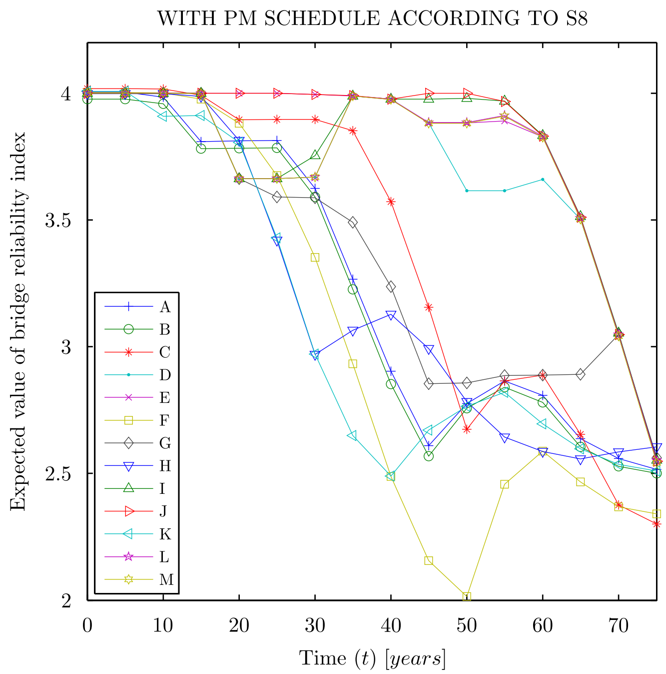

Fig. 17. Expected value of the bridge reliability index including PM and RM actions.

2. The effects of the correlation between the bridge states all across the network, which have been proved to have a strong impact on the network performance, are included and accounted for in the simulation procedure.
3. The PM interventions are modeled by superposition of four basic components. This results in the ability to describe virtually every possible real maintenance action. Moreover, the same computational tools can be used in future versions of the algorithm also to describe other environmental and/or mechanical stressors that modify $\beta(t)$, such as the sudden damage induced by extreme events that do not lead to total collapse, but still cause a significant reduction of the reliability.
4. All the PM actions are assumed to be the same, even if the actual effects on $\beta(t)$ are modeled as random. Nevertheless, the results in terms of network performance, total expected present cost and Pareto optimal front for different types of PM actions can be easily compared running the code several times. A possible future development will be to consider the combination of different PM actions in the same run of the optimization code.
5. The concept of RM has been introduced. The use of this type of intervention in the network maintenance model provides more realistic and conservative results than those obtained by considering EM only. In fact, EM requires accurate knowledge of the bridge time-dependent reliability, that is usually unavailable for most of the bridges of a network. If the information on the individual bridge performance indexes is unavailable, it is not possible to know when they cross an acceptable threshold. Therefore, it is not realistic to apply EM at that instant. If EM were included in the model and then actually not applied, the result would have been an unconservative PM schedule.

The numerical example presents a first application of the procedure, but further studies and development can be introduced.

One of the critical aspects of the methodology is the computational efficiency. In fact, the use of a MCS nested in a GAs optimization can be very time-consuming. The use of computational techniques such as lookup table and bookkeeping has dramatically reduced the computational cost with respect to the early versions of the code. A general fine tuning of the code has been necessary to keep the CPU time low. For instance, some tasks, such as the computation of the bivariate Normal cumulative distribution function for the simulation of the dichotomized Gaussian are called more than 3×10^{10} times in a run of the code, so it has been necessary to make their execution extremely efficient. The average time required by 10^{10} calls of the bivariate Normal cumulative distribution function has been reduced from about 67 h (with the original and most standard implementation) to 61 min by means of the lookup table technique, without any detectable reduction of accuracy.

The current version of the code allows to consider a number of samples on the order of 10^5 in a reasonable amount of time. Importance sampling techniques, Latin hypercube, or optimal stochastic reduced-order models could be proficiently employed to improve the representation of the stochastic space (or to reduce the number of samples and, therefore, the computational cost).

Finally, the proposed procedure certainly needs further calibration. For instance, the best values of the GAs parameters should be found for different network sizes and time horizons. Moreover, the costs and the effects of the interventions should be assessed more accurately based on real data. To this purpose, other realistic bridge networks and maintenance interventions have to be considered in future studies. Unfortunately, the authors are not aware of any existing data of a real cost of an entire bridge network. Since the interest in this topic is rapidly growing, it is very likely that this kind of data will become available in a near future and it will be very valuable for comparison, verification and improvement of the presented framework. However, the presented computational and theoretical approach appears complete and solid.

Notation

$\sim$	distributed as
A	scaling factor
a	application of PM/RM index
b	bridge index
c	secondary bridge index
$cost$	total maintenance present cost (PM and RM)
$costRM$	unit RM cost
$costRM_{i,b,a}$	cost of the a-th RM on bridge b of sample i
$costPM$	cost of the fixed PM intervention
d^{xy}	length of highway segment x–y
$E[\cdot]$	expected value, mean value
e	extreme event scenario index
$f_i^{xy}(T)$	traffic flow on highway segment x–y for sample i at time T
i	sample index
n_B	number of bridges of the network
n_E	number of considered extreme event scenarios
n_S	number of samples for MCS
n_{MPM}	maximum number of PM actions per bridge
K	shifting factor
$l_{b,e}$, $l_{c,e}$	damage level of bridge b/c in the e-th extreme event scenario
NP	lifetime network performance indicator
$\mathcal{P}(\cdot)$	probability of the event in brackets
Q	degradation parameter of the exponential model
r	discount rate
R^L, R^Q, R^S, R^E	slope parameters of the four reliability models
REL_{E_1}	bridge reliability, with respect to event E_1
S	shape parameter of the exponential model

TF_{E_1}	time to failure, with respect to event E_1
$TRM_{i,b,a}$	time of the a-th RM on bridge b of sample i
$TPM_{i,b,a}$	time of the a-th PM on bridge b of sample i
X	set of nodes of the transportation network (cities)
x	network node index
Y	subset of nodes of the network that are connected to node x
y	secondary network node index
$s_{bi}(t)$	in/out of service state of bridge b of sample i at time t
T^I	initiation time of the degradation process
$TTD_i(T)$	Total Travel Distance indicator for sample i at time T
$TTT_i(T)$	Total Travel Time indicator for sample i at time T
t	time
$\beta(t)$	reliability index profile
$\beta_{bi}(t)$	reliability index profile of bridge b of sample i without maintenance
$\hat{\beta}_{bi}(t)$	reliability index profile of bridge b of sample i with maintenance
β^0	initial value ($t=0$) of the reliability index
$\Gamma_i(T)$	network performance indicator for sample i at time T
Γ^0	network performance indicator when all the bridges are out of service
Γ^{100}	network performance indicator when all the bridges are in service
γ_D	balancing factor (cost) associated with the distance covered
γ_T	balancing factor (cost) associated with the time spent
Φ,Φ^{-1}	standard Gaussian cumulative distribution function and inverse
λ	correlation distance (or correlation length)
μ_b,μ_c	average value of damage for bridges b and c
ρ	correlation coefficient
σ_b,σ_c	standard deviation of damage for bridges b and c
$\tau^{xy}(f)$	time to cover highway segment x–y with traffic flow f
ξ_{bc}	distance between bridges b and c

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