

Incremental Nonlinear Model for Predicting Pavement Serviceability

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Abstract: A recursive nonlinear model was developed for the prediction of pavement performance as a function of traffic characteristics, pavement structural properties, and environmental conditions. The model highlights some of the advantages of relaxing the linear restriction that is usually placed on the specification form of pavement performance models. First, a functional form that better represents the physical deterioration process can be used. Second, the estimated parameters are unbiased, owing to a proper specification and the use of sound statistical techniques. Finally, the standard error of the prediction is reduced by half that of the equivalent existing linear model. This improved accuracy has important economic implications in the context of pavement management. The model developed as part of this research enables the determination of an unbiased exponent of the so-called power law and of the equivalent loads for different axle configurations. The estimated exponent confirms the value of 4.2 traditionally used. However, it should be noted that this exponent is only to be used for determining damage in terms of serviceability. On the other hand, equivalent loads estimated for different axle configurations tend to differ from traditionally used values.

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Introduction

The objective of this research was the development of sound flexible pavement performance models to be used primarily for the management of the road infrastructure because accurate prediction of pavement performance is important for effective management of the infrastructure.

Owing to the great complexity of the road deterioration process, performance models are, at best, only approximate predictors of expected conditions. Models that only produce a deterministic prediction of performance without any quantification of the accuracy of the prediction are unrealistic; hence, an estimation of the prediction error is essential. The absence of such an estimate imposes important limitations on the applicability of the performance prediction model.

The Present Serviceability Index (PSI) was developed in the early 1960s and constituted the first comprehensive effort to establish performance standards based upon considerations of riding quality [Carey and Irick 1960; Highway Research Board (HRB) 1962]. A panel of highway users from different backgrounds evaluated seventy-four flexible pavement sections and rated them on a discrete five-point scale (0 for poor, 5 for excellent). This

experiment gave origin to the Present Serviceability Rating (PSR). The PSR was found to correlate highly with surface roughness and, to a lesser extent, with rutting, cracking, and patching.

Models for the prediction of riding quality are very important to highway agencies for the purpose of managing their road network. The prediction of riding quality is also important for road pricing and regulation studies. Both the rate of riding quality deterioration over time and the contribution of the various factors to such deterioration are important inputs to these studies. Useful models are those that establish the contribution of pavement structure, traffic, environment and any other factors that are relevant for cost allocation.

Data Considerations

There are a number of possible data sources that have been used for the development of pavement performance models. Some of these sources are: (1) randomly selected in-service pavement sections; (2) in-service pavement sections selected according to an experimental design; (3) purposely built pavement test sections subjected to the action of actual traffic and the environment; and (4) purposely built pavement test sections subjected to the accelerated action of traffic and environmental conditions.

Data originating from in-service pavement sections subjected to the combined actions of highway traffic and environmental conditions are those that most closely represent the actual deterioration process of pavements in the field. All other data sources would produce models that would suffer from some kind of bias or restrictions unless special considerations are taken into account during the parameter estimation process. Some of these considerations are briefly described in the following paragraphs.

The most common problems encountered in models developed from randomly selected in-service pavement sections are caused by unobserved events typical of such data sets, and by the prob-

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lem of endogeneity bias generated by the use of endogenous variables as explanatory variables (Paterson 1987; Prozzi and Madanat 2000). These two issues are discussed in detail in the following paragraphs.

Data-gathering surveys during experimental tests are usually of limited duration. This may cause problems when such data are used for developing predictive models of pavement failure, or other discrete events (such as cracking initiation). If only the events observed during the survey are included in the statistical analysis (ignoring the information of the after and before events) the resulting models would suffer from truncation bias. If censoring of the events is not properly accounted for, the model may suffer from censoring bias (Paterson 1987; Prozzi and Madanat 2000).

Another common problem is that of endogeneity bias. Pavements that are expected to carry higher volumes of traffic during their design life are designed to higher standards. The bearing capacity of these pavements is higher than those of pavements designed to carry lower volumes of traffic. Thus, any variable which is an indicator of a higher bearing capacity, such as the structural number, will indeed be an endogenous variable which is determined within the system and cannot be used as an independent explanatory variable. If such a variable were incorporated into the model, the resulting models would suffer from endogeneity bias (Madanat et al. 1995).

These problems can be accounted for through the use of statistical techniques that take into account the presence of truncation or endogeneity or, alternatively, by developing models that are based on data originating from in-service pavement sections whose selection is based on an experimental design.

Purposely built pavement sections subjected to the action of actual traffic and the environment are often desirable sources of data possible. However, time and budget limitations constrain this type of experiment to a very limited number. Constructing pavement test sections and subjecting them to the accelerated action of traffic and the environment solves some of the budgetary and time constraints. However, this produces models that are dependent on the testing conditions. One way of overcoming some of these limitations is through the use of data from multiple sources. Archilla and Madanat (2001) have successfully developed models for the prediction of pavement rutting by combining two different experimental data sources.

Modeling Approaches

Pavement performance models can be classified into two groups: empirical models and mechanistic models, depending on the approach followed for the development of the performance function. A third group comprises the so-called mechanistic–empirical models that make use of both mechanistic concepts and empirical modelling. Some of the main characteristics of each type are described in the following sections.

Empirical Models

In these models, the dependent variable is some indicator of pavement performance. Both subjective indicators (riding quality, serviceability, condition index, etc.) and objective indicators (roughness, rutting, cracking, etc.) are used as dependent variables. These performance indicators are related to one or more explanatory variables, such as pavement structural strength, traffic loading and environmental conditions. These models are often devel-

oped based purely on statistical considerations without any attempt to represent the actual physical phenomenon underlying the performance process. Different researchers have approached the problem in different ways, especially in the way in which the form of the model specification is developed.

In the majority of empirical work found in the literature, explanatory variables are used and discarded solely on the basis of consideration of the statistics of their parameters. Often, relevant variables are discarded, owing to low statistical significance (as measured by t statistics). On the other hand, irrelevant variables are often incorporated into the model, based on the same considerations. Any models developed following such an approach will undoubtedly suffer from specification biases. Most of the specifications are a linear combination of the available regressors, and the criterion for the selection of the best specification among several alternatives is to obtain the best possible fit to the data (usually measured by the coefficient of determination, R^2).

A few researchers have used specification forms that simulate the actual physical process of deterioration. In their work, the form of the specification, even though relatively simple (by comparison with the actual physical phenomenon), is not constrained to linear equations. This approach is often referred to as empirical–mechanistic approach.

Mechanistic Models

These models are based on the use of material behavior and pavement response functions, which are believed to represent the actual behavior of the pavement structure under the combined actions of traffic and the environment. Although there are currently various attempts in this direction, a comprehensive and reliable mechanistic pavement model has yet to be developed. Material models presently used are simplifications and only represent material behavior under restricted conditions. They lack empirical validation under a wide range of traffic and environmental conditions. Owing to the complexity of the road deterioration process, this approach is, at present, unfeasible.

Mechanistic–Empirical Models

These models make use of material characterization (laboratory or in situ testing) and pavement response models (usually multilayer linear elastic or finite element type models) to determine pavement response. This response is, in turn, correlated to pavement performance and finally calibrated to an actual pavement structure. Both pavement test sections and in-service pavement sections are used for this purpose. The models are usually calibrated by applying a bias correction factor (usually referred to as the shift factor). The determination of this factor is performed following ad-hoc procedures that are not supported by rigorous statistical procedures. It is often found that these shift factors are not supported by actual performance data but by engineering judgment.

In the past 20 years there has been a tendency to direct their efforts toward mechanistic–empirical models because they are more appealing from the engineering point of view. It should be noted that mechanistic–empirical approaches have still to prove that they constitute an improvement over more traditional empirical models. The main advantage that mechanistic models claim is their ability to extrapolate predictions out of the original data range. This advantage constitutes, in turn, their main disadvantage, since it is impossible to assess the reliability of their predictions when they are used out of the original data range for which these models were calibrated.

Table 1. Axle Configuration and Loads During American Association of State Highway Officials Road Test

Loop	Lane	Axle configuration ^a	Weight (kN)		
			Front axle	Rear axle	Gross weight
2	1	1-1	8.9	8.6	17.8
	2	1-1	8.9	26.7	35.6
3	1	1-1-1	17.8	53.4	124.6
	2	1-2-2	26.7	106.8	240.3
4	1	1-1-1	26.7	80.1	186.9
	2	1-2-2	40.1	142.4	324.9
5	1	1-1-1	26.7	99.7	227.0
	2	1-2-2	40.1	178.0	396.1
6	1	1-1-1	40.1	133.5	307.1
	2	1-2-2	53.4	213.6	480.6

^a1-1-1 indicates single front axle and two single rear axles, while 1-2-2 indicates single front axle and two tandem rear axles.

Data Source: American Association of State Highway Officials Road Test

The American Association of State Highway Officials (AASHO) Road Test was sponsored by AASHO and took place in the late 1950s near Ottawa, Ill. (HRB 1962). The site was chosen because the soil in the area was uniform and representative of soils in large areas of the country. The climate was also considered to be representative of many states in the northern United States. The average annual precipitation in the region of the test was 864 mm (34 in.). The precipitation occurred throughout the year without any significant differentiation between dry and wet seasons. The soil remained mostly frozen during the winter months, with the depth of freezing depending on the length and severity of the cold season.

Only one subgrade material was evaluated during the experiment, as well as only one climatic region. Even though both conditions are typical of large areas of the United States, the use of the results outside these conditions should be subjected to detailed assessment of their applicability. Estimation of the effects of other subgrade material and/or environmental conditions cannot be attained with this data set. This limitation could be overcome by incorporating new data sets and by applying joint estimation (Archilla and Madanat 2001).

The test tracks consisted of four large loops, numbered 3–6, and two small loops, numbered 1 and 2. Each loop was a segment of a four-lane divided highway whose north tangents were surfaced with asphalt concrete (AC) and the south tangents with Portland cement concrete. Only loops 2–6 were subjected to traffic and all vehicles assigned to any one traffic lane had the same axle arrangement–axle load configuration. Table 1 shows a summary of the traffic-loading configuration applied to each loop and lane. A total of approximately 1,114,000 axle load repetitions were applied from November 1958 until December 1960. The number of axle repetitions is equal to the number of truck passes for loop 2, while, for the other loops, the number of truck passes is equal to one half of the number of axle repetitions. This is because of the axle configuration of the truck given in Table 1.

A total of 142 flexible pavement sections were built into the various loops. Each section covered the two lanes and each lane was subjected to different traffic loading, so the total number of test sections was 284. Out of these, there were 252 original designs and 32 duplicates.

Most of the sections on the flexible pavement tangents were part of a complete experimental design, the design factors of which were surface thickness, base thickness, and subbase thickness. In each loop, the dimensions of the main factorial designs were $3 \times 3 \times 3$, that is, three levels of surface thickness combined with three different base thicknesses and three subbase thicknesses. Typical layer thicknesses were selected. For the pavement structural sections comprising the main experimental design the surface thickness varied from 0 to 150 mm, in increments of 25 mm. The base layer varied in thickness from 0 (no base layer) to 225 mm, in increments of 75 mm. The subbase layer thickness varied from 0 (no subbase layer) to 400 mm, in increments of 100 mm. The materials used for the construction of the AC surface, base and subbase layers were the same in all sections. Hence, the effect of material properties on pavement performance cannot be assessed from the data of the main experimental design. Other experiments aimed at assessing different surface and base materials were also conducted during the AASHO Road Test but were not part of the main experimental design and were not therefore considered in the development of the models presented in this research.

Existing Models Based on American Association of State Highway Officials Road Test Data

The first pavement performance model was developed based on the data provided by the AASHO Road Test. The AASHO equation estimates deterioration based on a dimensionless parameter g referred to as damage. The damage parameter was defined as the loss in serviceability at any given time

$$g_t = \frac{p_0 - p_t}{p_0 - p_f} = \left(\frac{N_t}{\rho} \right)^b \quad (1)$$

where g_t =dimensionless damage parameter; p_0 =initial serviceability at time $t=0$; p_t =serviceability index at time t ; p_f =terminal serviceability index, i.e., serviceability at time of failure; N_t =cumulative number of equivalent 80 kN single axle loads applied until time t ; and ρ, b =regression parameters, which were found to be functions of axle configuration and load and pavement strength.

The original serviceability equation as developed from the AASHO Road Test was the following:

$$p_t = 5.03 - 1.9 \log(1 + SV_t) - 0.01 \sqrt{C_t + P_t} - 1.38 RD_t^2 \quad (2)$$

where SV_t =slope variance at time t ; C_t =crack length in feet per 1,000 ft²; P_t =patching in ft² per 1,000 ft²; and RD_t =average rut depth in inches.

A number of studies have indirectly incorporated mechanistic principles for serviceability prediction. They were not based on a mechanistically developed model for serviceability but they incorporated a mechanistic subsystem for the prediction of cracking and rutting, which, in turn, is used for the prediction of serviceability. This indirect approach has been used by Ullidtz (1979) and by Uzan and Lytton (1982). Uzan and Lytton developed the following model that is an updated version of the serviceability model developed during the AASHO Road Test

$$p_t = 4.4 - 1.7 \log[1 + 350 \text{Var}(RD_t)] - 0.9 RD_t^{2.5} - 0.03 \sqrt{C_t + P_t} \quad (3)$$

These equations are examples of statistical methods in which the specification was derived mainly on the basis of the best statistical fit to the data, without any physical considerations. Even though these models fit the data well, they suffer from important specification biases.

Model Specification

Two basic considerations were taken into account for the selection of the form of the specification: (1) for a given pavement structure, riding quality decreases as traffic increases, and (2) for a given traffic level, the riding quality of weaker pavements decreases more rapidly than that of stronger pavements. Therefore, the following general and flexible form was selected:

$$y = f(x) = a + bx^c \quad (4)$$

where y =variable representing a measure of riding quality; x =variable representing a measure of traffic; a =parameter or function that represents the initial condition; b =parameter or function that represents the rate at which quality deteriorates with traffic; and c =parameter that represents the curvature of the function.

The sign of the rate parameter b indicates whether the riding quality is increasing or decreasing with traffic. In addition, the magnitude of b gives an indication of how much quality varies with traffic. Finally, the parameter c represents the curvature of the function, i.e., for $c < 1$, $c = 1$, and $c > 1$, the curve is concave, linear or convex respectively.

The form of Eq. (4) is suitable for predicting some performance indicator at any time and, therefore, suitable for design and analysis purposes. However, from the pavement management perspective, an incremental form is more beneficial, since condition data are usually available on a regular basis and predictions are usually only desired for the next one or two time periods. By using a first order series approximation, the same specification can also be used in its incremental form

$$y_t = y_{t-1} + f'(x_t)(x_t - x_{t-1}) \quad (5)$$

In the present research, serviceability (measured in PSI) was selected as the riding quality indicator to be predicted. Hence, the recursive model for predicting serviceability has the following form:

$$p_t = p_{t-1} + \alpha N_{t-1}^\delta \Delta N_t \quad (6)$$

where p_t =pavement serviceability at time t ; N_t =cumulative traffic up to time t ; ΔN_t =traffic increment from time $t-1$ to time t ; and α, δ =parameters or functions to be estimated.

If Eq. (6) is applied from the beginning of the experiment, the following expression is obtained:

$$p_t = p_0 + \alpha \sum_{s=0}^{s=t-1} N_s^\delta \Delta N_{s+1} \quad (7)$$

Specification for Aggregated Traffic

An extension of the traditional approach of aggregating all traffic into its equivalent number of 80 kN (18,000 lb) single standard axle loads is used in this paper. This number is usually referred to as equivalent single axle load (ESALs) or E80s. All axle load configurations are converted into their equivalent number of ESALs by means of a load equivalence factor (LEF) (AASHTO 1981). The most widely used form for the determination of the LEF is the so-called power law

$$\text{LEF} = \left(\frac{P}{80} \right)^\eta \quad (8)$$

where LEF=load equivalence factor; P =axle load in kN; and η =parameter.

The LEF multiplied by the actual number of axles of the given load, P , represents the number of ESALs. The validity of the

power law is restricted to the conditions under which it was derived, a fact that is often forgotten by pavement engineers. The present serviceability index was the failure criteria, and the formula converts dual wheeled single axles of different loads into their equivalent number of standard axles. A dual wheel single axle of 80 kN was used as a standard axle. In addition, due to the use of inadequate statistics techniques the original estimate of the parameter η is biased. Hence, it was decided to define different power laws for the different axle configurations that are present in the experimental data set.

Different standard loads (denominator of the power law) are necessary to transform different axle configurations into ESALs. It should be noted that three axle configurations were used during the AASHTO Road Test: single axles with single wheels, single axles with dual wheels and tandem axles with dual wheels. These three configurations currently cover the vast majority of highway traffic in the U.S.

A number of studies have shown the close dependence of the exponent of the power law on the type of distress being considered and on the pavement structure [Christison 1986; Committee of State Road Authorities (CSRA) 1986; Prozzi and de Beer 1997; Archilla 2000]. The decision was therefore made to estimate the exponent from the data instead of using a precalibrated exponent. It was also decided to keep the exponent of the power law constant for the various axle configurations.

Based on the above considerations, the concept of the equivalent damage factor (EDF) is introduced. The equivalent damage factor is a dimensionless factor, which depends on the characteristics of the truck and, which, when multiplied by the number of trucks, yields up the equivalent number of standard axles. The following equation applies for the traffic configurations used during the AASHTO Road Test:

$$\text{EDF} = \left(\frac{\text{FA}}{\lambda_1 80} \right)^{\lambda_2} + n_1 \left(\frac{\text{SA}}{80} \right)^{\lambda_2} + n_2 \left(\frac{\text{TA}}{\lambda_3 80} \right)^{\lambda_2} \quad (9)$$

where EDF=equivalent damage factor; FA=load in kN of the front axle (single axle with single wheels); SA=load in kN of the single axle with dual wheels; TA=load in kN of the tandem axles with dual wheels; λ_i =parameters to be estimated; and n_1, n_2 =number of single axles and tandem axles per truck, respectively.

Specification for Pavement Strength

The function α in Eq. (7) is a decreasing function of the strength of the pavement. That is, the serviceability of stronger pavements decreases more slowly than that of weaker pavements. The specification of the function α is based on the concept of structural number (HRB 1962); however, an alternative designation is proposed in order to differentiate the present specification from the concept developed by the AASHTO researchers. Thus, α is specified as a decreasing function of the equivalent thickness (ET) according to the following expression:

$$\alpha = \alpha_0 \text{ET}^{\alpha_4} = \alpha_0 (1 + \alpha_1 T_a + \alpha_2 T_b + \alpha_3 T_s)^{\alpha_4} \quad (10)$$

where T_a, T_b, T_s =thickness of the surface, base, and subbase layers, respectively (in cm); $\alpha_0 - \alpha_4$ =set of parameters to be estimated; and ET=equivalent thickness (in cm).

The parameters α_1, α_2 , and α_3 represent the contribution of each layer, relative to the subgrade's contribution, to the resistance of the pavement to serviceability loss.

Environmental Considerations

Environmental conditions are of significant importance to pavement performance; in the hypothetical case that a pavement section is not subjected to traffic, deterioration will still take place. There are two main environmental effects to be taken into account: (1) the effect of high temperatures, which reduce the stiffness of the asphalt layer, and (2) the effect of moisture, which reduces the stiffness of the untreated granular layers.

The viscosity of the asphalt binder decreases as the temperature increases. Thus, the stiffness of the asphalt concrete also varies with temperature. At low temperatures the asphalt concrete layer becomes very stiff and brittle, thus, prone to cracking. As the temperature increases, the asphalt concrete softens, losing stiffness very rapidly and becoming more susceptible to rutting.

The presence of moisture reduces the inter-particle friction of the untreated materials, resulting in a significant loss of material shear strength and stiffness. This, in turn, results in loss of support of the asphalt concrete, producing increased stress and strain levels for the same applied load. As stresses and strains increase, so does the rate of deterioration of the pavement structure. In the context of the present data set, the presence of water in the pavement layers has a significant effect during the spring months because of thawing of the water that was captured and then froze during the winter months.

The effect of environmental conditions can be taken into account following either of two approaches: (1) by modifying the pavement strength accordingly, or (2) by weighing the traffic load repetitions with factors that account for the relative damage of each repetition. The latter approach was used during the initial analysis of the AASHO Road Test data (HRB 1962) by introducing weighting factors. The calculation of the weighting factors was based on the effect of environmental conditions on surface deflections. In the present study, however, the former approach will be followed because it is believed that it is a more accurate representation of the physical deterioration process.

From observation of the data, it was apparent that three phases could be distinguished in the serviceability trend of the pavement sections:

1. a normal phase, characteristic of the summer and fall periods, when pavement serviceability of the sections decreased slowly;
2. a stable phase, characteristic of the winter period, when serviceability remained stable; and
3. an accelerated phase, during which the rate of deterioration increased significantly; this phase corresponds to the spring months.

Furthermore, it was observed that the three phases described above were almost identical to the periods of (1) zero frost penetration, (2) increasing frost penetration, and (3) decreasing frost penetration, respectively. Consequently the frost gradient was the variable chosen to capture the effect of the environmental conditions on road serviceability. The frost gradient represents the daily change in the depth of frost penetration and is expressed in centimeters (cm) of frost penetration per day.

Final Specification

In the preceding sections the form of the specification was given as a function of the relevant variables for each pavement test section. In this section, the full specification is given taking into account that the data set consists of a panel data set, i.e. time series data and cross sectional data are available.

$$p_{it} = p_{i0} - \alpha_i \sum_{r=0}^{r=t-1} N_{ir}^{\beta_8} \Delta N_{i,r+1} \quad (11)$$

where the first subscript, i , indicates the pavement test section and the second subscript, t , indicates the time period. For the final formulation all the parameters are renamed as follows:

$$p_{it} = \beta_1 + \beta_2 e^{-T_1} + (1 + \beta_3 T_{ai} + \beta_4 T_{bi} + \beta_5 T_{si})^{\beta_6} \sum_{r=0}^{r=t-1} \exp(\beta_7 G_r) N_{ir}^{\beta_8} \Delta N_{i,r+1} \quad (12a)$$

where G_r = frost gradient in period r (in cm/day); ΔN_{ir} , represents the cumulative traffic in ESALs for section i in period r , obtained by multiplying the equivalent damage factor of each truck configuration by the actual number of truck passes over the pavement test section as follows:

$$\Delta N_{it} = n_{it} \left(\left(\frac{FA_i}{\beta_9 80} \right)^{\beta_{11}} + D_i \left(\frac{SA_i}{80} \right)^{\beta_{11}} + D_i \left(\frac{TA_i}{\beta_{10} 80} \right)^{\beta_{11}} \right) \quad (12b)$$

where β_i = parameters to be estimated; n_{it} = actual number of truck passes for section i at time period t ; and D_i = dummy variable ($D = 1$ for one rear axle, $D = 2$ for two rear axles).

The parameter β_2 was incorporated into the specification [Eq. (12a)] to take into account the dependence of the initial serviceability value on the thickness of the asphalt layer.

Estimation Using Ordinary Least-Squares

The data consist of a panel data set: time series data as well as cross-sectional data. Several approaches can be followed to address the estimation, the simplest being the estimation of one time series regression for each section or, alternatively, the estimation of one cross-sectional regression at each point in time. These techniques are commonly used, however, at a high cost. If the parameters in the model are believed to be constant across section and along time, more efficient parameters can be estimated by combining all the data into a single regression.

The most common and general technique used to estimate Eq. (12) is by combining all time series data and cross sectional data and carrying out ordinary least-squares (OLS) estimation. In this case, the intercept is assumed to be the same for all sections. This assumption is reasonable, as it considers that the serviceability of all pavements is the result of the same process for all sections, and it only depends on the variables that are observed. However, unobserved heterogeneity is often present as a result of unobserved section-specific variables. Some of the unobserved variables that may affect the performance of the various sections differently are: materials density, drainage conditions, foundation type, etc. This unobserved heterogeneity could be dealt with in a number of ways. In this study, both the fixed effect approach and the random effects approach were used. Some of the results of the estimation using random effects are reported in the following section.

Eq. (11) represents the conditional expectation of the level of serviceability at a given time t for a given section i . This expectation is dependent on a vector of parameters $\beta = (\beta_1, \dots, \beta_{11})$ and on a vector of explanatory variables $\mathbf{X} = (X_1 \dots X_{11})$ which take on a specific value for a given section i and at a given time t . These variables describe the combination of pavement properties, traffic characteristics and environmental conditions at a given time for a given section. By applying the specification to all observations (all sections $i = 1, \dots, S$ and all times periods $t = 1, \dots, T_i$) we obtain the following equation:

$$p_{it} = E(p_{it}/\mathbf{X}_i, \boldsymbol{\beta}) + \varepsilon_{it} \quad (13)$$

Eq. (13) is not linear in the parameters so the estimation does not have a closed-form solution; hence, a nonlinear minimization routine was applied to estimate the parameters. Two assumptions are necessary to proceed with the estimation:

1. the random error term, ε_{it} , is assumed to have mean zero and constant variance: $E(\varepsilon_{it})=0$, and σ_ε^2 is constant; and
2. the covariance of the error terms is zero across sections and along time: $\text{Cov}(\varepsilon_{it}, \varepsilon_{js})=0$, for $i \neq j$, or $t \neq s$.

Under the assumption of normality, the values of the parameters that minimize the sum of squares deviations will be the maximum likelihood estimators as well as the nonlinear least squares estimators (Green 2000). The sum of square deviations is given by

$$F(\boldsymbol{\beta}) = \sum_{i=1}^S \sum_{t=1}^{T_i} [p_{it} - E(p_{it}/\mathbf{X}_i, \boldsymbol{\beta})]^2 \quad (14)$$

where T_i represents the number of observation periods for section i . Since the panel data set is unbalanced, in general, $T_i \neq T_j$ for $i \neq j$. A consistent estimator of σ_ε^2 is also based on the residuals (note that the degree of freedom correction is not necessary here, since all the results are asymptotic)

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{n} \sum_{i=1}^S \sum_{t=1}^{T_i} [p_{it} - E(p_{it}/\mathbf{X}_i, \hat{\boldsymbol{\beta}})]^2 \quad (15)$$

where $\hat{\boldsymbol{\beta}}$ =OLS estimate of $\boldsymbol{\beta}$; and $n=\sum_i T_i$.

Estimation Using Random Effects

As indicated earlier, there are two commonly used methods to assess the effect of the unobserved heterogeneity: fixed effects and random effects. Both methods assume that the differences across sections can be captured by differences in the intercept term. The fixed effects method is based on the estimation of section specific intercepts, while the random effects method assumes that the intercept term is randomly distributed in the population. It should be borne in mind that the OLS approach assumes that the intercept is constant for all sections.

One of the disadvantages of the fixed effects approach is that the estimated intercepts are specific to the given sample and are not necessary applicable to the population. In addition, there is a high cost (reduction of degrees of freedom) associated with its estimation, especially when the number of sections is large. For these reasons, the random effects approach was used in this study. The condition that the unobserved effects are uncorrelated with the explanatory variables is satisfied because the data used for the estimation is originated from an experimentally designed test. Hence, using the random effects (RE) approach, the specification becomes

$$p_{it} = \beta_1 + u_i + f(\mathbf{X}_{it}, \beta_2, \dots, \beta_{11}) + \varepsilon_{it} \quad (16)$$

Estimation Results

The parameters of the proposed model were estimated using OLS and random effects with the data corresponding to the 252 test sections with original designs. The parameter estimates, as well as their asymptotic t values, are given in Table 2. The high values of the asymptotic t statistics that were obtained with the sample should be noted. The variance estimates are given in Table 3.

The nonlinear equation developed in this research fits the data of the AASHO Road test much better than the original model

Table 2. Estimated Parameters and Corresponding Statistics

Parameter	OLS estimates	t value	RE estimates	t value
β_1	4.13	267.0	4.33	159.4
β_2	-1.7	15.9	-2.2	15.4
β_3	0.59	18.2	0.58	18.3
β_4	0.20	15.1	0.21	14.7
β_5	0.15	15.0	0.13	14.9
β_6	-3.11	-42.9	-2.66	-39.6
β_7	-0.075	-48.0	-0.071	-57.0
β_8	-0.47	-39.0	-0.55	-58.3
β_9	0.57	26.2	0.55	21.7
β_{10}	1.8	104.0	1.9	67.9
β_{11}	3.8	48.6	4.2	44.9

based on multistep linear regressions. The variance of the estimated serviceability of the original linear model was approximately 0.51, while the variance of the estimate of the nonlinear model is 0.26. Most importantly, this better fit-to-the-data was achieved using the same data source and the same number of explanatory variables as a result of the flexibility of the nonlinear formulation.

Statistical testing was carried out using Lagrange multipliers (LMs) to test the hypothesis that $\sigma_u^2=0$, in other words, that the OLS estimation was appropriate. The resulting LM was 105.07, which is much larger than the corresponding value for a 5% significance level (3.84). Therefore, the null hypothesis was rejected and it was concluded that the ordinary least squares approach was inappropriate: the unobserved heterogeneity cannot be ignored.

Conclusions

In both models, the parameter of the asphalt concrete layer (layer coefficient) is approximately three times the parameter of the untreated granular base layer. This indicates that, per unit of layer thickness, the contribution of asphalt to the resistance against loss of serviceability is three times that of the granular base. The layer parameters of the base and subbase layers are of the same order of magnitude. However, the difference is between 33 and 62% for the OLS and RE approaches, respectively. This is an indication of the importance of using the correct approach when carrying out the parameter estimation. These differences are even more important in the case of the parameters corresponding to the load variables.

The parameter corresponding to the exponent of the power law is 3.8 for the OLS approach and 4.2 for the RE approach. This is significant, not only from the statistical point of view, but also for its impact on the allocation of the cost of deterioration produced by the various vehicle classes. This effect is twofold. On one hand the damaging effect of heavy axles (>80 kN) is underestimated by the OLS approach and therefore the damage usually attributed to heavy vehicles is greater than that which they actually cause. On the other hand, the damaging effect of lighter vehicles (those

Table 3. Estimates of Variance Components of Two Approaches

Estimate	Ordinary least squares	Random effects
σ_ε^2	0.263	0.150
σ_u^2	N/A ^a	0.106
$\sigma_w^2 = \sigma_\varepsilon^2 + \sigma_u^2$	N/A ^a	0.256

^aNot available.

with axle loads less than 80 kN) is overestimated with the OLS approach, thus overpredicting the contribution of lighter traffic in the estimation of the design traffic.

The formulation of the equivalent load in terms of the EDFs enabled equivalent loads for various axle/wheel configurations to be determined. The equivalent load of a single axle with single wheels is estimated to be 46 and 44 kN for the OLS and the RE approach, respectively. These values are smaller than those traditionally used. If it is borne in mind that the RE estimates are unbiased estimates, this value indicates that a single axle with single wheels with a 44 kN load has the same effect on serviceability as an 80 kN single axle with dual wheels. This finding has important implications on the allocation of the amount of damage that can be attributed to the front axles and it is recommended that further research be carried out in this area.

The equivalent load for a tandem axle with dual wheels is estimated to be 144 and 152 kN for the OLS and RE approach, respectively. The OLS estimate is close to the equivalent load determined during the original analysis of the AASHO Road Test data. However, the RE (unbiased) estimate is significantly different. The result of the RE estimation highlights the benefit of using tandem axles from the point of view of road damage in terms of serviceability.

The above discussion emphasizes the three most important aspects that need to be taken into account when pavement performance models are being developed:

1. a physically realistic model specification;
2. an adequate data source; and
3. statistically sound estimation techniques.

The model specification should be supported by engineering knowledge of the materials behavior under load and environmental conditions. The data should be obtained from a well-conceived experimentally designed test aimed at addressing all the important variables that have been identified during the development of the theory. Unfortunately, as this is seldom the case, it is up to the modeler to take into account these limitations in order to develop models that are statistically defensible.

This research proved the importance of these three steps: a model was formulated using the same data set and the same variables, however, the prediction error of the new model was reduced by half. By halving the prediction error agencies can affect significant budget savings by timely intervention and accurate planning.

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Notation

The following symbols are used in this paper:

- D = dummy variable;
- $E(\)$ = expectation operator;
- EDF = equivalent damage factor;
- ET = equivalent thickness;
- FA = front (steering) axle load;
- G = frost gradient;
- g = damage;

- LEF = load equivalency factor;
- N = cumulative traffic in equivalent single axle loads;
- n = number of axle loads;
- p = serviceability index;
- R^2 = coefficient of determination;
- RD = rutting depth;
- SA = single axle load;
- SV = slope variance;
- T = layer thickness;
- TA = tandem axle load;
- u, ε = error terms;
- \mathbf{X} = vector of explanatory variables;
- $\alpha, \beta, \delta, \eta, \rho, \lambda$ = regression parameters;
- $\boldsymbol{\beta}$ = vector of regression parameters; and
- ΔN = traffic increment in equivalent single axle loads.

Subscripts

- i = positive integer indicating section number; and
- t = positive integer indicating time period.

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