Use of Recurrent Markov Chains for Modeling the Crack Performance of Flexible Pavements

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Abstract: Accurate prediction of the pavement crack condition is vital for pavement rehabilitation budget allocation. Due to largely nonlinear surface layer properties, randomness in the cracking mechanism, and the complexities involved with deterministic modeling, deterioration of the pavement crack condition has been more efficiently characterized as a stochastic process with Markov chains being an appropriate and popular modeling technique. However, routine Markov modeling techniques suffer from the shortcomings that the transient probabilities have to be computed only implicitly from extensive historical statistics of crack performance and they are also insensitive to timely variations in pavement condition transition trends. This paper presents a new methodology that involves the use of a recurrent or dynamic Markov chain for modeling the pavement crack performance with time in which the transition probabilities are determined based on a logistic model. This model is compatible with basic Markovian concepts since it only uses the current crack condition data along with other relevant data. It is also capable of continuously updating the transition probabilities in an explicit manner. A case study is performed to compare the newly developed recurrent Markov chain with the currently popular static Markov chain. For this comparison, transition probabilities corresponding to both methods are derived from the State of Florida's pavement condition survey database. It is illustrated how the recurrent Markov chain clearly outperforms the static Markov chain in terms of the forecasting accuracy. Therefore it is concluded that by incorporating the dynamics of crack state transition and randomness experienced with the pavement cracking process, the recurrent Markov chain provides a more appropriate, applicable, and above all, a computationally efficient methodology for modeling the pavement deterioration process with respect to cracks.

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Introduction

Pavement management system (PMS) decision-making associated with (1) allocation of pavement rehabilitation funds at the network level and (2) rehabilitation strategies at the project level depends on the prediction of the future pavement condition. Hence accurate modeling of pavement condition deterioration is an essential component of PMS. Two broad categories of models have been used in modeling the pavement condition deterioration process: (1) deterministic models and (2) probabilistic models. The deterministic model assumes that the pavement behavior follows a predetermined pattern that can be formulated by a specific mathematical expression relating the considered pavement performance indicator to one or more explanatory variables. However, inherent variability of material properties, environmental conditions, and traffic characteristics cause the pavement performance to inherit random characteristics. Therefore by disregarding the uncertainty observed in pavement deterioration modeling, the deterministic models tend to oversimplify the process of pavement deterioration. On the other hand, probabilistic models treat pavement condition measures such as crack, ride, and rut indices as random variables, and therefore are able to incorporate the uncertainty associated with pavement deterioration. The most popular probabilistic modeling approach is the method of Markov chains. Markovian transition models have been employed extensively for modeling infrastructure performance (Kulkarni 1984; Butt et al. 1987; Jiang et al. 1988; Kleiner 2001). The key to modeling the condition deterioration process using a Markov chain is to establish a matrix of appropriate transition probabilities.

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Estimation of Transition Probabilities—The State of the Art

Historically, two methods have been employed for the derivation of the transition probabilities depending on the extent of the available pavement condition survey data. Due to the scarcity of data in the initial stages of a PMS, pavement expert knowledge is usually sought to construct a reasonably accurate transition probability matrix that is stationary or invariant with respect

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to the condition deterioration process. Considering the subjective nature of pavement expert knowledge and the wide variation of the impact of the associated variables on the pavement deterioration, the adequacy of the stationary and subjective transition probability matrix in representing the deterioration process is questionable. On the other hand, in an established and well-functioning PMS with a wealth of historical condition survey data, the transition probability matrix is usually deduced from statistics of pavement condition data. In this regard, an excellent case study has been reported by Wang et al. (1994), who developed transition probability matrices from statistics of survey data for the Arizona Department of Transportation. However, most highway agencies, which adopt the Markov-chain-based performance model in their PMS, still rely on static transition probabilities.

State-of-the-art literature review indicates a scarcity of techniques that model the state dependence of the condition deterioration process. Few econometric methods have been found in modeling pavement condition deterioration behavior over time. Some researchers have recently applied econometric methodologies in modeling infrastructure (bridges and sewer systems) deterioration using condition rating data. Combining well-established methodologies and accurate facility characteristics data, these models can be considered more appropriate than the Markov chains based on stationary transition probabilities. As an example, Madanat et al. (1995) introduced an ordered probit model for estimating transition probabilities from infrastructure inspection data. The above model assumes the existence of an underlying continuous random variable and therefore allows the latent nature of infrastructure performance to be captured. Then an ordered probit model is used to construct an incremental discrete deterioration model in which the difference in observed condition rating is an indicator of the underlying latent deterioration. Finally this model is used to compute a nonstationary (i.e., time dependent) transition matrix. Based on the previous work, Madanat et al. (1997) proposed an improved probit model with the specification of random effects to account for the heterogeneity and extend the model to investigate the state dependence. More recently, Ariaratnam et al. (2001) presented a new methodology for predicting the likelihood that a particular infrastructure system can be in a deficient state, using logistic regression models. The methodology is illustrated in a case study involving the evaluation of the local sewer system of Edmonton, Alberta, Canada. The outcome of the model does not produce the capability of prediction of condition but rather provides decision-makers with the means of evaluating sewer sections for the planning of future scheduled inspection, based on the probability of deficiency. However, as a totally different infrastructure, the mechanism of pavement condition deterioration may differ from that of bridges or sewer systems. The main objective of the research reported here is to establish a simple relationship between the transition probabilities of pavement crack condition and all relevant explanatory variables through a logistic model to facilitate the computation of dynamic transition probabilities that truly represent the state dependency of the pavement deterioration process. The issue of state dependency of transition probabilities was addressed by including the lagged pavement crack condition index itself as one predictor in the model specification. Then, a recurrent Markov chain is constructed based on the logistic model and a computationally simple procedure is established for crack condition forecasting.

Derivation of Recurrent Markov Chains

Theoretical Background

A discrete time Markov process is defined by Parzen (1962) as a stochastic process defined in terms of the state parameter $X(t)$ [i.e., crack condition at time t] with the conditional distribution of $X(t_n)$ given the series of values of $\{X(t_1), X(t_2), \ldots, X(t_{n-1})\}$ depends only on the immediately previous state value, i.e., $X(t_{n-1})$. This can be formulated as

$$P[X(t_n) \leqslant x_n | X(t_1) = x_1, X(t_2) = x_2, \ldots, X(t_{n-1}) = x_{n-1}] = P[X(t_n) \leqslant x_n | X(t_{n-1}) = x_{n-1}] \quad (1)$$

A Markov process in discrete state space is called a Markov chain. In an n-state Markov chain, the state of the process at any time t is defined by a probability mass function that can be expressed as

$$\mathbf{P}(t) = [p_1^t, p_2^t, \ldots, p_n^t]; \quad \sum p_i^t = 1 \quad (2)$$

where p_i^t=probability that the process is in state i at time t.

Given the process starting time of t, the probability mass function of the process at time $(t+k)$ can be derived by multiplying the probability matrices for each of k transitive steps. This can be formulated as follows:

$$\mathbf{P}(t+k) = \mathbf{P}(t)\mathbf{P}^{t,t+1}\mathbf{P}^{t+1,t+2} \cdots \mathbf{P}^{t+k-1,t+k} \quad (3)$$

where $\mathbf{P}(t)$=vector of probability mass function at any time t; $\mathbf{P}(t+k)$=vector of probability mass function at the kth step of the process; and $\mathbf{P}^{t+i,t+i+1}$=transition probability matrix from step $t+i$ to step $t+i+1$.

By assuming that the transition probability matrices depend only on the time difference, a stationary Markov chain process can be derived from Eq. (3) as shown in Eq. (4).

$$\mathbf{P}(t+k) = \mathbf{P}(t)(\mathbf{P}^{t,t+1})^k \quad (4)$$

The transition matrix $\mathbf{P}^{t,t+1}$ can be expressed as

$$\mathbf{P}^{t,t+1} = \begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & \cdots & p_{1(n-1)}^{t,t+1} & p_{1n}^{t,t+1} \\ p_{21}^{t,t+1} & p_{22}^{t,t+1} & \cdots & p_{2(n-1)}^{t,t+1} & p_{2n}^{t,t+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{(n-1)1}^{t,t+1} & p_{(n-1)2}^{t,t+1} & \cdots & p_{(n-1)(n-1)}^{t,t+1} & p_{(n-1)n}^{t,t+1} \\ p_{n1}^{t,t+1} & p_{n2}^{t,t+1} & \cdots & p_{n(n-1)}^{t,t+1} & p_{nn}^{t,t+1} \end{bmatrix} \quad (5)$$

However, during the pavement deterioration process it is impossible to upgrade the state parameter $X(t)$ (crack condition) unless maintenance or rehabilitation is implemented. Therefore the transition probability matrix in Eq. (5) is reduced to the simplified form in Eq. (6) which represents a semi-Markov process.

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$$\mathbf{P}^{t,t+1}=\begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & \ldots & p_{1(n-1)}^{t,t+1} & p_{1n}^{t,t+1} \\ 0 & p_{22}^{t,t+1} & \ldots & p_{2(n-1)}^{t,t+1} & p_{2n}^{t,t+1} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & \ldots & p_{(n-1)(n-1)}^{t,t+1} & p_{(n-1)n}^{t,t+1} \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \tag{6}$$

where $\Sigma_{j=i}^{n} p_{ij}^{t,t+1}=1,\ i=1,2,3,\ldots,n-1$.

The entry of 1 in the last row of the transition probability matrix corresponding to state n indicates a "trapping" state at which the pavement condition cannot deteriorate further unless maintenance or rehabilitation is performed. Obviously, it is a tedious exercise to estimate all the probabilities associated with the transition probability matrix. Hence, the assumption is made in practice that the condition can drop, at most, one state in a single duty cycle. Consequently, the transition probability matrix can be further simplified as in Eq. (7). Nevertheless, this simplification assumption does not present a critical constraint since either the duty cycle or the condition state $X(t)$ can be arbitrarily defined to satisfy it.

$$\mathbf{P}^{t,t+1}=\begin{bmatrix} p_{11}^{t,t+1} & p_{12}^{t,t+1} & \ldots & 0 & 0 \\ 0 & p_{22}^{t,t+1} & p_{22}^{t,t+1} & 0 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & \ldots & p_{(n-1)(n-1)}^{t,t+1} & p_{(n-1)n}^{t,t+1} \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \tag{7}$$

where $p_{ii}^{t,t+1}+p_{i(i+1)}^{t,t+1}=1,\ i=1,2,3,\ldots,n-1$.

Framework of Recurrent Markov Chain

In practice, the framework of the proposed recurrent Markov chain for the pavement crack condition represented by condition $X(t)$ is illustrated in Fig. 1. As shown in Fig. 1, the recurrent Markov chain uses the transition probabilities, which are functions of relevant explanatory variables and the lagged pavement crack condition index $X(t)$, to forecast pavement crack condition in the next duty cycle $X(t+1)$. The transition probabilities can be estimated through a discrete choice model. In this research, a logistic model is used for estimating the transition probabilities based on a set of explanatory variables and the lagged crack condition index. For multiple-step forecasting, a recurrent or an iterative process is applied, where the output of the process at one time step becomes the input at the following time step.

Estimation of Transition Probabilities Using a Logistic Model

Based on the assumption that for any pavement section the crack condition can only drop one state during a given duty cycle, a binary choice situation exists for the following duty cycle; either remaining in the current state or moving to the next worse state. Accordingly, the following logistic model can be defined to relate the transition probabilities to the variables associated with crack condition deterioration.

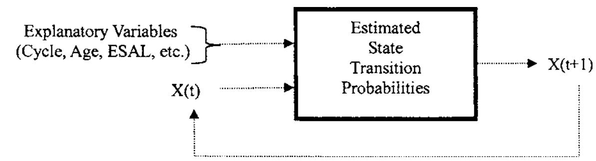

Fig. 1. Framework of recurrent Markov chain

$$P[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i]=\frac{1}{1+e^{\Sigma_{m=0}^{k}\beta_m X_m}} \tag{8}$$

$$P[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i]=\frac{1}{1+e^{-\Sigma_{m=0}^{k}\beta_m X_m}} \tag{9}$$

where $P[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i]$=probability of the crack condition remaining in state i for the next duty cycle given the current state is i; $P[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i]$=probability of the crack condition deteriorating to state $i-1$ for the next duty cycle given the current state is i; X_m=mth variable governing crack condition, β_m=parameter associated with the mth variable; and k=total number of variables governing crack condition, including the constant term.

The novelty of the current model is exemplified by Eqs. (8) and (9) which facilitate the expression of transition probabilities in an explicit form in terms of the current values of the explanatory variables. A major assumption involved in this model is that β_m are constants for the entire network of pavements or homogeneous subsets (or family) of the network. The model parameters β_m can be evaluated using a sample of pavement sections from the family with known condition states and the corresponding values of the explanatory variables.

Assuming that the observations in a statistical random sample are drawn independently and the variables X_m are nonstochastic, the logarithm likelihood function for the sample conditioned on the parameters $\boldsymbol{\beta}$ can be written as

$$L(\boldsymbol{\beta})=\prod_{n=1}^{N} P_n[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i]^{y_{in}} \times(1-P_n[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i])^{y_{(i-1)n}} \tag{10}$$

where n=pavement section number, $\boldsymbol{\beta}=[\beta_1,\beta_2,\ldots,\beta_k]$; N=sample size; $y_{in}=1$ if state i is actually chosen by the section n, otherwise 0; and $y_{(i-1)n}=1$ if state $i-1$ is actually chosen by the section n, otherwise 0.

By setting the first derivative of $L(\boldsymbol{\beta})$ with respect to β_m equal to 0, a system of k nonlinear simultaneous equations with k unknowns, $\beta_1,\beta_2,\ldots,\beta_k$ can be derived as follows:

$$\sum_{n=1}^{N}(y_{in}-P_n[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i])X_{mn}=0,\quad m=1,\ldots,k \tag{11}$$

where $\mathbf{X}_{mn}$=vector of contributing variables for pavement section n.

The maximum-likelihood estimates of β_m can be found by solving the system of k nonlinear simultaneous equations. Inspection of Eqs. (8) and (9) shows how one can recursively update the transitional probabilities for the family of pavements knowing the model parameters β_m and the updated values of the explanatory variable X_m.

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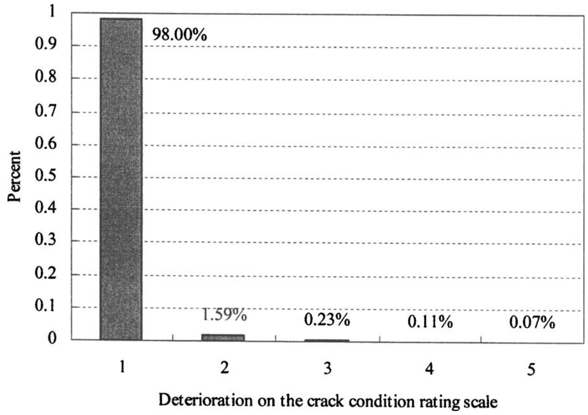

Fig. 2. Historical distribution of flexible pavement deterioration

Table 1. Definition of Crack Condition States

State	Range of crack rating
10	$9 < CI \leq 10$
9	$8 < CI \leq 9$
8	$7 < CI \leq 8$
7	$6 < CI \leq 7$
6	$5 < CI \leq 6$
5	$4 < CI \leq 5$
4	$3 < CI \leq 4$
3	$2 < CI \leq 3$
2	$1 < CI \leq 2$
1	$0 \leq CI \leq 1$

Preprocessing of Data

The model explanatory variables (X_m) comprised traffic, pavement crack condition, and pavement structural parameters. Hence two sources of data were utilized for the development of the new model. They are (1) Florida traffic information data, and (2) Florida pavement condition survey data on the roadways maintained by the Florida Department of Transportation (FDOT). The traffic information database consists of traffic characteristic information, such as peak season factors, *K*-factors, *D*-factors, vehicle classification, truck percentage, historical average annual daily traffic (AADT), etc. The Florida pavement condition survey database contains detailed roadway information, such as historical crack ratings, roadway identification, section begin mileage and section end mileage, roadway age, roadway type, number of lanes, district, system, maintenance cycle, asphalt overlay thickness, etc.

FDOT uses the crack index (CI) for evaluating the pavement crack condition based on visual rating of pavements by trained crew. CI is rated on a 0–10 scale where 10 indicates the best condition and 0 the worst. Hence CI was adopted as a convenient practical parameter of expressing the crack condition $X(t)$ in the model [Eqs. (8) and (9)]. Single-year degradation of the crack condition was examined in 7,434 flexible roadway sections from 1986 to 2003. The percent distribution of pavement sections with respect to the extent of single-year degradation on the crack condition rating scale is illustrated in Fig. 2.

In pavement management practices, a duty cycle is normally defined as 1 year since seasonal climate changes occur in 1 year cycles and traffic (AADT) is usually measured on an annual basis. As shown in Fig. 2, the majority of the flexible pavement sections in Florida (98%) are seen to deteriorate just by one integer rating on the condition rating scale within one duty cycle. This information certainly seems to support the assumption made in the proposed recurrent Markov model [Eq. (7)] that pavements deteriorate, at most, one state during one duty cycle under normal traffic conditions.

As identified in the state-of-the-art review, some current performance models include AADT as a predictor variable. However, AADT is not an appropriate representation of traffic loading because the traffic loading effect on the pavement condition deterioration is mainly caused by heavy vehicles such as trucks, and not passenger cars. An accurate representation of the traffic loading can be achieved using the equivalent single axle loads (ESAL). The traffic data and pavement condition data were integrated through a common roadway identification number and the milepost reference location number. Integration of databases allows AADT and the corresponding truck factors for each section to be identified and the ESAL per lane to be determined for each roadway section. Due to the large range of ESAL magnitude, the logarithm of ESAL is included as one predictor variable (X_m) in the logistic model in Eqs. (8) and (9).

The period between two consecutive rehabilitation actions on a given pavement section known as the pavement cycle, a parameter different from the duty cycle, which can also be associated with the number of overlays applied up to the evaluated instant, is also included in the model as a nominal variable. In cases where the nominal variable has more than two levels, multiple dummy variables need to be created to represent the nominal variable. The total number of dummy variables required is one less than the number of values of the original nominal variable since a value must be specified as the base case for reference of a nominal variable which does not appear in the model specification. In the current model, Cycle 1 is referred to as the base case and hence three additional dummy variables are defined with respect to the base case as follows:

- Cycle 2: 1 when pavement cycle=2, 0 otherwise;
- Cycle 3: 1 when pavement cycle=3, 0 otherwise; and
- Cycle 4: 1 when pavement cycle=4, 0 otherwise.

Since the crack index (CI) was adopted for modeling the pavement crack performance, the CI scale was categorized into 10 states with one integer interval representing each state, as shown in Table 1. As seen in Fig. 2 the 10-state pavement crack condition discretization scheme assures that for any given pavement the crack condition would not drop more than one state in a single duty cycle under normal traffic conditions.

For improved efficiency in data integration and preprocessing, codes were developed in *Visual Basic*, which can import traffic data and roadway condition data into a single MS *Access* database. Then, the integrated data set was imported into the SAS system and SAS codes were developed for data preprocessing. Due to the excessive size of the resultant database and the limitations of the modeling software, a sample data set was drawn for convenient data management. The data preprocessing exercise included (1) omitting of those pavement sections with critical data missing; (2) elimination of irrational condition rating data (improved condition perceived by raters occurring without actual rehabilitation); (3) data transformation; (4) data sampling; and (5) formation of the data set for model development and validation.

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Table 2. Variables Significant to the Model

Variable	Parameter estimate	Wald statistic	Significance
Constant	−8.4246	10.4599	0.0012
CI	−0.7134	28.3502	0.0000
Age	1.3485	16.3611	0.0000
log(ESAL)	2.0418	23.9186	0.0000
Cycle 2	1.5347	11.5328	0.0007
Cycle 3	1.0964	5.6401	0.0176
Cycle 4	1.5278	8.0936	0.0044
Age·age	−0.0337	31.4722	0.0000
CI·age	0.0503	14.7651	0.0001
log(ESAL)·age	−0.2191	18.5292	0.0000
Cycle·age	−0.1327	7.9318	0.0049
Summary statistics:			
No. of obs: 2,552			
L(C)=−1,220.468			
L(B)=−1,050.911			
L(M)=−1,074.575			

Note: CI=crack index and ESAL=equivalent single axle loads.

Formulation of Logistic Model

A backward stepwise elimination process was employed for selecting specific variables that merit inclusion in the model. The process starts with a comprehensive model with all relevant explanatory variables, and sequentially removing one variable at a time, based on a specific criterion, such as the statistical significance (example: 0.05 significance level) of the considered variable or the improvement in the explained variance. Three types of hypothesis tests were involved in the model selection process: (1) the significance test for each model parameter based on a Wald test; (2) determination of the significance of multiple parameters using a likelihood-ratio test; and (3) examination of the overall model fit using a Hosmer and Lemeshow goodness-of-fit test.

Wald tests are based on chi-square statistics that tests the null hypothesis that a particular variable has no significant effect given that the other variables are included in the model. In the formulation of the current model, the Wald test was performed first on each variable or model parameter to investigate its significance. Table 2 shows the variables that do meet the 0.05 significance level criterion and hence included in the final model specification. The new asphalt overlay thickness (Thickness) turns out to be insignificant. This is not a surprising finding since fatigue cracking of asphalt pavement is a long-term phenomenon that occurs with aging and traffic. On the other hand, from a structural mechanistic viewpoint, the model is expected to be improved if the thickness of the base is included in the model. However, this information was unavailable in a ready-to-use form. As shown in Table 2, all variables and interaction terms are significant at the 0.01 level except for Cycle 3, which is significant at the 0.05 level. The negative sign of the crack condition (CI) reveals that the better the current condition the lower the probability of deterioration is. In contrast, positive signs of age and logarithm of ESAL indicate that aged pavements with higher traffic loading tend to have a higher probability of deterioration. Furthermore, positive coefficients of the dummy variable for the second, third, and the fourth pavement cycles indicate higher deterioration propensity of pavements in latter cycles than those in the first cycle, which reflects the as-built pavement condition. Although the above results are expected intuitively, an unexpected result occurs when comparing the effects of different cycles on the deterioration. The magnitudes of coefficients of different cycles reveal that the pavement sections in the third cycle tend to deteriorate slower than those in the second cycle while pavement sections in the fourth cycle bounce back to more or less the same deterioration rate (probability) as those in the second cycle. This could be explained by realizing that change in the crack condition in a given construction cycle is a result of the interplay between two contrasting effects: (1) structural improvement (new surface condition and thicker pavement, resulting in a stiffer pavement) and (2) accumulation of further damage. A higher cycle implies an increased stiffness and also higher cumulative damage. Therefore the effect of a cycle on pavement deterioration is a resultant contribution of two competing factors. The pavement sections in the second cycle exhibit a higher deterioration probability than those in the first cycle because the pavements in the second cycle in general could have a more dominant contribution from the cumulative damage than from the improvements. The pavements in the third cycle still have a higher deterioration probability than those in the first cycle, but lower deterioration probability than those in the second cycle implying that, in the third cycle, the effect of additional cumulative damage has been overcome by the most recent improvements. This anomaly could be database-specific perhaps due to many pavement sections in this database which are in their third cycle possessing conservatively designed overlays that exceed the rehabilitation demands. Finally the pavements in the fourth cycle seem to have almost the same deterioration probability as those in the second cycle because once again the additional cumulative damage seems to have counteracted the increased stiffness due to the improvements.

On the other hand, the likelihood ratio test is used for joint testing of several parameters. It compares two different model specifications: a simple model and a relatively more complex model by testing whether the extra parameters in the latter model have any significance. The test begins with a comparison of the likelihood scores of the two models. The test statistic is denoted by Eq. (12), which approximately follows a chi-square distribution with k degrees of freedom where k=number of additional parameters in the more complex model.

$$-2\log\left(\frac{L_0}{L_1}\right) = -2(\log L_0 - \log L_1) \qquad (12)$$

where L_0=likelihood score of the simpler model, and L_1=likelihood score of the more complex model.

Significance of the Interaction Terms

All interaction terms turn out to be insignificant except for those between age and crack condition $[\mathrm{CI}(t)]$, age and log(ESAL), and age and cycle. To validate the significance of these interaction terms, the above likelihood ratio test was performed by comparing the predictions of the model with and without the above interaction terms. The respectively obtained likelihood values of $L(B)=-1{,}050.911$ and $L(M)=-1{,}074.575$ are shown in Table 2 and the likelihood ratio is computed as $L=-2[L(M)-L(B)]$ $=47.328$. Since this is greater than the critical chi-square value

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Table 3. Overall Model Goodness of Fit (Hosmer–Lemeshow Test)

		Dropping to the next state		Remaining in current state	
Group	Total	Observed	Expected	Observed	Expected
1	257	3	4.20	254	252.80
2	256	3	9.47	253	246.52
3	257	17	15.76	240	241.24
4	256	27	24.22	229	231.78
5	255	37	35.56	218	219.44
6	255	50	46.89	205	208.11
7	255	57	60.62	198	194.38
8	255	74	72.77	181	182.23
9	256	98	87.14	158	168.86
10	250	105	114.33	145	135.67
Summary statistics:					
Chi-square=9.4901					
Degree of freedom=8					
p-value=0.3027					

of 11.34 corresponding to three degrees of freedom at 0.01 significance level, the null hypothesis that the selected interaction terms are insignificant can be rejected.Traditionally, the likelihood ratio test is also performed to investigate if all the parameters other than the constant term are significant or not. As shown in Table 2, since the likelihood value for the simplest model, the one that involves only the constant term, $L(C)=-1{,}220.468$, it can be shown that for the null hypothesis all the parameters that are insignificant can also be rejected. The most plausible reason for the above observed significance of the interaction term is that they indirectly attempt to explain the impact of the actual rheological response of asphalt, base, subbase, and subgrade material, an aspect overlooked in this model.

Finally, the goodness-of-fit test is used to evaluate how well the model-based predictions coincide with the actual observations. However, in the case of logistic regression models, investigating the goodness-of-fit is often problematic when continuous covariates such as ESAL and crack index (CI) are modeled, since the approximate chi-squared null distributions for the Pearson test statistic are no longer valid. Hosmer and Lemeshow (1980) proposed a goodness-of-fit test that can be used for logistic regression models with continuous predictors

Table 4. Test of Multicollinearity

Variable	Tolerance	Variance inflation factor
CI	0.6054	1.6518
Age	0.5712	1.7507
log(ESAL)	0.9518	1.0506
Cycle 2	0.6095	1.6408
Cycle 3	0.5958	1.6783
Cycle 4	0.8274	1.2087

Note: CI=crack index and ESAL=equivalent single axle loads.

by using an alternative approach to group the predictions of a logistic regression model rather than predictor variable data of the model, which is the approach used in the Pearson test. The Hosmer–Lemeshow (HL) statistic is computed using the following equation:

$$\mathrm{HL}=\sum_{j=1}^{G}\frac{(o_j-n_j\pi_j)^2}{n_j\pi_j(1-\pi_j)} \tag{13}$$

where G=number of groups of predictions, o_j=total frequency of event outcomes in group j, n_j=total frequency of subjects in group j, and π_j=average estimated probability of an event outcome in group j. The HL statistic follows a chi-square distribution with $G-2$ degrees of freedom. The number of samples used in formulating the current logistic model was well above the specified limiting sample size of 400.

The Hosmer–Lemeshow goodness-of-fit test was used to test the fittingness of the overall model and the results are shown in Table 3. The null hypothesis for this test is that the observed data fits the specified model. In view of the high *p*-value (0.3027), the null hypothesis is not rejected. Thus the conclusion may be drawn that the actual data fits the specified model.

Consequent to the foregoing analysis, the logistic model is finally established as follows:

$$P_n[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i]=\frac{1}{1+e^{f[\mathrm{CI}(t).Age,Cycle,\mathrm{ESAL}]}} \tag{14}$$

$$P_n[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i]=\frac{1}{1+e^{-f[\mathrm{CI}(t).Age,Cycle,\mathrm{ESAL}]}} \tag{15}$$

where i=present crack condition state; t=present duty cycle, n=pavement section number; $P_n[\mathrm{CI}(t+1)\subset i|\mathrm{CI}(t)\subset i]$ =probability of crack condition remaining in state i given the present condition is in state i; $P_n[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i]$ =probability of crack condition deteriorating to state $i-1$ given the present condition is in state i; and

$$f[\mathrm{CI}(t),Cycle,Age,\mathrm{ESAL}]=-8.4246-0.7134\mathrm{CI}(t)+1.3485Age+2.0418\log(\mathrm{ESAL})+1.5347Cycle\ 2+1.0964Cycle\ 3+1.5278Cycle\ 4$$
$$-0.0337Age^2+0.0503Age\cdot\mathrm{CI}(t)-0.2191Age\cdot\log\mathrm{ESAL}-0.1327(Cycle\ 2+Cycle\ 3+Cycle\ 4)\cdot Age \tag{16}$$

Testing of Multicollinearity

The existence of strong multicollinearity inflates the variances of the parameter estimates, and sometimes results in highly unreliable coefficients. Therefore the multicollinearity of selected independent variables was tested using two commonly used diagnostic statistics: tolerance and variance inflation factor (VIF). The results are provided in Table 4 and it is seen that, for the current model, all VIFs are below the critical VIF of 2.5 (Allison 1999). Hence it can be concluded that multicollinearity is not an impediment for the developed model.

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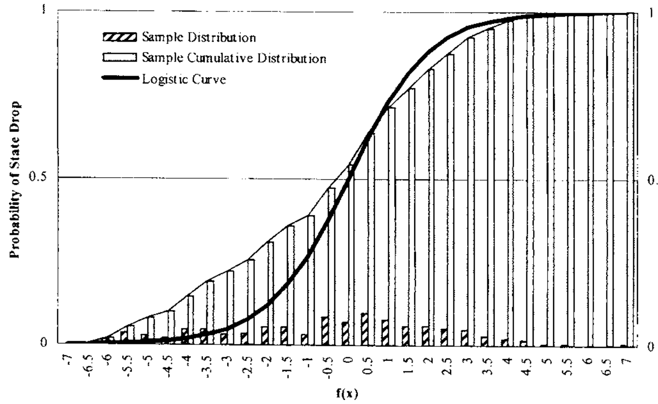

Fig. 3. Validation of the logistic distribution of $f(x)$

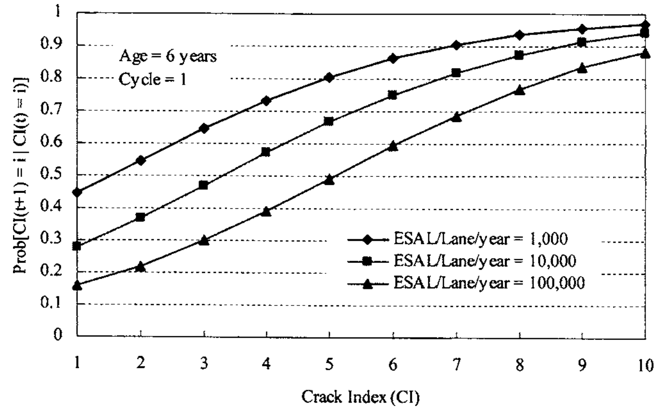

Fig. 5. Predicted variation of crack index with different levels of equivalent single axle loads

Verification of the Logistic Model Assumption

Use of a logistic regression model assumes that the difference of the utilities associated with each choice [index function $f(x)$ in Eq. (16)] follows a logistic distribution. In order to verify this assumption, both the frequency and cumulative distributions of the actual observations from the database with respect to "state drops" and "state holds" within consecutive duty cycles were plotted against the logistic distribution curve (Fig. 3). As seen in Fig. 3, there is reasonable agreement between the actual data and the theoretical logistic distribution.

Parametric Analysis of the Logistic Model

To further evaluate the applicability of the model, a parametric analysis was performed to assess the sole impact of each variable on the crack performance while holding the other variables constant at their mean values. Fig. 4 shows the probability of an arbitrary pavement section remaining in the current state with respect to the crack condition index. It can be seen that pavements in relatively better conditions have a higher probability of remaining in the current state than those in poorer conditions, a finding that indeed concurs with observations. Fig. 4 also shows that pavements in Cycle 1 have the highest probability of remaining in the current state, and pavements in Cycles 2 and 4 have almost the same probability of remaining in the current state. Pavements in Cycle 3 lie in between those in Cycles 1 and 4. Due to the complex interaction between the additional damages and improvements inherited in each cycle that was discussed in "Formulation of Logistic Model," pavements in Cycle 3 have a higher probability of remaining in the same state than those in Cycle 2. The variation of crack condition index at different levels of ESAL, representing the pavements with low, medium, and high traffic loading, respectively, is plotted in Fig. 5. Fig. 5 indicates that pavements with higher ESAL tend to have a lower probability of remaining in the current state.

The impact of pavement age in different cycles and levels of ESAL on the crack condition deterioration is illustrated in Figs. 6 and 7. It indicates that older pavements tend to have a lower probability of remaining in the current state. Similar patterns in the crack condition index across different cycles and levels of ESAL were observed with respect to the pavement age as shown in Figs. 4 and 5.

Analysis of Model Sensitivity

The objective of the sensitivity analysis is to test the reliability of the model structure by evaluating its sensitivity to minor changes in the data sets. In this effort, two additional logistic models were developed using two distinct data sets, i.e., randomly selected pavement sections containing 80 and 90% of the pavement sections in the original data set. The two corresponding models were subsequently compared to the original logistic

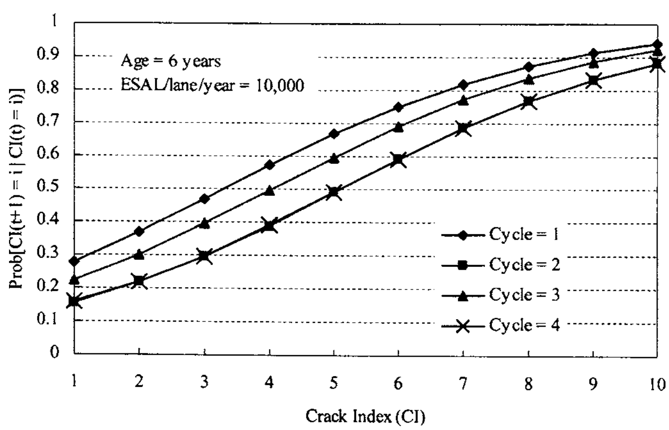

Fig. 4. Predicted variation of crack index in different pavement cycles

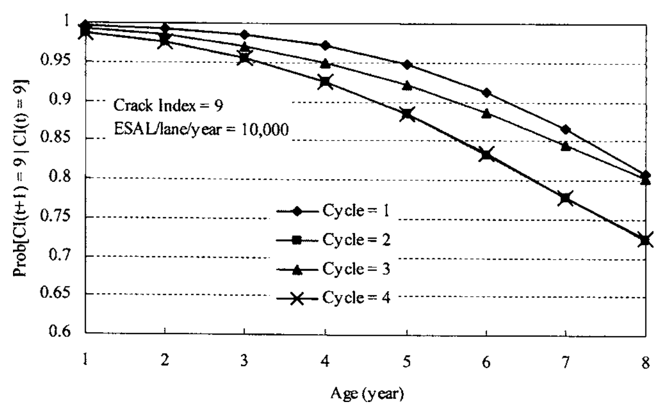

Fig. 6. Deterioration impact of pavement age with different cycles

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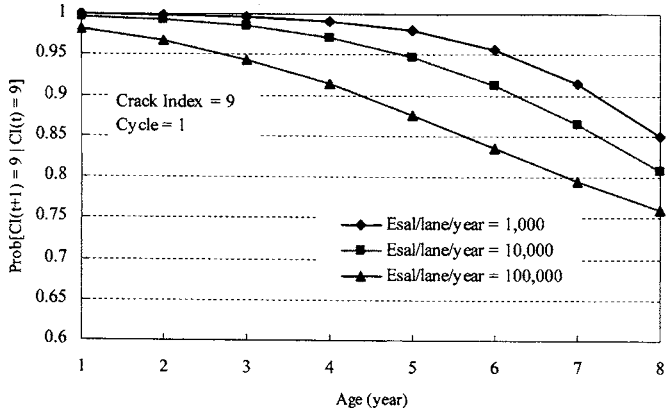

Fig. 7. Deterioration impact of pavement age with different levels of equivalent single axle loads

model. For comparison purposes, the estimated parameters corresponding to all three models are presented in Table 5.

It can be seen that the coefficients estimated from the three models agree reasonably well in terms of both the sign and the magnitude (within 10% of each other). The Wald statistics for the coefficients were significant at a relatively lower level for the models based on 80 and 90% sample sets. To support this finding and show that there is no difference among these three models from a statistical point of view, the Kruskal-Wallis test was performed under the following hypotheses:

- H_0: The models are equal (there is no significant difference between models).
- H_a: the models are different. The following procedure was followed in applying the Kruskal–Wallis (KW) test.

1. Combine all three samples into one large sample, sort the result in ascending order, and assign ranks.
2. Find r_i, the sum of the ranks of the observations in the ith $(i=1,2,3)$ sample.
3. Compute the test statistic KW using Eq. (17)

$$\mathrm{KW} = \frac{12}{N(N+1)}\sum_{i=1}^{3}\frac{r_i^2}{n_i} - 3(N-1) \tag{17}$$

where N=total sample size, and n_i=sample size for group i.

4. Under H_0, KW follows an approximate chi-square distribution with 2 degrees of freedom.
5. Reject the null hypothesis that all three models are the same if $\mathrm{KW} > \chi^2_{\alpha,2}$.

Projections of the probabilities of the pavement sections remaining in the current state and the corresponding rank measures across different ages for the three data sets are listed in Table 6. As shown in Table 6, the KW statistic [Eq. (17)] is calculated to be 0.195 and compared with the $\chi^2_{0.01,2}$ of 4.61. Therefore the null hypothesis cannot be rejected, indicating that no significant difference exists among the three models. Thus it can be concluded that the proposed model is stable and can be deemed as an appropriate representation of the data set.

Construction of Recurrent Markov Chain

Application of the Markov chain for forecasting the pavement condition requires a mechanism that can convert discrete states combined with transition probabilities back to the pavement condition rating. Crack condition state value expressed in terms of CI and probabilities associated with each condition state (probability mass function) can be used to compute the expected value of pavement crack condition in the next duty cycle using Eq. (18) given the present state i.

$$\mathrm{CI}(t+1) = \sum_{j=1}^{i} SI_j p_{ij}^{t,t+1} \tag{18}$$

where t=present duty cycle; $t+1$=next duty cycle; $\mathrm{CI}(t+1)$=crack index in next duty cycle; SI_j=mean value of crack conditions at state j; and $p_{ij}^{t,t+1}$=transition probabilities from state i to state j.

In the case where the states are equidistant, i.e., $SI_{j+1}-SI_j=d$ $(j=1,2,\ldots,n-1)$, Eq. (18) can be rewritten as

$$\mathrm{CI}(t+1) = SI_i - d\sum_{j=1}^{i-1}(i-j)p_{ij}^{t,t+1} \tag{19}$$

where SI_i=mean value of crack conditions at current state i; and d=uniform state distance.

As indicated in Eqs. (18) and (19), state value of the crack condition, usually the mean value of the crack condition index of the subject state, is used in calculating the forecasted crack

Table 5. Parameter Estimation of Different Data Sets

	100% sample		90% sample		80% sample	
Variable	Estimate	Significance	Estimate	Significance	Estimate	Significance
Constant	−8.4246	0.0012	−8.5291	0.0025	−8.4114	0.0060
CI	−0.7134	0.0000	−0.7497	0.0000	−0.7606	0.0000
Age	1.3485	0.0000	1.3844	0.0000	1.4113	0.0004
log(ESAL)	2.0418	0.0000	2.0049	0.0000	1.9747	0.0000
Cycle 2	1.5347	0.0007	1.5091	0.0014	1.5310	0.0025
Cycle 3	1.0964	0.0176	1.0771	0.0262	1.1527	0.0264
Cycle 4	1.5278	0.0044	1.4923	0.0086	0.5097	0.0137
Age·age	−0.0337	0.0000	−0.0326	0.0000	−0.0370	0.0000
CI·age	0.0503	0.0001	0.0519	0.0001	0.0510	0.0006
log(ESAL)·age	−0.2191	0.0000	−0.2186	0.0000	−0.2118	0.0002
Cycle·age	−0.1327	0.0049	−0.1259	0.0104	−0.1296	0.0147
Sample size	2,552		2,297		2,042	

Note: CI=crack index and ESAL=equivalent single axle loads.

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Table 6. Kruskal–Wallis Test

	Probability of remaining			Rank measure			
	Sample size			Sample size			
Age	100%	90%	80%	100%	90%	80%	Combined
1	0.9941	0.9964	0.9965	43	44	45	132
2	0.9873	0.9917	0.9918	40	41	42	123
3	0.9745	0.9824	0.9823	37	39	38	114
4	0.9528	0.9654	0.9645	34	36	35	105
5	0.9194	0.9368	0.9347	31	33	32	96
6	0.8733	0.8937	0.8906	28	30	29	87
7	0.8167	0.8359	0.8328	25	27	26	78
8	0.7550	0.7671	0.7665	22	24	23	69
9	0.6950	0.6944	0.6996	20	19	21	60
10	0.6433	0.6260	0.6402	18	16	17	51
11	0.6042	0.5682	0.5942	14	8	13	35
12	0.5802	0.5248	0.5647	10	4	7	21
13	0.5723	0.4972	0.5531	9	3	5	17
14	0.5810	0.4860	0.5597	11	1	6	18
15	0.6058	0.4910	0.5844	15	2	12	29
			Sum:	357	327	351	1,035
			KW:	0.195			

conditions. This introduces a serious limitation in the predictive capability of Markov chains since variation in crack conditions within a state is not accounted for. As discussed previously, the lagged crack index was introduced into the logistic model as a predictor for estimating transition probabilities. This results in a dynamic state distance, i.e., transition probabilities are functions of the present crack condition, and hence the state distance from the present crack condition state to the lower crack condition states depends on the present crack condition, which should be expressed as $d(t)=\mathrm{CI}(t)-SI_j$ ($j=1$ to i, $i=$current state). Accordingly, in Eq. (19), the actual present crack condition index $\mathrm{CI}(t)$ should be used instead of the state value SI_i. With these considerations, Eq. (19) is further modified to the following form:

$$\mathrm{CI}(t+1)=\mathrm{CI}(t)-\sum_{j=1}^{i-1}[\mathrm{CI}(t)-SI_j](i-j)p_{ij}^{t,t+1} \tag{20}$$

Eq. (20) defines a multistate transition process. Moreover, based on the assumption that pavement crack condition can drop only one state during one duty cycle, a binary-state transition process can be defined by Eq. (21):

$$\mathrm{CI}(t+1)=\mathrm{CI}(t)-[\mathrm{CI}(t)-SI_{i-1}]\times p_{i,(i-1)}^{t,t+1} \tag{21}$$

As indicated in Eq. (21), SI_{i-1} can be arbitrarily defined to meet the assumption that the condition drops at most one state for one duty cycle.

Substituting $P[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i]$ for $p_{i,(i-1)}^{t,t+1}$ in Eq. (21), we obtain

$$\mathrm{CI}(t+1)=\mathrm{CI}(t)-[\mathrm{CI}(t)-SI_{i-1}]\times P[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i] \tag{22}$$

Eq. (22) was employed in the recurrent Markov chain developed in this research for forecasting the evolution of pavement crack conditions over time. The mechanism of the recurrent Markov chain is illustrated in Fig. 8, where $d1$ represents the dynamic state distance of crack conditions which depends on the present crack conditions $\mathrm{CI}(t)$, and $d2$ represents the static state distance of crack conditions.

As implied in the specification of the logistic model [Eqs. (14) and (15)], the transition probabilities are functions of the present crack condition index $\mathrm{CI}(t)$, age, cycle, and ESAL. Use of such a logistic model in the recurrent Markov chain process is considered to be theoretically appropriate because it perfectly satisfies the Markov chain assumption that the condition in the current duty cycle depends only on the condition in the previous duty cycle. In addition, a recurrent Markov chain is computationally more efficient since the transition probabilities are dynamically linked to the relevant dependent variables and the variation of each explanatory variable is manifested in the transition probabilities. Therefore the recurrent Markov chain model is expected to outperform its static counterpart in terms of the pavement crack condition forecasting accuracy and ease of implementation.

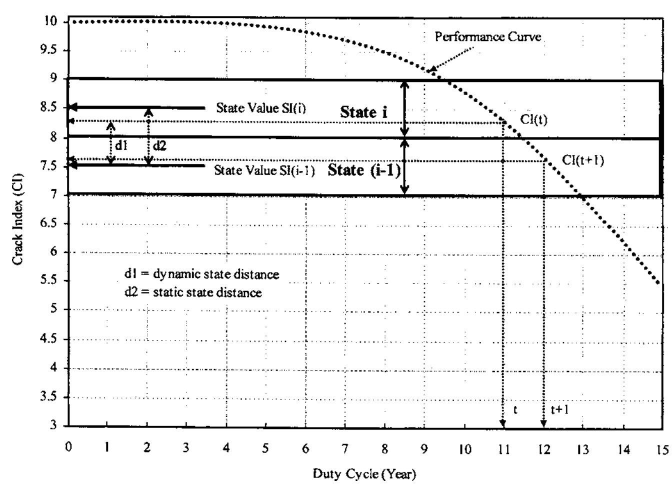

Fig. 8. Illustration of the recurrent Markov chain

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Table 7. Static Transition Probability Matrix

State	10	9	8	7	6	5	4	3	2	1
10	0.9012	0.0988								
9		0.6797	0.3203							
8			0.5833	0.4167						
7				0.6424	0.3576					
6					0.5273	0.4727				
5						0.6667	0.3333			
4							0.8250	0.1750		
3								0.7458	0.2542	
2									0.6667	0.3333
1										1.0000

Generalization of the Recurrent Markov Chain

Rearranging terms in Eq. (22), we obtain

$$\frac{\mathrm{CI}(t+1)-\mathrm{CI}(t)}{(t+1)-t} = -[\mathrm{CI}(t)-SI_{i-1}] \times P[\mathrm{CI}(t+1)\subset i-1|\mathrm{CI}(t)\subset i] \quad (23)$$

The left side of Eq. (23) defines the rate of deterioration of the crack condition. By substituting Eq. (15) into Eq. (23), and replacing CI(t) with the function $C(t)$ which represents continuous variation of the crack condition with time (dotted curve in Fig. 8) and SI_{i-1} with L, the lowest value the condition can be transferred within a given duty cycle, a generalized model is obtained in Eq. (24).

$$\frac{\partial C(t)}{\partial t} = \frac{-[C(t)-L]}{1+e^{-f[C(t),t,\mathrm{Cycle},\mathrm{ESAL}]}} \quad (24)$$

In Eq. (24) the rate of crack condition deterioration is represented by the partial derivative of $C(t)$ with respect to t since $C(t)$ is also a function of the pavement cycle and ESAL. In light of the complexity of the index function $f(\cdot)$ as seen in Eq. (16), the only practical solution for the differential equation in Eq. (24) is in fact obtained using its numerical counterpart in Eq. (22).

Model Performance Evaluation

To evaluate the performance of the model with respect to an independent data set, the model predictions were compared to the known FDOT pavement crack condition data for the year 2003, which were not utilized in the model development. To obtain unbiased evaluations, irrational data that erroneously showed an unrealistically improved pavement crack condition were discarded. In this evaluation, the predictions of the recurrent Markov chain are compared with those of the static Markov chain in terms of three criteria: average absolute error (AAE), root-mean-square error (RMSE), and goodness-of-fit measure (R^2). The three criteria are defined as follows:

$$\mathrm{AAE} = \frac{\sum_{i=1}^{n}|o_i-p_i|}{n} \quad (25)$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(o_i-p_i)^2}{n}} \quad (26)$$

$$R^2 = 1-\left[\sum(o_i-p_i)^2\Big/\sum(o_i-o_{\mathrm{avg}})^2\right] \quad (27)$$

where n=number of observations, o_i=actual value of observation i, p_i=predicted value of observation i, and o_{avg}=average of actual values.

Comparison between the Recurrent and the Static Markov Chains

To illustrate the superiority of the recurrent Markov chain relative to the corresponding static Markov chain, a homogenous static transition probability matrix was also developed and used in the prediction of the pavement crack condition deterioration over time. The transition probabilities were derived from crack condition statistics of the same FDOT pavement condition survey database. More specifically, these probabilities were calculated based on the time-based distribution of the frequencies of pavement sections in each condition state. The transition probability matrix obtained from this exercise is shown in Table 7. For comparison purposes, the crack condition of the pavements in 2003 was forecasted using both the recurrent Markov chain and the static Markov chain. Forecasting errors were computed based on the known actual values and compared in terms of the criteria in Eqs. (25) and (26) and the results are summarized in Table 8. As expected, the recurrent Markov chain produced more accurate forecasts than those of the static Markov chain across all condition states. Furthermore, results in Table 9 show that the

Table 8. Comparison of Single-Year Forecasting Errors of the Static Markov Chain and the Recurrent Markov Chain

Condition state	Static Markov chain		Recurrent Markov chain	
	AAE	RMSE	AAE	RMSE
10	0.6614	0.6850	0.1021	0.1265
9	0.7851	0.8093	0.2101	0.2282
8	0.6645	0.7098	0.2262	0.2464
7	0.7156	0.7576	0.2671	0.2947
6	0.7705	0.8095	0.3003	0.3282
5	0.4614	0.4939	0.2013	0.2417
4	0.3681	0.4083	0.2220	0.2638
3	0.8129	0.8129	0.3585	0.4343
2	0.7537	0.7716	0.1587	0.1733
1	0.5000	0.5000	0.1916	0.2603
Total	0.6715	0.7044	0.1566	0.1948

Note: RMSE=root-mean-square error and AAE=average absolute error.

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Table 9. Comparison of Two-Year Forecasting Errors of the Static Markov Chain and the Recurrent Markov Chain

Condition state	Static Markov chain		Recurrent Markov chain	
	AAE	RMSE	AAE	RMSE
10	0.5660	0.6078	0.4001	0.4519
9	0.8292	0.9280	0.4765	0.5524
8	0.7058	0.8349	0.4731	0.5956
7	0.7667	0.8601	0.4419	0.5745
6	0.7675	0.8836	0.4584	0.6321
5	0.5091	05499	0.2667	0.4064
4	0.4967	0.5332	0.3933	0.5788
3	0.7923	0.8784	0.2854	0.4688
2	1.1000	1.1000	0.0950	0.1366
1	0.5000	0.5000	0.4724	0.6354
Total	0.6721	0.7584	0.4295	0.5157

Note: RMSE=root-mean-square error and AAE=average absolute error. Calculated R-squares of two-year forecasts are 0.90 and 0.79 for the dynamic Markov chain and the static Markov chain, respectively.

same conclusion can be drawn regarding 2-year predictions as well although the prediction errors grow rapidly with longer prediction periods. Therefore it can be concluded that linking the transition probabilities to explanatory variables associated with the pavement crack condition deterioration provides an explicit, adaptive, and more accurate means of estimating those transition probabilities than the simple frequency-based approach.

Goodness of Fit

In this evaluation, the crack conditions forecasted for 2003 were plotted against the actually observed ones. The coefficient of determination (R^2) was computed using Eq. (27), which forces the regression line to be $y=x$ (predicted CI=observed CI). Based on the R^2 values shown in Figs. 9 and 10, the recurrent Markov chain outperforms the static Markov chain. In addition, the vertical scatter of the representative data points of the recurrent Markov chain indicates realistic predictions using probabilistic models since the predicted crack conditions distribute evenly around the correlation line. In contrast to the recurrent Markov chain, the horizontal scatter of the representative data points of the static Markov chain indicates nonresponse of the predictions

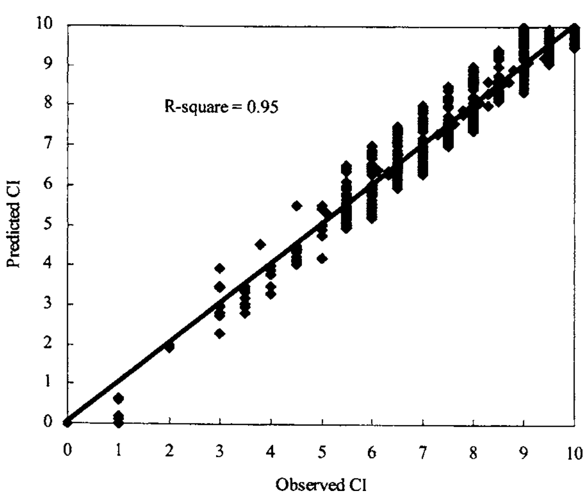

Fig. 9. Goodness of fit—recurrent Markov chain

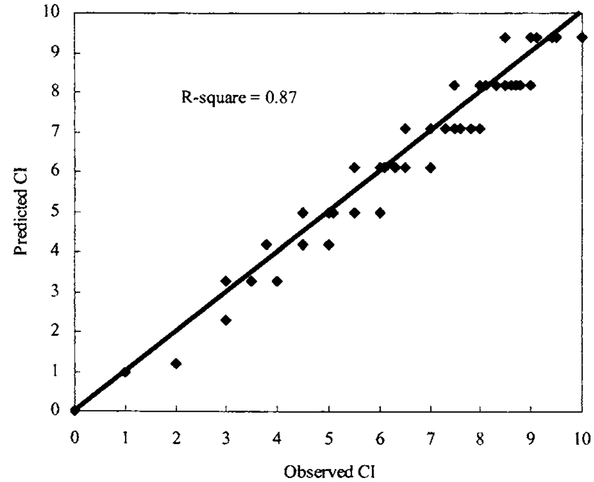

Fig. 10. Goodness of fit—static Markov chain

to the variation of the crack condition in one state, which is contradictory to the Markov property of state dependency. Therefore the use of the recurrent Markov chain would alleviate such inconsistencies in modeling pavement crack condition deterioration.

Conclusions

Due to its conceptual conciseness and ease of implementation, Markov chain has been applied extensively in modeling a variety of physical phenomena that are stochastic in nature. It has also been adopted by many highway agencies in their PMSs for condition prediction. However, major shortcomings encountered in routine Markov chains are the difficulties in modeling the time-dependent rates of condition deterioration and in estimating transition probabilities in an explicit manner. Various statistical methods used by agencies equipped with well-established and extensive pavement condition databases to estimate the transition probabilities result in transition probabilities that are invariant with time and do not incorporate current condition deterioration trends. This paper documents a study that was conducted to address the above deficiencies and develop an innovative pavement crack performance model based on a recurrent Markov chain. The uniqueness of this study is that a logistic model was created to associate the transition probabilities with a set of relevant explanatory variables including the current crack condition index. An extensive historical pavement condition database and statistical significance tests were used to select those variables that are most relevant to the logistic model of transition probabilities. Then, a recurrent Markov chain was constructed in such a way that the logistic model can be smoothly integrated into it with computational ease. It was also shown how the model can be extended to include degradation processes that are continuous with respect to time. Therefore the proposed methodology is not process specific and hence it can be employed for modeling the performance of other pavement distress types, such as ride quality and rutting as well. As an adaptive process, the recurrent Markov chain is able to realize the true dynamics of the crack condition deterioration process not only in the estimation of the transition probabilities but also in the application of them in realistic forecasting. It is also shown that the recurrent Markov chain outperforms the static Markov chain in terms of forecasting

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accuracy. Furthermore, a parametric analysis showed that the model predictions are intuitively acceptable; while the model stability was ascertained by a sensitivity study.

Most importantly, in comparison to commonplace static Markov chains, the recurrent Markov chain is considered theoretically more appropriate because the structure of the logistic model by its very nature satisfies the Markov property of state dependency. In addition, the logistic model would be more conveniently adaptable in transportation agencies since it explicitly uses the relevant explanatory variables in the estimation of transition probabilities. This is because once the model structure has been established and the dynamics of pavement degradation are seen not to change drastically, the same logistic model format could be only updated with newly acquired condition, traffic, and rehabilitation data. All in all, the new model has the potential to curtail pavement rehabilitation expenditure significantly due to its improved efficiency and condition prediction accuracy.

Limitations of the Methodology

Markovian models generally suffer from somewhat unrealistic assumptions of discrete transition time intervals and dependence of the future facility condition only on the current condition (Morcous 2002). In addition, the model outcomes are highly dependent on the quality of the data used. Especially, the manual data collection method currently adopted by FDOT is bound to introduce errors due to the raters' subjectivity. Therefore it is recommended that the pavement condition survey procedure be conducted in a consistent manner over time and the annual survey data be carefully examined for any irregularities before the PMS database is updated. In the future this issue would be better addressed by the automatic data collection systems that are currently under development. Furthermore, timely updates of the model parameters using fresh condition data are necessary to capture any marked changes in the deterioration pattern seen in the updated data. When the need arises even multiple-state transition probabilities can be derived from the binary-state transition probabilities, or alternatively, distinct states or duty cycles can be redefined to satisfy the binary-state transition condition assumed in this study. Nonetheless, for improved forecasting accuracy, it is recommended that the model be developed in terms of multiple-state transition probabilities when this trend is supported by the condition data.

As for this specific model, the environmental impact on the pavement crack condition is not included as Florida's PMS database does not provide the relevant geotechnical data needed to perform such a comprehensive analysis. Moreover, structure-specific variables such as stiffness of base or subgrade that would more appropriately represent the changing pavement structure than the variable of construction cycles used in this study, would certainly enhance the performance of the recurrent Markov chain.

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