Valuing Flexibility in Infrastructure Expansion

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Abstract: Infrastructure facilities are generally heavy, fixed, and normally irreversible once construction has been completed. As existing facilities, they may confront economic competition of an increased space demand and the need for future expansion. Due to economic-based irreversibility, the expansion of a constructed facility requires the foundation and, to a lesser degree, columns to be enhanced and options for expansion to be accounted for at the very beginning of construction. Enhancing the foundation and columns represents an up-front cost, but has a return in flexibility for future expansion. This trade-off can be viewed as an investment problem, in that a premium has to be paid first for an option that can be exercised later. A model of the foundations versus flexibility trade-off enables the competing options to be optimized by balancing the expected profits that may arise from future expansion, i.e., the value of flexibility, and the cost of enhancing the foundation. Use of the model is demonstrated for the construction of a public parking garage, with the optimal foundation size determined. The evolution of parking demand is modeled with a trinomial lattice. Stochastic dynamic programming is used to determine the optimal expansion process. A model that does not consider the value of flexibility is compared with two value-flexible models. The value of flexibility in this case study is so significant that failure to account for flexibility is not economical. Valuation modeling such as discounted cash flow analysis with uncertainty modeling is important to capitalize on the worth of flexibility.

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Introduction

A facility, such as a building, dam, bridge, or airport, is heavy, fixed, and normally irreversible once construction has been completed. A facility may face economic competition because of an increased space demand and the need for future expansion. Due to the economic-based irreversibility, the expansion of a constructed facility requires the foundation and, to a lesser degree, columns to be enhanced and options for expansion to be accounted for at the very beginning of construction. Enhancing the foundation beyond immediate needs requires a higher initial cost. In return, the facility increases its flexibility to expand whenever it is needed. This flexibility should be assigned a significant value. If it is not assigned value, then the cost of the additional enhancement of the foundation would not be marginally practical. Designers have increasingly preferred to design a facility with features that facilitate future expansion. Although the merits of flexibility are recognized, an objective method of valuing flexibility in an enhanced foundation project has not been available. Recent literature in the field of option theory recognizes that traditional valuation methods such as discounted cash flow analyses may not be proper for valuing flexibility (Trigeorgis 1996). This paper intends to develop a valuation model for facility expansion that accounts for the value of expandable flexibility.

A *flexible* system is one ''characterized by a ready capability to adapt to new, different, or changing requirements,'' according to *Webster's Dictionary*. In other words, it is a system that has the capability to cope with uncertainties associated with changing needs. Due to the massiveness of civil engineering facilities, fewer alternatives are available to provide expansion flexibility, compared to less costly mechanical systems.

In this paper, the foundation of a facility is used as the structural component through which concepts related to flexibility are introduced; however, the concepts can certainly be applied to any component. A foundation is referred to herein as the structural elements that connect the structure to the ground. Generally, a foundation is characterized by the depth to which structural loads can be transmitted to underground soils and/or rocks. Strength requirements, serviceability, constructability, and economic value are the major factors for designing the type and size of a foundation. If the foundation has been enhanced to support an expanded structure, the facility will have the flexibility to expand (vertically) later.

Enhancing the foundation at the time of initial construction involves a significant cost. However, the increased flexibility for future expansion may offset the initial cost of the extra enhancement. If the foundation of a facility were not designed to support expansion, it would be either technically prohibitive or extremely uneconomical to expand the facility. In fact, enhancing an original foundation normally takes place only when abnormal conditions of the foundation or structure, such as differential settlement, exist. Further, the purpose of this kind of enhancement is usually to remedy an anomaly, not to expand the structure. The foundation selection problem can be viewed as an investment problem, such that a premium has to be paid first for an option that can be exercised later. A model is presented that determines whether enhancing the foundation is favorable or not, and if so, to what

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extent the enhancement should be done. To achieve optimal decision making for this trade-off problem, the expected profits that may arise from future expansions (called the value of flexibility) must be balanced against the costs of enhancing the foundation.

Flexibility has value because of uncertainty. If a system were steady and every future outcome could be fully predicted, then flexibility would not be needed. To value flexibility in engineering design, uncertainty modeling is necessary for optimum assessment. A rich body of literature that addresses decision making under uncertainty, such as the De Neufville (1990) source, is available. A valuation method based on a trinomial lattice for valuing the flexibility in construction expansion is presented, with the construction of a public parking structure used to illustrate the methodology. Three demand model types are compared to describe parking demand evolution over a 15-year study period. A comparison of the three models clearly shows the value of flexibility. In fact, the value of flexibility may be so significant that it would be very costly to exclude it from the design.

The major contribution of this paper is to identify the value of flexibility of an enhanced foundation in infrastructure expansion and to propose a complete method to value the flexibility. Through a case study of the construction of a public parking garage, we also contribute to a better understanding of the benefits of applying flexibility in infrastructure design and construction.

Option and Flexibility

In finance, an *option* is defined as the right, but not the obligation, to buy (or sell) an asset under specified terms (Luenberger 1998; Hull 1999). For example, an option that gives the right to purchase something is called a *call option*; an option that gives the right to sell something is called a *put*. Normally an option has a price (called a *premium*). Consider a call option that allows you to buy a specified stock at $K(=\$50)$ at some future time T. This option will be valuable if the stock price at T, denoted by $S(T)$, turns out to be higher than K, say, \$70. The call option can be exercised by buying the stock at \$50 and then reselling the stock back to the market at a profit of \$20 $(=\$70-\$50)$. However, if the stock price falls below \$50 at T, this call option is virtually worthless and the owner loses the premium. Intuitively, at time t the expected payoff of such a call option is

$$\text{Expected option payoff} = E_{\xi}\{\max[S(T)-K,0]\}(1+r)^{-(T-t)} \tag{1}$$

where $E[\cdot]$ = expectation operator; ξ = some probability measure at time t; and r = corresponding discount rate. If the expected option payoff calculated by Eq. (1) is higher than the cost to acquire the option, then the call option is favorable (note that risk preference can be taken into account through either the discount rate or the probability measure).

Recently, the concept of options has been applied extensively in a variety of areas other than financial contracts. This subject is known as *real options* valuation. Real options can be defined as the options embedded in *real* operational processes, activities, or investment opportunities that are not financial instruments (Trigeorgis 1996). For example, a real option gives the option holder *the right but not the obligation* to take an action (not merely purchase something) in the future. An enhanced foundation is one immediate example of real options because it provides the flexibility for expansion. In other words, a real option gives the owner some flexibility. The two terms, option and flexibility, will be used interchangeably in this paper, and an option is distinguished from an *alternative*. To gain an option (or flexibility), a price may have to be paid up front. In reality, options may arise in varied applications and circumstances, and provide the ability to react to a changing environment. Examples of real options include the following.

- *Flexibility to defer* allows the decision maker to delay taking an action until uncertainties are favorable [e.g., one may gain some lease such as a right-of-way contract that grants him/her the right to defer; Dixit and Pindyck (1993)].
- *Volume flexibility* of a manufacturing system allows a firm to change the production rate, depending on market conditions.
- *Product flexibility* allows a varying product mix in different market conditions (Fine and Freund 1990).
- *Flexibility to expand or contract* allows the decision maker to increase or decrease the system capacity scale when a trend of higher or lower system demand is formed (Kumar 1995).
- *Flexibility to switch* allows a system operator to switch to different technologies or resources (Kulatilaka 1993).

Other applications of real options can also be found in investments of intelligent transport systems (Leviäkangas and Lähesmaa 2002) and construction projects (Ford et al. 2002). In general, an option provides an opportunity for the decision maker to take some action *after* uncertainties are revealed. For example, the owner of a call option will exercise the option only after learning that the stock price $S(T)$ is greater than K.

To value a real option, one should explore all possible scenarios under which exercising the option is favorable. This process may become nontrivial when the problem involves multiple options and multiple periods. In terms of the methodology for valuing real options, two major approaches have been proposed—the lattice approach (Hull and White 1993) and the Monte Carlo (MC) simulation (Hull 1999). Both approaches differ in how uncertainty evolution is handled. The lattice approach is, in general, more computationally efficient than the MC method, while the MC method has the flexibility to model multiple or complicated uncertainty processes. A review of the lattice approach can be found in Ho and Liu (2003).

In this paper, a systematic method based on stochastic dynamic programming will be presented, which falls into the category of the lattice approach. The proposed approach handles sequential decision making and variable outcomes over time. In the multiperiod cases, net present value (NPV) is calculated by repeating the single-period discounting for every period, starting from the final period and working backward toward the initial time.

Modeling Demand Uncertainty

Flexibility has value because of uncertainty associated with changes in future needs or conditions. If a system were steady and every future outcome could be fully predicted, then flexibility would not be needed. Therefore, uncertainty modeling is important for valuing flexibility.

Foundation design relies on assessments of the soil and rock conditions, as well as the load combination of the superstructure. These assessments always involve considerable uncertainty, such as characteristics of soil and rock and the underground water level. The owner may also face other operational uncertainties such as demand, labor costs, and rent. The type of uncertainty that evolves over time can have tremendous impacts on operational economics. To demonstrate the method, demand will be the only uncertainty modeled here.

In this paper, we propose using stochastic diffusion processes to describe uncertainty evolution. If data for the daily demand of an infrastructure service can be obtained and displayed as a function of time, a trajectory fluctuating up and down, much like some signal noise, would be evident. In theory, statistical techniques can be used to summarize the data to reflect a stochastic diffusion process. Indeed, diffusion processes have long been used to model uncertainties in natural sciences, including population growth (Bailey 1964; Newell 1982). The uncertainty model to be presented next can be viewed as a discrete approximate model for one type of stochastic diffusion process. By modeling the demand as a diffusion process, we are assuming that the demand does not depend on the supply, such as the availability of a new parking garage, which may not be the case in reality.

Given a demand uncertainty d, which evolves over time, consider the following multiplicative model (Luenberger 1998, Chapter 11):

$$d(t+1)=d(t)\cdot z(t) \tag{2}$$

for $t=0,1,\ldots,T$, where $z(t)$, $t=0,1,\ldots,T$ are random variables that may be a function of $d(t)$, and only take positive real values. For the multiplicative model in Eq. (2), once the demand process starts with a positive value, it will remain positive. The multiplicative model has been commonly used in finance because stock prices can never be negative and $z(t)$s can be interpreted as uncertain returns. If we take the natural logarithm of both sides of Eq. (2), the multiplicative model of Eq. (2) is transformed to an additive model, which is easier to optimize. Let $D(t)=\ln d(t)$ and $Z(t)=\ln z(t)$; Eq. (2) is equivalent to

$$D(t+1)=D(t)+Z(t) \tag{3}$$

for $t=0,1,\ldots,T$. Next, we will develop a model for $D(t)$. Since $d(t)=e^{D(t)}>0$, $D(t)$ can be any real number, including a negative one. We also assume that $Z(t)$s are normal random variables, with the following relations:

$$E[D(t+1)-D(t)]=E[Z(t)]=\mu[D(t),t] \tag{4}$$

$$\mathrm{var}[D(t+1)-D(t)]=\mathrm{var}[Z(t)]=\sigma^2 \tag{5}$$

In Eqs. (4) and (5), $\mu(\cdot)$ is a *drift* function of $D(t)$ and t, and σ is a constant *volatility*. The drift function describes the trend of the uncertainty movements, in the sense of the expected value. From Eq. (3), we have

$$D(t)=D(0)+\sum_{i=0}^{t-1}Z(i) \tag{6}$$

Therefore, we know that $D(t)$ is normal, because the sum of normal random variables is itself a normal. In other words, all demands, $d(t)$s, are lognormal.

While various approaches can be used to model uncertainties, uncertainty modeling should also be integrated with the solution procedures. To better implement dynamic decision making, a trinomial lattice model, which has been widely applied in the valuation of financial options (Luenberger 1998), is used to approximate the uncertainty characterized by Eqs. (4) and (5). For a trinomial lattice, each demand node is designed to branch out into three demand nodes corresponding to the following time period (Fig. 1). To form a lattice, discrete demand values (nodes), with a constant increment ΔD, are formed for each time period, such that branching is only allowed from nodes to nodes of the following time period. How to select the three nodes to map, as well as their corresponding (conditional) probabilities, is discussed next.

Assume that the value of the uncertainty is known to be $D(t)$ at time period t, and after one time period, $t+1$, the uncertainty $D(t+1)$ will have three possible values, $D(t)+(k+1)\Delta D$, $D(t)+k\Delta D$, and $D(t)+(k-1)\Delta D$, with probabilities p^u, p^m, and p^d, respectively. The superscripts u, m, and d specify three different branches, up, middle, and down. The demand increment ΔD is obtained using $\Delta D=\theta\sigma$, where θ is a constant. The choice of θ will be shown later in this section. The branching factor k is an integer, and the value of k is chosen such that $k\Delta D$ can best approximate the expected drift $\mu(D,t)$

$$k\equiv\left\lfloor\frac{\mu(D,t)}{\Delta D}+\frac{1}{2}\right\rfloor \tag{7}$$

where $\lfloor\cdot\rfloor=$ so-called floor function, which maps a real number to the nearest integer toward $-\infty$.

To solve for the branching probabilities p^u, p^m, and p^d, the following linear equations are formed:

$$E[D(t+1)-D(t)]=p^u(k+1)\Delta D+p^m k\Delta D+p^d(k-1)\Delta D$$
$$=\mu(D,t) \tag{8a}$$

$$E\{[D(t+1)-D(t)]^2\}$$
$$=p^u(k+1)^2\Delta D^2+p^m k^2\Delta D^2+p^d(k-1)^2\Delta D^2$$
$$=\mathrm{var}[D(t+1)-D(t)]+E[D(t+1)-D(t)]^2$$
$$=\sigma^2+\mu(D,t)^2 \tag{8b}$$

$$p^u+p^m+p^d=1 \tag{8c}$$

where Eqs. $(8a)$ and $(8b)$ intend to match the conditional mean and variance of $D(t+1)-D(t)$, respectively. Note that for p^u, p^m, and p^d to be between 0 and 1, θ must be chosen between $2/\sqrt{3}$ and 2. For details, see Hull and White (1993).

Repeating the foregoing process to the second period, third period, and so on produces a (trinomial) *lattice* of the uncertainty. An example of a lattice with two time periods is illustrated in Fig. 1. Note that the value of k, and probabilities p^u, p^m, and p^d are dependent on both time t and state D.

As an example, the lattice in Fig. 1 implies nine $(=3^2)$ possible paths for the evolution of $D(0)=D_0$ from $t=0$ to $t=2$

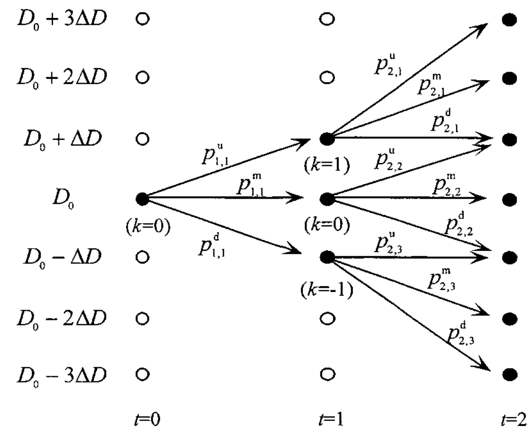

Fig. 1. A trinomial lattice evolves over three time periods (from $t=0$ to $t=2$).

$$\text{Path 1:} \quad \{D_0, D_0+\Delta D, D_0+3\Delta D\} \quad \text{with probability} \quad p_{1,1}^{u} p_{2,1}^{u}$$

$$\text{Path 2:} \quad \{D_0, D_0+\Delta D, D_0+2\Delta D\} \quad \text{with probability} \quad p_{1,1}^{u} p_{2,1}^{m}$$

$$\text{Path 3:} \quad \{D_0, D_0+\Delta D, D_0+\Delta D\} \quad \text{with probability} \quad p_{1,1}^{u} p_{2,1}^{d}$$

$$\vdots$$

$$\text{Path 9:} \quad \{D_0, D_0-\Delta D, D_0-3\Delta D\} \quad \text{with probability} \quad p_{1,1}^{d} p_{2,3}^{d}$$

where $p_{t,j}^{b}$ represents the probability for branching $b \in \{u,m,d\}$ at node $j \in \Omega(t)$ in time period $t \in \{1,2\}$. Note that $\Omega(t)$ is some ordered set of indices for those nodes that are mapped into at time t (from the top down, numbered starting from 1 in Fig. 1); e.g., $\Omega(0)=\{1\}$, $\Omega(1)=\{1,2,3\}$, and $\Omega(2)=\{1,2,\ldots,7\}$. The branching factor k, obtained using Eq. (7), may vary from node to node and is given underneath each node that branches out in Fig. 1.

Embedded on an uncertainty lattice, decision making can be performed at each node in the lattice. This is the basic idea of the stochastic dynamic programming approach. The differences between a lattice approach and a decision tree are as follows: (1) branches recombine with a lattice but not with a decision tree; and (2) a lattice node plays both roles—that of a chance node and that of a decision node of a decision tree. Therefore, the stochastic dynamic programming approach using lattices can be viewed as a generalized extension of the decision tree method.

Valuing Foundation

As mentioned previously, an enhanced foundation can be viewed as a real option for expansion. In this section, the expected value of this real option is appraised.

Constructing Public Parking Garage—Case Study

This case study pertains to the construction of a public parking garage by a county in the Washington, D.C. area. The cost data were compiled from the feasibility study done by the county. A daily demand of 250 units of parking space at the location where the garage is to be constructed is assumed. Assume that each parking space can generate revenue of $3,600 per year from parking fees, denoted by R, but requires $100 of maintenance cost, denoted by c_m. The site for the garage can accommodate 100 parking spaces per level, denoted by m, and the garage will have multiple levels. Assume that the life of the parking garage (T) is 15 years, and after that the facility will be obsolete, with no salvage value. Note that the notations with an uppercase C represent fixed costs. In addition, this case study is designed based on a discount rate of 8%, which was suggested by the feasibility study report of that county. Detailed demand models over the 15-year planning period will be presented in the next section.

Both profiting and providing satisfactory services are assumed to be important objectives for the public parking garage. For each unserviced demand, a penalty c_{pn} may be applied. The penalty can be viewed as either a loss of revenue or opportunity, or a potential cost for unsatisfied service, such as complaints that may aggravate and lead to other economic losses. We use the term "net profit" to represent the profit less penalty for unserviced demands, and assume that the decision maker maximizes the net profit when he/she makes expansion decisions. To be general, the penalty is assumed to be $c_{pn}=\alpha \cdot R$, $0 \leqslant \alpha \leqslant 1$, a fraction α of revenue R generated by each parking space per year. When $\alpha=0$, it corresponds to the case without penalty and the decision making is purely profit maximization. On the other hand, the value of α can be viewed as a weighting factor for the importance for meeting demands; the higher the value, the more important it is. In the rest of this section, for simplicity, only the results of two extreme cases, $\alpha=0$ and $\alpha=1$, are specified. Full comparison of the effect of α is deferred to a later section for discussion. Other costs related to construction are summarized in Table 1. All costs and revenues are subject to an annual increase of $f=5\%$ due to factors such as inflation.

Table 1. Summary of Construction Costs

Parameter	Value
Site preparation (C_s)	$300,000
Fixed cost for foundation (C_f)	$1 million
Variable cost for foundation (c_n)	$100,000/level
Superstructure and miscellaneous (c_u)	$800,000/level
Construction cost for expansion (c_e)	$850,000/level

For simplicity, the foundation size (or strength) of the facility is represented in terms of the maximum number of levels of superstructure, denoted by N, that the foundation can support safely. Let n_t be the number of levels of the facility in time period t. At time t, $N-n_t$ represents the "foundation reserve" that would be available for expansion in terms of additional levels. The unit of time t is in years, and the expansion decision is reviewed at the beginning of each year. The problem is to determine the optimal foundation size N and the initial number of levels to construct n_0 ($n_0 \leqslant N$) at time 0. Furthermore, an optimal expansion plan n_t, $t=1,,\ldots,T-1$ over the study period needs to be determined.

Assume that the demand for parking spaces over time period $[t,t+1]$ is denoted by d_t, $t=0,\ldots,T-1$. Given the initial foundation size N and an expansion plan of the facility n_t, $t=0,\ldots,T-1$, a cash flow stream $(x_0,x_1,\ldots,x_{T-1})$ for the net profit of the facility can be identified. The sequence of decision making is as follows. At time $t=0$, the foundation size N and the initial number of levels n_0 are decided. Construction incurs the following costs:

$$\begin{aligned} x_0 &= -\text{costs of site preparation, foundation,} \\ &\quad \text{and superstructure} \\ &= -C_s-(C_f+c_n N)-c_u n_0 \end{aligned} \tag{9}$$

The cost of the foundation is assumed to be a linear function of the foundation size N. Assume that the construction would take 1 year to complete. At any time $t>0$, the total net profit accumulated over time period $[t,t+1]$ is represented by x_t, where

$$\begin{aligned} x_t &= \text{Revenue}-\text{maintenance cost} \\ &\quad -\text{penalty for unmet demand}-\text{expansion cost} \\ &= [R \cdot \min(d_t, n_{t-1}m)-c_m n_{t-1}m-c_{pn}\max(0, d_t-n_{t-1}m) \\ &\quad -c_e(n_t-n_{t-1})](1+f)^t \end{aligned} \tag{10}$$

The revenue, maintenance cost, and penalty over time period $[t,t+1]$ are based on the capacity of the parking structure $n_{t-1}\cdot m$, determined one time period (year) earlier because of the one-year construction lead time assumed. At time t, the expansion decision n_t ($\geqslant n_{t-1}$) is made. If $n_t > n_{t-1}$, that means (n_t-n_{t-1}) additional level(s) of parking structure will be built during the year, which incurs an expansion cost of $c_e(n_t-n_{t-1})(1+f)^t$, represented by the last term of Eq. (10). Again, any expan-

Table 2. Ten-Year Historical Demand Data

Year j	d_j^H	$D_j^H = \ln d_j^H$	$D_{j+1}^H - D_j^H$
0	164	5.0999	—[a]
1	157	5.0562	−0.04362
2	181	5.1985	0.14225
3	175	5.1648	−0.03371
4	208	5.3375	0.17275
5	185	5.2204	−0.11718
6	174	5.1591	−0.06130
7	202	5.3083	0.14921
8	237	5.4681	0.15979
9	216	5.3753	−0.09278
10	250	5.5215	0.14618

Note: Mean $\mu = 0.04216$; standard deviation $\sigma = 0.12047$.

[a]Not applicable.

Table 3. Outlook of Future (Average) Demand $\tilde{d}_t$

Year	Time t	Average demand $\tilde{d}_t$ (units)
1	0	250.00
2	1	262.66
3	2	275.97
4	3	289.95
5	4	304.64
6	5	320.07
7	6	336.29
8	7	353.32
9	8	371.22
10	9	390.02
11	10	409.78
12	11	430.54
13	12	452.35
14	13	475.27
15	14	499.34

sion construction is assumed to take 1 year to complete. Note that the expansion decision n_t is subject to the following nondecreasing condition:

$$n_0 \leqslant n_1 \leqslant \cdots \leqslant n_{T-1} \leqslant N \tag{11}$$

Once the cash flow stream $(x_0, x_1, \ldots, x_{T-1})$ is obtained, its NPV (of the net profits) can be determined as follows:

$$\mathrm{NPV} = \sum_{t=0}^{T} \frac{x_t}{(1+r)^t} \tag{12}$$

where r = discount rate per time period. Note that tax should also be included in practice, which may reduce the NPV.

Note that the penalty term used in Eq. (10) may not represent a specific cash outflow. Using penalties is a common technique in decision making and economics to contrast alternative decisions. Here, we use the penalty to show the value of flexibility versus the potential loss of the profit if the expansion flexibility is not in place. Including such an opportunity cost (penalty) in a cash flow stream may raise concern. One immediate consequence is that the NPV obtained in Eq. (12) may not be 100% realizable. However, the term "cash flow stream" is still used in this paper because we focus on the aspect of decision making under uncertainty, rather than cost accounting. If the reader prefers, the cash flow stream $(x_0, x_1, \ldots, x_T)$ may be termed a "sequence of the net profit."

Three Demand Models

Assume that historical demand information is available for the past 10 years, denoted by d_j^H, $j = 1, \ldots, 10$, which is summarized in Table 2. Assume that the logarithm of demand $D_j^H = \ln d_j^H$ is subject to a process following Eqs. (4) and (5), where $Z(t)$, $t = 1, 2, \ldots, T$ are mutually independent, and the mean $\mu(\cdot)$ is assumed to be a constant μ, independent of state D_j^H and time j. Using the following formulas, with the historical data in Table 2, the best estimate of μ is 0.04216 and σ is 0.12047 units:

$$\mu = \frac{1}{10} \sum_{j=0}^{9} (D_{j+1}^H - D_j^H) \tag{13a}$$

$$\sigma^2 = \frac{1}{(10-1)} \sum_{j=0}^{9} (D_{j+1}^H - D_j^H - \mu)^2 \tag{13b}$$

From Eq. (8), the expected logarithm of annual parking demand of year t in the future is

$$E[D(t)] = E[\ln d(t)] = D_0 + \sum_{i=0}^{t-1} u = D_0 + t\mu \tag{14}$$

The expected annual parking demand of year t in the future, $E[d(t)]$, is, however, not $\exp\{E[D(t)]\} = d_0 \exp(t\mu)$, but

$$\tilde{d}_t \equiv E[d(t)] = d_0 \exp(t\mu + \tfrac{1}{2} t\sigma^2) \tag{15}$$

with some contribution from the variance σ^2. Note that Eq. (15) is obtained using the lognormal property of $d(t)$. Given the known demand at $t = 0$, $d_0 = d(0) = 250$, and the logarithm of demand $D_0 = D(0) = \ln 250$, the values of $\tilde{d}_t$, $t = 0, \ldots, 14$, using Eq. (15), are obtained and are summarized in Table 3.

The trinomial lattice introduced previously will be used to represent demand evolution. While the trinomial lattice model is capable of handling more general processes such as time and state dependent drift functions $\mu(D,t)$, the demand uncertainty model considered in this case study (by setting it to be a constant) is more simplistic due to lack of data. If data are sufficiently available, statistical methods can be used to estimate the drift function $\mu(D,t)$, as well as its relations to the state D and time t. Because we assume the drift function μ to be a constant for all time periods, the three branching probabilities (p^u, p^m, p^d) and the branching factor k will be the same for every node. The parking demands in the lattice may no longer be integers. The computation is still carried forward to maintain the integrity of the lattice.

Depending on the complexity of the available demand information, three models can be applied to determine the foundation size and the facility expansion plan. To produce a fair comparison, all three models are based on the same data source and are consistent.

- **Model 1**—deterministic annual parking demands, identical for every year over the entire study period. In this model, the demand is assumed to be fixed and is denoted by $\tilde{d}$. It is the simple arithmetic average of the annual demands $\tilde{d}_t$ given in Table 3

$$\tilde{d} = \frac{1}{T} \sum_{t=0}^{T-1} \tilde{d}_t = 361.4 \tag{16}$$

- **Model 2**—deterministic annual parking demands, with a trend described by $\tilde{d}_t$ from Eq. (15), $t = 0, \ldots, 14$, as given in Table 3. In this case, the demand trend is growing because $\mu > 0$.

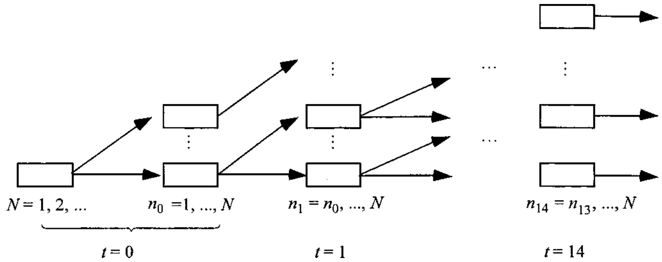

Fig. 2. Deterministic decision tree for solving Models 1 and 2

- **Model 3**—stochastic annual parking demands, described by the trinomial lattice. In this model, at each node and at each time t, the branching factor k and branching probabilities p^u, p^m, and p^d are determined by using Eq. (7) and solving Eqs. $(8a)$–$(8c)$, respectively. Let $\Delta D = \theta\sigma = \sqrt{3} \times 0.12047 = 0.2087$, where θ is set to be $\sqrt{3}$, as suggested by Hull and White (1993). Using Eq. (7), in this case study, the branching factor is

$$k = \left\lfloor \frac{\mu}{\Delta D} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{0.04216}{0.2087} + \frac{1}{2} \right\rfloor = 0 \tag{17}$$

and $(p^u, p^m, p^d) = (0.288, 0.626, 0.086)$, the same for each node at each time period. Since the lattice covers 15 time periods (years), from $t=0,\ldots,14$, equivalently it accounts for 3^{15} $(=14.348907 \text{ million})$ demand scenarios.

Model 1—Annual Parking Demand is a Constant $\tilde{d}$ for All Years

This model assumes that the annual parking demand is a constant $\tilde{d}$ for all years. Since uncertainty is not involved, a deterministic decision tree (De Neufville 1990), as shown in Fig. 2, is used to solve the problem. At the beginning of construction, $t=0$, the foundation size N and the number of levels of superstructure n_0 are decided. Assume that the construction will take 1 year to complete. After the first year, expansion decision n_t is reviewed, which is subject to Eq. (11).

Mathematically, the decision-making process in Fig. 2 can be summarized by a recursive relation. Extending from Eqs. (9) and (10), the cash flow x_t is now written as a function of decision variables N and n_t; i.e., $x_t(N, n_t)$

$$x_0 = \max_{N, n_0}[-C_s - (C_f + c_n N) - c_u n_0 + (1+r)^{-1} x_1(N, n_0)] \tag{18a}$$

and for $t = 1, \ldots, T-1$

$$\begin{aligned} x_t(N, n_{t-1}) = {} & [R \cdot \min(d_t, n_{t-1}m) - c_m n_{t-1} m \\ & - c_{pn} \max(0, d_t - n_{t-1}m)](1+f)^t \\ & + \max_{n_{t-1} \le n_t \le N} [(1+r)^{-1} x_{t+1}(N, n_t) \\ & - c_e(n_t - n_{t-1})(1+f)^t] \end{aligned} \tag{18b}$$

with boundary conditions

$$x_T(N, n_{T-1}) = 0; \quad \forall n_{T-1} \tag{18c}$$

Assuming $d_t = \tilde{d} = 361.4$ for all t, the optimal solution obtained for this model is $N = n_0 = 4$, regardless of the penalty factor α. That is, a foundation reserve is not made, nor is an expansion plan. The corresponding optimal NPV of the model is \$9.5 million. This optimal solution seems intuitive. Since the demand is assumed to be 361.4 units every year, the optimal solution is to build a parking structure with 400 parking spaces. As mentioned previously, if every future outcome could be perfectly forecast, it is unnecessary to reserve any flexibility. Detailed results for the case with the penalty factor of $\alpha = 1$ are given in Table 4.

In real options literature, a risk-free interest rate is usually used as the discount rate, which, however, only applies to the cases where real options can be hedged and all risks can be eliminated. Since all risks are eliminated, under no arbitrage argument, the discount rate has to be the "risk-free" rate. In reality, there is nothing tradable in the market such that the risk of holding an enhanced foundation can be hedged. Therefore, the discount rate is determined based on the riskiness of the project (Copeland et al. 1990). To obtain a risk-adjusted discount rate, a pricing model must be assumed. An example based on the capital asset pricing model (CAPM) can be found in Leviäkangas and Lähesmaa (2002).

Model 2—Annual Parking Demands Are Constants, $\tilde{d}_t$, $t = 0, \ldots, 14$

The same deterministic decision tree used in Fig. 2, or Eqs. $(18a)$–$(18c)$, is used to solve this model, with the optimal design $N=4$, $n_0=3$ obtained for the case without penalty $(\alpha=0)$, and $N=5$, $n_0=3$ for the case with maximal penalty $(\alpha=1)$. In the optimal solution of the case without penalty, the foundation is enhanced to support expansion for one additional level in the future with an NPV of \$8.9 million, and the expansion decision will be made at the beginning of Year 4 $(n_3=3)$. Since the demand exceeds 300 spaces in the 4th year, the structure is expanded one additional level at $t=3$, and the expansion construction is completed at $t=4$. A similar observation can be made for the case with maximal penalty, in which the optimal NPV is \$8.7

Table 4. NPV (\$1,000) of Alternatives in Expansion Planning for Model 1 $(\alpha = 1)$

	Initial foundation reserve (levels), $N - n_0$						
Initial number of levels n_0	0	1	2	3	4	5	6
1	−8,943.1	−1,794.2	5,354.7	9,335.7	9,235.7	9,135.7	9,035.7
2	−1,744.2	5,404.7	9,385.7	9,285.7	9,185.7	9,085.7	8,985.7
3	5,454.7	9,435.7	9,335.7	9,235.7	9,135.7	9,035.7	8,935.7
4	**9,485.7**	9,385.7	9,285.7	9,185.7	9,085.7	8,985.7	8,885.7
5	8,471.6	8,371.6	8,271.6	8,171.6	8,071.6	7,971.6	7,871.6
6	7,457.5	7,357.5	7,257.5	7,157.5	7,057.5	6,957.5	6,857.5
7	6,443.5	6,343.5	6,243.5	6,143.5	6,043.5	5,943.5	5,843.5

Table 5. NPV (\$1,000) of Alternatives in Expansion Planning for Model 2 ($\alpha=1$)

Initial number of levels n_0	Initial foundation reserve (levels), $N-n_0$						
	0	1	2	3	4	5	6
1	−8,931.9	−2,449.7	3,802.5	7,009.4	7,555.2	7,455.2	7,355.2
2	−1,733.0	4,519.2	7,726.1	8,271.9	8,171.9	8,071.9	7,971.9
3	4,974.5	8,181.4	**8,727.2**	8,627.2	8,527.2	8,427.2	8,327.2
4	8,133.3[a]	8,679.2	8,579.2	8,479.2	8,379.2	8,279.2	8,179.2
5	8,460.4	8,360.4	8,260.4	8,160.4	8,060.4	7,960.4	7,860.4
6	7,446.3	7,346.3	7,246.3	7,146.3	7,046.3	6,946.3	6,846.3
7	6,432.3	6,332.3	6,232.3	6,132.3	6,032.3	5,932.3	5,832.3

[a]Model 1 optimum.

million and one additional level is expanded at $t=3$ and $t=9$, respectively. Detailed results for the case with maximal penalty are given in Table 5. Note that directly comparing the optimal NPV in Model 2 with the optimal NPV obtained in Model 1 is meaningless, since they are obtained based on different demand models. The optimal decision $N=n_0=4$ obtained in Model 1 now corresponds to suboptimal solutions with an NPV equal to \$8.8 million without penalty and \$8.1 million with maximal penalty, respectively. One can attribute the difference between both models to the overestimation of the net profit due to the crudeness of Demand Model 1.

Since $N-n_0$ represents the foundation reserve at the beginning of construction, the column in Table 5 under $N-n_0=0$ corresponds to the inflexible case. Consider the optimal expansion plan $N=5$, $n_0=3$, with an NPV of \$8.7 million, for the case with maximal penalty. Comparing this optimal solution with the corresponding inflexible plan $N=n_0=3$, with an NPV of \$4.97 million in Table 5, the difference, \$3.75 million, can be viewed as the *value of flexibility* by reserving foundation size $N-n_0=2$, which accounts for 43% of the optimal NPV in this case. This value indicates the potential profit to be capitalized on if expansion is possible. Given n_0, it can be seen that the value of flexibility increases as $N-n_0$ increases until some point is reached where reserving too much flexibility becomes uneconomical. That is, with too much flexibility, the potential profit due to expansion cannot offset the increased cost for laying a deeper foundation.

Model 3—Demand Evolution Follows Trinomial Lattice

To incorporate decision making into a trinomial lattice, stochastic dynamic programming is used. Since demand scenarios are used at each time period, the cash flow function used in Model 1 will be further extended to include the demand outcomes. Consider a demand branching in the trinomial lattice, as shown in Fig. 3(a). At time t, a logarithmic demand outcome D_t branches into D_{t+1}^u, D_{t+1}^m, and D_{t+1}^d, where $D_{t+1}^u=D_t+(k+1)\Delta D=\ln d_{t+1}^{u}$, $D_{t+1}^m=D_t+k\Delta D=\ln d_{t+1}^{m}$, and $D_{t+1}^d=D_t+(k-1)\Delta D=\ln d_{t+1}^{d}$, and the corresponding cash flow functions are $x_t(N,n_{t-1},d_t)$,

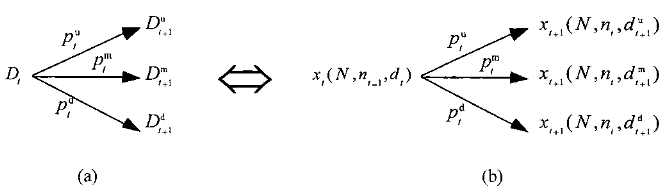

Fig. 3. Demand branching and corresponding cash flow functions, where $d_t=\exp(D_t)$ and $d_{t+1}^{b}=\exp(D_{t+1}^b)$, $b\in\{u,m,d\}$

$x_{t+1}(N,n_t,d_{t+1}^{u})$, $x_{t+1}(N,n_t,d_{t+1}^{m})$, and $x_{t+1}(N,n_t,d_{t+1}^{d})$, as shown in Fig. 3(b). Note that the cash flow function x_t now also depends on the demand outcome. The recursive relation between the three cash flow functions in Fig. 3(b) is as follows:

$$\begin{aligned} x_t(N,n_{t-1},d_t)=&[R\min(d_t,n_{t-1}m)-c_m n_{t-1}m\\ &-c_{pn}\max(0,d_t-n_{t-1}m)](1+f)^t\\ &+\max_{n_{t-1}\leq n_t\leq N}(1+r)^{-1}[p_t^u x_{t+1}(N,n_t,d_{t+1}^{u})\\ &+p_t^m x_{t+1}(N,n_t,d_{t+1}^{m})+p_t^d x_{t+1}(N,n_t,d_{t+1}^{d})\\ &-c_e(n_t-n_{t-1})(1+f)^t] \end{aligned} \quad (19a)$$

with boundary conditions

$$x_T(N,n_{T-1},d_T)=0;\quad \forall n_{T-1},\quad \forall d_T \quad (19b)$$

It is obvious that Eqs. $(19a)$ and $(19b)$ are directly extended from Eqs. $(18b)$ and $(18c)$, respectively. The only difference is that the recursive function x_{t+1} is replaced by the discrete expectation of x_{t+1}, since now the three possible outcomes of x_{t+1}, given x_t and D_t, are possible.

Running the recursive functions in a backward manner, from $t=T$ to $t=0$, using Eqs. $(19a)$ and $(19b)$, at the last step

$$\begin{aligned} x_0=\max_{N,n_0}\{&-C_s-(C_f+c_nN)-c_u n_0+(1+r)^{-1}[p_0^u x_1(N,n_0,d_{1}^{u})\\ &+p_0^m x_1(N,n_0,d_{1}^{m})+p_0^d x_1(N,n_0,d_{1}^{d})]\}. \end{aligned} \quad (19c)$$

The optimal decision in Eq. $(19c)$ is $N=6$, $n_0=3$ in the case without penalty, and $N=7$, $n_0=3$ in the case with maximal penalty. The foundation is enhanced to support the expansion of three (without penalty) or four (with maximal penalty) additional levels during the initial construction, with optimal NPVs of \$8.1 million and \$7.6 million, respectively. Detailed results for other design alternatives are given in Table 6 for the case with maximal penalty.

Consider the value of flexibility in this model by taking the optimal (expected) NPV of \$7.6 million ($N=7$, $n_0=3$ with maximal penalty), subtracting the NPV corresponding to the inflexible plan, \$3.7 million $(N=n_0=3)$. This value of flexibility by reserving the foundation size $N-n_0=4$ now accounts for 51.6% of the optimal (expected) NPV, higher than 43% obtained in Model 2.

Directly comparing Models 2 and 3, the optimal (expected) NPV obtained by Model 3 is 7.6% (with maximal penalty), or 3.3% (without penalty) higher than that obtained by Model 2. This difference of the expected NPV between Models 2 and 3 can be interpreted as the *value of uncertainty modeling*, i.e., a deter-

Table 6. Expected NPV ($1,000) of Alternatives in Expansion Planning for Model 3 ($\alpha = 1$)

	Initial foundation reserve (levels), $N - n_0$						
Initial number of levels n_0	0	1	2	3	4	5	6
1	−8,930.2	−2,516.7	2,525.8	4,919.2	5,913.3	6,317.2	6,452.0
2	−1,800.0	3,242.5	5,635.8	6,629.9	7,033.8	7,168.7	7,150.3
3	3,681.6	6,075.0	7,069.1[b]	7,473.0	**7,607.8**	7,589.5	7,541.3
4	5,989.4[a]	6,983.4	7,387.3	7,522.2	7,503.8	7,455.7	7,373.5
5	6,636.0	7,039.9	7,174.7	7,156.4	7,108.2	7,026.0	6,938.7
6	6,495.3	6,630.1	6,611.8	6,563.6	6,481.5	6,394.2	6,297.6
7	5,932.0	5,913.7	5,865.5	5,783.4	5,696.1	5,599.5	5,502.9

[a]Model 1 optimum.

[b]Model 2 optimum.

ministic demand $\tilde{d}_t$ for time t in Model 2 versus a set of demand scenarios in Model 3.

Discussion

The optimal decisions from all three models are summarized in Table 7. The following observations can be made.

First, from Table 7, it may seem that Model 3 obtained a poorer optimal NPV than did Model 2, which is poorer than Model 1. Because all three models are different, comparisons among these models should be done with caution. Basically, a crude demand model, such as Model 1 versus Model 2, and Model 2 versus Model 3, may overestimate profits or expected profits. In this case study, the overestimates seem very significant.

Second, this case study justifies the claim in the literature of real options that traditional NPV methods tend to underestimate the flexible alternatives. To see this, in Model 1 the design alternatives that reserve foundations are strongly discouraged. The fact is that the value of flexibility generally increases as the volatility of the uncertainty increases. A flexible design alternative can increase the expected value of the profit. This reiterates the nature of flexibility, which is the capability to cope with uncertainties.

Third, it seems fair to say that if flexibility could be correctly valued, the optimal design tends to be more aggressive than those that do not consider flexibility. Overall, the aggressiveness of the design is proportional to the penalty factor α considered in the objective function for each unfulfilled demand. In Fig. 4, the value of flexibility (in terms of percentage of optimal NPV) is displayed as the value of α changes. Intuitively, the value of flexibility increases as α increases for both Models 2 and 3, because (1) the need to satisfy increasing demand is more emphasized; and (2) the underservice of the inflexible counterpart is more discouraged. In the worst case with $\alpha = 0$, the value of flexibility accounts for at least 12% of the optimal NPV. In a good scenario path of the uncertainty, that the demand increases every year, the value of the flexibility may exceed 100% of the optimal NPV. As mentioned previously, reserving flexibility in design can be viewed as an investment. The additional cost for laying a deeper foundation can be thought of as a premium for the purchase of a real option. While an investment is not a guarantee of success, a chance exists that the real option may never be exercised and therefore may be worthless. For example, a structure with an enhanced foundation may never have a chance to expand, even if the designer expects it to happen. This probably corresponds to a bad scenario path of uncertainty realization, one of the many paths considered in a lattice. This bad scenario path may be associated with a small probability of occurrence, but it may still prevail. For example, in the bad scenario that the demand does not increase but decrease every year, the optimal NPV is $-\$16.5$ million, which, however, only has a very small probability of occurrence (2.36×10^{-16}). The proposed valuation procedure, however, selects the optimal design based on the expected value accounting for *all* possible scenario paths.

Table 7. Summary of Results of Three Models ($1,000)

(N, n_0)	Model 1 NPV	Model 2 NPV	Model 3 E[NPV]	Penalty factor α
Model 1 optimum (4, 4)	9,485.7	8,133.3	5,989.4	$\alpha = 1$
Model 2 optimum (5, 3)	9,335.7	8,727.2	7,069.1	$\alpha = 1$
Model 3 optimum (7, 3)	9,135.7	8,527.2	7,607.8	$\alpha = 1$
Model 1 optimum (4, 4)	9,485.7	8,803.9	7,731.0	$\alpha = 0$
Model 2 optimum (4, 3)	9,435.7	8,866.9	7,858.7	$\alpha = 0$
Model 3 optimum (6, 3)	9,235.7	8,666.9	8,115.8	$\alpha = 0$

Implications to Infrastructure Development

The proposed model has broader implications to infrastructure development. The benefits of an effective infrastructure system are undoubtedly significant for economic development, employment, and competitiveness. Provision of these systems requires investments that are normally huge, irreversible, and highly risky, and have a very long economic return period. Planning infrastructure systems to satisfy both present and future demands with adequate flexibility should be a primary consideration in making infrastructure decisions. Because that demand varies with time and many other factors, planning infrastructure capacity in order

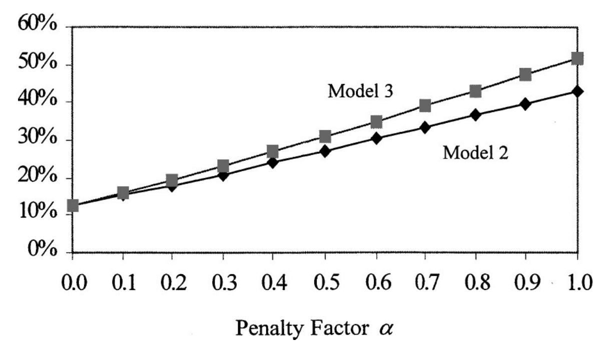

Fig. 4. Value of flexibility (in terms of percentage of optimal NPV) versus α

to maintain sustainability should be an ongoing process, rather than a one-time effort. As discussed previously, every planning decision has an option value, because the decision maker can always elect to delay the decision. Therefore, the proposed model obtains the optimal *option value* of each expansion decision. In this model, since the decision is constantly reviewed at each time period, once a decision (or an alternative) is selected, it must be the optimal decision (alternative), taken at the optimal timing. That also means the option (flexibility) value of decision making is fully captured. When applying the proposed approach, the uncertainty model should be updated constantly, taking new information into account. That is, the proposed multistage model should be applied repeatedly with a rolling planning horizon and up-to-date outlook for uncertainty. While the planning of many infrastructure projects focuses more on short-term consideration than long-term demand evolution, the proposed model clearly addresses both perspectives.

In a complex infrastructure system, multiple uncertainties may exist. In this situation, the proposed stochastic dynamic programming approach can still be integrated with either multifactor lattices [see Tseng (2001) for an application using a two-factor lattice] or Monte Carlo simulation (Longstaff and Schwartz 2001; Tseng and Barz 2002) to value flexibility. In fact, how to properly model the evolution of the underlying uncertainties remains an important and challenging task in real options valuation.

Conclusions

Flexibility has value, which has long been understood but rarely used. If a design alternative can provide flexibility, the value of flexibility should be incorporated into the design process and decisions. In this paper, it has been demonstrated how to select the foundation size in order to maximize the expected profits from potential expansions. Through a case study, numerical results show that the omission of the value of flexibility from an analysis may result in the rejection of alternatives and opportunities that should not have occurred otherwise. The value of flexibility in this case study is so significant that failure to account for flexibility is not economical. Test results also suggest a need to integrate common valuation techniques such as the discounted cash flow analysis with uncertainty modeling so as to capitalize on the value of flexibility.

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