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# Optimal intervention strategies for multiple objects affected by manifest and latent deterioration processes

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In the existing infrastructure management systems, optimal interventions strategies (OISs) are determined for objects that deteriorate gradually (manifest deterioration process, MDPs), under the assumption that with appropriate inspection and intervention strategies the probability of failure of object can be neglected. Objects that deteriorate suddenly (latent deterioration process, LDPs), for example, due to scouring during a flood or earth movements during an earthquake are not considered. The determination of OISs for an object that deteriorates due to both MDPs and LDPs requires the consideration of both. The latter, however, means that the probability of failure of the object must be considered. In this article, a Markov model is presented that can be used to determine OISs for multiple objects of multiple types affected by uncorrelated MDPs and LDPs. The model is an extension of the model proposed by Mayet and Madanat (Incorporation of seismic considerations in bridge management systems. *Computer-Aided Civil and Infrastructure Engineering*, 17:185–193, 2002). In the model, a set of condition states (CSs) is used to describe the condition of objects of each type, where each set is composed of non-failure CSs and failure CSs. The probabilities of going from each non-failure CS to each failure CS are estimated using normalised fragility curves, and the probabilities of going from each non-failure CS to each non-failure CS are initially estimated using the Markov deterioration prediction model of Kobayashi, Kaito, and Lethanh (A Bayesian estimation method to improve deterioration prediction for infrastructure system with Markov chain model. *International Journal of Architecture, Engineering and Construction*, 1:1–13, 2012a) and later adjusted taking into consideration the probabilities of entering the failure CSs. The use of the model is demonstrated using a road link comprising one road section and one bridge.

**Keywords:** Markov model; bridge management; road management; latent deterioration process; hazard risks; optimisation

## 1. Introduction

One of the main tasks of infrastructure managers is to determine the optimal intervention strategies (OISs) to follow for their infrastructure. In order to do so, they rely on the use of optimisation models, which can be either deterministic or probabilistic. In deterministic models, it is often assumed that initiation and development of indicators (e.g. roughness, crack, bearing capacity) follow deterministic empirical functions, which take continuous forms (Fwa, Chan, & Hoque, 2000; Ouyang, 2007; Sathaye & Madanat, 2011). One limitation of the use of deterministic models is that it does not consider the uncertainties (e.g. the probabilistic changes in the deterioration, changes in demand of usages, interventions). This limitation can be overcome by using probabilistic models, although discrete condition states (CSs) are normally used rather than continuous function in order to make the problems tractable (Madanat, 1993; Madanat, Mishalani, & Ibrahim, 1995; Tsuda, Kaito, Aoki, & Kobayashi, 2006).

In the field of infrastructure management, Markov models are one of the most widely used probabilistic models (Golabi & Shepard, 1997; Kobayashi, Kaito, & Lethanh, 2012b; Thompson, Small, Johnson, & Marshall, 1998). In discrete time Markov chain, the deterioration of an object is modelled by determining the probability that the object will pass from one discrete CS to another in a fixed period of time, where the CSs are defined using values of performance indicators (e.g. visual indicators of corrosion, roughness and cracking of the road surface) (Kuhn, 2010; Kobayashi et al. 2012a, 2012b).

An advantage of Markov models is that, once discrete CSs are defined, they can be used to take into consideration the uncertainty of an object being in each of these CSs in the future. Markov models are currently used, however, to model only the manifest deterioration processes (MDPs)[3] (Kobayashi et al., 2012b; Lethanh, 2009; Tsuda et al., 2006). They are not used to model latent deterioration processes (LDPs).[4]

Examples of using Markov models based on the MDP to determine the OISs for infrastructure objects can be with the existing bridge management systems (BMSs) such as the PONTIS (American Association of State

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Highway and Transportation Officials [AASHTO], 2004; Fruguglietti, Pasqualato, & Spallarossa, 2012; Golabi & Shepard, 1997) and the KUBA-MS (Hajdin, 2006). Technical guidelines for using the Markov models and national specifications with regard to monitoring and quantification of the status and impacts associated with each CS as well as agency rules in the Markov models have been well established and used for managing the deteriorating civil structures (Hajdin, 2003, 2006; Robert, Marshall, Lin, Shepard, & Aldayuz, 2002).

However, as the existing infrastructure management systems (IMSs) using Markov models do not incorporate the LDP, the intervention strategies that are derived from the models might not be the optimal strategies, especially for civil structures located in regions with high probability of hazard occurrences. It is a solid piece of experiential knowledge that when a bridge, as an example, is in good CS, the failure probability due to hazard occurrence is less than that when it is in worst CS (Korup & Clague, 2009; Schubert, Faber, Jacquemoud, & Straub, 2010). In other words, a bridge with a proper preventive maintenance and repair will have less chance of collapsing than a bridge with no or poor maintenance and repair. Thus, in any case, it should constantly be remembered that the determination of the OISs should depend on both the MDPs and the LDPs. This phenomenon has actually been demonstrated in numerous literatures, particularly with the development of block- and age-replacement models used in the field of facilities management (Berg & Epstein, 1978; Chen & Savits, 1992; Gertsbakh, 1997, 2000; Kaio & Osaki, 1984).

As a result of the increasing need of our society to have proactive responses to natural disasters, assessing and managing hazard risks to civil infrastructures and methods to incorporate hazards into existing IMSs have become a high-profile matter in the agenda of infrastructure managers (Biringer, Vugrin, & Warren, 2013; Taylor, Werner, & Graf, 2006; Thompson, Rogers, & Thomas, 2012). Significant works have been archived, especially in the application of probabilistic methods to model the occurrences of hazard events and the expected impacts incurred (Castelli & Scavia, 2008; Faber & Stewart, 2003; Korup & Clague, 2009; Schubert et al. 2010).

Models used to quantify the occurrence of natural hazards or disaster can be assumed to follow a certain probabilistic distribution. One of the most widely used probabilistic distributions is Poisson distribution for quantifying the occurrence of extreme events such as earthquake based on historical record as well as the seismic force (Korup & Clague, 2009; Zucchini & MacDonald, 2009). Not limited to earthquake occurrence, other methods such as the use of GIS-tool (Barufini, 2010) or Bayesian network modelling (Bayraktarli & Faber, 2011; Graf, Nishijima, & Faber, 2009) have also been proposed to perform risk assessment and calculate the probabilistic occurrence of other natural hazards such as floods, avalanches, rockfalls and landslides along the infrastructure network.

After acquiring the probabilistic occurrence of natural hazards, the next step is to use it to estimate the probability of failure of the civil structures. To date, one of the popular methodologies that is used to estimate the failure probability of infrastructure objects due to hazard risks is the method of using fragility curves (Choi, DesRoches, & Nielson, 2004; Graf et al. 2009; Karim & Yamazaki, 2001). To the best of our knowledge, despite the rapid development and improvement in the area of structural analysis associated with natural hazard risks using the fragility curves, application of using fragility curve is more or less in the design stage of civil structures, but not in the operational stage, where preventive and corrective interventions need to be executed at some point in time to reduce the negative impacts of both the MDPs and the LDPs.

Mayet and Madanat (2002) were the first to attempt the consideration of both MDPs and LDPs simultaneously in determining the OIS for objects of one type, for example bridges, using the Markov model. The LDPs were incorporated into the Markov model by adding a CS to represent the failure of the object. Fragility curves, together with the hazard risk curves, were used to estimate the failure probability of the object when it is in the failure state. Fragility curves, together with hazard curves, were used to estimate the probability of the object entering the failure CS from all other CSs.

In this article, the model proposed by Mayet and Madanat (2002) is extended and generalised to cover multiple objects of different types, instead of multiple objects of one type. In order to do this, a set of CSs is used to describe the condition of objects of each type, where each set is composed of the following:

(1) Non-failure CSs: The CSs used to describe the condition of the object when the object is considered to provide an adequate level of service, that is not failed. These CSs can occur only due to MDPs (as described by Mayet and Madanat (2002)).
(2) Failure CSs: The CSs used to describe the condition of the object when the object is considered to provide an inadequate level of service, that is failed. It is considered that these CSs can occur only due to LDPs (multiple-failure CSs are used instead of single-failure CS as described by Mayet and Madanat (2002)).

The probabilities of passing from each non-failure CS to each failure CS are estimated using normalised fragility curves, and the probabilities of passing from each non-failure CS to each non-failure CS are initially estimated using the Markov model of Tsuda et al. (2006) and Kobayashi et al. (2012a) and later adjusted taking into consideration the probabilities of entering the failure CSs.

As Mayet and Madanat (2002) only had one-failure CS, it was possible for them to use point estimates of the probability of failure from the fragility curves to estimate the transition probabilities, instead of intervals.

The model allows the consideration of multiple objects with different range of discrete CSs, multiple states of failures and also the imposition of a global budget constraint, that is not per object type but for all object types simultaneously, in the determination of the OISs. This budget constraint is assumed to be on average over a long period.

In existing BMSs, such global constraints are only introduced in the construction of work programs, that is the exact interventions to be executed over a normally relatively short period of time. These work programs are often constructed using incremental cost–benefit analysis (Farid, Johnston, Rihani, & Chen, 1994) or by using the knapsack algorithm to determine the interventions that can be executed in a specific time period to maximise benefit (Holst, 2006).

The remainder of the article is structured as follows. In Section 2, a model normally used to predict the future condition of objects due to MDPs is explained. In Section 3, a model to be used to predict the future condition of objects due to both MDPs and LDPs is proposed. The estimation of probability of entering failure CSs using fragility curves and hazard curves is explained. In Section 4, a linear optimisation model for the determination of OISs of a road link composed of objects of multiple types is explained. In Section 5, the use of the proposed model is demonstrated using a road link composed of a road section and a bridge, each of which is affected by an MDP and an LDP. Section 6 describes the conclusions of the article and suggested directions for future research.

## 2. A model for manifest processes

When objects are affected only by MDPs, Markov models, as they are currently used in BMSs, can be used to predict deterioration. In these models, it is assumed that an object passes from one discrete CS to another with a certain probability in each unit of time (Hajdin, 2006). The probability of passing from one non-failure CS to another in each unit of time, that is a transition probability (t.p.), is determined using past inspection data and can be estimated as described in Tsuda et al. (2006) and Kobayashi et al. (2012a).

The determination of transition probabilities can be explained with reference to Figure 1, where time is represented by $\tau$, the discrete CSs by $i (i = 1, \ldots, I)$, with $i = 1$ as initial CS (when structure is new) and $I$ as absorbing CS, $\tau_A$ and $\tau_B$ represents inspection times, and $\tau_i$ represents any arbitrary time between inspections and $Z$ represents the length of time between two inspections.

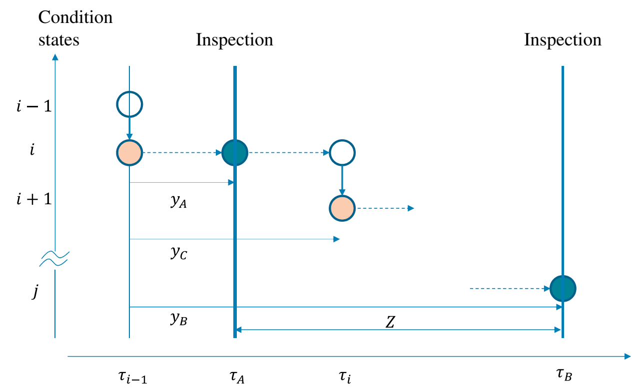


Figure 1. Illustration of the relationship between actual transitions between CSs and observed transitions between CSs (Adopted from Lethanh (2009)).

With the data collected at inspection times $\tau_A$ and $\tau_B$, the transition probabilities can be described as follows:

$$\mathrm{Prob}[h(\tau_B) = j | [h(\tau_A) = i] = p_{ij}, \tag{1}$$

where $h(\tau_A)$ and $h(\tau_B)$ are representations of the CSs observed at time $\tau_A$ and $\tau_B$, respectively. In cardinal form, the transition probabilities can be written as

$$p_{ij} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1I} \\ 0 & p_{22} & \cdots & p_{2I} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}. \tag{2}$$

If the CS of the object changes from $i$ to $i+1$ at time $\tau_i$ (timing $y_C$), then the length of time in CS $i$ can be expressed by

$$\zeta_i = \tau_i - \tau_{i-1} = y_C, \tag{3}$$

where $\zeta_i$ is a random variable, valid on a domain of $[0, \infty]$ with the pdf $f_i(\zeta_i)$, and the cdf $F_i(\zeta)$. In this case,

$$F_i(y_i) = \int_0^{y_i} f_i(\zeta_i) d\zeta_i, \tag{4}$$

where $F_i(y_i)$ represents the cumulative probability of the change from CS $i$ to CS $i+1$ in the period from time $y_i = 0$ (time $\tau_{i-1}$), at which the CS has become CS $i$, to $y_i$ (time $\tau_i + y_i$). Accordingly, $\tilde{F}_i(y_i)$ is the probability that the object remains in CS $i$ from time $y_i = 0$ to $y_i \in [0, \infty]$. It can be expressed using the cdf of the change from CS $i$ to CS $i+1$ up to time $y_i$:

$$\mathrm{Prob}[\zeta_i \geq y_i] = \tilde{F}_i(y_i) = 1 - F_i(y_i). \tag{5}$$

The conditional probability of the event that the object remains in CS $i$ until $y_i$ and transitions to CS $i+1$ in the period $[y_i, y_i + \Delta y_i)$ is given by

$$\lambda_i(y_i)\Delta y_i = \frac{f_i(y_i)\Delta y_i}{\tilde{F}_i(y_i)}, \tag{6}$$

where the instantaneous hazard rate $\lambda_i(y_i)$ of the change in CS of the object from CS $i$ to CS $i+1$ at $y_i$ is called a hazard function.

If the hazard function is independent of the value of $y_i$, then the hazard rate $\theta_i = \lambda_i(y_i)$ is used. The probability that the object remains in CS $i$ up to $y_i$, with $\theta_i$, is given by

$$\tilde{F}_i(y_i) = \exp\left[-\int_0^{y_i} \lambda_i(u)du\right] = \exp(-\theta_i y_i). \tag{7}$$

This survival probability function is identical to the equation required to estimate the t.p. $p_{ii}$ when the time interval $[0, y_i)$ equals $Z$. By defining the subsequent conditional probability of going from CS $i$ to $j$, with respect to $Z$, a general mathematical formula for estimating $p_{ij}$ can be obtained:

$$\begin{aligned} p_{ij}(z) &= \mathrm{Prob}[h(\tau_B) = j | h(\tau_A) = i] \\ &= \sum_{k=i}^{j} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \prod_{m=k}^{j-1} \frac{\theta_m}{\theta_{m+1} - \theta_k} \exp(-\theta_k z), \end{aligned} \tag{8}$$

where $i$, $j$, $k$ and $m$ are running index of CS.

The hazard rate is often a function of characteristic variables such as traffic volume, thickness of pavement, bridge slab area and ambient temperature (Lethanh, 2009; Tsuda et al., 2006). In addition, discrete CSs can be defined or converted from a certain range in the value of indicators (e.g. roughness value used in pavement management system or percentage of crack of concrete deck of a bridge).

## 3. A model for manifest and latent processes

### *3.1. Model description*

The model of MDPs and LDPs is similar to that for MDPs. The extension of the model of MDPs to include LDPs, however, requires the consideration of failure CSs, that is CSs in which inadequate levels of service are provided, as they have a non-negligible probability of occurrence due to, for example, the ground shaking during an earthquake. These failure CSs, $l = (1, \ldots, L)$, are added to the non-failure CSs, $i = (1, \ldots, I)$ (Equation (2)), as shown in

$$Q = \left(\begin{array}{cccc|cccc} p_{11} & p_{12} & \cdots & p_{1I} & e^p_{11} & e^p_{12} & \cdots & e^p_{1L} \\ 0 & p_{22} & \cdots & p_{2I} & e^p_{21} & e^p_{22} & \cdots & e^p_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_{II} & e^p_{I1} & e^p_{I2} & \cdots & e^p_{IL} \\ \hline 0 & 0 & \cdots & 0 & e_{11} & e_{12} & \cdots & e_{1L} \\ 0 & 0 & \cdots & 0 & 0 & e_{22} & \cdots & e_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{array}\right), \tag{9}$$

where $p$ represents the t.p.s between the non-failure CSs, $e^p$ represents the t.p.s from the non-failure CSs to the failure CSs and $e$ represents the t.p.s between the failure CSs.

The summation of the t.p.s between all CSs, that is non-failure and failure CSs, equals 1.

$$\sum_{j=1}^{I} p_{ij} + \sum_{l=1}^{L} e^p_{il} = 1. \tag{10}$$

The estimation of the t.p.s when objects are affected by both MDPs and LDPs is more difficult than when they are affected by only MDPs due to lack of data, since the failure CSs happen so infrequently. The estimation can, however, still be carried out by adjusting the values of $p_{ij}$ based on the values of $e^p_{il}$ as explained in Section 3.2.

### *3.2. Steps to estimate transition probabilities*

The three steps proposed to estimate the t.p.s in $Q$ (Equation (9)) are as follows:

- Step 1: Estimate $p_{ij}$ based on past data (Section 2);
- Step 2: Estimate $e^p_{ij}$ and $e_{ij}$ taking into consideration the probable damage to an object when subjected to hazards of different magnitudes [using condition-based fragility curves (Section 3.4)] and the probability of occurrence of each magnitude of the hazard (using hazard curves).
- Step 3: Adjust $p_{ij}$ proportionally so as to satisfy the condition in Equation (10).

### *3.3. Estimate transition probabilities due to MDPs alone*

The t.p.s due to MDPs are first estimated as is normally explained in Tsuda et al. (2006), Lethanh (2009) and Kobayashi et al. (2012a). As this is a well-established process, this step is not explained here.

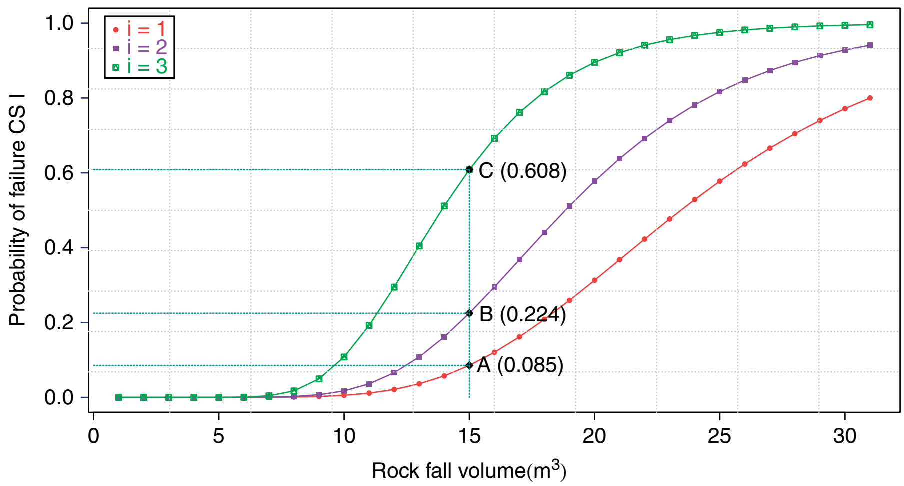


Figure 2. Fragility curves for a bridge deck subjected to rockfalls.

### 3.4. Estimate transition probabilities due to LDPs alone

The probabilities of entering failure CSs ($e^p_{ij}$ and $e_{ij}$) are estimated using fragility curves and hazard curves; both concepts that have been used extensively in the field of structural engineering and hazard risk modelling (Choi et al., 2004; Graf et al., 2009; Karim & Yamazaki, 2001; Mayet & Madanat, 2002). A fragility curve gives the probability of an object exceeding a failure CSs when subjected to a hazard of a given intensities. An example is given for the rockfall hazard in Figure 2 and more can be found in Karim and Yamazaki (2001), Choi et al. (2004) and Schultz, Gouldby, Simm, and Wibowo (2010).

The fragility curves shown in Figure 2 represent the probability (vertical axis) of one level of bridge deck failure, failure CSs taking into consideration the initial condition of the bridge deck, that is the non-failure CS of the bridge deck ($i = 1$, 2, and 3) prior to hazard occurrence given the volume of rockfall that comes in contact with the deck. As can be observed in the figure, if 15 $m^3$ of rock comes in contact with a bridge deck in CS1, there is a 0.085 probability (point A) that the bridge deck would fail. It can also be observed that the higher the initial CS the higher the probability of bridge deck failure(e.g. when the bridge deck is initially in CS2 and CS3 the probability that the bridge deck would fail is 0.224 and 0.608, respectively).

With an estimation of the volume of rock that will come in contact with the bridge deck, it is possible to estimate the probability of each failure CS from each non-failure CS (Equation (11)). Fragility curves are object dependent.

$$\Omega^l_{s,i} = \text{prob}[\text{CS} > l|s, i], \tag{11}$$

where the probability $\Omega^l_{s,i}$ of exceeding CS $l$ is conditional on the intensity $s$ of the hazard and the CS $i$ of the object, immediately prior to hazard occurrence.

The probability of the occurrence of a hazard with an intensity exceeding a certain level (e.g. rockfall volume greater than 15 $m^3$), which is a function of the intensity $s$ and time $t$, is defined using a hazard curve (Equation (12)). Hazard curves are location dependent.

$$H_s(t) = \text{Prob}[\text{intensity} > s, (0, t)]. \tag{12}$$

From Equations (11) and (12), the probability of occurrence of each level of object failure within a period of time $t$, that is $e$ or $e^p_{il}$ in Equation (9) can be estimated.

$$\left\{e^p_{il}, e\right\} = \text{prob}[\text{CS} = l|s, i, t] = \begin{cases} \int_0^t H_s(t) \cdot d\Omega^{L-1}_{s,i} & l = L \\ \int_s^t H_s(t) \cdot d\Omega^{l-1}_{s,i} - \int_0^t H_s(t) \cdot d\Omega^l_{s,i} & l \leq L - 1 \end{cases}, \tag{13}$$

where $L$ is the highest failure CS.

The current formulation is carried out assuming that at most one hazard event can occur on the smallest time interval investigated, for example only one rockfall can occur per year in the 50-year investigated time period. It is assumed that the probability of multiple hazards occurring in one time period is negligible; something that is defendable with relatively well-constructed infrastructure and short analysis time periods.

### 3.5. Adjust the transition probabilities due to MDPs

Once the t.p.s due to LDPs are determined, the t.p.s due to MDPs, $p_{ij}$, are adjusted (Equation (10)). This is done by multiplying the $p_{ij}$ estimated in Equation (9) with a proportional factor $\Delta_i$, as

$$\Delta_i = \left(1 - \sum_{l=1}^{L} e_{il}^{p}\right). \tag{14}$$

## 4. Optimisation model

The model used to determine the OISs is a linear optimisation model. It is an extension of the model proposed by Mayet and Madanat (2002) for determining the OIS for a single bridge affected by earthquakes and similar to those used in many existing BMSs (ASTRA, 2010; FHWA, 2002; Fruguglietti et al., 2012). Adaptations have been made so that multiple objects of multiple object types can be considered, that is different objects whose deterioration is modelled using different CS definitions and different t.p.s. The model is as follows:

Objective function:

$$MIN \sum_{n=1}^{N} \sum_{a_n=1}^{A_n} \sum_{i_n}^{I_n+L_n} \pi_{n,a_n,i_n} \cdot S_{n,a_n,i_n}, \tag{15}$$

where $n = (1, \ldots, N)$ represents the objects; $a_n = (1, \ldots, A_n)$ represents the interventions on object $n$; $i_n = (1, \ldots, I_n + L_n)$ are indices of object $n$, if $i_n = (1, \ldots, I_n)$ then $i_n$ represents the non-failure CSs, if $i_n = (I_n + 1, \ldots, I_n + L_n)$ then $i_n$ represents the failure CS; $\pi_{n,a_n,i_n}$ represents the steady-state probability of object $n$, that is if an IS is consistently followed, the probability of object $n$ being in CS $i_n$ in one time interval at an infinite point of time in the future, and intervention $a_n$ being executed; $S_{n,a_n,i_n}$ represents the total impacts incurred due to the execution of intervention $a_n$ on object $n$ when it is in CS $i_n$.

Subject to the following constraints:

$$S_{n,a_n,i_n} = \sum_{s=1}^{S} c^{s}_{n,a_n,i_n}, \tag{16}$$

where $s$ is an index of stakeholders (e.g. owner, users, public,), $c^{s}_{n,a_n,i_n}$ is the impact incurred by stakeholder $s$ when intervention $a_n$ is executed on object $n$ when it is in state $i_n$.

$$\sum_{n=1}^{N} \sum_{a_n=1}^{A} \sum_{i_n=1}^{I_n} \pi_{n,a_n,i_n} c^{s}_{n,a_n,i_n} \leq B^{s}, \tag{17}$$

$B^{s}$ is the maximum allowable impact to be incurred by stakeholder $s$ (e.g. the limit of the amount of resources that can be used for the execution of interventions).

The steady-state probability of object $n$ is estimated as

$$\begin{cases} \pi_{n,a_n,i_n} \geq 0, \quad \forall n, \forall a_n, \forall i_n, \\ \sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} \cdot Q_{n,a_n,i_n,j_n} = \sum_{a_n=1}^{A_n} \pi_{n,a_n,j_n} \quad \forall n, \forall j_n \end{cases}, \tag{18}$$

where $Q_{n,a_n,i_n,j_n}$ is the t.p. matrix (from $i$ to $j$) when the set of interventions $a_n$ in the appropriate CSs are executed.

$$\sum_{a_n=1}^{A_n} \sum_{i_n=1}^{I_n+L_n} \pi_{n,a_n,i_n} = 1, \quad \forall n. \tag{19}$$

This constraint ensures that only one intervention is executed per object per time interval.

## 5. Example

### 5.1. *Problem description*

In this example, the OIS is determined for a road link comprising a road section and a bridge. Three budget scenarios are investigated

- Scenario 1: Sufficient financial resources are available to execute all interventions, when required, on objects that are in either non-failure or failure CSs.
- Scenarios 2 and 3: Sufficient financial resources are not available to execute all interventions, when required, on objects that are in non-failure CSs, but sufficient financial resources are available to execute all interventions, when required, on objects that are in failure CSs.

The road section and the bridge are both affected by uncorrelated MDPs and LDPs. The CSs used to classify the condition of the road section and the bridge are given in Tables 1 and 2, respectively.

The effectiveness of the interventions executed when the road section and the bridge are in non-failure CSs is given in Tables 3 and 4. For example, if an intervention is executed when a road section at time $t$ is in CS $i = 2$, there is a probability of 0.1 that the road section will be in CS $i = 1$ and

Table 1. Road section CSs.

| CS type | Level of service: Number | Level of service: Description | CSs | Definition |
|---|---|---|---|---|
| Non-failure | 0 | Full service | $i = 1$ | New, no crack, no potholes |
| | | | $i = 2$ | $<10\%$ crack, no potholes |
| | | | $i = 3$ | 10–40% crack, potholes occur |
| | | | $i = 4$ | Badly deteriorated, $>40\%$ crack, potholes |
| Failure | 1 | Reduced service | $l = 1$ | Up to half of the road width is unusable and traffic must be restricted to one lane |
| | 2 | No service | $l = 2$ | The complete road is blocked |

Table 2. CS definitions for bridges.

| CS type | Level of service: Number | Level of service: Description | CS | Definition |
|---|---|---|---|---|
| Non-failure | 0 | Full service | $i = 1$ | Deck is new or near new, almost no sign of deterioration |
| | | | $i = 2$ | Leakage is occurring over $<10\%$ of the deck surface area |
| | | | $i = 3$ | Leakage is occurring over $<25\%$ of the deck surface area |
| | | | $i = 4$ | Leakage is occurring over $\geq 25\%$ of the deck surface area, some spalling is occurring, substantial efflorescence |
| | | | $i = 5$ | Heavy spalling, heavy efflorescence, deck saturated to a point that concrete is rubble |
| Failure | 1 | Reduced service | $l = 1$ | There are no visual indications of a reduction in the load-carrying capacity of the bridge but there is material on the bridge that restricts use to one lane |
| | 2 | No service | $l = 2$ | The bridge is not useable |

Table 3. Effectiveness of road section interventions.

| Maintenance interventions | | | | | Renovation interventions | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Year $(t + 1)$ | | | | | Year $(t + 1)$ | | | |
| CS | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | CS | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ |
| Year $(t)$ | | | | | | | | | |
| $i = 1$ | 1 | 0 | 0 | 0 | $i = 1$ | 1 | 0 | 0 | 0 |
| $i = 2$ | 0.1 | 0.9 | 0 | 0 | $i = 2$ | 1 | 0 | 0 | 0 |
| $i = 3$ | 0 | 0.2 | 0.8 | 0 | $i = 3$ | 1 | 0 | 0 | 0 |
| $i = 4$ | 0 | 0 | 0.49 | 0.51 | $i = 4$ | 1 | 0 | 0 | 0 |

Table 4. Effectiveness of bridge interventions.

| Maintenance interventions | | | | | | Rehabilitation interventions | | | | | | Renovation interventions | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Year $(t + 1)$ | | | | | | Year $(t + 1)$ | | | | | | Year $(t + 1)$ | | | | |
| CSs | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $i = 5$ | CSs | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $i = 5$ | CSs | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $i = 5$ |
| Year $(t)$ | | | | | | | | | | | | | | | | | |
| $i = 1$ | 1 | 0 | 0 | 0 | 0 | $i = 1$ | 1 | 0 | 0 | 0 | 0 | $i = 1$ | 1 | 0 | 0 | 0 | 0 |
| $i = 2$ | 0.2 | 0.8 | 0 | 0 | 0 | $i = 2$ | 0.3 | 0.7 | 0 | 0 | 0 | $i = 2$ | 1 | 0 | 0 | 0 | 0 |
| $i = 3$ | 0 | 0.3 | 0.7 | 0 | 0 | $i = 3$ | 0 | 0.4 | 0.6 | 0 | 0 | $i = 3$ | 1 | 0 | 0 | 0 | 0 |
| $i = 4$ | 0 | 0 | 0.4 | 0.6 | 0 | $i = 4$ | 0 | 0 | 0.5 | 0.5 | 0 | $i = 4$ | 1 | 0 | 0 | 0 | 0 |
| $i = 5$ | 0 | 0 | 0 | 0.5 | 0.5 | $i = 5$ | 0 | 0 | 0 | 0.6 | 0.4 | $i = 5$ | 1 | 0 | 0 | 0 | 0 |

a probability of 0.9 that it will remain in CS $i = 2$ at time $t + 1$. The effectiveness of interventions, executed when objects are in failure CSs $l = 1$ and 2 are assumed to restore the object with a probability of 1 to CS $i = 1$. The impacts incurred by the owner and other stakeholders due to the execution of interventions are given in Tables 5 and 6. For example, a renovation intervention executed when a road section is in CS $i = 1$ will result in impacts to the owner worth 35 mu and impacts to others worth 60 mu.

## 5.2. *Transition probabilities*

### 5.2.1. *Step 1*

The t.p.s for each object were estimated using the Equation (8), assuming that the objects were affected only by MDP (Table 7). The resulting distributions of CSs over 50 years, if no interventions are executed, are shown in Figure 3(a),(b).

### 5.2.2. *Step 2*

The fragility curves for each object (Figure 4) were estimated based on the research of Schubert et al. (2010), where the rockfall intensity was estimated using Equation (12).

$$H_s(t = 1) = \alpha \cdot S^{-\gamma}, \tag{20}$$

where $\alpha$ and $\gamma$ are the parameters associated with the local condition of the cliff from which the rockfall may occur. The values of the parameters used are given in Table 8.

The fragility curves were used to estimate the t.p.s of going from each non-failure CS to each failure CS, as well

Table 5. Impacts incurred during the execution of road interventions.

| | Owner | | | Other stakeholders | | |
|---|---|---|---|---|---|---|
| CSs | Do nothing | Maintenance intervention | Renovation intervention | Do nothing | Maintenance intervention | Renovation intervention |
| $i=1$ | 0 | 4 | 35 | 0 | 0 | 60 |
| $i=2$ | 0 | 4 | 35 | 4 | 4 | 60 |
| $i=3$ | 0 | 4 | 35 | 6 | 6 | 60 |
| $i=4$ | 0 | 4 | 35 | 8 | 8 | 60 |
| $l=1$ | 75 | 75 | 75 | 125 | 125 | 125 |
| $l=2$ | 100 | 100 | 100 | 150 | 150 | 150 |

Table 6. Impacts incurred during the execution of bridge interventions.

| | Owner | | | | Other stakeholders | | | |
|---|---|---|---|---|---|---|---|---|
| CSs | Do nothing | Maintenance intervention | Rehabilitation intervention | Renovation intervention | Do nothing | Maintenance intervention | Rehabilitation intervention | Renovation intervention |
| $i=1$ | 0 | 6 | 7 | 68 | 0 | 0 | 0 | 97 |
| $i=2$ | 0 | 6 | 7 | 68 | 8 | 8 | 8 | 97 |
| $i=3$ | 0 | 6 | 7 | 68 | 10 | 10 | 10 | 97 |
| $i=4$ | 0 | 6 | 7 | 68 | 12 | 12 | 12 | 97 |
| $i=5$ | 0 | 6 | 7 | 68 | 15 | 15 | 15 | 97 |
| $l=1$ | 108 | 108 | 108 | 108 | 162 | 162 | 162 | 162 |
| $l=2$ | 138 | 138 | 138 | 138 | 210 | 210 | 210 | 210 |

Table 7. Transition probabilities assuming objects affected only by MDPs.

| (a) Road section | | | | | (b) Bridge | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Year $(t+1)$ | | | | | Year $(t+1)$ | | | | |
| CSs | $i=1$ | $i=2$ | $i=3$ | $i=4$ | CSs | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $i=5$ |
| Year $(t)$ | | | | | | | | | | |
| $i=1$ | 0.9231 | 0.0706 | 0.0057 | 0.0006 | $i=1$ | 0.9418 | 0.0557 | 0.0024 | 0.0001 | 0.0000 |
| $i=2$ | 0 | 0.8437 | 0.1345 | 0.0218 | $i=2$ | 0 | 0.9139 | 0.0783 | 0.0072 | 0.0006 |
| $i=3$ | 0 | 0 | 0.7408 | 0.2592 | $i=3$ | 0 | 0 | 0.8270 | 0.1525 | 0.0205 |
| $i=4$ | 0 | 0 | 0 | 1 | $i=4$ | 0 | 0 | 0 | 0.7788 | 0.2212 |
| | | | | | $i=5$ | 0 | 0 | 0 | 0 | 1 |

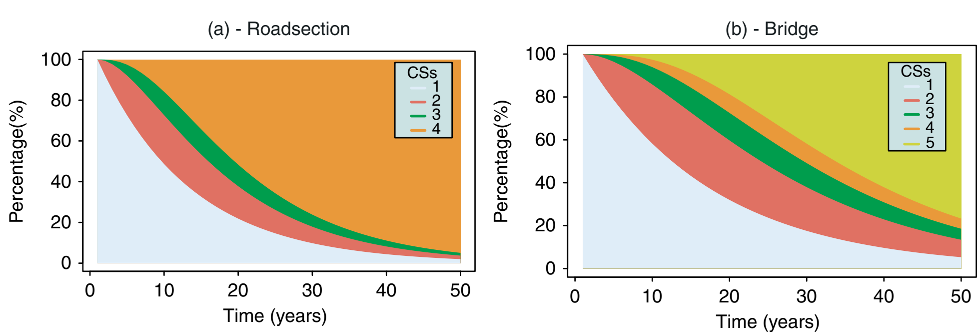


Figure 3. CSs distribution.

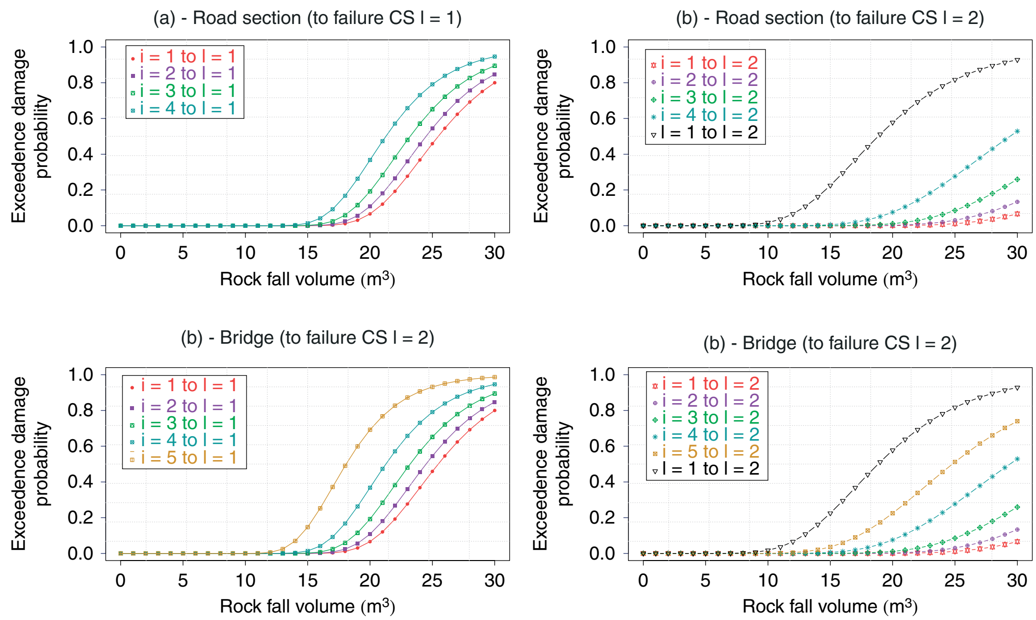


Figure 4. Fragility curve for objects subjected to rockfall hazards.

as between failure CSs. The estimated t.p.s are given in the last two columns of Tables 9 and 10 for road section and bridge respectively. Tables 9 and 10 are the deterioration matrices taking into consideration both MDPs and LDPs.

### 5.2.3. Step 3

The t.p.s between non-failure CSs are proportionally adjusted by multiplying its original values with a factor defined in Equation (14). By doing so, the condition of Equation (10) is satisfied. The modified t.p.s are given in the first four and five columns in Tables 9 and 10, respectively.

Table 8. Parameter values.

| | to CS5 ($l = 1$) | | to CS6 ($l = 2$) | |
|---|---|---|---|---|
| CSs | $\alpha$ | $\gamma$ | $\alpha$ | $\gamma$ |
| Road | | | | |
| $i = 1$ | 0.40 | 2.80 | 0.20 | 2.80 |
| $i = 2$ | 0.45 | 2.70 | 0.22 | 2.70 |
| $i = 3$ | 0.50 | 2.50 | 0.28 | 2.50 |
| $i = 4$ | 0.60 | 2.40 | 0.35 | 2.40 |
| CS5($l = 1$) | NA | NA | 0.41 | 2.20 |
| | to CS6 ($l = 1$) | | to CS7 ($l = 2$) | |
| CSs | $\alpha$ | $\gamma$ | $\alpha$ | $\gamma$ |
| Bridge | | | | |
| $i = 1$ | 0.30 | 3.00 | 0.15 | 3.00 |
| $i = 2$ | 0.30 | 3.00 | 0.15 | 3.00 |
| $i = 3$ | 0.30 | 3.00 | 0.15 | 3.00 |
| $i = 4$ | 0.45 | 3.20 | 0.20 | 3.20 |
| $i = 5$ | 0.60 | 3.50 | 0.30 | 3.50 |
| CS6($l = 1$) | NA | NA | 0.42 | 3.60 |

### 5.3. Results

The t.p.s estimated as described earlier were then used as input for the optimisation model, programmed in AMPL. The steady-state probabilities $\pi_{n,a_n,i_n}$ for the three scenarios were estimated. In Scenarios 2 and 3, the budget was 8.5 and 8.0 mu, respectively, whereas in Scenario 1 the budget was unlimited as shown in Tables 11–16. The budget constraints are assumed to limit the average expenditure per year over a long period of time.

- Under Scenario 1, it is possible to execute the theoretically optimal interventions when the objects are in the non-failure CSs. The do-nothing intervention will be executed on the road section when it is in non-failure CSs 1 and 2, which will occur with probabilities of 0.68256 and 0.23885, respectively. Renovation interventions will be executed when the road section is in non-failure CSs 3 and 4, which will occur with probabilities of 0.03443 and 0.00537, respectively. The do-nothing intervention will be executed on the bridge when it is in non-failure CSs 1, 2 and 3, which will occur with probabilities of 0.55998, 0.27758 and 0.11565, respectively. Renovation interventions will be executed when the bridge is in non-failure CSs 4 and 5. The total average value of impacts under this scenario is 27.14 mu, in which 12.56 mu is due to the execution of interventions on the road section and 14.58 mu is due to the execution of interventions on the bridge.
- Under Scenario 2, it is not possible to always execute the theoretically optimal interventions when the objects are in the non-failure CSs. The do-

Table 9. Transition probabilities for road sections due to manifest-latent processes.

| | Year $(t+1)$ | | | | | |
|---|---|---|---|---|---|---|
| CSs | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $l=1$ | $l=2$ |
| Year $(t)$ | | | | | | |
| $i=1$ | 0.88818 | 0.06793 | 0.00548 | 0.00058 | 0.02041 | 0.01741 |
| $i=2$ | 0 | 0.80588 | 0.12847 | 0.02082 | 0.02445 | 0.02038 |
| $i=3$ | 0 | 0 | 0.69999 | 0.24492 | 0.02555 | 0.02954 |
| $i=4$ | 0 | 0 | 0 | 0.92976 | 0.03048 | 0.03975 |
| $l=1$ | 0 | 0 | 0 | 0 | 0.94986 | 0.05014 |
| $l=2$ | 0 | 0 | 0 | 0 | 0 | 1 |

Table 10. Transition probabilities for bridges due to manifest-latent processes.

| | Year $(t+1)$ | | | | | | |
|---|---|---|---|---|---|---|---|
| CSs | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $i=5$ | $l=1$ | $l=2$ |
| Year $(t)$ | | | | | | | |
| $i=1$ | 0.91787 | 0.05419 | 0.00224 | 0.00019 | 0.00010 | 0.01420 | 0.01120 |
| $i=2$ | 0 | 0.89068 | 0.07631 | 0.00702 | 0.00058 | 0.01420 | 0.01120 |
| $i=3$ | 0 | 0 | 0.80599 | 0.14863 | 0.01998 | 0.01420 | 0.01120 |
| $i=4$ | 0 | 0 | 0 | 0.75015 | 0.21306 | 0.02211 | 0.01468 |
| $i=5$ | 0 | 0 | 0 | 0 | 0.96321 | 0.01562 | 0.02117 |
| $l=1$ | 0 | 0 | 0 | 0 | 0 | 0.97012 | 0.02988 |
| $l=2$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Table 11. Results of Scenario 1 (road section).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | |
|---|---|---|---|---|---|---|
| Intervention type | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $l=1$ | $l=2$ |
| Do nothing | 0.68256 | 0.23885 | – | – | – | – |
| Rehabilitation intervention | – | – | – | – | – | – |
| Renovation intervention | – | – | 0.03443 | 0.00537 | 0.02081 | 0.01798 |

Table 12. Results of scenario 1 (bridge).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Intervention type | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $i=5$ | $l=1$ | $l=2$ |
| Do nothing | 0.55998 | 0.27758 | 0.11565 | – | – | – | – |
| Maintenance intervention | – | – | – | – | – | – | – |
| Rehabilitation intervention | – | – | – | – | – | – | – |
| Renovation intervention | – | – | – | 0.01925 | 0.00253 | 0.01400 | 0.01101 |

Table 13. Results of scenario 2 (road section).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | |
|---|---|---|---|---|---|---|
| Intervention type | $i=1$ | $i=2$ | $i=3$ | $i=4$ | $l=1$ | $l=2$ |
| Do nothing | 0.61225 | 0.21425 | 0.10294 | – | – | – |
| Rehabilitation intervention | – | – | – | – | – | – |
| Renovation intervention | | | | 0.03002 | 0.02128 | 0.01926 |

Table 14. Results of scenario 2 (bridge).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Intervention type | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $i = 5$ | $l = 1$ | $l = 2$ |
| Do nothing | 0.43433 | 0.21529 | 0.08970 | 0.05974 | 0.16496 | – | – |
| Maintenance intervention | – | – | – | – | – | – | – |
| Rehabilitation intervention | – | – | – | – | – | – | – |
| Renovation intervention | – | – | – | – | 0.00862 | 0.01453 | 0.01283 |

Table 15. Results of scenario 3 (road section).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | |
|---|---|---|---|---|---|---|
| Intervention type | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $l = 1$ | $l = 2$ |
| Do nothing | 0.54901 | 0.19212 | 0.09230 | 0.10381 | – | – |
| Rehabilitation intervention | – | – | – | – | – | – |
| Renovation intervention | – | – | – | 0.01963 | 0.02202 | 0.02111 |

Table 16. Results of scenario 3 (bridge).

| | Steady-state probabilities $\pi_{a,i}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| Intervention type | $i = 1$ | $i = 2$ | $i = 3$ | $i = 4$ | $i = 5$ | $l = 1$ | $l = 2$ |
| Do nothing | 0.35197 | 0.17446 | 0.07269 | 0.04842 | 0.32356 | – | – |
| Maintenance intervention | – | – | – | – | – | – | – |
| Rehabilitation intervention | – | – | – | – | – | – | – |
| Renovation intervention | – | – | – | – | – | 0.01463 | 0.01427 |



nothing intervention will be executed on the road section when it is in non-failure CSs 1, 2 and 3, which will occur with probabilities 0.61225, 0.21425 and 0.10294, respectively. Renovation interventions will be executed when the road section is in non-failure CS 4, which will occur with a probability of 0.03002. The do-nothing intervention will be executed on the bridge when it is in non-failure CSs 1–4, which will occur with probabilities of 0.43433, 0.21529, 0.08970 and 0.05974. When the bridge is in non-failure CS 5 either a do-nothing intervention or a renovation intervention will be executed. This will happen with probabilities of 0.16496 and 0.00862, respectively. The imposition of the budget constraint has resulted in a change in the OIS and a change in the probability of being in each CS. It has also resulted in it not being possible to always execute the same intervention on the bridge when it is in a specific non-failure CSs and indicates that when the bridge is in non-failure CS 5 an infrastructure manager should approximately 95% of the time do nothing, and 5% of the time execute a renovation intervention. The total expected value of impacts under this scenario is 28.16 mu, in which 12.54 mu is due to the execution of interventions on the road section and 15.62 mu is due to execution of interventions on the bridge.

- Under Scenario 3, it is not possible to always execute the theoretically optimal interventions when the objects are in the non-failure CSs. The do-nothing intervention will be executed on the road when it is in the non-failure CSs 1, 2 and 3, which will occur with a probability of 0.54901, 0.19212 and 0.09230, respectively. When the road section is in non-failure CS 4 either a do-nothing intervention or a renovation intervention will be executed. This will happen with probabilities of 0.10381 and 0.01963, respectively. The do-nothing intervention will be executed on the bridge when it is in all non-failure CSs. The imposition of the budget constraint has resulted in a change in the OIS and a change in the probability of being in each CS. It has resulted in it not being possible to always execute a specific intervention on the road section when it is in non-failure CSs and indicates that when the road section is in non-failure CS 4 an infrastructure manager should approximately 84% of the time do nothing and 16% of the time execute a renovation intervention. It has also resulted in it not being possible to execute any interventions on the bridge unless it is in a failure CSs. The total expected value of impacts under this scenario is 29.29 mu, in which, 12.81 mu is due to the execution of interventions on road sections and 16.48 mu is due to the execution of intervention on bridges.

## 6. Conclusions

In this article, a model to be used to determine OISs for multiple objects of two types that are affected by uncorrelated MDPs and LDPs is presented. The model is an extension of the model proposed by Mayet and Madanat (2002). In the model, a set of CSs is used to describe the condition of objects of each type is used, where each set is composed of non-failure CSs, that is where the object provides an adequate level of service, and failure CSs, where the object provides an inadequate level of service. The probabilities of going from each non-failure CSs to each failure CS are estimated using normalised fragility curves, and the probabilities of going from each non-failure CS to each non-failure CSs are initially estimated using the Markov deterioration prediction model of Tsuda et al. (2006) and Kobayashi et al. (2012a) and later adjusted taking into consideration the probabilities of entering the failure CSs.

The model is demonstrated by determining the OIS for a road link comprising one road section and one bridge, affected by both MDPs and LDPs. One weakness of the model, as with all Markov models that are currently used in BMSs, is that the determination of the OIS is based on the steady-state probabilities, that is the probabilities of being in each CS at an infinite point of time in the future. This is considerably different than what happens in reality, when interventions are executed due the failure of an object. The object will most often be improved so that it is more resistant to failure than the initial design.

Another weakness is that the model is currently focused on objects, such as road sections and bridges, and not on elements, such as abutments and columns. This is not entirely compatible with the models used in existing BMSs which are used to make predictions on the element level. An adaptation of the presented model to the element level, however, requires considerable additional work in the estimation of the t.p.s, in order to take into consideration the fact that elements that comprise an object are interconnected, and the probability of failure of each element cannot be determined without taking into consideration the probability of failure of other elements.

Future research topics should include the adaptation of the model so that it can be used to determine the OISs on the element level, and the investigation of models that do not require the determination of the OIS using steady-state probabilities.

## Notes

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3. MDPs are processes whose progression over time is followed in a way that a condition of the object triggers the execution of an intervention early enough so that it can be assumed that the object will never provide an unexpected inadequate level of service. An example of a typical MDP is chloride-induced corrosion of reinforced concrete.
4. LDPs are processes whose progression over time is not followed in a way that a condition of the object triggers the execution of an intervention early enough so that it can be assumed that the object will never provide an unexpected inadequate level of service. An example of a typical LDP is ground accelerations due to an earthquake. This definition is different from the ones used in some of the past research articles in which the word was used to describe Markov models (Ben-Akiva & Ramaswamy, 1993; Ben-Akiva, Humplick, Madanat & Ramaswamy, 1993). In these cases, it was used to mean that the processes could not be modelled directly. Latent processes are often modelled as stochastic point processes.

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