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Developing optimal work programs for concrete bridges using three preventive intervention models

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Abstract

The infrastructure in Switzerland is more or less built and the focus gets more and more in the direction of maintaining the available infrastructure. The propose of the following master thesis is to develop a program that allows the determination of optimal work programs using the input from one of three different deterioration models. The three different models are called MUSTEM model, E-MEPI model and WEAR-OUT model. Each of the model has his own input data. The input data can be collected through inspections and tests on existing bridges so that the physical parameters for the models can be defined. Some of the input data can be estimated through expert opinions due to the fact that bridges are built in an area where already others were built. To compare the different models to each other a case study is made on a bridge in the Canton of Valais. For the case study the bridge is divided into three objects, which are deck, pier and abutment and the deterioration process is model for each of them separately. For the bridge two different "optimal" work programs are built. The first one is based on the optimal intervention strategies for each objects (object level) and the second one is based on a so called "discount factor" which considers the fact that if two objects have an intervention in the same year the cost of them can partially be shared and it results sort of a discount on the price (bridge level). At the end the different model where compared and their advantages and disadvantages were discussed. It has to be mentioned that according to this work it can not be said which model does the most accurate predictions. They just can be compared with each other and it can be said that it depends on the boundary conditions which of the approaches does make the most sense to use. One other big part of this thesis was the coding for the different model in R. The output are three different codes to estimate the transition probability for each model and then calculate the optimal work program based on the transition probability.

Die Infrastruktur in der Schweiz ist mehr oder weniger fertiggestellt und der Fokus liegt immer mehr in der Erhaltung der vorhandenen Infrastruktur. Das Ziel dieser Master-Arbeit ist es, ein Programm zu entwickeln, welches Anhand von drei verschiedenen Abnutzungsmodellen das optimale Arbeitsprogram bestimmt. Die drei verschiedenen Modelle sind das MUSTEM Modell, das E-MEPI Modell und das WEAR-OUT Modell. Jedes dieser Modelle hat seine eigenen Eingangsdaten. Diese Eingabedaten knnen durch Inspektionen oder durch Test an bestehenden Brcken gesammelt werden, so dass die physikalischen Parameter fr die Modelle definiert werden knnen. Einige der Eingabedaten knnen auch durch Erfahrungswerte definiert werden, dies aufgrund der Tatsache, dass einige Brcken in einem Bereich erstellt werden, in dem bereits andere Brcken erstellt wurden. Um die verschiedenen Modelle miteinander zu vergleichen wurde eine Fallstudie, basierend auf einer Brcke im Kanton Wallis, durchgefhrt. Fr die Fallstudie wurde die Brcke in drei Objekte unterteilt, die Fahrbahn, die Pfeiler und die Widerlager. Der Korrosionsprozess wurde fr jedes Objekt getrennt modelliert. Fr die Brcke wurden zwei verschiedene "optimale" Arbeitsprogramme erstellt. Das erste besteht aus den optimalen Interventionsstrategien fr die einzelnen Objekte (Objektlevel) und das zweite basiert auf der Grundlage eines so genannten "Rabatt-Faktor", welcher die Tatsache bercksichtigt, dass wenn zwei Objekte eine Intervention im selben Jahr haben die Kosten fr die beiden Interventionen teilweise geteilt werden knnen (Brckenlevel). Somit erhlt man eine Art Rabatt auf den ursprnglichen Preis der beiden Interventionen. Am Ende werden die verschiedenen Modelle verglichen und deren Vor- und Nachteile diskutiert. Es muss gesagt werden, dass nach dieser Arbeit nicht gesagt werden kann welches der Modelle die genauesten Vorhersagen trifft. Sie knnen lediglich miteinander verglichen werden und es kann gesagt werden, dass es von den Randbedingungen abhngt, welches der Modelle am besten verwendet werden sollte. Ein weiterer grosser Teil dieser Arbeit war die Codierung fr die verschiedenen Modelle in R. Das Ergebnis sind drei verschiedene Codes, um die bergangswahrscheinlichkeit fr jedes Korrosionsmodell zu bestimmen und anhand dieser dann das optimale Arbeitsprogramm.

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This project consumed huge amount of work, research and dedication. Still, implementation would not have been possible if I did not have a support of some individuals. Therefore we would like to extend my sincere gratitude to all of them.

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1. Introduction

1.1. Rationale

Infrastructure managers try to achieve that infrastructure provides an adequate level of service (LOS) over the according life time. On the other hand they are forced to spent the minimal amount of money or "negative impact", like time which the road section has to be closed, to ensure the adequate level of service of the infrastructure. This tradeoff can be achieved by investigation the optimal work program (OWP) for the infrastructure. One big challenge for the infrastructure managers is the use of the appropriate model to predict the change of the infrastructure over time. There are many different models to estimate the deterioration process of an objects and it is up to the infrastructure manager which model he applies and sometimes the infrastructure manager does not really know about the advantages and disadvantages of the different models to determine the deterioration process of the infrastructure.

Infrastructure management has a number of different decision situations where managers need to use deterioration models for predicting future condition states of the infrastructures. It is identified that infrastructure managers have in general three decision situations (DSs) where deterioration predictions are useful.

1. Estimation of required financial needs in the future, e.g. if intervention strategy i is applied how much money is required over the investigated time period.
2. Determination of work programs, e.g. if intervention strategy i is applied which object has an intervention in the next 10 years and what kind of intervention will be executed.
3. Determination of the exact intervention to execute on an object, e.g. if it is known that an intervention needs to be executed on a concrete bridge should the deck that is currently in condition state 2 also have an intervention.

To support the infrastructure managers, in making those decisions, Switzerland developed a bridge management system (BMS) called KUBA. KUBA uses the Markov model to estimate the work programs for the bridge. However the Markov transition probability, which is one of the main input for the Markov model, can be estimated through different deterioration models, like mentioned above. According to that three different models are examined to give an idea about the advantages and disadvantages to the infrastructure manager.

In order to estimate the Markov transition probability (m.t.p.), different deterioration models can be used. For concrete bridges the three following models are generally used:

1. The MUSTEM model, which was developed by TSUDA et al. [2006] and uses the hazard rates to estimate the m.t.p.
2. The E-MEPI model, which was developed by DuraCrete [2000] and uses the chloride induced corrosion to estimate the state probability of the object and in a further step via a minimization problem the m.t.p.
3. The WEAR-OUT model, which is used in the Moscow Bridge Management System (Brodsky et al. [2006]) and uses the life time of the object to estimate the m.t.p.

1.2. Problem statements

This Master Thesis deals with some points which influence the estimation of optimal work programs in the Bridge Management Systems used in practice. There are ways shown which could be included at some point in the future or at least it gives an idea to the infrastructure manager about which other models could be used to estimate the transition probability matrix and what could be done to estimate the optimal work program on bridge level. The main problems are listed below.

- Current Bridge Management Systems use the unit jump transition probability matrix (WEAR-OUT model).
- The optimal work program is determined at object level and without consideration of bridge level interventions.
- The transition probability matrix is calculated using historical inspection data, but in many situations, there is no such data. Thus it is difficult to determine more or less a suitable deterioration model to be used.

1.3. Objectives

The goals of this Master's Thesis are as follows:

- Estimate the transition probability using the three deterioration models mentioned above.
- Develop a program for determination of optimal work programs for concrete bridges.
- Compare the three OWPs obtained from the three different deterioration models using the same optimization model.

1.4. Methodologies

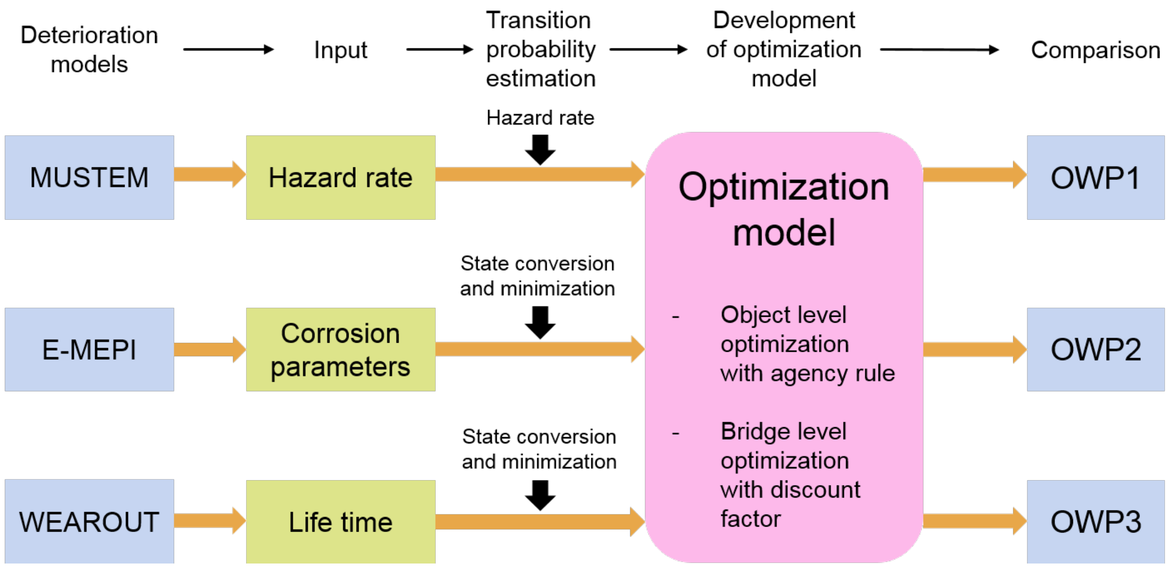


Figure 1.1.: Methodology

The methodology of this master thesis is the development of an optimal work program based on three different models to estimate the deterioration process, compare Fig. 1.1. The three different deterioration models are named in section 1.1. One big difference between the three deterioration models is the input, the MUSTEM model needs only the hazard rates θ_i of the object. Where the E-MEPI model needs input related to chloride induced corrosion and each input parameter has to be defined through mean value, standard deviation and distribution type. The WEAR-OUT model needs the life time T_c of the object as input. According to the different deterioration models the m.t.p. can be calculated and with the m.t.p. the state probability (condition state distribution) can be defined for any time t. It can be seen that only for the MUSTEM model it is possible to estimate the m.t.p. directly out of the models output. The other two deterioration models (E-MEPI and WEAR-OUT) need an additional step. This step is in both cases a minimization problem to estimate the m.t.p. out of the models output. After the deterioration process is model the evaluation of the optimization model can estimate the optimal intervention strategies and according to that the optimal work programs. If the optimal work program on bridge level is determined the costs for the infrastructure manager are displayed over the investigated time period as well as the condition state distribution for the different objects.

1.5. Scope of study

For this study the Getwing bridge was used as a case study to compare the different deterioration models with each other. The Getwing bridge is located in the canton of Valais in Switzerland and is a reinforced concrete bridge. According to the study an optimal work program is generated for the Getwing bridge over the next 100 years.

One of the main part is the derivation of the three deterioration models. All of them need different input data and it strongly depends on the available inspection data and knowledge of the infrastructure manager which model does the best predictions. In the end the three different deterioration models are compared with each other and their strength and weaknesses are discussed.

An other part is the development optimization model which calculates the optimal work program on bridge level with consideration of a so-called discount factor, which considers the fact the if two or more interventions are executed in the same year the costs can partially be shared between them and according to this fact money can be saved.

1.6. Contribution

According to the study the input parameter for the different models will be displayed so that an infrastructure manager has an idea about the different inputs for the different models and he can choose the right model depending on his available information.

One big part of the study is the development of three different computer supported programs to estimate the deterioration process. This programs are lined up so that they are not only made for the Getwing bridge. Which means that they can be used for different objects which have the same main deterioration process.

As well as an optimization model which estimates the optimal work program on bridge level. This program is also written in the R environment and has the big advantage that it can easily be expanded in further researches.

This study should give an idea to the infrastructure manager of which model he should use for the different cases. What means that it depends always on the specific problem which model does fit best.

2. Literature review

2.1. Bridge management

If public authorities talk about bridge cost they can not only consider the costs to built the bridge they also have to consider the medium to long-term costs which belong to the bridge. After the bridge is built the financial resources required to maintain the bridge is depending on the followed intervention strategy, assuming that there are no changes in use. It is not only the total costs over the investigated time period which changes with the different intervention strategies it is also the time when financial resources are needed and the amount which has to be invested at an instant of time in the future. This means that there could be two different intervention strategies which lead to the same total costs over the investigated time period but one of them has the better costs distribution over the investigated time period. Which means that the costs are uniformly distributed over the investigated time period. The evaluation of the different intervention strategies strongly depends on the deterioration process according to the bridge. As well as on the effectiveness of the different interventions and of course on the costs according to the different interventions. The task of infrastructure management is to determine, with these information, the optimal work program. It can be said that the criterion for an optimal intervention strategy can be different, i.e. can the optimal intervention strategy be defined through the lowest annual costs over the investigated time period or the optimal intervention strategy can be the one which results in the best average condition state of the bridge over the investigated time period and the costs only play a secondary role. When the bridge is to be kept in service in the long-term any deviation from the optimal intervention strategy results in an increase in the total required financial resources, Adey and Hajdin [2011].

With the nearing completion of the federal highway network in Switzerland the main effort of public authorities shifted from construction of new roads toward preservation of existing facilities. In order to that Switzerland started to development a bridge Management Systems in 1991 and in 1995 a master concept was released, which guidelines all subsequent work.

The annual spending on road infrastructure takes a significant part of the limited public funds. Due to the facts that infrastructure is getting older and traffic increases it requires even more funds to keep the current level of service to road users. Being under permanent pressure to cut spending, the road authorities are searching for ways to increase the efficiency of available resources. Higher efficiency means not only lower agency costs but optimum balance between costs and benefit from available road infrastructure (Hajdin [2000]).

Preservation actions are executed to counteract the deterioration effects on structures. The planning of preservation actions is a complex task wherein partly contradictory aspects have to be considered. In order to cope with these challenges, the Swiss Federal Roads Office has launched a systematic development of a computer-supported system, which would enable consistent, unified and comparable decision processes for road structures on the federal network. The result of this development is a system called KUBA-MS.

According to Hajdin [2001] the preservation model in the Swiss bridge management (KUBA-MS) encompasses condition assessment and forecast of the condition state, specification of technical actions and acquiring of work programs. During the inspection the existing deterioration process is identified and rated according to a scale of five condition states (1 represents like no conditions and 5 represents alarming conditions). Based on optimization on object level the infrastructure manager can obtain the long-term preservation needs. The generated preservation projects consists of optimal actions for each object. This means that the total costs are the sum of the object costs.

2.2. Deterioration process

A bridge experiences two different deterioration processes, one is the manifest or gradual deterioration and the other one is called sudden deterioration which occurs due to an external event such as an earthquake. In the long-term the majority of intervention costs can be attributed to gradual deterioration processes. Bridge deterioration due to a gradual deterioration process can be modeled in a reliable systematic manner using the presented methodologies of the deterioration processes at work meet the following criteria:

- Deterioration is continuous and predictable and permits classification of bridges into condition states.
- Damage due to the deterioration process can be discovered and inspected with relatively little effort.
- Interventions can be assigned to each object based on its condition state.
- Relationships between the extent of intervention and the extent of damage are easily discernible.

As there is an increasing need to make the above mentioned predictions with only basic data, numerous researchers and consultants have developed various methodologies with which to make them, in particular Adey and Hajdin [2005].

Before an optimal bridge management strategy can be determined in the existing BMSs, there first must be a determination of the type and schedule of interventions to be performed on each object of the bridge to achieve a minimal-cost long-term steady state (object level). Once the different object level strategies are known, they are bundled together to develop a management strategy for the bridge. This is currently done with a some agency rules without considering if the bundle of interventions is optimal. Interventions are bundled because it is not desirable to perform interventions on the same bridge in successive years. The work discussed in Adey and Hajdin [2005] uses inventory theory to bundle interventions on bridges optimally with multiple objects.

Condition assessment in the Swiss bridge management system (KUBA-MS) is performed on the object level. Five condition states are defined based on visual appearance. To predict the condition states of a given object at any time t a relationship between the age of the object and its condition state has to be established. This relationship, which describes the state distribution over time, can be described empirically from statistical analysis of pairs of consecutive condition assessments (inspections). In KUBA-MS Markov chains are used to represent condition evolution. And to determine the transition probabilities regression analysis is used. Unfortunately there are almost no inspection data for the worst and second worst condition states. The predictions made using Markov chains are therefore not always reliable. In the paper of Guido Roelfstra [2004] an alternative model is suggested, which takes the physical phenomena into consideration which describes the deterioration process of the object. This model is applied to chloride induced corrosion of reinforced concrete, the most common deterioration mechanism in Switzerland. The chloride induced corrosion is modeled mathematically and numerical simulations of the condition evolution for different values of model parameters are performed. The simulation results have been mapped to condition states as defined in KUBA-MS and Markov transition matrices have been calibrated to fit simulation results.

Corrosion is one of the most critical problems that affect the durability of reinforced concrete (RC) structures. Both carbonation induced and chloride induced corrosion widely prevail in civil infrastructure around the globe. Expansive products are formed due to corrosion at the interface between concrete and reinforcing bar (rebar). The cracking and spelling due to the expanding corrosion products and the reduction in the cross-sectional area of rebar affects the safety and serviceability of RC structures. The paper from Yuan Zhou [2014] focus on the mechanisms of the two types of corrosion, mechanical degradation in RC structures as a result of these mechanisms, the analytical methods to predict the basic parameters most related to corrosion, and the available laboratory and field corrosion measurement techniques.

The paper of George Morcous [2007] presents two approaches (probabilistic and mechanistic) to determine the deterioration process of bridges which can lead to an optimal maintenance management strategy. Probabilistic models are used to predict the macro-response of bridge components, while reliability-based mechanistic models are used to predict the micro-response of bridge components. Probabilistic (state-based/time-based) models use qualitative performance indicators which are determined through visual inspections to identify the overall condition. Reliability-based mechanistic models on the other hand use quantitative performance indicators which are determined through detailed condition surveys and empirical investigations to identify the extent and severity of specific deterioration mechanisms.

An example for these condition-based models are Markov models. These models might be used if infrastructure managers require information about the overall condition of the infrastructure in the future in order to prioritize necessary interventions (DS2). TSUDA et al. [2006] presents a methodology to estimate the Markov transition probability model to predict the deterioration process of bridge objects. The deterioration states of the different objects are divided into different condition states, and the deterioration process is defined through the hazard models.

Other researchers have investigated the exact physio-chemical behavior of specific deterioration processes such as corrosion induced deterioration of reinforced concrete, and have developed complex analytical deterioration models (e.g. Andrade et al. [1993], Molina et al. [1993] and Cabrera [1996]). These models, in contrast to condition-based models, are usually used in the situations when only the behavior of concrete at a local material level or bridge level is of interest. According to Kirkpatrick et al. [2002] chloride induced corrosion of the reinforcing steel is known to be a major cause of premature rehabilitation of bridge decks accelerated by the used of deicing salts. In this work, therefore, this process is considered as the only cause of deterioration.

As a result these models variate with respect to the amount of detail required to make predictions. The amount of detail could vary due to the intervention strategy, the exactness of the information required for budgets, and the cost of making the deterioration predictions.

2.3. Optimization

To achieve an optimal management strategy different types of maintenance techniques are available. One o the earliest maintenance strategies is breakdown management, which means that an intervention is executed when the infrastructure fails (also called unplanned maintenance, or run-to-failure maintenance). The time-based preventive maintenance, which is the maintenance strategy after the breakdown management, is based on regular interventions on the infrastructure. This leads to the fact that an infrastructure has an intervention with no respect to the condition state the infrastructure has at this specific time (also called planned maintenance). According to the development of the modern technology, infrastructure has become more and more complex and those model does not satisfy the claims of infrastructure management. Therefore, approaches like condition-based maintenance are implemented to handle the situation, Jardine et al. [2006].

According to Adey and Hajdin [2011] the development of an optimal intervention strategy for a bridge requires the determination of possible intervention types in each condition state, their unit cost and their effectiveness. The activities that are likely to be performed with respect to each intervention type should be explicitly noted in order to increase the accuracy of the estimation of the unit costs.

The construction of an intervention strategy requires the determination of condition states in which the object of the bridge can be in and interventions that are to be executed when the object is in this state. An object is said to be in a condition state when it has at least one of the predefined damage indicators and nothing more serious, i.e. listed as a damage indicator of a higher condition state. An object does not need to have all of the listed damage indicators to be in the condition state. Although it would be nice to be able to construct all possible intervention strategies, evaluate them and then determine the optimal one to say definitively which intervention strategy should be followed, this is only possible in a few simple cases. Usually expert opinions are used to whittle down the infinite number of intervention strategies to a few feasible ones. The intervention strategies are determined taking into consideration standards and engineering judgment. The more detail that is included in the models the easier it is to convert the intervention strategy into a work program, and the more accuracy in the cost estimates can be expected. Simply making the model helps with the management of infrastructure as it provides less experienced engineers with guidelines, though its development agreement is reached on the aspects that are important for decision making and considerably more objectivity is brought into the planning of interventions.

The evaluation of an intervention strategy requires the estimation of the probability, compare section 2.2, of the object being in each of the possible states in the future and the costs related to these states, over a specified period of time. When this is done for multiple intervention strategies the costs and probable states of the item in each time interval in the investigated time period can be compared and the optimal intervention strategy selected. This may be the intervention strategy which is likely to result in the lowest costs or one that minimizes the likelihood of being in a poor condition state. Finally according to those optimal interventions strategies an optimal work program can be built for the hole bridge.

According to Lethanh et al. [2015] and Fernando et al. [2014], in the existing infrastructure management systems, optimal interventions strategies (OISs) are determined for objects that deteriorate gradually (manifest deterioration process, MDPs). Objects that deteriorate suddenly (latent deterioration process, LDPs) are not considered. For estimating the OISs, for an object which deteriorates due to MDPs and LDPs, it requires the consideration of both. Which means that the probability of failure must be considered. They presents a Markov model that can be used to determine OISs such objects. A set of CSs is used to describe the condition, where each set is divided into non-failure CSs and failure CSs. The probabilities of moving from non-failure CS to failure CS are estimated using normalized fragility curves.

3. Methodology

3.1. Deterioration model

3.1.1. The MUSTEM model

In most of the cases inspection intervals, to determine the transition probabilities for the different objects, are not uniform. Which means that a bridge can be inspected at time $t = 0$, then at $t = 2.5$, and again at $t = 4$. For such cases, the transition probabilities can be determined by a model developed by TSUDA et al. [2006].

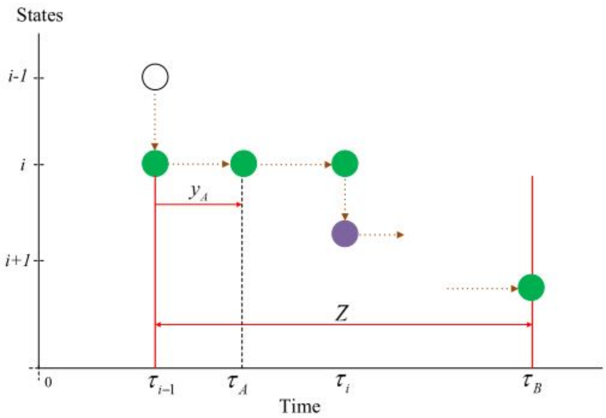


Figure 3.1.: Model of deterioration process (TSUDA et al. [2006])

τ_A and τ_B in Fig. 3.1 are two inspection times. The time between these two inspections is Z. The condition state of the object at time τ_A is i. The element entered the condition state i at some previous time τ_{i-1}. In this case the time the element has been in state i is $Y_A = \tau_A - \tau_{i-1}$. At time τ_i the state changes from i to $i+1$, where $\tau_A \leq \tau_i \leq \tau_B$.

The time an element spends in state i is represented by a random variable. The random variable by itself is represented by a probabilistic density function and a distribution function $F_i(\zeta_i)$. The relationship between the two functions can be seen in Eq. (3.1).

$$F_i(y_i) = \int_0^{y_i} f_i(\zeta_i) d\zeta_i \tag{3.1}$$

The upper bound $y_i \in [0, \inf)$ represents the total length of time that the element spends in state i counted from τ_{i-1}. Eq. (3.1) represents the cumulative probability of the transition from state i to state $i+1$ from the time of entering state i. The probability that the element remains in state i in the time $[0, y_i]$ is then:

$$Prob\{\zeta_i \geq y_i\} = \tilde{F}_i(y_i) = F_i(y_i) \tag{3.2}$$

Eq. (3.2) is often called the survival function. The conditional probability of an element entering state $i+1$ from state i during a increment of time y_i, i.e. $[y_i, y_i + \Delta y_i]$ is:

$$\lambda_i(y_i)\Delta y_i = \frac{f_i(y_i)\Delta y_i}{\tilde{F}_i(y_i)} \tag{3.3}$$

Where the probability density function $\lambda_i(y_i)$ is the hazard function. The hazard function, in the case of exponential distribution, becomes the hazard rate and can be further simplified to Eq. (3.4).

$$\lambda_i(y_i) = \theta_i \tag{3.4}$$

According to Eq. (3.4) there is a constant rate for an element moving through state i.

If Eq. (3.2) is differentiated on both sides, it becomes to Eq. (3.5).

$$\frac{\Delta \tilde{F}_i(y_i)}{\Delta y_i} = f_i(y_i) \tag{3.5}$$

And Eq. (3.3) then becomes to Eq. (3.6).

$$\lambda_i(y_i) = \frac{f_i(y_i)}{\tilde{F}_i(y_i)} = \frac{\frac{\Delta \tilde{F}_i(y_i)}{\Delta y_i}}{\tilde{F}_i(y_i)} = \frac{d}{dy_i}(-\log \tilde{F}_i(y_i)) \tag{3.6}$$

Knowing that $\tilde{F}_i(0) = 1 - F_i(0) = -\log \tilde{F}_i(y_i)$ and by integrating Eq. (3.6) the following equation can be obtained.

$$\int_0^{y_i} \lambda_i(u)du = [-\log \tilde{F}_i(u)]|_0^{y_i} = -\log \tilde{F}_i(y_i) \tag{3.7}$$

By combining Eq. (3.4) and (3.7), the probability that the element stays at least y_i units of time in state i is:

$$\tilde{F}_i(y_i) = \exp[-\int_0^{y_i} \lambda_i(u)du] = \exp(-\theta_i y_i) \tag{3.8}$$

The probability density function $f_i(\zeta_i)$ is:

$$f_i(y_i) = \theta_i \exp(-\theta_i y_i) \tag{3.9}$$

3.1.2. The E-MEPI model

Chloride induced corrosion is composed of two deterioration phases: the first is called initiation phase, which covers the entire time from construction to the time a critical chloride content is reached at the reinforcement bar (rebar) level. In this phase the chloride starts to penetrate into the concrete cover, reach the rebar until a sufficient concentration is reached to start corrosion. The second phase of chloride induced corrosion then starts with the actual begin of the corrosion at the rebars and includes the continued widening of the cracks in the concrete due to the corrosion of the rebars. The entire deterioration process can be seen in Fig. 3.2.

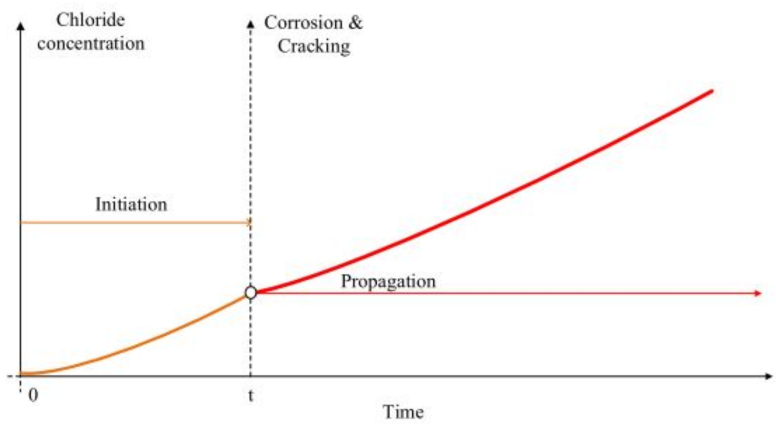


Figure 3.2.: Deterioration process of reinforced concrete bridge due to chloride-induced corrosion (IMP [HS2014])

In order to predict the deterioration due to chloride induced corrosion it is considered that each object of the bridge could be in different CS. The first of the CS are associated with the initiation phase and last of them with the propagation phase. In the initiation phase each CS is described by the amount of chlorides in the concrete at rebar level C_{cl}. In the propagation phase the CS are described by the crack with w.

Initiation phase

Condition measures should be represented probabilistically because they contain considerable uncertainty, like mentioned in section 2.2. A simplified approach is the use of the so called survival functions. A survival function represents the probability that an object stays in its condition state for at least time t, versus the time in this condition. The method is simple so it is easy to make intervention decisions.

For the initiation phase, according to DuraCrete [2000], the following limit state can be considered:

$$P_f = P(C(d_c, t) > C_{crit}) \leq \Phi(-\beta_t) \tag{3.10}$$

with

C_{crit}, critical chloride content at which corrosion will start $[wt. - \%Cl^-/binder]$.
$C(d_c)$, chloride content at the reinforcement $[wt. - \%Cl^-/binder]$.
d_c, concrete cover $[mm]$.

Eq. (3.10) states that the probability that the chloride content C, at the rebar level (cover depth) d_c, is larger than the critical chloride content C_{crit} should be smaller than some predefined value.

Deterioration model

To cause corrosion of the rebars, the chloride concentration at the rebar level has to overcome the threshold-level c_{crit}. This depends primarily on the environmental conditions and on the concrete parameters. It is established that the onset of chlorides into concrete is a complex interaction of physical and chemical processes, but it is also acknowledged that diffusion is one of the principal mechanisms. Therefore the diffusion theory (Eq. (3.11)) and the error function equation can be used to model the onset of chlorides.

According to DuraCrete [2000] the following deterioration model has been proposed:

$$\frac{\delta C_{cl}}{\delta t} = D_{cl} \frac{\delta^2 C_{cl}}{\delta x_{cl}^2} \tag{3.11}$$

with

C_{cl}, chloride ion concentration at the depth $[wt. - \%Cl^-/binder]$.
x_{cl}, surface of concrete after exposes to chlorides till time t $[mm]$.
D_{cl}, chloride diffusion coefficient $[mm^2/yr]$.

The solution for partial differential of Eq. (3.11) gives the explicit form to calculate the chloride concentration as a function of cover depth x_{cl} and time t.

$$C_{cl}(x_{cl}, t) = C_s \left(1 - erf\left(\frac{x_{cl}}{2\sqrt{D_{cl}(t)t}}\right)\right) \tag{3.12}$$

where

$erf(\bullet)$ denotes the error function.

It's useful to split the chloride diffusion coefficient $D_{cl}(t)$ into different measurable and quantifiable parameters to make accurate predictions. This leads to the separation of the different effects according to the material, the environment and the execution.

For potential environmental conditions, the apparent chloride diffusion coefficient at time t_0 is equal to a measured chloride migration coefficient if a test method factor k_t is introduced:

$$D_{cl}(t_0) = k_t \cdot D_0(t_0) \tag{3.13}$$

For other environmental conditions Eq. (3.13) should be extended by an environmental factor k_e. The executional influences will be considered by an executional factor k_c. In addition to that the material and environmental induced decrease of the apparent diffusion coefficient will be covered by a time dependent exponential function.

$$D_{cl}(t) = D_{cl}(t_0) \cdot \left(\frac{t_0}{t}\right)^n = k_e \cdot k_t \cdot k_c \cdot D_0 \cdot \left(\frac{t_0}{t}\right)^n \tag{3.14}$$

Which leads to the following equation:

$$C_{cl}(t) = C_s \cdot \left(1 - erf\left(\frac{x}{2\sqrt{k_e \cdot k_t \cdot k_c \cdot D_0 \cdot \left(\frac{t_0}{t}\right)^n \cdot t}}\right)\right) \tag{3.15}$$

with

D_0, chloride migration coefficient at defined compaction, curing and environmental conditions, measured at time t_0 $[mm^2/yr]$ - material variable.
n, age factor $[-]$ - environmental and material variable.
C_s, surface chlorilde level $[wt. - \%Cl^-/binder]$ - environmental and material variable.
t, exposure periode $[yr]$.
t_0, reference period $[yr]$.
k_t, constant parameter which considers the influence of test method on measured D_0 $[-]$ - test method variable.
k_c, constant parameter which considers the influence of curing on D_0 $[-]$ - execution variable.

k_e, constant parameter which considers the influence of environment on D_0 $[-]$ - environmental variable.
x, depth $[mm]$; if the onset of corrosion is considered, x is equal to the concrete cover depth d_c.
C_{crit}, critical chloride content $[wt. - \%Cl^{-}/binder]$.

For any bridge object, values of variables C_{cl}, C_s and D_{cl} in the equation can be considered as random, each one is associated with its own statistical distribution. Thus, estimation of time to arrive at certain value of chloride concentration is probabilistic in nature.

Propagation phase

The propagation period is defined as the period that extends from the moment at which the critical chloride content C_{crit} at rebar level is reached and corrosion, known as "rust", develops progressively until the structure reaches an unacceptable deterioration (failure). This period can be determined from the attack penetration function. This attack penetration function, $P_x(t)$, represents the loss of rebar diameter at the time t and it can be later implemented in the damage functions of the structural performance.

In a similar way as for the onset of corrosion, according to DuraCrete [2000], the following limit state can be considered:

$$P_f = P(P(t) > P_{crit}) \leq \Phi(-\beta_t) \tag{3.16}$$

with

P_{crit}, critical loss of rebar $[mm]$.

Deterioration model

The attack penetration function, $P_x(t)$, can be expressed as:

$$P_x(t) = \int_{t_i}^{t} V_{corr}(\tau) d\tau \tag{3.17}$$

with

$V_{corr}(\tau)$, corrosion rate at the instant τ $[mm/yr]$.
t_i, initiation period $[yr]$.

Eq. (3.17) gives the progress of corrosion with time as a function of the corrosion rate V_{corr}.

Considering a uniform corrosion rate (Fig. 3.3) the progressive loss of rebar diameter (attack penetration) becomes:

$$P_x(t) = V_{corr} \cdot \alpha \cdot t \tag{3.18}$$

with

α, pitting factor; taking non-uniform corrosion of the rebars into account [−].

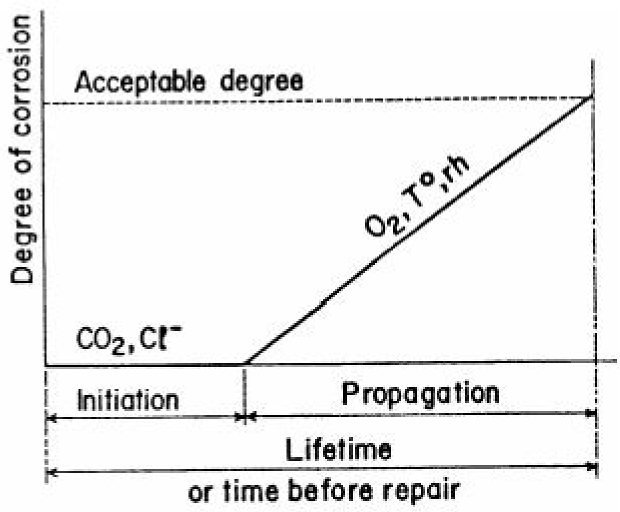


Figure 3.3.: Service life model of reinforcement corrosion (DuraCreta [2000])

The corrosion rate is usually not constant, compare Fig. 3.2, but develops due to the corrosion process itself and due to climate differences. Therefore, a representative, or average, value of the corrosion rate, $V_{corr,a}$, has to be determined.

The main difficulty regarding the modeling of the damage function $P_x(t)$ lies in the estimation of the value of $V_{corr,a}$. There are three possibilities to estimate $V_{corr,a}$:

1. To assume values on function only of the exposure classes (and not of the type of attack).
2. To estimate $V_{corr,a}$ from direct measurements of I_{corr} (in specimens for new structures or on-site for existing ones).
3. To use empirical expressions based on a variable governing the process (resistivity was selected, it is a positive decision, because it also governs the Initiation period, but it might be selected other one).

For our model we used the first model to establish the value of $V_{corr,a}$:

$$V_{corr} = V_{corr,a} \cdot w_t \tag{3.19}$$

with

$V_{corr,a}$, mean corrosion rate when corrosion is active $[mm/yr]$.
w_t, wetness period, i.e. the fraction of the year that corrosion is active $[-]$.

Taking account of the variables with relevant influence on the initiation and propagation of concrete cracking, the following relationship between crack width and attack penetration due to corrosion was proposed by DuraCrete [2000]:

$$w = w_0 + \beta(P(x) - P(x_0)) \tag{3.20}$$

with

w, crack width over time $[mm]$.
β, parameter that controls the propagation $[-]$.
w_0, crack width when it's visible (≈ 0.05) $[mm]$.
$P(x_0)$, amount of loss of rebar diameter when first crack is visible $[mm]$.
$P(x)$, the amount of loss of rebar diameter at time t $[mm]$.

According to the experimental work carried out (Institute E. Torroja and Geocisa in the past Brite project BE92-4062), it was considered that cracking initiation mainly depends on the cover/diameter ration (c/ϕ in $[mm/mm]$) and the tensile splitting strength of the concrete ($f_{t,spl}$ in $[MPa]$). The following linear relationship was proposed:

$$P(x_0) = a_1 + a_2 \cdot c/\phi + a_3 \cdot f_{t,spl} \tag{3.21}$$

with

a_1, regression parameter 1 $[mm]$.
a_2, regression parameter 2 $[mm]$.
a_3, regression parameter 3 $[mm/MPa]$.
c, cover depth $[mm]$.
ϕ, rebar diameter $[mm]$.
$f_{t,spl}$, tesile splitting strength $[MPa]$.

After the value of chloride concentration reaches the threshold-level, corrosion starts on the rebars and after it reaches another limit the cracking process starts. The crack initiation phase is referred as the propagation phase and the width of the cracks over time can be determined by using following equations:

$$w(t) = w_0 + \beta \cdot (P(t) - P_0) \tag{3.22}$$

with

$w(t)$, crack width over time $[mm]$.
β, parameter that controls the propagation $[-]$.
w_0, crack width when it's visible (≈ 0.05) $[mm]$.
P_0, amount of loss of rebar diameter when first crack is visible $[mm]$.
$P(t)$, amount of loss of rebar diameter at time t $[mm]$.

The reinforcement loss function can be represented as:

$$P(t) = V_{corr,a} \cdot \alpha \cdot w_t \cdot t \tag{3.23}$$

with

$V_{corr,a}$, mean corrosion rate when corrosion is active $[mm/yr]$.
w_t, wet period in a year (equal to the ratio of rainy days in a year) $[-]$.
α, pitting factor that takes non-uniform corrosion of the rebars into consideration $[-]$.

Combining Eq. (3.21), (3.23) and (3.22) the following equation can be obtained.

$$w(t) = w_0 + \beta \cdot ((V_{corr,a} \cdot \alpha \cdot w_t \cdot t) - (a_1 + a_2 \cdot c/\phi + a_3 \cdot f_{t,spl})) \tag{3.24}$$

with

$w(t)$, crack width over time $[mm]$.
β, parameter that controls the propagation $[-]$.
w_0, crack width when it's visible (≈ 0.05) $[mm]$.
$V_{corr,a}$, mean corrosion rate when corrosion is active $[mm/yr]$.
w_t, wet period in a year (equal to the ratio of rainy days in a year) $[-]$.
α, pitting factor that takes non-uniform corrosion of the rebars into consideration $[-]$.
a_1, regression parameter 1 $[mm]$.
a_2, regression parameter 2 $[mm]$.
a_3, regression parameter 3 $[mm/MPa]$.
c, cover depth $[mm]$.
ϕ, rebar diameter $[mm]$.
$f_{t,spl}$, tesile splitting strength $[MPa]$.

3.1.3. The WEAR-OUT model

In the WEAR-OUT model the deterioration process of each object is described as an exponential relationship of wear in relation to the time (Brodsky et al. [2006]).

$$y(t) = e^{\lambda t} - 1 \tag{3.25}$$

with

λ, rate coefficient $[-]$.

The rating coefficient is determined based on the following boundary condition.

$$\lambda = \frac{Ln2}{T_c} \tag{3.26}$$

with

T_c, average life time of the object.

3.2. Markov Transition probability

To model the continuous deterioration process of the different bridge objects Markov deterioration matrices are used. The deterioration matrix is filled with transition probabilities p_{ij}, which can be seen in Fig. 3.4b. The different transition probabilities p_{ij} show the probability that a bridge or a part of it will chance from condition state i at time t into condition state j at time $t+1$, compare Fig. 3.4a.

In Fig. 3.4b can be observed that $p_{ij} = 0$ for $i > j$. This means that bridges cannot improve in condition state. The probability to be in condition state j at time $t+1$, is the summation of all the transitions probabilities like in Eq.(3.27):

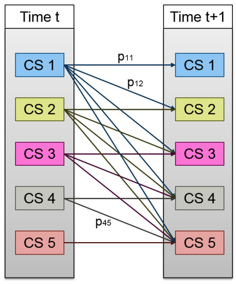
$$\pi_j^{t+1} = \sum_{i=1}^{j} p_{ij} \cdot \pi_i^t \tag{3.27}$$

with

π_i^t, probability to be in condition state i at time t $[-]$.

An important property of the Markov model is that the condition state distribution at time $t+1$ only depends on the distribution at time t and no longer on any stages before t. In other words can be said that the Markov process is memory less.

The Markov transition probability (m.t.p.) matrix (Fig.3.4b) is the basic input for the optimization model.



(a)

./figures/tpm.jpg

(b)

Figure 3.4.: Transition probability: (a) Illustration of the transition probability (b) Transition probability matrix (m.t.p.)

3.2.1. The MUSTEM model

According to Eq. (3.9), if the object is in state i at time τ_A, then the probability that it will still be in state i at a subsequent time $z_i \geq 0$ is:

$$\tilde{F}_i(y_A + z|\zeta_i \geq y_A) = Prob\left\{\zeta_i \geq y_A + z_i|\zeta_i \geq y_A\right\} \tag{3.28}$$

By dividing both sides of Eq. (3.28) by (Eq. (3.2)) the conditional probability of being in state i at z_i.

$$\frac{\tilde{F}_i(y_A + z_i)}{\tilde{F}_i(y_A)} = \frac{\exp\left\{-\theta_i(y_a + z_i)\right\}}{\exp(-\theta_i z_i)} = \exp(-\theta_i z_i) \tag{3.29}$$

If z_i is equal to $\tau_B - \tau_A$:

$$Prob\left[X_{\tau_B} = i|X_{\tau_A} = i\right] = \exp(-\theta_i Z) \tag{3.30}$$

With similar steps Eq. (3.31) can be obtained which can be used to estimate the transition probabilities from each state i to all states greater than i, TSUDA et al. [2006]:

$$p_{ij}(z) = Prob\left[X(\tau_B) = j|X(\tau_A) = i\right] = \sum_{k=i}^{j}\prod_{m=i}^{k-1}\frac{\theta_m}{\theta_m - \theta_k}\prod_{m=k}^{j-1}\frac{\theta_m}{\theta_{m+1} - \theta_k}\exp(-\theta_k z) \tag{3.31}$$

where i, j, m and k are running indexes of the state from 1 to I.

With Eq. (3.31), if the θ_i is known for every state and the time interval Z is known, the transition probability can be estimated.

3.2.2. The E-MEPI model

For the E-MEPI model the input parameter where defined with their mean value and standard deviation. To calculate the state probability for this model Monte Carlo simulations are used. The result of the Monte Carlo simulations are density functions for each time step t, comparing Fig. 3.5.

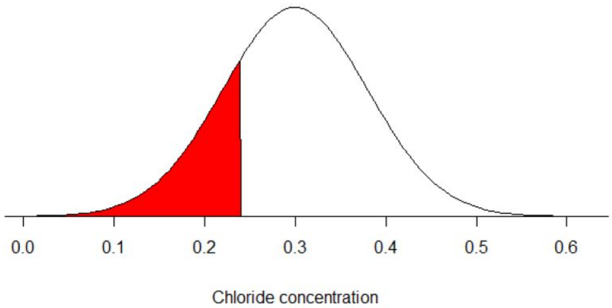


Figure 3.5.: Density function for any time step t (IMP [HS2014])

To calculate the optimal work program the objects needs to be defined through condition states. According to the E-MEPI model the output is a distribution of the chloride content (initiation phase) or the crack width (propagation phase) at time t. To estimate the state probability of the object (condition state distribution at any time t) the model output needs to be transfer into condition states. This is done via Eq. (3.33). This equation show the probability of being in the predefined range of a condition state. In other words through Eq. (3.33) the probability of being in CS i at time t is calculated.

If the density function for each time step is known the state probability can be calculated as shown in Eq. (3.33).

$$\pi_i(t) = Prob[CS = i] = Prob[LBC \leq C \leq UBC] = \int_{LBC}^{UBC} f(x)dx \tag{3.32}$$

with

$\pi_i(t)$, state probability of being in CS i at time t $[-]$.
C, criterion to determine the CS of an object $[dependingC]$.
LBC, lower boundary condition for CS i $[dependingC]$.
UBC, upper boundary condition for CS i $[dependingC]$.
$f(x)$, density function for time t $[dependingC]$.

For the given example in Fig. 3.5 can be seen that the range for the condition state (CS 1) is a chloride concentration between 0 and 0.24. This would lead to the following equation.

$$\pi_1(t) = Prob[CS = 1] = Prob[0 \leq C_{cl} \leq 0.24] = \int_0^{0.24} f(x)dx = 0.227 \tag{3.33}$$

Which means that the state probability at time t of being in CS 1 is 0.227. According to this the state probability for the object over the investigated time period can be estimated. In this case $f(x)$ is the density function of the normal distribution. But if the density function cannot be described, the state probability simply can be estimated by counting the simulations which are in the defined boundary conditions for each CS.

According to Jones [2005] it is possible to calculate the transition probability p_{ij} out of the state probability $\pi_j(t)$. The paper demonstrates how to use proportions data to estimate the transition probability. A stochastic relation relating the actual and estimated occurrence of $\pi_j(t)$ can be written:

$$\pi_j(t) = \sum_i \pi_i(t-1)p_{ij} + u_j(t) \tag{3.34}$$

with

$\pi_j(t)$, state probability of the object at time t [$-$].
$\pi_i(t-1)$, state probability of the object at time $t-1$ [$-$].
p_{ij}, transition probability [$-$].
$u_j(t)$, error term at time t [$-$].

If Eq. (3.34) is solved for each time step in the investigated time period and minimize the sum of the error term the transition probability matrix for the deterioration process can be found, compare (Eq. (3.35)).

$$Z = min\left(\sum_t^T \left(\pi_j(t) - \sum_i \pi_i(t-1)p_{ij}\right)\right) \tag{3.35}$$

3.2.3. The WEAR-OUT model

The transition probability matrix for the WEAR-OUT model can be found by minimize the difference between the deterministic prediction (Eq. (3.25)) and the proposed transition probability matrix. To do so the following objective function can be used.

$$Z = min \sum_{t}^{T} (y(t) - \overline{y}(t))^2 \tag{3.36}$$

with

$y(t)$, deterministic prediction.
$\overline{y}(t)$, average state prediction using Markov model.

The average state prediction using the Markov model can be estimated through Eq. (3.37).

$$\overline{y}(t) = \sum_{i}^{I} x_i(t) \cdot \overline{u}_i \tag{3.37}$$

with

$x_i(t)$, state probability at time t for condition state i.
$\overline{u}_i$, pre-defined continuous value to which the condition state is referred to.

$$x_i(t) = \begin{cases} x_i(t-1) \cdot p_{ii} \text{ ; when i} = 1 \\ x_{i-1}(t-1) \cdot p_{i-1,i} + x_i(t-1) \cdot p_{ii} \text{ ; when i} > 1 \end{cases} \tag{3.38}$$

Like in Eq. (3.38) can be seen, the transition probability matrix for the WEAR-OUT model does only consider unit jumps. Which means that it is not possible for the object to jump in one time step from CS i to CS i+2, compare Eq. (3.39). This is a big simplification due to the minimization problem to estimate the transition probability matrix.

$$\begin{array}{c} \\ CS_1 \\ CS_2 \\ CS_3 \\ CS_4 \\ CS_5 \end{array} \begin{array}{c} \begin{array}{ccccc} CS_1 & CS_2 & CS_3 & CS_4 & CS_5 \end{array} \\ \begin{pmatrix} p_{11} & p_{12} & & & \\ & p_{22} & p_{23} & & \\ & & p_{33} & p_{34} & \\ & & & p_{44} & p_{45} \\ & & & & 1 \end{pmatrix} \end{array} \tag{3.39}$$

3.3. Optimization model

After the transition probability p_{ij} is determined the optimization for the work program can start.

3.3.1. General procedure

Object level

In a first step the costs for all different intervention strategies $C_k^o(t)$ is calculated over the investigated time period, according to Eq. (3.40).

$$C_k^o(t) = \sum_{i=1}^{I} \pi_{k,i}^o(t) \cdot c_{k,i}^o \cdot a^o \tag{3.40}$$

with

$C_k^o(t)$, intervention strategy costs over the investigated time period $[mu]$.
$\pi_{k,i}^o(t)$, state probability for the objects at time t $[-]$.
$c_{k,i}^o$, intervention costs per area $[mu/m^2]$.
a^o, area of the object $[m^2]$.

In the next step the total costs for each intervention strategy is calculated.

$$C_k^o = \sum_{t=1}^{T} C_k^o(t) \tag{3.41}$$

Finally the optimal intervention strategy (OIS) for each object can be defined.

$$OIS^o = argmin_{k \in IS^o} C_k^o \tag{3.42}$$

The result of Eq. (3.42) is the number of the optimal intervention strategy.

Bridge level

If the work program on object level is found the optimal work program on bridge level can be estimated. In a first step the costs for all possible work programs can be calculated, according to Eq. (3.44).

$$S_n = \sum_{t=1}^{T} \sum_{o=1}^{O} C^o_{k(n)}(t) \tag{3.43}$$

with

S_n, cost for the different work programs $[mu]$.

Where n represents all possible combinations of different interventions strategies for the different work programs.

$$n = \prod_{o=1}^{O} IS(o) = \begin{Bmatrix} 1 & 1 & 1 & 1 & ... & 5 & 5 \\ 1 & 1 & 2 & 2 & ... & 4 & 4 \\ 1 & 2 & 1 & 2 & ... & 1 & 2 \end{Bmatrix} \tag{3.44}$$

Where row one represents the intervention strategy of the deck (object 1), row two the intervention strategy of the pier (object 2) and row three the intervention strategy of the abutment (object 3).

Finally the optimal work program (OWP) can be found by using Eq. (3.45).

$$OWP = argmin_n S_n \tag{3.45}$$

3.3.2. Input

The optimization model needs some general input in addition to the transition probability matrices P for the different objects:

- Intervention strategy matrix I for each object. An intervention strategy is comprised of the interventions that are to be performed as soon as the object is in a specific condition state. The interventions that comprise the optimal intervention strategies are the ones that if done when a part of the object is in each condition state will result in the lowest long term costs. Including to the different interventions for the intervention strategy an agency rule has to be defined for each intervention strategy.
- Effectiveness and cost matrix R for each object, this matrix describes the different intervention types of the intervention strategies and their effectiveness and cost for each condition state. Due to the fact that intervention do not always result in a like new structure, the effectiveness of the intervention types needs to be modeled. When no data is available to determine interventions effectiveness, which is often the case, it is determined based on expert opinion, referring to Adey and Hajdin [2011].
- Initial condition state vector ICS for the different objects, this vector defines the condition state distribution of the objects at the beginning of the investigated time period.

3.3.3. Agency rule

Since it is not desired to perform interventions on the same object every year, and it is not desired to perform interventions on very small parts of the bridge, agency rules are used to built work programs. An agency rule defines when an intervention has to be performed. Which means that it is not always necessary to perform an intervention immediately when an object reaches the defined condition state to perform an intervention. In the optimization model used in this study an agency rule is defined as follows, if $x\%$ of the object are in CS i or less the planned interventions will be executed.

3.3.4. Optimization

The steps for the determination of the financial needs and object conditions for the optimal intervention strategies using the basis presented above are shown in Table 3.1.

Table 3.1.: Steps for the development of financial needs and object conditions on object level

Step	Description
1	Define the initial condition states of the objects (condition state distribution at time $t = 0$).
2	Determine all possible intervention strategies (ISs) for each object of the bridge.
3	Select the intervention strategy i to be performed.
4	Determine the interventions to be performed at time t (for IS i).
5	Sum the financial needs of all necessary interventions at time t (for IS i).
6	Estimate the distribution of structural condition for each object at time $t = t + 1$, if IS i if followed.
7	Repeat steps 4 to 7 (with $t = t + 1$) until the structural condition prediction and financial needs for the implementation of intervention strategy i over the investigated time period is determined.
8	Repeat steps 3 to 8 until the structural condition predictions and financial needs for each of the intervention strategies over the investigated period are determined.
9	Evaluate the structural condition predictions and the financial needs for each intervention strategy over the investigated time period.
10	Select the optimal intervention strategy.

Spezific transition probabilities

In the first step the specific transition probability matrix Q is defined. It represents the combination of the Intervention strategy matrix I and the effectiveness matrix R which means that for each objects and intervention strategy an own transition probability matrix is made due to the fact that according to the different interventions the transition probability changes. The result is the array $Q^o_{CS(o),CS(o),IS(o)}$, where CS represents the number of different condition states, IS the number of different intervention strategies and o the number of object of the bridge. It can be seen that the number of condition states and intervention strategies depend on the object, which means that not all objects have to have the same number of condition states or intervention strategies.

The number of different probability matrices depends on the number of different objects and the number of intervention strategies per object.

$$\text{Number of Q} = \sum_{i=1}^{O} \text{Number of Intervention strategies for object i} \tag{3.46}$$

State probability for the different intervention strategies

The state probability $\pi^{o}_{IS(o),CS(o),T}$ can be calculated according to Eq. (3.47) and (3.48) under consideration of the agency rule:

If $t = 1$:

$$\pi_{r,,1,o} = ICS_{,o} \tag{3.47}$$

If $t > 1$:

$$\pi_{r,,t,o} = \begin{cases} \pi_{r,,(t-1),o} \cdot Q_{,,r,o} \text{ ; if agency rule fulfilled} \\ \pi_{r,,(t-1),o} \cdot P_{,,o} \text{ ; if agency rule not fulfilled} \end{cases} \tag{3.48}$$

where $r = (IS(1), ..., IS(o))$ represents the intervention strategies according to the different objects, $t = (1, 2, ..., T)$ represents the time and o represents the object. And π represents the state probability for the different objects and interventions strategies, ICS the initial condition state of the objects, Q the specific transition probability matrix for the different intervention strategies and objects and P the initial transition probability matrix (doing nothing).

Costs for the different intervention strategies

To calculate the costs for the different intervention strategies for the different objects the following equation (Eq. (3.49)) was used:

$$C_{r,i,t,o} = \begin{cases} \pi_{r,i,t,o} \cdot c_{r,i,o} \cdot a_o \text{ ; if agency rule fulfilled} \\ 0 \text{ ; if agency rule not fulfilled} \end{cases} \tag{3.49}$$

where $i = (CS1, ..., CS(o))$. And C represents the intervention costs for each intervention strategy and object, c the intervention costs depending on the condition state where the intervention is executed and a the surface area of the different objects.

The summation of C over the different condition states each year gives the total costs for each intervention strategy for every object, according to Eq. (3.50).

$$TC_{r,t,o} = \sum_{i}^{CS(o)} C_{r,i,t,o} \tag{3.50}$$

In the next step the total costs over the investigated time period were calculated (Eq. (3.51)) to compare the different costs and define the optimal intervention strategy.

$$SC_{r,o} = \sum_{t}^{T} TC_{r,t,o} \tag{3.51}$$

Work program based on object level

Optimal intervention strategies

The optimal intervention strategy for each object can be found by looking for the minimum of SC.

$$OIS_o = \min SC_{r,o} \tag{3.52}$$

According to the optimal intervention strategies (Eq. (3.52)) the work program based on those optimal intervention strategies can be calculated. The output of this work program is the distribution of the costs as well as the condition state distribution over the investigated time period.

Optimal work program based on bridge level

To calculate the optimal work program based on all different interventions strategies an so called "discount factor" is introduced. This factor considers the fact that the intervention cost consists of two different parts. The first one is the fix part (infrastructure, etc.) and a variable part for the specific work which has to be done. This leads to the fact that if two interventions, on different objects, are executed in the same year the fix part of the costs can be partially shared and the cost for those two interventions is not exactly the summation of the two intervention costs at their own. This can result in an optimal work program which does not include the optimal intervention strategies for each object, because if there is an other intervention strategy where more interventions take place in the same year the costs can be reduced by this factor.

All possible work programs

In order to estimate the optimal work program all possible work programs, combinations of the different intervention strategies, have to be taken into consideration.

$$\text{number of different work programs} = \prod^{o} IS(o) \tag{3.53}$$

The number of different work program can be calculated according to Eq. (3.53).

In the next step the costs for each work program can be estimated, according to the costs of the different intervention strategies which are included in the work program. If the costs for each year are determined the "discount factor" can be taken into consideration, due to the fact that we now know how many interventions take place each year. After that step the work program with the minimal costs can be found and displayed as the optimal work program.

All these steps mentioned above are used in the R-code to calculate the work program based on the optimal intervention strategies and the optimal work program and can be seen in appendix F.

4. Case study

4.1. General information

Figure 4.1.: Getwing bridge

For this work the Getwing bridge (Fig. 4.1) was used as a case study to compare the different approaches and the resulting work programs.

The Getwing bridge is located in the Canton of Valais in Switzerland (Fig. 4.2a) and combines the two villages Getwing and Turtmann. The bridge crosses the river Rhone, as can be seen in Fig. 4.2b, and is about 302m long.

It is a reinforced concrete bridge with nine, respectively 18, piers, compare Fig. 4.1, and the deck is about 7m wide.

The location of the bridge is important for the E-MEPI model due to the fact that the input parameters depend on the local climate, compare section 3.1.2.

(a) (b)

Figure 4.2.: Location of the Getwing bridge: (a) Situation of the Getwing bridge in Switzerlang. (b) Local location of the Getwing bridge between Getwing and Turtmann.

In order to define the condition state of the hole bridge, the bridge is divided into three different objects, which are deck, piers and abutments.

For each of those objects the condition state distribution over time will be estimated. Out of this the optimal intervention strategy and in the end the optimal work program for the hole bridge according to the three objects can be estimated.

4.2. General input for the model

Areas

In this section there is a lot of input data and in terms of tidiness the general input parameter for the object "deck" are in the report and the ones for the pier and abutment can be found in appendix A.

The first step for the calculations is to define the area of the three different objects.

According to Eq. (4.1) the area of the deck is $2'114m^2$.

$$A_{deck} = \text{length of the bridge} * \text{width of the deck} = 302m * 7m = 2'114m^2 \tag{4.1}$$

According to the drawings the different pier dimensions are shown in Tab. 4.1. The summation of all piers gives an area of $326.06m^2$. But due to the fact, that each pier occurs twice, compare Fig. 4.1, the area needs to be doubled as well. The final area of the piers is $652.12m^2$.

For the abutment it is more sophisticated to calculate the areas. According to the drawing the abutment on the Getwing side has an area of approximately $50m^2$ and the abutment of the Turtmann side of about $45m^2$. Which leads to a area of $95m^2$ for the abutment.

Table 4.1.: Piers of the Getwing bridge

Pier	Hight $[m]$	Width $[m]$	Depth $[m]$	Area $[m^2]$
2	3.8	1.8	0.6	18.24
3	5.4	1.8	0.6	25.92
4	6.7	1.8	0.6	32.16
5	7.6	1.8	0.6	36.48
6	8.6	1.8	0.6	41.28
7	9.7	1.8	0.8	50.44
8	9.5	1.8	0.9	51.30
9	7.6	1.8	0.8	39.52
10	6.4	1.8	0.6	30.72

In Tab. 4.2 is the summation of the different areas which are used for the further calculations.

Table 4.2.: Areas for the different objects

Object	Area
Deck	$2114m^2$
Pier	$652m^2$
Abutment	$95m^2$

Initial condition states

According to the inspections of an infrastructure manger the initial condition state of the bridge has to be taken into account. Which means that the bridge does not have to be in like new conditions at the beginning of the investigated time period. Due to the fact that we do not have inspection data for the Getwing bridge we assumed initial condition states for the different objects which can be seen in Tab. 4.3. As well as the number of condition states for each object.

Table 4.3.: Initial condition states for the different objects

	Condition state				
Object	1	2	3	4	5
Deck	0.40	0.30	0.15	0.10	0.05
Pier	0.50	0.30	0.15	0.05	-
Abutment	0.60	0.30	0.10	-	-

In Tab. 4.3 can be seen that the deck has five different condition states, the pier has four different condition states and the abutment has three different condition states. It has to be mentioned that in Switzerland normally five different condition states are used. The fact that different number of condition states are used is just that it can be seen that the objects does not have to have the same number of CS and that the developed optimization problem does not care about the number of condition states of the different objects.

Like mentioned before the different condition states needs to be defined or described so that it is always clear for an infrastructure manager in which condition state an object is according to his inspections or measurements. It cannot be the case that an object can be in two condition states at the same time. The definitions of the condition states for the deck can be seen in Tab. 4.4, the ones for the pier and abutment can be seen in appendix A.1.

Table 4.4.: Condition state definition for the deck

CS	Description	Indicator	Criterion
1	New/Partially new	chloride content in the concrete at reinforcement bar level C_{cl}	$0 < C_{cl} \leq 0.24$
2	Concrete contaminated	chloride content in the concrete at reinforcement bar level C_{cl}	$0.24 < C_{cl} \leq 0.48$
3	Corrosion has initiated, no visible cracking has occurred	Width of cracks (w)	$C_{cl} > 0.48$, $w \leq 0.25$
4	Visible cracking has occurred	Width of cracks (w)	$0.25 < w \leq 0.50$
5	Visible cracking has occurred and cover has spelled off	Width of cracks (w)	$w > 0.50$

Intervention strategies

As a next step the different intervention strategies for each object should be defined. Each intervention strategy can be one of the interventions mentioned in Tab. 4.5 or a combination of them. The possible activities which are linked with each intervention can be seen in Tab. 4.5 as well. The different intervention strategies for the deck can be seen in Tab. 4.6, for the pier and abutment in appendix A.2. It needs to be said that there is an indefinite number of different intervention strategies for an object and it is up to an infrastructure manager to get them down to a reasonable number.

Table 4.5.: Examples activities for each intervention type for concrete bridges (IMP [HS2014])

Preservation intervention	Description of activities
Do nothing	-
Rehabilitation	• Replacement of the waterproofing • Restoration of parts of the drainage system • Improvement or replacement of construction joints • Repair of the reinforced concrete • Renewal of any finishing
Renewal	• Improvement or replacement of construction joints • Replacement total or partial of the supports • Renewal of the drainage system • Replacement of the guard rails • Strengthening of one or more structural elements
Replacement	• Replacement of part or all of the structure

According to the intervention strategy the agency rule (AR) for each intervention strategy with boundary condition (BC) needs to be defined, like explained in section 3.3.3.

Table 4.6.: Intervention strategy matrix with agency rule I^{De} for the deck

	Condition state						
IS	1	2	3	4	5	AR	BC
1	Do nothing	Do nothing	Rehabilitation	Rehabilitation	Replacement	0.40	3
2	Do nothing	Do nothing	Renewal	Renewal	Replacement	0.40	3
3	Do nothing	Do nothing	Do nothing	Rehabilitation	Replacement	0.30	4
4	Do nothing	Do nothing	Do nothing	Renewal	Replacement	0.30	4
5	Do nothing	Do nothing	Do nothing	Do nothing	Replacement	0.15	5

The meaning of i.e. the agency rule for intervention strategy 3 for the deck, compare Tab. 4.6, is that if 30% of the object are in condition state 4 or less the planned interventions will be executed and if it is not the case no intervention will take place in that year.

Effectiveness and costs for the different interventions

The last general input is the effectiveness of the interventions and the cost according to the interventions. It needs to be said that the effectiveness and cost is not only depending on the object and intervention. It is also depending on the condition state in which the object is before performing the intervention. The cost is given in mu (monetary units), which means that it can be transformed in any kind of costs. The most important fact for the estimation of the optimal work program is the ratio between the different interventions and not the absolute value.

It has to be mentioned that for all the three different models these input data stays the same. Which means that in terms of comparison the same costs and the same rule are applied for the different models. The only difference for the different optimal work program is estimation of the transition probabilities, according to the three different deterioration models.

Table 4.7.: Effectiveness and unit cost matrix R^{De} for the deck

		Effectiveness					
CS	Intervention	1	2	3	4	5	Unit costs mu/m^2
3	Rehabilitation	0.80	0.20	0	0	0	215
	Renewal	0.95	0.05	0	0	0	380
	Replacement	1	0	0	0	0	515
4	Rehabilitation	0.75	0.25	0	0	0	250
	Renewal	0.90	0.10	0	0	0	400
	Replacement	1	0	0	0	0	515
5	Rehabilitation	0.60	0.40	0	0	0	320
	Renewal	0.85	0.15	0	0	0	480
	Replacement	1	0	0	0	0	515

4.3. Model specific input

4.3.1. The MUSTEM model

Like in section 3.2.1 mentioned the only model specific inputs for the MUSTEM model are the hazard rates for the different objects. The hazard rates θ_i^o can be estimated via two different data bases. The first data base is historical data and the second is through a well defined deterioration function.

The estimated hazard rates for the deck can be seen in Tab. 4.8, the ones for the pier and abutment in appendix B.1.

Table 4.8.: Hazard rates for the deck

CS	Hazard rate θ_i
1	0.29
2	0.21
3	0.09
4	0.06
5	0.00

4.3.2. The E-MEPI model

The input for the E-MEPI model is based on the data from DuraCrete [2000]. A compilation of the different parameters can be seen in Tab. 4.9 and 4.10, where μ represents the mean value, σ the standard deviation and D the type of distribution.

Table 4.9.: Input parameters for the initiation phase of the deck for the E-MEPI model

	d	D_0	C_s	k_e	k_t	k_c	t_0	n
μ	20	220.9	2.4	0.265	0.832	1	0.0767	0.37
σ	–	28.5	0.2	0.045	0.024	–	–	0.07
D	discrete	normal	normal	gamma	normal	discrete	discrete	beta

Table 4.10.: Input parameters for the propagation phase of the deck for the E-MEPI model

	w_0	β	V_0	wet	α	a_1	a_2	a_3	ϕ
μ	0.05	0.01	0.3	0.75	9.28	74.4	7.3	-17.4	20
σ	0.005	0.001	0.04	0.2	4.04	5.7	0.06	5.7	–
D	normal	normal	normal	normal	normal	normal	normal	normal	discrete

It has to be said, that according to DuraCrete [2000] those input data is based on a very few comparable objects (so called road environment in DuraCrete [2000]). Most of the investigations are made in the marine environment. This means that those data should be expanded so that the input data for the E-MEPI model is based on a more reliable data base.

4.3.3. The WEAR-OUT model

Like mentioned in section 3.1.3, the only model specific input for the WEAR-OUT model are the different life times for the objects. The life times for the different objects can be seen in Tab. 4.11.

Table 4.11.: Lifte times for the different objects

Object	Lifte time T_c
Deck	35 yr
Pier	60 yr
Abutment	75 yr

5. Results

Like in chapter 4 the results were mainly shown for the deck. The others can all be found in appendix C for the deterioration models and in appendix D for the optimization model on object level and in appendix E for bridge level.

5.1. Deterioration model

5.1.1. Transition probability

The first model output is the Markov transition probability matrix (m.t.p.) for the different deterioration models.

The m.t.p. for the MUSTEM model is calculated out of the hazard rates, according to Eq. (3.31). For the E-MEPI, the m.t.p. is calculated through the minimization problem, compare Eq. (3.35). As well as for the WEAR-OUT model (Eq. (3.36)).

The transition probability matrices (m.t.p.) for the deck can be seen in Tab. 5.1 to Tab. 5.3.

Table 5.1.: Transition probability P^{De} for the MUSTEM for the deck

	Condition state at $t+1$				
CS	1	2	3	4	5
1	0.748	0.226	0.025	0.001	0
2		0.811	0.181	0.008	0
3			0.914	0.084	0.002
4				0.942	0.058
5					1

Table 5.2.: Transition probability P^{De} for the E-MEPI for the deck

	Condition state at $t+1$				
CS	1	2	3	4	5
1	0.909	0.087	0.004	0.000	0.000
2		0.939	0.062	0.000	0.000
3			0.934	0.059	0.007
4				0.908	0.092
5					1

Table 5.3.: Transition probability P^{De} for the WEAR-OUT for the deck

	Condition state at $t+1$				
CS	1	2	3	4	5
1	0.889	0.111	0	0	0
2		0.862	0.138	0	0
3			0.815	0.185	0
4				0.716	0.284
5					1

According to the different m.t.p. (Tab. 5.1 to Tab. 5.3) and the initial conditions of the different objects (Tab. 4.3) the condition state distribution over the investigated time period can be estimated (Fig. 5.1 to Fig. 5.3). This estimation corresponds to the so called do nothing intervention strategy and represents the case that no intervention is executed over the hole investigated time period.

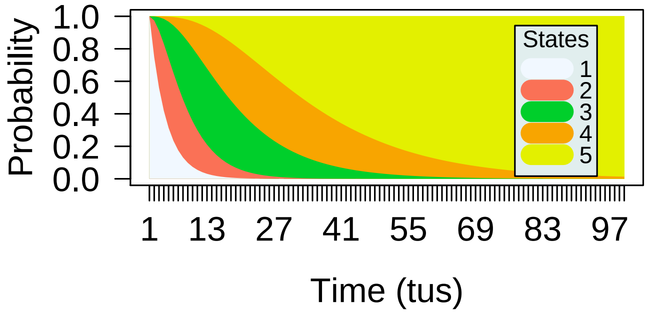


Figure 5.1.: Condition state distribution for the MUSTEM for the deck

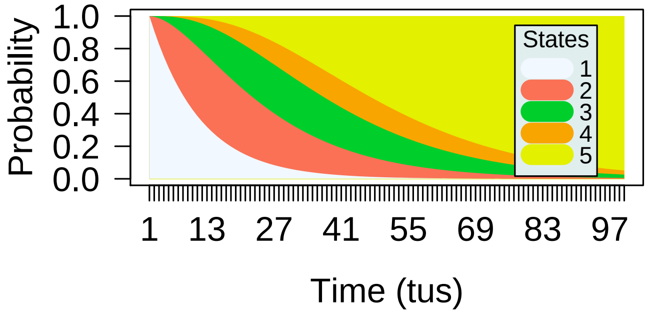


Figure 5.2.: Condition state distribution for the E-MEPI for the deck

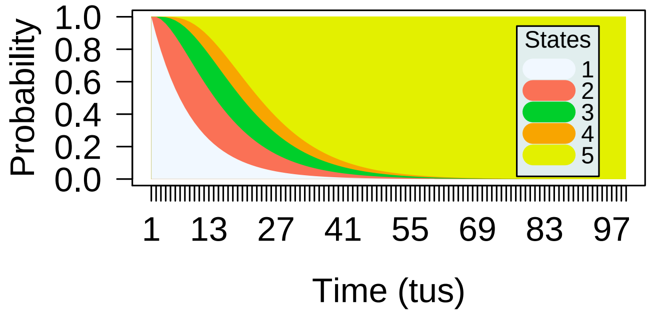


Figure 5.3.: Condition state distribution for the WEAR-OUT for the deck

5.1.2. Transition probability for the different Intervention strategies

In the next step the transition probabilities according to the different intervention strategies for the different objects are calculated. This is due to the fact that the transition probability changes if an intervention is executed, compare Tab. 4.7 where the effectiveness of the different interventions are shown. This means that each intervention strategy has their own transition probability matrix.

Table 5.4.: Transition probabilities Q_i^{De} for the different ISs for the MUSTEM for the deck

		Condition state at $t+1$				
IS	CS	1	2	3	4	5
1	1	0.748	0.226	0.025	0.001	0
	2	0.000	0.811	0.181	0.008	0
	3	0.800	0.200	0	0	0
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
2	1	0.748	0.226	0.025	0.001	0
	2	0	0.811	0.181	0.008	0
	3	0.950	0.050	0	0	0
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
3	1	0.748	0.226	0.025	0.001	0
	2	0	0.811	0.181	0.008	0
	3	0	0	0.914	0.084	0.002
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
4	1	0.748	0.226	0.025	0.001	0
	2	0	0.811	0.181	0.008	0
	3	0	0	0.914	0.084	0.002
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
5	1	0.748	0.226	0.025	0.001	0
	2	0	0.811	0.181	0.008	0
	3	0	0	0.914	0.084	0.002
	4	0	0	0	0.942	0.058
	5	1	0	0	0	0

Table 5.5.: Transition probabilities Q_i^{De} for the different ISs for the E-MEPI for the deck

		Condition state at $t+1$				
IS	CS	1	2	3	4	5
1	1	0.909	0.087	0.004	0	0
	2	0	0.939	0.062	0.000	0
	3	0.800	0.200	0	0	0
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
2	1	0.909	0.087	0.004	0	0
	2	0	0.939	0.062	0	0
	3	0.950	0.050	0	0	0
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
3	1	0.909	0.087	0.004	0	0
	2	0	0.939	0.062	0	0
	3	0	0	0.934	0.059	0.007
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
4	1	0.909	0.087	0.004	0	0
	2	0	0.939	0.062	0	0
	3	0	0	0.934	0.059	0.007
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
5	1	0.909	0.087	0.004	0	0
	2	0	0.939	0.062	0	0
	3	0	0	0.934	0.059	0.007
	4	0	0	0	0.908	0.092
	5	1	0	0	0	0

Table 5.6.: Transition probabilities Q_i^{De} for the different ISs for the WEAR-OUT for the deck

		Condition state at $t+1$				
IS	CS	1	2	3	4	5
1	1	0.889	0.111	0	0	0
	2	0	0.862	0.138	0	0
	3	0.800	0.200	0	0	0
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
2	1	0.889	0.111	0	0	0
	2	0	0.862	0.138	0	0
	3	0.950	0.050	0	0	0
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
3	1	0.889	0.111	0	0	0
	2	0	0.862	0.138	0	0
	3	0	0	0.815	0.185	0
	4	0.750	0.250	0	0	0
	5	1	0	0	0	0
4	1	0.889	0.111	0	0	0
	2	0	0.862	0.138	0	0
	3	0	0	0.815	0.185	0
	4	0.900	0.100	0	0	0
	5	1	0	0	0	0
5	1	0.889	0.111	0	0	0
	2	0	0.862	0.138	0	0
	3	0	0	0.815	0.185	0
	4	0	0	0	0.716	0.284
	5	1	0	0	0	0

5.2. Optimization

5.2.1. Work program on object level

To calculated the work program on object level under consideration of the agency rule, compare section 3.3.3, the input data of Tab. 4.6 is needed. According to this input the program differentiates the two cases:

- There is a probability, for the object, of being in the condition states where interventions are planned but the probability is too small, according to the defined agency rule, and the intervention will not be executed (agency rule not fulfilled).
- There is a probability, for the object, of being in the condition states where interventions are planned and the probability is high enough, according to the defined agency rule, and the intervention will be executed (agency rule fulfilled).

Referring to those two cases the two different transition probabilities (transition probabilities for doing nothing "case 1" (Tab. 5.1, 5.2 and 5.3) or transition probabilities for the different intervention strategies "'case 2" (Tab. 5.4, 5.5 and 5.6)) are used:

$$\pi_j^{t+1} = P \cdot \pi_j^t \tag{5.1}$$

$$\pi_j^{t+1} = Q \cdot \pi_j^t \tag{5.2}$$

where π_j^{t+1} is the state probability at time $t+1$, π_j^t is the state probability at time t, P is the transition probability matrix for "doing nothing" and Q is the transition probability matrix for one of the different intervention strategies.

In the next step the costs for the different intervention strategies over the investigated time period were calculated, according to Tab. 4.7, and the total costs for each intervention strategy.

Table 5.7.: Costs for the different intervention strategies for the deck

	MUSTEM	E-MEPI	WEAR-OUT
IS	Annual cost mu/yr	Annual cost mu/yr	Annual cost mu/yr
1	43'850	18'217	30'975
2	71'204	24'944	42'029
3	26'039	15'254	32'577
4	35'499	18'017	36'773
5	24'241	17'368	38'460

Table 5.8.: Costs for the different intervention strategies for the pier

	MUSTEM	E-MEPI	WEAR-OUT
IS	Annual cost mu/yr	Annual cost mu/yr	Annual cost mu/yr
1	7'260	6'396	8'089
2	6'097	5'360	7'290
3	6'848	4'484	7'691
4	7'300	6'039	9'595

Table 5.9.: Costs for the different intervention strategies for the abutment

	MUSTEM	E-MEPI	WEAR-OUT
IS	Annual cost mu/yr	Annual cost mu/yr	Annual cost mu/yr
1	2'225	1'979	1'469
2	1'408	1'049	1'631

According to the Tab. 5.7 to 5.9 the optimal intervention strategy can be evaluated. It needs to be said that in this work only the costs are considered to determine the optimal intervention strategy.

The optimal intervention strategies for the different models and objects can be seen in Tab. 5.10.

Table 5.10.: Optimal intervention strategies for the different objects

	MUSTEM		E-MEPI		WEAR-OUT	
Object	IS	Annual cost mu/yr	IS	Annual cost mu/yr	IS	Annual cost mu/yr
Deck	5	24'241	3	15'254	1	30'675
Pier	2	6'097	3	4'484	2	7'290
Abutment	2	1'408	2	1'049	1	1'469
Total		**31'746**		**20'787**		**39'434**

The costs distribution over the investigated time period for the different objects can be seen in appendix D.1. The costs for the hole bridge for the three different approaches can be seen in Fig. 5.4 to 5.6.

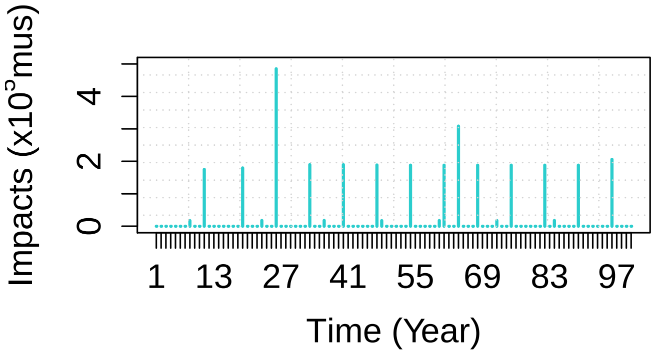


Figure 5.4.: Costs distribution for the OIS's (MUSTEM)

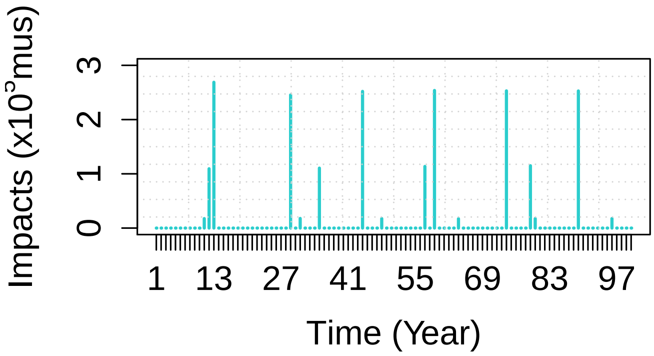


Figure 5.5.: Costs distribution for the OIS's (E-MEPI)

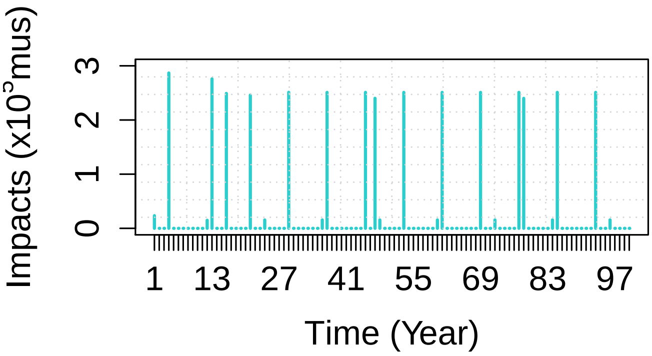


Figure 5.6.: Costs distribution for the OIS's (WEAR-OUT)

The condition state distribution for the different intervention strategies can also be seen in appendix D.2.

5.2.2. Work program based on bridge level

In this section the optimal work program on bridge level, with agency rule and discount factor, is built. The discount factor is a factor which considers the fact that the intervention costs are based on a fix part and a variable part which depends on the intervention type. If now two objects or more have an intervention in the same year the fix part of the costs can be shared and it results a kind of discount on the total costs for the interventions. This is considered in the optimization program with the discount factor, Tab. 5.11.

Table 5.11.: Discount factor for the bridge

Interventions in the same year	Discount factor
1	0.00
2	0.20
3	0.30

Table 5.12.: all possible work programs (1-20)

	Intervention strategy			MUSTEM	E-MEPI	WEAR-OUT
Work program	Deck	Pier	Abutment	annual costs mu/yr		
1	1	1	1	51’166	25’255	39’369
2	1	1	2	49’922	24’328	39’856
3	1	2	1	50’787	24’102	*39’434*
4	1	2	2	49’961	23’641	39’057
5	1	3	1	50’316	23’960	39’113
6	1	3	2	50’021	23’750	**38’859**
7	1	4	1	49’687	24’677	41’060
8	1	4	2	48’989	24’744	40’534
9	2	1	1	77’553	31’855	49’549
10	2	1	2	76’182	31’924	50’575
11	2	2	1	77’313	30’818	50’788
12	2	2	2	76’499	31’352	50’950
13	2	3	1	76’703	28’764	49’218
14	2	3	2	76’282	29’550	49’124
15	2	4	1	75’123	30’153	47’821
16	2	4	2	74’569	31’216	47’833
17	3	1	1	33’288	23’070	41’271
18	3	1	2	33’836	22’233	41’712
19	3	2	1	32’519	22’034	40’312
20	3	2	2	33’068	21’662	39’893

Table 5.13.: all possible work programs (21-40)

	Intervention strategy			MUSTEM	E-MEPI	WEAR-OUT
Work program	Deck	Pier	Abutment	annual costs mu/yr		
21	3	3	1	32'035	20'906	40'972
22	3	3	2	33'103	***20'787***	40'297
23	3	4	1	33'032	22'184	41'006
24	3	4	2	33'432	22'342	40'785
25	4	1	1	42'138	25'755	45'467
26	4	1	2	43'133	24'996	45'843
27	4	2	1	41'368	24'718	44'438
28	4	2	2	42'366	24'425	43'950
29	4	3	1	40'804	23'591	45'098
30	4	3	2	42'239	23'550	44'354
31	4	4	1	41'721	24'869	44'924
32	4	4	2	42'568	25'105	44'668
33	5	1	1	31'809	23'492	45'942
34	5	1	2	31'363	22'815	46'138
35	5	2	1	30'807	23'238	46'601
36	5	2	2	*30'363*	23'026	46'134
37	5	3	1	31'640	22'111	47'202
38	5	3	2	32'084	22'215	46'278
39	5	4	1	27'717	23'389	47'136
40	5	4	2	**27'127**	23'706	46'586

In Tab. 5.12 and 5.13 are all possible work programs listed with their annual costs. It can be seen that in two of the three models the optimal work program based on the discount factor (**fat in the table**) is not the same one like if only the optimal intervention strategies were combined (*cursive in the table*). In Fig. 5.7 to 5.9 the distribution of the costs for the optimal work program is illustrated.

In appendix E.1 are the costs distribution for the objects as well as the condition state distribution shown.

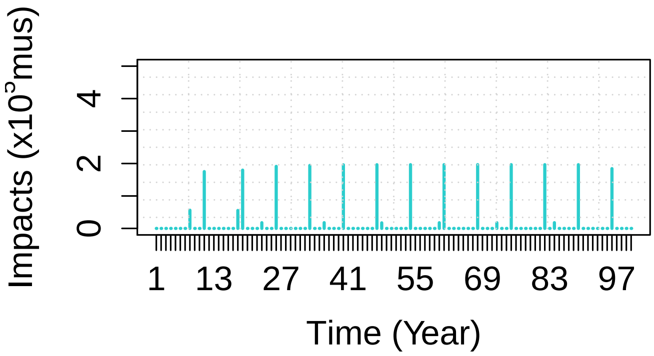


Figure 5.7.: Costs distribution for the OWP (MUSTEM)

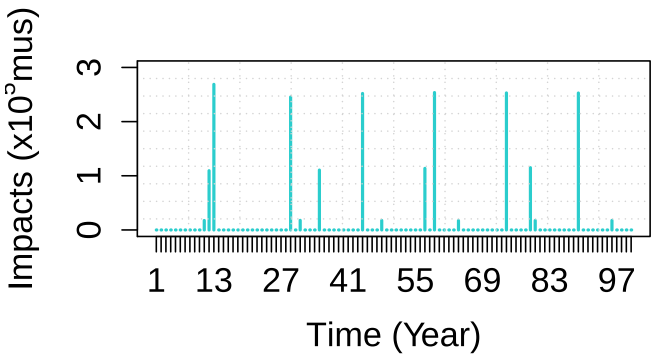


Figure 5.8.: Costs distribution for the OWP (E-MEPI)

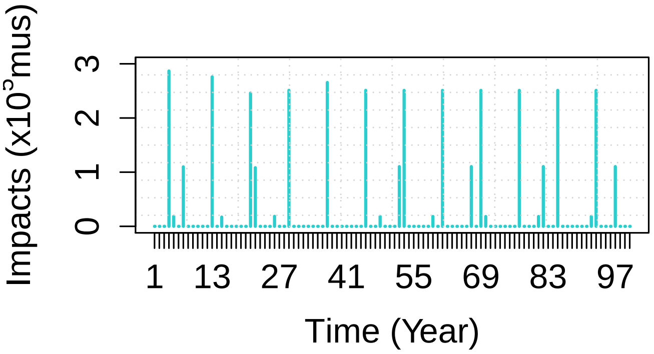


Figure 5.9.: Costs distribution for the OWP (WEAR-OUT)

The condition state distribution for the different intervention strategies can also be seen in appendix E.2.

6. Discussion

Table 6.1.: Results

Model	Level	Intervention strategy			annual costs mu/yr
		Deck	Pier	Abutment	
MUSTEM	Object	5	2	2	31'745
	Bridge	5	4	2	27'127
E-MEPI	Object	3	3	2	20'787
	Bridge	3	3	2	20'787
WEAR-OUT	Object	1	2	1	39'434
	Bridge	1	3	2	38'859

The input data from section 4.3 for the three deterioration models differentiate in some case. The input data for the MUSTEM model is simple. It is just the hazard rates for the object. With only this model specific input the hole work program can be calculated.

For the E-MEPI model much more information are needed. There are eight different parameters for the initiation phase and another ten for the propagation phase. As well as the standard deviation and the type of distribution has to be described, which has to be known for each of the parameters. This leads to the fact, that for the E-MEPI models lots of data has to be collected to predict an accurate work program.

The simplest model, according to the input data, is the WEAR-OUT model. This model needs just the expected life time of the object to calculate its transition probability matrix. But it can be said that the WEAR-OUT model should not be used in practice due its simplification in the estimation of the transition probability matrix. Like mentioned before the transition probability matrix of the WEAR-OUT model only considers unit jumps, which is due to the minimization problem. The fact that only unit jumps where considers leads to a simplification of the reality. In other words can be said that the WEAR-OUT model does not consider uncertainties of nature. The WEAR-OUT model can be used to get a rough idea about the costs of a bridge over an investigated time period, but it is not accurate enough to make founded predictions.

If we look at the different optimal work programs for the MUSTEM and E-MEPI models we can see that they differentiate in costs. The OWP for the MUSTEM has annual costs of $27'127 mu/yr$ and the E-MEPI of $20'787 mu/yr$, compare Tab. 5.13. This is a difference of about 30%. To explain this difference in costs we have to look at the different deterioration processes of the two models. In Fig. 6.1 can we see the deterioration process of the deck for the MUSTEM model and in Fig. 6.2 the deterioration process for the E-MEPI model.

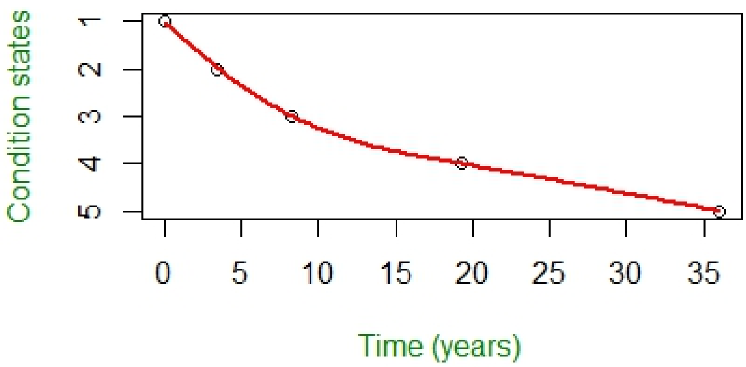


Figure 6.1.: Deterioration curve for the deck (MUSTEM)

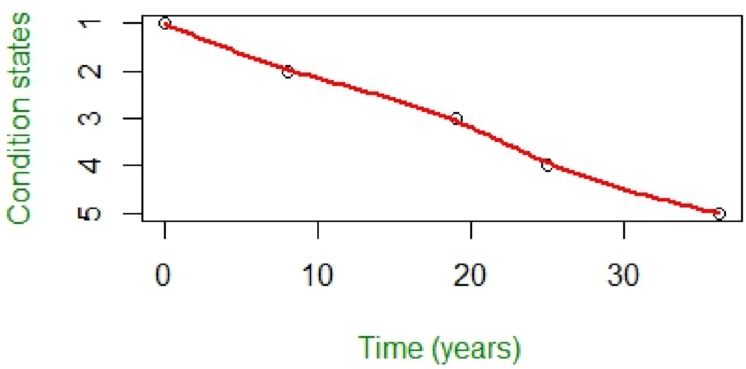


Figure 6.2.: Deterioration curve for the deck (E-MEPI)

We can see that the two deterioration processes are different, which leads to the difference in the annual costs. The deterioration curve for the MUSTEM model (Fig. 6.1) has an exponential process but the E-MEPI deteriorates more or less linear. Those two deterioration processes are the results of the two different deterioration models. The life time of the object is more or less the same for both deterioration models, about 36 years. The big difference in the two deterioration curves is the process. The exponential process for the MUSTEM model leads to the fact, that the objects, compared to the E-MEPI, relatively quick moves through the first two condition states but then stays relatively long in condition state three and four before getting into condition state five. For the E-MEPI the development of the condition states is relatively constant, which means that the object spends more or less the same amount of time in each condition state before getting into condition state five. This leads to the fact that for the MUSTEM model more interventions have to be executed because the objects gets faster into weak condition states. According to this case can be said that it is not only important for an infrastructure manger to know about the life time of an object it is also very important to know about its deterioration process. It cannot be said that if an object has a life time of about 35 years that this specific intervention strategy is the best. The deterioration process has always to be taken into account.

If we look at the benefit of the consideration of bridge level for estimating the optimal work program can be seen that the benefit is not the same for all models. It depends on the costs-ratio between the different intervention strategies. It can be seen that for the E-MEPI there is no difference between the two levels. Which means that the optimal interventions strategies for the different objects are much cheaper than the others and it do not make sense to change them even if costs can be saved. On the other hand for the MUSTEM model the bridge level OWP is about 15% cheaper than the OWP on object level.

./figures/obowp.jpg

Figure 6.3.: optimal work program on object level (MUSTEM)

./figures/browp.jpg

Figure 6.4.: optimal work program on bridge level (MUSTEM)

This can be explained through Fig. 6.3 and 6.4. It can be seen that on object level for the pier only two big interventions are planned over the investigated time period. But under consideration of the bridge level the intervention strategy for the pier changed and now many smaller interventions are planned. They mostly take place when an intervention on the deck is planned as well, which means that the costs for those interventions can partially be shared. Due to the change in the intervention strategy for the pier the optimal work program on bridge level has lower annual costs than the work program on object level.

For the E-MEPI model can be said that it is the most ornate model out of those three. It needs a huge amount of input data. It is not enough to have only one value for each corrosion parameter. It has to know its mean value and standard deviation as well as the statistical distribution to describe each parameter by itself. It has the advantage that the input parameters have a physical meaning. And this fact makes it easier, in my opinion, to understand the hole process behind the model. It helps to avoid rough errors in terms of wrong understanding of the input parameters, because the infrastructure manager has at least kind of a feeling in which scale the different parameters should be in.

The MUSTEM model on the other hand has the big advantage of the small amount of input parameters, which makes the model clear for the infrastructure manager. An other big advantage of the MUSTEM model is that it is an input driven model, which means that at any time new information can be added to the model to make it more sophisticated.

It has to be mentioned that for an optimal work program not only the costs can be considered. It could be sometimes the case that for example the condition state distribution is a criterion for the estimation of the optimal work program and not only the costs. The criterion for the optimal work program have always to be adopted to the boundary conditions. This means that if perhaps the infrastructure mangers has enough money but has an infrastructure which has to have in terms of use most of the time an acceptable condition state because if the infrastructure fails the consequences are enormous (i.e. for a dam).

As a last point I would like to mention that in the beginning of this master thesis it was planned to calculate the optimal work program once in the Markov world and once in the mechanistic-empirical world (E-MEPI), without changing into the Markov world after calculating the Markov transition probability matrix. But it was relatively quick clear that this is not possible or lets say not very easy to do. The E-MEPI model uses, like mentioned in the thesis, Monte Carlo simulations to estimate the state probability over the investigated time period. If the object would always get back into like new conditions after an intervention is done the same Monte Carlo simulations could be used after the intervention to estimate the state probability of the object until the next intervention. But due to the fact of the effectiveness matrix, which tells us that an object does not always get back into like new conditions, it is not possible to use the same Monte Carlos simulations. The fact the object could as well get into condition state two leads to the case that an other Monte Carlo simulation has to be done for those objects which are getting back into condition state two and in the end the different Monte Carlo simulations should be combined to get the state probability until the next interventions is executed. This fact makes it very hard to develop a computer supported optimization model which is not in the Markov world.

7. Conclusion

First of all it has to be mentioned that according to this study infrastructure managers should get kind of a guideline when and under which circumstances each of the three different deterioration models can be used. It has been done comparisons between the three models to discuss their advantages and disadvantages.

There are two big outcomes of this master thesis. The first output is the development of a computer based program, in the R environment, to estimate the transition probability from proportions data (output of the E-MEPI model). The second output is the development of a optimization model to determine the optimal work program which considers both object and bridge level interventions.

The comparison of the three models showed those two main points about the three different models to calculate the deterioration process of a concrete bridge:

- It can be said that the WEAR-OUT model shouldn't be used in practice due its simplifications.
- The last point is that both, the MUSTEM model and the E-MEPI model, can be used with a thoughtful consideration of the availability of inspection data (MUSTEM) and the assumptions used to represent the probabilistic distributions of parameters (E-MEPI). It really depends on the information an infrastructure manager has available which model should be used.

Reference

B. T. Adey and R. Hajdin. Methodology for determination of financial needs of gradually deteriorating bridges with only structure level data. *Structure and Infrastructure Engineering*, 7(7-8):645–660, 2011. doi: 10.1080/15732479.2010.501568. URL http://dx.doi.org/10.1080/15732479.2010.501568.

Bryan Adey and Rade Hajdin. Potential use of inventory theory to bundle interventions in bridge management systems. *Transportation Research Record: Journal of the Transportation Research Board*, 1933(-1):44–49, January 2005. URL http://dx.doi.org/10.3141/1933-06.

C. Andrade, C. Alonso, and F.J. Molina. Cover cracking as a function of bar corrosion: Part i-experimental test. *Materials and Structures*, 26(8):453–464, 1993. ISSN 1359-5997. doi: 10.1007/BF02472805. URL http://dx.doi.org/10.1007/BF02472805.

G. Brodsky, R. Muzykin, E. S. Brodskaia, Yu. A. Ponomarev, Yu. A. Yenyutin, and M. S. Vlasova. Analysis of parameters of structure deterioration models within the moscow bridge management system. *Structure and Infrastructure Engineering*, 2(1):13–21, 2006. doi: 10.1080/15732470500030950. URL http://www.tandfonline.com/doi/abs/10.1080/15732470500030950.

J.G. Cabrera. Deterioration of concrete due to reinforcement steel corrosion. *Cement and Concrete Composites*, 18(1):47 – 59, 1996. ISSN 0958-9465. doi: http://dx.doi.org/10.1016/0958-9465(95)00043-7. URL http://www.sciencedirect.com/science/article/pii/0958946595000437.

DuraCrete. *Statistical Quantification of the Variables in the Limit State Functions*. The European Union - Brite EuRam III, 2000. ISBN 9789037603743. URL https://books.google.ch/books?id=eiEtQAAACAAJ.

Dilum Fernando, Bryan T. Adey, and Nam Lethanh. A model for the evaluation of intervention strategies for bridges affected by manifest and latent deterioration processes. *Structure and Infrastructure Engineering*, 0(0):1–18, 2014. doi: 10.1080/15732479.2014.976576. URL http://dx.doi.org/10.1080/15732479.2014.976576.

Z. Louins George Morcous. Probabilistic and mechanistic deterioration models for bridge management. *Computing in Civil Engineering*, 2007.

Bryan Adey Eugen Brhwiler Guido Roelfstra, Rade Hajdin. Condition evolution in bridge management systems and corrosion-induced deterioration. *Journal of Bridge Engineering*, 2004.

Rade Hajdin. Bms development in switzerland. *Advanced Technology in Structural Engineering*, 2000.

Rade Hajdin. Kuba-ms: The swiss bridge management system. *Structure 2001*, 2001.

Andrew K.S. Jardine, Daming Lin, and Dragan Banjevic. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical Systems and Signal Processing*, 20(7):1483 – 1510, 2006. ISSN 0888-3270. doi: http://dx.doi.org/10.1016/j.ymssp.2005.09.012. URL http://www.sciencedirect.com/science/article/pii/S0888327005001512.

Matthew T. Jones. Estimating markov transition matrices using proportions data: An application to credit risk. *International Monetary Fund*, 2005.

Trevor J Kirkpatrick, Richard E Weyers, Christine M Anderson-Cook, and Michael M Sprinkel. Probabilistic model for the chloride-induced corrosion service life of bridge decks. *Cement and Concrete Research*, 32(12):1943 – 1960, 2002. ISSN 0008-8846. doi: http://dx.doi.org/10.1016/S0008-8846(02)00905-5. URL http://www.sciencedirect.com/science/article/pii/S0008884602009055.

Nam Lethanh, Bryan T. Adey, and Dilum N. Fernando. Optimal intervention strategies for multiple objects affected by manifest and latent deterioration processes. *Structure and Infrastructure Engineering*, 11(3):389–401, 2015. doi: 10.1080/15732479.2014.889178. URL http://dx.doi.org/10.1080/15732479.2014.889178.

F.J. Molina, C. Alonso, and C. Andrade. Cover cracking as a function of rebar corrosion: Part 2??numerical model. *Materials and Structures*, 26(9):532–548, 1993. ISSN 1359-5997. doi: 10.1007/BF02472864. URL http://dx.doi.org/10.1007/BF02472864.

Yoshitane TSUDA, Kiyoyuki KAITO, Kazuya AOKI, and Kiyoshi KOBAYASHI. Estimating markovian transition probabilities for bridge deterioration forecasting. *STRUCTURAL ENGINEERING / EARTHQUAKE ENGINEERING*, 23(2):241s–256s, 2006. doi: 10.2208/jsceseee.23.241s.

K. William A. Attar Yuan Zhou, Bora Gencturk. Carbonation-induced and chloride-induced corrosion in reinforced concrete structures. *Journal of Materials in Civil Engineering*, 2014.

A. General input

A.1. Condition state definition

Table A.1.: Condition state definition for the pier

CS	Description	Indicator	Criterion
1	New/Partially new	chloride content in the concrete at reinforcement bar level C_{cl}	$0 < C_{cl} \leq 0.48$
2	Corrosion has initiated, no visible cracking has occurred	Width of cracks (w)	$C_{cl} > 0.48$, $w \leq 0.25$
3	Visible cracking has occurred	Width of cracks (w)	$0.25 < w \leq 0.50$
4	Visible cracking has occurred and cover has spelled off	Width of cracks (w)	$w > 0.50$

Table A.2.: Condition state definition for the abutment

CS	Description	Indicator	Criterion
1	New/Partially new	chloride content in the concrete at reinforcement bar level C_{cl}	$0 < C_{cl} \leq 0.48$
2	Corrosion has initiated, no visible cracking has occurred	Width of cracks (w)	$C_{cl} > 0.48$, $w \leq 0.25$
3	Visible cracking has occurred and cover has spelled off	Width of cracks (w)	$w > 0.50$

A.2. Intervention strategies

Table A.3.: Intervention strategy matrix with agency rule I^{Pi} for the pier

	Condition state					
Intervention strategy	1	2	3	4	AR	BC
1	Do nothing	Rehabilitation	Rehabilitation	Replacement	0.40	3
2	Do nothing	Rehabilitation	Renewal	Replacement	0.30	4
3	Do nothing	Do nothing	Rehabilitation	Replacement	0.15	4
4	Do nothing	Do nothing	Do nothing	Replacement	0.10	4

Table A.4.: Intervention strategy matrix with agency rule I^{Ab} for the abutment

	Condition state				
Intervention strategy	1	2	3	AR	BC
1	Do nothing	Rehabilitation	Replacement	0.25	2
2	Do nothing	Do nothing	Replacement	0.15	3

A.3. Effectiveness and cost

Table A.5.: Effectiveness and unit cost matrix R^{Pi} for the pier

		Effectiveness				
CS	Interventions	1	2	3	4	Unit costs mu/m^2
2	Rehabilitation	0.85	0.15	0	0	240
	Renewal	0.95	0.05	0	0	420
	Replacement	1	0	0	0	800
3	Rehabilitation	0.80	0.20	0	0	280
	Renewal	0.90	0.10	0	0	460
	Replacement	1	0	0	0	800
4	Rehabilitation	0.75	0.25	0	0	320
	Renewal	0.85	0.15	0	0	500
	Replacement	1	0	0	0	800

Table A.6.: Effectiveness and unit cost matrix R^{Ab} for the abutment

		Effectiveness			
CS	Interventions	1	2	3	Unit costs mu/m^2
2	Rehabilitation	0.85	0.15	0	420
	Replacement	1	0	0	1200
3	Rehabilitation	0.80	0.20	0	440
	Replacement	1	0	0	1200

B. Model specific input

B.1. The MUSTEM model

Table B.1.: Hazard rates for the pier

CS	Hazard rate θ_i
1	0.07
2	0.05
3	0.04
4	0.00

Table B.2.: Hazard rates for the abutment

CS	Hazard rate θ_i
1	0.05
2	0.02
3	0.00

B.2. The E-MEPI model

Table B.3.: Input parameters for the initiation phase of the pier for the E-MEPI model

	d	D_0	C_s	k_e	k_t	k_c	t_0	n
μ	20	220.9	2.4	0.265	0.832	1	0.0767	0.37
σ	–	28.5	0.2	0.045	0.024	–	–	0.07
D	discrete	normal	normal	gamma	normal	discrete	discrete	beta

Table B.4.: Input parameters for the propagation phase of the pier for the E-MEPI model

	w_0	β	V_0	wet	α	a_1	a_2	a_3	ϕ
μ	0.05	0.01	0.2	0.75	9.28	74.4	7.3	-17.4	20
σ	0.005	0.001	0.04	0.2	4.04	5.7	0.06	5.7	–
D	normal	normal	normal	normal	normal	normal	normal	normal	discrete

Table B.5.: Input parameters for the initiation phase of the abutment for the E-MEPI model

	d	D_0	C_s	k_e	k_t	k_c	t_0	n
μ	20	220.9	2.4	0.265	0.832	1	0.0767	0.37
σ	–	28.5	0.2	0.045	0.024	–	–	0.07
D	discrete	normal	normal	gamma	normal	discrete	discrete	beta

Table B.6.: Input parameters for the propagation phase of the abutment for the E-MEPI model

	w_0	β	V_0	wet	α	a_1	a_2	a_3	ϕ
μ	0.05	0.01	0.15	0.75	9.28	74.4	7.3	-17.4	20
σ	0.005	0.001	0.04	0.2	4.04	5.7	0.06	5.7	–
D	normal	normal	normal	normal	normal	normal	normal	normal	discrete

C. Deterioration model

C.1. The MUSTEM model

C.1.1. Transition probabilities

Table C.1.: Transition probability P^{Pi} for the MUSTEM for the pier

	Condition state at $t+1$			
CS	1	2	3	4
1	0.932	0.066	0.002	0
2		0.951	0.048	0.001
3			0.961	0.039
4				1

Table C.2.: Transition probability P^{Ab} for the MUSTEM for the abutment

	Condition state at $t+1$		
CS	1	2	3
1	0.951	0.048	0
2		0.980	0.020
3			1

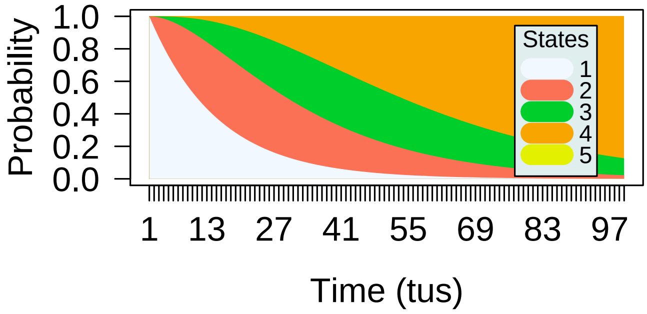


Figure C.1.: Condition state distribution for the MUSTEM for the pier

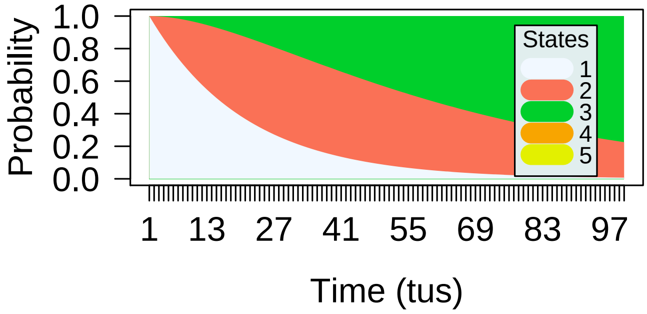


Figure C.2.: Condition state distribution for the MUSTEM for the abutment

C.1.2. Transition probability for the different Intervention strategies

Table C.3.: Transition probability Q_i^{Pi} for the different IS's for the MUSTEM for the pier

		Condition state at $t+1$			
IS	CS	1	2	3	4
1	1	0.932	0.066	0.002	0
	2	0.850	0.150	0	0
	3	0.800	0.200	0	0
	4	1	0	0	0
2	1	0.932	0.066	0.002	0
	2	0.850	0.150	0	0
	3	0.900	0.100	0	0
	4	1	0	0	0
3	1	0.932	0.066	0.002	0
	2	0	0.951	0.048	0.001
	3	0.800	0.200	0.000	0
	4	1	0	0	0
4	1	0.932	0.066	0.002	0
	2	0	0.951	0.048	0.001
	3	0	0	0.961	0.039
	4	1	0	0	0

Table C.4.: Transition probability Q_i^{Ab} for the different IS's for the MUSTEM for the abutment

		Condition state at $t+1$		
IS	CS	1	2	3
1	1	0.951	0.048	0
	2	0.850	0.150	0
	3	1	0	0
2	1	0.951	0.048	0
	2	0	0.980	0.020
	3	1	0	0

C.2. The E-MEPI model

C.2.1. Transition probabilities

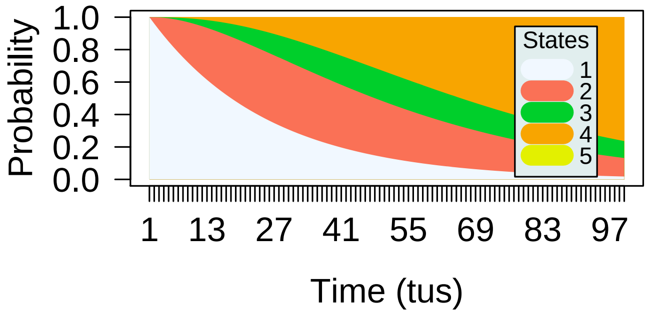


Figure C.3.: Condition state distribution for the pier

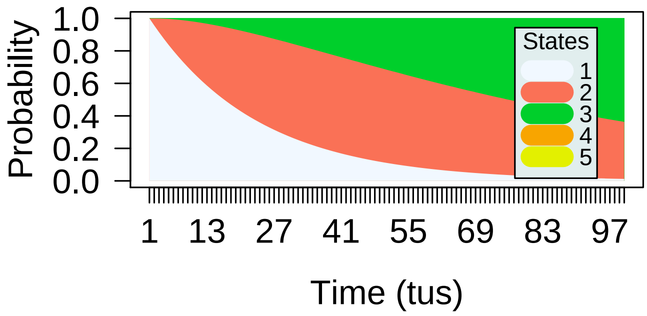


Figure C.4.: Condition state distribution for the abutment

Table C.5.: Transition probabilities P^{Pi} for the E-MEPI for the pier

	Condition state at $t+1$			
CS	1	2	3	4
1	0.960	0.040	0.000	0
2		0.968	0.027	0.005
3			0.951	0.049
4				1

Table C.6.: Transition probabilities P^{Ab} for the E-MEPI for the abutment

	Condition state at $t+1$		
CS	1	2	3
1	0.956	0.044	0
2		0.986	0.014
3			1

C.2.2. Transition probability for the different Intervention strategies

Table C.7.: Transition probability Q_i^{Pi} for the different IS's for the E-MEPI for the pier

		Condition state at $t+1$			
IS	CS	1	2	3	4
1	1	0.960	0.040	0	0
	2	0.850	0.150	0	0
	3	0.800	0.200	0	0
	4	1	0	0	0
2	1	0.960	0.040	0	0
	2	0.850	0.150	0	0
	3	0.900	0.1	0	0
	4	1	0	0	0
3	1	0.960	0.040	0	0
	2	0	0.968	0.027	0.005
	3	0.800	0.200	0	0
	4	1	0	0	0
4	1	0.960	0.040	0	0
	2	0	0.968	0.027	0.005
	3	0	0	0.951	0.049
	4	1	0	0	0

Table C.8.: Transition probability Q_i^{Ab} for the different IS's for the E-MEPI for the abutment

		Condition state at $t+1$		
IS	CS	1	2	3
1	1	0.956	0.044	0
	2	0.850	0.150	0
	3	1	0	0
2	1	0.956	0.044	0
	2	0	0.986	0.014
	3	1	0	0

C.3. The WEAR-OUT model

Transition probabilities

Table C.9.: Transition probability P^{Pi} for the WEAR-OUT for the pier

	Condition state at $t+1$			
CS	1	2	3	4
1	0.964	0.036	0	0
2		0.935	0.065	0
3			0.888	0.112
4				1

Table C.10.: Transition probability P^{Ab} for the WEAR-OUT for the abutment

	Condition state at $t+1$		
CS	1	2	3
1	0.977	0.023	0
2		0.948	0.052
3			1

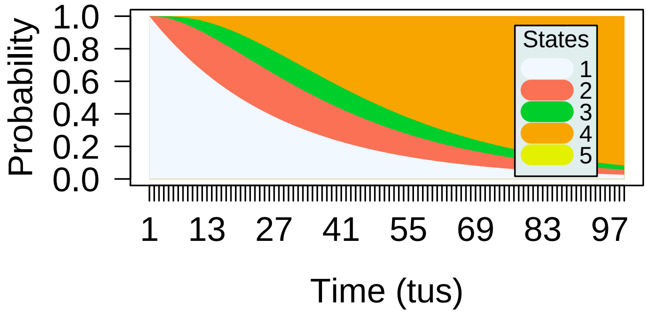


Figure C.5.: Condition state distribution for the WEAR-OUT for the pier

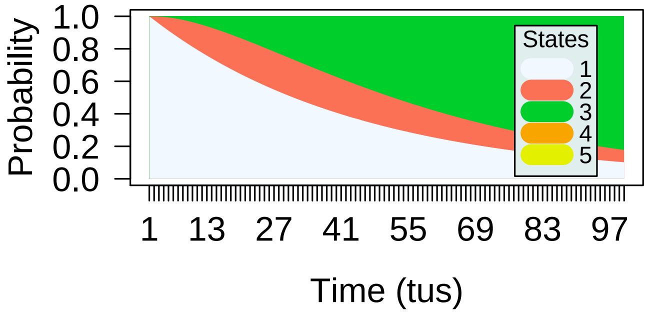


Figure C.6.: Condition state distribution for the WEAR-OUT for the abutment

C.3.1. Transition probability for the different Intervention strategies

Table C.11.: Transition probability Q_i^{Pi} for the different IS's for the WEAR-OUT for the pier

		Condition state at $t+1$			
IS	CS	1	2	3	4
1	1	0.964	0.036	0	0
	2	0.850	0.150	0	0
	3	0.800	0.200	0	0
	4	1	0	0	0
2	1	0.964	0.036	0	0
	2	0.850	0.150	0	0
	3	0.900	0.100	0	0
	4	1	0	0	0
3	1	0.964	0.036	0	0
	2	0	0.935	0.065	0
	3	0.800	0.200	0.000	0
	4	1	0	0	0
4	1	0.964	0.036	0	0
	2	0	0.935	0.065	0
	3	0	0	0.888	0.112
	4	1	0	0	0

Table C.12.: Transition probability Q_i^{Ab} for the different IS's for the WEAR-OUT for the abutment

		Condition state at $t+1$		
IS	CS	1	2	3
1	1	0.977	0.023	0
	2	0.850	0.150	0
	3	1	0	0
2	1	0.977	0.023	0
	2	0	0.948	0.052
	3	1	0	0

D. Optimization model on object level level

D.1. Costs

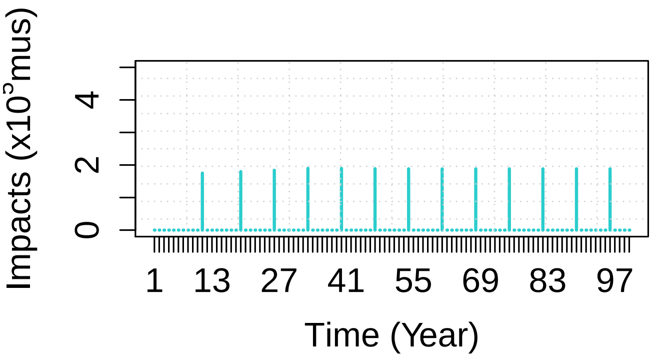


Figure D.1.: Costs distribution for the deck (MUSTEM)

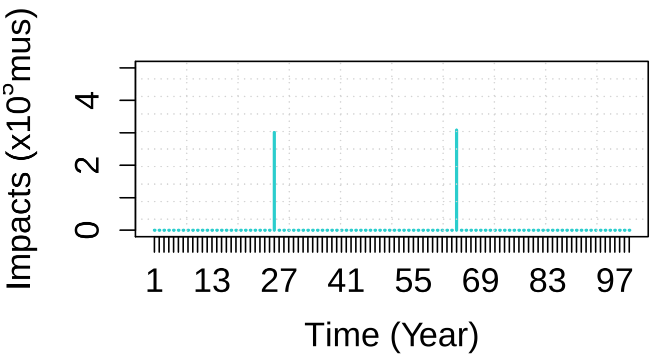


Figure D.2.: Costs distribution for the pier (MUSTEM)

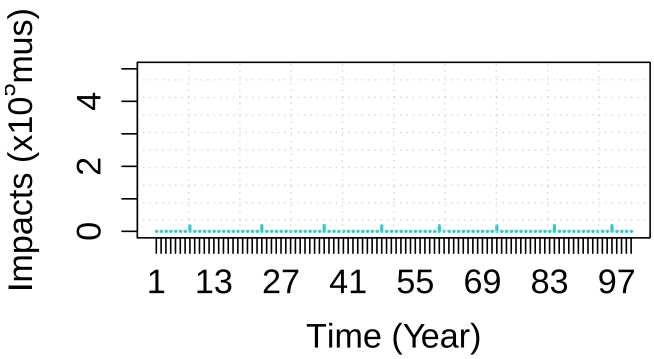


Figure D.3.: Costs distribution for the abutment (MUSTEM)

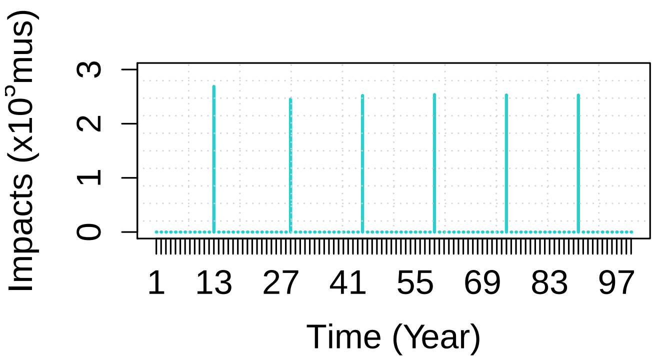


Figure D.4.: Costs distribution for the deck (E-MEPI)

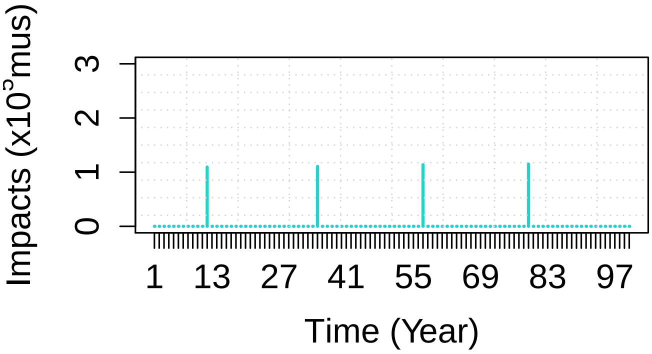


Figure D.5.: Costs distribution for the pier (E-MEPI)

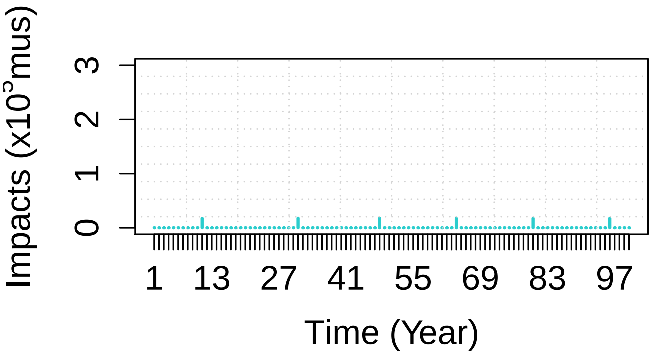


Figure D.6.: Costs distribution for the abutment (E-MEPI)

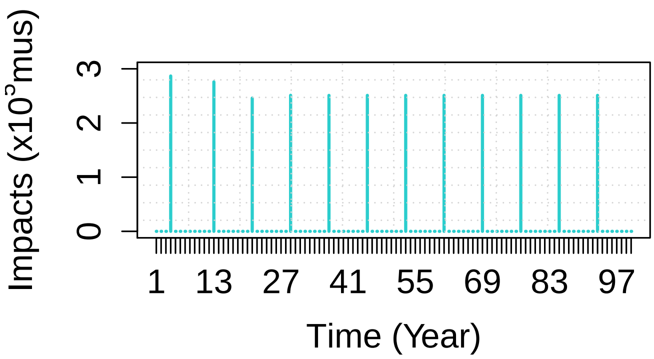


Figure D.7.: Costs distribution for the deck (WEAR-OUT)

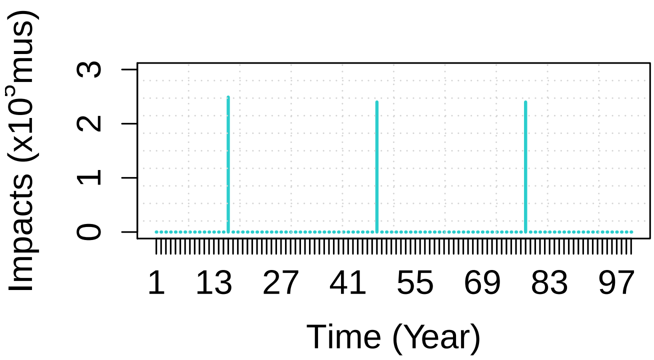


Figure D.8.: Costs distribution for the pier (WEAR-OUT)

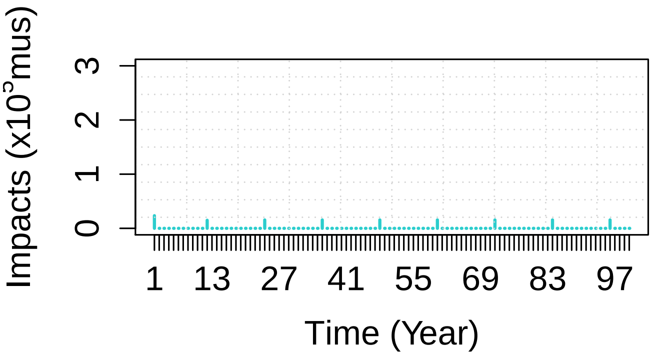


Figure D.9.: Costs distribution for the abutment (WEAR-OUT)

D.2. Condition state

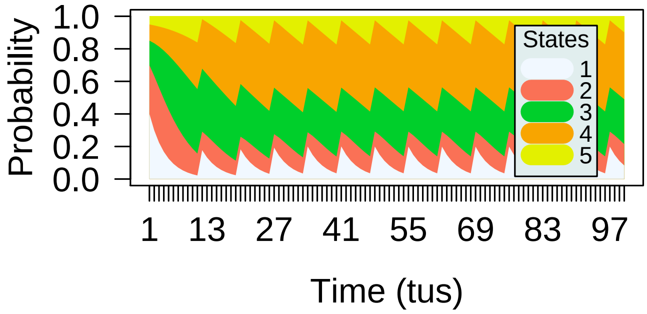


Figure D.10.: Condition state distribution for the deck (MUSTEM)

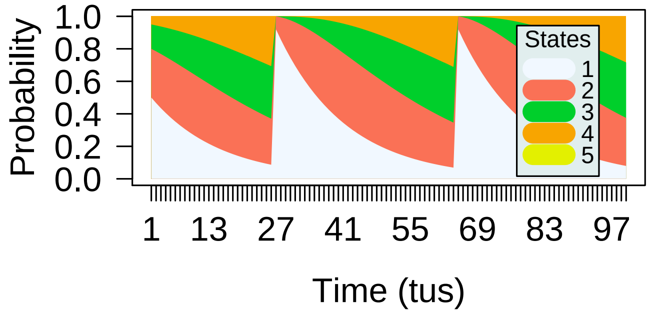


Figure D.11.: Condition state distribution for the pier (MUSTEM)

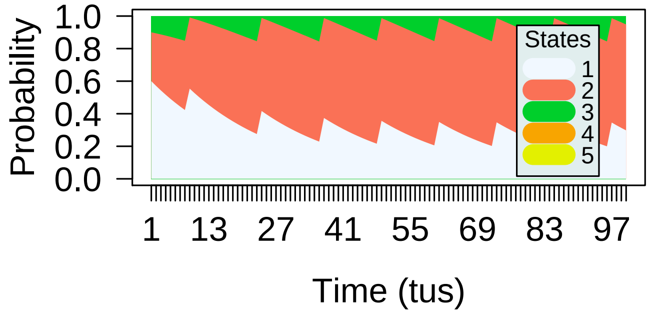


Figure D.12.: Condition state distribution for the abutment (MUSTEM)

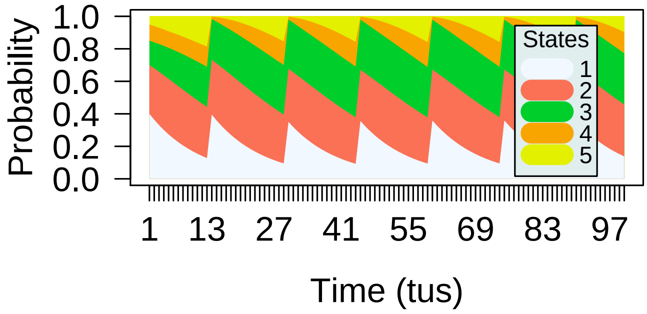


Figure D.13.: Condition state distribution for the deck (E-MEPI)

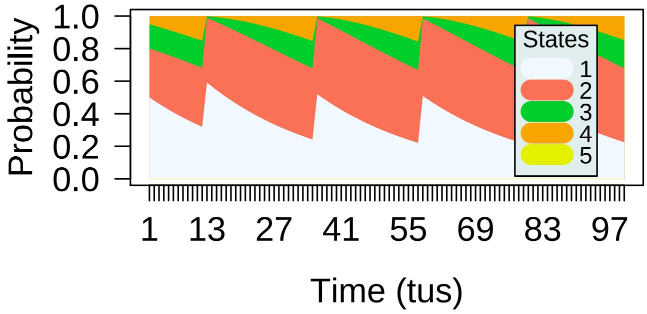


Figure D.14.: Condition state distribution for the pier (E-MEPI)

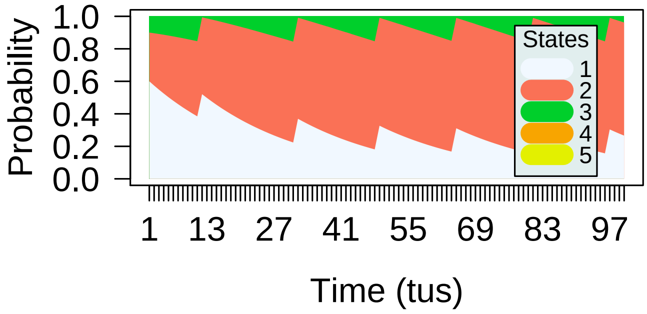


Figure D.15.: Condition state distribution for the abutment (E-MEPI)

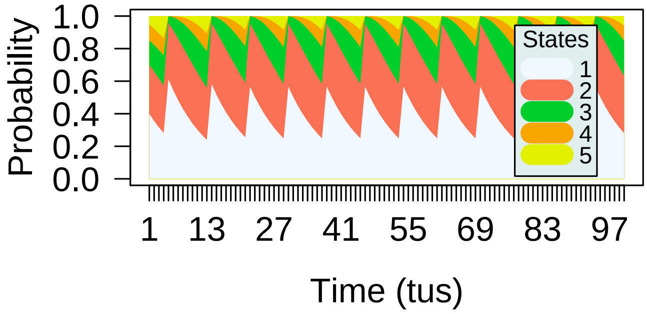


Figure D.16.: Condition state distribution for the deck (WEAR-OUT)

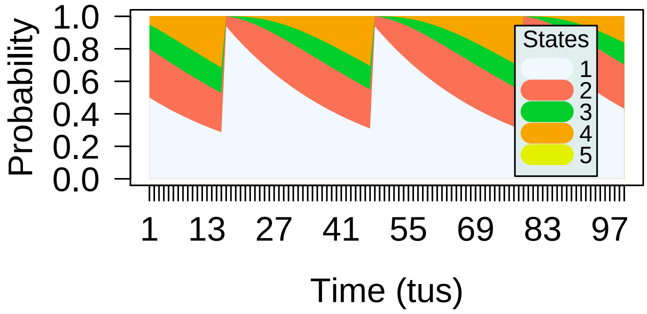


Figure D.17.: Condition state distribution for the pier (WEAR-OUT)

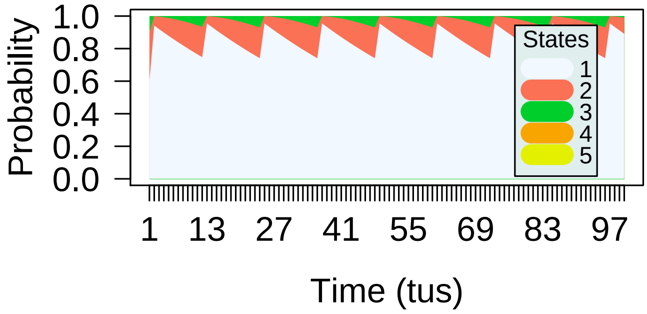


Figure D.18.: Condition state distribution for the abutment (WEAR-OUT)

E. Optimization model on bridge level

E.1. Costs

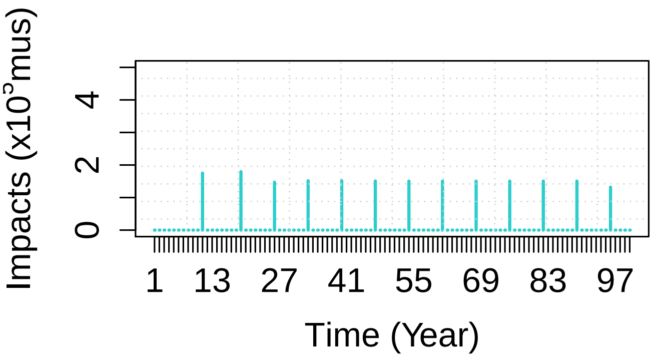


Figure E.1.: Costs distribution for the deck (MUSTEM)

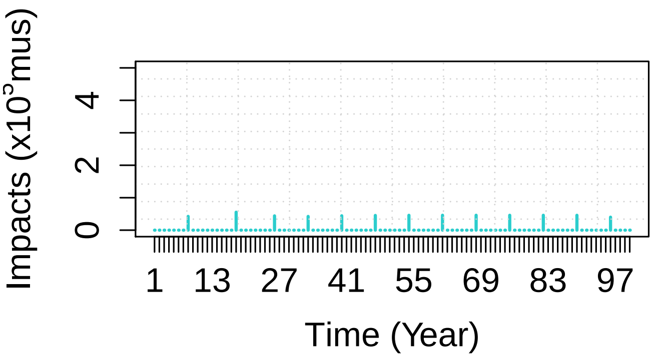


Figure E.2.: Costs distribution for the pier (MUSTEM)

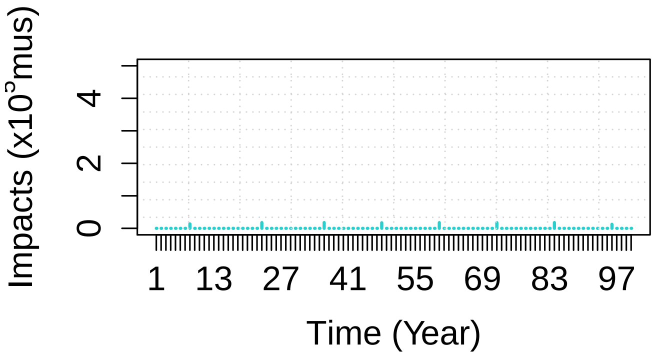


Figure E.3.: Costs distribution for the abutment (MUSTEM)

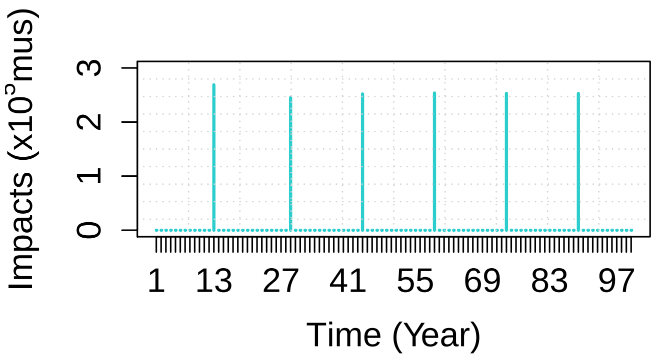


Figure E.4.: Costs distribution for the deck (E-MEPI)

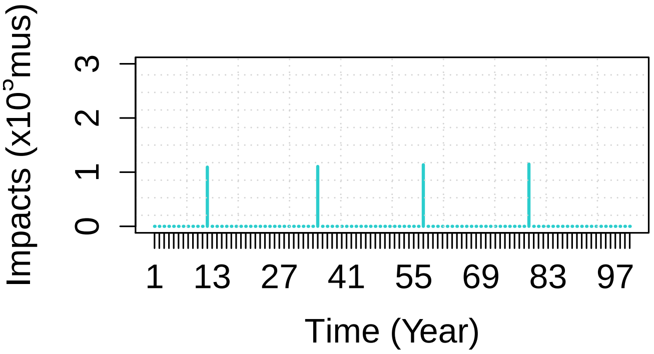


Figure E.5.: Costs distribution for the pier (E-MEPI)

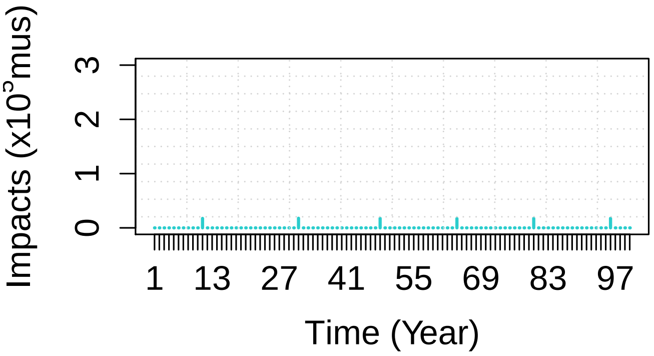


Figure E.6.: Costs distribution for the abutment (E-MEPI)

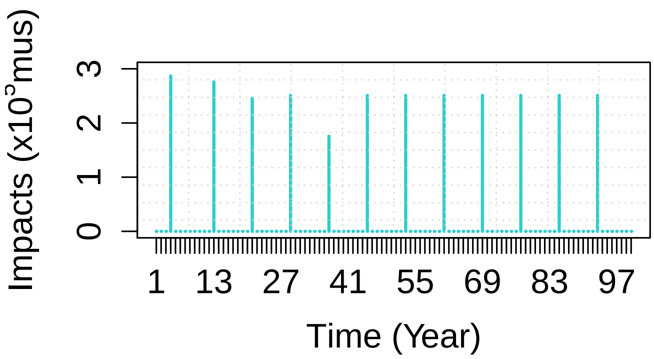


Figure E.7.: Costs distribution for the deck (WEAR-OUT)

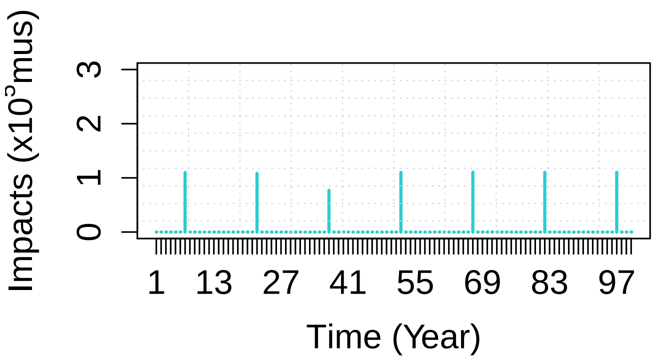


Figure E.8.: Costs distribution for the pier (WEAR-OUT)

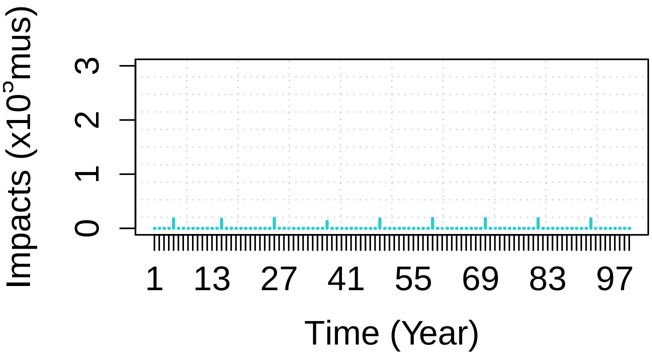


Figure E.9.: Costs distribution for the abutment (WEAR-OUT)

E.2. Condition state

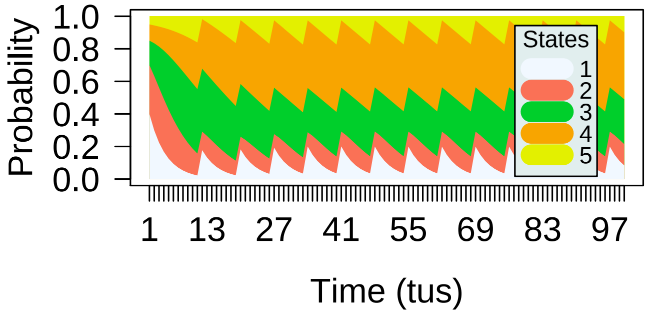


Figure E.10.: Condition state distribution for the deck (MUSTEM)

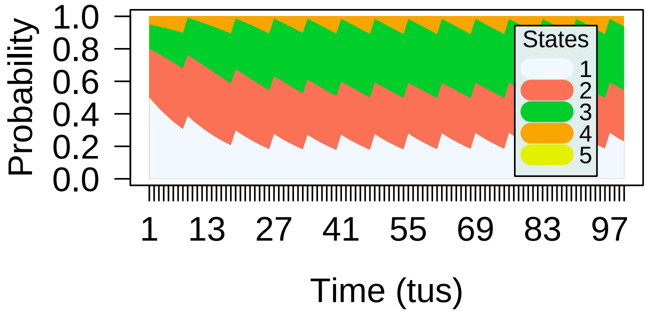


Figure E.11.: Condition state distribution for the pier (MUSTEM)

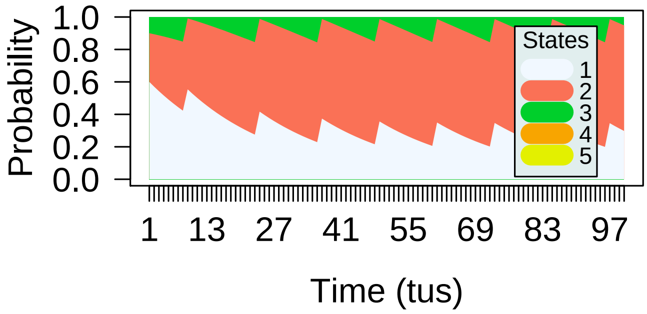


Figure E.12.: Condition state distribution for the abutment (MUSTEM)

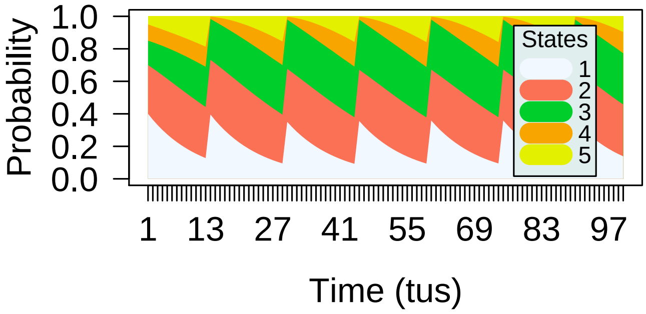


Figure E.13.: Condition state distribution for the deck (E-MEPI)

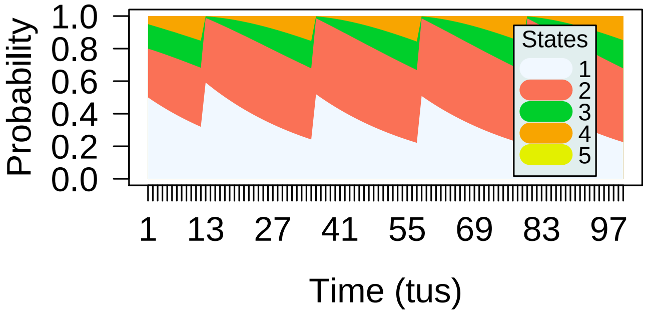


Figure E.14.: Condition state distribution for the pier (E-MEPI)

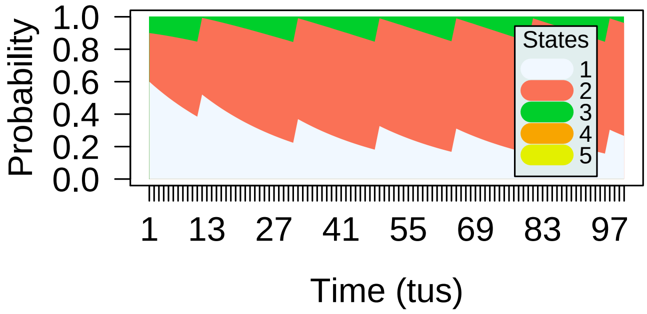


Figure E.15.: Condition state distribution for the abutment (E-MEPI)

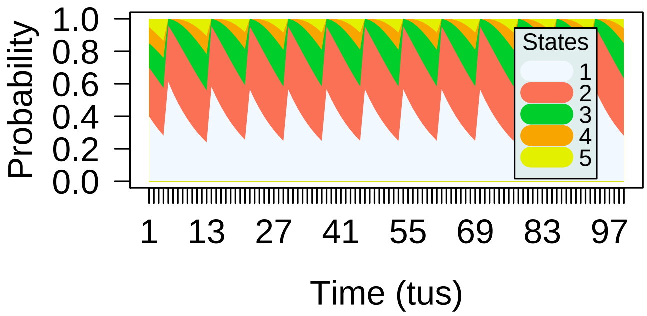


Figure E.16.: Condition state distribution for the deck (WEAR-OUT)

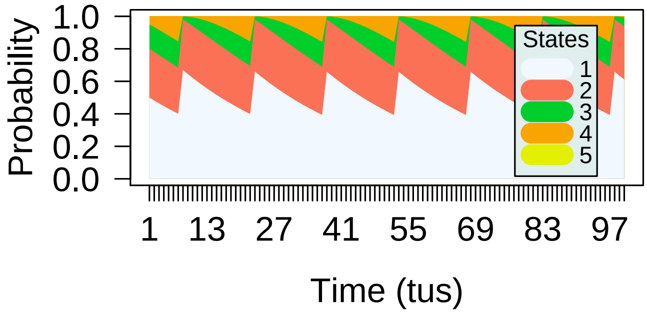


Figure E.17.: Condition state distribution for the pier (WEAR-OUT)

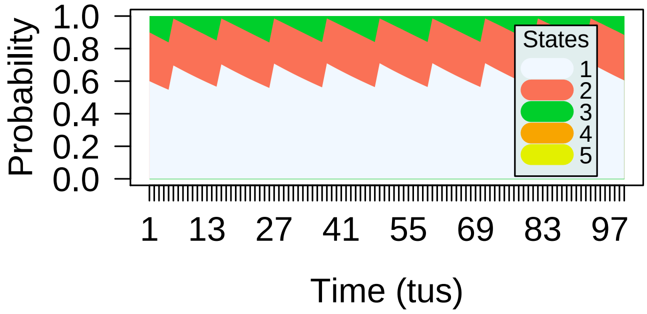


Figure E.18.: Condition state distribution for the abutment (WEAR-OUT)

F. R-codes

F.1. General codes

F.1.1. Reading input data and generate graphes

```
# This program was coded by Christoph Schlegel based on a program code by Nam Lethanh
# This program reads the general input needed for the construction of the optimal work programs and generates the graphs as output of the calculations
# This program is used for all three different deterioration processes
#-----------------------------------------------------------

stackedPlot <- function(data, time=NULL, col=1:length(data),...) {
  if (is.null(time)) {
    time <- 1:length(data[[1]])
    plot(0,0, xlim = range(time), ylim = c(0,max(rowSums(data))), axes=FALSE,t="n" , ...)
    axis(2, ylim=c(0,1),col="black",las=1)
    axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
    box()

    for (i in length(data):1) {
      #..... the summup to the current collumn
      prep.data <- rowSums(data[1:i]);
      #..... the polygon must have his first and last point on the zero line
      prep.y <- c(0, prep.data,0)
      prep.x <- c(time[1], time, time[length(time)] )
      polygon(prep.x, prep.y, col=col[i], border = NA);
    }
  }
}

colors <- c("aliceblue","coral1","limegreen","orange","yellow2")

#-----------------------------------------------------------

stp <- array(dim=c(Omax,Nmax)) #: define for each object initial condition state equal to 1
for (o in 1:Omax){
  for (i in 1:Nmax){
    if (i==1){
      stp[o,i] = 1
    } else {
        stp[o,i] = 0
    }
  }
```

```
}

#..... state probability in every year
csstate <- array(dim=c(YearMax,Nmax,Omax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (i in 1:Nmax){
      csstate[t,i,o] = 0
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    if (t==1) {
      csstate[t,,o] = stp[o,]
    } else {
        csstate[t,,o] = csstate[(t-1),,o]%*%P[,,o]
    }
  }
}
#---------------------------------------------------------------

#..... state probability over time for the different objects
plot.new()
stackedPlot(data.frame(csstate[,,1]),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Deck Condition state distribution - life time", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(csstate[,,2]),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Pier Condition state distribution - life time", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(csstate[,,3]),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Abutment Condition state distribution - life time", col.main="red", font.main=4)

#---------------------------------------------------------------

#..... read the intervention strategies, agency rule and condition for the agency rule for each interventions strategy
I1 <- read.csv("00-INPUT/00.1-I1.csv",header=TRUE,sep=";") #: Intervention strategies for object 1
I2 <- read.csv("00-INPUT/00.2-I2.csv",header=TRUE,sep=";") #: Intervention strategies for object 2
I3 <- read.csv("00-INPUT/00.3-I3.csv",header=TRUE,sep=";") #: Intervention strategies for object 3
```

```
IS1 <- length(I1[,1]) #: number of intervention strategies for object 1
IS2 <- length(I2[,1]) #: number of intervention strategies for object 2
IS3 <- length(I3[,1]) #: number of intervention strategies for object 3

IS <- cbind(IS1,IS2,IS3)
ISmax <- max(IS) #: maximum number of intervention strategies of all objects

I <- array(dim=c(ISmax,Nmax,Omax)) #: prepare intervention informations for
    calculations
for (r in 1:ISmax){
  for (j in 1:Nmax){
    for (o in 1:Omax){
      I[r,j,o] = 0
    }
  }
}
for (o in 1:Omax){
  for (r in 1:IS[o]){
    for (j in 1:NCS[o]){
      if (o==1){
        I[r,j,o] = I1[r,j+1]
      } else if (o==2){
        I[r,j,o] = I2[r,j+1]
      } else {
        I[r,j,o] = I3[r,j+1]
      }
    }
  }
}

AR <- array(dim=c(Omax,ISmax)) #: agency rule for each object and
    intervention strategy
for (o in 1:Omax){
  for (r in 1:ISmax){
    AR[o,r] = 0
  }
}
for (o in 1:Omax){
  for (r in 1:IS[o]){
    if (o==1){
      AR[o,r] = I1[r,NCS[o]+2]
    } else if (o==2){
      AR[o,r] = I2[r,NCS[o]+2]
    } else {
      AR[o,r] = I3[r,NCS[o]+2]
    }
  }
}

SAR <- array(dim=c(Omax,ISmax)) #: condition of the agency rule for each
    object and intervention strategy
for (o in 1:Omax){
  for (r in 1:ISmax){
    SAR[o,r] = 0
  }
}
```

```
for (o in 1:Omax){
  for (r in 1:IS[o]){
    if (o==1){
      SAR[o,r] = I1[r,NCS[o]+3]
    } else if (o==2){
      SAR[o,r] = I2[r,NCS[o]+3]
    } else {
      SAR[o,r] = I3[r,NCS[o]+3]
    }
  }
}

#...... read effectiveness matrix and intervention cost for each object
R1 <- read.csv("00-INPUT/00.1-R1.csv",header=TRUE,sep=";")
R2 <- read.csv("00-INPUT/00.2-R2.csv",header=TRUE,sep=";")
R3 <- read.csv("00-INPUT/00.3-R3.csv",header=TRUE,sep=";")

IT1 <- length(R1[,1]) #: number of different intervention types for object 1
IT2 <- length(R2[,1]) #: number of different intervention types for object 2
IT3 <- length(R3[,1]) #: number of different intervention types for object 3

IT <- cbind(IT1,IT2,IT3)
ITmax <- max(IT) #: maximum number of intervention types for all objects

R <- array(dim=c(ITmax,Nmax+2,Nmax,Omax)) #: effectiveness matrix and Intervention cost for each object and any intervention
for (o in 1:Omax){
  for (c in 1:Nmax){
    for (i in 1:ITmax){
      for (j in 1:(Nmax+2)){
        R[i,j,c,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (c in 1:NCS[o]){
    for (i in 1:IT[o]){
      for (j in 1:(NCS[o]+2)){
        if (o==1){
          R[i,j,c,o] = R1[i,j+(c-1)*(NCS[o]+2)] #: read effectiveness and costs for object 1
        } else if (o==2){
          R[i,j,c,o] = R2[i,j+(c-1)*(NCS[o]+2)] #: read effectiveness and costs for object 2
        } else {
          R[i,j,c,o] = R3[i,j+(c-1)*(NCS[o]+2)] #: read effectiveness and costs for object 3
        }
      }
    }
  }
}

ITU <- array(dim=c(ITmax,Omax)) #: Intervention types used for each object
for (o in 1:Omax){
  ITU[,o] = R[,1,1,o]
```

```
}

#..... transition probability matrix for each intervention strategy and
    object
Q <- array(dim=c(Nmax,Nmax,ISmax,Omax))
for (o in 1:Omax){
  for (r in 1:ISmax){
    for (i in 1:Nmax){
      for (j in 1:Nmax){
        Q[i,j,r,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (r in 1:IS[o]){
    for (i in 1:NCS[o]){
      if (I[r,i,o]==0){
        for (j in 1:NCS[o]){
          Q[i,j,r,o] = P[i,j,o]
        }
      } else {
        for (l in 1:IT[o]){
          if (ITU[l,o]==I[r,i,o]){
            for (j in 1:NCS[o]){
              Q[i,j,r,o] = R[l,j+1,i,o]
            }
          }
        }
      }
    }
  }
}

#..... cost for each intervention strategy in each condition state for each
    object
C <- array(dim=c(ISmax,Nmax,Omax))
for (r in 1:ISmax){
  for (i in 1:Nmax){
    for (o in 1:Omax){
      C[r,i,o] = 0
    }
  }
}
for (o in 1:Omax){
  for (r in 1:IS[o]){
    for (i in 1:NCS[o]){
      if (I[r,i,o]==0){
        C[r,i,o] = 0
      } else {
        for (l in 1:IT[o]){
          if (ITU[l,o]==I[r,i,o]){
            C[r,i,o] = R[l,NCS[o]+2,i,o]
          }
        }
      }
    }
  }
```

```
  }
}

#----------------------------------------------------------------------------------------
print(P)
print(I)
print(R)
print(Q)
print(C)
#----------------------------------------------------------------------------------------

#-----------------------------------------------RUN CALCULATIONS-----------------------------------------------

source("00.3-markov.R")

#----------------------------------------------------------------------------------------

#-----------------------------------------------DETERIORATION DISTRIBUTION-----------------------------------------------

#..... state probability for IS "do nothing" for the different objects
plot.new()
stackedPlot(data.frame(t(pidn[1,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Deck Condition state distribution - Do nothing", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(t(pidn[2,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Pier Condition state distribution - Do nothing", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(t(pidn[3,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Abutment Condition state distribution - Do nothing", col.main="red", font.main=4)

#..... state probability for optimal intervention strategy for the different objects based on costs and agency rule
plot.new()
stackedPlot(data.frame(t(Wpiar[1,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Deck Condition state distribution - OIS", col.main="red", font.main=4)

plot.new()
```

```
stackedPlot(data.frame(t(Wpiar[2,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Pier Condition state distribution - OIS", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(t(Wpiar[3,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Abutment Condition state distribution - OIS", col.main="red", font.main=4)

#..... state probability for optimal intervention strategy for the different objects based on costs agency rule and discount
plot.new()
stackedPlot(data.frame(t(OWPpiard[1,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Deck Condition state distribution - OWP", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(t(OWPpiard[2,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Pier Condition state distribution - OWP", col.main="red", font.main=4)

plot.new()
stackedPlot(data.frame(t(OWPpiard[3,,c(1:YearMax)])),col=colors,xlab="Time (tus)",ylab="Probability")
legend("topright", inset=0.09, title="States",col=colors,lty=2,lwd=13,legend=c(1:Nmax),bg="azure2",cex=0.7)
title(main="Abutment Condition state distribution - OWP", col.main="red", font.main=4)

#----------------------------------------------------------------------

#------------------------------------INTERVENTIONCOST DISTRIBUTION------------------------------------

color <- c("aliceblue","limegreen","orange")
par(mar=c(5,4,4,6)+0.3)
scale <- 1000000
limy <- c(0,round_any(max(TWcostar),100000, f=ceiling))
limx <- c(1,YearMax)

#..... interventioncost for optimal intervention for each object with agency rule
plot.new()
plot(Wcostar[1,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy,axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
```

```
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Deck Intervention cost - OIS", col.main="red", font.main=4)

plot.new()
plot(Wcostar[2,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy,axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Pier Intervention cost - OIS", col.main="red", font.main=4)

plot.new()
plot(Wcostar[3,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy,axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Abutment Intervention cost - OIS", col.main="red", font.main=4)

plot.new()
plot(TWcostar[],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy,axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Bridge Intervention cost - OIS", col.main="red", font.main=4)

#..... interventioncost for optimal work program for each object with agency rule and discount
plot.new()
plot(OWPcostard[1,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy,axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line
```

```
=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Deck Intervention cost - OWP", col.main="red", font.main=4)

plot.new()
plot(OWPcostard[2,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy, axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Pier Intervention cost - OWP", col.main="red", font.main=4)

plot.new()
plot(OWPcostard[3,],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy, axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Abutment Intervention cost - OWP", col.main="red", font.main=4)

plot.new()
plot(TOWPcostard[],lwd=2,col="cyan3",ylab="",xlab="",xlim=limx,ylim=limy, axes=FALSE,lty=1,type="h",pch=5,cex=0.3)
axis(2,c(seq(0,scale,by=100000)),c(seq(0,scale/100000,by=1)))
axis(1,c(seq(1,YearMax,by=1)),c(seq(1,YearMax,by=1)))
mtext(expression(paste('Impacts (',x10^5, 'mus)')),side=2,col="black",line=3)
mtext(expression(paste('Time (Year)')),side=1,col="black",line=3)
grid(10, 10, col = "lightgray", lty = "dotted",lwd = par("lwd"), equilogs = TRUE)
box()
title(main="Bridge Intervention cost - OWP", col.main="red", font.main=4)

#--------------------------------------THE END--------------------------------------
```

F.1.2. Generate optimal work program on object and bridge level

```
# This program was coded by Christoph Schlegel based on a code by Nam Lethanh
# This program calculates the different optimal work programs, based on the input of the different deterioration models, on object and bridge level
#------------------------------------------------------------------------

```

```r
#--------------------------------------------STATE PROBABILITY--------------------------------------------

#..... define state probability
pi <- array(dim=c(ISmax,Nmax,YearMax,Omax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:ISmax){
      for (i in 1:Nmax){
        pi[r,i,t,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:IS[o]){
      if (t==1){
        pi[r,,t,o] = ICO[o,]
      } else {
          pi[r,,t,o] = pi[r,,t-1,o]%*%Q[,,r,o]
        }
    }
  }
}

#..... define state probability for doing nothing
pidn <- array(dim=c(Omax,Nmax,YearMax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (i in 1:Nmax){
      pidn[o,i,t] = 0
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
      if (t==1){
        pidn[o,,t] = ICO[o,]
      } else {
          pidn[o,,t] = pidn[o,,t-1]%*%P[,,o]
        }
  }
}

#--------------------------------------------------------------------------------
print(pi[,,YearMax,])
print(pidn[,,YearMax])
#--------------------------------------------------------------------------------

#--------------------------------------------COST FOR EACH INTERVENTIONSTRATEGY--------------------------------------------

#..... calculate the evoluation of cost over the investigated period of time for each object and each intervention strategy
cost <- array(dim=c(ISmax,Nmax+1,YearMax,Omax))
for (o in 1:Omax){
```

```
  for (t in 1:YearMax){
    for (r in 1:ISmax){
      for (i in 1:(Nmax+1)){
        cost[r,i,t,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:IS[o]){
      for (i in 1:(NCS[o]+1)){
        if (i<(NCS[o]+1)){
          cost[r,i,t,o] = pi[r,i,t,o]%*%C[r,i,o]%*%area[o]
        } else{
            cost[r,i,t,o] = sum(cost[r,c(1:NCS[o]),t,o])
          }
      }
    }
  }
}

#----------------------------------------------------------------------------------
print(cost)
#----------------------------------------------------------------------------------

#---------------------------------------------CONSTRUCTION OF WORKPRGRAM WITH AGENCY RULE---------------------------------------------

piar <- array(dim=c(ISmax,Nmax,YearMax,Omax)) #: this is the state probability for each object in every year with agency rule
sumar <- array(dim=c(ISmax,YearMax,Omax)) #: this is the probability of beeing in the conditionstates where the condition for the agency rule is fulfilled
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:ISmax){
      sumar[r,t,o] = 0
      for (i in 1:Nmax){
        piar[r,i,t,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:IS[o]){
      for (i in 1:NCS[o]){
        sumar[r,t,o] = sum(piar[r,c(SAR[o,r]:NCS[o]),t,o])
        if (t==1){
          piar[r,,t,o] = ICO[o,]
        } else {
            if (sumar[r,t-1,o]>=AR[o,r]){
              piar[r,,t,o] = piar[r,,t-1,o]%*%Q[,,r,o]
            } else {
                piar[r,,t,o] = piar[r,,t-1,o]%*%P[,,o]
              }
```

```
          }

        }
      }
    }
}

#--------------------------------------------------------------
print(piar)
print(sumar)
#--------------------------------------------------------------

#..... cost for each object and each intervention strategy in every year
costar <- array(dim=c(ISmax,Nmax+1,YearMax,Omax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:ISmax){
      for (i in 1:(Nmax+1)){
        costar[r,i,t,o] = 0
      }
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:IS[o]){
      for (i in 1:(NCS[o]+1)){
        if (i < (NCS[o]+1)){
          if (sumar[r,t,o] >= AR[o,r]){
            costar[r,i,t,o] = piar[r,i,t,o]%*%C[r,i,o]%*%area[o]
          } else {
              costar[r,i,t,o] = 0
            }
        } else {
            costar[r,i,t,o] = sum(costar[r,c(1:NCS[o]),t,o])
          }
      }
    }
  }
}

#--------------------------------------------------------------
print(costar)
#--------------------------------------------------------------

#..... summary of the cost for the hole object for each intervention strategy every year
Tcostar <- array(dim=c(ISmax,YearMax,Omax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:ISmax){
      Tcostar[r,t,o] = 0
    }
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
```

```
      for (r in 1:ISmax){
        Tcostar[r,t,o] = costar[r,NCS[o]+1,t,o]
      }
    }
}

#..... summary of the cost for the hole object for each intervention
    strategy over all
Scostar <- array(dim=c(ISmax,Omax))
for (o in 1:Omax){
  for (r in 1:ISmax){
    Scostar[r,o] = 0
  }
}
for (o in 1:Omax){
  for (r in 1:ISmax){
    Scostar[r,o] = sum(Tcostar[r,,o])
  }
}

#--------------------------------------------------------------
print(Tcostar)
print(Scostar)
#--------------------------------------------------------------

#..... optimal intervention strategy for each object (based on costs)
OIScostar <- array(dim=c(Omax))
for (o in 1:Omax){
  OIScostar[o] = min(Scostar[c(1:IS[o]),o])
}

OISnrar <- array(dim=c(Omax))
for (o in 1:Omax){
  for (i in 1:IS[o]){
    if (OIScostar[o]==Scostar[i,o]){
      OISnrar[o] = i
    }
  }
}

#--------------------------------------------------------------
print(OIScostar)
print(OISnrar)
#--------------------------------------------------------------

#..... work program based on the different optimal intervention strategies
Wcostar <- array(dim=c(Omax,YearMax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    Wcostar[o,t] = 0
  }
}
for (o in 1:Omax){
  for (t in 1:YearMax){
    for (r in 1:IS[o]){
      if (OIScostar[o]==Scostar[r,o]){
        Wcostar[o,t] = Tcostar[r,t,o]
```

```
        }
      }
    }
  }

#..... total cost for the optimal workprogram based on the different OIS's
TWcostar <- array(dim=c(YearMax))
for (t in 1:YearMax){
  TWcostar[t] = sum(Wcostar[c(1:Omax),t])
}

#------------------------------------------------------------
print(Wcostar)
print(TWcostar)
#------------------------------------------------------------

#..... condition state distribution of the work program based on the
    different optimal IS's
Wpiar <- array(dim=c(Omax,Nmax,YearMax))
for (t in 1:YearMax){
  for (o in 1:Omax){
    for (i in 1:Nmax){
      Wpiar[o,i,t] = 0
    }
  }
}
for (t in 1:YearMax){
  for (o in 1:Omax){
    for (i in 1:NCS[o]){
      Wpiar[o,i,t] = piar[OISnrar[o],i,t,o]
    }
  }
}

#------------------------------------------------------------
print(Wpiar)
#------------------------------------------------------------

#----------------------------------CONSTRUCTION OF WORKPRGRAM WITH AGENCY RULE
    AND "DISCOUNT"------------------------------------

#..... maximum number of different work programs
W <- array(dim=c(Omax))
for (o in 1:Omax){
  W[o] = 0
}
for (o in 1:Omax){
  if (o==1){
    W[o] = IS[o]
  } else {
    W[o] =W[o-1]%*%IS[o]
    }
}

Wmax <- W[Omax] #: maximum number of different work programs

WN <- array(dim=c(Omax)) # help for the differerent work programs
```

```
WNN <- array(dim=c(Omax)) # help for the differerent work programs
for (o in 1:Omax){
  WNN[o] = 0
}
for (o in 1:Omax){
  if (o==1){
    WNN[o] = IS[Omax]
  } else {
    WNN[o] = WNN[o-1]%*%IS[Omax-(o-1)]
  }
}
for (o in 1:Omax){
  WN[o] = WNN[Omax-(o-1)]
}

#..... generate all different combinations of intervention strategies for
    the work programs
WPC <- array(dim=c(Omax,Wmax))
for (o in 1:Omax){
  for (w in 1:Wmax){
    WPC[o,w] = 0
  }
}
for (w in 1:Wmax){
  m = Omax
  if (w==1){
      WPC[,w] = 1
  } else {
      while (m>=1){
        if (m==Omax){
          if (WPC[m,(w-1)]<IS[m]){
            WPC[m,w] = WPC[m,(w-1)] +1
          } else {
              WPC[m,w] = 1
            }
        m = m-1
        } else {
            if (w<=WN[m+1]){
              WPC[m,w] = 1
            } else {
                if (mean(WPC[m,c((w-WN[m+1]):(w-1))])==IS[m]){
                  WPC[m,w] = 1
                } else if (mean(WPC[m,c((w-WN[m+1]):(w-1))])==WPC[m,(w-1)]){
                    WPC[m,w] = WPC[m,(w-1)] + 1
                  } else {
                      WPC[m,w] = WPC[m,(w-1)]
                    }
              }
          m = m - 1
          }
      }
    }
}

#-------------------------------------------------------------------------------
print(WPC)
#-------------------------------------------------------------------------------
```

```
#..... cost for all possible work programs
WPcostar <- array(dim=c(Omax+1,YearMax,Wmax))
for (w in 1:Wmax){
  for (t in 1:YearMax){
    for (o in 1:(Omax+1)){
      WPcostar[o,t,w] = 0
    }
  }
}
for (w in 1:Wmax){
  for (t in 1:YearMax){
    for (o in 1:(Omax+1)){
      if (o<=Omax){
        WPcostar[o,t,w] = Tcostar[WPC[o,w],t,o]
      } else {
          WPcostar[o,t,w] = sum(WPcostar[c(1:Omax),t,w])
        }
    }
  }
}

Rabcost <- array(dim=c(YearMax,Wmax))
for (w in 1:Wmax){
  for (t in 1:YearMax){
    Rabcost[t,w] = 0
  }
}
for (w in 1:Wmax){
  for ( t in 1:YearMax){
    Rabcost[t,w] = length(which(WPcostar[c(1:Omax),t,w]>0))
  }
}

#..... generate work program costs with discount due to the fact, that more
    than one object has to do an intervention
WPcostard <- array(dim=c(Omax+1,YearMax,Wmax))
for (w in 1:Wmax){
  for (t in 1:YearMax){
    for (o in 1:(Omax+1)){
      WPcostard[o,t,w] = 0
    }
  }
}
for (w in 1:Wmax){
  for (t in 1:YearMax){
    for (o in 1:(Omax+1)){
      if (o<=Omax){
        if (Rabcost[t,w]==0){
          WPcostard[o,t,w] = WPcostar[o,t,w]
        } else {
            WPcostard[o,t,w] = WPcostar[o,t,w]%*%(1-Discount[Rabcost[t,w]])
          }
      } else {
          WPcostard[o,t,w] = sum(WPcostard[c(1:Omax),t,w])
        }
    }
```

```
  }
}

#---------------------------------------------------------------
print(WPcostar)
print(WPcostard)
#---------------------------------------------------------------

#..... total cost for each work program
TWPcostard <- array(dim=c(Wmax))
for (w in 1:Wmax){
  TWPcostard[w] = 0
}
for (w in 1:Wmax){
  TWPcostard[w] = sum(WPcostard[(Omax+1),,w])
}

#---------------------------------------------------------------
print(TWPcostard)
#---------------------------------------------------------------

#..... find optimal work program with discount
OWPcost <- min(TWPcostard)

OWPnr <- array(dim=c(1))
for (w in 1:Wmax){
  if (OWPcost==TWPcostard[w]){
    OWPnr = w
  }
}

OISnrard <- array(dim=c(Omax))
for (o in 1:Omax){
    OISnrard[o] = WPC[o,OWPnr]
}

#---------------------------------------------------------------
print(OWPcost)
print(OWPnr)
print(OISnrard)
#---------------------------------------------------------------

#..... optimal work program with discount
OWPcostard <- array(dim=c(Omax,YearMax))
for (o in 1:Omax){
  for (t in 1:YearMax){
    OWPcostard[o,t] = WPcostard[o,t,OWPnr]
  }
}

OWPpiard <- array(dim=c(Omax,Nmax,YearMax))
for (t in 1:YearMax){
  for (o in 1:Omax){
    for (i in 1:Nmax){
      OWPpiard[o,i,t] = 0
    }
  }
```

```
}
for (t in 1:YearMax){
  for (o in 1:Omax){
    for (i in 1:NCS[o]){
      OWPpiard[o,i,t] = piar[OISnrard[o],i,t,o]
    }
  }
}

#.... total intervention cost for every year based on the OWP
TOWPcostard <- array(dim=c(YearMax))
for (t in 1:YearMax){
  TOWPcostard[t] = sum(OWPcostard[c(1:Omax),t])
}

#-------------------------------------------------------------------
print(OWPcostard)
print(OWPpiard)
print(TOWPcostard)
#-------------------------------------------------------------------

#-------------------------------------------------------------------
#..... anual cost for the work program based on the different optimal IS's
AWcostar <- sum(TWcostar)/YearMax

#..... anual cost for the work program based on the different optimal IS's
AOWPcostard <- sum(TOWPcostard)/YearMax

#-------------------------------------THE END-------------------------------------
```

F.2. MUSTEM codes

F.2.1. Reading model specific input data

```r
# This program was coded by Christoph Schlegel based on a program coded by Nam Lethanh
# This program generates the optimal work program on object and bridge level based on the MUSTEM deterioration process

# Use of the program:
# 1) Define general inputs for the optimization model:
#     00.0-objects, define area & number of condition states for each object and the discount factor for bridge level
#     00.x-Ix, define the different intervention strategies for the objects (x), intervention strategy has to include the intervention strategy be followed in each condition state and the agency rule
#     00.x-Rx, define the effectiveness matrix and the according costs for each intervention type at each condtion state
# 2) Define model specific input:
#     03.x-Px, define the hazard rates and the initial condition state for the objects
# 3) Run this program to get the output (OWPs)

#-----------------------------------------------------------
rm(list=ls()) #: clear the memory and objects in R console
#-----------------------------------------------------------
library(plyr)
#-----------------------------------------------------------

#-----------------------------INPUT-----------------------------

YearMax <- 100 #: number of year

#..... read objects, their area, their initial condition states and the discount rate
obj <- read.csv("00-INPUT/00.0-objects.csv",header=TRUE,sep=";")

Omax <- length(obj[,1]) #: number of objects

NCS <- obj[,3] #: numer of condition states for each object
Nmax <- max(NCS) #: maximum numer of condition stats of all objects
area <- obj[,2] #: surface area of each object
Discount <- obj [,4] #: discount due to the fact that more than one object has an intervention

#..... read hazard rate and the initial condition state for the different objects
P1 <- read.csv("01-MUSTEM/01.1-P1.csv",header=TRUE,sep=";") #: hazard rate for object 1
P2 <- read.csv("01-MUSTEM/01.2-P2.csv",header=TRUE,sep=";") #: hazard rate for object 2
P3 <- read.csv("01-MUSTEM/01.3-P3.csv",header=TRUE,sep=";") #: hazard rate for object 3

H <- array(dim=c(Omax,Nmax)) #: hazard rate for each object
for (o in 1:Omax){
```

```
    for (i in 1:Nmax){
        H[o,i] = 0
    }
}
for (o in 1:Omax){
    for (i in 1:NCS[o]){
        if (o==1){
        H[o,i] = P1[i,1] #: read hazard rate for object 1
        } else if (o==2){
            H[o,i] = P2[i,1] #: read hazard rate for object 2
            } else {
                H[o,i] = P3[i,1] #: read hazard rate for object 3
                }
    }
}

#..... if there are some of the hazard rates the same the program can't
    calculate the transition probability, if there are some the same they're
    change by 0.00001
for (o in 1:Omax){
    for (i in 1:(NCS[o]-1)){
        if (i==1){
        H[o,i] = H[o,i]
        } else {
            for (j in 1:(i-1)){
                if (H[o,i]==H[o,j]){
                H[o,i] = H[o,i] + 0.00001
                } else {
                    H[o,i] = H[o,i]
                }
            }
        }
    }
}

#----------------------------------------TRANSITION PROBABILITY
    ----------------------------------------

z <- 1 #: time intervall

source("00.1-mtp.R") #: source the program to calculate the transition
    probability

P <- mtp(NCS,z,H) #: transition probability for each object

RMD <- array(dim=c(Omax,(Nmax-1))) #: duration of each condition state
CRMD <- array(dim=c(Omax,Nmax)) #: cummulative life time of condition state

#..... subroutine to compute the life expectancy based on hazard rate.
for (o in 1:Omax){
    for (i in 1:(Nmax-1)){
        RMD[o,i] = 0
    }
}
for (o in 1:Omax){
    for (i in 1:(NCS[o]-1)){
        RMD[o,i] = 1/H[o,i]
```

```
  }
}

#..... subroutine to compute the cummulative life expectancy for plotting
for (o in 1:Omax){
  for (i in 1:Nmax){
    CRMD[o,i] = 0
  }
}
for (o in 1:Omax){
  for (i in 1:NCS[o]){
    if (i==1){
      CRMD[o,i] = 0
    } else {
        CRMD[o,i]=CRMD[o,(i-1)] + RMD[o,(i-1)]
      }
  }
}

#-----------------------------------------------------------

#..... plot the life time for object 1
plot.new()
plot(CRMD[1,],c(1:NCS[1]),xlim=c(0,CRMD[1,NCS[1]]),ylim=rev(range(c(1,NCS
    [1]))),xlab="",ylab="",type="p")
predict(smooth.spline(CRMD[1,],c(1:NCS[1]),df=4.5))
lines(predict(smooth.spline(CRMD[1,],c(1:NCS[1]),df=4.5),x=seq(0,CRMD[1,NCS
    [1]],length=100)),col="red",lwd=2)
title(main="Deterioration curve Deck", col.main="red", font.main=4)
title(xlab="Time (years)", col.lab=rgb(0,0.5,0))
title(ylab="Condition states", col.lab=rgb(0,0.5,0))

#..... plot the life time for object 2
plot.new()
plot(CRMD[2,c(1:NCS[2])],c(1:NCS[2]),xlim=c(0,CRMD[2,NCS[2]]),ylim=rev(range
    (c(1,NCS[2]))),xlab="",ylab="",type="p")
predict(smooth.spline(CRMD[2,c(1:NCS[2])],c(1:NCS[2]),df=3.5))
lines(predict(smooth.spline(CRMD[2,c(1:NCS[2])],c(1:NCS[2]),df=3.5),x=seq(0,
    CRMD[2,NCS[2]],length=100)),col="red",lwd=2)
title(main="Deterioration curve Pier", col.main="red", font.main=4)
title(xlab="Time (years)", col.lab=rgb(0,0.5,0))
title(ylab="Condition states", col.lab=rgb(0,0.5,0))

#..... plot the life time for object 3
#plot.new()
#plot(CRMD[3,c(1:NCS[3])],c(1:NCS[3]),xlim=c(0,CRMD[3,NCS[3]]),ylim=rev(
    range(c(1,NCS[3]))),xlab="",ylab="",type="p")
#predict(smooth.spline(CRMD[2,c(1:NCS[2])],c(1:NCS[2]),df=3.5))
#lines(predict(smooth.spline(CRMD[3,c(1:NCS[3])],c(1:NCS[3]),df=3.5),x=seq
    (0,CRMD[3,NCS[3]],length=100)),col="red",lwd=2)
#title(main="Deterioration curve Abutment", col.main="red", font.main=4)
#title(xlab="Time (years)", col.lab=rgb(0,0.5,0))
#title(ylab="Condition states", col.lab=rgb(0,0.5,0))

#-----------------------------------------------------------

#..... read the initial condition states for each object
```

```
ICO <- array(dim=c(Omax,Nmax))
for (o in 1:Omax){
  for (i in 1:Nmax){
    ICO[o,i] = 0
  }
}
for (o in 1:Omax){
  for (i in 1:NCS[o]){
    if (o==1){
      ICO[o,i] = P1[i,2] #: read initial condition state for object 1
    } else if (o==2){
        ICO[o,i] = P2[i,2] #: read initial condition state for object 2
      } else {
          ICO[o,i] = P3[i,2] #: read initial condition state for object 3
        }
  }
}

source("00.2-inp.R") #: source the program to calculate the work programs

#----------------------------------------SAVE THE RESULTS----------------------------------------

file.remove("01-MUSTEM/01.1-resultsOWP.csv")
file.create("01-MUSTEM/01.1-resultsOWP.csv")

file.remove("01-MUSTEM/01.2-resultsOIS.csv")
file.create("01-MUSTEM/01.2-resultsOIS.csv")

write.table(OWPcostard, file="01-MUSTEM/01.1-resultsOWP.csv", sep = ";", append = TRUE, col.names = FALSE)

write.table(Wcostar, file="01-MUSTEM/01.2-resultsOIS.csv", sep = ";", append = TRUE, col.names = FALSE)

#------------------------------------------------------------------------------------------

cat("annual costs for optimal work program on object level \n")
print(AWcostar)

cat("annual costs for optimal work program on bridge level \n")
print(AOWPcostard)

#----------------------------------------THE END----------------------------------------
```

F.2.2. Generate transition probability out of proportions data

```
# This program is coded by Christoph Schlegl
# This program calculates the transition probability matrix out of the hazard rates (MUSTEM model), according to TSUDA et al. [2006]
#------------------------------------------------------------------------------------------

mtp <- function(NCS,z,theta){

probb <- array(dim=c(Nmax,Nmax,Omax))
for (o in 1:Omax){
  for (i in 1:Nmax){
```

```
    for (j in 1:Nmax){
      probb[i,j,o] = 0
    }
  }
}

#------------------------------------------------------------

for (o in 1:Omax){
  for (i in 1:NCS[o]){
    for (j in 1:NCS[o]){
      k = i
      p = 0
      while (k<=j){
        m = i
        prod1 = 1
        while (m<=(k-1)){
          prod11 = theta[o,m]/(theta[o,m]-theta[o,k])
          prod1 = prod1*prod11
          m = m+1
        }
        n = k
        prod2 = 1
        while (n<=(j-1)){
          prod22 = theta[o,n]/(theta[o,n+1]-theta[o,k])
          prod2 = prod2*prod22
          n = n+1
        }
        pp = prod1*prod2*exp(-theta[o,k]*z)
        p = p + pp
        k = k+1
      }
      probb[i,j,o] = p
    }
  }
}
return(probb)
}

#---------------------------------THE END---------------------------------
```

F.3. E-MEPI codes

F.3.1. Estimate state probability based on E-MEPI

```r
# This program was coded by Christoph Schlegel based on a code by Nam
    Lethanh
# This program calculates the state probability for the E-MEPI model based
    on DURACRETE2000
#-----------------------------------------------------------------------
rm(list=ls()) #: clear the memory and objects in R console
#-----------------------------------------------------------------------
#.....read inputparameters from file

input <- read.csv("02-EMEPI/02.0-input.csv", header =TRUE, sep = ";") #:
    import data from excel

data <- data.frame(input) #: making data to be in frame

Omax <- length(data[,1])/2 #: number of objects

#---------------------------------------------------------------------

#.....STEP 1: Define model parameter

T <- 100 #: years of investigation

limit_chrolide <- 0.48 #: limit concentration of chloride [wt.-%Cl/binder]
limit_crack <- 0.50 #: limit crack width [mm]

N <- 20000 #: maximum number used for random sampling with Monte Carlo or
    sampling techniques [-]

ftspl <- 2.6 #: tensil splitting strength [MPa]

epsilon <- 10e-16 #: this is used to prevent value of simulated standard
    deviation to be less than 0.005, which gives a distribution with value
    less than 10^(-16), and in R, if value is less than 10^(-16), the PC
    cannot understand, basically when that situation happens the result
    returns to 0, which is not true.

#.....Define the number of condition state

I <- array(dim=c(Omax,1)) #: number of CS related to chloride-induced onset
    of corrosion (inition phase), the last condition state related to failure
     state
for (o in 1:Omax){
  I[o] = data[(o-1)*2+1,1]
}

J <- array(dim=c(Omax,1)) #: number of CS related to propagation of
    corrosion (propagation phase "cracking")
for (o in 1:Omax){
  J[o] = data[(o-1)*2+2,1]
}

IM <- max(I)
```

```
a <- array(dim=c(Omax,IM)) #: range value of condition state for chloride [wt.-%Cl/binder]
for (o in 1:Omax){
  for (i in 1:IM){
    a[o,i] = 0
  }
}
for (o in 1:Omax){
  a[o,(1:I[o])] = seq(from=0, to=limit_chrolide,length.out=I[o])
}

JM <- max(J)

b <- array(dim=c(Omax,JM)) #: range value of condition state for crack [mm]
for (o in 1:Omax){
  for (i in 1:JM){
    b[o,i] = 0
  }
}
for (o in 1:Omax){
  b[o,(1:J[o])] = seq(from=0, to=limit_crack,length.out=J[o])
}

#.....Define parameter for initiation phase

d <- array(dim=c(Omax,1)) #: distance from the concrete surface to rebar [mm] (if the onset of corrosion is considered, d is equal to the concrete cover depth dc)
for (o in 1:Omax){
  d[o] = data[(o-1)*2+1,2]
}

D0 <- array(dim=c(Omax,2)) #: mean value and standard deviation of chloride migration coefficient at defined compaction, measured at time t0 [mm2/yr]
for (o in 1:Omax){
  for (v in 1:2){
    D0[o,v] = data[(o-1)*2+v,3]
  }
}

Cs <- array(dim=c(Omax,2)) #: mean value and standard deviation of the surface chloride level [wt.-%Cl/binder]
for (o in 1:Omax){
  for (v in 1:2){
    Cs[o,v] = data[(o-1)*2+v,4]
  }
}

ke <- array(dim=c(Omax,2)) #: shape and scale value of the parameter which considers the influence of environment on D0 [-]
for (o in 1:Omax){
  for (v in 1:2){
    ke[o,v] = data[(o-1)*2+v,5]
  }
}
```

```
kt <- array(dim=c(Omax,2)) #: mean value and standard deviation of the parameter which considers the influence of test method on D0 [-]
for (o in 1:Omax){
  for (v in 1:2){
    kt[o,v] = data[(o-1)*2+v,6]
  }
}

kc <- array(dim=c(Omax,1)) #: parameter which considers the influence of curing on D0 [-]
for (o in 1:Omax){
  kc[o] = data[(o-1)*2+1,7]
}

t0 <- array(dim=c(Omax,1)) #: reference periode [yr]
for (o in 1:Omax){
  t0[o] = data[(o-1)*2+1,8]
}

nn <- array(dim=c(Omax,2)) #: shape value 1 and 2 of the age factor [-]
for (o in 1:Omax){
  for (v in 1:2){
    nn[o,v] = data[(o-1)*2+v,9]
  }
}

#.....Define parameter for propagation phase

w0 <- array(dim=c(Omax,2)) #: mean value and standard deviation of the crack width when it is visible
for (o in 1:Omax){
  for (v in 1:2){
    w0[o,v] = data[(o-1)*2+v,10]
  }
}

beta <- array(dim=c(Omax,2)) #: mean value and standard deviation of the propagation controlling parameter [-]
for (o in 1:Omax){
  for (v in 1:2){
    beta[o,v] = data[(o-1)*2+v,11]
  }
}

V0 <- array(dim=c(Omax,2)) #: mean value and standard deviation of the corrosion rate when corrosion is active [mm/yr]
for (o in 1:Omax){
  for (v in 1:2){
    V0[o,v] = data[(o-1)*2+v,12]
  }
}

wet <- array(dim=c(Omax,2)) #: mean value and standard deviation of the wetness period (sum of all raining days in a year) [-]
for (o in 1:Omax){
  for (v in 1:2){
    wet[o,v] = data[(o-1)*2+v,13]
```

```
  }
}

alpha <- array(dim=c(Omax,2)) #: mean value and standard deviation of the
    pitting factor [-]
for (o in 1:Omax){
  for (v in 1:2){
    alpha[o,v] = data[(o-1)*2+v,14]
  }
}

a1 <- array(dim=c(Omax,2)) #: mean value and standard deviation of the
    regression parameter 1 [mm]
for (o in 1:Omax){
  for (v in 1:2){
    a1[o,v] = data[(o-1)*2+v,15]
  }
}

a2 <- array(dim=c(Omax,2)) #: mean value and standard deviation of the
    regression parameter 2 [mm]
for (o in 1:Omax){
  for (v in 1:2){
    a2[o,v] = data[(o-1)*2+v,16]
  }
}

a3 <- array(dim=c(Omax,2)) #: mean value and standard deviation of the
    regression parameter 3 [mm/MPa]
for (o in 1:Omax){
  for (v in 1:2){
    a3[o,v] = data[(o-1)*2+v,17]
  }
}

phi <- array(dim=c(Omax,1)) #: rebar diameter [mm]
for (o in 1:Omax){
  phi[o] = data[(o-1)*2+1,18]
}

#-----------------------------------------------------------------

#.....STEP 2: Define functions to be used

#.....Define error function
erf <- function(x) 2 * pnorm(x * sqrt(2)) - 1

#.....Model parameters

#......Define the function of chloride concentration in initiation phase
Ccl <- function(Cs,x,ke,kt,kc,D0,t0,n,t){
  var=ke*kt*kc*D0*(t0/t)^n*t
  if (var >  epsilon) {
    Cs*(1-erf(x/(2*sqrt(var))))
  }
  else {
    Cs*(1-erf(x/(2*sqrt(epsilon))))
```

```
  }
}

#.....Cs: surface chloride level [wt.-%Cl/binder] (probabilistic, ND (LN))
#.....x: cover depth [mm]
#.....ke: influence of environment on D0 [-] (probabilistic, Gamma (ND))
#.....kt: influence of test method on D0 [-] (probabilistic, ND)
#.....kc: influence of curing on D0 [-] (D (probabilistic, Beta))
#.....D0: chloride migration coefficient [mm2/yr] (probabilistic, ND)
#.....t0: reference periode [yr]
#.....t: exposure periode [yr]
#.....n: age factor [-] (probabilistic, Beta (ND))

#-----------------------------------------------------------------

#.....Define the dimension

pi_cl <- array(dim=c(T,IM,Omax)) #: state probability for initiation stage
for (o in 1:Omax){
  for (t in 1:T){
    for (i in 1:IM){
      pi_cl[t,i,o] = 0
    }
  }
}

pi_cr <- array(dim=c(T,JM,Omax)) #: state probability for propagation stage
for (o in 1:Omax){
  for (t in 1:T){
    for (j in 1:JM){
      pi_cr[t,j,o] = 0
    }
  }
}

pi <- array(dim=c(T,(IM+JM-1),Omax)) #: state probability of the entire
    process -1 mean we consider the last state I as failure state, therefore,
     the total state of the system is I+J-1
for (o in 1:Omax){
  for (t in 1:T){
    for (i in 1:(IM+JM-1)){
      pi[t,i,o] = 0
    }
  }
}

Cclt <- array(dim=c(T,N,Omax))
Ccrt <- array(dim=c(T,N,Omax))

Ccltd <- array(dim=c(T,Omax))
Ccrtd <- array(dim=c(T,Omax))

#-----------------------------------------------------------------

#......Define the function for the crack width (propagation phase)
crackwidth <- function(w0,beta,V0,alpha,wet,a1,a2,a3,c,phi,ftspl,t){
  w0+beta*(V0*wet*alpha*t-(a1+a2*c/phi+a3*ftspl))
```

```
}

# .....w(x): crack width at time x [mm]
# .....w(0): crack width when it is visible 0.05 [mm] (probabilistic, ND)
# .....beta: propagation controlling parameter [-] (probabilistic, ND)
# .....V0: mean corrosion rate when corrosion is active [mm/yr] (
    probabilistic, ND (Weibull))
# .....V: corrosion rate [mm/yr]
# .....wet: wetness period [-] (sum of all raining days in a year) (
    probabilistic, ND)
# .....alpha: pitting factor taking non-uniform corrosion of the rebars into
     account [-] (probabilistic, ND)
# .....a1: regression parameter 1 [mm] (probabilistic, ND)
# .....a2: regression parameter 2 [mm] (probabilistic, ND)
# .....a3: regression parameter 3 [mm/MPa] (probabilistic, ND)
# .....c: cover depth [mm] (D)
# .....phi: rebar diameter [mm] (D)
# .....ftspl: tensile splitting strength [MPa] (D)

#-------------------------------------------------------------------

#.....STEP 3: ESTIMATION

  #.....for parameter concerning the initiation phase
  i1 <- array(dim=c(N,Omax)) #: i1 represent for surface Cl concentration Cs
  for (o in 1:Omax){
    i1[,o] = rnorm(N,Cs[o,1],Cs[o,2])
  }
  i2 <- array(dim=c(N,Omax)) #: i2 represent for diffusion coefficient D0
  for (o in 1:Omax){
    i2[,o] = rnorm(N,D0[o,1],D0[o,2])
  }
  i3 <- array(dim=c(N,Omax)) #: i3 represent the environmental factor ke
  for (o in 1:Omax){
    i3[,o] = rgamma(N,ke[o,1],scale = ke[o,2])
  }
  i4 <- array(dim=c(N,Omax)) #: i4 represent the test method factor kt
  for (o in 1:Omax){
    i4[,o] = rnorm(N,kt[o,1],kt[o,2])
  }
  i5 <- array(dim=c(N,Omax)) #: i5 represent the age factor n
  for (o in 1:Omax){
    i5[,o] = rbeta(N,nn[o,1],nn[o,2],ncp = 0)
  }
  #.....for parameter concerning the propagation phase
  p1 <- array(dim=c(N,Omax)) #: p1 represent the crack width when it is
      visible w0
  for (o in 1:Omax){
    p1[,o] = rnorm(N,w0[o,1],w0[o,2])
  }
  p2 <- array(dim=c(N,Omax)) #: p2 represent the propagation controlling
      parameter beta
  for (o in 1:Omax){
    p2[,o] = rnorm(N,beta[o,1],beta[o,2])
  }
  p3 <- array(dim=c(N,Omax)) #: p3 represent the mean corrosion rate V0
  for (o in 1:Omax){
```

```
      p3[,o] = rnorm(N,V0[o,1],V0[o,2])
    }
    p4 <- array(dim=c(N,Omax)) #: p4 represent the pitting factor alpha
    for (o in 1:Omax){
      p4[,o] = rnorm(N,alpha[o,1],alpha[o,2])
    }
    p5 <- array(dim=c(N,Omax)) #: p5 represent the wetness period wet
    for (o in 1:Omax){
      p5[,o] = rnorm(N,wet[o,1],wet[o,2])
    }
    p6 <- array(dim=c(N,Omax)) #: p6 represent the regression parameter 1 a1
    for (o in 1:Omax){
      p6[,o] = rnorm(N,a1[o,1],a1[o,2])
    }
    p7 <- array(dim=c(N,Omax)) #: p7 represent the regression parameter 2 a2
    for (o in 1:Omax){
      p7[,o] = rnorm(N,a2[o,1],a2[o,2])
    }
    p8 <- array(dim=c(N,Omax)) #: r8 represent the regression parameter 3 a3
    for (o in 1:Omax){
      p8[,o] = rnorm(N,a3[o,1],a3[o,2])
    }

for (o in 1:Omax){
    for (t in 1:T){
      #.....STEP 3.1: estimate the statistical properties of chloride Ccl at
          any time t using random generation (e.g. Monte Carlo simulation or
          random sampling)
      for (n in 1:N){
        Cclt[t,n,o] = Ccl(i1[n,o],d[o],i3[n,o],i4[n,o],kc[o],i2[n,o],t0[o],i5[
            n,o],t)
        Ccrt[t,n,o] = crackwidth(p1[n,o],p2[n,o],p3[n,o],p4[n,o],p5[n,o],p6[n,
            o],p7[n,o],p8[n,o],d[o],phi[o],ftspl,t)
      }

      Ccltd[t,o] = Ccl(mean(i1[,o]),d[o],mean(i3[,o]),mean(i4[,o]),kc[o],mean(
          i2[,o]),t0[o],mean(i5[,o]),t)
      Ccrtd[t,o] = crackwidth(mean(p1[,o]),mean(p2[,o]),mean(p3[,o]),mean(p4[,
          o]),mean(p5[,o]),mean(p6[,o]),mean(p7[,o]),mean(p8[,o]),d[o],phi[o],
          ftspl,t)

      #.....STEP 3.2: Calculating the state probability for each phase

      #.....for chloride
        #...............................................
      for (k in 1:I[o]){
        if (k==1){
          pi_cl[t,k,o] = (length(which(Cclt[t,,o]<a[o,(k+1)])))/N
        } else if (k<I[o]) {
            pi_cl[t,k,o] = (length(which(Cclt[t,,o]<a[o,(k+1)])))/N - sum(pi_
                cl[t,(1:(k-1)),o])
          } else if (k==I[o]) {
              pi_cl[t,k,o] = 1 - sum(pi_cl[t,(1:(k-1)),o])
            }
      }

      #for propagation
```

```
    #.................................................
    for (k in 1:J[o]){
      if (k==1){
        pi_cr[t,k,o] = (length(which(Ccrt[t,,o]<b[o,(k+1)])))/N
      } else if (k<J[o]) {
          pi_cr[t,k,o] = (length(which(Ccrt[t,,o]<b[o,k+1])))/N - sum(pi_cr[t,(1:(k-1)),o])
        } else if (k==J[o]) {
            pi_cr[t,k,o] = 1 - sum(pi_cr[t,(1:(k-1)),o])
          }
    }

    #.....STEP 3.3: calculating the state probability for the entire system

    #.....combining two probability
    for (k in 1:(I[o]+J[o]-1)){
      if (k<I[o]){
        pi[t,k,o] = pi_cl[t,k,o]
      } else {
          pi[t,k,o] = pi_cl[t,I[o],o]*pi_cr[t,(k-I[o]+1),o]
        }
    }
  }
}
#####################################################
detl <- array(dim=c(T,Omax)) #: deterioration process for the inition phase
for (o in 1:Omax){
  for (t in 1:T){
    detl[t,o] = 0
  }
}
for (o in 1:Omax){
  for (t in 1:T){
    if (I[o]>2){
      for (i in 1:(I[o]-1)){
        if (i==1){
          if (Ccltd[t,o]<a[o,2]){
            detl[t,o] = 1
          }
        } else {
          if (detl[t,o]==0){
            if (Ccltd[t,o]<a[o,(i+1)]){
              detl[t,o] = i
            } else {
              detl[t,o] = I[o]
            }
          } else {
          }
        }
      }
    } else {
      for (i in 1:(I[o]-1)){
        if (Ccltd[t,o]<a[o,(i+1)]){
          detl[t,o] = i
        } else {
          detl[t,o] = I[o]
        }
```

```
        }
      }
    }
  }

  detr <- array(dim=c(T,Omax)) #: deterioration process for the propagation
      phase
  for (o in 1:Omax){
    for (t in 1:T){
      detr[t,o] = 0
    }
  }
  for (o in 1:Omax){
    for (t in 1:T){
      if (J[o]>2){
        for (j in 1:(J[o]-1)){
          if (j==1){
            if (Ccrtd[t,o]<b[o,2]){
              detr[t,o] = 1
            }
          } else {
            if (detr[t,o]==0){
              if (Ccrtd[t,o]<b[o,(j+1)]){
                detr[t,o] = j
              } else {
                detr[t,o] = J[o]
              }
            } else {
            }
          }
        }
      } else {
        for (j in 1:(J[o]-1)){
          if (Ccrtd[t,o]<b[o,(j+1)]){
            detr[t,o] = j
          } else {
            detr[t,o] = J[o]
          }
        }
      }
    }
  }
  det <- array(dim=c(T,Omax)) #: combined deterioration process
  for (o in 1:Omax){
    for (t in 1:T){
      if (detl[t,o]==I[o]){
        det[t,o] = detr[t,o] + (I[o]-1)
      } else {
        det[t,o] = detl[t,o]
      }
    }
  }

  haz <- array(dim=c((IM+JM-1),Omax)) #: calculating the hazard rate
  for (o in 1:Omax){
    for (i in 1:(IM+JM-1)){
      haz[i,o] = 0
```

```
  }
}
for (o in 1:Omax){
  for (i in 1:(I[o]+J[o]-1)){
    haz[i,o] = length(which(det[,o]==i))
  }
}
for (o in 1:Omax){
  for (i in 1:(I[o]+J[o]-1)){
    haz[i,o] = 1/haz[i,o]
  }
}
##########################################################
cat("state probability of the initiation phase \n")
print(pi_cl)
cat("state probability of the propagation phase \n")
print(pi_cr)
cat("state probability of the entire process \n")
print(pi)

#-----------------------------------SAVE THE RESULTS-----------------------------------

file.remove("02-EMEPI/02.1-pi1.csv")
file.remove("02-EMEPI/02.2-pi2.csv")
file.remove("02-EMEPI/02.3-pi3.csv")
file.remove("02-EMEPI/02.0-haz.csv")

file.create("02-EMEPI/02.1-pi1.csv")
file.create("02-EMEPI/02.2-pi2.csv")
file.create("02-EMEPI/02.3-pi3.csv")
file.create("02-EMEPI/02.0-haz.csv")

write.table(pi[,,1], file = "02-EMEPI/02.1-pi1.csv", sep = ";", append =
    TRUE, col.names = FALSE)
write.table(pi[,,2], file = "02-EMEPI/02.2-pi2.csv", sep = ";", append =
    TRUE, col.names = FALSE)
write.table(pi[,,3], file = "02-EMEPI/02.3-pi3.csv", sep = ";", append =
    TRUE, col.names = FALSE)
write.table(haz, file = "02-EMEPI/02.0-haz.csv", sep = ";", append = TRUE,
    col.names = FALSE)
#-----------------------------------THE END-----------------------------------
```

F.3.2. Reading model specific input data

```
# This program is coded by Christoph Schlegel
# This program generates the optimal work program on object and bridge level
     based on the E-MEPI deterioration process
# Before sourcing this program the transition probability needs to be
    calculated via the excel files 02.x-Linear_optimizationx

# Use of the program:
# 1) Define general inputs for the optimization model:
#      00.0-objects, define area & number of condition states for each object
     and the discount factor for bridge level
#      00.x-Ix, define the different intervention strategies for the objects
    (x), intervention strategy has to include the intervention strategy be
```

```r
followed in each condition state and the agency rule
#     00.x-Rx, define the effectiveness matrix and the according costs for each intervention type at each condtion state
# 2) Define model specific input:
#     02.0-input, define parameters for the chloride induced corrosion (mean value and standard deviation) for each object
# 3) Run E-MEPIgen to get the state probability for the different objects according to the chloride induced corrosion
# 4) Copy the state probability of the different obecjts (02.x-pix) into the linearization program 02.x-Linear_optimizationx
# 5) 02.x-Linear_optimizationx, use solver to calculate the Markov transition probability
# 6) 02.x-Px, copy the Markov transition probability into this file and define the initial condition states of the object
# 7) Run this program to get the output (OWPs)

#---------------------------------------------------------------------
library(plyr)
#---------------------------------------------------------------------

#----------------------------------INPUT----------------------------------

YearMax <- 100 #: number of year

#..... read objects, their area, their initial condition states and the discount rate
obj <- read.csv("00-INPUT/00.0-objects.csv",header=TRUE,sep=";")

Omax <- length(obj[,1]) #: number of objects

NCS <- obj[,3] #: numer of condition states for each object
Nmax <- max(NCS) #: maximum numer of condition stats of all objects
area <- obj[,2] #: surface area of each object
Discount <- obj [,4] #: discount due to the fact that more than one object has an intervention

#..... read transition probability and the initial condition state for the different objects
P1 <- read.csv("02-EMEPI/02.1-P1.csv",header=TRUE,sep=";") #: transition probability for object 1
P2 <- read.csv("02-EMEPI/02.2-P2.csv",header=TRUE,sep=";") #: transition probability for object 2
P3 <- read.csv("02-EMEPI/02.3-P3.csv",header=TRUE,sep=";") #: transition probability for object 3

#----------------------------------TRANSITION PROBABILITY----------------------------------

P <- array(dim=c(Nmax,Nmax,Omax)) #: transition probability for each object
for (o in 1:Omax){
  for (i in 1:Nmax){
    for (j in 1:Nmax){
      P[i,j,o] = 0
    }
  }
}
for (o in 1:Omax){
```

```
  for (i in 1:NCS[o]){
    for (j in 1:NCS[o]){
      if (o==1){
        P[i,j,o] = P1[i,j] #: read transition probability for object 1
      } else if (o==2){
          P[i,j,o] = P2[i,j] #: read transition probability for object 2
        } else {
            P[i,j,o] = P3[i,j] #: read transition probability for object 3
          }
    }
  }
}

#------------------------------------------------------------------

#..... read the initial condition states for each object
ICO <- array(dim=c(Omax,Nmax))
for (o in 1:Omax){
  for (i in 1:Nmax){
    ICO[o,i] = 0
  }
}
for (o in 1:Omax){
  for (i in 1:NCS[o]){
    if (o==1){
      ICO[o,i] = P1[i,NCS[o]+1] #: read initial condition state for object 1
    } else if (o==2){
        ICO[o,i] = P2[i,NCS[o]+1] #: read initial condition state for object 2
      } else {
          ICO[o,i] = P3[i,NCS[o]+1] #: read initial condition state for object 3
        }
  }
}

source("00.2-inp.R") #: source the program to calculate the work programs

#-----------------------------------SAVE THE RESULTS-----------------------------------

file.remove("02-EMEPI/02.1-resultsOWP.csv")
file.create("02-EMEPI/02.1-resultsOWP.csv")

file.remove("02-EMEPI/02.2-resultsOIS.csv")
file.create("02-EMEPI/02.2-resultsOIS.csv")

write.table(OWPcostard, file="02-EMEPI/02.1-resultsOWP.csv", sep = ";", append = TRUE, col.names = FALSE)

write.table(Wcostar, file="02-EMEPI/02.2-resultsOIS.csv", sep = ";", append = TRUE, col.names = FALSE)

#------------------------------------------------------------------

cat("annual costs for optimal work program on object level \n")
print(AWcostar)
```

```

cat("annual costs for optimal work program on bridge level \n")
print(AOWPcostard)
#————————————————————THE END————————————————————
```

F.4. WEAR-OUT codes

F.4.1. Reading model specific input data

```r
# This program was coded by Christoph Schlegel
# This program generates the optimal work program on object and bridge level based on the WEAR-OUT deterioration process
#--------------------------------------------------------------------------------
rm(list=ls()) #: clear the memory and objects in R console
#--------------------------------------------------------------------------------
library(plyr)
#--------------------------------------------------------------------------------

#---------------------------------------------INPUT---------------------------------------------

YearMax <- 100 #: number of year

#..... read objects, their area, their initial condition states and the discount rate
obj <- read.csv("00-INPUT/00.0-objects.csv",header=TRUE,sep=";")

Omax <- length(obj[,1]) #: number of objects

NCS <- obj[,3] #: numer of condition states for each object
Nmax <- max(NCS) #: maximum numer of condition stats of all objects
area <- obj[,2] #: surface area of each object
Discount <- obj [,4] #: discount due to the fact that more than one object has an intervention

#..... read transition probability and the initial condition state for the different objects
P1 <- read.csv("03-WEAROUT/03.1-P1.csv",header=TRUE,sep=";") #: transition probability for object 1
P2 <- read.csv("03-WEAROUT/03.2-P2.csv",header=TRUE,sep=";") #: transition probability for object 2
P3 <- read.csv("03-WEAROUT/03.3-P3.csv",header=TRUE,sep=";") #: transition probability for object 3

#---------------------------------------------TRANSITION PROBABILITY---------------------------------------------

P <- array(dim=c(Nmax,Nmax,Omax)) #: transition probability for each object
for (o in 1:Omax){
  for (i in 1:Nmax){
    for (j in 1:Nmax){
      P[i,j,o] = 0
    }
  }
}
for (o in 1:Omax){
  for (i in 1:NCS[o]){
    for (j in 1:NCS[o]){
      if (o==1){
        P[i,j,o] = P1[i,j] #: read transition probability for object 1
      } else if (o==2){
          P[i,j,o] = P2[i,j] #: read transition probability for object 2
```

```
        } else {
          P[i,j,o] = P3[i,j] #: read transition probability for object 3
        }
      }
    }
}

#------------------------------------------------------------------------

#..... read the initial condition states for each object
ICO <- array(dim=c(Omax,Nmax))
for (o in 1:Omax){
  for (i in 1:Nmax){
    ICO[o,i] = 0
  }
}
for (o in 1:Omax){
  for (i in 1:NCS[o]){
    if (o==1){
      ICO[o,i] = P1[i,NCS[o]+1] #: read initial condition state for object 1
    } else if (o==2){
        ICO[o,i] = P2[i,NCS[o]+1] #: read initial condition state for object 2
      } else {
          ICO[o,i] = P3[i,NCS[o]+1] #: read initial condition state for object 3
        }
  }
}

source("00.2-inp.R") #: source the program to calculate the work programs

#------------------------------------SAVE THE RESULTS------------------------------------

file.remove("03-WEAROUT/03.1-resultsOWP.csv")
file.create("03-WEAROUT/03.1-resultsOWP.csv")

file.remove("03-WEAROUT/03.2-resultsOIS.csv")
file.create("03-WEAROUT/03.2-resultsOIS.csv")

write.table(OWPcostard, file="03-WEAROUT/03.1-resultsOWP.csv", sep = ";", append = TRUE, col.names = FALSE)

write.table(Wcostar, file="03-WEAROUT/03.2-resultsOIS.csv", sep = ";", append = TRUE, col.names = FALSE)

#------------------------------------------------------------------------

cat("annual costs for optimal work program on object level \n")
print(AWcostar)

cat("annual costs for optimal work program on bridge level \n")
print(AOWPcostard)

#------------------------------------THE END------------------------------------
```