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Use of exponential hidden Markov models for modelling pavement deterioration

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In this paper, the potential of using an exponential hidden Markov model to model an indicator of pavement condition as a hidden pavement deterioration process, i.e. one that is not directly measurable, is investigated. It is assumed that the evolution of the values of the pavement condition indices can be adequately described using discrete condition states and modelled as a Markov process. It is also assumed that the values of the indices can be measured over time and represented continuously using exponential distributions. The potential advantage of using such a model is illustrated using a real-world example.

Keywords: deterioration modelling; exponential distribution; hidden Markov model; pavement management

1. Introduction

1.1 *Insufficient data*

The prediction of future condition is necessary in the determination of optimal pavement intervention strategies (OISs). Accurate prediction requires that prediction models be validated using condition data. The quantity and quality of these data are, however, not always sufficient to directly validate the prediction models and therefore to determine the OIS. Two such examples, which result either due to a desire to reduce the number of inspections performed or due to poor planning, are as follows:

- There exist extensive data on the visual appearance of the pavement surface, e.g. percentage of surface area that is cracked, but there are sparse data on the longitudinal and transversal unevenness, friction or load-bearing capacity of the road section.
- There exist extensive data on the road roughness, e.g. the International Roughness Index (IRI), but there are sparse data on the degree of rutting or bearing capacity.

Determining OISs in either one of these cases may result in the exclusion of certain types of interventions, as appropriate triggers cannot be assigned which would allow interventions of certain types to be selected. In the first case, the optimal time to execute an intervention that alleviates a longitudinal unevenness problem cannot be determined by knowing only the evolution over time of the cracking or texture depth of the pavement surface. In the second case, the optimal time to execute an intervention that improves pavement friction cannot be determined by knowing only the evolution over time of the roughness of the road.

The reasons for insufficient data include the lack of sufficient resources to conduct the necessary inspections, e.g. to purchase required equipment or to hire an adequate number of inspectors, and a perceived negative net benefit, i.e. it is not worth the effort to acquire the additional data.

1.2 *Modelling deterioration and determining optimal intervention strategies*

The modelling of pavement deterioration in pavement management systems (PMSs) can be done either deterministically (Paterson 1990b, Fwa *et al.* 2000, Ferreira *et al.* 2002a, Ouyang and Madanat 2004, Ouyang and Madanat 2006, Sathaye and Madanat 2011) or probabilistically (Ferreira *et al.* 2002b, Hong and Prozzi 2006, Khaled and Maher 2010). One of the probabilistic models used is the Markov model (Shin and Madanat 2003, Tsuda *et al.* 2006, Nakat and Madanat 2008, Lethanh *et al.* 2009). The use of Markov models is also evident in research on the management of other infrastructure objects, such as bridges (Golabi and Shepard 1997, Roelfstra *et al.* 2004, Tsuda *et al.* 2006) and pipelines (Sinha and Knight 2004, Sinha and McKim 2007). Two advantages of the use of Markovian models in situations in which it is desired to determine high level optimal intervention strategies, i.e. for many road sections grouped as one type, e.g. with the same climatic exposure, the same traffic volume and the same basic construction, are as follows:

- They allow generalisation of the deterioration process into the transition pattern among condition states, which, in this case, is suitable for representing pavement performance.
- They can be used in the absence of historical data, as the probability of observing future state depends

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only on the probability of observed condition states at present. Thus, with a minimum of two visual inspections, deterioration progress of the pavement can be predicted.

With the Markov model, the deterioration of road sections is described using the probabilities of transition between discrete condition states. The probabilities of being in each condition state in the Markov transition matrix multiplied by the predetermined or initial condition state probabilities give an overall assessment of pavement condition in respective time frames. In developed countries such as Japan, Switzerland and the USA, the definition of condition states (composite pavement indices) has been standardised to facilitate pavement management, for example the pavement condition index (PCI) developed by the United States of Army Corps of Engineers (Shahin 2005) or the management criteria index (MCI) used in Japan (Lethanh 2009). These composite pavement indices are normally calculated using a deterministic relational function between several important pavement condition indices, such as cracking, texture depth and roughness.

These deterioration models are used in PMSs to determine the condition state (often represented by the value of an aggregate composite index) in which a road section of each type needs to be to trigger the execution of an intervention and the type of intervention to be executed once an intervention is triggered, i.e. the OISs. When inaccurate deterioration models exist, the future condition state of the road section cannot be accurately estimated, and therefore neither can the OISs.

A potential way to overcome this problem is to estimate the transition probabilities using a hidden Markov model (HMM). HMMs have been significantly studied and broadly applied in the field of image processing and speech recognition (Ephraim and Merhav 2002). There has, however, been very little investigation of the use of HMMs in the field of infrastructure management, and the research that has been conducted has focused principally on the elimination of measurement errors (e.g. Lethanh 2009).

In order to use HMMs to model road deterioration, it is necessary to consider that the physical deterioration of the road is something that cannot be observed directly, but that can be deduced through the evolution over time of one aspect related to the physical deterioration of the road, e.g. roughness. In this case, the physical deterioration of the road is considered to be a hidden process, and the aspect related to the physical deterioration of the road is considered to be an observable process. Assuming that the probability of the observable and hidden processes passing between discrete states in fixed time intervals can be represented with exponential distributions, as suggested by Lin and Folias (1975), Paterson (1990a) and assumed in this article, Markov models can then be used to model their evolution over time.

In this case, the transition probabilities of the Markov model for the observable process can be determined by analysing the existing data, and the transition probabilities of the Markov model for the hidden process can be determined by exploiting the statistical correlation between the two processes. The Markov model used to model the hidden process is officially referred to as an exponential hidden Markov model (EHMM).

The use of EHMMs to model road deterioration is explored in this article. The article is divided as follows: Section 2 contains a discussion on the relationship between the observed process and the hidden process and an explanation of the EHMM. Sections 3 and 4 contain the model formulation and estimation methodology, respectively. Section 5 contains a case study, using information from the national road network of Vietnam. Section 6 contains the conclusions of the work and gives indication of future research needs.

2. Observed and hidden processes

A road is normally considered to be in perfect condition immediately after it is built and to deteriorate over time. The condition of the road can be described in different ways, for example discrete condition states may be used, $i (i = 1, \ldots, I)$, where condition state 1 may be considered perfect or like perfect and I alarming, i.e. the condition state where it is simply no longer acceptable and an intervention is to be executed. These condition states are often described with attributes, or indices, e.g. the amount of cracked surface area (m^2) or the value of texture depth (mm). Although these indices do represent the real condition, they are only an indicator of the real condition. The relationship between the values of an indicator and the physical condition is depicted in Figure 1. In Figure 1, the values of the indices increase over time and the physical condition described by condition states $i, j (i \leq j \leq I)$ are shown.

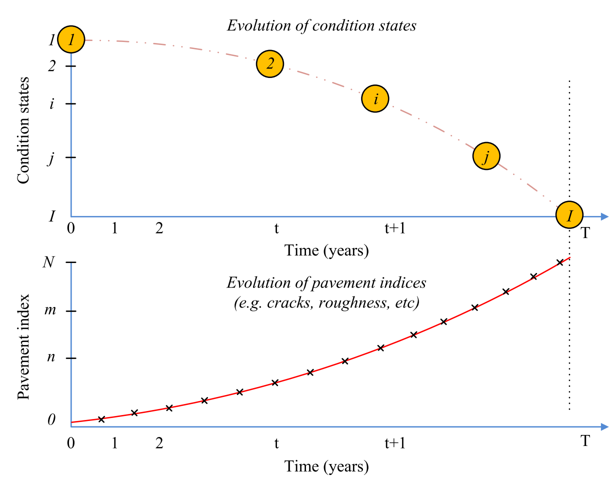

Figure 1. Relationship between road section condition and a condition indicator.

In order to model the deterioration process using a Markov model, it is necessary to estimate the probability of the process passing between condition states in set periods of time. Accurate estimation of these transition probabilities requires that significant data are available on past condition, which is not always available. If it is not possible to accurately estimate the transition probabilities, significant uncertainty is introduced into the deterioration model, and therefore there is a significant lack of confidence that the theoretically OIS can be determined.

If it is assumed that the deterioration processes can be accurately represented using the values of multiple condition indicators, then it is imaginable that the values of all these indicators over time are not available. By exploiting the statistical correlation between the values of the multiple indicators, it is, however, possible to approximate the transition probabilities of the deterioration process using the values of the condition indicators that are available. In such a case, a Markov model can be used to model the evolution of the condition indicator for which data exist, i.e. for the observable process, an EHMM can be used to model the deterioration process, i.e. the hidden process. An illustration of the relationship between the observable and the hidden process is given in Figure 2, where the transition between condition states $g^l(t_r)$ (l of the road section l at respective times $t_r(r = 1, \ldots, T)$ can be deduced from the values of the condition indicator that corresponds to each condition state $h^l(t_r)$. Knowing the condition state transitions over time, the transition probabilities π_{ij} to be used in a Markov model of deterioration can be estimated.

3. The model

3.1 *Transition probabilities*

The model used in this paper to estimate the transition probabilities of the EHMM is an extension of the maximum-likelihood estimation (MLE) method proposed by Tsuda *et al.* (2006). The explicit mathematical formula for estimating Markov transition probability π_{ij} is given in Equation (1). A short description of the model's

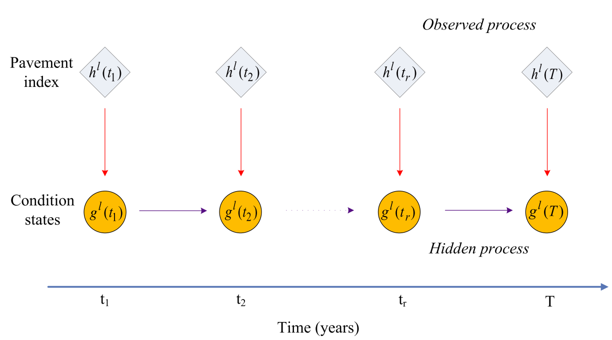

Figure 2. Illustration of the observed and hidden process.

formulation is given in Appendix.

$$\pi^{ij}(z) = \sum_{k=i}^{j} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \prod_{m=k}^{j-1} \frac{\theta_m}{\theta_{m+1} - \theta_k} \exp(-\theta_k z), \quad (1)$$

In Equation (1), the condition states of the road section are described by indices i, j, k, m, $(i \le k \le m \le j)$. The condition states i and j are the priori and posteriori condition states, respectively, m and k are the variables to allow appropriate consideration of condition states between i and j, θ is the deterioration rate, or hazard rate, from respective condition state i, i.e. the rate at which an object goes from condition state i to worse condition states and z is the time interval between two inspections. Statistically, the hazard rate θ can be expressed in the multiplicative form $\theta = x \cdot \beta$ (refer to Equation (A11) in Appendix), where x is a vector of characteristic variables (traffic volume, change in pavement thickness, temperature, etc.) that affect the deterioration process, and β is to be estimated by the given information on the evolution of condition states and the characteristic variables.

3.2 *Exponential hidden Markov model*

In the EHMM, it is assumed that the Markov transition probabilities π_{ij} are used to model the hidden deterioration process, and the available data are the values of a condition indicator. The occurrence of the values of the condition indicator can be represented as the random variable $h(t)$, with t being time.

The probability distribution of $h(t)$ is considered to be dependent on the condition state $g(t) = i(1, \ldots, I)$ at time t. The probability of the condition indicator having a specific value $h(t) = n$, given $g(t) = i$ at time t, is assumed to follow an exponential probability distribution (Equation (2)).

$$\gamma_{\text{in}} = \text{Prob}[h(t) = n | g(t) = i] = \lambda_i \exp(-\lambda_i t), \quad (2)$$

where n is the value of the condition indicator at time t and λ_i is the rate parameter (or parameter of distribution) associated with condition state i. In other words, it is the rate of leaving condition state i.

Equation (2) is also constrained in that the summation of all possible probabilities γ_{in} must be equal to 1 ($\int_{n=1}^{N} \gamma_{\text{in}} = 1$), where N is the worst possible value of the condition indicator.

The conditional probability distribution of $h(t)$ with respect to condition state i can then be determined using the joint probability distribution function of condition state and value of condition indicator.

$$\begin{aligned} &\text{Prob}[h(t) = n] = \text{Prob}[h(t) = n | g(t) = i] \\ &\text{Prob}[g(t) = i] = \gamma_{\text{in}} \rho_i, \end{aligned} \quad (3)$$

where ρ_i is a conditional state probability of the object being in state i. ρ_i can be defined as $\rho_{i,t} = \rho_{i,t-1}\pi_{ij}$.

Since the values of condition indices are affected by both deterioration and the improvement resulting from interventions, the likelihood function to be used to determine the values of the condition indicator is

$$\tilde{\mathcal{L}}\{h(t)=n\} = \sum_{i_1=1}^{I}\sum_{i_2=1}^{I}\cdots\sum_{i_T=1}^{I}\rho_{i_1}\gamma_{i_1,n_1}\prod_{t=2}^{T}\pi_{i_{t-1},i_t}\gamma_{i_t,n_t}. \quad (4)$$

Given the exponential distribution representing the relationship between the condition states and the value of the condition indicator in Equation (2), Equation (4) can be rewritten as

$$\tilde{\mathcal{L}}\{h(t)=n\} = \sum_{i_1=1}^{I}\sum_{i_2=1}^{I}\cdots\sum_{i_T=1}^{I}\rho_{i_1}\exp(-\lambda_{i_1}1) \cdot \prod_{t=2}^{T}\pi_{i_{t-1},i_t}\exp(-\lambda_{i_t}t). \quad (5)$$

It is noted that Equations (4) and (5) are in their explicit forms. The equations have embedded model's parameters (variables) β, θ and λ, which can be read out from Equations (1), (2) and (A11). Similar types of expressions for Equations (4) and (5) can be found in the recent literature (Paroli *et al.* 2000, Lethanh *et al.* 2011).

4. Methodology

In order to explain the methodology, it is convenient to introduce the following terminology. A set of observed values of the condition indicator for a road section l in a road network with L sections are denoted as $\tilde{\Xi}_t^l[g(t),h(t),x],(t=1,\ldots,T^l)$, where T^l is the time of the most recent inspection for the road section l. The sign $\sim$ indicates the measurable vector. The hazard rate of a road section l is influenced by the changes in the values of characteristic variables, such as traffic volume, thickness of overlay and weather, which is shown in Equation (A11) in Appendix, and is referred to as θ^l.

In order to use the observed data, it is also necessary to introduce the following two dummy variables:

$$\delta_i^{tl} = \begin{cases} 1 & \text{if } g^l(t)=i \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

and

$$\delta_{ij}^{tl} = \begin{cases} 1 & \text{if } g^l(t-1)=i \text{ and } g^l(t)=j \\ 0 & \text{otherwise} \end{cases} \quad (t=1,\ldots,T; i=1,\ldots,I). \quad (7)$$

By using these dummy variables and by taking the natural logarithm on both sides of the likelihood function in Equation (5) with respect to the complete set of data, i.e. all information related to the observed and hidden processes, the following complete likelihood function can be obtained:

$$\ln\tilde{\mathcal{L}} = \sum_{l=1}^{L} = \left\{\sum_{i=1}^{I}\delta_i^{1l}\ln(\rho_i^l) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(\sum_{t=1}^{T}\delta_{ij}^{tl}\right)\ln\pi_{ij}^l - \sum_{i=1}^{I}\sum_{t=1}^{T}\delta_i^{tl}\lambda_i^{t,l}n_t\right\}. \quad (8)$$

The values of the parameters in the set $\Theta = (\rho,\pi,\lambda)$ are then estimated by determining the optimal solution to Equation (8), i.e. the most likely values of the hidden process knowing the values of the observable process.

This is done using the MLE method, a popular statistical method used to fit statistical models to observed data, and the Baum–Welch algorithm, a particular case of expectation-maximum (EM) algorithm (McLachlan and Krishnan 1997), which is suitable for determining optimal solutions for HMMs when there are incomplete data. Using the Baum–Welch algorithm, the likelihood in Equation (8) can be simplified as

$$L_c = \text{Prob}\{g(t)=i|h(t)=n\}\cdot\text{Prob}\{h(t)=n\}. \quad (9)$$

The maximisation of Equation (9) is done by performing the EM algorithm.[2] The steps of EM algorithm are outlined in Table 1.

5. Example

5.1 *Problem*

In this example, it is assumed that it is necessary to know the evolution of a composite condition index (CCI) over time in order to determine the OIS, but that is not desired to collect the data required on all road sections. In this example, the CCI may be thought of as a better approximation of the physical condition of a road section than that provided by either the roughness of the road

Table 1. Outline of EM algorithm.

1.	Set the initial values for $\Theta = (\rho,\pi,\lambda)$
2.	Start the LOOP
3.	Do the E-step to estimate the values of the dummy variables δ_i^t and δ_{ij}^t given the initial values of Θ
4.	Do the M-step to compute the values of Θ by maximising the likelihood function Equation (9)
5.	Check the convergent condition for the values of Θ a. if not satisfied, the iteration goes back to step 2 b. if satisfied, stop the iteration

section or the texture depth of the road section alone. The value of the CCI is estimated using the weighted values of the roughness and the texture depth as shown in the following equation:

$$\mathrm{CCI} = w_r \cdot r + w_{\mathrm{td}} \cdot \mathrm{td}, \tag{10}$$

where r is the roughness (mm/m), td is the texture depth,[3] and w is the weighting factors, where $w_r = 0.5$ and $w_{\mathrm{te}} = 0.5$.

The data used in the example were collected in the years 2001 and 2004 on the condition of 1′237 asphalt concrete road sections of 1 km in length in Vietnam (Table 2).

The condition states of the road section were defined as shown in Table 3.

It is considered that four inspection scenarios are possible (Table 4).

With

- *inspection scenario 1*: both roughness and texture depth measurements are made on all 1′237 road sections, and therefore, the CCI can be calculated directly. Also, the multi-stage exponential Markov model (refer to Appendix) can be used directly given the historical evolution of CCI over time.
- *inspection scenario 2:* roughness measurements are made on all 1′237 road sections but texture depth measurements are made on only 618 road sections (equivalent to 50% of the original data).
- *inspection scenario 3:* roughness measurements are made on all 1′237 road sections but texture depth measurements are made on only 309 road sections (equivalent to 25% of the original data).
- *inspection scenario 4:* roughness measurements are made on all 1′237 road sections but texture depth measurements are made on only 124 road sections (equivalent to 10% of the original data).

The four scenarios were selected to demonstrate the relationship between the amount of available data and the accuracy of deterioration prediction, when using the HMM.

Table 2. Overview of data.

Condition indicator	Unit	Numbers of data 2001	2004
Roughness	mm/m	6′510	6′510
Texture	mm	1′237	1′237

Table 3. Notations of condition states.

Condition states	Equivalent CCI	Remark
1	0–1.5	Very good
2	1.5–3	Good
3	3–4.5	Fair
4	4.5–6	Poor
5	>6	Very poor

5.2 *Results*

For scenario 1, the hazard rate θ_i, i.e. the speed at which the CCI leaves each condition state, was defined as a function of a constant term x_1 and a traffic volume x_2 (Equation (A11) in Appendix). Traffic volume was chosen, as it is considered as a significant factor influencing the deterioration process. It is important to note that more characteristic variables such as thickness of road sections, ambient temperatures or properties of materials should be included in the functional form of Equation (A11). However, due to lack of data concerning these variables, and without loss of generality of the

Table 4. Investigated inspection scenarios and modelling characteristics.

Scenarios	1	2	3	4
1. Data-set				
1.1. No. of road sections	1′237	1′237	1′237	1′237
1.2. No. of road sections with roughness data	1′237	1′237	1′237	1′237
1.3. No. of road sections with texture depth data	1′237	618(50%)	309(25%)	124(10%)
2. Process				
2.1. CCI	Obs.	Hidden	Hidden	Hidden
2.2. Roughness	Obs.	Obs.	Obs.	Obs.
2.3. Texture depth	Obs.	NA	NA	NA
3. Initial values of the exponential rate λ_i on each condition state i				
λ_1		0.333	0.333	0.333
λ_2		0.111	0.111	0.111
λ_3		0.100	0.100	0.100
λ_4		0.067	0.067	0.067
λ_5		0.064	0.064	0.064

model's application, in this example, only traffic volume was selected as the representative one, although effects of other variables are thought to be truncated in the values of x_1 attached to constant term. Using the multi-stage exponential Markov model of Tsuda *et al.* (2006), values of the unknown parameters β, the hazard rate θ_i and the expected Markov transition probability π_{ij} were obtained, as shown in Tables 5 and 6, respectively.

As can be seen from Table 5, the values of the unknown parameter β_i and their corresponding t-values (in brackets) infer that the traffic volume has a strong impact on the deterioration speed, especially on the last two condition states 3 and 4. Reasons behind the higher impact of traffic volume on condition states 3 and 4 can be due to the following reasons:

- Poor routine maintenance on condition states 1 and 2 might contribute to the acceleration of deterioration speed in condition states 3 and 4, especially under the increase of traffic volume.
- After a road section enters condition state 3, its bearing capacity becomes weaker, and under higher traffic volume, a faster deterioration speed occurs.

It is noted that for more precise conclusion on the influences of traffic volume and other factors on deterioration speed, an in-depth study should be carried out.

For the three scenarios in which there are incomplete texture depth data (scenarios 2–4), the initial values of the exponential rate parameters λ_i for each condition state i of the hidden process are estimated by mapping the values of the roughness to each condition state, using the MLE approach (Equation (11)).

$$\bar{\eta}_i = \frac{1}{n}\sum_{e=1}^{n} \eta_{i,e}, \tag{11}$$

where $\eta_{i,e}$ is the observed value and $\bar{\eta}_i$ is its mean.

The rate parameter $\bar{\lambda}_i$ can then be defined as

$$\bar{\lambda}_i = \frac{1}{\bar{\eta}_i}. \tag{12}$$

The obtained values of λ_i for each condition state i ($\lambda_i = 0.333, 0.111, 0.100, 0.067, 0.064$). These values are the same for all scenarios, as the total number of road sections with roughness measurements for each scenario is $1'237$ (Table 4).

The values of the Markov transition probabilities and the hazard rates for scenario 2 are given in Table 7.

To verify the confidence in the estimation results, the distribution of residuals, i.e. the value representing the unexplained variation after fitting the data to the regression model, or in other words the difference between the observed value and the value determined using the regression model, has a normal distribution with a nice bell shape (Figure 3) and their sum around the mean is 0, indicating that the regression model is appropriate for the data (Montgomery and Runger 2007).

Estimation results for scenarios 3 and 4 are further shown in Tables 8 and 9.

5.3 *Comparison*

Given the obtained values of hazard rates θ_i for four scenarios (Tables 6–9), the differences with regard to deterioration speed among the four scenarios can be compared. Using Equations (A12) and (A13) in Appendix, the duration of stay in each condition state i is determined, and as a result, total time for a typical road section from new condition to condition state 5 can be estimated (Figure 4).

As can be interpreted from Figure 4, the expected deterioration for all road sections (Scenarios 2–4) is almost linear from condition states 1 to 3, with almost equal amounts of time spent in each condition states 1 and 2, and there is a sharp decrease in condition with respect to time when the condition reaches condition state 3. The average time for the road section to stay in condition states

Table 5. Values of the β parameters in the multi-stage exponential Markov model (scenario 1).

Condition states	Absolute $\beta_{i,1}$	Traffic volume $\beta_{i,2}$
1	0.3493	–
	(54.9292)	
2	0.1747	–
	(29.0005)	
3	0.2600	2.8944
	(9.7360)	(2.7631)
4	0.2321	2.9035
	(3.0572)	(1.2187)

Table 6. Scenario 1: Markov transition probabilities and hazard rates.

Condition states	1	2	3	4	5	Hazard rate θ_i
1	0.7052	0.2691	0.0225	0.0029	0.0003	0.3493
2	0	0.8397	0.1314	0.0255	0.0034	0.1747
3	0	0	0.6705	0.2718	0.0578	0.3998
4	0	0	0	0.6893	0.3107	0.3721
5	0	0	0	0	1	–

Table 7. Scenario 2: Markov transition probabilities and hazard rates.

Condition states	1	2	3	4	5	Hazard rate θ_i
1	0.7300	0.2461	0.0215	0.0022	0.0002	0.3147
2	0	0.8361	0.1395	0.0216	0.0028	0.1790
3	0	0	0.7256	0.2281	0.0463	0.3208
4	0	0	0	0.6968	0.3032	0.3612
5	0	0	0	0	1	–

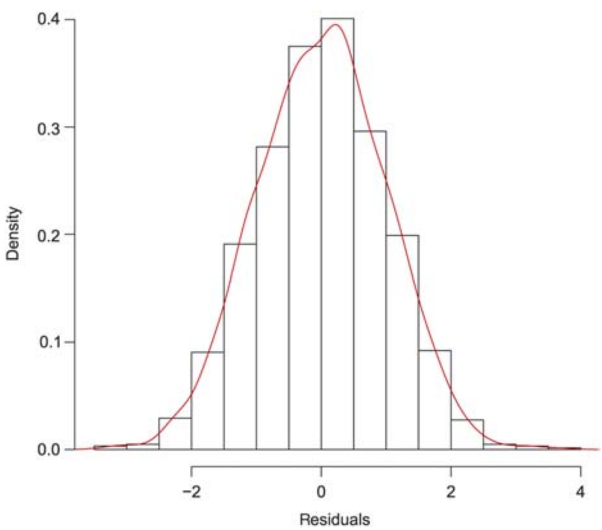

Figure 3. Distribution of residuals (scenario 2).

3 and 4 is approximately 3 years. As a result, it takes on average 14 years for a typical road section to go from new to condition state 5. This speed of deterioration is normal in Vietnam. For a better understanding of deterioration, a visual representation of condition state distribution over time for scenario 1 is shown in Figure 5.

Comparing scenarios 1 and 2, it can be seen that the predicted deterioration is similar, especially when the road section is in condition states 1 and 2. Using scenario 1 as the reference case, the deviation in predictions is small (less than 3% with respect to the time required to enter condition state 5 and even less with respect to entering each condition state before condition state 5).

Comparing scenarios 1 and 3, it can be seen that the predicted deterioration is similar when the road section is in condition states 1 and 2. Also using scenario 1 as the reference case, the deviation in predictions is getting slightly larger, especially when the road section is in condition states 3–5 (less than 6.4% with respect to the time required to enter condition state 5 and proportionally less with respect to entering each condition state before condition state 5).

Comparing scenarios 1 and 4, it can be seen that the predicted deterioration is similar when the road section is in condition states 1 and 2. Also using scenario 1 as the reference case, the deviation in predictions increases with worsening condition and is up to 11.6% with respect to the time required to enter condition state 5.

Comparing scenarios 3 and 4 with scenario 1, it can be seen that the deterioration predicted

- is slightly slower,
- has a different form and
- indicates the same deterioration speed between condition states 1 and 2 but slower between condition states 2 and 5.

Table 8. Scenario 3: Markov transition probabilities and hazard rates.

Condition states	1	2	3	4	5	Hazard rate θ_i
1	0.7300	0.2460	0.0216	0.0022	0.0002	0.3147
2	0	0.8357	0.1402	0.0216	0.0025	0.1796
3	0	0	0.7289	0.2295	0.0416	0.3128
4	0	0	0	0.7226	0.2774	0.3251
5	0	0	0	0	1	–

Table 9. Scenario 4: Markov transition probabilities and hazard rates.

Condition states	1	2	3	4	5	Hazard rate θ_i
1	0.7300	0.2462	0.0215	0.0021	0.0002	0.3147
2	0	0.8369	0.1402	0.0206	0.0023	0.1780
3	0	0	0.7406	0.2216	0.0378	0.3003
4	0	0	0	0.7354	0.2646	0.3074
5	0	0	0	0	1	–

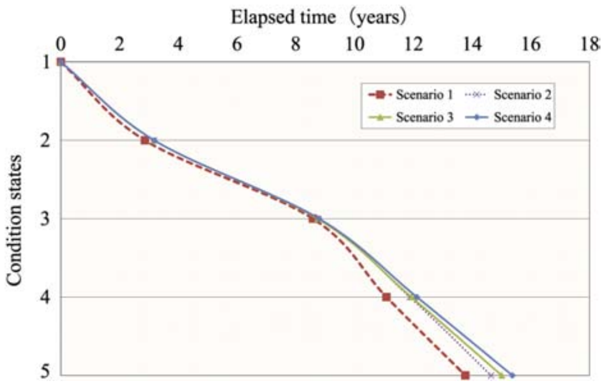

Figure 4. Performance curves.

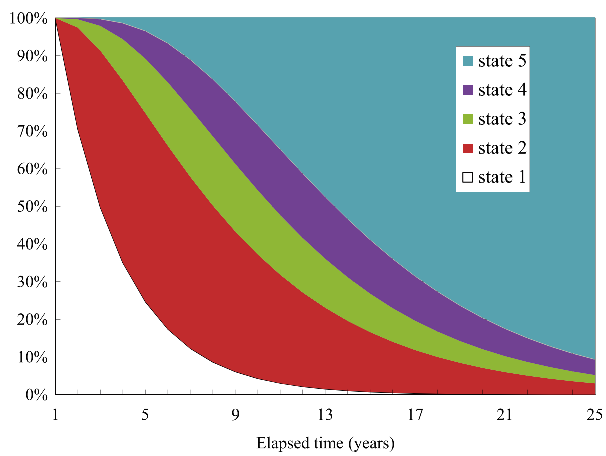

Figure 5. Condition state distribution.

The reasons for these differences are that scenario 1 is a better representation of the behaviour of the road sections than the other scenarios.

In general, it can be seen, not surprisingly, that the more texture depth information available, the better the correlation between the deterioration predicted using incomplete monitoring data (scenario 2 – 3% deviation, scenario 3 – 6.4% deviation and scenario 4 – 11.6% deviation) and the deterioration predicted using complete monitoring data.

6. Conclusion

In this paper, the potential use of the EHMM to predict the deterioration of road sections when incomplete inspection data are available is demonstrated. It was demonstrated for the scenarios in which the values of an overall performance indicator, obtained from two indices, are required to determine the optimal intervention strategy, but there are less data available with respect to one of the indices than the other indices. The evolution of the overall performance indicator is described as an exponential distribution, and is considered to be hidden. The exponential hidden Markov model was used to estimate the evolution over time of the overall performance indicator from the incomplete data.

The model was tested using the data collected on 1′237 road sections in the Vietnamese national road network. A CCI representing discrete condition states in Markov model was built using roughness and texture depth indices. Four scenarios were investigated. It was found that the more the texture depth data available, the closer the deterioration prediction between the multi-stage exponential Markov model and the proposed hidden Markov model. Perhaps one of the most interesting results from the example is that even if texture depth data were available for only 50% of the road sections, the time of entry into condition state 5 could be predicted with only a 3% deviation from that predicted if texture depth data were available for 100% of the road sections. This indicates that using this methodology it might be possible to reduce the number of inspections required on road networks in cases in which all indices are inspected for all road sections. In addition to allowing road authorities to deal with incomplete data in the past, it is expected that the use of the EHMM may allow road authorities to reduce the number of inspections that need to be made in order to make accurate predictions in the future.

The methodology presented in this paper can be applied not only for prediction of the deterioration of road sections but also for other types of infrastructure objects.

Future work on improving this model could be focused on:

- The use of the Bayesian estimation approach instead of the MLE method with EM algorithm. Bayesian estimation is considered as more advantageous than the MLE method since it increases the chances of determining the globally optimal solution.
- Using the model to develop an inspection strategy for a road authority taking into consideration the costs of intervention and the use of the results of the inspection information in determining the intervention strategies to be followed and the work programmes to be executed.
- Testing the model on a much more extensive data-set. Ideally, such a data-set would include the values of many different condition indicators over many years and on many different road sections. It would then be possible to evaluate in great detail the effectiveness of the model.
- Investigating different functional forms of the deterioration models, e.g. the use of Euler's confined space growth function to represent the initiation and evolution of cracking and other indices.

Notes

1. Email: adey@ibi.baug.ethz.ch
2. A detailed explanation for the M-step and E-step in the EM algorithm can be found in Dempster *et al.* (1977) and McLachlan and Krishnan (1997).
3. Time series data of texture depth were transformed from decreasing values to increasing values.

References

Dempster, A., Laird, N.M., and Rubin, D.B., 1977. Maximum likelihood from incomplete data via the EM Algorithm. *Journal of the Royal Statistical Society: Series B*, 39 (1), 1–38.

Ephraim, Y. and Merhav, N., 2002. Hidden Markov processes. *IEEE Transactions on Information Theory*, 48 (6), 1518–1569.

Ferreira, A., *et al.*, 2002a. A segment-linked optimization model for deterministic pavement management systems. *The International Journal of Pavement Engineering*, 3 (6), 95–105.

Ferreira, A., *et al.*, 2002b. Probabilistic segment-linked pavement management optimization model. *Journal of Transportation Engineering*, 128 (6), 568–577.

Fwa, T., Chan, W., and Hoque, K., 2000. Multiobjective optimization for pavement maintenance programing. *Journal of Transportation Engineering*, 126 (5), 367–374.

Golabi, K. and Shepard, R., 1997. Pontis: a system for maintenance optimization and improvement of US bridge networks. *Interfaces*, 27 (1), 71–88.

Hong, F. and Prozzi, J., 2006. Estimation of pavement performance deterioration using Bayesian approach. *Journal of Infrastructure System*, 12 (2), 77–86.

Khaled, A. and Maher, M., 2010. *Pavement Rehabilitation Project Ranking Approach Using Probabilistic Long-term Performance Indicators*. Vol. 2153. Washington, DC: Transportation Research Board, National Academy Press, 3–12.

Lethanh, N., 2009. Stochastic Optimization Methods for Infrastructure Management with Incomplete Monitoring Data. Thesis (Ph.D.). Graduate School of Engineering, Kyoto University.

Lethanh, N., *et al.*, 2009. A benchmarking approach to pavement management: lessons from Vietnam. *Infrastructure Planning Review*, 26 (1), 101–112.

Lethanh, N., *et al.*, 2011. Modelling pavement deterioration process by poisson hidden Markov model. *In*: *Proceedings of JSCE, F4 (in Japanese)*.

Lin, S.T. and Folias, E., 1975. On the fracture of highway pavements. *International Journal of Fracture*, 11, 93–106, doi:10.1007/BF00034716.

McLachlan, G.J. and Krishnan, T., 1997. *The EM Algorithm and Extensions*. Vol. 274 John Wiley & Sons, USA.

Montgomery, D. and Runger, G., 2007. *Applied Statistics and Probability for Engineer*. Wiley India Pvt. Ltd, John Wiley & Sons, USA.

Nakat, Z. and Madanat, S., 2008. Stochastic duration modeling of pavement overlay crack initiation. *Journal of Infrastructure Systems (ASCE)*, 14 (3), 185–192.

Ouyang, Y. and Madanat, S., 2004. Optimal scheduling of rehabilitation activities for multiple pavement facilities: exact and approximate solutions. *Transportation Research Part A: Policy and Practice*, 38 (5), 347–365.

Ouyang, Y. and Madanat, S., 2006. An analytical solution for the finite-horizon pavement resurfacing planning problem. *Transportation Research Part B: Methodological*, 40 (9), 767–778.

Paroli, R., Redaelli, G., and Spezia, L., 2000. Poisson hidden Markov models for time series of overdispersed insurance counts. *ASTIN Colloquium International Actuarial Association-Brussels, Belgium*, 461–472.

Paterson, W., 1990a. International roughness index: relationship to other measures of roughness and riding quality. *Transportation Research Record*, 1084, 49–59.

Paterson, W., 1990b. Quantifying the effectiveness of pavement maintenance and rehabilitation. *In*: *Proceedings at the 6th REAAA Conference*, Kuala Lumpur, Malaysia.

Roelfstra, G., *et al.*, 2004. Condition evolution in bridge management systems and corrosion-included deterioration. *Journal of Bridge Engineering*, 9 (3), 268–277.

Sathaye, N. and Madanat, S., 2011. A bottom-up solution for the multi-facility optimal pavement resurfacing problem. *Transportation Research Part B: Methodological*, 45 (7), 1004–1017.

Shahin, M., 2005. *Pavement Management for Airports, Roads, and Parking Lots*. Springer, USA.

Shin, H.C. and Madanat, S.M., 2003. Development of stochastic model of pavement distress initation. *Journal of Infrastructure Planning and Management*, IV-61 (744), 61–67.

Sinha, S.K. and Knight, M.A., 2004. Intelligent system for condition monitoring of underground pipelines. *Computer-Aided Civil and Infrastructure Engineering*, 19 (1), 42–53.

Sinha, S.K. and McKim, R.A., 2007. Probabilistic based integrated pipeline management system. *Tunnelling and Underground Space Technology*, 22 (5–6), 543–552, Trenchless Technology.

Tsuda, Y., *et al.*, 2006. Estimating Markovian transition probabilities for bridge deterioration forecasting. *Journal of Structural Engineering and Earthquake Engineering*, 23 (2), 241–256.

Appendix: Mathematical formulation of the multi-stage exponential Markov model Tsuda *et al.* (2006)

From condition data from two temporally consecutive inspections t and $t+1$, the Markov transition probability can be described as follows:

$$\mathrm{Prob}[g(t+1)=j|g(t)=i]=\pi^{ij}. \tag{A1}$$

Markov transition probability matrix can be written in the following form:

$$\Pi=\begin{pmatrix}\pi^{11} & \cdots & \pi^{1I}\\ \vdots & \ddots & \vdots \\ 0 & \cdots & \pi^{II}\end{pmatrix}, \tag{A2}$$

where

$$\left.\begin{array}{l}\pi^{ij}\geq 0 \quad (i,j=1,\ldots,I)\\ \pi^{ij}=0 \quad (\text{when } i>j)\\ \sum_{j=i}^{I}\pi^{ij}=1\end{array}\right\}. \tag{A3}$$

The time that an object is expected to stay in condition state i is assumed to be a stochastic variable, with the probability density function $f_i(\zeta_i)$ and distribution function $F_i(\zeta_i)$. The conditional probability, to which condition state i at time y_i reaches condition state $i+1$ at $y_i+\Delta_i$, can be expressed as hazard function $\lambda_i(y_i)\Delta y_i$:

$$\lambda_i(y_i)\Delta y_i=\frac{f_i(y_i)\Delta y_i}{\tilde{F}_i(y_i)}, \tag{A4}$$

where $\tilde{F}_i(y_i)=1-F_i(y_i)$ is referred as the survival function of the object in condition state i during the time interval from $y_i=0$ to y_i.

Since it is assumed that the deterioration process satisfies the Markov property and the hazard function is independent of the time instance y_i the hazard rate is constant and positive:

$$\lambda_i(y_i)=\theta_i. \tag{A5}$$

The time that an object is in condition state i longer than the time instance y_i is referred as the value of the survival function $\tilde{F}_i(y_i)$ and can be expressed in the exponential form as

$$\tilde{F}_i(y_i)=\exp(-\theta_i y_i). \tag{A6}$$

When y_i equals z of the inspection period z_i between $[t, \text{and} t+1)$, the value of the survival function is identical to the transition probability π^{ii}, therefore

$$\begin{aligned}\tilde{F}_i(t+z|\zeta_i\geq t)&=\mathrm{Prob}\{\zeta_i\geq t+z|\zeta_i\geq t\}\\ &=\frac{\exp\{-\theta_i(t+z)\}}{\exp(-\theta_i t)}=\exp(-\theta_i z),\end{aligned} \tag{A7}$$

$$\mathrm{Prob}[g(t+1)=i|g(t)=i]=\exp(-\theta_i z). \tag{A8}$$

By defining the subsequent conditional probability of condition state j to i, with respect to the actual interval time z of inspection, a general mathematical formula for estimating the Markov transition probability can be defined as

$$\begin{aligned}\pi^{ij}(z)&=\mathrm{Prob}[h(t+1)=j|h(t)=i]\\ &=\sum_{k=i}^{j}\prod_{m=i,\neq k}^{k-1}\frac{\theta_m}{\theta_m-\theta_k}\exp(-\theta_k z),\end{aligned} \tag{A9}$$

where

$$\begin{aligned}&\prod_{m=i,\neq k}^{k-1}\frac{\theta_m}{\theta_m-\theta_k}\exp(-\theta_k z)\\ &=\prod_{m=i}^{k-1}\frac{\theta_m}{\theta_m-\theta_k}\prod_{m=k}^{j-1}\frac{\theta_m}{\theta_{m+1}-\theta_k}\exp(-\theta_k z),\end{aligned}$$

and

$$\begin{cases}\prod_{m=i}^{k-1}\frac{\theta_m}{\theta_m-\theta_k}=1 \quad (k=i)\\ \prod_{m=k}^{j-1}\frac{\theta_m}{\theta_{m+1}-\theta_k}=1 \quad (k=j)\end{cases}$$

$$(i=1,\ldots,I-1; j=i+1,\ldots,I).$$

Transition probability from condition state i to absorbing condition state I is eventually defined in the following equation:

$$\pi^{iI}(z)=1-\sum_{j=i}^{I-1}\pi^{ij}(z) \quad (i=1,\ldots,I-1). \tag{A10}$$

The likelihood function of hazard rate θ_i can be expressed in multiplicative form with characteristic variable x and unknown parameter β_i'.

$$\theta_i=\theta_i(x)=x\beta_i'. \tag{A11}$$

The remaining time of the object in condition state i $\mathrm{RMD}_i(x)$ is then given by the survival probability of condition state i over continuous time.

$$\mathrm{RMD}_i(x)=\int_0^\infty \tilde{F}_i(y_i|\theta_i(x))\mathrm{d}y_i=\int_0^\infty \exp\{-\theta_i(x)y_i\}\mathrm{d}y_i=\frac{1}{x\beta_i'}. \tag{A12}$$

The average time of the object in condition state $j(>1)$ can be defined by the summation of the times over the range of condition states counted from $i=1$:

$$\mathrm{ET}_j(x)=\sum_{i=1}^{j}\frac{1}{x\beta_i'}, \tag{A13}$$

where ET_j stands for average time of the object being in condition state j.