

UNIVERSITY
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Master's Thesis

NEUROMORPHIC COMPUTING

With Spiking Neural Networks

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ABSTRACT

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ACKNOWLEDGEMENTS

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Wordcount: 15569

GLOSSARY

AI – Artificial Intelligence 5, 5, 6, 6, 7, 7, 31

CNN – Convolutional Neural Network 12

MLP – Multi Layer Perceptron 9, 10

SNN – Spiking Neural Network 32

1 • INTRODUCTION

The development of intelligent machines represents a significant objective in modern science and engineering. While the concept has historical roots in philosophy and early automata, the field has transitioned from speculative theory to practical application. Currently, artificial intelligence is central to technological and economic development. Understanding the mechanisms of intelligence and reproducing them in synthetic systems offers the potential for improved analysis of biological minds and the creation of tools for applications ranging from personalized medicine to automated scientific discovery. Some have argued that general-purpose Artificial Intelligence (AI) could act as a terminal invention for humanity [1].

In recent years, progress toward this goal has increased. Deep Learning, which utilizes multilayered neural networks, has exceeded previous performance benchmarks. These systems have demonstrated high proficiency in tasks previously limited to human capability. For example, AlphaFold has addressed complex problems in protein folding [1], reinforcement learning agents have managed the complexity of games such as Go [1], and Large Language Models have shown capabilities in text generation that approach human fluency. Consequently, AI is increasingly viewed as a general-purpose technology that may influence societal infrastructure.

However, despite these advances, there are significant limitations to the current approach. The success of modern deep learning relies heavily on scaling, which involves increasing data volume and computational power. This strategy is approaching physical and economic boundaries. Training state-of-the-art models consumes substantial energy and results in a large carbon footprint [1]. Although specialized hardware provides some efficiency, the underlying architecture remains computationally expensive. Furthermore, the requirement for massive datasets presents challenges in sourcing and curation. Additionally, evidence suggests that this scaling approach yields diminishing returns. Models often function as statistical correlation engines; they lack common-sense reasoning, struggle with out-of-distribution generalization, and are prone to brittle failure modes.

These limitations are evident when comparing artificial systems to biological intelligence. The human brain demonstrates that high-level intelligence is possible without massive energy consumption or dataset sizes. The brain operates on approximately 20 watts [1]. With this limited energy budget, it manages biological functions, processes real-time multi-sensory data, and performs abstract reasoning. In contrast, deep learning models require GPU clusters with significantly higher power requirements to match a fraction of these capabilities. There is also a discrepancy in learning efficiency. Deep learning models are sample-inefficient, often requiring vast numbers of examples to learn a representation. Biological systems, however, are capable of “one-shot” or “few-shot” learning and can acquire

new information without catastrophic forgetting. This suggests the inefficiency of current AI is a paradigmatic issue rather than just an engineering problem.

The proposed direction for addressing these issues involves biological inspiration in hardware design, specifically Neuromorphic Computing. This field attempts to engineer computer architectures that mimic the biological structure of the nervous system. Unlike traditional AI, which runs as software on general-purpose hardware, neuromorphic engineering aims to align the algorithm with the physical substrate. It moves away from clock-driven processing toward asynchronous, event-driven systems. In this paradigm, information is encoded as sparse, discrete events or “spikes.” Similar to biological neurons, a neuromorphic processor consumes minimal energy when inactive, processing information only when triggered. This approach is being pursued by both industrial groups, such as Intel’s Loihi [1], and academic projects like SpiNNaker [1]. These systems represent a shift from calculation-based machines to those capable of real-time adaptation.

Although neuromorphic systems achieve optimal performance on cross-stack optimized platforms—where the algorithm is embedded directly into the hardware—there is remaining utility in executing neuromorphic algorithms on traditional von Neumann architectures. In this thesis, we explore biologically inspired algorithms deployed on traditional CPU and GPU hardware. We examine how event-driven, biologically plausible computation can address limitations in scalability, data efficiency, and energy consumption, even when simulated on standard processors. We present approaches for efficient information coding and learning algorithms inspired by neural mechanisms. However, biological inspiration is constrained by engineering realities. We cannot strictly replicate the brain’s 3D connectivity due to the planar limitations of CMOS manufacturing. Therefore, a pragmatic approach is necessary. We must identify and abstract the functional principles of the brain, separating core computational mechanisms from biological details that may be evolutionary artifacts. Concretely, this thesis aims to:

- **Investigate Sparse Information Flow:** Explore how information can be encoded and processed using sparse, asynchronous events (spikes) within a neural network.
- **Develop Biologically Plausible Learning:** Explore and evaluate learning algorithms that are suitable for such networks, adhering to the constraints of locality and efficiency.

The remainder of this thesis is organized as follows: The succeeding chapter lays the theoretical foundation, covering early neuroscience and the development of artificial neural networks based on simple models of the brain. Following this, we review relevant modern neuroscience literature, extracting key concepts that will inform the methodology. Finally, we detail the implementation of these principles in a neuromorphic context and evaluate their performance against standard benchmarks.

2 • BACKGROUND

The aim of this section is to shed light on the history and developments of AI, we will also go through key concepts in modern neuroscience that will be important to understand the motivation for the methods used in this thesis.

We will begin at that shared origin point, a time when AI research and neuroscience were one and the same. We will then follow a diverging path that leads to modern deep learning and understand why it has little in common with modern neuroscience but also why it is so powerful but also so inefficient. Finally, we will explore the neuromorphic path, the modern neuroscience it is built upon. We will end this section with a review of similar frameworks and methods—showcasing their strengths and weaknesses and how we can improve upon them.

2.1 • NEURON DOCTRINE

Historically, the understanding of neural tissue was dominated by the reticular theory, which posited that the brain consisted of a continuous, fused network. This paradigm was fundamentally challenged by the work of Santiago Ramón y Cajal. Through the application of novel staining techniques, Cajal established the neuron doctrine, demonstrating that the nervous system is composed of discrete, individual cells [1]. Subsequent analysis using electron microscopy has provided irrefutable validation of this discrete cellular structure.

The conceptualization of the brain as a collection of discrete units facilitated the development of mathematical models describing neural function. In 1943, Warren McCulloch and Walter Pitts published A Logical Calculus of the Ideas Immanent in Nervous Activity, introducing the first formal model of the neuron.

The McCulloch-Pitts (M-P) neuron abstracted biological complexity into a binary decision device governed by the following logic:

- **Integration:** The neuron receives multiple binary inputs, weighted as either excitatory or inhibitory.
- **Summation:** The unit calculates the linear sum of these weighted inputs.
- **Thresholding:** If the aggregate sum exceeds a fixed threshold, the neuron outputs a 1 (firing); otherwise, it outputs a 0 (silence).

McCulloch and Pitts demonstrated that networks of these units could theoretically compute any logical operation (AND, OR, NOT) [1]. This abstraction established the foundational link between biological processes and digital logic, suggesting that neural function could be replicated in electronic hardware. Consequently, the M-P

neuron serves as the common ancestor for both computational neuroscience and artificial intelligence.

However, despite its theoretical utility, the original M-P model presented significant functional limitations. The connectivity was static, requiring circuits to be manually designed rather than learned. Furthermore, the restriction to binary weights precluded the modeling of graded signal intensity, preventing the system from capturing the nuance of real-world sensory input.

In 1949, Donald Hebb addressed the critical issue of plasticity in his work *The Organization of Behavior*. He proposed a theoretical mechanism for synaptic modification, now known as Hebbian learning, which provided a biological basis for how neural networks could adapt over time. Hebb postulated:

“Let us assume that the persistence or repetition of a reverberatory activity (or “trace”) tends to induce lasting cellular changes that add to its stability. ... When an axon of cell A is near enough to excite a cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased” [1].

This principle is colloquially summarized as “neurons that fire together, wire together” [1]. Crucially, this describes a local and decentralized learning rule; a synapse does not require a global error signal or external supervision to adjust. It requires only the correlation between the pre-synaptic input and the post-synaptic output. The convergence of the M-P architectural model and the Hebbian plasticity framework established the prerequisite conditions for the development of modern neural networks.

2.2 • THE PERCEPTRON

In 1957, Frank Rosenblatt advanced these theoretical concepts by engineering the Perceptron. The “Mark I Perceptron” was a hardware implementation of the neural model, distinguished by a crucial innovation: a weight-adjustment mechanism based on Hebbian principles. Rosenblatt introduced the perceptron learning rule, an iterative algorithm capable of minimizing error automatically. The system processed an input pattern (e.g., a pixelated character) and produced a binary classification. When the output deviated from the target, the algorithm adjusted the weights proportional to the error: strengthening connections that should have contributed to a correct firing and weakening those that led to false positives.

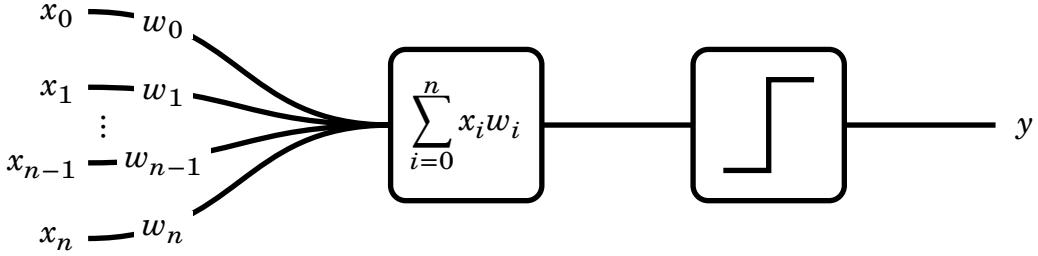


Figure 1: The perceptron model. Inputs x_i are multiplied by weights w_i and summed. If the linear combination $\sum x_i w_i$ exceeds the bias b , the neuron activates.

Consequently, the Perceptron was capable of converging on a solution for any problem where the data was linearly separable. This success generated significant enthusiasm, with contemporary reports suggesting that such machines would soon mimic human consciousness [1]. However, these expectations were tempered by theoretical limitations. In 1969, Marvin Minsky and Seymour Papert published *Perceptrons*, a rigorous mathematical analysis of the architecture. They demonstrated that a single-layer perceptron is fundamentally a linear classifier. While capable of learning operations like AND or OR, it is mathematically incapable of solving the XOR (Exclusive OR) problem. In the XOR case, the classes cannot be separated by a single hyperplane. This proof highlighted a severe boundary on the utility of single-layer networks for complex, non-linear tasks.

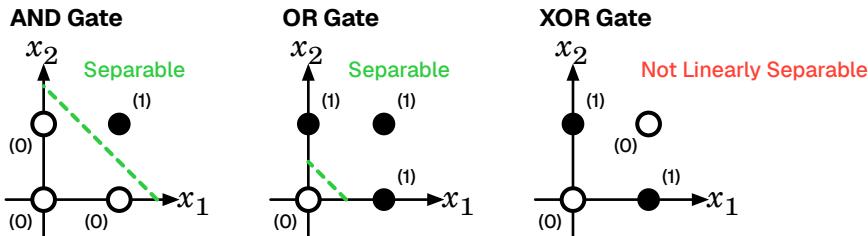


Figure 2: The XOR problem. Unlike AND/OR, the data points for XOR cannot be separated by a single linear boundary.

The publication of *Perceptrons* coincided with a significant reduction in neural network research funding, a period retrospectively termed the “First AI Winter.” It is worth noting that Minsky and Papert acknowledged that a Multi Layer Perceptron (MLP), a network stacking multiple layers of neurons, could theoretically solve the XOR problem by creating complex, non-linear decision boundaries. However, a critical algorithmic gap remained: the “credit assignment problem.” While researchers knew that hidden layers could represent complex features, there was no known method to propagate error signals back through the layers to adjust the weights of hidden neurons effectively. Rosenblatt’s rule was mathematically valid only for the output layer. The field remained stagnant until a method for training multi-layer networks could be formalized.

2.3 • THE DEEP LEARNING PATH

The critique presented by Minsky and Papert precipitated a contraction in funding; however, theoretical inquiry persisted. It was widely hypothesized that the limitations of the single perceptron could be overcome by a Multi-Layer Perceptron (MLP). By organizing neurons into hierarchical layers, the network could theoretically perform successive non-linear transformations on the input space, enabling the formation of complex decision boundaries. The primary impediment was not the architecture itself, but the absence of a viable learning algorithm.

In a single-layer perceptron, error attribution is immediate: if the output deviates from the target, the error is directly derived from the weights of the output layer. However, in a multi-layer architecture, quantifying the contribution of a specific neuron within the “hidden” layers to the final output error presents a significant challenge. This is formally known as the Credit Assignment Problem [1], and it remained the central theoretical obstacle for over a decade.

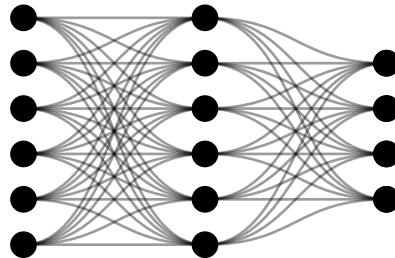


Figure 3: A Multi-Layer Perceptron (MLP). By inserting “hidden layers” between input and output, the network can approximate non-linear functions such as XOR. The historical challenge lay in deriving a method to train these intermediate layers.

2.3.1 • FORWARD AND BACKWARD PROPAGATION

To understand the mechanism of modern neural networks, we must first formalize the objective. The goal of a neural network is to approximate a function $f * (\mathbf{x})$ by defining a parameterized mapping $y = f(\mathbf{x}\theta)$, where θ represents the learnable parameters (weights and biases).

The Forward Pass The process begins with the forward pass, where information flows from the input layer through the hidden layers to the output. Mathematically, a deep network is a composition of non-linear functions. For a network with L layers, the activation of the l -th layer, denoted as $\text{bold}(a)((l))$, is computed as:

$$\begin{aligned} \mathbf{z}^{(l)} &= \mathbf{W}^{(l)} \mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \\ \mathbf{a}^{(l)} &= \sigma(\mathbf{z}^{(l)}) \end{aligned}$$

Here, $\text{bold}(W)((l))$ is the weight matrix, $\text{bold}(b)((l))$ is the bias vector, and $\sigma(\cdot)$ is the non-linear activation function. This recursion continues until the final output $\text{bold}(\hat{y})$ is produced.

The Loss Function To evaluate the quality of this prediction, we introduce a Loss Function (or Cost Function), denoted as $\text{cal}(L)(\boldsymbol{\theta}, \hat{\mathbf{y}}, \mathbf{y})$. This scalar function quantifies the divergence between the network's prediction $\boldsymbol{\theta}(\hat{\mathbf{y}})$ and the ground truth \mathbf{y} . Common examples include Mean Squared Error (MSE) for regression or Cross-Entropy for classification. The training objective is to minimize this loss with respect to the parameters:

$$\min(\boldsymbol{\theta}) \mathcal{L}(f(\boldsymbol{x}\boldsymbol{\theta}), \mathbf{y})$$

Backpropagation and Automatic Differentiation Since the parameters $\boldsymbol{\theta}$ determine the output, they are responsible for the error. To minimize the loss, we must calculate the contribution of every single parameter to that error. This requires computing the gradient of the loss function with respect to every weight: $\nabla_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta})$.

Because the network is a nested composite function, we utilize the Chain Rule of calculus. This process is known as Backpropagation. We compute the gradients starting from the output layer and propagate them backward toward the input. For a specific weight w connecting neuron j to i in layer l , the gradient is decomposed as:

$$\frac{\partial \mathcal{L}}{\partial w_{ij}^{(l)}} = \frac{\partial \mathcal{L}}{\partial a_i^{(l)}} \cdot \frac{\partial a_i^{(l)}}{\partial z_i^{(l)}} \cdot \frac{\partial z_i^{(l)}}{\partial w_{ij}^{(l)}}$$

In modern deep learning frameworks, this is implemented via Automatic Differentiation (AutoDiff). Unlike symbolic differentiation (which can lead to expression swell) or numerical differentiation (which is computationally expensive and prone to approximation errors), AutoDiff constructs a computational graph of elementary operations during the forward pass. It then traverses this graph in reverse to compute exact gradients efficiently.

Gradient Dynamics and Activation Functions The choice of the activation function $\sigma(\cdot)$ is critical not only for the network's expressivity but for its trainability. The “Universal Approximation Theorem” states that a feedforward network with a single hidden layer containing a finite number of neurons can approximate any continuous function, provided the activation function is non-linear [1].

However, the specific choice of non-linearity dictates the stability of the gradients. During backpropagation, gradients are multiplied primarily by the derivative of the activation function $\sigma'(z)$.

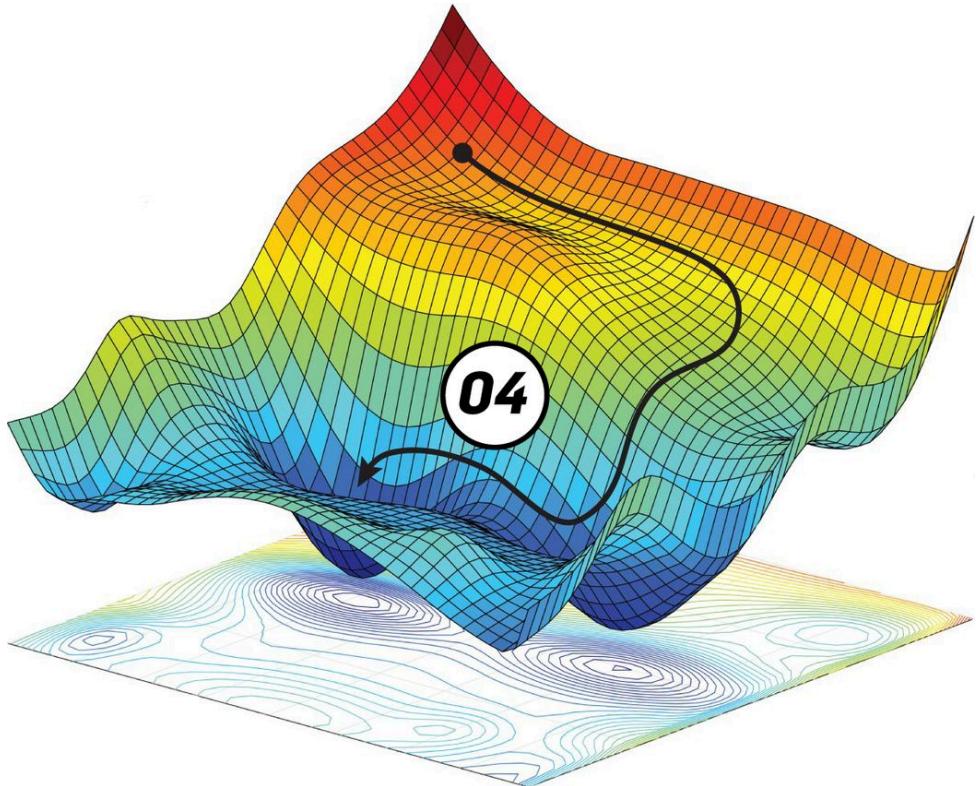


Figure 4: Common activation functions. The Sigmoid function (left) saturates at the extremes, causing gradients to vanish. The ReLU function (right) maintains a constant gradient for positive inputs.

If we utilize the traditional Sigmoid or Tanh functions, the derivative is strictly less than 1 (max 0.25 for Sigmoid). As we multiply these small numbers across many layers, the gradient exponentially decays toward zero. This is known as the Vanishing Gradient Problem, effectively preventing deep networks from learning. Conversely, if weights are initialized poorly, gradients can accumulate to become infinitely large, known as Gradient Explosion.

To mitigate this, modern networks often employ the Rectified Linear Unit (ReLU), defined as $f(x)=\max(0,x)$. Since the derivative of ReLU is either 0 or 1, it preserves the gradient magnitude for active neurons, allowing for the successful training of significantly deeper architectures.

2.3.2 • ACHIEVEMENTS

With the training mechanism solved, the field exploded. The combination of Back-propagation, massive datasets, and GPU hardware led to a “Cambrian Explosion” of neural architectures, each solving domains previously thought impossible for computers.

The revolution began in earnest with computer vision. Convolutional Neural Networks (Convolutional Neural Network (CNN)), such as AlexNet (2012) [1] and later ResNet [1], introduced the idea of learning hierarchical features—detecting

edges, then shapes, then objects—much like the human visual cortex. This allowed machines to classify images with superhuman accuracy.

Soon after, the focus shifted to sequence data. Recurrent Neural Networks (RNNs) and LSTMs gave machines a short-term memory, enabling breakthroughs in speech recognition and machine translation. However, the true paradigm shift occurred with the introduction of the Transformer architecture in 2017. By utilizing an “attention mechanism” to parallelize the processing of language, Transformers allowed for the training of massive Large Language Models (LLMs) like GPT.

These techniques have even transcended media generation. Deep Learning has solved fundamental scientific problems; notably, DeepMind’s AlphaFold utilized these architectures to predict the 3D structure of proteins from their amino acid sequences, a 50-year-old grand challenge in biology.

2.3.3 • SHORTCOMINGS

The “engineering path” was undeniably successful. By ignoring biological constraints, we created models that could master Go, fold proteins, and generate human-like text. However, this success was achieved through brute force. By effectively simulating neural networks on hardware that was never designed for them, we have run into a new set of fundamental walls. The very divergence that allowed Deep Learning to thrive—separating the software from the hardware—has now become its greatest liability.

- Memory bottleneck

The brain is a “compute-in-memory” architecture; synapses store memory and perform processing in the exact same physical location. Modern computers, however, are built on the Von Neumann architecture, which physically separates the Processing Unit (CPU/GPU) from the Memory (RAM). For Deep Learning, this is catastrophic. A neural network is essentially a massive collection of weights (memory) that must be multiplied by inputs (compute). To run a modern LLM, the hardware must constantly shuttle billions of weights back and forth between the memory chips and the processor cores for every single token generated.

- Energy usage

The bottleneck is not just about speed; it is about energy. In modern silicon, the act of moving data is far more expensive than processing it. Fetching a single byte of data from off-chip memory consumes roughly 1,000 times more energy than performing a floating-point operation on that data. This has led to the era of “Megawatt Models.” While the human brain operates on \approx 20 watts (roughly the power of a dim lightbulb), our artificial equivalents require entire power plants.

- Data inefficiency and opacity

Finally, Backpropagation is incredibly sample-inefficient compared to biology. Deep Learning models often require millions of examples to learn concepts that a human child can grasp from a single observation (“one-shot learning”). Furthermore, the resulting models are opaque “Black Boxes.” We know that they work, but due to the distributed nature of their representations, we often cannot explain how or why specific decisions are made. Backpropagation is also dependent on a global error signal, training and inference are two separate operations to the network and information has to flow forwards and backwards for each training run. Every neuron has to be synchronized and need to know everything about every other neuron in order for the algorithm to work.

This brings us to the root cause of these shortcomings. While the Perceptron was born from a desire to mimic the brain, the success of Backpropagation drove the field toward mathematical pragmatism and away from biological realism. To solve the training problem, mainstream AI accepted mechanisms that are fundamentally impossible in biological tissue. In backpropagation, a synapse in the first layer changes its strength based on an error calculation that occurred at the very end of the network. In the brain, learning is widely believed to be local (Hebbian plasticity)—synapses change based only on the immediate activity of the two neurons they connect, not a global error signal. To calculate the error gradient backward, the algorithm requires the backward pass to use the exact same synaptic weights as the forward pass. In the brain, synapses are unidirectional chemical bridges; there is no known biological mechanism that allows a neuron to “read” the strength of a downstream synapse to calculate an error derivative.

This divergence created a paradox: we achieved Artificial Intelligence, but at the cost of efficiency and explainability. We built software that behaves somewhat like a brain, but forced it to run on hardware that functions nothing like one.

To break through the energy and efficiency walls, researchers are now asking a new question: What if we stop forcing neural networks into the Von Neumann architecture? What if, instead of ignoring the biological constraints that Minsky and Papert critiqued, we embraced them? This line of thinking points toward a return to the original inspiration: Neuromorphic Computing.

2.4 • THE NEUROMORPHIC PATH

While the artificial intelligence community was struggling through the AI Winter, debating the merits of symbolic logic versus connectionism, a parallel revolution was brewing in the field of hardware physics. In the late 1980s at Caltech, physicist and engineer Carver Mead—already a legend for pioneering VLSI (Very Large Scale Integration) chip design—began to question the fundamental trajectory of digital computing.

Mead observed that while digital computers were becoming exponentially faster (following Moore's Law), they were also becoming exponentially less efficient in terms of energy per operation. He realized that the prevailing method of building computers—using transistors as rigid, high-power “on/off” switches to perform boolean logic—was incredibly wasteful compared to the biological brains they were trying to emulate.

In 1990, Mead published his seminal paper, Neuromorphic Electronic Systems, coining the term “neuromorphic” (combining neuro for nerve and morph for form). His thesis was radical: rather than writing software to simulate the equations of a neuron on a digital computer, we should build physical hardware that relies on the same laws of physics as the biological nervous system.

2.4.1 • THE SUBTHRESHOLD INSIGHT

The core insight that launched the field was a physical analogy between silicon and biology. In standard digital electronics, transistors are operated in “strong inversion.” They are driven with high voltages to act as binary switches—either fully open (1) or fully closed (0). This ignores the complex physics that happens in between those states.

Mead, however, looked at the transistor in the “subthreshold” (or weak inversion) region—the tiny trickle of current that flows when the transistor is technically “off.” He discovered that in this low-power regime, the current flowing through a transistor I_{ds} is an exponential function of the gate voltage V_{gs}

$$I_{ds} \propto e k \frac{V_{gs}}{U} T$$

Crucially, this is the exact same Boltzmann-distribution physics that governs the flow of ions through protein channels in a biological neuronal membrane.

This realization was profound. It meant that a single transistor, operating in its natural subthreshold state, could physically compute the same exponential, non-linear functions that biological neurons use, but at a distinct speed advantage and with microscopic power consumption. We did not need thousands of logic gates to simulate a synapse; a synapse could be implemented by a single transistor.

2.4.2 • THE SILICON RETINA

To prove this concept, Mead and his doctoral student Misha Mahowald developed the Silicon Retina in 1991. It was the first true neuromorphic system.

Unlike a standard camera, which takes a snapshot of the entire world frame-by-frame (creating a massive stream of redundant data), the Silicon Retina mimicked the human eye. It used analog circuits to compute spatial and temporal derivatives

directly on the chip. It did not output “frames”; it output asynchronous “events” only when the light intensity changed at a specific pixel.

If the retina stared at a static wall, it transmitted zero data and consumed almost zero energy. This mimicked the efficiency of biology and solved the redundancy problem inherent in digital sampling. It demonstrated that by “listening to the silicon”—by letting the physics of the device perform the computation—machines could process sensory information with a fraction of the power of a digital computer.

2.4.3 • THE FUTURE OF NEUROMORPHIC

Since the invention of neuromorphic computing, the field of neuroscience has undergone its own revolution. While Mead provided the physical intuition—that silicon physics could emulate ion channel physics—he was working with a limited map of the brain’s circuitry. We now possess a much richer understanding of neuronal behavior, synaptic plasticity, and brain mechanics.

To build better neuromorphic systems, we must look beyond the transistor and examine the biological machinery it intends to emulate. The fundamental ideas from Mead can now be refined by the specific architectural principles of modern mechanistic neuroscience.

2.5 • MODERN NEUROSCIENCE

The biological brain remains the gold standard for energy-efficient, robust, and adaptive computation. Modern neuroscience has revealed that the brain’s efficiency stems from its specific physical mechanisms, not just its connectivity. To engineer systems that truly rival biological performance, we cannot simply mimic the brain’s output; we must emulate its internal dynamics. We must move away from the spherical cow abstractions of early cybernetics and look at the neuron as it actually functions: a complex, time-dependent, and event-driven processor.

This section provides a mechanistic overview of the nervous system, translating biological observations into the computational primitives required for neuromorphic engineering. It explores the structural hierarchy of the neuron, the physics of the action potential, and the mathematical models used to capture these dynamics in silicon.

2.5.1 • NEURON STRUCTURE & FUNCTION

The neuron is the fundamental computational unit of the brain as outlined in section 2.1 - neuron doctrine. While it shares standard cellular machinery (like mitochondria and a nucleus) with other cells, it is morphologically specialized for information transmission. A neuron consists of three functional zones:

- The Input (Dendrites): A branching tree structure that collects signals from thousands of upstream neurons. This is where inputs are integrated.
- The Integration Zone (Soma): The cell body where electrical potentials from the dendrites summate.
- The Output (Axon): A long, cable-like structure that transmits the neuron's own signal to downstream targets.

These structures allow neurons to form complex networks connected by synapses. Unlike the weighted lines in an artificial neural network, a biological synapse is a complex chemical junction. When a pre-synaptic neuron activates, it releases neurotransmitters (packets of chemicals) that bind to the post-synaptic dendrite, causing ion channels to open and current to flow.

To successfully emulate the brain, one must first understand the physical “chassis” of biological intelligence. The neuron is the fundamental computational unit of the nervous system. While it shares standard cellular machinery with other cell types—such as a nucleus and mitochondria—it is morphologically specialized for the rapid reception, integration, and transmission of information.

The structure of a neuron is polarized, meaning information flows in a specific direction through three distinct functional zones. The process begins at the dendrites, a vast, tree-like structure branching out from the cell body. These dendrites act as the input stage, covered in thousands of receptor sites that capture chemical signals from upstream neurons. These signals are conducted toward the central cell body, or soma. The soma serves as the integration center; it collects the minute electrical currents generated by the dendrites and sums them together. Finally, the processed signal is transmitted via the axon, a long, cable-like projection that extends toward other cells. Many axons are wrapped in myelin, a fatty insulating layer that functions similarly to wire cladding, ensuring the signal travels efficiently over long distances without loss.

Functionally, the neuron operates as an electrochemical battery. Unlike digital circuits that rely on the flow of electrons, the brain computes by moving ions—specifically sodium (Na^+) and potassium (K^+)—across a liquid membrane. By actively pumping positive ions out of the cell, the neuron maintains a stable voltage difference known as the resting potential, typically around -70 mV. In this state, the cell is polarized, effectively storing potential energy like a compressed spring, waiting for an input to trigger a release. The computational decision-making process occurs when the neuron receives input. As neurotransmitters bind to the dendrites, ion channels open, allowing positive ions to flow back into the cell. This raises the internal voltage, a process called depolarization. These voltage changes accumulate at the soma. If the sum of these inputs causes the membrane potential to cross a critical threshold (approximately -55 mV), the neuron undergoes a rapid, non-linear reaction. Voltage-gated channels snap open, causing a massive influx of ions and generating an action potential—a sharp electrical pulse that travels down the axon. Immediately after firing, the neuron pumps the ions out to reset its voltage, entering a brief refractory period where it cannot fire again. This “all-or-nothing” mechanism ensures that noise is filtered out, transmitting only significant, integrated information to the network. Throughout the brain there are several types of neurons such as GLIA cells, which act as supporting structure and energy delivery. The vast majority of neurons in the brain are the spiking neurons which all communicate via a uniform action potential. There are some neurons that emit graded potentials where the shape of the output signal can vary in shape and form.

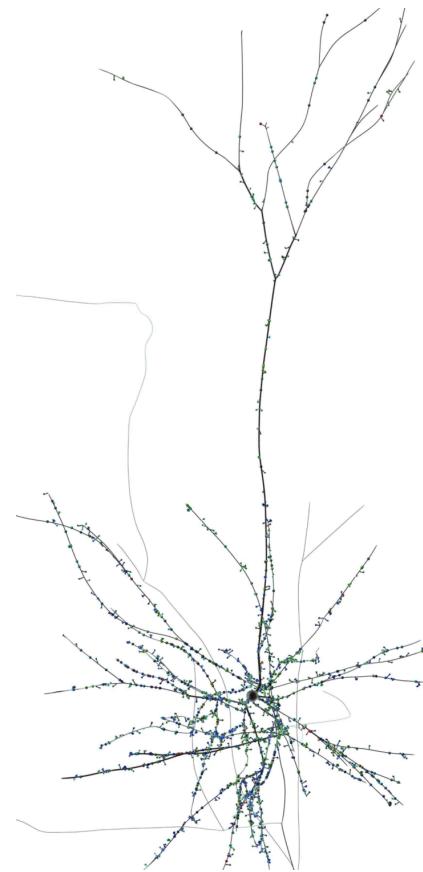


Figure 5: Rendering of a morphologically plausible neuron

2.5.2 • ACTION POTENTIAL

The defining feature of biological computation is the Action Potential or spike. In standard Artificial Neural Networks (ANNs), neurons communicate using continuous values (e.g., 0.75) representing an average firing rate. In contrast, biological neurons communicate using discrete, binary events.

The mechanism is all-or-nothing. The neuron maintains a voltage difference across its membrane (Membrane Potential). As inputs arrive, this potential fluctuates. If the potential crosses a specific Threshold, the neuron fires a spike—a rapid, uniform pulse of voltage that travels down the axon.

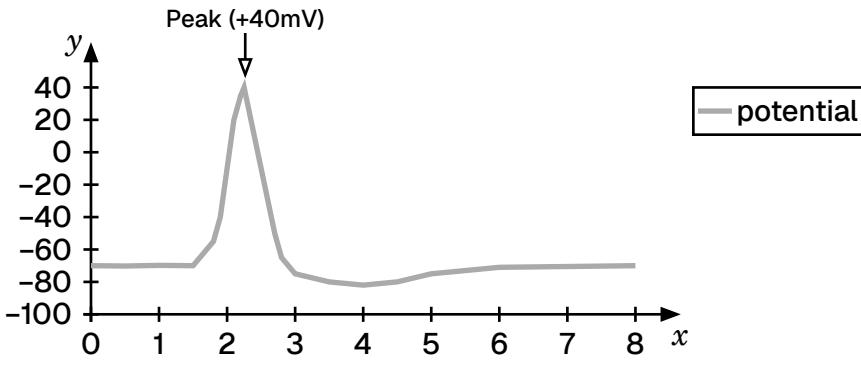


Figure 6: action potential

This “Spiking” paradigm shifts the nature of computation from Spatial (what represents the data) to Spatio-Temporal (when the data happens):

- Event-Driven: Computation only occurs when data changes (a spike arrives). This creates inherent sparsity and energy efficiency.
- Temporal Coding: Information can be encoded in the precise timing of spikes (e.g., time-to-first-spike) rather than just the frequency.

Action potentials are in general uniform and look the same for every neuron every time they spike, Graded potentials are primarily generated by sensory input

The artificial neurons used in most deep learning models (like ReLU or sigmoid units) are static. They compute a weighted sum of their inputs, apply an activation function, and output a single, continuous value (like 0.83 or 5.2). This value is assumed to represent the neuron’s firing rate. Biological neurons don’t work this way. They are spiking neurons, and their computation is:

- Temporal: They integrate inputs over time. A neuron’s internal state (its membrane potential) rises and falls based on when inputs arrive.
- Event-Driven: They do not communicate with continuous values. They communicate using discrete, all-or-nothing electrical pulses called action potentials, or “spikes.” A neuron only fires a spike when its internal potential crosses a specific threshold.
- Efficient: Because they are event-driven, they are sparse. A neuron spends most of its time silent, only consuming energy when it receives or sends a spike.

In this model, information is not just in how many spikes there are (a rate code), but when they occur (a temporal code). A spike arriving a few milliseconds earlier or later can completely change the computational outcome.

While the biological action potential is a complex, continuous voltage fluctuation that lasts approximately one to two milliseconds, it is remarkably consistent in shape. For a given neuron, every spike looks nearly identical. This observation leads to a crucial simplification in neuromorphic engineering: if the shape of the spike is constant, it carries no information. The information must therefore be contained entirely in the timing of the event.

To model this mathematically, we abstract the physical voltage pulse into a dimensionless point process. We treat the spike as an event that occurs at a precise instant in time, t_f , with zero duration. The mathematical tool used to represent this is the Dirac delta function, $\delta(t)$.

The Dirac delta is an idealized function defined by two properties: it is zero everywhere except at $t=0$ (where it is infinitely high), and its integral over time is exactly equal to 1.

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases} \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

Equation 1: The defining properties of the Dirac delta function.

In the context of a neural network, we represent the output of a neuron not as a continuous voltage curve, but as a “spike train”—a sequence of these impulses summed together. If a neuron fires spikes at specific times t_1, t_2, \dots, t_n , the resulting signal $S(t)$ is modeled as:

$$S(t) = \sum f_i \delta(t - t_i)$$

Equation 2: A spike train represented as a sum of Dirac delta functions.

This abstraction is powerful because it simplifies the interaction between neurons. In the Leaky Integrate-and-Fire (LIF) model, when this delta function arrives at a synapse, it acts as an instantaneous “kick” to the current. Because the integral of the delta function is 1, integrating this input simply adds a discrete step-change to the post-synaptic neuron’s membrane potential, perfectly mimicking the rapid charge deposition of a real biological synapse without requiring us to simulate the complex curve of the voltage pulse.

2.5.3 • GENERALIZED LEAKY INTEGRATE AND FIRE

Real neurons exhibit complex non-linear dynamics, often modeled by the Hodgkin-Huxley equations which simulate individual ion channel conductances. However, these are too computationally expensive for large-scale neuromorphic systems.

Instead, we use the Generalized Leaky Integrate-and-Fire (GLIF) model. It captures the essential behavior of a neuron—integration of inputs and threshold firing—without simulating the chemical micro-physics.

The dynamics of the membrane potential $u(t)$ are governed by a linear differential equation:

Where τ_m is the membrane time constant (how fast the neuron “forgets” input), u_{rest} is the resting potential, and $I(t)$ is the input current. In a spiking network, the input current $I(t)$ is not continuous, but a series of discrete spikes arriving at specific times. We can model this input as a sum of Dirac delta functions, scaled by the synaptic weight q :

Because the system is linear below the threshold, we can solve this differential equation analytically. The membrane potential at any time t is the sum of the decaying effects of all previous incoming spikes:

This equation represents the core of most neuromorphic algorithms. It defines a system that integrates information over time, leaks it to ensure temporal relevance, and resets upon firing. While it simplifies away complex bifurcation dynamics (like Hopf bifurcations or chaotic attractors found in biology), the GLIF model provides the necessary abstraction to build scalable, event-driven computing architectures.

$$\tau_m \frac{du}{dt} = -(u - u_{\text{rest}}) + R(u)I(t)$$

Equation 3: Generalized leaky integrate and fire differential equation governing the dynamics of a neurons membrane potential

Considering all input currents to be uniform packets (spikes), the dirac delta function fits well into the mathematical framework

$$\tau_m \frac{du}{dt} = -(u - u_{\text{rest}}) + R(u)q\delta(t)$$

Equation 4: Generalized leaky integrate and fire differential equation governing the dynamics of a neurons membrane potential

solving the equation

$$u(t) = u_{\text{rest}} + \sum_f (u_r - \varphi) \exp\left(-\frac{t-t(f)}{\tau_m}\right) + \frac{R}{\tau_m} \int_0^\infty \exp\left(-\frac{s}{\tau_m}\right) I(t-s) ds$$

Equation 5: Generalized leaky integrate and fire differential equation governing the dynamics of a neurons membrane potential

2.5.4 • BIOPHYSICAL REALISTIC NEURONS & LIMITATION OF THE GLIF MODEL

In the quest to simulate the brain, there exists a fundamental trade-off between biological realism and computational efficiency. At the high end of the spectrum lie conductance-based models, most notably the Hodgkin-Huxley model (1952). This model describes the neuron not as a simple integrator, but as an electrical circuit with variable resistors representing the precise opening and closing dynamics of sodium and potassium ion channels.

Large-scale initiatives, such as the Blue Brain Project, utilize these highly detailed “multi-compartment” models. They simulate the neuron not as a single point, but as

a complex 3D structure, breaking the dendrites and axon into hundreds of distinct segments to model how current flows through the physical geometry of the cell. While these simulations provide invaluable insights into pharmaceutical interactions and pathology, they are computationally prohibitive for real-time engineering. Simulating a single second of brain activity using these models can require supercomputers and massive energy expenditure.

To build practical neuromorphic hardware, engineers must simplify these equations into the Generalized Leaky Integrate-and-Fire (GLIF) or Izhikevich models. However, this abstraction comes at a cost in behavioral fidelity.

A standard LIF neuron is a “one-dimensional” system. Its state is defined entirely by a single variable: the membrane potential. Consequently, it supports only a limited set of firing modes. It can handle tonic spiking (regular, repeated firing under constant input) and phasic spiking (firing once at the onset of input).

However, a standard LIF neuron cannot replicate complex behaviors such as bursting (clusters of rapid spikes followed by silence) or chattering. These behaviors require a “memory” of previous activity that lasts longer than the simple membrane time constant.

To capture these complex dynamics without reverting to the heavy Hodgkin-Huxley equations, the model must be augmented with a second variable, typically called the adaptation variable (w).

$$\tau_m \frac{du}{dt} = -(u - u_{\text{rest}}) + RI(t) - w\tau_w \frac{dw}{dt} = a(u - u_{\text{rest}}) - w$$

Equation 6: The Adaptive LIF model. The variable w provides negative feedback, allowing for bursting dynamics.

In this “Adaptive” formulation, every time the neuron spikes, the variable w increases, acting as a drag or “fatigue” on the membrane potential. This negative feedback loop allows the simple linear model to exhibit sophisticated neuro-computational properties, including spike-frequency adaptation and bursting, bridging the gap between silicon efficiency and biological complexity.

2.5.5 • NEURONAL CODING

In digital representation, a piece of information can be represented in various different ways, all using bits as the lowest information quanta. Combining bits we get a richer representation. float is a popular choice of combining bits. For instance representing a luminance value of a pixel in an image is straight forward, each pixel gets its own float. Representing information using analog systems offers far more precision, in electronic systems, values can be represented with currents or voltages which can be any value. The brain uses action potentials to communicate, action potentials are analog voltages so we might expect that information gets packaged into the shape of the action potential, however as we have discussed the vast

majority of neurons in the brain are spiking neurons, and their action potentials are uniform in size, looking more like individual bits in time. The information stored in brain signals are much more similar to the discrete digital representation than analog. The information must be encoded in the relative timing between spikes.

It is observed that neurons fire in short bursts called spikes. Experiments show that neurons fire repeatably. A sequence of spikes is called a spike train, and exactly how information is encoded in a spike train is a topic of hot debate in neuroscience. A popular idea is that information is encoded in the average value of spikes per time called rate encoding. Temporal encoding the brain most likely uses a combination of all. The time to first spike encoding could be understood like this it is not about the absolute timing of the neurons rather a race of which spikes come first. the first connections would excite the post-synaptic neurons first and they should inhibit the others (lateral inhibition)

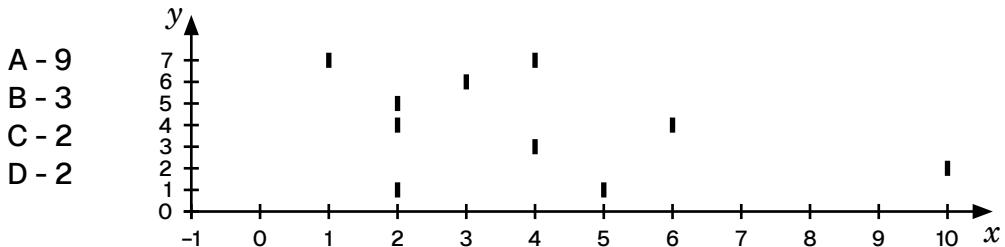


Figure 7: The perceptron—a simple model of how a neuron operates. Inputs x_i get multiplied by weights w_i and summed. If the sum $\sum w_i x_i$ surpasses a threshold (or “bias” b), the neuron fires.

Understanding how the brain processes information requires deciphering the “neural code”—the set of rules by which sensory stimuli and internal states are translated into sequences of action potentials. Unlike a digital bus where voltage levels map directly to binary values, the brain utilizes a more complex, multi-faceted signaling scheme.

The most traditional interpretation of neural activity is Rate Coding. In this paradigm, information is conveyed by the average frequency of action potentials over a specific window of time. A strong stimulus, such as a bright light or heavy pressure, results in a high firing rate, while a weak stimulus results in few or no spikes. This model treats the neuron essentially as an analog-to-digital converter where the precise timing of individual spikes is considered noise, and only the mean activity carries the signal. While rate coding is robust and easily observed in motor neurons, it suffers from significant latency; the system must wait to count spikes over a period of time before a value can be determined.

However, the rapid reaction times observed in biological systems suggest that rate coding is insufficient for time-critical processing. This has led to the prominence of Temporal Coding theories in neuromorphic engineering. In temporal coding, the precise timing of a spike carries significant information. A primary example is “time-to-first-spike” coding, where the intensity of a stimulus is mapped to the

latency of the response; a stronger stimulus causes the neuron to fire earlier relative to a stimulus onset.

This shift from rate to timing fundamentally changes the computational model. By utilizing the exact arrival time of an action potential, neural circuits can process information asynchronously and sparsely. A decision can be made as soon as the first few salient spikes arrive, rather than waiting to average activity over a long duration. This temporal precision allows the brain to maximize information transmission per spike, achieving the extreme energy efficiency that neuromorphic hardware aims to replicate.

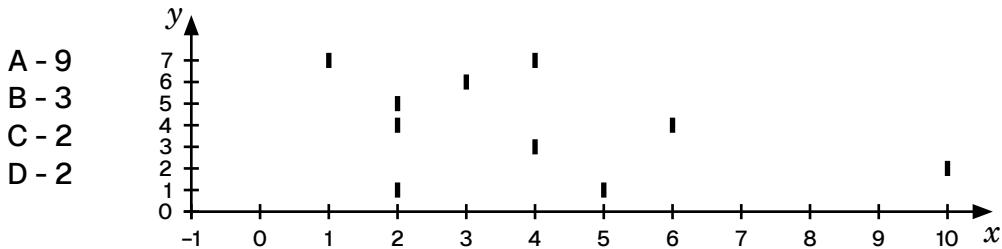


Figure 8: The perceptron—a simple model of how a neuron operates. Inputs x_i get multiplied by weights w_i and summed. If the sum $\sum w_i x_i$ surpasses a threshold (or “bias” b), the neuron fires.

However, relying solely on single neurons to represent complex data is risky due to biological noise and component failure. To ensure robustness, the nervous system employs Population Coding. In this scheme, variables—such as the direction of arm movement or the orientation of a visual edge—are not represented by a single neuron, but by the joint activity of a large ensemble. Each neuron in the population has a “tuning curve,” responding maximally to a specific value but also firing weakly for similar values. By averaging the vectors of activity across the entire population, the brain can extract a precise signal from noisy individual components. This distributed representation ensures that the failure of a few neurons does not corrupt the message, a principle of fault tolerance that is highly desirable in neuromorphic hardware.

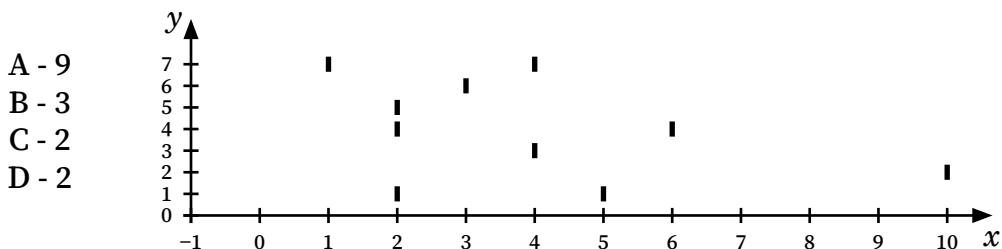


Figure 9: The perceptron—a simple model of how a neuron operates. Inputs x_i get multiplied by weights w_i and summed. If the sum $\sum w_i x_i$ surpasses a threshold (or “bias” b), the neuron fires.

Finally, the brain optimizes for metabolic efficiency through Sparse Coding. This theory posits that neural systems minimize the number of active neurons required to represent a stimulus. At any given moment, out of billions of neurons, only a tiny

fraction are firing. This stands in contrast to “dense” coding (where many units participate) or “local” coding (the hypothetical “grandmother cell” where one unique unit represents one unique object). Sparse coding strikes a mathematical balance: it creates a representation that has a high capacity for information but consumes minimal energy. For neuromorphic engineers, this is the holy grail: mimicking the brain’s ability to process massive sensory streams while leaving the vast majority of the chip in a dormant, zero-power state.

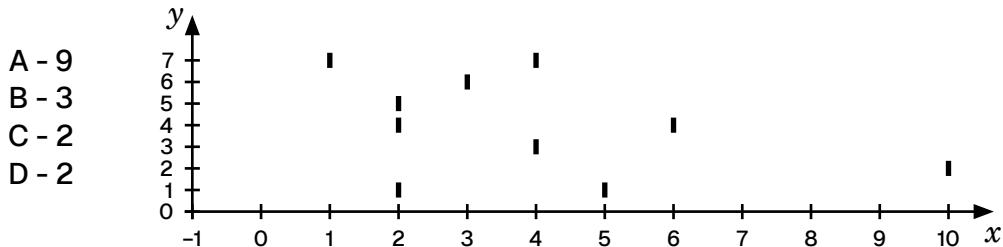


Figure 10: The perceptron—a simple model of how a neuron operates. Inputs x_i get multiplied by weights w_i and summed. If the sum $\sum w_i x_i$ surpasses a threshold (or “bias” b), the neuron fires.

2.5.6 • NERONAL CIRCUITS & DYNAMICS

By now we have covered the neuron behaviour on its own. The neuron on its own is not very useful and needs to connect to other neurons to form functional circuits. In the brain we find distinct regions with specific network configurations that form circuits that do specific tasks

While the individual neuron acts as the computational unit, complex behavior emerges only when these units are organized into functional circuits. The brain is not a random web of connections; it is structured into specific architectural “motifs” that appear repeatedly across different cortical areas. Understanding these motifs is essential for designing neuromorphic architectures that go beyond simple feed-forward processing.

The most fundamental distinction in neural circuitry is between Feed-Forward and Recurrent connections.

Feed-Forward: In sensory areas (like the early visual cortex), information flows unidirectionally from input to output. This allows for rapid, reflex-like feature extraction.

Recurrent: In higher cognitive areas, neurons form feedback loops, connecting back to themselves or their neighbors. This recurrence introduces a time component to the computation, allowing the network to maintain a “state” or short-term memory even after the input stimulus has vanished.

- Lateral Inhibition and Winner-Take-All

A ubiquitous circuit motif in the brain is Lateral Inhibition. In this configuration, an active neuron excites itself while inhibiting its neighbors via interneurons. This competition results in a “Winner-Take-All” (WTA) dynamic, where the neuron with the strongest input suppresses all others.

In neuromorphic engineering, WTA circuits are crucial. They provide a mechanism for noise reduction and categorical decision-making (e.g., deciding which specific pixel is the “edge”) without requiring a central processor to compare values. The decision emerges physically from the circuit’s competition.

2.5.7 • ATTRACTOR DYNAMICS

When recurrent circuits interact with inhibition, stable patterns of activity called Attractors emerge. These are specific states toward which the system naturally evolves, acting as the neural basis for memory and spatial orientation.

Point Attractors: The network settles into a specific static firing pattern. This is analogous to an energy minimum in physics and is believed to be the mechanism behind associative memory (e.g., recognizing a face from a partial clue).

Continuous Attractors (Line/Ring): Instead of settling to a single point, the network maintains a bump of activity that can move continuously along a manifold.

Ring Attractors: Found in the *Drosophila* (fruit fly) heading-direction system. A “bump” of activity moves around a ring of neurons to represent the fly’s compass heading.

Line Attractors: Used in the oculomotor system to maintain eye position.

These dynamics highlight a major shift from standard digital logic. In a digital memory, data is static and must be read; in an attractor network, the memory is an active, self-sustaining process.

2.5.8 • EXCITATORY-INHIBITORY (E/I) BALANCE

Finally, for any of these dynamics to function, the network must maintain a precise Excitation-Inhibition (E/I) Balance. The brain operates at a critical point of instability.

Too much excitation leads to runaway feedback (similar to an epileptic seizure).

Too much inhibition leads to signal death (quiescence).

Modern neuromorphic chips increasingly implement “homeostatic plasticity”—algorithms that automatically adjust synaptic weights to keep the network in this critical regime, ensuring that signals can propagate through deep layers without fading out or exploding.

2.5.9 • MACRO-ARCHITECTURE: THE CORTICAL COLUMN

Moving beyond local circuits, the neocortex—the seat of higher intelligence—displays a remarkable uniformity in its structure. In 1957, Vernon Mountcastle discovered that the cortex is organized into vertical structures called Cortical Columns.

A cortical column is a cylinder of tissue approximately 0.5 mm in diameter containing roughly 6 distinct layers of neurons. It acts as the fundamental “functional unit” or “microchip” of the brain. Whether the cortex is processing visual input, auditory signals, or motor commands, the internal circuitry of the column remains largely the same.

This suggests a profound principle for neuromorphic engineering: the brain uses a single, universal algorithm to solve many different problems. If we can reverse-engineer the operation of a single column, we can potentially scale that architecture to solve any modality of data.

2.5.10 • THE THOUSAND BRAINS THEORY

Building on the concept of the cortical column, Jeff Hawkins (founder of Numenta) proposed the Thousand Brains Theory of Intelligence. This theory challenges the traditional view that the brain builds a single, global model of the world.

Instead, Hawkins argues that every single cortical column learns a complete, independent model of the object it is sensing (a “reference frame”).

Distributed Modeling: If you touch a coffee cup with five fingers, your brain creates five separate models of the cup simultaneously.

Voting: These columns communicate laterally to vote on the object’s identity. If one column thinks it feels a “curved handle” and another feels “warm ceramic,” they reach a consensus that the object is a “Coffee Cup.”

This theory is highly influential in modern neuromorphic design because it implies that intelligence is massively parallel and decentralized. It moves away from the fragile, deep hierarchies of Deep Learning toward robust, distributed consensus networks.

2.5.11 • THE VISUAL HIERARCHY

While columns provide the local processing power, they are arranged globally into a hierarchy. The most studied example—and the blueprint for almost all modern computer vision—is the Primate Visual Cortex.

Information from the retina travels to the Primary Visual Cortex (V1) at the back of the brain. From there, it splits into two distinct pathways:

The Ventral Stream (“What” Pathway): Travels to the temporal lobe. It processes object identity. V1 detects edges; V2 detects textures; V4 detects shapes; and the Inferior Temporal (IT) cortex detects full objects (like faces).

The Dorsal Stream (“Where” Pathway): Travels to the parietal lobe. It processes spatial location and movement, guiding motor actions (e.g., reaching for a glass).

Neuromorphic cameras (like the Silicon Retina mentioned earlier) are often paired with spiking neural networks designed to mimic this specific split, separating the “flow” of motion from the “recognition” of objects to process dynamic scenes efficiently.

2.5.12 • ASSOCIATIVE MEMORY

In this biological architecture, memory is not stored in a separate “hard drive” or register. Instead, memory is Associative and Content-Addressable.

Memory is stored physically in the synaptic weights between neurons. To retrieve a memory, the brain does not look up an address; it re-activates the specific pattern of neurons associated with that memory. Because of the attractor dynamics mentioned earlier, the brain allows for Pattern Completion: if a network receives a noisy or partial input (e.g., seeing half of a familiar face), the energy landscape of the network naturally “slides” the activity back into the learned state of the full face. This robustness to missing data is a key advantage of neuromorphic memory architectures over traditional RAM.

Write about dynamical models and hoph bifucations, write about modes of firing depending on bifurcations ...

The biological neuron is the fundamental building block of the brain. The biological neuron consists of a cell body also called the soma, which contains all of the core machinery that other cells have, like the nucleus and mitochondria. However, it also has distinct structures:

- Dendrites: A branching “input” tree that receives signals from other neurons.
- Axon: A long “output” cable (which can be over a meter long in humans) that transmits the neuron’s own signal.
- Synapses: The junctions where an axon’s terminal meets another neuron’s dendrite to pass the signal.

When pre-synaptic neurons fire, they release neurotransmitters (like glutamate or dopamine) across the synapse. These chemicals open ion channels on the post-synaptic neuron, causing its internal electrical potential (the “membrane potential”) to increase. If this potential, which is integrated over time, reaches a critical threshold, the neuron itself fires an action potential (a spike) down its axon.

Although the perceptron captures the basic idea of “integrating inputs to make a decision,” a lot is left on the table. A lot of progress and new ideas have surfaced since its invention. The neuron, once thought to be a simple switch, turns out to be a complex computational device.

This forces us to rethink everything:

- **Information Encoding:** How is information represented? Is it in the rate of spikes, the precise timing of the first spike, or in correlations between patterns of spikes? This is a key research topic not explored by older models.
- **Learning Rules:** How does the brain learn? It is entirely different from what deep learning uses. The brain’s learning is local (like STDP) and continuous.
- **Network Architecture:** The brain isn’t a simple feed-forward stack of layers. It is a deeply recurrent, complex, and fully asynchronous graph where signals propagate at their own speed.

A lot of work has been put into modeling the neuron. The most biologically realistic models, like the Hodgkin-Huxley model, are incredibly complex and computationally expensive, simulating the precise dynamics of multiple ion channels.

For most neuromorphic computing, a balance is struck with simpler, more efficient models. The most popular are the Leaky Integrate-and-Fire (LIF) models.

- **Integrate:** The model’s potential rises as it receives input spikes (summing their weights).
- **Leaky:** The potential “leaks” away over time, modeling the neuron’s tendency to return to its resting state.
- **Fire:** If the potential crosses the threshold, it fires a spike and its potential is reset.

The Generalized Leaky Integrate-and-Fire (GLIF) and other variants (like the Izhikevich model) are the best models we have today for large-scale, efficient simulation. They add more biological realism (like adaptation, where a neuron’s firing rate slows over time) without the extreme computational cost of full biophysical models.

2.5.13 • BIOLOGICAL LEARNING

This different model of computation requires a different model of learning. Deep learning’s backpropagation is a “global” algorithm. To update a synapse in the first layer, you need an error signal calculated at the very last layer and propagated all the way back. This is highly effective but biologically implausible; there is no known mechanism for such a precise, “backward” error signal in the brain.

Instead, the brain appears to use local learning rules. The most famous is Hebb’s rule: “Neurons that fire together, wire together.”

A modern, measurable version of this principle is Spike-Timing-Dependent Plasticity (STDP). STDP is a learning rule that adjusts the strength (the “weight”) of a

synapse based purely on the relative timing of spikes between the pre-synaptic neuron (the sender) and the post-synaptic neuron (the receiver).

The rule is simple and local:

- LTP (Long-Term Potentiation): If the pre-synaptic neuron fires just before the post-synaptic neuron (meaning it likely contributed to the firing), the connection between them is strengthened.
- LTD (Long-Term Depression): If the pre-synaptic neuron fires just after the post-synaptic neuron (meaning it fired too late and did not contribute), the connection is weakened.

This mechanism allows the network to learn correlations, causal relationships, and temporal patterns directly from the stream of incoming spikes, without any “supervisor” or global error signal. It is the biological alternative to backpropagation and a cornerstone of modern neuromorphic learning.

- no evidence that brain does not use global error signal like backpropagation uses
- each neuron weight gets updated by a local rule
- spikes are discrete and does not have a gradient

While spiking neural networks offer greater biological plausibility and potential advantages in processing temporal information and energy efficiency, their adoption faces significant challenges, primarily stemming from the nature of their core computational element: the discrete spike.

A cornerstone of the success of modern deep learning, particularly with Multi-Layer Perceptrons (MLPs) and related architectures, is the backpropagation algorithm. Backpropagation relies fundamentally on the network’s components being differentiable; specifically, the activation functions mapping a neuron’s weighted input sum to its output must have a well-defined gradient. This allows the chain rule of calculus to efficiently compute how small changes in network weights affect the final output error, enabling effective gradient-based optimization (like Stochastic Gradient Descent and its variants). These techniques have proven exceptionally powerful for training deep networks on large datasets.

However, when we transition from the continuous-valued, rate-coded signals typical of MLPs to the binary, event-based spikes used in SNNs, this differentiability is lost. The spiking mechanism itself—where a neuron fires an all-or-none spike only when its internal state (e.g., membrane potential) crosses a threshold—is inherently discontinuous. Mathematically, this firing decision is often represented by a step function (like the Heaviside step function), whose derivative is zero almost everywhere and undefined (or infinite) at the threshold.

Consequently, standard backpropagation cannot be directly applied to SNNs. Gradients calculated using the chain rule become zero or undefined at the spiking neurons, preventing error signals from flowing backward through the network to

update the weights effectively. This incompatibility represents a substantial obstacle, as it seemingly precludes the use of the highly successful and well-understood gradient-based optimization toolkit that underpins much of modern AI.

Surrogate Gradients: A popular approach involves using a “surrogate” function during the backward pass of training. While the forward pass uses the discontinuous spike generation, the backward pass replaces the step function’s derivative with a smooth, differentiable approximation (e.g., a fast sigmoid or a clipped linear function). This allows backpropagation-like algorithms (often termed “spatio-temporal backpropagation” or similar) to estimate gradients and train deep SNNs, albeit with approximations.

2.6 • NEUROMORPHIC STATE OF THE ART & RESEARCH GAPS

Disentangling core computational mechanisms from biological implementation details is a major ongoing challenge in neuroscience and neuromorphic engineering. Some complex molecular processes might be essential for learning or adaptation, while others might primarily serve metabolic or structural roles not directly involved in the instantaneous computation being modeled. The principles of neuromorphic computing, born from Carver Mead’s vision and informed by modern neuroscience, have matured from theoretical concepts into a vibrant field of applied research. This progress is best seen in two key areas: the development of specialized, brain-inspired hardware and the creation of sophisticated software frameworks for simulating and deploying spiking neural networks (SNNs).

2.6.1 • KEY CONCEPTS IN NEUROMORPHIC ENGINEERING

From this foundation, the field of Neuromorphic Engineering was established on three core principles that directly oppose the Von Neumann architecture:

- Analog Computation:

Summing two numbers is very efficient to do with analog computers, you simply add two currents. If you can

- Asynchronous Communication:

Removing the global clock. Parts of the chip only operate when there is data to process, eliminating the massive power drain of “clock distribution” seen in modern CPUs.

- Co-location of Memory and Compute:

Eliminating the separation between the processor and the RAM. In a neuromorphic chip, the “weight” of a neural connection is stored physically within the circuit that does the processing, effectively destroying the Von Neumann bottleneck. Why did

it not take off immediately? Despite the brilliance of Mead's insight, neuromorphic computing remained a niche academic curiosity for decades. The reason was economic: Moore's Law. For thirty years, it was simply cheaper and easier to simulate neural networks on rapidly improving, general-purpose digital CPUs than it was to design difficult, noisy, custom analog hardware. However, as discussed in the previous chapter, Moore's Law is now slowing, and the energy demands of Deep Learning are exploding. The “free lunch” of digital scaling is over. This has forced the AI community to look back at Mead's original vision. We are now seeing a renaissance of these ideas, evolving from purely analog circuits to modern Spiking Neural Networks (Spiking Neural Network (SNN)) implemented on digital-neuromorphic hybrids like Intel's Loihi and IBM's TrueNorth.

2.6.2 • INFORMATION CODING: RANK ORDER & N-OF-M

A central challenge in neuromorphic engineering is encoding information into spikes. The most “standard” method is Rate Coding (frequency of spikes = intensity), but this is slow and energy-inefficient. To solve this, Simon Thorpe proposed Rank Order Coding.

Thorpe observed that the human visual system processes images far too quickly for neurons to average spikes over time. Instead, he proposed that information is encoded in the order of firing. The most strongly activated neurons fire first.

The “N-of-M” Strategy: To implement this efficiently in hardware, engineers often use an “N-of-M” coding scheme.

M (Population): A large pool of potential neurons (e.g., 1000).

N (Active): Only the first N neurons (e.g., 50) to spike are transmitted.

Mechanism: Once N spikes are received, the system inhibits the rest. This guarantees extreme sparsity (low power) and filters out “noise” (late spikes).

This approach transforms time into a priority queue. A downstream neuron does not wait for a “frame” to finish; it begins computing as soon as the earliest, most salient spikes arrive.

2.6.3 • UNSUPERVISED LEARNING: STDP

The most biologically plausible learning algorithm is Spike-Timing-Dependent Plasticity (STDP). Unlike Deep Learning, which updates weights based on a global error calculated at the output, STDP updates weights based on local causality between two connected neurons.

$$\Delta w = \begin{cases} A_+ \exp(-\Delta \frac{t}{\tau_+}) & \text{if } \Delta t > 0 \\ -A_- \exp(\Delta \frac{t}{\tau_-}) & \text{if } \Delta t < 0 \end{cases}$$

Equation 7: Standard weight update rule for STDP.

Where $\Delta t = t^{\text{post}} - t^{\text{pre}}$.

Causal (LTP): If the input spike (pre) arrives before the output spike (post), the synapse is strengthened (Long-Term Potentiation). The input “caused” the firing.

Acausal (LTD): If the input spike arrives after the output spike, the synapse is weakened (Long-Term Depression). The input was irrelevant to the decision.

STDP allows networks to self-organize and detect repeating patterns in data without labeled supervision. However, on its own, it struggles to reach the high accuracy of modern supervised classifiers.

2.6.4 • MODERN SOTA: SURROGATE GRADIENTS

To achieve “State-of-the-Art” (SOTA) performance on complex tasks (like ImageNet or Speech Recognition), modern neuromorphic engineers often hybridize biology with Deep Learning.

The core problem is that spikes are non-differentiable (a step function has zero derivative everywhere), which breaks standard Backpropagation. The solution is Surrogate Gradient Learning.

Forward Pass: The hardware uses the true, crisp spiking physics (non-differentiable).

Backward Pass: The learning algorithm substitutes the spike with a smooth “surrogate” function (like a sigmoid) to calculate gradients.

This allows us to train SNNs using powerful optimizers (like Adam) and frameworks (like PyTorch), transferring the trained weights to the neuromorphic chip for efficient inference.

2.6.5 • COMPLEX DYNAMICAL MODELS

Beyond these standard algorithms, there is a rich landscape of more complex theoretical models. While we will not utilize them in this specific implementation, they represent the frontier of the field:

Reservoir Computing (Liquid State Machines): Using a chaotic, randomly connected “tank” of neurons to project inputs into high-dimensional space before a simple readout layer.

Equilibrium Propagation: A physics-based learning rule that relaxes a network to an energy minimum, avoiding the need for a separate backward pass.

Attractor Networks: Recurrent networks (Ring or Line attractors) that maintain stable states (memories) even in the absence of input, crucial for spatial navigation and working memory.

2.6.6 • NEUROMORPHIC HARDWARE

The primary goal of neuromorphic hardware is to escape the von Neumann bottleneck and emulate the power efficiency and massive parallelism of the brain. Two landmark systems define the state of the art:

IBM TrueNorth: A prominent early example, TrueNorth is a fully digital, real-time, event-driven chip. It consists of 4,096 “neurosynaptic cores,” collectively housing one million digital neurons and 256 million synapses. Its architecture is explicitly non-von Neumann; processing and memory are tightly integrated within each core. TrueNorth’s key achievement is its staggering power efficiency: it can perform complex SNN inference tasks (like real-time video object detection) while consuming only tens of milliwatts—orders of magnitude less than a CPU or GPU performing a similar task. However, its architecture is largely fixed, making it a powerful “inference engine” but less flexible for researching novel, on-chip learning rules.

Intel Loihi (and Loihi 2): Intel’s line of neuromorphic research chips, starting with Loihi in 2017, represents a significant step towards flexible, on-chip learning. Like TrueNorth, Loihi is an asynchronous, event-driven digital chip, but with a key difference: it features programmable “learning engines” within each of its 128 neuromorphic cores. This allows researchers to implement and test dynamic learning rules, such as STDP and its variants, directly on the hardware in real-time. The second generation, Loihi 2, further refines this with greater scalability, improved performance, and more advanced, programmable neuron models, positioning it as a leading platform for cutting-edge neuromorphic algorithm research.

2.6.7 • SIMULATION AND SOFTWARE FRAMEWORKS

Before algorithms can be deployed on specialized hardware, they must be designed, tested, and validated. This is the role of SNN simulators, which function as the “TensorFlow” or “PyTorch” of the neuromorphic world.

Brian: A highly flexible and popular SNN simulator used extensively in the computational neuroscience community. Its strength lies in its intuitive syntax, which allows researchers to define neuron models and network rules directly as a set of mathematical equations (e.g., the differential equations of a Leaky Integrate-and-Fire neuron). This makes it an ideal tool for exploring the detailed dynamics of biologically realistic models.

Nengo: A powerful, high-level framework that functions as a “neural compiler.” Nengo is built on a strong theoretical foundation (the Neural Engineering Framework) that allows users to define complex computations and dynamical systems in high-level Python code. Nengo then “compiles” this functional description into an equivalent SNN. Its key advantage is its backend-agnostic nature; the same Nengo-defined network can be run on a standard CPU, a GPU, or deployed directly to neuromorphic hardware like Intel’s Loihi.

2.6.8 • THE RESEARCH GAP: THE LEARNING PROBLEM

Despite this immense progress in hardware and software, a fundamental challenge remains, creating a critical research gap: the training problem.

Mainstream deep learning has a powerful, universal tool: backpropagation. Neuromorphic computing does not yet have a clear equivalent. While these systems exist, they still struggle with finding an efficient, scalable, and powerful learning algorithm that is both biologically plausible and computationally effective.

This gap manifests in several ways:

- Limited Supervised Learning: “Local” rules like STDP are fundamentally unsupervised. They are excellent for finding patterns and correlations but struggle with complex, task-driven “supervised” problems (e.g., “classify this audio signal into one of ten specific words”).
- The “Conversion” Compromise: A popular workaround is to first train a conventional, non-spiking ANN using backpropagation, and then “convert” its weights to an SNN for efficient inference. This method, while practical, is a compromise. It discards the rich temporal dynamics SNNs are capable of and does not represent true “neuromorphic learning.”
- The “Surrogate Gradient” Challenge: The “firing” of a spiking neuron is a non-differentiable event, which makes it incompatible with standard backpropagation. New methods, like “surrogate gradient” learning, attempt to approximate this spike event with a smooth function to enable gradient-based learning, but this is an area of intense and ongoing research.

This thesis confronts this central challenge: How to effectively and efficiently train spiking neural networks for complex, real-world temporal tasks. While hardware like Loihi provides the platform for on-chip learning, it still requires a robust and scalable algorithm. The existing approaches of ANN-to-SNN conversion or simple Hebbian rules are insufficient.

This thesis proposes a method to bridge this gap by... (...for example: “...developing a novel, event-driven surrogate gradient algorithm capable of training deep SNNs directly in the temporal domain,” or “...introducing a hybrid learning rule that combines the efficiency of STDP with the task-driven power of error-based feedback,” or “...proposing a new architecture for temporal credit assignment that is both hardware-friendly and scalable.”)

3 • METHOD

3.1 • NEUROMORPHIC FRAMEWORK

(e.g., “A Novel Framework for On-Chip Synaptic Plasticity”) This chapter is your first contribution. It’s where you present your new idea. Derivation: Start from first principles. Walk the reader through the math, the logic, or the formal model you developed. The Model: Formally present your new algorithm, equation, or architecture. This is the “pure” theoretical part. Hypotheses: End the chapter by explicitly stating the hypotheses your theory predicts. Example: “Based on this framework, it is hypothesized that an SNN implementing this rule (1) will be able to learn the N-MNIST dataset... (2) will do so with a lower average spike rate than a baseline model... and (3) will be robust to synaptic noise...” By doing this, you’ve already “banked” a major contribution before you even show a single graph.

3.2 • NEURONS AND ENCODING

The neuron models and encoding are intertwined and cannot be developed separately, the same goes for learning. Encoding where the inputs get increasingly smaller as the threshold increases can encode order of the elements, evidence for bioplauibility can be found from eq 1 where the $R(u)$ is dependent on the membrane potential

A neuron should have the following characteristics

- Accumulate spikes in some form (integrate)
- Fire when some threshold has been reached
- Leak the potential over time

A neural code should

- be fast
- be efficient in terms of neurons used
- able to encode a wide range of stimuli
- robust to noise

3.2.1 • RATE CODE

A rate code uses the firing rate of a single neuron as carrier of information

3.2.2 • TEMPORAL CODE

In an temporal code information is stored in time time to first spike, each neuron/ input sends out one spike and the imformation is stored in the delay, shorter delay means stronger input, in such a sceme the computation could work with a balance of excitatiry and inhibitory neurons, the first to arrive (the stronges ones) win a “race” and inhibits others.

3.2.3 • POPULATION CODE

Both rate codes and temporal codes is most likely also combined with population codes in the brain

3.2.4 • PHASE AMBIGUITY FOR TEMPORAL ENCODINGS

For rate coding phase is a non issue as we can find the instantanious firing rate at any phase, for time to first spike encoding we need a reference signal. If the reference signal starts at time t_0 we have started the phase and if the pattern does not match up with the reference signal we could miss it. Evidence suggests that brain waves could play the role of a global reference signal. This is the fundamental trade off between the two. A learning scheme where inputs have to occur in the same episode repeatedly two inputs that happen at the same time yields a stronger response. If the pattern is random then they would sometimes occur in the same episode and sometimes not. Two strong responses should occur in the same time frame.

If the post neuron fires then we should strengthen the weights

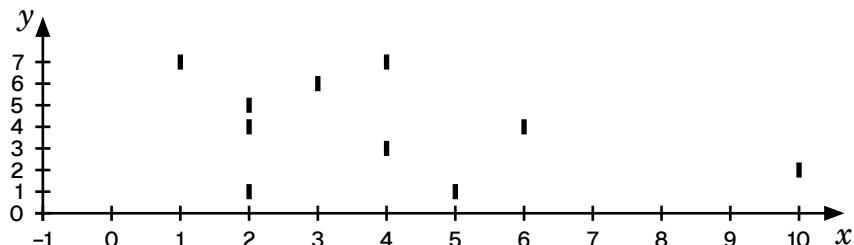


Figure 11: Proposed simplified layout of a SNN. The neurons are connected with hirearchical busses that allow for the network to be configured as a *small world network*

Footnote digression for longer patterns Longer patterns require a latched state such as neurons entering repeated firing like a state machine

An important point is to declare whether a mechanism is bio plausible. An engineer might not care wether or not a mechanism is used by the brain and is crucial to the brains function. An engineer might only care about if the mechanics works and is effective for the system that is created in the engineers vision. An engineer might just use the brain as an inspiration. Evolution althoug achieved remarkable feats is not guaranteed to foster up the most optimal solution, only good enough to survive to the next generation. However discussing wether or not a mechanism is bio

plausible is still useful for understanding our own brain. And creating bio plausible artificial systems can contribute to more than one field.

Some neurons have other properties like bursting modes or continuous firing once the threshold has been reached.

Inhibition should make a neuron not fire

$$T = \sum \frac{w}{t}$$

Equation 8: Weighted sum

In a time to first spike scheme of we care about the order (the relative values since information is stored in time and order) we have to use weights and a neuron model that distinguish between inputs arriving earlier than others. I present a scheme where the first neuron that arrives starts a linear count where the slope of the counter is the weight additional inputs will increase or decrease the slope according to their weight. We can see that neurons arriving earlier will get more time to increase the counter and thus will carry a higher value. If the counter reaches a threshold the neuron will fire. The astute will notice that in this scheme the neuron will fire even for the smallest stimulus since the counter will count up a non zero value and eventually reach the threshold, to mitigate this we can simply say that if the counter is too slow the neuron will not fire we will see later that this scheme satisfies the criteria above.

The problem with this decoding is for strong stimuli we would ideally make the neuron respond immediately and fire, but it has to wait until the counter has reached the threshold to fix this we can also add the weight of the input directly to the potential while also starting a counter. Now if early strong inputs arrive they will fill up the potential and make the neuron fire almost immediately. Small inputs will take some time

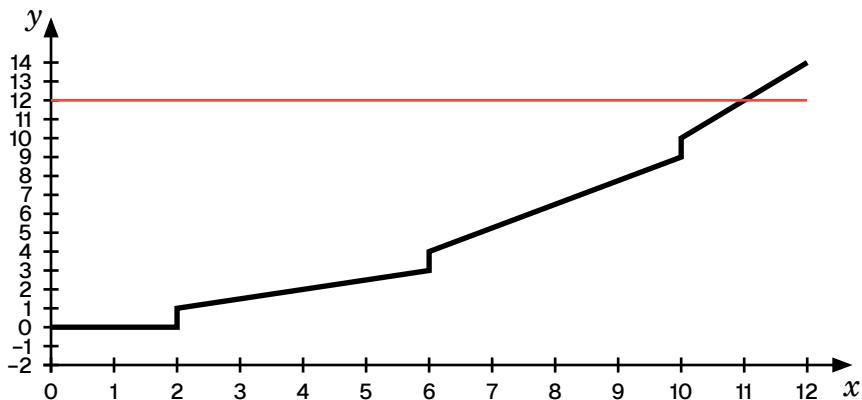


Figure 12: Proposed simplified layout of a SNN. The neurons are connected with hierarchical busses that allow for the network to be configured as a *small world network*

Leaky integrate and fire models seem the best bet, however complex dynamics like exponential decay and analog weights and potentials seem excessive, we might do

without. Binary weights 1 for excitatory and 0 for inhibitory. Stronger weights can be modeled with multiple parallel synapses

Another way which is also based on relative firing order of single spikes could be a passcode encoding. Such an encoding could work by having neurons only react to a sequence. It has an internal state machine of sorts and will only advance to the next state if receives the correct input in the correct order. This encoding does only care about relative order not relative timings.

3.3 • LEARNING

learning with binary weights learning with more steps of quantized or continuous weights Say we want to detect the pattern ABC and the pattern ABD. First of all if the order does not matter set all the weights equal. If the order does matter the weights determine the order. Now if a neuron learns pattern ABC so well that it learns to fire on only AB then it can fire faster. However if a second neuron wants to learn ABD then inhibition from the AB neuron prohibits it. A solution can be that if a neuron originally learned ABC but now fires on AB but still has a strong weight on C it should remember this and if it fires on AB but then C does not arrive it should be like “oh, C did not show maybe I am wrong to fire early” eg. Decrease weights for A and B It predicts!

A second way is to have a hierarchy with bypass. So one layer detects only AB then the next layer has bypass of the first layer and the second combining AB and C or D

A second problem is how to decode order. When do we start the decreasing timer, how fast, should it be in time or in amount of spikes, what to do with phase? The phase should correct itself. The weights need to be as precise as the timing of the spikes? Or we could make the neuron sensitivity proportional to its inverse potential and add leaking

```
1 start with a collection of neurons with arbitrary connections
2 if a pre-synaptic neuron fires then
3   | it has a chance to grow a synapse to a random post-synaptic neuron
4 if a post-synaptic neuron fires then
5   | strengthen all connections to pre-synaptic neurons that fired before
6   | remove connections to pre-synaptic neurons that did not fire or fired
    | after
    | i) a neuron can be both pre-synaptic and post-synaptic
```

Algorithm 1: Unsupervised local learning rule for individual neurons. Based on STDP

-
- ¹ probability of growing a synapse is inversely proportional to the amount it already has
² earlier firings should get a better chance to grow synapses,
 although this is regulated by inhibitory action
-

Algorithm 2: Growing rules for synapses

3.4 • NETWORK

The nwtnetwork archiecture should be paralizeable allowing for each neuron to operate independently with only a local influence from other neurons, some neurons can act as hubs allowing clusters with specialized tasks to communicate.

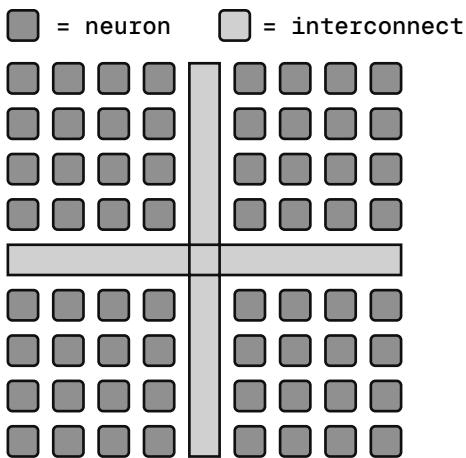


Figure 13: Proposed simplified layout of a SNN. The neurons are connected with hirearchical busses that allow for the network to be configured as a *small world network*

3.5 • SIMULATOR

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4 • RESULTS

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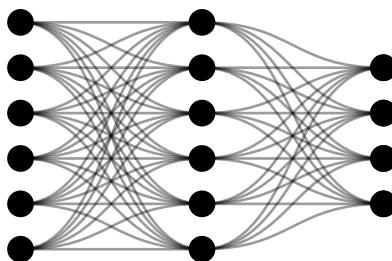


Figure 14: Neural network before learning

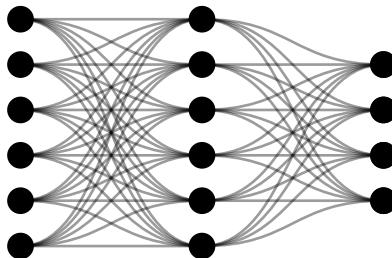


Figure 15: Neural network during learning

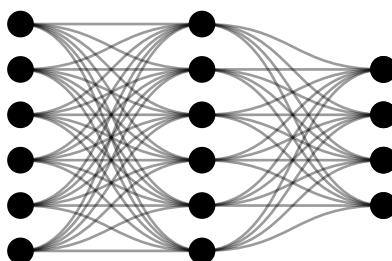


Figure 16: Neural network after learning

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0	0	1	1
1	1	1	1

Table 1: Number of operations

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5 • DISCUSSION

In this section we discuss the

We see a trade-off between the ability to learn and speed of learning and forgetting. Synaptic plasticity must be tuned in order for the right learning environment to form

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6 • CONCLUSION

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