

Quiz 8

1, We assume the system of qubits has the corresponding density matrix ρ and finite Hilbert space. The Von Neumann entropy is

$$S(\rho) = -\text{Tr}(\rho \ln(\rho))$$

In Computational basis $\{|0\rangle, |1\rangle\}$, the eigenvalues of $\rho \ln \rho$ are $p_x \ln p_x$ and $S(\rho) = -\sum_x p_x \ln p_x = H(x)$

With a pure state $|\varphi\rangle$, the density matrix is $\rho = |\varphi\rangle\langle\varphi|$
the eigenvalues are 1 and 0; thus,

$$S|\varphi\rangle\langle\varphi| = 0$$

$$2) \quad |\varphi\rangle = \alpha |10\rangle + \beta |11\rangle$$

$$|0\rangle \longrightarrow |000\rangle$$

$$|1\rangle \longrightarrow |111\rangle$$

$$\rightarrow |\varphi\rangle = \alpha (|0\rangle \otimes |0\rangle \otimes |0\rangle) + \beta ($$

$$|001\rangle =$$

$$|000\rangle = |0\rangle$$

$$|010\rangle = |1\rangle$$

$$|100\rangle = |2\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |2\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$