$$|\psi(A,B)\rangle = \frac{1}{8}\sum_{a=0}^{7}\sum_{l=0}^{7} e^{\frac{i2\pi al}{L}} |l, f(a)\rangle$$

$$= \frac{1}{8}|0\rangle\left(|f(0)\rangle + \dots + |f(7)\rangle\right)$$

$$+ \frac{1}{8}|1\rangle\left(e^{i0}|f(0)\rangle + e^{\frac{i2\pi.1.1}{8}}|f(1)\rangle + e^{\frac{i2\pi.1.2}{8}}|f(2)\rangle\right.$$

$$\left. + e^{\frac{i2\pi.1.3}{8}}|f(3)\rangle + \dots + e^{\frac{i2\pi.1.7}{8}}|f(7)\rangle\right)$$

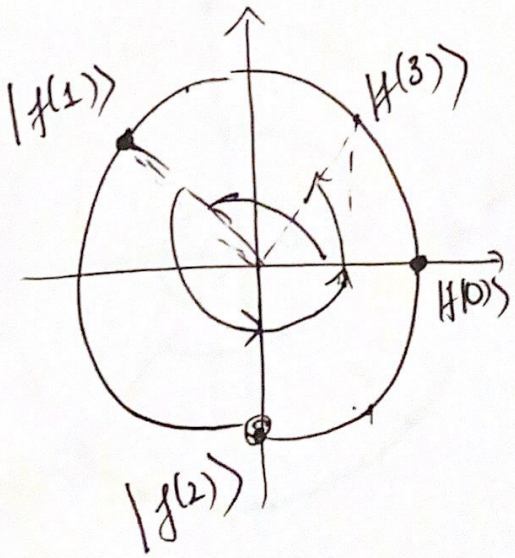
For $a = 3$

$$\rightarrow \frac{1}{8}|3\rangle\left(e^{i0}|f(0)\rangle + e^{\frac{i2\pi.3.1}{8}}|f(1)\rangle + e^{\frac{i2\pi.3.2}{8}}|f(2)\rangle\right.$$

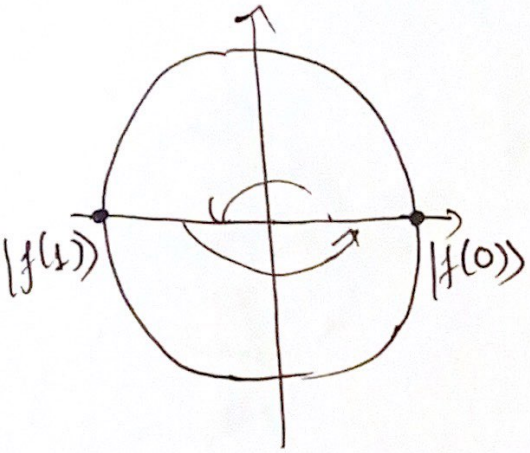
$$e^{i\frac{3\pi}{4}}|f(1)\rangle + e^{i\frac{6\pi}{4}}|f(2)\rangle$$

$$\left. + e^{i\frac{9\pi}{4}}|f(3)\rangle + \dots + e^{i\frac{21\pi}{4}}|f(7)\rangle\right)$$

$$e^{i\pi/4}$$

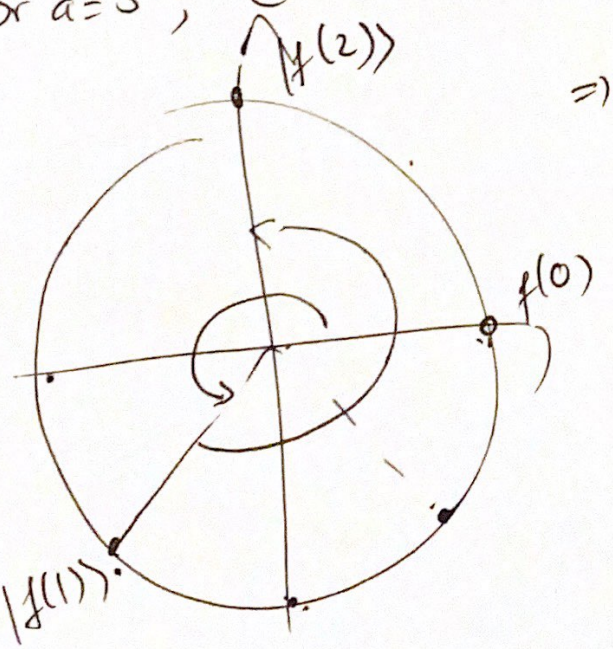


For $a=4 \rightarrow e^{\frac{i2\pi 4k}{8}} = e^{i\pi k}$ ; $k = 0, \ldots 7$



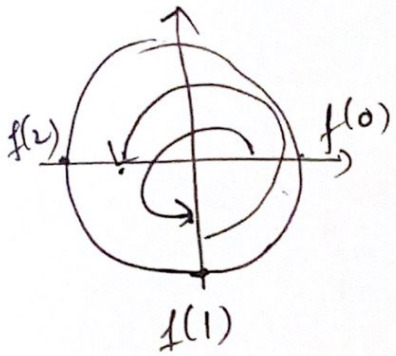

$$\frac{1}{8}|4\rangle \left( e^{i0}|f(0)\rangle + e^{i\pi}|f(1)\rangle + e^{i0}|f(2)\rangle + e^{i\pi}|f(3)\rangle + \ldots \right.$$
$$\left. \ldots e^{i0}|f(6)\rangle + e^{i\pi}|f(7)\rangle \right)$$

For $a=5$; $e^{\frac{i2\pi 5k}{8}} = e^{i\frac{5\pi}{4}k}$ ; $k = 0, \ldots 7$.




$$\Rightarrow \frac{1}{8}|5\rangle \left( e^{i0}f(0) + e^{i\frac{5\pi}{4}}|f(1)\rangle + e^{i\frac{\pi}{2}}f(2)\rangle \right.$$
$$e^{i\frac{-\pi}{4}}|f(3)\rangle + e^{i\pi}|f(4)\rangle$$
$$+ e^{i\frac{\pi}{4}}|f(5)\rangle + e^{-i\frac{\pi}{2}}|f(6)\rangle$$
$$\left. + e^{i\frac{3\pi}{4}}|f(7)\rangle \right)$$

$a=6$ ; $e^{\frac{i2\pi\, 6k}{8}} = e^{i\frac{6k}{4}\pi} = e^{i\frac{3\pi}{2}k}$.




$$\frac{1}{8}|6\rangle\Big(e^{i0}|f(0)\rangle + e^{-i\frac{\pi}{2}}|f(1)\rangle + e^{i\pi}|f(2)\rangle$$
$$e^{i\frac{\pi}{2}}|f(3)\rangle + e^{i0}|f(4)\rangle + e^{-i\frac{\pi}{4}}|f(5)\rangle$$
$$e^{i\pi}|f(6)\rangle + e^{i\frac{\pi}{2}}|f(7)\rangle\Big)$$