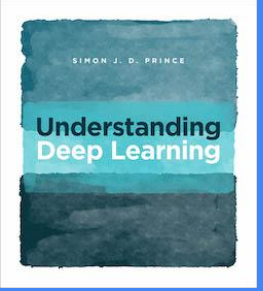
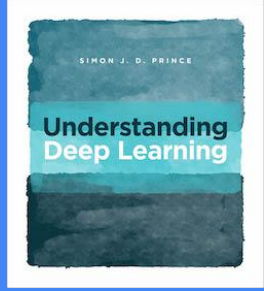
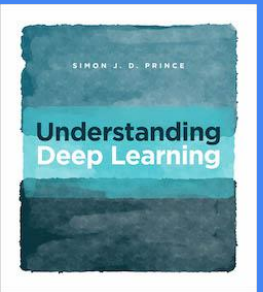
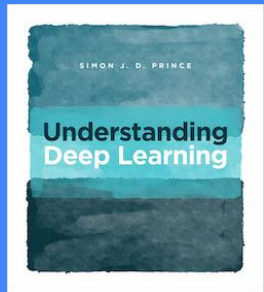




Shallow Neural Networks



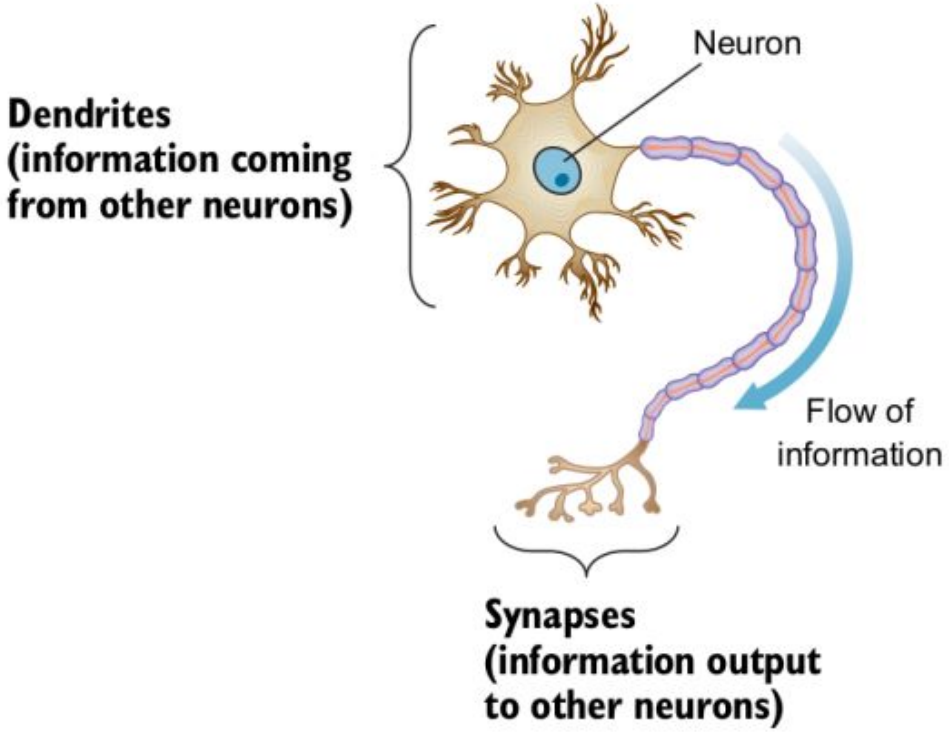


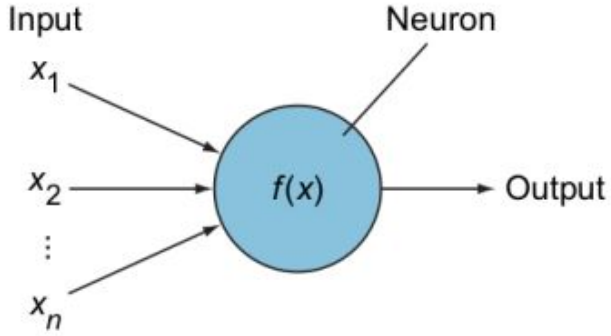
Building Complex Concept from Simple Components

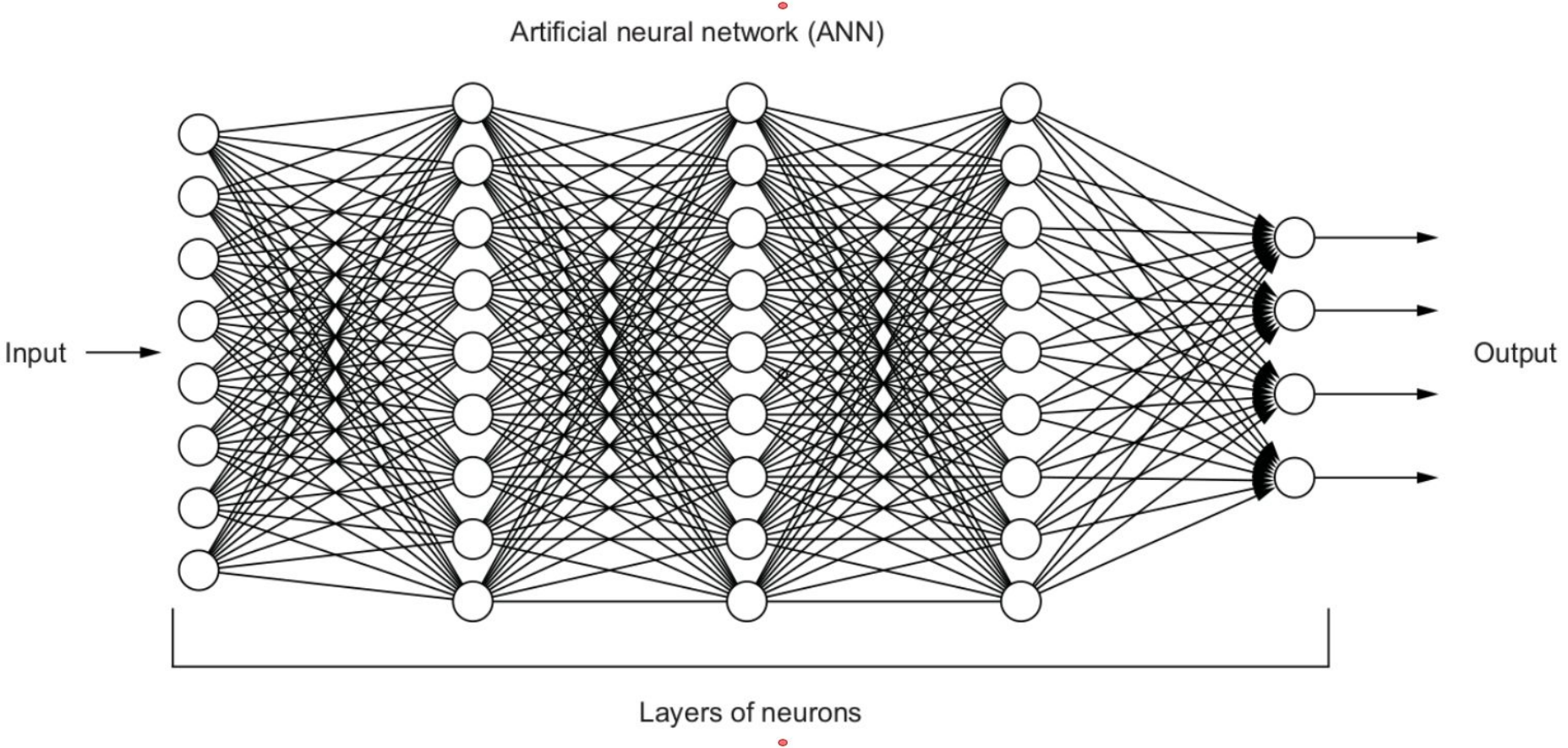

SINGAPORE AIRLINES

Biological neuron

Artificial neuron

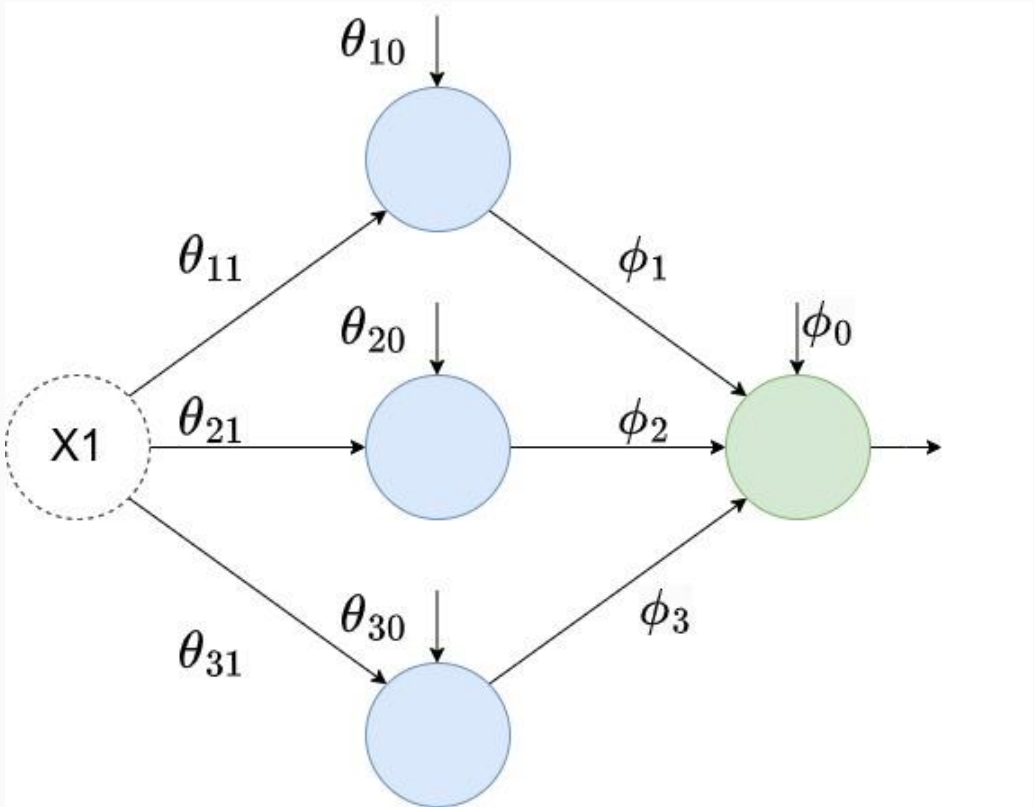





Artificial neural network (ANN)
Input
Output
Layers of neurons

Intelligence

Can intelligence emerge from a complex network of simple functions?



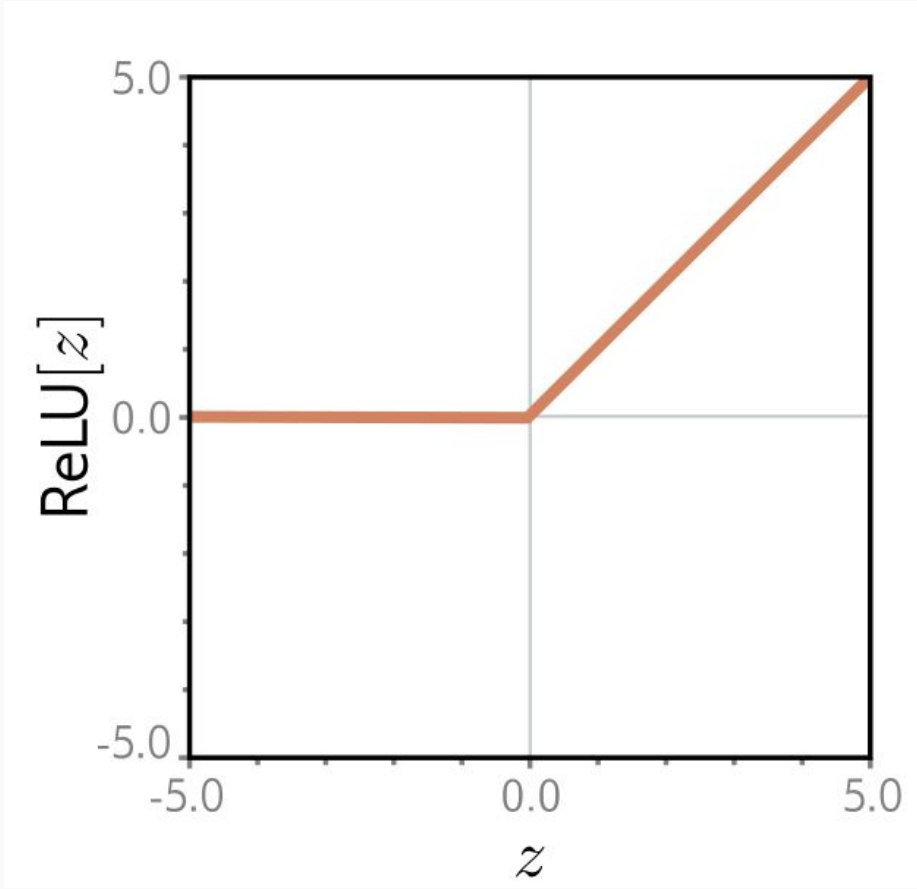
$$
\begin{aligned}
y &= \mathrm{f}[x, \boldsymbol{\phi}] \\
&= \phi_0 + \phi_1 \mathrm{a}[\theta_{10} + \theta_{11} x] + \phi_2 \mathrm{a}[\theta_{20} + \theta_{21} x] + \phi_3 \mathrm{a}[\theta_{30} + \theta_{31} x]
\end{aligned}
$$

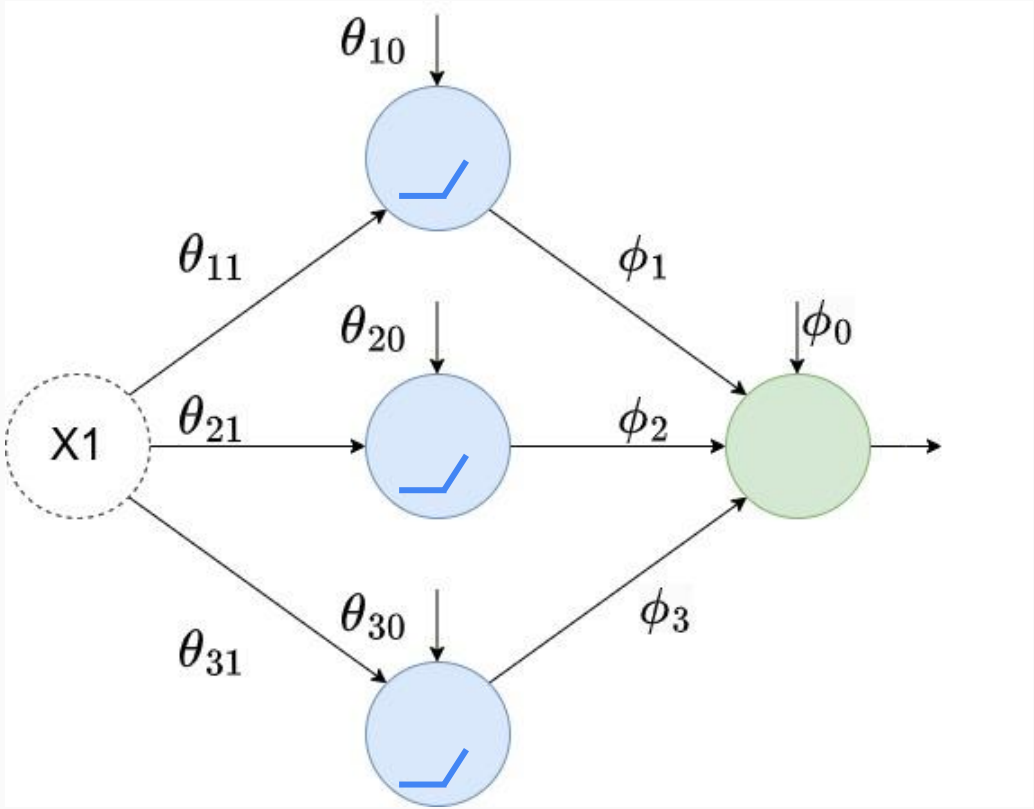
Is it possible to build complex functions in this manner?

We Need to Add Nonlinearity!

ReLU: Rectified Linear Unit

$$\mathrm{a}[z] = \mathrm{ReLU}[z] = \begin{cases} 0 & z < 0 \\ z & z \geq 0 \end{cases}$$



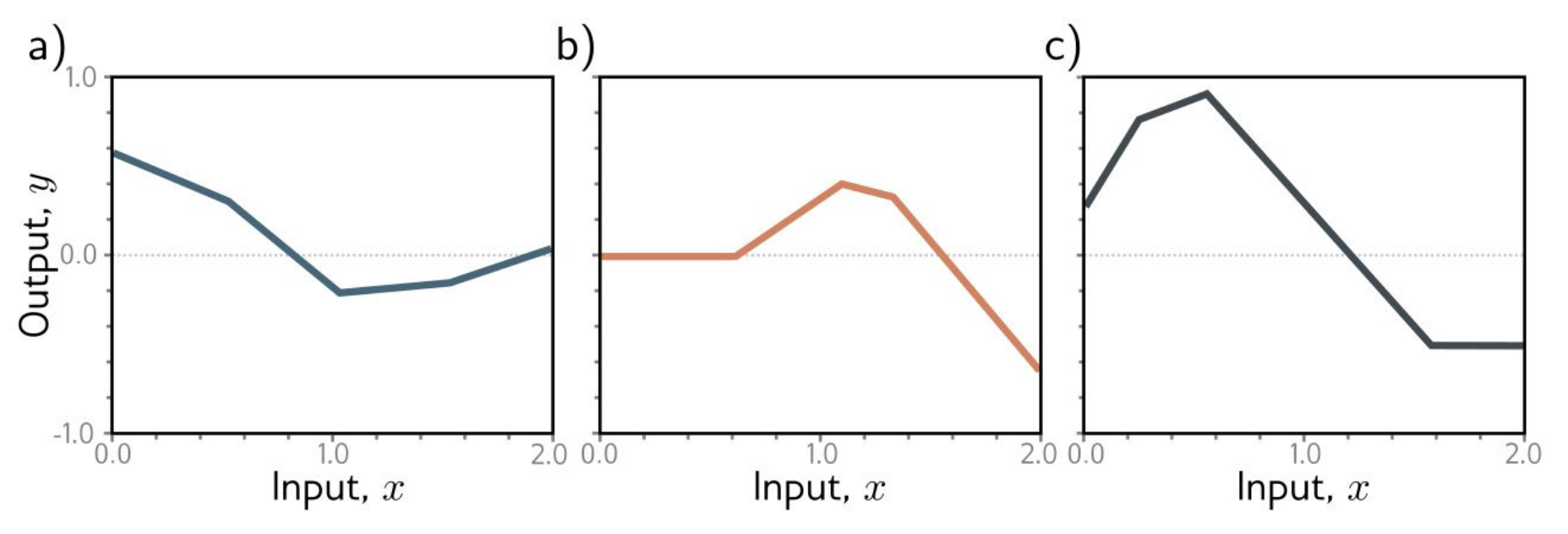


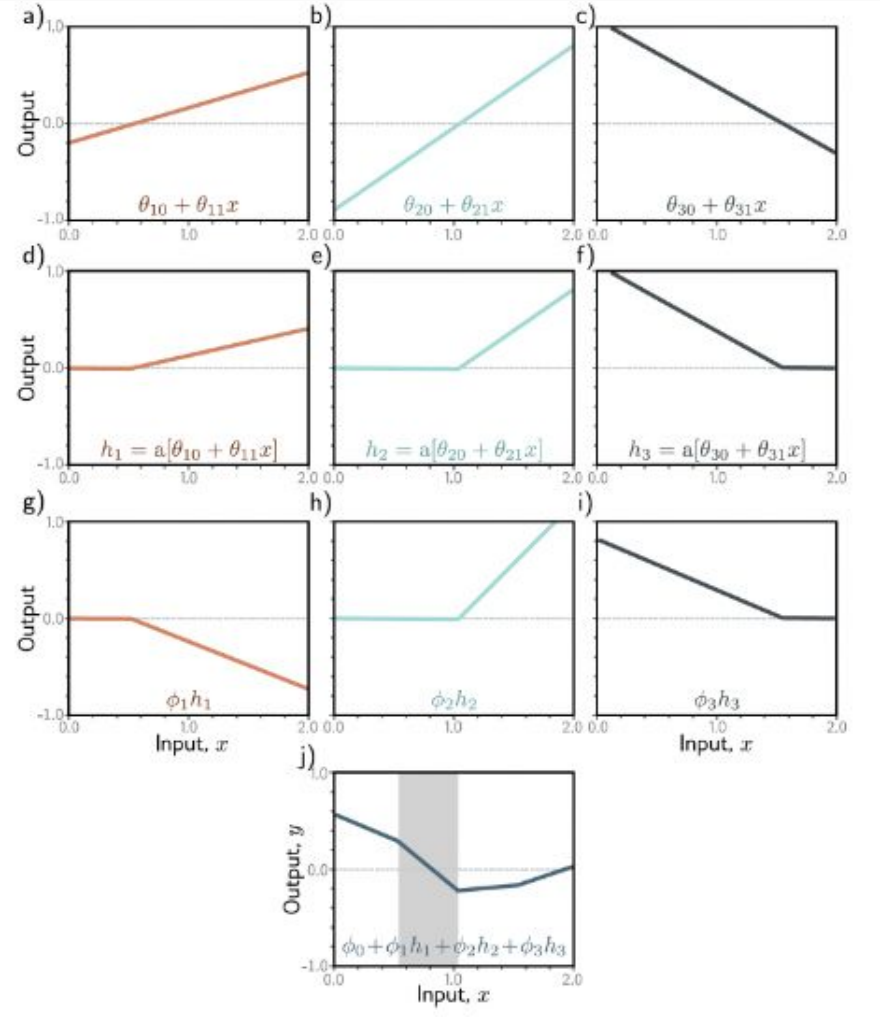
ReLU: 

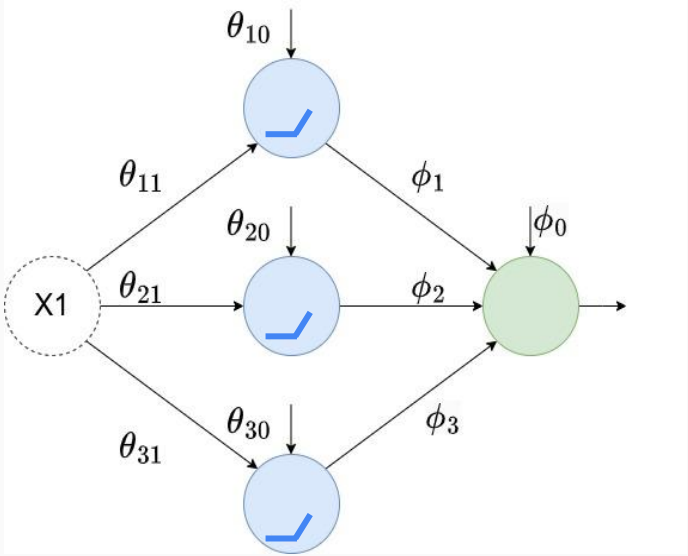
$$
\begin{aligned}
h_1 &= \mathrm{a}[\theta_{10} + \theta_{11}x] \\
h_2 &= \mathrm{a}[\theta_{20} + \theta_{21}x] \\
h_3 &= \mathrm{a}[\theta_{30} + \theta_{31}x]
\end{aligned}
$$

$$y = \phi_0 + \phi_1 h_1 + \phi_2 h_2 + \phi_3 h_3$$

Piecewise Linear Functions




a)
b)
c)
d)
e)
f)
g)
h)
i)
j)
Output
Output, y
Input, x
$\theta_{10} + \theta_{11}x$
$\theta_{20} + \theta_{21}x$
$\theta_{30} + \theta_{31}x$
$h_1 = a[\theta_{10} + \theta_{11}x]$
$h_2 = a[\theta_{20} + \theta_{21}x]$
$h_3 = a[\theta_{30} + \theta_{31}x]$
$\phi_1 h_1$
$\phi_2 h_2$
$\phi_3 h_3$
$\phi_0 + \phi_1 h_1 + \phi_2 h_2 + \phi_3 h_3$


θ_{10}
θ_{11}
θ_{20}
θ_{21}
θ_{30}
θ_{31}
ϕ_0
ϕ_1
ϕ_2
ϕ_3
X1

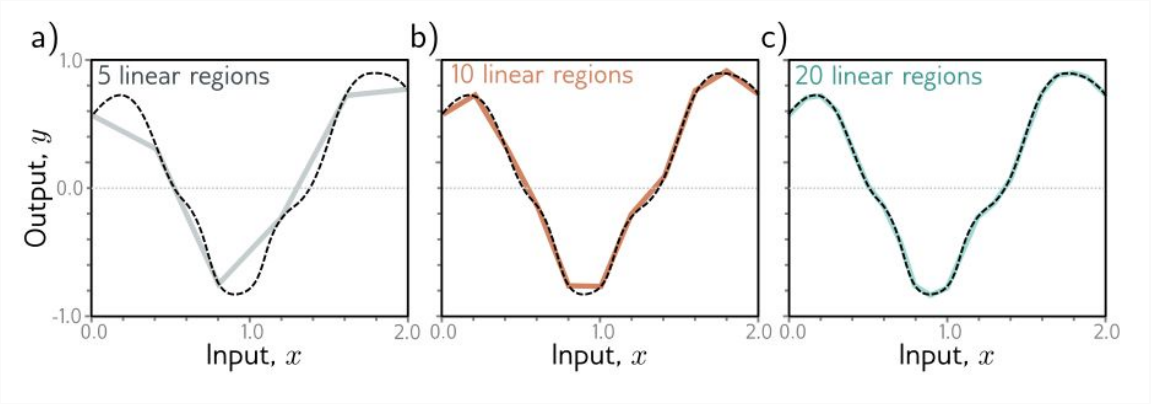
ReLU:

Universal Approximation Theorem

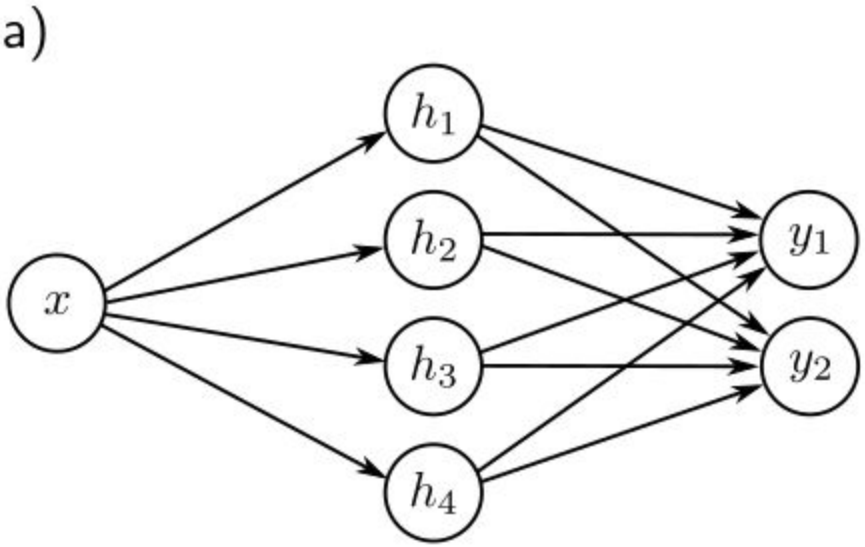
$$h_d = \mathrm{a}[\theta_{d0} + \theta_{d1}x]$$

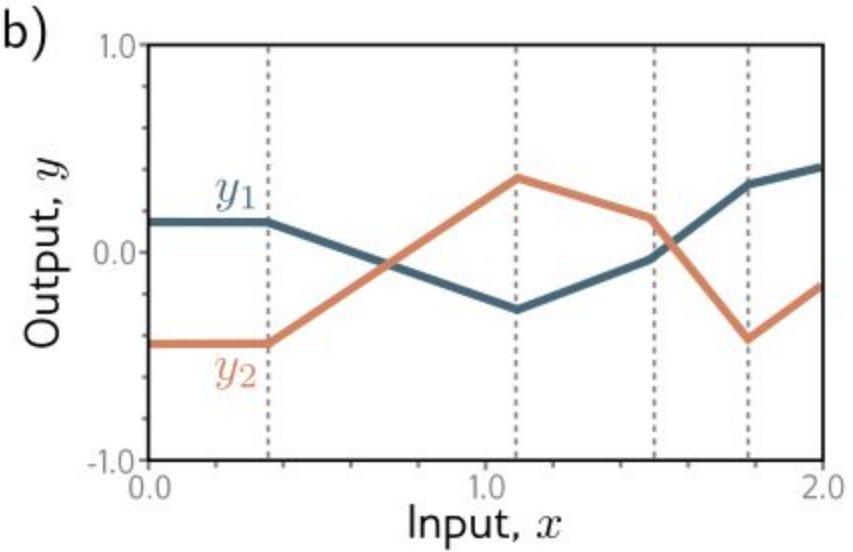
Piecewise Linear Functions

$$y = \phi_0 + \sum_{d=1}^{D} \phi_d h_d$$



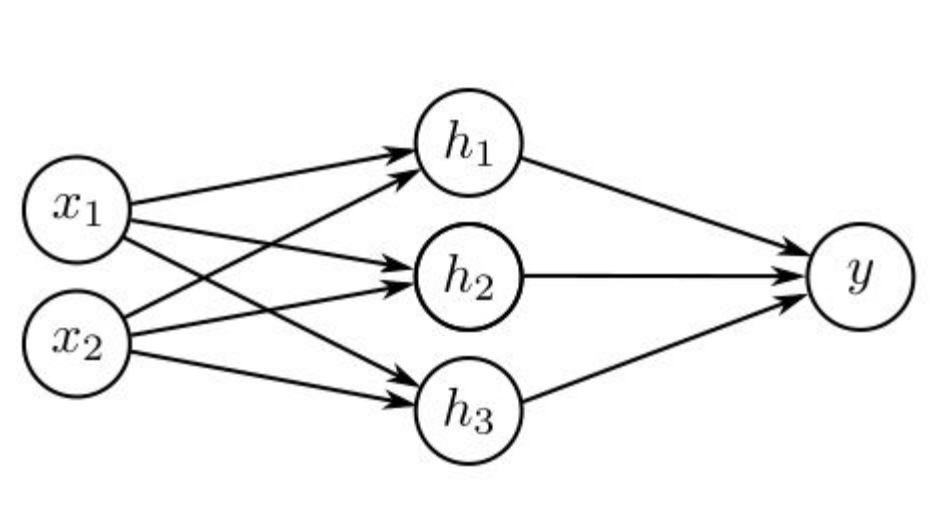
Multivariate Outputs

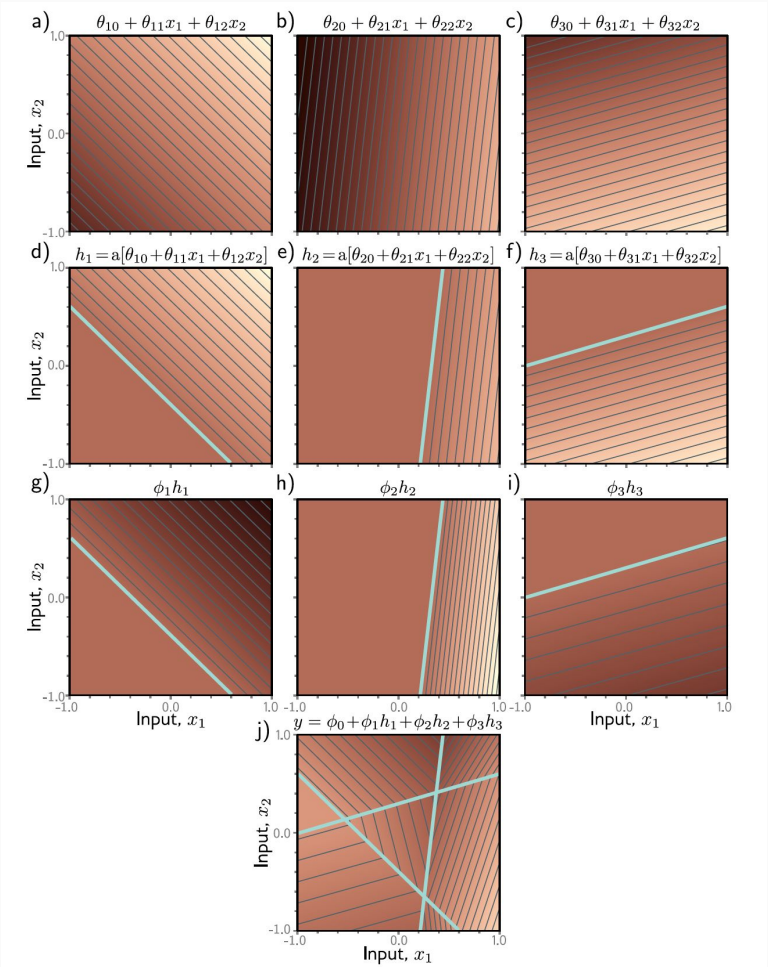




$$
\begin{aligned}
y_1 &= \phi_{10} + \phi_{11}h_1 + \phi_{12}h_2 + \phi_{13}h_3 + \phi_{14}h_4 \\
y_2 &= \phi_{20} + \phi_{21}h_1 + \phi_{22}h_2 + \phi_{23}h_3 + \phi_{24}h_4
\end{aligned}
$$

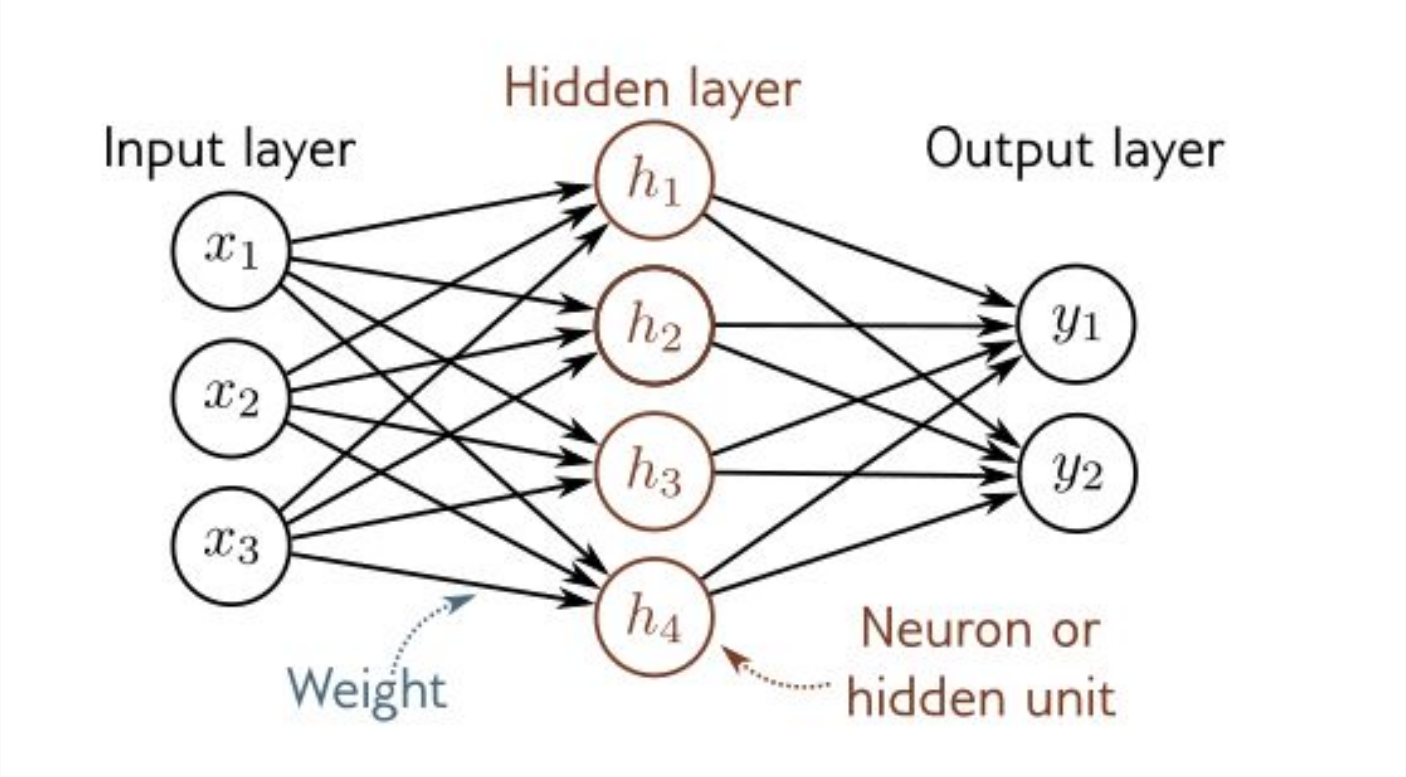
Multivariate Inputs





Visualizing Multivariate Inputs

Building Artificial Neural Networks



Chapters 4, 5, & 6 of UDL

Reading for next week