Prof. Andreas Orth High Integrity Systems, 2nd semester: Data Mining Assignment 5

week 5

Exercise 1:

For the shoesize/bodysize example in Assignment 4 construct the following matrices:

```
> shoesize<-c(38,38,39,39,40,40,41,41,42,42)
> bodysize<-c(153,161,167,169,173,176,182,181,188,189)</pre>
```

X is a matrix with 2 columns, one of which is filled with 1's, and the other is the vector of x-values (shoesize-values),

 X^{T} , which then has two rows (the first of which is just 1's) consisting of 10 values, $X^{T}X$, which is a 2 by 2 matrix,

 $(X^{T}X)^{-1}$, which of course is also 2 by 2,

 $\mathbf{X}^{\mathsf{T}}\mathbf{y}$ where \mathbf{y} is the vector of y-values (bodysize-values) – $\mathbf{X}^{\mathsf{T}}\mathbf{y}$ will be a 2-vector, $\mathbf{b} = (\mathbf{X}^{\mathsf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathsf{T}}\mathbf{y}$, this will also be a 2-vector of coefficients,

 $y_{pred} = X(X^TX)^{-1} X^Ty$, this will of course be a 10-vector of bodysize-predictions.

Now calculate the residual standard error using

```
ss_res<-sum((ypred - y)^2)
ms_res<-ss_res / 8
rse<-sqrt(ms res)</pre>
```

Using the two diagonal elements of $(X^TX)^{-1}$, calculate the standard error of each of the two coefficients (don't forget to take squre roots).

Now calculate a t-value for each coefficient (i.e coefficient / its standard error), and from this a p-value, using the t-distribution, pt(...,8,lower.tail=FALSE) for 8 degrees of freedom.

Exercise 2:

Replay the above example using the lm()-function.

Do the p-values coincide? Why or why not?

Do the following diagnostic plots:

- (a) x-values on the x-axis and residuals on the y-axis,
- (b) predictions on the x axis and residuls on the y-axis,

```
plot(fitted(fm), resid(fm), main="Residuals vs Fitted",
xlab="Fitted values", ylab="Residuals")
```

- (c) predictions on the x axis and observed values on the y-axis,
- (d) histogram of the residuals,
- (e) normal scores plot to check for skewness, kurtosis and outliers qqnorm(resid(fm), main="Residuals Rankit Plot")