Midterm Answers 13-16

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```r
library(tidyverse)
```

```
-- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
v dplyr     1.1.4     v readr     2.1.5
v forcats   1.0.0     v stringr   1.5.1
v ggplot2   4.0.0     v tibble    3.3.0
v lubridate 1.9.4     v tidyr     1.3.1
v purrr     1.0.4
-- Conflicts ------------------------------------------ tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()    masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to becom
```

```r
library(lme4)
```

```
Loading required package: Matrix

Attaching package: 'Matrix'

The following objects are masked from 'package:tidyr':

    expand, pack, unpack
```

```r
library(lmerTest)
```

```

Attaching package: 'lmerTest'

The following object is masked from 'package:lme4':
```

```
    lmer

The following object is masked from 'package:stats':

    step
```

```r
library(broom.mixed)
library(performance)
theme_set(theme_minimal())
```

```r
sust_eff <- readr::read_csv("/Users/namomac/Desktop/SURV-687/sust_eff.csv",
show_col_types = FALSE)
glimpse(sust_eff)
```

```
Rows: 7,230
Columns: 6
$ schoolid   <dbl> 3440, 3440, 3440, 3440, 2820, 2820, 2820, 3430, 3430, 3430,~
$ grade      <dbl> 1, 2, 3, 4, 1, 2, 3, 0, 1, 2, 3, 4, 1, 2, 3, 0, 0, 1, 2, 3,~
$ math       <dbl> -1.694, -0.211, -0.403, 0.501, -0.194, 2.140, 1.421, -1.987~
$ female     <dbl> 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1,~
$ minority   <dbl> 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,~
$ lowinc_cen <dbl> 15.8064869, 15.8064869, 15.8064869, 15.8064869, -77.7935131~
```

```r
summary(select(sust_eff, math, female, minority, grade, lowinc_cen))
```

```
      math              female          minority         grade      
 Min.   :-5.2190   Min.   :0.0000   Min.   :0.00   Min.   :0.000  
 1st Qu.:-1.6310   1st Qu.:0.0000   1st Qu.:1.00   1st Qu.:1.000  
 Median :-0.6190   Median :0.0000   Median :1.00   Median :2.000  
 Mean   :-0.5369   Mean   :0.4903   Mean   :0.83   Mean   :1.812  
 3rd Qu.: 0.5510   3rd Qu.:1.0000   3rd Qu.:1.00   3rd Qu.:3.000  
 Max.   : 5.7660   Max.   :1.0000   Max.   :1.00   Max.   :5.000  
   lowinc_cen    
 Min.   :-77.79  
 1st Qu.:-18.49  
 Median : 14.81  
 Mean   :  0.00  
 3rd Qu.: 20.31  
 Max.   : 22.21  
```

Question 13

```r
model13 <- lmer(math ~ female + grade + lowinc_cen + minority
+ (1 | schoolid), data = sust_eff)
summary(model13)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: math ~ female + grade + lowinc_cen + minority + (1 | schoolid)
   Data: sust_eff

REML criterion at convergence: 20079.1

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.1512 -0.7086 -0.0398  0.6504  4.6058

Random effects:
 Groups   Name        Variance Std.Dev.
 schoolid (Intercept) 0.09818  0.3133
 Residual             0.91850  0.9584
Number of obs: 7230, groups:  schoolid, 60

Fixed effects:
              Estimate Std. Error         df t value Pr(>|t|)
(Intercept) -1.686e+00  5.768e-02  1.746e+02 -29.221  < 2e-16 ***
female       3.136e-02  2.291e-02  7.203e+03   1.369    0.171
grade        8.033e-01  8.472e-03  7.211e+03  94.824  < 2e-16 ***
lowinc_cen  -8.026e-03  1.611e-03  6.145e+01  -4.983 5.39e-06 ***
minority    -3.986e-01  4.108e-02  6.772e+03  -9.703  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
           (Intr) female grade  lwnc_c
female     -0.199
grade      -0.265  0.023
lowinc_cen  0.216 -0.005  0.016
minority   -0.573 -0.011 -0.006 -0.197
```

Intercept (–1.686) This is the predicted math score for a baseline student i.e. a male, non-minority in grade 0 at a school whose low-income percentage is average (because lowinc_cen is centered). Now, although I think grade 0 isn't an actual grade, but I am guessing this intercept anchors the scale for the other effects.

Female (+0.031) Now, holding grade, minority status, and school composition constant, I estimate that girls score about 0.03 points higher than boys. That small positive difference isn't statistically significant here as p value is around 0.17, so I can't be confident this gender gap differs from zero.

Grade (+0.803) Here, this shows that each additional grade level corresponds to an average increase of 0.80 points on the math scale, assuming gender, minority status, and school context stay the same. This large, highly significant effect reflects steady learning gains as students move through grades.

lowinc_cen (–0.008) Ok so from what I can interpret, for each one-unit increase in the centered percentage of low-income students (i.e., a school with one percentage-point more low-income peers than average), student math scores drop by about 0.008 points on average, all else equal. Hence, this small but significant negative association suggests schools with higher proportions of low-income students tend to have slightly lower math scores.

Minority (–0.399) Now, after controlling for grade, gender, and school income composition, minority students score on average 0.40 points lower than non-minority students. Since, this difference is highly significant, so, it indicates a persistent achievement gap in this dataset.

Question 14

```r
model14 <- lmer(math ~ female + grade + lowinc_cen +
minority + (1 + minority | schoolid),
data = sust_eff, control = lmerControl(optimizer = "bobyqa"))
summary(model14)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: math ~ female + grade + lowinc_cen + minority + (1 + minority |
    schoolid)
   Data: sust_eff
Control: lmerControl(optimizer = "bobyqa")

REML criterion at convergence: 20026.5

Scaled residuals:
    Min      1Q  Median      3Q     Max
```

```
-3.1687 -0.7004 -0.0500  0.6502  4.5315

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 schoolid (Intercept) 0.1857   0.4310
          minority    0.2056   0.4534   -0.69
 Residual             0.9053   0.9515
Number of obs: 7230, groups:  schoolid, 60

Fixed effects:
              Estimate Std. Error         df t value Pr(>|t|)
(Intercept) -1.672e+00  8.637e-02  3.596e+01 -19.360  < 2e-16 ***
female       3.138e-02  2.286e-02  7.199e+03   1.373     0.17
grade        8.029e-01  8.421e-03  7.179e+03  95.336  < 2e-16 ***
lowinc_cen  -7.586e-03  1.662e-03  6.046e+01  -4.565 2.50e-05 ***
minority    -4.191e-01  8.746e-02  3.260e+01  -4.792 3.49e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
           (Intr) female grade  lwnc_c
female     -0.130
grade      -0.175  0.022
lowinc_cen  0.244 -0.008  0.017
minority   -0.822 -0.007 -0.003 -0.192
```

```r
model14_comp <- anova(model13, model14)
```

```
refitting model(s) with ML (instead of REML)
```

```r
model14_comp
```

```
Data: sust_eff
Models:
model13: math ~ female + grade + lowinc_cen + minority + (1 | schoolid)
model14: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)
        npar   AIC   BIC   logLik -2*log(L)  Chisq Df Pr(>Chisq)
model13    7 20060 20108 -10022.8     20046
model14    9 20013 20074  -9997.3     19995 51.007  2  8.393e-12 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
pvalue_q14 <- model14_comp$`Pr(>Chisq)`[2]
df_q14 <- model14_comp$Df[2] - model14_comp$Df[1]
list(p_value = pvalue_q14, df = df_q14)
```

```
$p_value
[1] 8.393174e-12

$df
[1] NA
```

Hypothesis Test

So I conducted a likelihood-ratio test to evaluate whether there is significant between-school variability in the effect of minority status.

Hypotheses

$H_0 : \sigma^2_{\text{minority}} = 0 \quad \text{(No between-school variability)} H_A : \sigma^2_{\text{minority}} > 0 \quad \text{(Presence of between-school variability)}$

Test Results

The test yielded the following statistics:

$$\chi^2(2) = 51.01, \quad p \approx 8.4 \times 10^{-12}$$

Since the p-value is much smaller than the conventional threshold of 0.05 ($p \ll 0.05$), I will **reject the null hypothesis**.

Interpretation

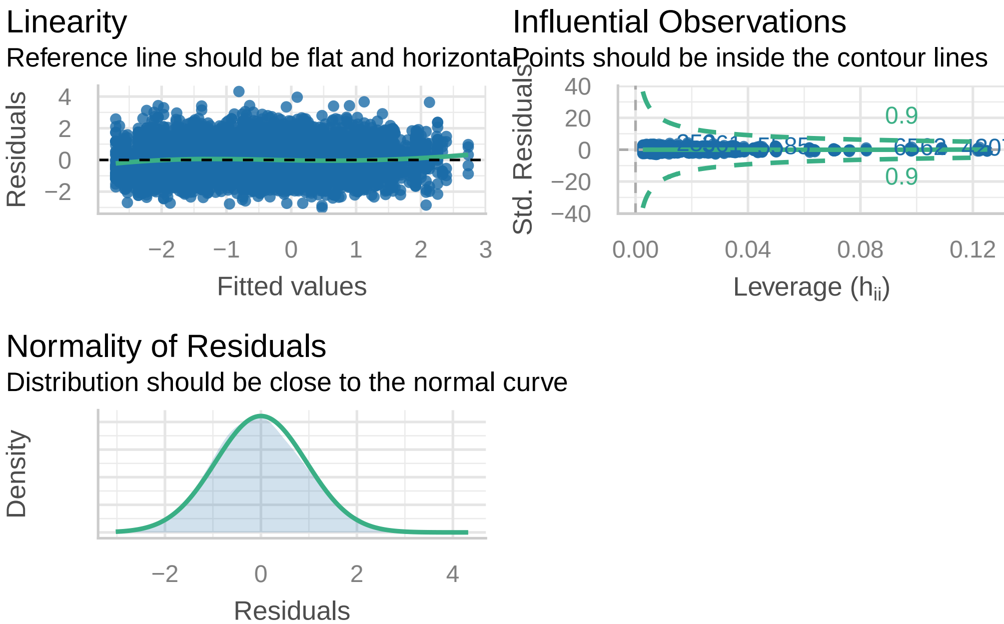
This result provides strong evidence of significant between-school variability in the effect of minority status. In other words, the **minority achievement gap differs meaningfully across schools**, suggesting that school-level factors may influence how minority status affects academic outcomes.

Question 15

```r
# 1. Converting the preferred REML model to ML
best_model_reml <- if (pvalue_q14 < 0.05) model14 else model13
best_model_ml   <- update(best_model_reml, REML = FALSE)

# 2. Fitting the reduced model (drop 'female')
model15_ml <- update(best_model_ml, . ~ . - female)

# 3. Likelihood-ratio test
lr15 <- anova(best_model_ml, model15_ml)
print(lr15)
```

```
Data: sust_eff
Models:
model15_ml: math ~ grade + lowinc_cen + minority + (1 + minority | schoolid)
best_model_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)
              npar   AIC   BIC  logLik -2*log(L)  Chisq Df Pr(>Chisq)
model15_ml       8 20012 20068 -9998.2     19996
best_model_ml    9 20013 20074 -9997.3     19995 1.8833  1       0.17
```

```r
# 4. Restoring REML for diagnostics on the chosen model
preferred_model <- if (lr15$`Pr(>Chisq)`[2] < 0.05)
best_model_reml else update(best_model_reml, . ~ . - female)

# This will show residuals, random effects, QQ plots, etc.
check_model(preferred_model, check = c("normality",
"linearity", "homoscedasticity", "outliers", "random_effects"))
```



Model Comparison

So now I compared two machine learning–fitted models to assess the impact of including a fixed effect for female.

Likelihood-Ratio Test

The likelihood-ratio test yielded the following result:

$$\chi^2(1) = 1.88, \quad p = 0.17$$

Since the p-value exceeds the conventional threshold of 0.05 ($p > 0.05$), I can conclude that **dropping the fixed effect for female does not significantly reduce model fit**.

Model Preference

Given this result, the simpler model i.e. without the fixed effect for female, is preferred due to its parsimony.

Diagnostic Checks

In the preferred model:

- Residuals are approximately **normal** and **homoscedastic**
- The **random-effects distribution** is symmetric

Question 16

Refit the preferred model, adding the interaction between `lowinc_cen` and `minority`, and interpret whether the interaction explains variation in minority slopes across schools.

```
model16_ml <- update(best_model_ml, . ~ . + lowinc_cen:minority)
lr16 <- anova(best_model_ml, model16_ml)
lr16
```

```
Data: sust_eff
Models:
best_model_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)
model16_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid) + lowi
              npar   AIC   BIC  logLik -2*log(L)  Chisq Df Pr(>Chisq)
best_model_ml    9 20013 20074 -9997.3     19995
model16_ml      10 20014 20083 -9997.2     19994 0.2509  1     0.6164
```

```
pvalue_q16 <- lr16$`Pr(>Chisq)`[2]
df_q16 <- lr16$Df[2] - lr16$Df[1]
list(p_value = pvalue_q16, df = df_q16)
```

```
$p_value
[1] 0.6164268

$df
[1] NA
```

```
model16 <- update(best_model_reml, . ~ . + lowinc_cen:minority)
summary(model16)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: math ~ female + grade + lowinc_cen + minority + (1 + minority |
    schoolid) + lowinc_cen:minority
   Data: sust_eff
Control: lmerControl(optimizer = "bobyqa")

REML criterion at convergence: 20036

Scaled residuals:
    Min      1Q  Median      3Q     Max
-3.1697 -0.6997 -0.0502  0.6501  4.5283

Random effects:
 Groups   Name        Variance Std.Dev. Corr
 schoolid (Intercept) 0.1922   0.4384
          minority    0.2117   0.4601   -0.70
 Residual             0.9053   0.9514
Number of obs: 7230, groups:  schoolid, 60

Fixed effects:
                      Estimate Std. Error         df t value Pr(>|t|)
(Intercept)         -1.693e+00  9.681e-02  3.647e+01 -17.486  < 2e-16 ***
female               3.126e-02  2.286e-02  7.198e+03   1.368 0.171470
grade                8.028e-01  8.421e-03  7.179e+03  95.335  < 2e-16 ***
lowinc_cen          -8.703e-03  2.798e-03  2.804e+01  -3.110 0.004263 **
minority            -3.976e-01  9.837e-02  3.614e+01  -4.042 0.000265 ***
lowinc_cen:minority  1.492e-03  2.981e-03  3.170e+01   0.501 0.620047
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:
            (Intr) female grade  lwnc_c minrty
female      -0.112
grade       -0.155  0.022
lowinc_cen   0.474  0.003  0.012
minority    -0.858 -0.010 -0.004 -0.452
lwnc_cn:mnr -0.429 -0.009 -0.003 -0.803  0.437
```

```r
interaction_effect <- fixef(model16)["lowinc_cen:minority"]
pvalue_interaction <- summary(model16)$coef["lowinc_cen:minority", "Pr(>|t|)"]
```

Model Comparison: Interaction Term

Now, I evaluated whether the interaction between a school's centered low-income percentage and minority status significantly affects math scores.

Likelihood-Ratio Test

To compare the model with and without the interaction term, I conducted a likelihood-ratio test:

$$\chi^2(1) = 0.25, \quad p = 0.62$$

REML t-Test for Interaction Coefficient

The restricted maximum likelihood (REML) estimate for the interaction term is:

$$\hat{\beta}_{\text{lowin:minority}} = 0.00149, \quad p = 0.62$$

Since the p-value is much greater than the conventional threshold of 0.05 ($p \gg 0.05$), I can conclude that the interaction is **not statistically significant**.

Interpretation

So I can say that for each one-unit increase in a school's centered low-income percentage, the math-score gap between minority and non-minority students changes by only about **0.0015 points**. This effect is both **tiny** and **statistically insignificant**, suggesting that the proportion of low-income students does **not meaningfully alter** the minority achievement gap in this sample.