

Midterm Answers 13-16

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```
library(tidyverse)
```

```
-- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
v dplyr      1.1.4      v readr      2.1.5
v forcats    1.0.0      v stringr    1.5.1
v ggplot2    4.0.0      v tibble     3.3.0
v lubridate  1.9.4      v tidyr      1.3.1
v purrr      1.0.4
-- Conflicts ----- tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()     masks stats::lag()
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
```

```
library(lme4)
```

Loading required package: Matrix

Attaching package: 'Matrix'

The following objects are masked from 'package:tidyr':

expand, pack, unpack

```
library(lmerTest)
```

Attaching package: 'lmerTest'

The following object is masked from 'package:lme4':

```
lmer
```

The following object is masked from 'package:stats':

```
step
```

```
library(broom.mixed)
library(performance)
theme_set(theme_minimal())
```

```
sust_eff <- readr::read_csv("/Users/namomac/Desktop/SURV-687/sust_eff.csv",
show_col_types = FALSE)
glimpse(sust_eff)
```

Rows: 7,230

Columns: 6

```
$ schoolid   <dbl> 3440, 3440, 3440, 3440, 2820, 2820, 2820, 3430, 3430, 3430,~
$ grade      <dbl> 1, 2, 3, 4, 1, 2, 3, 0, 1, 2, 3, 4, 1, 2, 3, 0, 0, 1, 2, 3,~
$ math       <dbl> -1.694, -0.211, -0.403, 0.501, -0.194, 2.140, 1.421, -1.987~
$ female     <dbl> 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1,~
$ minority   <dbl> 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,~
$ lowinc_cen <dbl> 15.8064869, 15.8064869, 15.8064869, 15.8064869, -77.7935131~
```

```
summary(select(sust_eff, math, female, minority, grade, lowinc_cen))
```

math	female	minority	grade
Min. : -5.2190	Min. : 0.0000	Min. : 0.00	Min. : 0.000
1st Qu.: -1.6310	1st Qu.: 0.0000	1st Qu.: 1.00	1st Qu.: 1.000
Median : -0.6190	Median : 0.0000	Median : 1.00	Median : 2.000
Mean : -0.5369	Mean : 0.4903	Mean : 0.83	Mean : 1.812
3rd Qu.: 0.5510	3rd Qu.: 1.0000	3rd Qu.: 1.00	3rd Qu.: 3.000
Max. : 5.7660	Max. : 1.0000	Max. : 1.00	Max. : 5.000

lowinc_cen
Min. : -77.79
1st Qu.: -18.49
Median : 14.81
Mean : 0.00
3rd Qu.: 20.31
Max. : 22.21

Question 13

```
model13 <- lmer(math ~ female + grade + lowinc_cen + minority
+ (1 | schoolid), data = sust_eff)
summary(model13)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]

Formula: math ~ female + grade + lowinc_cen + minority + (1 | schoolid)
Data: sust_eff

REML criterion at convergence: 20079.1

Scaled residuals:

Min	1Q	Median	3Q	Max
-3.1512	-0.7086	-0.0398	0.6504	4.6058

Random effects:

Groups	Name	Variance	Std.Dev.
schoolid	(Intercept)	0.09818	0.3133
	Residual	0.91850	0.9584

Number of obs: 7230, groups: schoolid, 60

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	-1.686e+00	5.768e-02	1.746e+02	-29.221	< 2e-16 ***
female	3.136e-02	2.291e-02	7.203e+03	1.369	0.171
grade	8.033e-01	8.472e-03	7.211e+03	94.824	< 2e-16 ***
lowinc_cen	-8.026e-03	1.611e-03	6.145e+01	-4.983	5.39e-06 ***
minority	-3.986e-01	4.108e-02	6.772e+03	-9.703	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	female	grade	lwnc_c
female	-0.199			
grade	-0.265	0.023		
lowinc_cen	0.216	-0.005	0.016	
minority	-0.573	-0.011	-0.006	-0.197

Intercept (-1.686) This is the predicted math score for a baseline student i.e. a male, non-

minority in grade 0 at a school whose low-income percentage is average (because `lowinc_cen` is centered). Now, although I think grade 0 isn't an actual grade, but I am guessing this intercept anchors the scale for the other effects.

Female (+0.031) Now, holding grade, minority status, and school composition constant, I estimate that girls score about 0.03 points higher than boys. That small positive difference isn't statistically significant here as p value is around 0.17, so I can't be confident this gender gap differs from zero.

Grade (+0.803) Here, this shows that each additional grade level corresponds to an average increase of 0.80 points on the math scale, assuming gender, minority status, and school context stay the same. This large, highly significant effect reflects steady learning gains as students move through grades.

lowinc_cen (-0.008) Ok so from what I can interpret, for each one-unit increase in the centered percentage of low-income students (i.e., a school with one percentage-point more low-income peers than average), student math scores drop by about 0.008 points on average, all else equal. Hence, this small but significant negative association suggests schools with higher proportions of low-income students tend to have slightly lower math scores.

Minority (-0.399) Now, after controlling for grade, gender, and school income composition, minority students score on average 0.40 points lower than non-minority students. Since, this difference is highly significant, so, it indicates a persistent achievement gap in this dataset.

Question 14

```
model14 <- lmer(math ~ female + grade + lowinc_cen +  
minority + (1 + minority | schoolid),  
data = sust_eff, control = lmerControl(optimizer = "bobyqa"))  
summary(model14)
```

```
Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]
Formula: math ~ female + grade + lowinc_cen + minority + (1 + minority |
      schoolid)
Data: sust_eff
Control: lmerControl(optimizer = "bobyqa")

REML criterion at convergence: 20026.5
```

```
Scaled residuals:
      Min       1Q   Median       3Q      Max
```

-3.1687 -0.7004 -0.0500 0.6502 4.5315

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
schoolid	(Intercept)	0.1857	0.4310	
	minority	0.2056	0.4534	-0.69
Residual		0.9053	0.9515	

Number of obs: 7230, groups: schoolid, 60

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	-1.672e+00	8.637e-02	3.596e+01	-19.360	< 2e-16 ***
female	3.138e-02	2.286e-02	7.199e+03	1.373	0.17
grade	8.029e-01	8.421e-03	7.179e+03	95.336	< 2e-16 ***
lowinc_cen	-7.586e-03	1.662e-03	6.046e+01	-4.565	2.50e-05 ***
minority	-4.191e-01	8.746e-02	3.260e+01	-4.792	3.49e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	female	grade	lwnc_c
female	-0.130			
grade	-0.175	0.022		
lowinc_cen	0.244	-0.008	0.017	
minority	-0.822	-0.007	-0.003	-0.192

```
model14_comp <- anova(model13, model14)
```

refitting model(s) with ML (instead of REML)

```
model14_comp
```

Data: sust_eff

Models:

model13: math ~ female + grade + lowinc_cen + minority + (1 | schoolid)
model14: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)

	npar	AIC	BIC	logLik	-2*log(L)	Chisq	Df	Pr(>Chisq)
model13	7	20060	20108	-10022.8	20046			
model14	9	20013	20074	-9997.3	19995	51.007	2	8.393e-12 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
pvalue_q14 <- model14_comp$`Pr(>Chisq)`[2]
df_q14 <- model14_comp$Df[2] - model14_comp$Df[1]
list(p_value = pvalue_q14, df = df_q14)
```

```
$p_value
[1] 8.393174e-12
```

```
$df
[1] NA
```

Hypothesis Test

So I conducted a likelihood-ratio test to evaluate whether there is significant between-school variability in the effect of minority status.

Hypotheses

$H_0 : \sigma_{\text{minority}}^2 = 0$ (No between-school variability) $H_A : \sigma_{\text{minority}}^2 > 0$ (Presence of between-school variability)

Test Results

The test yielded the following statistics:

$$\chi^2(2) = 51.01, \quad p \approx 8.4 \times 10^{-12}$$

Since the p-value is much smaller than the conventional threshold of 0.05 ($p \ll 0.05$), I will **reject the null hypothesis**.

Interpretation

This result provides strong evidence of significant between-school variability in the effect of minority status. In other words, the **minority achievement gap differs meaningfully across schools**, which I guess is suggesting that school-level factors may influence how minority status affects academic outcomes.

Question 15

```
# 1. Converting the preferred REML model to ML
best_model_reml <- if (pvalue_q14 < 0.05) model14 else model13
best_model_ml <- update(best_model_reml, REML = FALSE)

# 2. Fitting the reduced model (drop 'female')
model15_ml <- update(best_model_ml, . ~ . - female)

# 3. Likelihood-ratio test
lr15 <- anova(best_model_ml, model15_ml)
print(lr15)
```

Data: sust_eff

Models:

model15_ml: math ~ grade + lowinc_cen + minority + (1 + minority | schoolid)

best_model_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)

	npars	AIC	BIC	logLik	-2*log(L)	Chisq	Df	Pr(>Chisq)
model15_ml	8	20012	20068	-9998.2	19996			
best_model_ml	9	20013	20074	-9997.3	19995	1.8833	1	0.17

```
# 4. Restoring REML for diagnostics on the chosen model
```

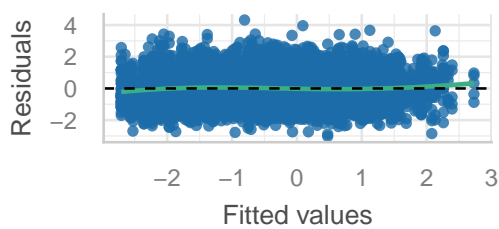
```
preferred_model <- if (lr15$`Pr(>Chisq)`[2] < 0.05)
best_model_reml else update(best_model_reml, . ~ . - female)
```

```
# This will show residuals, random effects, QQ plots, etc.
```

```
check_model(preferred_model, check = c("normality",
"linearity", "homoscedasticity", "outliers", "random_effects"))
```

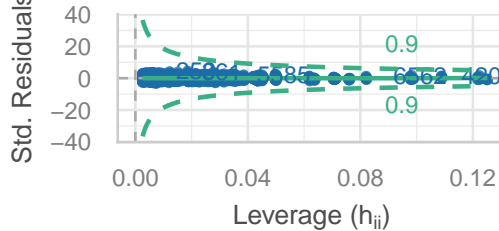
Linearity

Reference line should be flat and horizontal



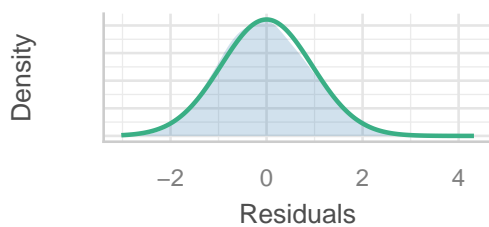
Influential Observations

Points should be inside the contour lines



Normality of Residuals

Distribution should be close to the normal curve



Model Comparison

So now I compared two fitted models to assess the impact of including a fixed effect for female.

Likelihood-Ratio Test

The likelihood-ratio test yielded the following result:

$$\chi^2(1) = 1.88, \quad p = 0.17$$

Since the p-value exceeds the conventional threshold of 0.05 ($p > 0.05$), I can conclude that **dropping the fixed effect for female does not significantly reduce model fit.**

Model Preference

Given this result, occam's razor, the simpler model i.e. without the fixed effect for female, is preferred due to its parsimony.

Diagnostic Checks

In the preferred model:

- Residuals are approximately **normal** and **homoscedastic**
- The **random-effects distribution** is symmetric

Question 16

```
model16_ml <- update(best_model_ml, . ~ . + lowinc_cen:minority)
lr16 <- anova(best_model_ml, model16_ml)
lr16
```

Data: sust_eff

Models:

best_model_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid)

model16_ml: math ~ female + grade + lowinc_cen + minority + (1 + minority | schoolid) + lowinc_cen:minority

	npar	AIC	BIC	logLik	-2*log(L)	Chisq	Df	Pr(>Chisq)
best_model_ml	9	20013	20074	-9997.3	19995			
model16_ml	10	20014	20083	-9997.2	19994	0.2509	1	0.6164

```
pvalue_q16 <- lr16$`Pr(>Chisq)`[2]
df_q16 <- lr16$Df[2] - lr16$Df[1]
list(p_value = pvalue_q16, df = df_q16)
```

\$p_value

[1] 0.6164268

\$df

[1] NA

```
model16 <- update(best_model_reml, . ~ . + lowinc_cen:minority)
summary(model16)
```

Linear mixed model fit by REML. t-tests use Satterthwaite's method [
lmerModLmerTest]

Formula: math ~ female + grade + lowinc_cen + minority + (1 + minority |
schoolid) + lowinc_cen:minority

```
Data: sust_eff
Control: lmerControl(optimizer = "bobyqa")
```

REML criterion at convergence: 20036

Scaled residuals:

	Min	1Q	Median	3Q	Max
	-3.1697	-0.6997	-0.0502	0.6501	4.5283

Random effects:

Groups	Name	Variance	Std.Dev.	Corr
	schoolid (Intercept)	0.1922	0.4384	
	minority	0.2117	0.4601	-0.70
	Residual	0.9053	0.9514	

Number of obs: 7230, groups: schoolid, 60

Fixed effects:

	Estimate	Std. Error	df	t value	Pr(> t)
(Intercept)	-1.693e+00	9.681e-02	3.647e+01	-17.486	< 2e-16 ***
female	3.126e-02	2.286e-02	7.198e+03	1.368	0.171470
grade	8.028e-01	8.421e-03	7.179e+03	95.335	< 2e-16 ***
lowinc_cen	-8.703e-03	2.798e-03	2.804e+01	-3.110	0.004263 **
minority	-3.976e-01	9.837e-02	3.614e+01	-4.042	0.000265 ***
lowinc_cen:minority	1.492e-03	2.981e-03	3.170e+01	0.501	0.620047

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation of Fixed Effects:

	(Intr)	female	grade	lwnc_c	minrty
female	-0.112				
grade	-0.155	0.022			
lowinc_cen	0.474	0.003	0.012		
minority	-0.858	-0.010	-0.004	-0.452	
lwnc_cen:mnr	-0.429	-0.009	-0.003	-0.803	0.437

```
interaction_effect <- fixef(model16)["lowinc_cen:minority"]
pvalue_interaction <- summary(model16)$coef["lowinc_cen:minority", "Pr(>|t|)"]
```

Model Comparison: Interaction Term

Now, I evaluated whether the interaction between a school's centered low-income percentage and minority status significantly affects math scores.

Likelihood-Ratio Test

To compare the model with and without the interaction term, I conducted a likelihood-ratio test:

$$\chi^2(1) = 0.25, \quad p = 0.62$$

REML t-Test for Interaction Coefficient

The restricted maximum likelihood (REML) estimate for the interaction term is:

$$\hat{\beta}_{\text{lowin:minority}} = 0.00149, \quad p = 0.62$$

Since the p-value is much greater than the conventional threshold of 0.05 ($p \gg 0.05$), I can conclude that the interaction is **not statistically significant**.

Interpretation

So I can say that for each one-unit increase in a school's centered low-income percentage, the math-score gap between minority and non-minority students changes by only about **0.0015 points**. This effect is both **tiny** and **statistically insignificant**, suggesting that the proportion of low-income students does **not meaningfully alter** the minority achievement gap in this sample.