Teacher Scheduling Constraints and Cost Function

Let:

- T = number of teachers (45 teachers)
- D = number of days (6 days)
- S = number of shifts per day (9 shifts, including 1 break shift)
- b = index of the break shift (4)
- M = maximum allowed shifts per teacher per week (12)
- $\text{schedule}[d, s] \in \{0, 1, \ldots, T-1\} \cup \{-1\}$ be the teacher assigned to day d and shift s, or -1 if unassigned.

1. Break Shift Must Be Empty

$$\text{schedule}[d, b] = -1, \quad \forall d \in \{0, \ldots, D-1\}$$

Penalty if violated:

$$\text{Cost}_{break} = \sum_{d=0}^{D-1} \mathbf{1}(\text{schedule}[d, b] \neq -1) \times P_{break}$$

where $P_{break} = 1000$.

2. No Teacher Teaches More Than One Shift per Day

$$\sum_{\substack{s=0 \\ s\neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t) \leq 1, \quad \forall d \in \{0, \ldots, D-1\}, \forall t \in \{0, \ldots, T-1\}$$

Penalty if violated:

$$\text{Cost}_{conflict} = \sum_{d=0}^{D-1} \sum_{t=0}^{T-1} \max\left(0, \left(\sum_{\substack{s=0 \\ s\neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t)\right) - 1\right) \times P_{conflict}$$

where $P_{conflict} = 50$.

3. Maximum Number of Shifts per Teacher per Week

$$\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t) \leq M, \quad \forall t \in \{0, \ldots, T-1\}$$

Penalty if violated:

$$\text{Cost}_{overload} = \sum_{t=0}^{T-1} \max\left(0, \left(\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t)\right) - M\right) \times P_{overload}$$

where $P_{overload} = 10$.

4. Every Teacher Must Teach at Least One Shift per Week

$$\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t) \geq 1, \quad \forall t \in \{0, \ldots, T-1\}$$

Penalty if violated:

$$\text{Cost}_{inactive} = \sum_{t=0}^{T-1} \mathbf{1}\left(\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t) < 1\right) \times P_{inactive}$$

where $P_{inactive} = 2000$.

Total Cost Function

$$\begin{aligned}
\text{Cost}(schedule) = \quad & \sum_{d=0}^{D-1} \mathbf{1}(\text{schedule}[d, b] \neq -1) \times 1000 \\
& + \sum_{d=0}^{D-1} \sum_{t=0}^{T-1} \max\left(0, \left(\sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t)\right) - 1\right) \times 50 \\
& + \sum_{t=0}^{T-1} \max\left(0, \left(\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t)\right) - M\right) \times 10 \\
& + \sum_{t=0}^{T-1} \mathbf{1}\left(\sum_{d=0}^{D-1} \sum_{\substack{s=0 \\ s \neq b}}^{S-1} \mathbf{1}(\text{schedule}[d, s] = t) < 1\right) \times 2000
\end{aligned}$$