Mathematical Foundations of Multi-objective Ant Colony Optimization for TSP

1. Haversine Formula (Geographical Distance)

$$
\begin{aligned}
a &= \sin^2\left(\frac{\Delta\phi}{2}\right) + \cos(\phi_1) \cdot \cos(\phi_2) \cdot \sin^2\left(\frac{\Delta\lambda}{2}\right) \\
c &= 2 \cdot \arcsin\left(\sqrt{a}\right) \\
d &= R \cdot c
\end{aligned}
$$

Where:

- ϕ: latitude, λ: longitude (in radians)
- $R = 6371$ km (Earth's radius)
- d: distance between two cities

2. Ant Transition Probability

$$
P_{ij} = \frac{\left[\tau_{ij}^{(\text{dist})}\right]^{\alpha} \cdot \left[\tau_{ij}^{(\text{cost})}\right]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum_{k \in \text{unvisited}} \left[\tau_{ik}^{(\text{dist})}\right]^{\alpha} \cdot \left[\tau_{ik}^{(\text{cost})}\right]^{\alpha} \cdot [\eta_{ik}]^{\beta}}
$$

$$
\eta_{ij} = w_{\text{dist}} \cdot \left(\frac{1}{d_{ij}}\right) + w_{\text{cost}} \cdot \left(\frac{1}{c_{ij}}\right)
$$

Where:

- $\tau_{ij}^{(\text{dist})}$: pheromone for distance
- $\tau_{ij}^{(\text{cost})}$: pheromone for cost
- α: pheromone influence
- β: heuristic influence
- d_{ij}: distance between city i and j

- c_{ij}: cost between city i and j

3. Pheromone Update Rules

Evaporation

$$\tau_{ij} \leftarrow (1 - \rho) \cdot \tau_{ij}$$

Deposition

$$\tau_{ij} \leftarrow \tau_{ij} + \frac{Q}{f}$$

- ρ: evaporation rate
- Q: pheromone deposit constant
- f: multi-objective score of the route

4. 2-opt Local Search

- Select two non-adjacent edges
- Reverse the segment between them
- Accept the change if the route becomes shorter
- Repeat until no further improvement

5. Multi-objective Score Function

$$\text{score} = w_{\text{dist}} \cdot \text{length} + w_{\text{cost}} \cdot \text{cost}$$

Where:

- $w_{\text{dist}}, w_{\text{cost}}$: weights for distance and cost
- length: total route distance
- cost: total route cost