

Bài tập 1

2. $T(n) = 1$ for $n \leq 2$

$$T(n) = 3T(n-1) + 2$$

$$\Rightarrow T(n) = 3[3T(n-2) + 2] + 2$$

$$= 3^2 [3T(n-3) + 2] + 3 \cdot 2 + 2$$

$$= 3^3 [3T(n-4) + 2] + 3^2 \cdot 2 + 3 \cdot 2 + 2$$

$$\dots$$

$$= 3^{n-1} [3T(n-n+1) + 2] + 3^{n-2} \cdot 2 + \dots + 2$$

$$= 3^n T(1) + 3^{n-1} \cdot 2 + 3^{n-2} \cdot 2 + \dots + 2$$

$$= 3^n + 3^{n-1} \cdot 2 + 3^{n-2} \cdot 2 + \dots + 2$$

$$= 3^n + 2(3^{n-1} + 3^{n-2} + \dots + 1)$$

$$= 3^n + 2 \cdot (3^n - 1)$$

$$= 3 \cdot 3^n - 2$$

$$\Rightarrow T(n) = \log_3(n)$$

$$\bullet \begin{cases} T(n) = 1 & n \leq 1 \\ T(n) = 6T\left(\frac{n}{6}\right) + 2n + 3 \end{cases}$$

$$\Rightarrow a = 6, b = 6, d = 1$$

$$\Rightarrow a = b^d = 6 \Rightarrow T(n) = O(n \log n)$$

1.2.4.

$$X(n) = X(n/5) + n \text{ for } n > 1 \quad X(1) = 1 \quad \text{solve } n = 5^k$$

$$X(5^k) = X(5^{k-1}) + 5^k$$

$$= [X(5^{k-2}) + 5^{k-1}] + 5^k$$

$$= [X(5^{k-3}) + 5^{k-2}] + 5^{k-1} + 5^k$$

$$\begin{aligned}
 &= (5^k - 1) + 5^1 + 5^2 + \dots + 5^k \\
 &= 1 + 5^1 + 5^2 + \dots + 5^k \\
 &= 5 \cdot 5^k - 1
 \end{aligned}$$

$$= 5n - 1 \rightarrow \log T(n) = O(n)$$

$$\begin{aligned}
 &• T(n) = T\left(\frac{n}{2}\right) - 2 \quad \text{for } n > 1, T(1) = 0 \\
 &\quad n = 4^k \rightarrow k = \log_4 n
 \end{aligned}$$

$$\begin{aligned}
 T(n) &= T(2^{2k-1}) - 2 \\
 &= T(2^{2k-2}) - 2 - 2 \\
 &= T(2^{2k-2k}) - 2 \cdot (2k) \\
 &= -2 - 2(2k) \\
 &= -2 - 4k \\
 &= -2 - 4 \log_4 n
 \end{aligned}$$

$$\rightarrow T(n) = O(\log_4 n)$$

2.4.5 Tower of Hanoi

- Algorithm:
1. Chuyển đĩa $n-1$ trên cây sang cột trung tâm
 2. Chuyển đĩa của trên cột đĩa
 3. Chuyển đĩa trung gian \rightarrow cột đích.

$$A(n) \rightarrow B$$

$$A(n-1) \rightarrow C \quad (\text{Chuyển 2 đĩa mới 3}$$

$$A(n) \rightarrow C \quad \text{buổi})$$

$$a_1 = 1 \quad a_2 = 3 \quad a_n = 2a_{n-1} + 1$$

$$a_n = 2a_{n-1} + 1 \quad (n \geq 2)$$

$$= 2(2a_{n-2} + 1) + 1$$

$$= 2^{n-1} \cdot a_1 - (n-1) + 2^{n-2} + \dots + 1 = 2^{n-1} - 1$$