> **"...It seems to me now that mathematics is capable of an artistic excellence as great as that of any music, perhaps greater; because it gives in absolute perfection that combination, characteristic of great art, of godlike freedom, with the sense of inevitable destiny."**
>
> \- Bertrand Russell

Term Paper on: Mathematics as Arts

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Mathematics and art are related endeavours? The view of most people is that art and mathematics could not be more different. One is left brain, the other right brain. One is creative, the other analytical. While this prevailing view is a partial truth, what mathematician has not marvelled at the beauty of an elegant proof, and what serious artist has not been aware of the importance of form and composition to a successful art work? Both disciplines are creative endeavours with analytical components that are essential elements of contemporary civilization.

Considering similarities between mathematics and art, it is clear that both disciplines share a wonderful *creative* aspect. While the external expressions and techniques of art and mathematics could not be more different, the fundamental creativity required to be practitioners is central to both disciplines. Both mathematicians and artists find that our vocations take us away from the mundane world of everyday life, and transform us into grown-up creative children of today's world. And the mysterious and inexplicable contribution of creative inspiration is at the heart of every great work of art and mathematics.

Another concept common to both art and mathematics is *Beauty*. Here we are not talking about beauty as the culturally defined idea of pleasing or attractive. By beauty we mean Aquinas's term *Constantia*, as harmony with or conformance to the eternal laws of form. For mathematics, this would be the adherence to the logic of the language of mathematics and the emphasis on elegance: that the best proof is the one that is most direct requiring the least assumptions. For art, it would be the acknowledged importance of form and composition; that the best art shows a clear harmony between form and subject.

An example where art and mathematics combine to create Harmony would be the artistic use of the *Golden Section* (ratio, or proportion), called phi, **Φ**, and approximated by the irrational number 1.618034.... Knowledge that represents a fundamental structural element of the physical universe has been known since before the dawn of civilization. This ratio occurs throughout nature at all size scales. The Egyptians and Greeks celebrated and applied this sacred metaphorical number and ancient as well as modern artists have used the Golden Rectangle and as a fundamental compositional guide.

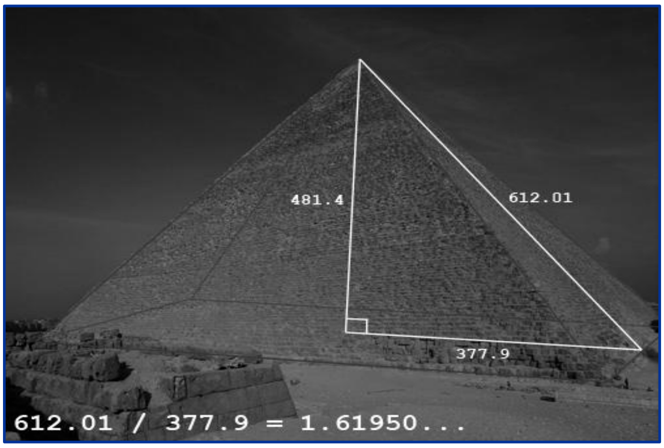


Fig 1.1: Presence of golden ratio in great pyramid of Giza

The past several decades has seen a renewed interest in the mathematics of the natural world in areas such as minimal surfaces, fractals, symmetries, mirror reflections, chaos theory, and complexity. Numbers and mathematics are intrinsically present in natural objects, and both artists and mathematicians have used these ideas in their respective disciplines. Mathematical formulas and geometric figures can be aesthetically pleasing in and of themselves. Physicist Wolfgang Pauli said, "It is more important for an equation to be beautiful than exact," suggesting an aesthetic appreciation of what is implied by the mathematical statement of cause and effect. As an example, Einstein's gravitation equation from general relativity speaks of a universal mystery in elegant form. One aspect not yet mentioned that is common to both proper art and mathematics is their deeper purpose to reveal truth about our world and ourselves. Both disciplines strive to discover and represent the unknown and in this process the question of the meaning of our lives is never too far away.

Here, we will discuss some of the ways that mathematics and art are both similar and different; how they express complementary aspects of life and culture; and how they both share a fundamental unity on a deeper level. Mathematics can indeed be defined as the general science of pattern and structure. Because art also involves patterns and structures, art and math relate to each other in many natural ways, which is the basis for this exploration.

Golden Sequence

The frequent appearance of the Golden Ratio in the arts over thousands of years presents us with an interesting question:

Do we surround ourselves with the Golden Ratio because we find it aesthetically pleasing, or do we find it aesthetically pleasing because we are surrounded by it?

In the 1930's, New York's Pratt Institute laid out rectangular frames of different proportions, and asked several hundred art students to choose which they found most pleasing. The winner? The one with Golden Ratio proportions.

Fibonacci numbers:

$$f(n) = f(n-1) + f(n-2)$$

Generated by adding the previous two numbers in the list together to form the next.

Golden ratio:

So resulting sequence from the above generator function:

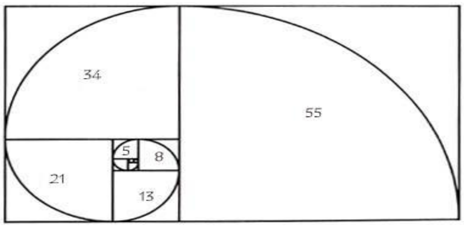
1, 1, 2, 3, 5, 8, 13, 21, 34, 55.....

Divide any number in the Fibonacci sequence by the one before it, for example 55/34, or 21/13, and the answer is always close to 1.61803. If we keep on reproducing we will reach more accurate result, which is known as Golden ratio or Golden section and hence sequence is also known as Golden sequence.

Although golden ratio is a mathematically derived term but it has great physical importance right since the moment of creation i.e. the rings of Saturn, presence of Fibonacci spiral in Sunflower etc.

Starting from the fundamental program of all life on earth i.e. **DNA molecule**, It measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral. 34 and 21, of course, are numbers in the Fibonacci series and their ratio, 1.6190476 closely approximates phi, 1.6180339.

A **Fibonacci spiral** is created by drawing circular arcs connecting the opposite corners of squares in the Fibonacci tiling. Side of consecutive squares represents Fibonacci or golden sequence.



During the Renaissance, large amount of ideas emerged on the aesthetics of the *golden ratio* were written and developed. *De Divina Proportione* by Luca Pacioli explored the proportions and mathematics of how this golden ratio is related to art and artistic proportions. It is widely noted and believed that Leonardo Da Vinci's ***Vitruvian Man*** was proportioned according to this golden ratio: i.e. the ratio of length of *foot-to-navel* to the length *navel-to-head* is approx. equal to **1.6180339**.

"**The Last Supper**," by Da Vinci, to which by simply viewing the painting you can note the proportions held within the painting and notice how eye catching it is. This beauty could only be obtained by such a ratio as the golden ratio.

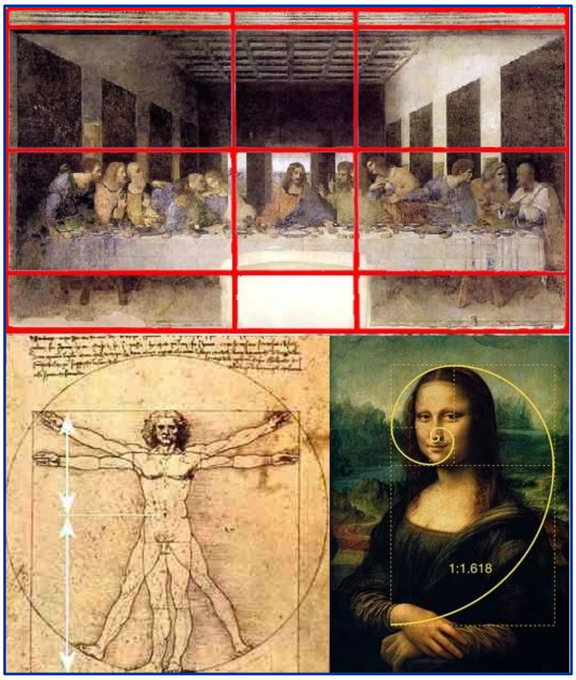


Fig 1.2: (a) The Last Supper divided into golden rectangles, (b) The Vitruvian man (c) The famous Monalisa by Da vinci proportionated appropriately in golden ratio.

The use of golden ratio is not limited to only paintings but is also employed in various architectures and in sculptures all around the globe.

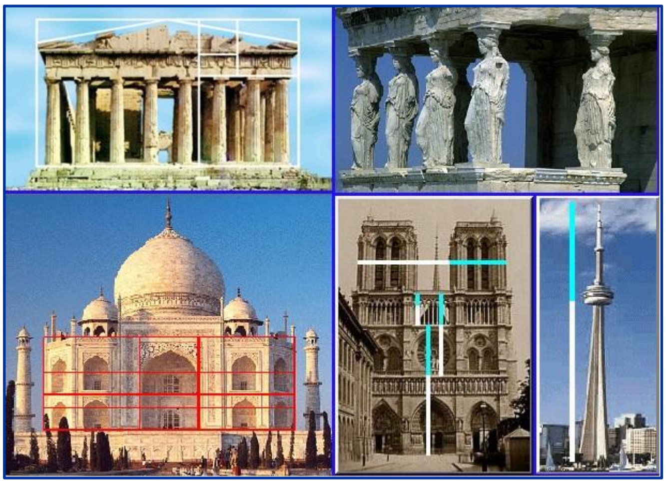


Fig 1.3: (a) The Greeks recognized it as dividing a line in the extreme and mean ratio, Parthenon, Acropolis, Athens. (b) Phidias (500 BC - 432 BC), a Greek sculptor and mathematician, studied phi and applied it to the design of sculptures for the Parthenon. Porch of Maidens, Acropolis, Athens. (c) In India, it was used in the construction of the Taj Mahal, which was completed in 1648. (d) Used in the design of Notre Dame in Paris, which was built in the 1163 and 1250? (e) The CN Tower in Toronto, one of the tallest tower and freestanding structure in the world, has the golden ratio in its design.

The golden sequence appears in the foundation of aspects of art, beauty and life. Even music has a foundation in the series, as:

- There are **13** notes in the span of any note through its octave.
- A scale is composed of **8** notes, of which the **5th** and **3rd** notes create the basic foundation of all chords, and are based on whole tone which is **2** steps from the root tone, that is the 1st note of the scale.

Another aspect of the golden ratio in music is illustrated in compositions by Mozart. Mozart's piano sonnets have been observed to display use of the golden ratio through the arrangement of sections of measures that make up the whole of the piece. In Mozart's time, piano sonatas were made up of two sections, the exposition and the recapitulation. In a one hundred measure composition it has been noted that Mozart divided the sections between the thirty-eighth and sixty-second measures. This is the closest approximation that can be made to the Golden Ratio within the confines of a one-hundred measure composition. Other music composers such as Schubert, Beethoven, Debussy, Bartôk also used golden ratio in their compositions.

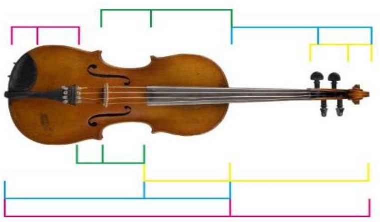


Fig 1.4: Fibonacci and phi are used in the design of violins and even in the design of high quality speaker wire.

Fractal Art

A **fractal** is an object or quantity that displays self-similarity, in a somewhat technical sense, on all scales. The object need not exhibit exactly the same structure at all scales, but the same "type" of structures must appear on all scales.

In 1979 the mathematician Benoit Mandelbrot was the first to create pictures that illustrate the dynamics of functions with domain and range in the complex plane. His very first plot was so complicated that he thought it due to a bug in his program. The function he was studying was just too simple to give rise to such a complex pattern. The fractal he created now bears his name, the **Mandelbrot set**.

The Mandelbrot set has the remarkable property that distorted copies of the whole set appear at all levels of magnification.

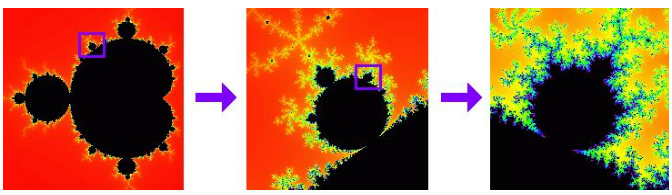


Fig 1.5: The self-similar property of Mandelbrot set

This type of infinite complexity is a characteristic of mathematical objects such as fractals. The irregularities often seem almost organic, and indeed, much of the physical universe has approximate fractal characteristics (like coastlines, mountains, clouds, nervous systems, circulatory systems, etc.). Mandelbrot and many others have applied fractal techniques to many real-world phenomena, with great success.

One of the most remarkable properties of the Mandelbrot set, and indeed fractals in general, are that despite their infinite complexity, they are created using very simple algebraic formulas using recursion.

Recursion is a feedback process, whereby the output of a function is used as the next input to the same function. For the Mandelbrot set, the function used is simply:

$$f(z) = z^2 + C$$

It may seem impossible that such a simple formula could produce such an infinitely complex creature as the Mandelbrot set. Recursion is what makes this possible. Consider the first few iterates of this recursion process for this function:

$$f(z) = z^2 + C$$
$$f(f(z)) = (z^2 + C)^2 + C$$
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Fractal art is a form of algorithmic art created by calculating fractal objects and representing the calculation results as still images, animations, and media. Fractal art developed from the mid 1980s onwards. It is a genre of computer art and digital art which are part of new media art. The Julia set and Mandelbrot sets can be considered as icons of fractal art.

Fractals show its face in many aspects of this ancient civilization. From the complex temples to their astonishingly accurate calendar, these ancient people do not cease to amaze. Even today in their intricate indigenous weaving, we see examples of fractals and even more concepts of math beyond our knowledge and technology today. The Mayan Calendar is one example of fractals in Mayan art. Mayan architecture according to archaeologists have discovered that the buildings form fractal-like, repeated, complex patterns of clusters.

Fig 1.6: An example of fractal art in Mayan architecture

Islamic art also has relation with this complex mathematical pattern. Their art ties in to their belief that the patterns of math and deterministic life patterns of God are so beautiful that their architecture and tiles reflect that pattern.

Fig 1.7: Mandelbrot Shape in Islamic religious art

Indian and Southeast Asian temples and monuments exhibit a fractal structure: a tower surrounded by smaller towers, surrounded by still smaller towers, and so on, for eight or more levels, **William Jackson** goes on to assert that the whole religious vision of Hinduism has a fractal character: " The ideal form gracefully artifice suggests the infinite rising levels of existence and consciousness, expanding sizes rising toward transcendence above, and at the same time housing the sacred deep within*"*.

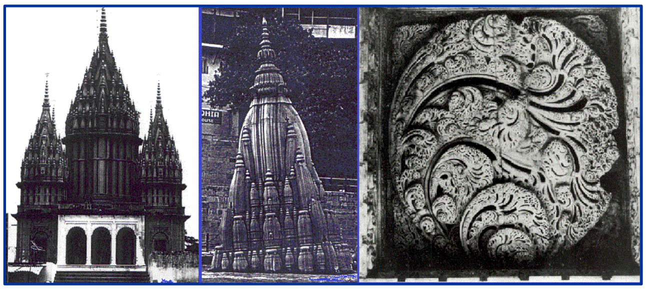

Fig 1.8: Design of various temples in India, stunning and intricate Fractal Arabesque on a ceiling panel at Delwara Jain Temple, Mt. Abu, India, 1031AD.

At nearly the same time in Italy, we can also see one more fractal based architectural marvel, the cathedral of Anagni there is a floor which is adorned with dozens of mosaics, each in the form of a Sierpinski gasket fractal.

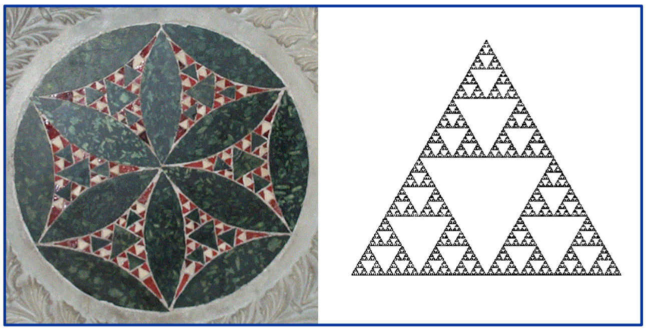

Fig 1.9: Floor of the cathedral of Anagni and the Sierpinski gasket.

Reims' Cathedral (France) has a rising fractal structure which represents the elevation. Every tower has: a big arcade, two windows, four ogives and their dimensions decrease in each floor. In Saint Paul church in Strasbourg (France), we have observed the presence of self-similar shapes on each two towers. In Venice there are many palaces that have a rising fractal structure (e.g., Ca' Foscari, Ca' d'Oro, Duke Palace, Giustinian Palace).

Artistic virtues of fractals were also used by various painters, such as Salvador Dali. Consider Dali's *Visage of War* (1940) note the eyes and mouth each contain a face, whose eyes and mouth each contain a face, many of whose eyes and mouth each contain a face, an obvious, if gruesome, example of fractals in art.

Fig 1.10: First "fractal" in visual arts. Left: Dali, Visage of War, 1940, oil n canvas; right: Cantor dust fractal set with Hausdorff-Besicovitch.

The Book of Kells is one of the finest and most famous of a group of manuscripts in what is known as the Insular style, produced from the late 6th through the early 9th centuries in monasteries in Ireland.

This cultural treasure contains every variety of design typical of Irish art at its best. The most characteristic ornaments of the Book of Kells, as of other illuminated Irish manuscripts of the period, are the closely coiled spirals connected with each other by a number of curves and terminating in the so-called "trumpet fractal pattern".

Fig 1.11: Medieval Celtic Book of Kells (597 A.D.)

Good art has always strived to incorporate the elements of self-similarity although this is generally done subtly. In a great work of art each image must related to the others in terms of its geometry and metaphorical themes. Artists and sculptors have always been inspired by the complex forms of nature. For example the fractal vortices in Van Gogh's famous painting, "Starry Night" appear to be taken directly from the meandering stream winding through separate vortices. Trains of fractal vortices also appear in the knurled cypress trees found in many of Van Gogh's late paintings such as "St Paul's Hospital, (1889)".

Polyhedra & Platonic Solids as Art Motifs

Platonic solids and the polyhedra have been connected with the world of art and architecture in different cultures and through many centuries. For some Renaissance artists, for example Leonardo da Vinci (1452 -1519), Albrecht Dürer (1471-1528), Piero Della Francesca (around 1420-1492), this solids provided the models to demonstrate the properties of the symmetry and to apply the laws of the perspective. For others, the polyhedra were the symbols of deep religious or philosophical truths. In the last centuries, they have also influenced the architecture and the design. For example, Bruno Munari (1907- 1998) and the husband and wife design team composed by Charles Eames (1907-78) and Ray Eames (1912-88) were designers fascinated by the polyhedral shapes.

The regular polyhedra, also known as the Platonic solids, are the three-dimensional bodies whose surfaces consist of identical, regular polygons (e.g., equilateral triangles, squares, and pentagons) which meet in equal angles at the corners. There are five Platonic solids: the Tetrahedron, the Octahedron, the Hexahedron (Cube), the Icosahedrons and the Dodecahedron.

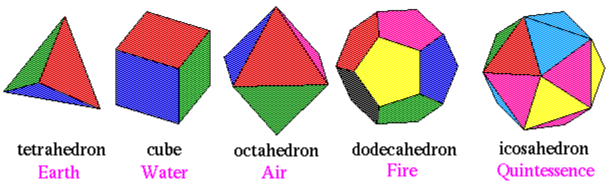

Fig 1.12: The Platonic Solids

The adjective "Platonic" derived by the Greek philosopher Plato (427-348 B. C.), that has described the five regular solids in his *Timaeus*, The Greek philosopher found an important association in the Timaeus between the Platonic solids and the Empedocles' Four Elements: the fire, the earth, the air, and the water (and the universe).

This association has influenced the philosophers, the artists, and the mathematicians of the Renaissance period. Plato constructed the five solids using simple rules and simple polygons: "*The first will be the simplest*" (Timaeus, 54e–55a) the tetrahedron "*which is the original element and seed of fire*" (Timaeus, 56b). "*The second species of solid is formed out of the same triangles…*" (Timaeus, 55a). The octahedron "*let assign the element which was next in the order of generation to air*" (Timaeus, 56b) and "*the third to water*" (Timaeus, 56b) the icosahedron. The cube: "*To earth let us assign the cubical form…to earth is the most immovable of the four*" (Timaeus, 55d- 55e), and the dodecahedron "*There was yet a fifth combination which God used in the delineation of the Universe*".

Polyhedra appeared as artistic motifs long before Greeks wrote about it. Nearly around 2000 BC hundreds of carved stone spheres, roughly three inches in diameter, have been found in Scotland.

However, there is a serious debate about these polyhedrons and some archaeologists even doubt that these represent platonic solids, but the fact that these

Fig 1.13: Neolithic Carved Stone Polyhedra

Stones are from Neolithic era and represents some kind of polyhedral is undeniable. In the age of the Renaissance, the artists used the Platonic solids and the polyhedra in their productions, and to study the properties of the perspective. For example, the Italian artist Paolo Uccello (1397-1475) has applied the Platonic solids and the polyhedra in his works. Leonardo da Vinci (1452 -1519) was also fascinated by the Polyhedra. Leonardo drawn the illustrations for the book entitled *De Divina Proportione* (1509), written by the Franciscan friar Luca Pacioli (1445-1514).In the figures 1.14(b) and 1.14(c), there are two examples of Leonardo's polyhedra illustrated for the De Divina Proportione.

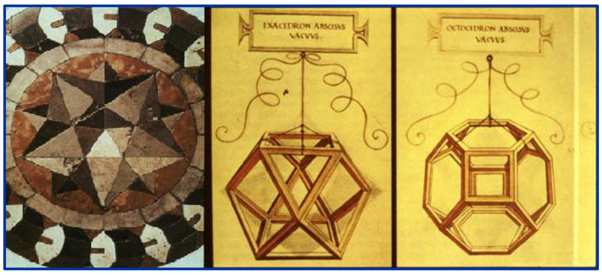

Fig 1.14: (a) Paolo Uccello, Basilica of San Marco, Venice (1425 – 1430) (b), (c) Leonardo's Illustrations for Luca's book.

The most important engraving realized by Dürer was Melancholia I (1519). In this work we can find some interesting symbolism. For example, the mysterious polyhedron truncated and the magic square associated to the Planet of Jupiter (and connected to the blood humour). The German astronomer Johannes Kepler (1571-1630) in his book entitled: *Mysterium Cosmograficum* (1595) has conceived a representation of our Solar system where the Platonic solids play a central role. He has associated the Platonic solids to the six planets known in that period (Saturn, Jupiter, Mars, Earth, Venus and Mercury).

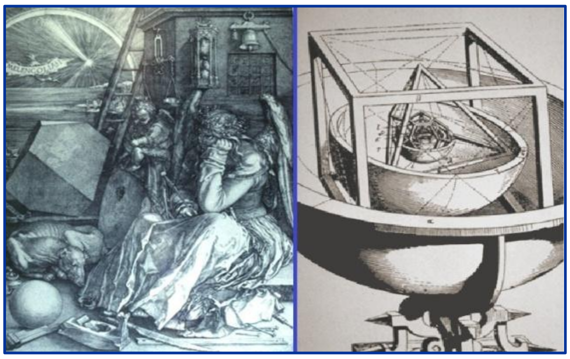

Fig 1.15: (a) Melancholia I (b) Kepler's model of Universe

In the last century, dutch artist Maurits Cornelis Escher (1898-1972) has also applied the polyhedral shapes in his works. For example, in Reptiles, lithograph (1943), there is the presence of a dodecahedron ("Quinta Essentia").

The wood engraving *Stars* (1948) includes the compound of different vacuum polyhedra (octahedra and cubes) that contain some chameleons. The Spanish surrealist painter Salvador Dalì (1904-1989) was fascinated by the Platonic solids. In Crucifixion (*Corpus Hypercubus*) (1954) he represented the cross using eight cubes in and his *The Sacrament of the Last Supper* (1955) a vacuum dodecahedron (as symbol of "God") is located over Jesus and his disciples, like a "virtual" embrace.

Fig 1.16: Crucifixion (Corpus Hypercubus, 1954), Dali's The Sacrament of the Last Supper (1955).

The Italian designer Bruno Munari (1907- 1998) has studied the shapes of the Platonic solids and the polyhedra to apply them in his product of design (ashtrays, lamps). His Cubic ashtray (1957) represents a famous example of Italian design. Munari did not smoke but he has created a functional ashtray.

Egyptian and Maya pyramids are good examples of polyhedra applied in the ancient architecture. These shapes, their symmetry and their properties, had guaranteed the structural equilibrium.

In the 20th century, polyhedral geometry has been found to be the basis for a wide range of designs, such as Fuller's geodesic domes, deployable buildings and many other types of "nonstandard architecture". Geodesic Domes are designed by the American architect engineer Richard Buckminster Fuller (1895-1983). Fuller invented the Geodesic Dome, to demonstrate his ideas about housing and "energetic-synergetic geometry" which he had developed during WWII. A Geodesic Dome is shaped like a molecule of carbon. Other modern architects have used Platonic solids and polyhedral shapes in their buildings.

The Swiss Charles-Edouard Jeanneret known as Le Corbusier (1887-1965) is one of the architects that has influenced the modern architecture of the 20th century. In his project realized for the *Convent of Saint Marie de la Tourette* (1957 to 1960) at *Eveux-sur-Arbresle*, near Lyon (France), Le Corbusier has applied the polyhedral shapes to emphasize the "rigour" of the Religious order of the Dominican.

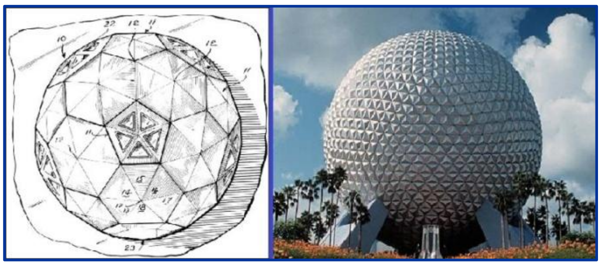

Fig 1.16: Fuller's geodesic dome.

Mathematics and Music

Throughout history, many mathematicians were interested in music. Most of us wonder that if many musicians were interested in mathematics. Certainly there are musicians who are interested in maths, but when a comparison is made mathematicians are more than the musicians. Examining the relationship between music and mathematics goes back to the ancient Greeks. In ancient Greece, music was as one of 4 main branches of mathematics. The school of Pythagoras accepted music, arithmetic, geometry and astronomy at the same level. Wires with different lengths were obtained reveal the different sounds were found by, Pythagoras, in BC 6th Century. Pythagoras created the basics of the music that is still used today.

Pythagoras divided a 12 unit wire and the octave has been achieved. Pythagoras found 5- intervalled system with 8 unit length of a wire and 4-intervalled system with 9 units of length of a wire. In ancient times, four voices heard together are called tetra-chord and it is considered as the basic rule of the music theory. Thus tetra-cord, were obtained from 6, 8, 9 and 12. These numbers have a relationship with golden ratio. According to Pythagoras rate, 5-intervalled system and 4-intervalled system differs by a full ton.

2 / 3 : 3 / 4 = 8 / 9 (5T-4T=2M)

So, full sound 8 / 9 with the multiplication gives us the shrill tone of a voice. If we continue; 8/9.8/9 = 64/81 (2M +2 M =3M). Pythagoras obtained 1 full tone with 8/9 of the wire, full tone, but when added 6 more notes he could obtain an octave of the note, which is called the coma of Pythagoras. In this case, Pythagoras needed some changes in order that 12 equal half-ton tampere a system had been developed. 1 full ton was not shown as 8 /9 but with twelve half-ton.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610.......

When we look at the sequence, the sum of last two numbers will give us the next number. The most important thing here is the ratio between the numbers. When we divide one of the consequent number to the other one we will obtain 0.61803398......This ratio was used in some period of music history. The ratio is known as the golden ratio.

After some studies, golden ratio was used in some compositions in parts of melody, and rhythm. Bella Bartok is one of the composers who use the golden ratio most. Bella Bartok constituted a Fibonacci sequence and used the terms of the sequence as the notes of his compositions. In his composition, "*Music for strings, percussion and celeste*" the most important part is the 55th measure of 89 measures. Both numbers are the elements of Fibonacci sequence. Another widely known composition is Hallelujah whose composer is Haendel. In the composition the most important part is "King of kings" which starts at 57th and 58th measure. It is approximately the 8/13 of the total measures. 8 and 13 are the Fibonacci numbers. When we calculate 57. 8/13 it will give us an approximate value of 34. At the 34th measure, the theme "The Kingdom of Glory" starts. 34 is again a Fibonacci number.

Even though, no one knows that whether Haendel used these numbers with knowing the Fibonacci numbers or not, this composition is one of the most popular compositions. There are some opinions that Mozart also used the golden ratio. According to John F. Putz, the compositions of Mozart are genius works and they are the works that was done by a person who loves numbers. He concluded that Mozart knew the golden ratio and used it in his works.

Fig 1.17: Pythagoras performing harmonics experiments with stretched vibrating strings. From Franchino Gafori, Theorica Musice (Milan, 1492).

Conclusion

Through centuries, mathematics have fascinated the artists, the architects, the engineers, and the designers. Mathematics is a science that contains amazing objects, and their interrelations and symbolism are sources of endless beauty, and they help to create an aesthetic sense. As Bertrand Russel affirmed: "Mathematics possesses not only truth but supreme beauty, a beauty cold and austere, like that a sculpture, sublimely pure and capable of a stern perfection, such as only the greatest art can show".

Patterns, designs, and forms that are the "automatic" product of purely mathematical processes (such as those described in "Mathematics generates art") are usually too precise, too symmetrical, too mechanical, or too repetitive to hold the art viewer's attention. They can be pleasing and interesting, and are fun to create (and provide much "hobby-art") but are mostly devoid of the subtlety, spontaneity, and deviation from precision that artistic intuition and creativity provide. In the hands of an artist, mathematically-produced art is only a beginning, a skeleton or a template to which the artist brings imagination, training, and a personal vision that can transform the mathematically perfect to an image or form that is truly inspired.

In conclusion, there is obviously a strong link between mathematics and the arts. Music, fine art, and literature wouldn't be the same without mathematics. From Mozart to Escher to Crichton, musicians, artists and novelists have used mathematics to highlight, improve and develop their work. This doesn't mean you need a degree in maths to be a great artist - however, it does mean that an understanding of certain aspects can make you better.

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