Prob. - Namra Sanjay Patel (20011070)

#Q1

a). Prob. of choosing 3 aces = ${}^4C_3$

Prob. of choosing non ace card = ${}^{48}C_4$

Total ways to choose 7 cards = ${}^{52}C_7$

$\therefore$ The Prob that the 7 cards include exactly 3 aces = $\dfrac{({}^4C_3 * {}^{48}C_4)}{{}^{52}C_7}$

$$= \frac{\left(\frac{4!}{3!}\right) * \left(\frac{48!}{44! * 4!}\right)}{\frac{52!}{45! * 7!}} \quad * \quad \cancel{\frac{(4) * \left(\frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2}\right)}{\frac{52 \times 51 \times 50 \times 49 \times 48 \times 47 \times 46}{7 \times 6 \times 5 \times 4 \times 3 \times 2}}}$$

$$= \frac{4 * 1945 8\cancel{0}}{13378456\cancel{0}} = \frac{77832}{13378456} \simeq \underline{0.0058} \text{ — Ans } (P1)$$

b). Prob(7 cards which include 2 kings)

$$= \frac{{}^4C_2 \times {}^{48}C_5}{{}^{52}C_7} = \underline{0.0768} \text{ — Ans } (P_2)$$

c). Req Prob = $(P1) + (P_2) - P(7 \text{ cards} \rightarrow 3 \text{ aces } \& \text{ } 2 \text{ kings})$

$$= 0.0058 + 0.0768 - 0$$

$$= \underline{0.0826} \text{ — Ans.}$$

Q2]. Event A = Bob's last n tosses minus the number of heads in Alice's n tosses.

$\therefore P(A>0) = P(A<0)$

Hence $\Rightarrow P(A>0) + \frac{1}{2} P(A=0) = \frac{1}{2}$

$\therefore$ P(Bob gets more heads than Alice) $= P(B=0)P(A>0) + P(B=1)P(A\geq 0)$

$$= \frac{1}{2}\left(P(A>0) + P(A\geq 0)\right)$$

$$= \frac{1}{2}\left[\left(P(A>0)\right) + \left(P(A>0) + P(A=0)\right)\right]$$

$$= P(A>0) + \frac{1}{2} P(A=0)$$

$$= \boxed{\frac{1}{2}} \text{ — Ans}$$

Q3]. Case I → coin with both head
Case II → Coin with both tail
Case III → fair coin is selected

$\therefore$ $P(\text{case I}) = P(\text{case II}) = P(\text{case III}) = 1/3$

$P(\text{Heads} | \text{Case I}) = 1$

$P(\text{Heads} | \text{Case II}) = 0$

$P(\text{Heads} | \text{Case III}) = 1/2$

$\therefore$ By using Baye's theorm :-

$$\therefore P(\text{Case III} | \text{Heads}) = \frac{P(\text{Heads}|\text{Case III})\, P(\text{Case III})}{\left[P(\text{Heads}|\text{Case I})\, P(\text{Case I}) + P(\text{Heads}|\text{Case II}) * P(\text{C II}) + P(\text{Heads}|\text{Case III})\, P(\text{Case III})\right]}$$

$$= \frac{(0.5)(\cancel{1/3})}{\cancel{1/3} + 0 + 1/2\,(\cancel{1/3})} = \frac{1/2}{1 + 0 + 1/2}$$

$$= \frac{1/2}{\frac{2+1}{2}} = \frac{1}{3} = \boxed{0.333}$$

Ans

Q4].

$$P(\text{White}) = \frac{m}{m+n} \Rightarrow$$ Now, lets assume statement is valid till (K-1) jar & check if its correct for K jars.

$$P(\text{White}_K / \text{White}_{K-1}) = \frac{m+1}{m+n+1}$$

$$P(\text{White}_K | \text{Black}_{K-1}) = \frac{m}{m+n+1}$$

∴ By using Total Prob. th.

$$P(\text{White}_K) = \left(\frac{m}{m+1}\right) * \left(\frac{m+1}{m+n+1}\right) + \left(\frac{n}{m+n}\right)\left(\frac{m}{m+n+1}\right)$$

$$= \frac{m\,\cancel{(m+n+1)}}{(m+n)\cancel{(m+n+1)}}$$

$$= \frac{m}{m+n}$$

∴ statement proved

Q5].

*). n = Different power plants

i = fails with prob p_i

a). $P(\text{Black out}) = (p_i)^4$ $\left\{ \text{as its independent events} \right\}$

b). $P(\text{black out}) = {}^nC_n \, p_i^n (1-p_i)^{n-n} + {}^nC_{n-1} \, p_i^{n-1} (1-p_i)^{n-(n-1)}$

$= p_i^n + (n)(p_i)^{n-1}(1-p_i)$

$= p_i^n \left[1 + \frac{n}{p_i}(1-p_i) \right]$

$= p_i^n \left[1 + \frac{n}{p_i} - n \right]$

$= p_i^n \left[1 + \frac{n}{p_i} - n \right]$