Homework - 4 Normal Distribution

Q33].

a). Let the max speed be X

$$\therefore P(X \le 50) = P\left(\frac{X-46.8}{1.75} \le \frac{50-46.8}{1.75}\right)$$

$$= P(Z \le 1.83)$$

$$= \phi(1.83)$$

$$= 0.9664 \quad \text{— Ans}$$

b). $$P(X \ge 48) = P\left(\frac{X-46.8}{1.75} \ge \frac{48-46.8}{1.75}\right)$$

$$= P(Z \ge 0.69)$$

$$= 1 - \phi(0.69)$$

$$= 1 - 0.7549$$

$$= 0.2451 \quad \text{— Ans}$$

c). $$P(|X-\mu| \le 1.5\sigma) = P\left(-1.5 \le \frac{X-\mu}{\sigma} \le 1.5\right)$$

$$= P(-1.5 \le Z \le 1.5)$$

$$= \phi(1.5) - \phi(-1.5)$$

$$= 0.9332 - 0.0668$$

$$= 0.8664 \quad \text{— Ans}$$

35].

a).

⇒ Standardized variable $z = \dfrac{x - \mu}{\sigma}$

$$= \frac{10 - 8.8}{2.8} = \underline{0.428}$$

⇒ Proportion of trees whose diameter $\geq$ 10 in.

= 1 − Area under z curve left of 0.428

= 1 − 0.6628

= $\underline{0.3372}$

⇒ Proportion of trees that have diameter at least 10 in.

$\therefore$ $\underline{z = 0.428}$

$\therefore$ Proportion of trees = $\underline{0.3372}$

b).

$$Z = \frac{x - \mu}{\sigma} = \frac{5 - 8.8}{2.8} = \underline{\underline{-1.36}}$$

Proportion of trees have diameter of most 5 in.

= Area under z curve to the left of (-1.36)

= $\underline{\underline{0.0869}}$

Z_1 (diameter between 5 & 10 in) $= \frac{5-8.8}{2.8} = \underline{\underline{-1.36}}$

Z_2 $= \frac{10-8.8}{2.8} = \underline{\underline{0.428}}$

∴ Proportion of x where $5 < x < 10$

$= 0.6628 - 0.0869$

$= \underline{\underline{0.5759}}$

c).

$$Z = \frac{20 - 8.8}{2.8} = \underline{\underline{4}}$$

∴ Proportion of trees with diameter > 20 in is zero (nearly) because mostly the value of z lies between -3 to +3.

d).

$$Z_1 = \frac{x-\mu}{\sigma} \Rightarrow -2.33 = \frac{8.8 - c_1 - \mu}{2.8}$$

$$\Rightarrow \boxed{c_1 = 6.524}$$

$$Z_2 = \frac{8.8 + c_2 - \mu}{2.8} \Rightarrow 2.33 = \frac{8.8 + c_2 - 8.8}{2.8}$$

$$\Rightarrow \underline{c_2 = 6.524}$$

Hence 98-99% of diameter lies within (2.276, 15.324)

e). $P(Y \geq 1) = 1 - P(Y=0)$

$$= P(T_1 < 10) * P(T_2 < 10) * P(T_3 < 10) * P(T_4 < 10)$$

$$= (1 - 0.3336)^4$$

$$P(Y \geq 1) = 1 - (0.6664)^4$$

$$= 1 - 0.1972$$

$$= \underline{0.8028} \text{ Ans}$$

49].

$$P(X > 4000) = P\left(Z > \frac{4000-3432}{482}\right)$$

$$= P\left(Z > \frac{568}{482}\right)$$

$$= P(Z > 1.1)$$

$$\therefore P(Z > a) = 1 - \phi(a)$$

$$= 1 - 0.8810$$

$$= \underline{0.119}$$ —— Ans. 1

$$\therefore P(3000 < X < 4000) = P\left(\frac{3000-3432}{482} < \frac{X-\mu}{\sigma} < \frac{4000-3432}{482}\right)$$

$$= P\left(\frac{-432}{482} < Z < \frac{568}{482}\right)$$

$$= P(-0.9 < Z < 1.18)$$

$$\therefore P(-0.9 < Z < 1.18) = \phi(1.18) - [1 - \phi(0.9)]$$

$$= 0.8810 - 1 + 0.8159$$

$$= \underline{0.6969}$$ —— Ans 2.