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Data Science, 2022
Tut 5: Evaluation and Measurement- Hypothesis Testing

Make Assumptions about values when it is necessary in consistent manner. Refer necessary table from following link when necessary.

https://www.sheffield.ac.uk/polopoly_fs/1.43999!/file/tutorial-10-reading-tables.pdf

Testing a Proportion of small samples

1. H_0: $p = p_0$

2. One of the alternatives H\: $p < p_n$, $p > p_o$, or $p \neq p_o$

3. Choose a level of significance equal to a.

4. Test statistic: Binomial variable X with $p = p_0$.

5. Computations: Find x, the number of successes, and compute the appropriate P-value.

6. Decision: Draw appropriate conclusions based on the P-value

Ex. 1

A builder claims that air-conditions are installed in 70% of all homes being constructed today in the city of Mumbai. Would you agree with this claim if a random survey of new homes in this city shows that 8 out of 15 had air-conditions installed? Use a 0.10 level of significance

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Tut 5: Evaluation and Measurement - Hypothesis Testing

Q1 $H_0 : p = 0.7$

$H_1 : p \neq 0.7$

Level of significance = $\alpha = 0.10$

Test Statistic: Binomial Variable X with $p = 0.7$ and $n = 15$

$X = 8$ and $np_0 = 15 \times 0.7 = 10.5$

$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$

$= 2 \sum_{x=0}^{8} b(x; 15, 0.7)$

$= 2 \times 0.1311$ (From Binomial Prob. Table)

$= 0.2622$

$\therefore P > 0.10$ i.e $P > \alpha$

$\therefore$ Do not reject H_0. Conclude that there is insufficient reason to doubt the builder's claim.

Ex.2

A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use a 0.05 level of significance.

Q.2 $H_0 : p = 0.6$

$H_1 : p > 0.6$

level of significance $= \alpha = 0.05$

Given : $x = 70$, $n = 100$, $p_0 = 0.6$

$$\therefore Z = \frac{x - np_0}{\sqrt{np_0q_0}}$$

$$\therefore Z = \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}}$$

$\therefore Z = 2.04$

$P = P(Z > 2.04)$

$= \cancel{0.120}$

$\cancel{P = 0.0207}$ $P = 0.0207$ (from table)

As $P < \alpha$, reject H_0 and conclude that new drug is superior.

Ex.3

A vote is to be taken among the residents of a Mumbai and the surrounding area to determine whether a proposed Nuclear plant should be constructed. The construction site is within the Mumbai limits, and for this reason many voters in the surrounding area feel that the proposal will pass because of the large proportion of Mumbai voters who favor the construction. To determine if there is a significant difference in the proportion of Mumbai voters and surrounding area voters favoring the proposal, a poll is taken. If 120 of 200 Mumbai voters favor the proposal and 240 of 500 surrounding area residents favor it, would you agree that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters? Use an a = 0.05 level of significance.

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Q.3 Let P_1 be the proportion of Mumbai voters and P_2 be the proportion of surrounding area residents

$\hat{P}_1 = \frac{120}{200} = 0.6$

$\alpha = 5\% = 0.05$

$\hat{P}_2 = \frac{240}{500} = 0.48$

$\hat{P}_P = \frac{120+240}{200+500} = 0.514$

Hypothesis

$H_0: P_1 \leq P_2$

$H_1: P_1 > P_2$

$$Z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_P(1-\hat{P}_P)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$\therefore Z = \frac{0.6 - 0.48}{\sqrt{0.514(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$\therefore Z = 2.869$

$\therefore P = P(Z > 2.869) = 0.0044$

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As $P < \alpha$, reject H_0 and conclude that the proportion of Mumbai voters favoring the proposal is higher than the proportion of surrounding area voters.

Ex.4

State the null and alternative hypotheses to be used in testing the following claims, and determine generally where the critical region is located:

(a) At most, 20% of next year's wheat, crop will be exported to the Russia.

(b) On the average, Indian homemakers drink 3 cups of tea per day.

(c) The proportion of graduates in engineering this year majoring in the computer sciences is at least. 0.15.

(d) The average donation to the Indian Autism Association is no more than 500 INR.

(e) Residents in suburban Mumbai commute, on the average, 15 kilometers to their place of employment.

Q. 4

a) Null Hypothesis

$H_0 : p = 0.20$

Alternative Hypothesis

$H_1 : p > 0.20$

The Critical region is in right tail

b) Null Hypothesis

$H_0 : \mu = 3$

Alternative Hypothesis:

$H_1 : \mu \neq 3$

The critical region is in both tails

c) Null Hypothesis

$H_0 : p = 0.15$

Alternative Hypothesis

$H_1 : p < 0.15$

$\longrightarrow$

The critical region is in left tail.

d) Null Hypothesis

$H_0 : \mu = 500$

Alternative Hypothesis

$H_1 : \mu > 500$

The critical region is in right tail

e) Null Hypothesis

$H_0 : \mu = 15$

Alternative Hypothesis

$H_1 : \mu \neq 15$

The critical region is in both tails.

Ex.5

In a study conducted by the Department of computer Engineering and analyzed by the Statistics Consulting Center at SPIT the laptops supplied by two different companies were compared. Ten sample laptops were made out of the Intel chips supplied by each company and the "robustness" was studied. The data are as follows:

Company A: 9.3 8.8 6.8, 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2,10.1 9.7 11.0 11.1 10.2 9.6

Can you conclude that there is virtually no difference in means between the laptops supplied by the two companies? Use a P-value to reach your conclusion. Should variances be pooled here?

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Q.5 Let μ_1 and μ_2 be the population mean 'robustness" of laptops supplied by Company A and company B respectively

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Significance level = α = 0.05

$$\overline{X_1} = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i}$$

$$= \frac{9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8.0 + 6.5 + 9.2 + 7.0}{10}$$

$\therefore \overline{X_1} = 7.95$

$$\overline{X_2} = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i}$$

$$= \frac{11.0 + 9.8 + 9.9 + 10.2 + 10.1 + 9.7 + 11.0 + 11.1 + 10.2 + 9.6}{10}$$

$\therefore \overline{X_2} = 10.26$

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$$S_1^2 = \frac{1}{n_1-1}\left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1 \bar{x}_1^2\right]$$

$$\therefore S_1^2 = \frac{10.865}{9} = 1.207$$

$$S_2^2 = \frac{1}{n_2-1}\left[\sum_{i=1}^{n_2} x_{2i}^2 - n_2 \bar{x}_2^2\right]$$

$$\therefore S_2^2 = \frac{2.924}{9} = 0.325$$

Since sample variances are quite different, we cannot assume that population variances are equal; so we will use the unpooled t-test

The degrees of freedom for this test are calculated as

$$V = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\cdot\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\cdot\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{10-1}\left(\frac{1.207}{10}\right)^2 + \frac{1}{10-1}\left(\frac{0.325}{10}\right)^2}$$

$$= 10.30$$

$$V \simeq 10$$

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The test statistics used to test these hypothesis is

$$T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

which under the null hypothesis, follows approximately t-distribution with $v=10$ degrees of freedom.
Also, under the null hypothesis we have $\mu_1 - \mu_2 = 0$, so the value of test statistic is

$$T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$$

Since the test is two sided, then the value of test is the doubled area under the density curve of t-distribution with 10 degrees of freedom, right of the absolute value of test statistic

$|t| = |-5.90| = 5.90$ i.e. the p-value is

p-value $= 2 \cdot P(T \geq |t|) = 2 \cdot P(T \geq 5.90)$

$t_{0.0005}(10) = 4.587$ and since $|t| = 5.90$ is even greater than $P(T \geq 5.90) < 0.0005$, so

p-value < 0.001

~~P~~-value $\approx$ ~~0.00~~

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As $p < \alpha$, we can reject the null hypothesis in favor of the alternative hypothesis and conclude that the mean robustness of laptops is not the same for the two companies.