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Class: BE COMPS

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Data Science, 2022

Tut 6: Machine Learning 1

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ .

- What is the probability of obtaining the first head at the $(k + 1)$ -th toss?
- What is the expected number of tosses needed to get the first head?

The image shows a handwritten solution on a lined notebook page. At the top, it is titled 'Data Science, 2022' and 'Tut 6: Machine Learning 1'. There is a small box for 'Page No.' and 'Date'. The solution is written in blue ink. It starts with 'Q.1 a)' and defines $P(H) = \lambda$ and $P(T) = 1 - \lambda$. It then calculates the probability of the first head at the $(k+1)$ -th toss as $P(H \text{ at } k+1 \text{ th toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } (k+1) \text{ th}) = (1-\lambda)^k \lambda$. For part b), it defines M as the number of tosses required to get the first head and $S = E[M]$. It then uses the property of independent tosses to set up the equation $S = \lambda \times 1 + (1-\lambda) \times (S+1)$, which simplifies to $S = \lambda + S + 1 - \lambda S - \lambda$, leading to $S\lambda = 1$ and finally $S = \frac{1}{\lambda}$.

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Q.1 a) $P(H) = \lambda$
 $\therefore P(T) = 1 - \lambda$

$P(H \text{ at } k \text{ th})$
 $P(H \text{ at } k+1 \text{ th toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } (k+1) \text{ th})$
 $= (1-\lambda)^k \lambda$

b) let M be no. of tosses required to get the first head and let $S = E[M]$

As tosses are independent and expectation is additive

$$S = \lambda \times 1 + (1-\lambda) \times (S+1)$$
$$\therefore S = \lambda + S + 1 - \lambda S - \lambda$$
$$\therefore S\lambda = 1$$
$$\therefore S = \frac{1}{\lambda}$$

2. [Probability] Assume X is a random variable.

a. We define the variance of X as: $\text{Var}(X) = E[(X - E[X])^2]$. Prove that $\text{Var}(X) = E[X^2] - E[X]^2$.

b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X ? If $Y = a + bX$, what is the variance of Y ?

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Q.2 $X \rightarrow$ random variable

a. Variance of X : $\text{Var}(X) = E[(X - E[X])^2]$

To prove. $\text{Var}(X) = E[X^2] - E[X]^2$

Given that

$$\begin{aligned}\text{Var}(X) &= E[(X - E[X])^2] \\&= E[X^2 - 2XE[X] + E[X]^2] \\&= E[X^2] - 2E[XE[X]] + E[X]^2 \\&= E[X^2] - 2E[X]^2 + E[X]^2 \\&= E[X^2] - E[X]^2\end{aligned}$$

b. $E[X] = 0$ and $E[X^2] = 1$

To find : ① Variance of X

② If $Y = a + bX$, $\text{Var}(Y) = ?$

① $\text{Var}(X) = E[X^2] - E[X]^2$ (from I)

$$= 1 - 0^2$$

$\therefore \text{Var}(X) = 1$

$$(2) \quad Y = a + bX$$

$$E[Y^2] = E[(a + bX)^2]$$

$$= E[a^2 + 2abX + b^2X^2]$$

$$= a^2 + 2abE[X] + b^2E[X^2]$$

$$= a^2 + 2ab(0) + b^2(1)$$

$$\therefore E[Y^2] = a^2 + b^2$$

$$E[Y] = E[a + bX] = a + bE[X]$$

$$= a + b(0)$$

$$\therefore E[Y] = a$$

$$\text{Var}(Y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2 = b^2$$

$$\therefore \text{Var}(Y) = b^2$$

3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5} , Aku predicts that a black beauty is a winning horse. This also means that with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.
- Given a horse, what is the probability that it wins?
 - What is the probability that Aku correctly predicts a black beauty is winning?

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Q.3 Let A be the event that "Aku predicts that a given horse is a winning horse"

let $\neg A$ be the event "Aku predicts that the given horse is not a winning horse"

Similarly let B be the event that the given horse wins and $\neg B$ be the event that the given horse does not win

a. Given a horse, the probability that it wins

$$\begin{aligned}
 P(B) &= P(B, A) + P(B, \neg A) \\
 &= P(B|A)P(A) + P(B|\neg A)P(\neg A) \\
 &= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5}) \\
 &\approx P(B) \approx 1.99 \times 10^{-5} \quad \text{--- (I)}
 \end{aligned}$$

b. Probability that Aku predicts a black beauty is winning

$$\begin{aligned}
 P(A|B) &= \frac{P(A, B)}{P(B)} = \frac{P(A|B)P(A)}{P(B)} \\
 &= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}} \quad (\text{from I}) \\
 \therefore P(A|B) &= 0.497
 \end{aligned}$$