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Data Science, 2022
Tut 6: Machine Learning 1

1. [Probability] Assume that the probability of obtaining heads when tossing a coin is λ.
a. What is the probability of obtaining the first head at the (k + 1)-th toss?
b. What is the expected number of tosses needed to get the first head?

Data Science, 2022
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Page No.
Date

Q.1 a) $P(H) = \lambda$

$\therefore P(T) = 1-\lambda$

~~P(H at k th~~

$P(H \text{ at } k+1 \text{ th toss}) = P(T \text{ at } k \text{ toss and } H \text{ at } k+1 \text{ th})$

$= (1-\lambda)^k \lambda$

b) Let M be no. of tosses required to get the first head and let $S = E[M]$

As tosses are independent and expectation is additive

$S = \lambda \times 1 + (1-\lambda) \times (S+1)$

$\therefore S = \lambda + S + 1 - \lambda S - \lambda$

$\therefore S\lambda = 1$

$\therefore S = \frac{1}{\lambda}$

2. [Probability] Assume X is a random variable.

a. We define the variance of X as: $Var(X) = E[(X - E[X])^2]$. Prove that $Var(X) = E[X^2] - E[X]^2$.

b. If $E[X] = 0$ and $E[X^2] = 1$, what is the variance of X? If $Y = a + bX$, what is the variance of Y ?

Page No.
Date

Q.2 $X \rightarrow$ random variable

a. Variance of X : $Var(x) = E[(x - E[x])^2]$

To prove. $Var(x) = E[x^2] - E[x]^2$

Given that

$Var(x) = E[(x - E[x])^2]$

$= E[x^2 - 2xE[x] + E[x]^2]$

$= E[x^2] - 2E[xE[x]] + E[x]^2$

$= E[x^2] - 2E[x]^2 + E[x]^2$

$= E[x^2] - E[x]^2$ — I

b. $E[x] = 0$ and $E[x^2] = 1$

To find : ① Variance of X

② If $Y = a + bX$, $Var(Y) = ?$

① $Var(x) = E[x^2] - E[x]^2$ (∵ from I)

$= 1 - 0^2$

$\therefore Var(x) = 1$

Page No.
Date

(2) $Y = a + bX$

$$E[Y^2] = E[(a+bX)^2]$$

$$= E[a^2 + 2abX + b^2X^2]$$

$$= a^2 + 2abE[X] + b^2E[X^2]$$

$$= a^2 + 2ab(0) + b^2(1)$$

$\therefore$ $\underline{E[Y^2] = a^2 + b^2}$

$$E[Y] = E[a + b\cancel{X}] = a + bE[X]$$

$$= a + b(0)$$

$\therefore$ $E[Y] = a$

$$Var(Y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2$$

$\therefore$ $\underline{Var(Y) = b^2}$

3. [Probability] Your friend Aku is a great predictor about winning horse race. Assume that we know three facts: 1) If Aku tells you that a horse name black beauty will win, it will win with probability 0.99. 2) If Aku tells you that a black beauty will not win, it will not win with probability 0.99999. 3) With probability 10^{-5}, Aku predicts that a black beauty is a winning horse. This also means that with probability $1 - 10^{-5}$, Aku predicts that a black beauty will not win.
a. Given a horse, what is the probability that it wins?
b. What is the probability that Aku correctly predicts a black beauty is winning ?

Page No.
Date / /

Q.3 Let A be the event that "Aku predicts that a given horse is a winning horse"

let $\neg A$ be the event "Aku predicts that the given horse is not a winning horse".

Similarly let B be the event that the given horse wins and $\neg B$ be the event that the given horse does not win

a. Given a horse, the probability that it wins

$$P(B) = P(B, A) + P(B, \neg A)$$

$$= P(B/A)\,P(A) + P(B/\neg A)\,P(\neg A)$$

$$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$$

$$\therefore P(B) \approx 1.99 \times 10^{-5} \quad \text{--- (I)}$$

b. Probability that Aku predicts a black beauty is winning.

$$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A|B)\,P(A)}{P(B)}$$

$$= \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}} \quad (\text{from I})$$

$$\therefore P(A|B) = 0.497$$