TAD_SoftwareProject_ML_Part1_Final

May 18, 2023

[1]:
```
"""
Created on Sat Apr 29 10:08:36 2023
@author: TAD
"""
import matplotlib as plt
%matplotlib inline
import numpy as np
import pandas as pd
import scipy,scipy.spatial
import sklearn
import seaborn as sns
import math
from sklearn.manifold import MDS
import random
import warnings
warnings.filterwarnings('ignore')
import seaborn as sns
from sklearn.manifold import TSNE
import matplotlib.pyplot as plt
from sklearn.cluster import KMeans
```

[2]:
```
#Reading the Dataset file

data=pd.read_csv("Wholesale customers data.csv")
```

[3]:
```
data.head()
```

[3]:

	Channel	Region	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicassen
0	2	3	12669	9656	7561	214	2674	1338
1	2	3	7057	9810	9568	1762	3293	1776
2	2	3	6353	8808	7684	2405	3516	7844
3	1	3	13265	1196	4221	6404	507	1788
4	2	3	22615	5410	7198	3915	1777	5185

[4]:
```
# Define Utiliy Functions

def data_insights_report(data=pd.DataFrame()):
```

```
    report = pd.DataFrame()
    report['Columns'] = data.columns
    report['Data_Types'] = data.dtypes.values
    report['Unique_Count'] = data.nunique().values
    report['NAN_Count'] = data.isna().sum().values

    tempdf = data.describe().apply(lambda x: round(x,2), axis=1).T
    pd.merge(report,tempdf.reset_index().rename(columns={'index':'Columns'}),␣
 ↪on='Columns')

    return report.join(tempdf, on='Columns')
```

[5]:
```
print('Descriptive Statastics of our Data including Channel & Region:')
data_insights_report(data)
```

```
Descriptive Statastics of our Data including Channel & Region:
```

[5]:
```
            Columns Data_Types  Unique_Count  NAN_Count  count      mean  \
0           Channel      int64             2          0  440.0      1.32
1            Region      int64             3          0  440.0      2.54
2             Fresh      int64           433          0  440.0  12000.30
3              Milk      int64           421          0  440.0   5796.27
4           Grocery      int64           430          0  440.0   7951.28
5            Frozen      int64           426          0  440.0   3071.93
6  Detergents_Paper      int64           417          0  440.0   2881.49
7        Delicassen      int64           403          0  440.0   1524.87

        std   min      25%     50%       75%       max
0      0.47   1.0     1.00     1.0      2.00       2.0
1      0.77   1.0     2.00     3.0      3.00       3.0
2  12647.33   3.0  3127.75  8504.0  16933.75  112151.0
3   7380.38  55.0  1533.00  3627.0   7190.25   73498.0
4   9503.16   3.0  2153.00  4755.5  10655.75   92780.0
5   4854.67  25.0   742.25  1526.0   3554.25   60869.0
6   4767.85   3.0   256.75   816.5   3922.00   40827.0
7   2820.11   3.0   408.25   965.5   1820.25   47943.0
```

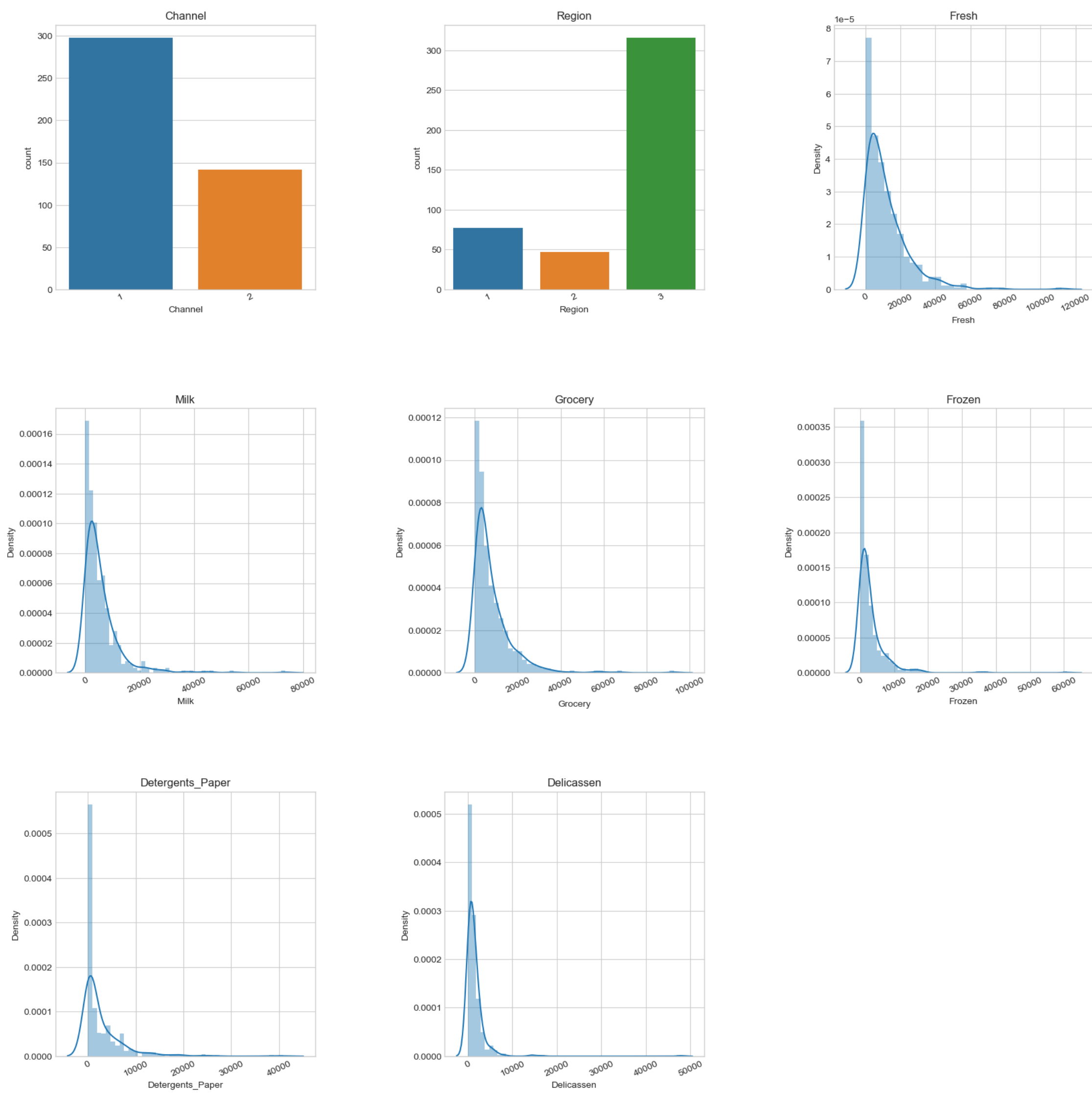
[6]:
```
# The percentage of different types of channels in dataset
data.Channel.value_counts()/len(data.Channel)*100
```

[6]:
```
1    67.727273
2    32.272727
Name: Channel, dtype: float64
```

[7]:
```
# The percentage of different types of channels in dataset

data.Region.value_counts()/len(data.Region)*100
```

[7]:
```
3    71.818182
1    17.500000
2    10.681818
Name: Region, dtype: float64
```

[8]:
```
#Plotting Univariate scatterplots and countplots, depending on the type of␣
↪column

def plot_distribution(df, cols=5, width=20, height=15, hspace=0.2, wspace=0.5):
    plt.style.use('seaborn-whitegrid')
    fig = plt.figure(figsize=(width,height))
    fig.subplots_adjust(left=None, bottom=None, right=None, top=None,␣
↪wspace=wspace, hspace=hspace)
    rows = math.ceil(float(df.shape[1]) / cols)
    for i, column in enumerate(df.columns):
        ax = fig.add_subplot(rows, cols, i + 1)
        ax.set_title(column)
        #if df.dtypes[column] == np.object:
        if column in ['Channel','Region']:
            g = sns.countplot(x=column, data=df)
            substrings = [s.get_text()[:18] for s in g.get_yticklabels()]
            g.set(yticklabels=substrings)
            plt.xticks(rotation=25)
        else:
            g = sns.distplot(df[column])
            plt.xticks(rotation=25)

plot_distribution(data, cols=3, width=20, height=20, hspace=0.45, wspace=0.5)
```



[9]:
```
# def plot_histograms(dataframe, features, rows, cols):
#     fig = plt.figure(figsize=(20, 20))
#     for i, feature in enumerate(features):
#         ax = fig.add_subplot(rows, cols, i+1)
#         dataframe[feature].hist(bins=20, ax=ax)
#         ax.set_title(feature )
#     fig.tight_layout()
#     plt.show()
```

[10]:
```
#drop channel and region
df=data.copy()
df = df.drop(columns = ['Channel','Region'])
```

```
#plot_distribution(df, cols=3, width=20, height=20, hspace=0.45, wspace=0.5)
```

Which Region and Channel seem to spend more (or less)?

[11]:
```
#Having the total expenditure for each customer

data['Total']=data.Fresh+data.Milk+data.Grocery+data.Frozen+data.
 ↪Detergents_Paper+data.Delicassen
```

[12]:
```
data.head()
```

[12]:
```
   Channel  Region  Fresh  Milk  Grocery  Frozen  Detergents_Paper  \
0        2       3  12669  9656     7561     214              2674
1        2       3   7057  9810     9568    1762              3293
2        2       3   6353  8808     7684    2405              3516
3        1       3  13265  1196     4221    6404               507
4        2       3  22615  5410     7198    3915              1777

   Delicassen  Total
0        1338  34112
1        1776  33266
2        7844  36610
3        1788  27381
4        5185  46100
```

[13]:
```
data.groupby(['Region','Channel'])['Total'].sum()
```

[13]:
```
Region  Channel
1       1          1538342
        2           848471
2       1           719150
        2           835938
3       1          5742077
        2          4935522
Name: Total, dtype: int64
```

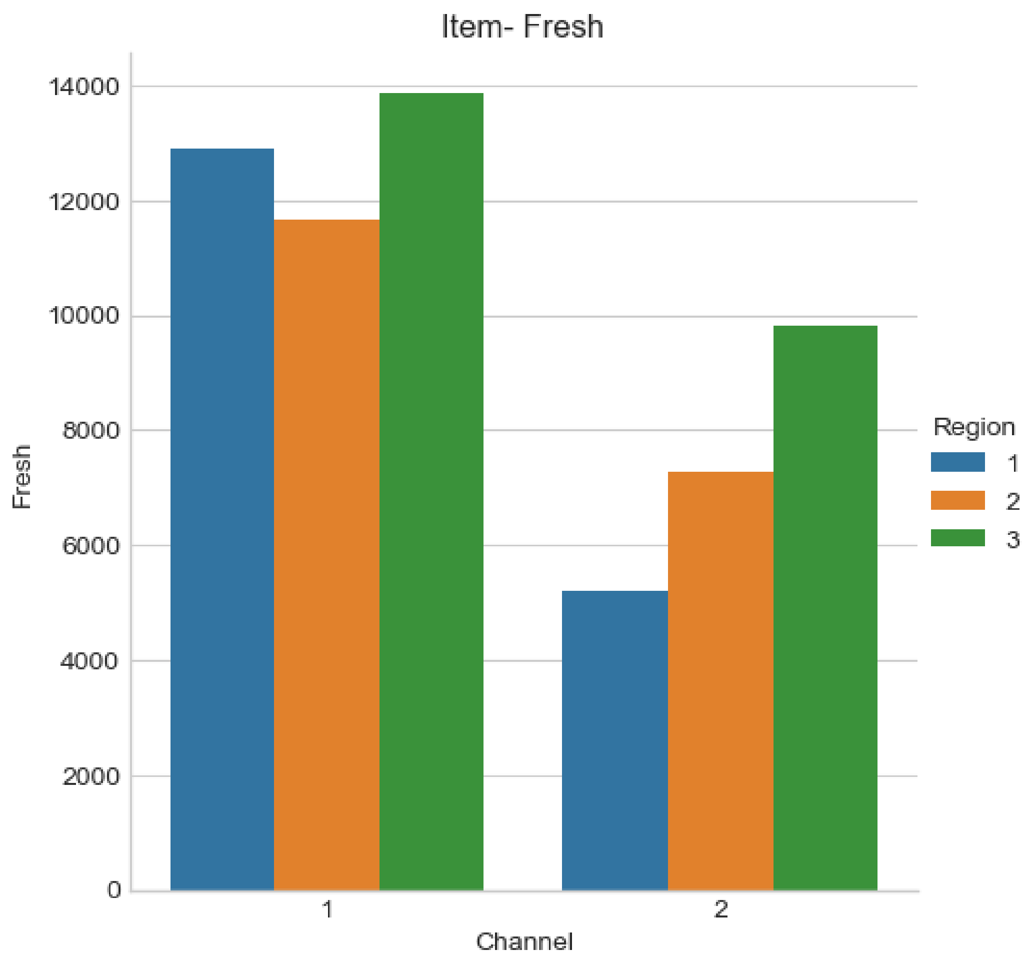
Highest Spending is from Region 3 and Channel 1.

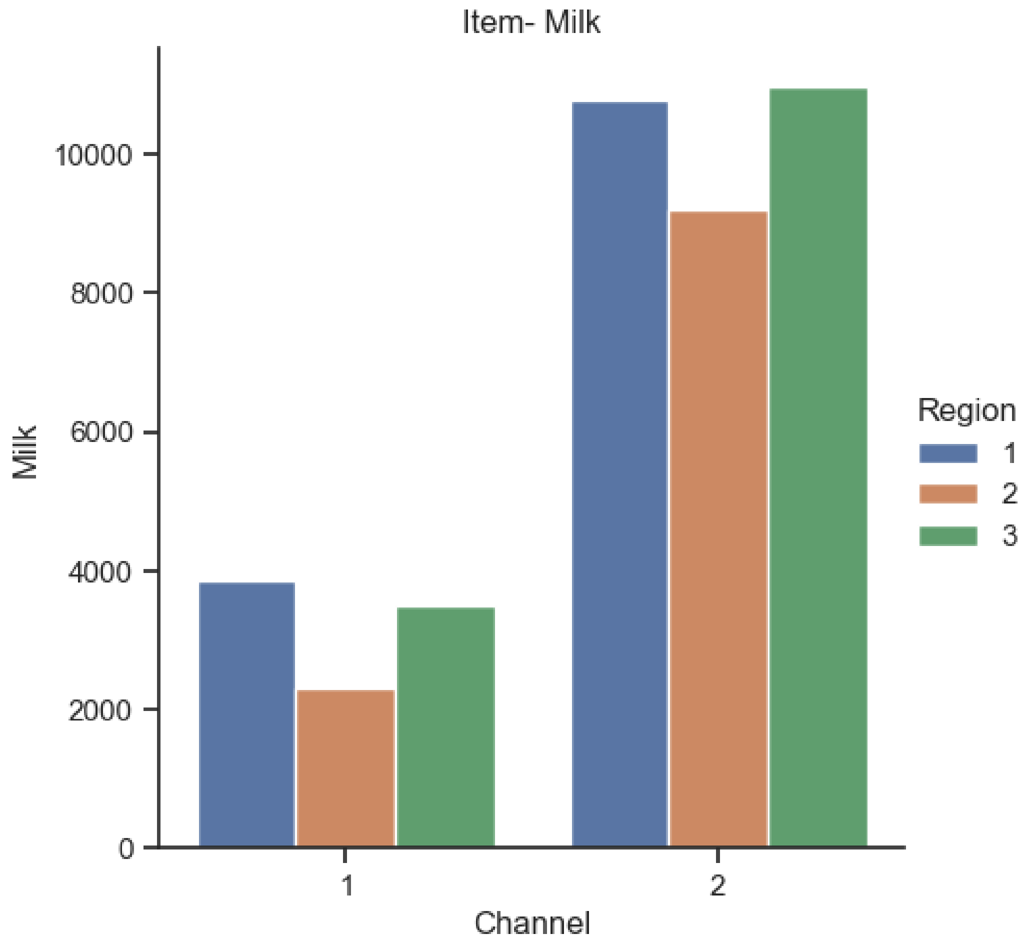
Lowest Spending is from Region 2 and Channel 1.

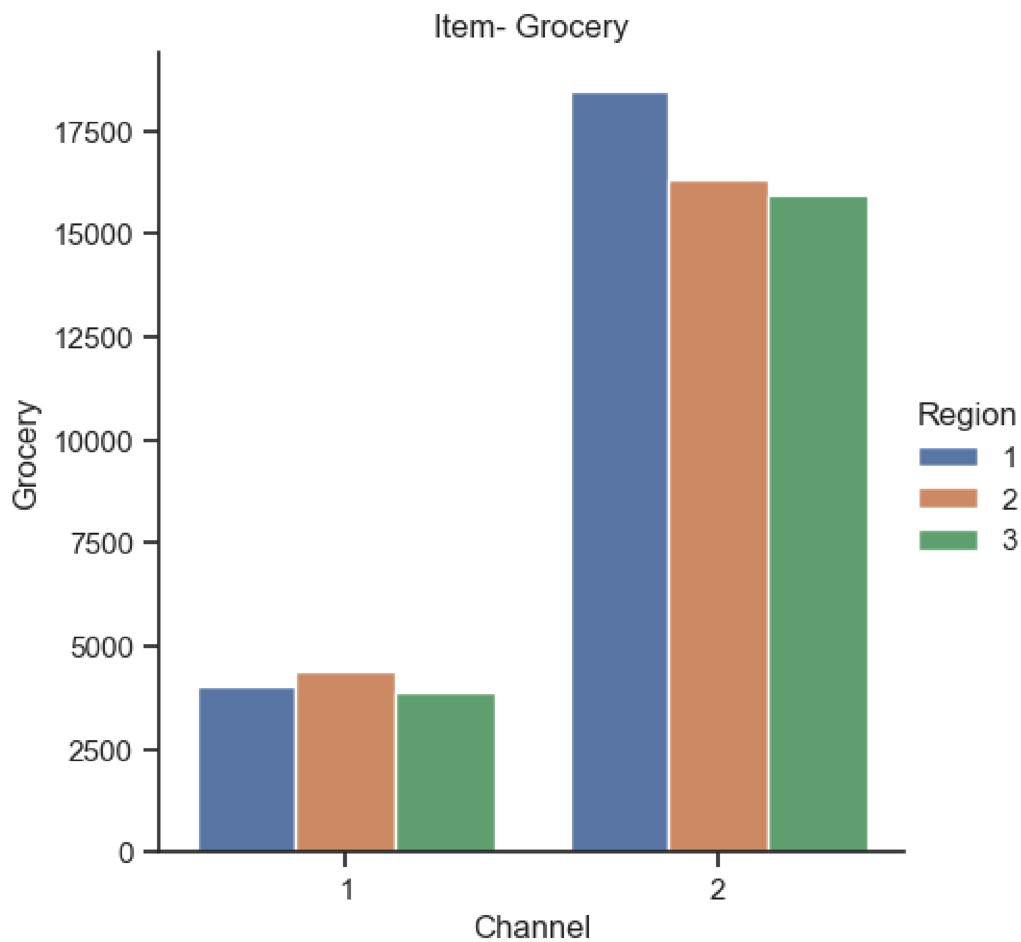
There are 6 different varieties of items. Do all varieties show similar behaviour across Region and Channel?

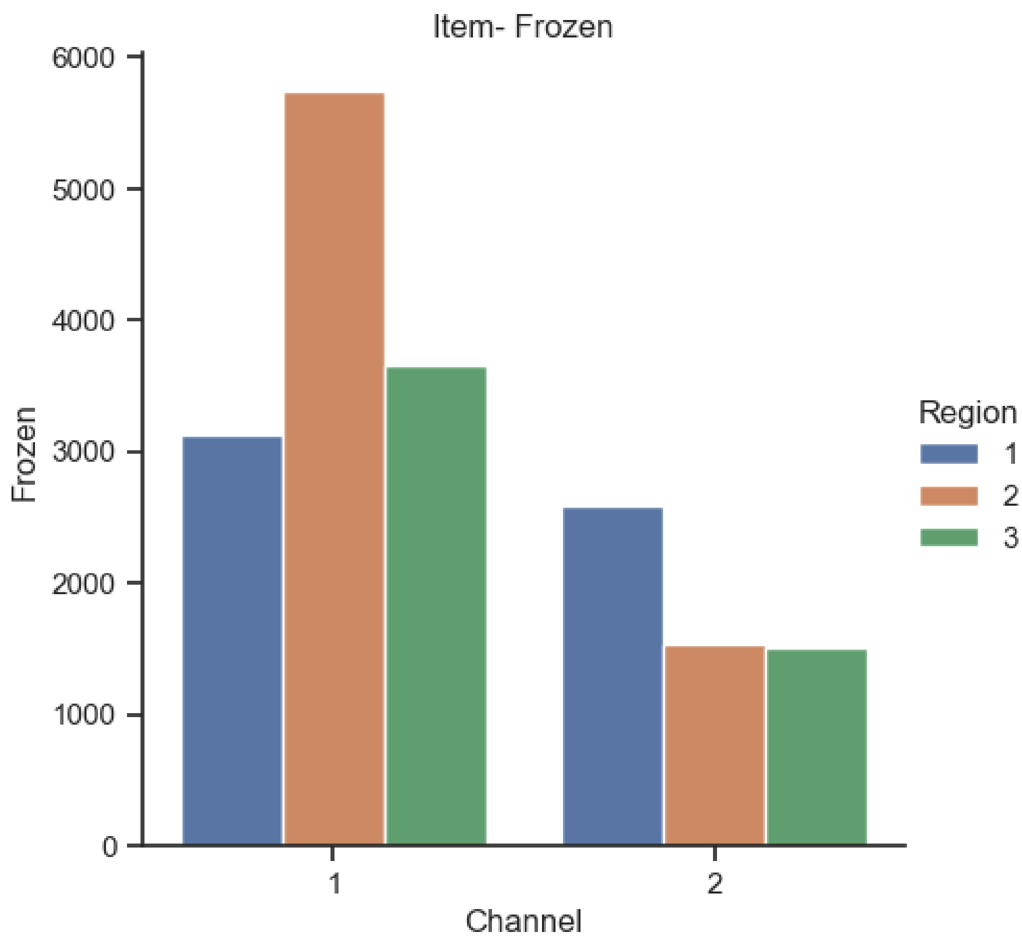
[14]:
```
#Histogram for general trend for each item, based on channel and region.
plt.style.use('seaborn-whitegrid')
for i, column in enumerate(df.columns):
```

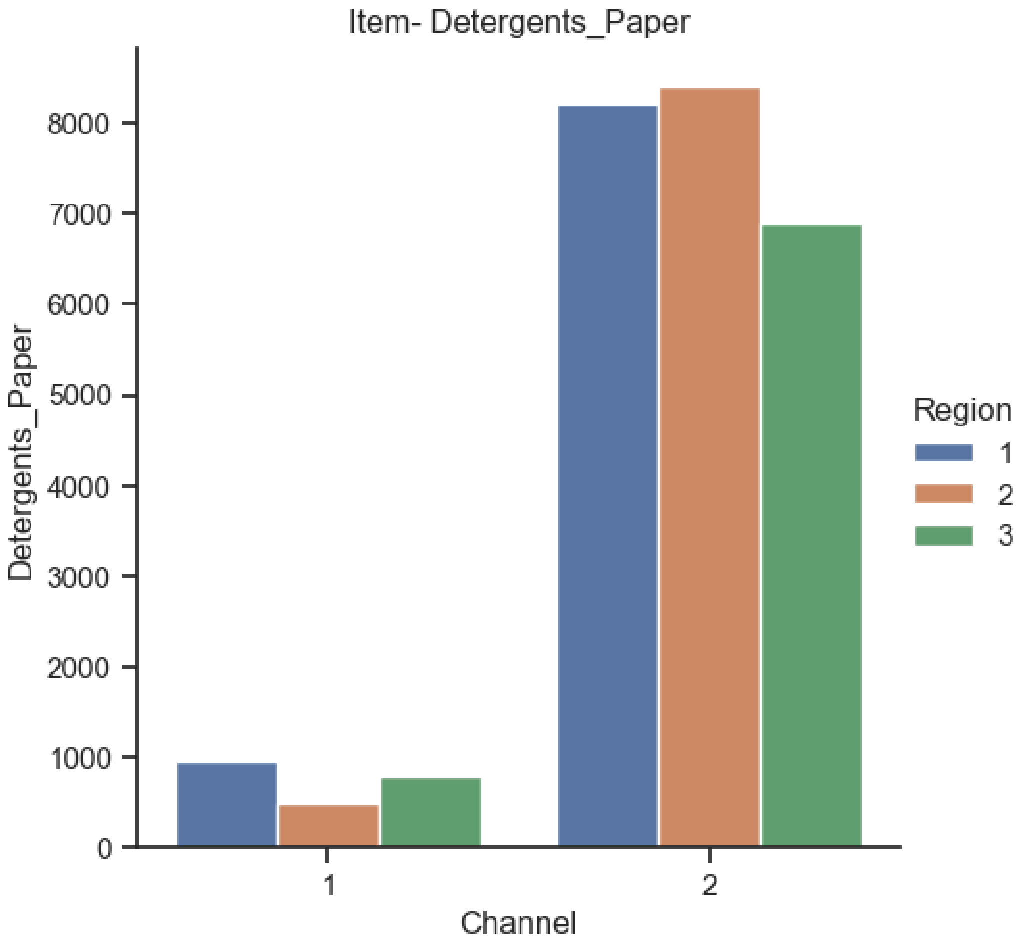
```
        sns.catplot(x="Channel", y=column, hue ="Region", kind="bar", ci=None,␣
↪data=data)
        sns.set(style="ticks", color_codes=True)
        plt.title('Item- '+column)
```



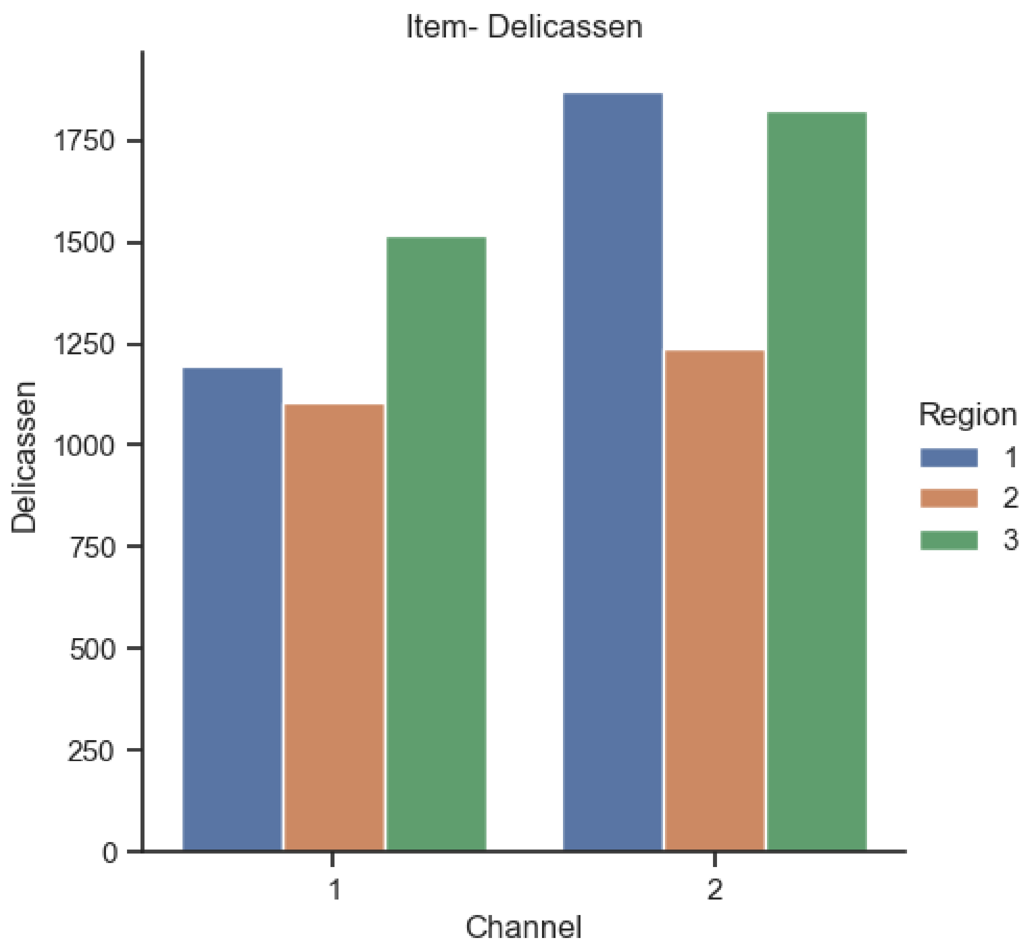

Item- Milk
Milk
10000
8000
6000
4000
2000
0
1
2
Channel
Region
1
2
3


Item- Grocery
Grocery
17500
15000
12500
10000
7500
5000
2500
0
1
2
Channel
Region
1
2
3


Item- Frozen
6000
5000
4000
3000
2000
1000
0
Frozen
1
2
Channel
Region
1
2
3



Item- Detergents_Paper
Detergents_Paper
8000
7000
6000
5000
4000
3000
2000
1000
0
1
2
Channel
Region
1
2
3



For Milk, Grocery and Detergents_paper, Channel 2 seems to spend more than Channel 1.

Region 2 seems to behave differently for Frozen items for Channel 1, as compared to channel 2.

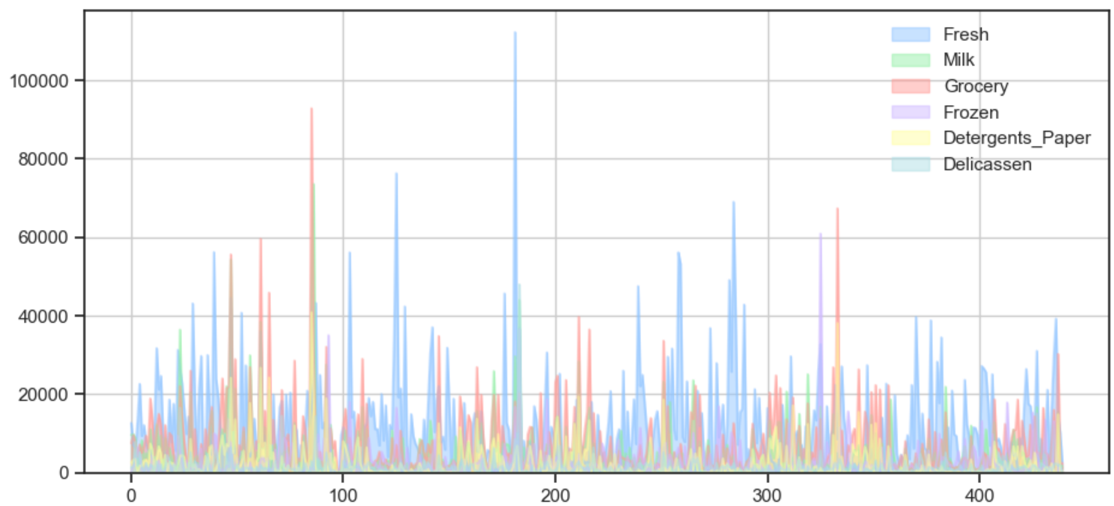
On the basis of the descriptive measure of variability, which items show the most and the least inconsistent behaviour resp.?

[15]:
```
std_items = df.std() #use standard deviation to check the measure of variabilty
std_items.round(2)
```

[15]:
```
Fresh               12647.33
Milk                 7380.38
Grocery              9503.16
Frozen               4854.67
Detergents_Paper     4767.85
Delicassen           2820.11
dtype: float64
```

Fresh Items are the most inconsistent as they have the highest standard deviation whereas Delicassen is the most consistent

[16]:
```
import pylab
pylab.style.use('seaborn-pastel')
df.plot.area(stacked=False,figsize=(11,5))
pylab.grid(); pylab.show()

#X axis: each data point (440 datapoints)
#Y axis: value of each item in every data point
```



[17]:
```
df['Fresh'].nlargest(3).index
```

[17]:
```
Int64Index([181, 125, 284], dtype='int64')
```

[18]:
```
df['Grocery'].nlargest(3).index
```

[18]:
```
Int64Index([85, 333, 61], dtype='int64')
```

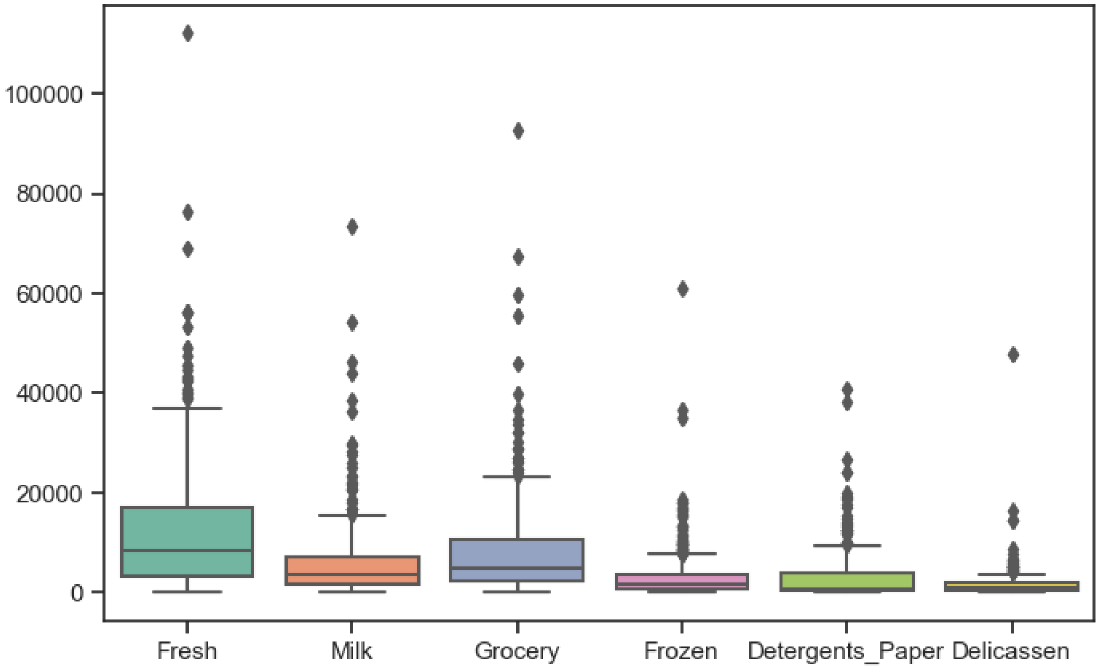
We see highest values for two sample features, e.g., Fresh and Grocery are for datapoints [181, 125, 284] and [85, 333, 61] respectively, which seems to match the spikes in our graph above.

The spikes could possibly represent anomalous points.

Outliers

[19]:
```
#Boxplots for spread of each feature. There seems to be outliers at the higher␣
↪end.

plt.figure(figsize=(8,5))
sns.boxplot(data=df, orient="v", palette="Set2")
```

[19]: <Axes: >



[20]:
```
df.agg(['skew', 'kurtosis']).transpose()
```

[20]:

	skew	kurtosis
Fresh	2.561323	11.536408
Milk	4.053755	24.669398
Grocery	3.587429	20.914670
Frozen	5.907986	54.689281
Detergents_Paper	3.631851	19.009464
Delicassen	11.151586	170.694939

As a rule of thumb, skewness can be interpreted like this:

Skewness

Fairly Symmetrical -0.5 to 0.5

Moderate Skewed -0.5 to -1.0 and 0.5 to 1.0

Highly Skewed < -1.0 and > 1.0

Hence our features are highly skewed. Apply Log to reduce the Skewness.

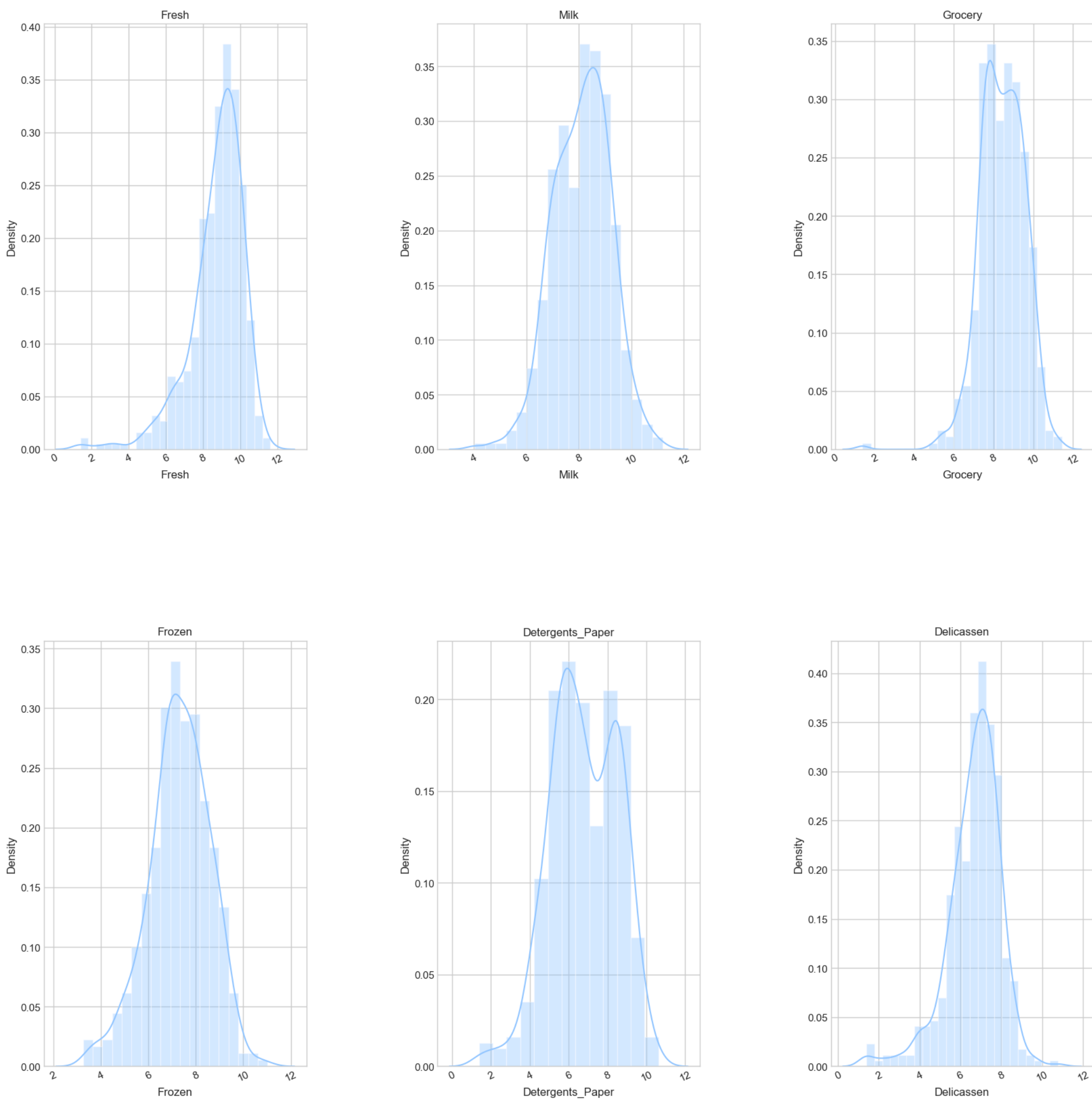
[21]:
```
df_log =df.add(1).applymap(np.log)
```

[22]:
```
df_log.agg(['skew', 'kurtosis']).transpose()
```

[22]:
```
                      skew  kurtosis
Fresh            -1.575326  4.052833
Milk             -0.224063  0.210842
Grocery          -0.674938  3.161866
Frozen           -0.352655  0.269540
Detergents_Paper -0.235961 -0.301082
Delicassen       -1.091827  2.748784
```

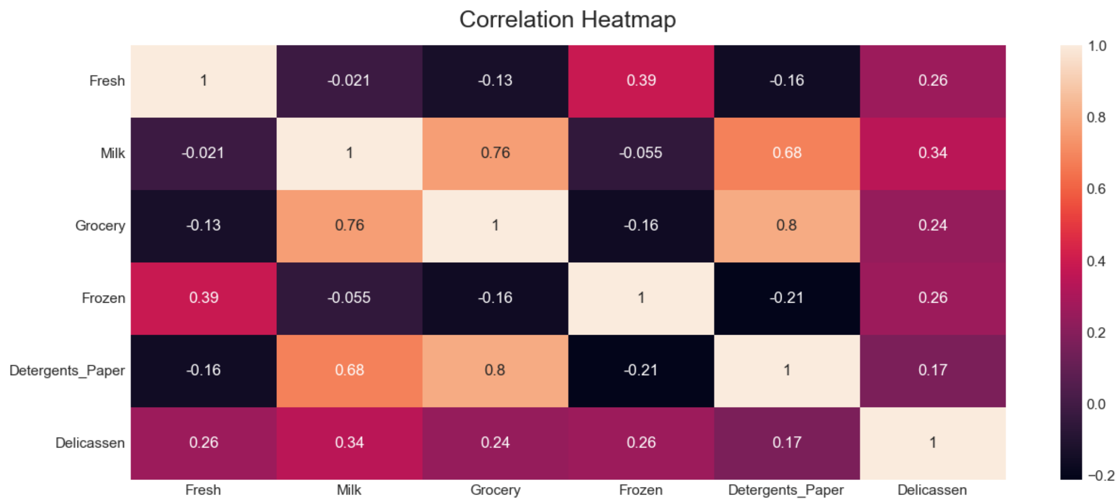
After the log transformation of data, we observe the data is more symmetric, based on the skewness scores. But “Fresh” and “Delicassen” are still highly skewed.

[23]:
```
plot_distribution(df_log, cols=3, width=20, height=20, hspace=0.45, wspace=0.5)
```



Is there a correlation between any of the items?

[24]:
```
plt.figure(figsize=(15, 6))
heatmap = sns.heatmap(df_log.corr(), annot=True)
heatmap.set_title('Correlation Heatmap', fontdict={'fontsize':18}, pad=14);
plt.show()
```



There is a high correlation between Detergents_Paper and Grocery. Also, Milk and Grocery are considerably correlated.

[25]:
```
data_insights_report(df_log)
```

[25]:

	Columns	Data_Types	Unique_Count	NAN_Count	count	mean	std
0	Fresh	float64	433	0	440.0	8.73	1.47
1	Milk	float64	421	0	440.0	8.12	1.08
2	Grocery	float64	430	0	440.0	8.44	1.11
3	Frozen	float64	426	0	440.0	7.30	1.28
4	Detergents_Paper	float64	417	0	440.0	6.79	1.71
5	Delicassen	float64	403	0	440.0	6.67	1.29

	min	25%	50%	75%	max
0	1.39	8.05	9.05	9.74	11.63
1	4.03	7.34	8.20	8.88	11.21
2	1.39	7.68	8.47	9.27	11.44
3	3.26	6.61	7.33	8.18	11.02
4	1.39	5.55	6.71	8.27	10.62
5	1.39	6.01	6.87	7.51	10.78

[26]:
```
df_log.head()
```

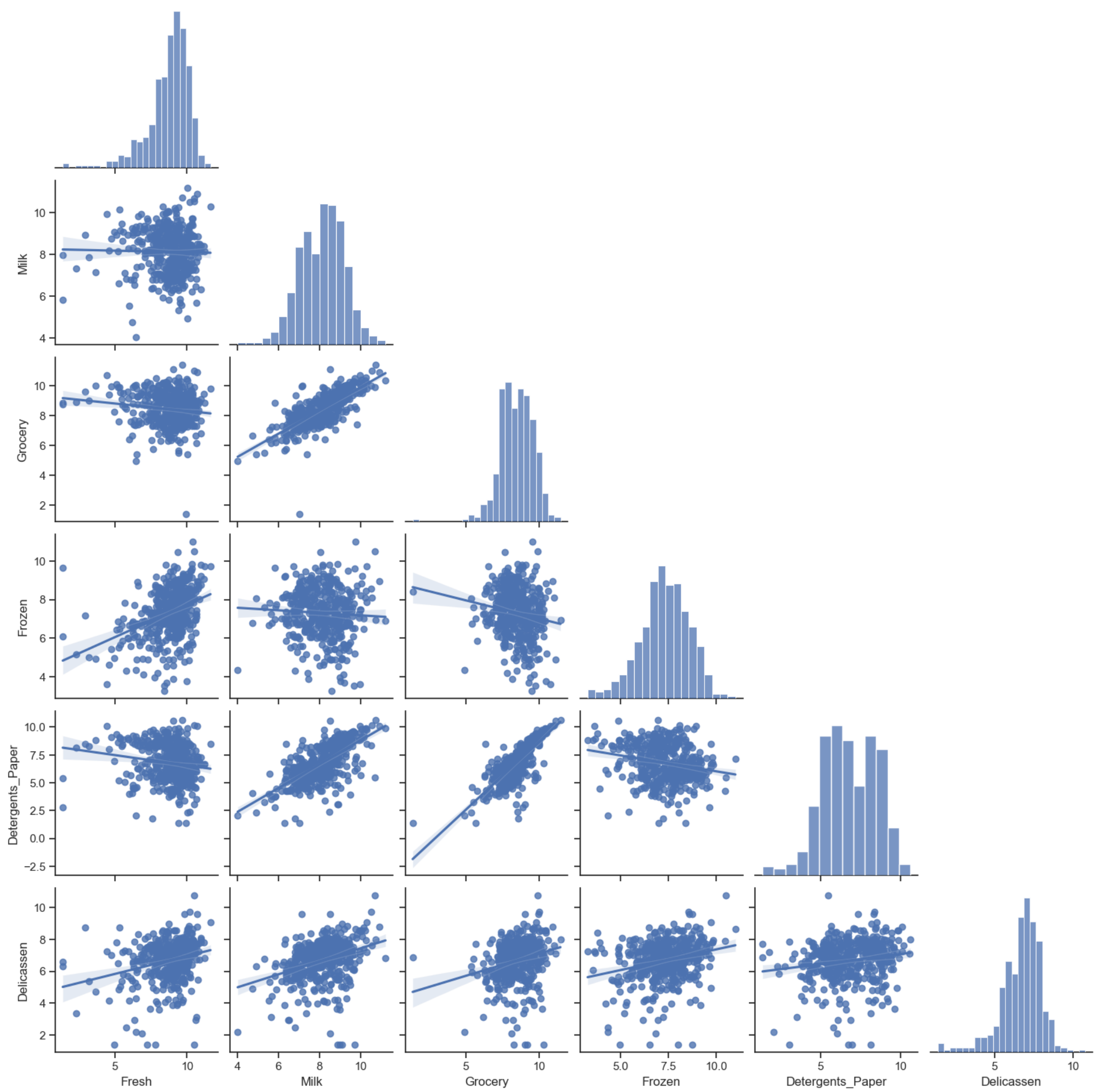
[26]:

	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicassen
0	9.446992	9.175438	8.930891	5.370638	7.891705	7.199678
1	8.861917	9.191259	9.166284	7.474772	8.099858	7.482682
2	8.756840	9.083529	8.947026	7.785721	8.165364	8.967632
3	9.492960	7.087574	8.348064	8.764834	6.230481	7.489412
4	10.026413	8.596189	8.881697	8.272826	7.483244	8.553718

Correlation between features:

[27]:

```
sns.set(style="ticks")
g = sns.pairplot(df_log,corner=True,kind='reg')
g.fig.set_size_inches(15,15)
```



1 Part 2

[28]:
```python
'''
Function to calculate Distance Matrix.
Input Parameter: Dataframe of size a*b
Output: Distance matrix of size a*a
'''

def calculate_dist_matrix(dataframe):
    X=dataframe.to_numpy()
    D=scipy.spatial.distance.cdist(X,X)
    Dist=pd.DataFrame(D)
    return Dist
```

[29]:
```python
dist_matrix = calculate_dist_matrix(df_log)
dist_matrix
```

[29]:
```
          0         1         2         3         4         5         6    \
0    0.000000  2.224584  3.085157  4.366366  3.331161  1.307943  1.645392
1    2.224584  0.000000  1.541680  3.263750  1.988811  1.362131  2.174861
2    3.085157  1.541680  0.000000  3.431039  1.651504  2.294735  3.340546
3    4.366366  3.263750  3.431039  0.000000  2.406136  3.262152  3.557725
4    3.331161  1.988811  1.651504  2.406136  0.000000  2.411643  3.233237
..        …       …        …       …       …       …       …
435  5.069015  3.848109  4.030637  3.060203  2.953376  4.127355  4.921473
436  5.564116  5.161151  5.187257  2.677452  4.048232  4.616688  5.134254
437  2.390978  2.502427  3.148681  5.387722  3.702175  2.904145  2.933326
438  3.789719  3.735002  3.950812  2.296451  3.330583  2.943401  3.540550
439  4.616926  5.708060  6.821026  6.012160  6.799926  4.748969  4.066376

          7         8         9    …       430       431       432  \
0    2.330232  1.597930  2.345042  …  3.195456  4.266183  2.642341
1    0.780828  2.092733  1.169105  …  3.126408  2.383102  3.582755
2    1.337177  3.137447  1.921839  …  3.550237  2.656078  4.519018
3    2.905404  3.407516  4.238947  …  3.455373  2.036131  3.668271
4    1.699094  3.257427  2.746759  …  3.630900  2.048335  4.048988
..        …       …       …  …       …       …       …
435  3.951802  4.778894  4.711158  …  4.008258  2.771055  5.387995
436  4.925099  4.909501  6.240254  …  4.710604  4.049138  4.677306
437  2.685797  3.298699  1.598576  …  4.633749  4.682201  4.327725
438  3.484226  2.993919  4.704888  …  2.547515  3.588645  3.204004
439  5.757638  3.757454  6.120177  …  4.797128  6.614062  3.424337

          433       434       435       436       437       438       439
0    3.901349  1.575182  5.069015  5.564116  2.390978  3.789719  4.616926
1    3.361482  1.780825  3.848109  5.161151  2.502427  3.735002  5.708060
2    3.651499  2.882593  4.030637  5.187257  3.148681  3.950812  6.821026
```

```
3    2.804716  3.128861  3.060203  2.677452  5.387722  2.296451  6.012160
4    3.706398  2.573983  2.953376  4.048232  3.702175  3.330583  6.799926
..        …         …         …         …         …         …         …
435  4.462180  4.273217  0.000000  3.931938  5.647413  3.835549  7.464192
436  3.630164  4.717930  3.931938  0.000000  7.161765  2.356582  6.579825
437  5.569987  2.807914  5.647413  7.161765  0.000000  5.650544  6.471399
438  1.992430  3.215658  3.835549  2.356582  5.650544  0.000000  4.904101
439  4.664636  4.576984  7.464192  6.579825  6.471399  4.904101  0.000000

[440 rows x 440 columns]
```

[30]:
```python
'''
Function to detect anomalous points by calculating minimum distane of each␣
 ↪point to any other point (i.e. taking the
minimum value for each row, denoting each data point).

Input Parameter:
    1. dist_matrix: Distance Matrix generated in the cell above.
Output:
    hardmin: Minimum value of each row for all the rows.

'''

def hardmin(dist_matrix):
    hardmin=dist_matrix.apply(lambda row: row.nsmallest(2).iloc[-1], axis=1)
    return hardmin

Df_HardMin=hardmin(dist_matrix)
Df_HardMin
```

[30]:
```
0      1.086943
1      0.507120
2      1.141368
3      0.975531
4      0.912961
         …
435    1.573612
436    1.320107
437    0.983008
438    0.883881
439    2.375114
Length: 440, dtype: float64
```

[31]:
```python
'''
We then choose the points which has the largest minimum distance with other␣
 ↪points.
In this example, 10 most anomalous points have been shown.
```

```
'''

Df_HardMin.nlargest(10)
```

[31]:
```
338    4.945913
75     4.647444
154    4.201955
142    3.774788
95     3.746081
187    3.001067
128    2.973513
183    2.956699
204    2.920883
109    2.721484
dtype: float64
```

[32]:
```
'''
Function to detect anomalous points, but instead of directly taking the minimum␣
 ↪value for each row, we use given
softmin function and apply it for each row (i.e. data point).

Input Parameter:
    1. dist_matrix: Distance Matrix generated in the cell above.
    2. gamma: Gamma Value (default value 0.5)
Output:
    softmin: Softmin value of each row for all the rows.

'''



def softmin(dist_matrix_row,gamma=0.5):
    sumexp=0
    N=len(dist_matrix_row)
    for i in dist_matrix_row:
        sumexp+=(math.exp((-gamma)*i))

    softmin= math.log((sumexp-1)/(N-1)) * (-1/gamma)
    return softmin


gamma=0.5
Df_SoftMin=dist_matrix.apply(softmin,args=(gamma,), axis=1)
Df_SoftMin
```

[32]:
```
0      3.523866
1      2.996242
2      3.578611
```

```
3      3.069036
4      3.367632
         …
435    4.244221
436    3.960418
437    4.143021
438    3.114930
439    4.939460
Length: 440, dtype: float64
```

[33]:
```
'''
Similarly, we then choose the points which has the largest softmin distance␣
↪with other points.
In this example, 10 most anomalous points have been shown.
'''

Df_SoftMin.nlargest(10)
```

[33]:
```
338    9.151482
154    8.881429
75     8.698126
95     7.730110
66     7.595647
128    7.111647
142    7.073153
65     6.834986
218    6.659030
357    6.521975
dtype: float64
```

[34]:
```
'''
Function to use bootstrapping method to randomly sample half of the instances␣
↪from the original dataset.
We then apply softmin() to getmost anomalous points from the bootstrapped data.

Input Parameter:
    1. dataframe: Distance Matrix generated in the cell above.
    2. number_of_bootstraps: Number of times bootstrapping is to be performed␣
↪(default value 3).
    3. gamma: Gamma Value (default value 0.5)
Output:
    data: Dataframe containing anomalous points and their anomaly score for␣
↪each bootstrap.

'''
```

```
def bootstrap(dataframe,
              number_of_bootstraps=3,
              gamma=0.5):


    n_samples = len(dataframe) // 2
    n_largest = n_samples

    data = pd.DataFrame()

    for i in range(number_of_bootstraps):

        # Draw random samples with replacement from the original data
        sample_data = dataframe.sample(n=n_samples, replace=True)

        #calculate distance matrix of sampled data
        sample_dist_matrix = calculate_dist_matrix(sample_data)

        #apply softmin to the distance matrix
        sample_softmin=sample_dist_matrix.apply(softmin,args=(gamma,), axis=1)

        #sort points according to their anomaly score for different bootstrap␣
 ↪results and add them to a table
        data[str(i+1)+'_gamma_'+str(round(gamma,2))] = sample_softmin.
 ↪nlargest(n_largest).index
        data[str(i+1)+'score_'+str(round(gamma,2))] = sample_softmin.
 ↪nlargest(n_largest).values

    return data


bootstrap_data=bootstrap(df_log,5,0.5)
bootstrap_data[:10]
```

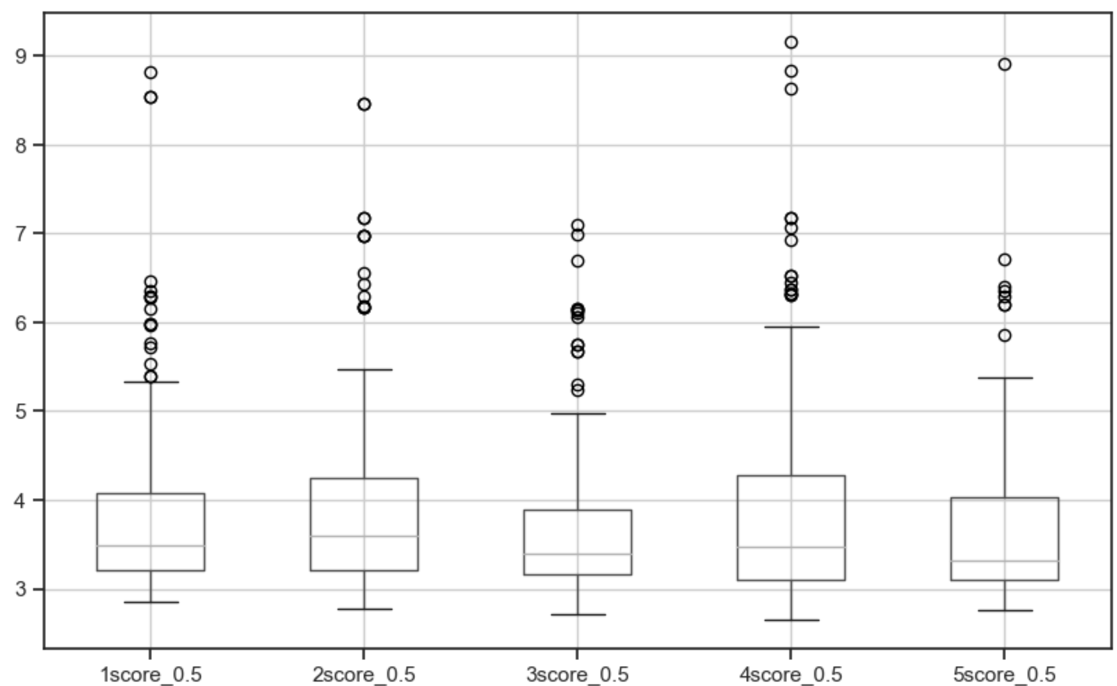
[34]:

```
   1_gamma_0.5  1score_0.5  2_gamma_0.5  2score_0.5  3_gamma_0.5  3score_0.5  \
0          138    8.820652          117    8.462460          134    7.092085
1           27    8.534227          122    8.462460           67    6.996711
2          115    8.534227           39    7.180956          115    6.699056
3           85    6.471290          194    7.169793          208    6.154864
4           17    6.361243          113    6.982584           58    6.154864
5          168    6.291427           34    6.982584           75    6.139291
6          171    6.291427           59    6.982584          123    6.139291
7          217    6.284391           44    6.561160           60    6.106953
8          150    6.148879          108    6.437400          159    6.055305
9          163    5.983124           28    6.289279           31    5.745166

   4_gamma_0.5  4score_0.5  5_gamma_0.5  5score_0.5
```

0	41	9.165425	140	8.909708
1	211	8.828446	32	6.709896
2	37	8.634154	110	6.408965
3	199	7.180902	215	6.350770
4	148	7.180902	47	6.298090
5	106	7.064719	112	6.206199
6	201	6.932884	94	6.206199
7	60	6.532134	198	5.854642
8	64	6.532134	79	5.372812
9	134	6.450658	61	5.302570

[35]:
```
# Boxplot with spread of anomaly score in each bootstrap
plt.figure(figsize=(10,6))

bootstrap_data.filter(like='score').boxplot()
```

[35]: <Axes: >



[36]:
```
'''
Function to use different gamma values while bootstrapping. Gamma values are␣
↪generated randomly within a range
provided by user.
```

```
Input Parameter:
    1. dataframe: Original Data
    2. n_diff_gammas: Number of gammas we want to experiment with (default␣
 ↪value 3).
    3. gamma_low: Lower bound of Gamma value (default value 0.1).
    4. gamma_high: Upper bound of Gamma value (default value 5).
Output:
    data: Dataframe containing anomalous points and their anomaly score for␣
 ↪each bootstrap for each gamma.

'''



def bootstrap_gamma(dataframe,
                    n_diff_gammas=5,
                    gamma_low=0.1,
                    gamma_high=0.5):

    data = pd.DataFrame()

    for i in range(n_diff_gammas):
        gamma=random.uniform(gamma_low,gamma_high)
        print('gamma:', gamma,'------------')
        bootstrap_gamma_outliers = bootstrap(dataframe,1,gamma)
#         print(bootstrap_gamma_outliers)
        data=pd.concat([data, bootstrap_gamma_outliers], axis=1)
        data.columns = data.columns.str.lstrip('1_')
    return data


outliers=bootstrap_gamma(df_log,
                         n_diff_gammas=5,
                         gamma_low=0.5,
                         gamma_high=5 )
```

```
gamma: 1.145850528586645 ------------
gamma: 3.2018784174948447 ------------
gamma: 4.160309073862101 ------------
gamma: 1.0139079887659532 ------------
gamma: 4.493397781517854 ------------
```

[37]:
```
outliers  #output of function bootstrap_gamma
```

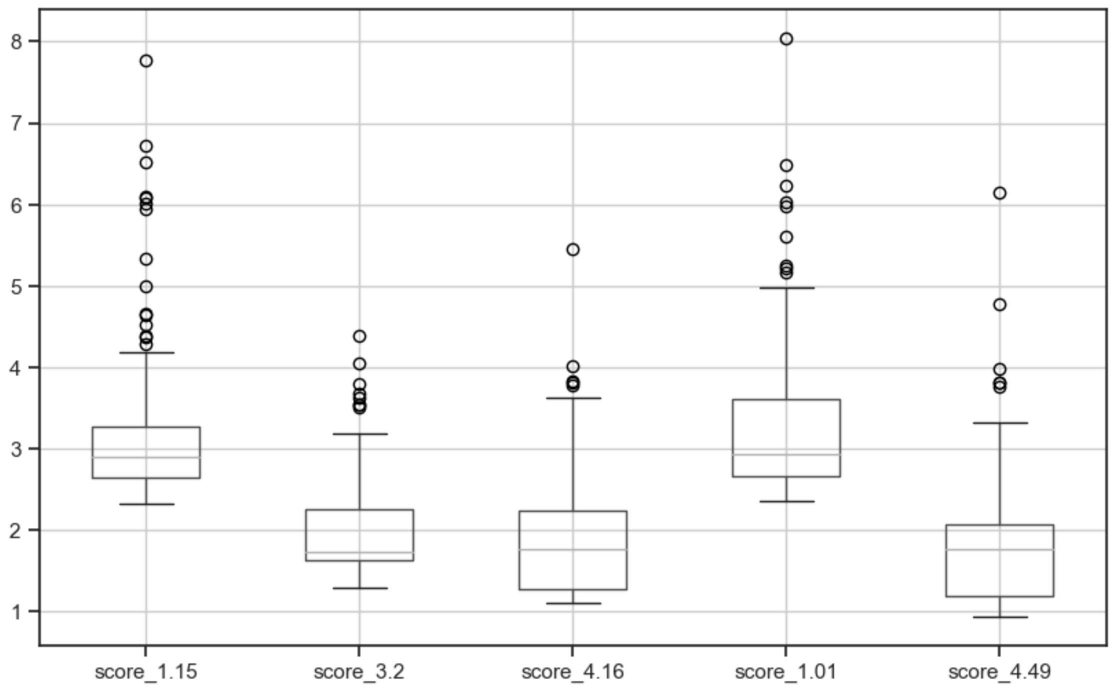
[37]:

	gamma_1.15	score_1.15	gamma_3.2	score_3.2	gamma_4.16	score_4.16	\
0	87	7.763259	140	4.387418	149	5.449131	
1	39	6.717649	124	4.047204	102	4.012690	
2	5	6.523850	96	3.802172	99	3.832200	

```
3            133    6.090311        166    3.673793           74     3.818150
4            139    6.072819         34    3.630190           14     3.778956
..             …           …       …          …          …         …
215          153    2.383754         98    1.301191          133     1.119498
216          142    2.364854         16    1.286866          125     1.119498
217           24    2.351204         17    1.286866           98     1.115676
218          123    2.329081         41    1.286866           80     1.115676
219           80    2.329081        180    1.286866          169     1.115676

     gamma_1.01  score_1.01  gamma_4.49  score_4.49
0           204    8.045576          64    6.144808
1           112    6.489130          49    4.781466
2            63    6.239136         203    3.977557
3           212    6.033537         109    3.818612
4           208    5.982512         199    3.815482
..            …           …         …          …
215         195    2.392022         159    0.952010
216          16    2.392022          36    0.945501
217         207    2.366019         209    0.945501
218         134    2.366019          55    0.945501
219          33    2.363781          79    0.945501

[220 rows x 10 columns]
```

[38]:
```python
'''
Boxplot with spread of anomaly score in each bootstrap for each gamma.

Observation: Different Gamma values usually leads to different spreads in the␣
↪anomaly scores.
'''
plt.figure(figsize=(10,6))

outliers.filter(like='score').boxplot()
```

[38]: <Axes: >



Since we observe different range of scores for different values of gamma, how do we choose gamma?

[39]:
```
'''
Function to check how gradually increasing Gamma values affects anomaly scores␣
 ↪while bootstrapping.

Input Parameter:
    1. dataframe: Dataset
    2. n_diff_gammas: Number of gammas we want to experiment with (default␣
 ↪value 3).
    3. gamma_init: Lower bound of Gamma value (default value 0.1).
    4. increase_gamma_by: Amount by which previous gamma value is to be␣
 ↪increased in each step (default value 0.5).
Output:
    data: Dataframe containing anomalous points and their anomaly score for␣
 ↪each bootstrap for each gamma.


'''


def bootstrap_gamma(dataframe,
                    n_diff_gammas=3,
                    gamma_init=0.1,
                    increase_gamma_by=0.5):
```

```
    data = pd.DataFrame()
    gamma=gamma_init

    for i in range(n_diff_gammas):
        bootstrap_gamma_outliers = bootstrap(dataframe,1,gamma)
#         print(bootstrap_gamma_outliers)
        data=pd.concat([data, bootstrap_gamma_outliers], axis=1)

        gamma+=increase_gamma_by

    data.columns = data.columns.str.lstrip('1_')

    return data


outliers_increasing_gamma=bootstrap_gamma(df_log,
                                          n_diff_gammas=1000,
                                          gamma_init=0.01,
                                          increase_gamma_by=0.005)
```

[40]:
```
outliers_boxplot = outliers_increasing_gamma.filter(like='score')
outliers_boxplot.columns = outliers_boxplot.columns.str.lstrip('score_')

outliers_boxplot
```

[40]:
```
         0.01      0.01      0.02      0.03      0.03      0.04      0.04  \
0    9.613845  8.200121  9.690076  9.555392  9.417074  9.630238  7.552890
1    9.613845  8.114947  9.690076  8.226959  9.417074  9.630238  7.394224
2    8.262643  7.698791  8.300161  7.619961  8.238438  7.697700  6.835907
3    7.400310  7.129159  8.254530  7.136393  7.731285  7.240910  6.433346
4    6.809749  7.082247  7.640914  7.136393  7.483699  6.763026  6.340437
..        …         …         …         …         …         …         …
215  3.114807  3.302475  3.339715  3.056631  3.252527  3.093070  3.024815
216  3.077798  3.272786  3.324909  3.052706  3.252527  3.084988  3.024815
217  3.077798  3.261821  3.278925  3.047257  3.251848  3.043829  3.007199
218  3.068080  3.223535  3.244716  3.047257  3.226257  3.038146  2.963045
219  2.993510  3.221539  3.198591  3.018763  3.226257  2.999932  2.963045

         0.04      0.05      0.05  …      4.96      4.96      4.97  \
0    7.275582  8.170048  9.160283  …  6.452576  5.310878  5.890585
1    6.889271  7.456460  8.334971  …  4.829493  5.286954  5.756489
2    6.860687  7.417589  7.252700  …  4.359823  4.000406  4.386919
3    6.860687  7.021558  7.226965  …  3.811639  3.994027  4.207148
4    6.748351  7.021558  6.581224  …  3.810722  3.814771  4.143002
..        …         …         …  …         …         …         …
215  3.293699  3.074379  2.883225  …  0.943325  0.863878  0.939397
```

```
216  3.291412  3.072551  2.883225  …  0.943325  0.861504  0.860109
217  3.291412  3.054428  2.874158  …  0.942135  0.861504  0.860109
218  3.271539  3.054428  2.874158  …  0.942135  0.861504  0.860109
219  3.235941  2.993997  2.835736  …  0.942135  0.861504  0.860109

         4.97      4.98      4.98      4.99      4.99       5.0       5.0
0    5.284534  5.128057  4.851046  5.525356  4.814352  6.494195  6.119436
1    4.620039  4.619280  4.048009  4.545916  4.016858  5.713386  4.848896
2    4.137305  4.141984  4.025355  3.731829  3.806746  5.247541  4.271151
3    4.136896  3.433994  3.651109  3.464637  3.794671  4.909900  4.003782
4    3.484035  3.230835  3.461094  3.443736  3.445855  3.994031  3.768332
..        …         …         …         …         …         …         …
215  0.937268  0.938720  0.928553  0.898703  0.801280  0.858077  0.917696
216  0.937268  0.856392  0.928553  0.859041  0.801280  0.857831  0.857235
217  0.912982  0.856392  0.924690  0.859041  0.801280  0.857831  0.857235
218  0.912982  0.856392  0.924690  0.859041  0.801280  0.857831  0.857235
219  0.912982  0.856392  0.924690  0.859041  0.801280  0.857831  0.857235

[220 rows x 1000 columns]
```

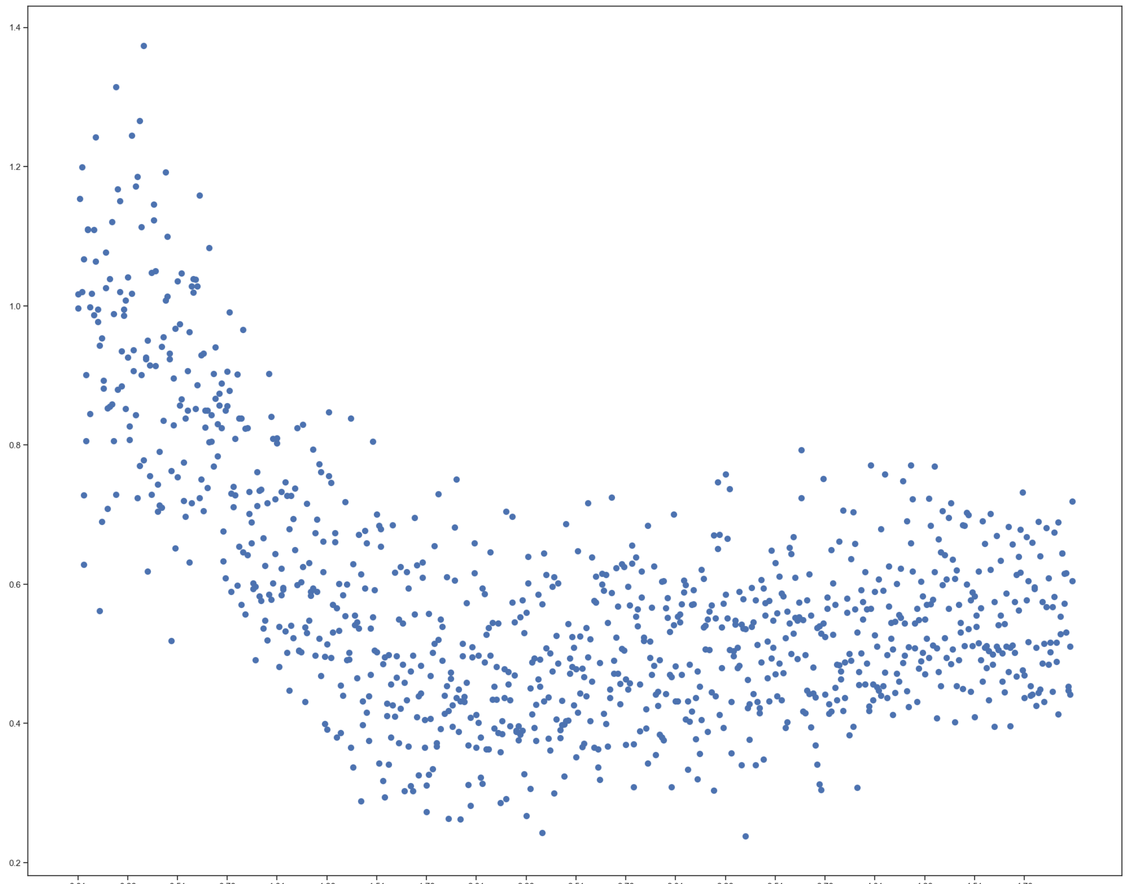
[41]:
```
variance = outliers_boxplot.var()
# variance
median = outliers_boxplot.median()
```

[42]:
```
variance
```

[42]:
```
0.01    1.017228
0.01    0.996442
0.02    1.153689
0.03    1.019886
0.03    1.199465
          …
4.98    0.453118
4.99    0.510753
4.99    0.441325
5.0     0.719093
5.0     0.605004
Length: 1000, dtype: float64
```

[43]:
```
import matplotlib.pyplot as plt

x1=[float(x) for x in variance.index]
y1=[float(x) for x in variance.values]

# Define the interval for x-axis values
x_interval = 50
```

```
plt.figure(figsize=(25,20))

# Create the scatter plot
plt.scatter(x1, y1,s=50)



plt.xticks(x1[::x_interval])

# Display the plot
plt.show()
```



We choose gamma to be 1.5!

We now repeat the softmin and see anomalous points!

[44]:
```
gamma=1.5
Df_SoftMin=dist_matrix.apply(softmin,args=(gamma,), axis=1)
Df_SoftMin
```

[44]:
```
0      2.922418
1      2.334077
2      2.929370
3      2.480738
4      2.737929
         …
435    3.585402
436    3.177422
437    3.203080
438    2.527323
439    4.363823
Length: 440, dtype: float64
```

[45]:
```python
Df_SoftMin.nlargest(10) #supposed to be most anomalous
```

[45]:
```
338    8.183188
75     7.518569
154    7.432491
95     6.684082
142    6.422580
128    5.861355
187    5.590151
66     5.585870
109    5.573200
183    5.560977
dtype: float64
```

[46]:
```python
Df_SoftMin.nsmallest(10) #supposed to be in a cluster
```

[46]:
```
246    2.231108
119    2.238386
389    2.239246
26     2.255172
374    2.255263
118    2.270828
162    2.271704
241    2.273313
415    2.279522
307    2.291392
dtype: float64
```

[47]:
```python
s=Df_SoftMin.nlargest(10).index #visualizing data points with high anomaly␣
 ↪scores
df.loc[s]
```

[47]:

	Fresh	Milk	Grocery	Frozen	Detergents_Paper	Delicassen
338	3	333	7021	15601	15	550

```
75   20398   1137       3   4407                    3         975
154    622     55     137     75                    7           8
95       3   2920    6252    440                  223         709
142  37036   7152    8253   2995                   20           3
128    140   8847    3823    142                 1062           3
187   2438   8002    9819   6269                 3459           3
66       9   1534    7417    175                 3468          27
109   1406  16729   28986    673                  836           3
183  36847  43950   20170  36534                  239       47943
```

2 Part 3

[48]:
```python
gamma = 1.5
```

[49]:
```python
def calculate_dist_matrix(dataframe):
    X=dataframe.to_numpy()
    D=scipy.spatial.distance.cdist(X,X, 'sqeuclidean')
    Dist=pd.DataFrame(D)
    return Dist

dist_matrix=calculate_dist_matrix(df_log)
dist_matrix
```

[49]:
```
             0          1          2          3          4          5  \
0     0.000000   4.948773   9.518191  19.065155  11.096636   1.710715
1     4.948773   0.000000   2.376777  10.652066   3.955369   1.855401
2     9.518191   2.376777   0.000000  11.772031   2.727466   5.265807
3    19.065155  10.652066  11.772031   0.000000   5.789492  10.641634
4    11.096636   3.955369   2.727466   5.789492   0.000000   5.816020
..         …          …          …          …          …          …
435  25.694910  14.807943  16.246037   9.364841   8.722427  17.035060
436  30.959383  26.637482  26.907633   7.168748  16.388185  21.313810
437   5.716774   6.262140   9.914189  29.027548  13.706102   8.434060
438  14.361973  13.950237  15.608919   5.273686  11.092780   8.663607
439  21.316007  32.581946  46.526389  36.146073  46.238995  22.552703

             6          7          8          9  …        430        431  \
0     2.707316   5.429980   2.553380   5.499223  …  10.210938  18.200315
1     4.730022   0.609692   4.379533   1.366806  …   9.774427   5.679174
2    11.159248   1.788044   9.843574   3.693466  …  12.604180   7.054751
3    12.657410   8.441374  11.611165  17.968674  …  11.939602   4.145827
4    10.453823   2.886920  10.610833   7.544686  …  13.183436   4.195675
..         …          …          …          …  …          …          …
435  24.220897  15.616737  22.837826  22.195014  …  16.066132   7.678746
436  26.360568  24.256605  24.103205  38.940770  …  22.189790  16.395520
437   8.604404   7.213507  10.881412   2.555446  …  21.471633  21.923008
```

```
438  12.535495  12.139833   8.963553  22.135972  …   6.489833  12.878374
439  16.535412  33.150398  14.118457  37.456572  …  23.012440  43.745821

           432        433        434        435        436        437  \
0     6.981967  15.220520   2.481199  25.694910  30.959383   5.716774
1    12.836134  11.299559   3.171338  14.807943  26.637482   6.262140
2    20.421522  13.333447   8.309340  16.246037  26.907633   9.914189
3    13.456209   7.866430   9.789773   9.364841   7.168748  29.027548
4    16.394306  13.737388   6.625387   8.722427  16.388185  13.706102
..         …          …          …          …          …          …
435  29.030486  19.911053  18.260381   0.000000  15.460136  31.893274
436  21.877194  13.178090  22.258862  15.460136   0.000000  51.290882
437  18.729204  31.024761   7.884380  31.893274  51.290882   0.000000
438  10.265641   3.969778  10.340456  14.711436   5.553480  31.928648
439  11.726081  21.758831  20.948783  55.714157  43.294098  41.878999

           438        439
0    14.361973  21.316007
1    13.950237  32.581946
2    15.608919  46.526389
3     5.273686  36.146073
4    11.092780  46.238995
..         …          …
435  14.711436  55.714157
436   5.553480  43.294098
437  31.928648  41.878999
438   0.000000  24.050205
439  24.050205   0.000000

[440 rows x 440 columns]
```

[50]:
```python
'''
Alternate function to calculate softmin
'''

def rowsum_except_diagonal_value(df: pd.DataFrame):
    row_sum_df = df.sum(axis=1)
    diag = np.diag(df)
    row_sum_df_ex_diag = row_sum_df - diag
    return row_sum_df_ex_diag

def softmin_2(dist_matrix:pd.DataFrame,gamma:float):
    gamma_exp_dist_matrix = dist_matrix.apply(lambda x: np.exp(-gamma * x))
    exp_row_sum = rowsum_except_diagonal_value(gamma_exp_dist_matrix)
    # softmin
    df_softmin = np.log(exp_row_sum / (len(exp_row_sum) - 1)) * (-1 / gamma)
    return df_softmin
```

```
df_softmin = softmin_2(dist_matrix,gamma)
df_softmin
```

[50]:
```
0      4.112392
1      2.753624
2      4.161826
3      3.259922
4      3.603235
         …
435    5.618648
436    4.829319
437    4.361749
438    3.374563
439    9.124005
Length: 440, dtype: float64
```

[51]:
```
'''
calculating the relevance of each datapoint:
the contribution of 439 other customers to one customer ,while calculating the␣
 ↪softmin,
it will be an NxN matrix where N is the number of datapoints/customers,
'''
def relevance_of_each_data_point(dist_matrix:pd.DataFrame,softmin:pd.
 ↪Series,gamma:float):
    gamma_exp_dist_matrix = dist_matrix.apply(lambda x: np.exp(-gamma * x))
    exp_row_sum = gamma_exp_dist_matrix.sum(axis=1)-1
    relevance_matrix_by_datapoint = gamma_exp_dist_matrix.
 ↪div(exp_row_sum,axis=0)
    relevance_matrix_by_datapoint = relevance_matrix_by_datapoint.
 ↪mul(softmin,axis = 0)
    df_array = relevance_matrix_by_datapoint.values
    np.fill_diagonal(df_array, 0)
    relevance_matrix_by_datapoint = pd.DataFrame(df_array,␣
 ↪columns=relevance_matrix_by_datapoint.columns)
    return relevance_matrix_by_datapoint

relevance_matrix_by_customer  =␣
 ↪relevance_of_each_data_point(dist_matrix=dist_matrix,
                                                         softmin=df_softmin,
                                                         gamma = gamma)



relevance_matrix_by_customer
```

[51]:
```
              0             1             2             3             4    \
0  0.000000e+00  2.671634e-03  2.818816e-06  1.701283e-12  2.641206e-07
1  2.330389e-04  0.000000e+00  1.103909e-02  4.488127e-08  1.034124e-03
```

2	3.072270e-06	1.379350e-01	0.000000e+00	1.045236e-07	8.151190e-02
3	3.754520e-13	1.135511e-07	2.116409e-08	0.000000e+00	1.670611e-04
4	1.078237e-07	4.839867e-03	3.053097e-02	3.090364e-04	0.000000e+00
..	…	…	…	…	…
435	1.068199e-15	1.320921e-08	1.527715e-09	4.642375e-05	1.216846e-04
436	1.045218e-19	6.833979e-17	4.557073e-17	3.291693e-04	3.247086e-10
437	1.301597e-03	5.743844e-04	2.399415e-06	8.489178e-19	8.126344e-09
438	5.346820e-10	9.915559e-10	8.237261e-11	4.452303e-04	7.207491e-08
439	2.373244e-10	1.087025e-17	8.958749e-27	5.181062e-20	1.378695e-26

	5	6	7	8	9 \
0	3.437024e-01	7.708246e-02	1.298071e-03	9.710369e-02	1.170013e-03
1	2.413125e-02	3.235429e-04	1.563458e-01	5.473370e-04	5.021935e-02
2	1.809839e-03	2.620634e-07	3.335802e-01	1.885782e-06	1.913940e-02
3	1.153419e-07	5.608237e-09	3.128444e-06	2.693978e-08	1.944679e-12
4	2.969804e-04	2.827938e-07	2.403619e-02	2.234534e-07	2.221279e-05
..	…	…	…	…	…
435	4.677782e-10	9.747337e-15	3.926401e-09	7.760250e-14	2.035311e-13
436	2.007871e-13	1.035302e-16	2.430392e-15	3.059199e-15	6.603872e-25
437	2.209650e-05	1.711411e-05	1.378613e-04	5.623617e-07	1.492442e-01
438	2.755808e-06	8.278268e-09	1.498615e-08	1.757323e-06	4.610923e-15
439	3.712847e-11	3.087601e-07	4.633684e-18	1.159115e-05	7.256126e-21

	…	430	431	432	433 \
0	…	9.972019e-07	6.225434e-12	1.265522e-04	5.436657e-10
1	…	1.674156e-07	7.791378e-05	1.695398e-09	1.699266e-08
2	…	2.999974e-08	1.236652e-04	2.424235e-13	1.004715e-08
3	…	1.646026e-08	1.966195e-03	1.692216e-09	7.410900e-06
4	…	4.712875e-09	3.375114e-03	3.815851e-11	2.053138e-09
..	…	…	…	…	…
435	…	2.000966e-09	5.822814e-04	7.173300e-18	6.258901e-12
436	…	5.396174e-14	3.211552e-10	8.624308e-14	4.005746e-08
437	…	7.097483e-14	3.606281e-14	4.341469e-12	4.244224e-20
438	…	7.183499e-05	4.949639e-09	2.492396e-07	3.147784e-03
439	…	1.863010e-11	5.802635e-25	4.193917e-04	1.221430e-10

	434	435	436	437	438 \
0	1.082075e-01	8.163519e-17	3.036547e-20	8.442449e-04	1.970723e-09
1	3.352148e-03	8.805491e-11	1.731801e-18	3.249722e-05	3.187855e-10
2	1.883456e-05	1.272507e-10	1.442951e-17	1.696250e-06	3.309066e-10
3	4.139284e-07	7.829665e-07	2.110425e-05	1.215166e-19	3.621533e-04
4	8.820077e-05	3.796410e-06	3.851048e-11	2.151788e-09	1.084492e-07
..	…	…	…	…	…
435	7.444141e-11	0.000000e+00	4.966047e-09	9.789928e-20	1.526671e-08
436	4.865072e-14	1.306360e-09	0.000000e+00	5.948675e-33	3.712551e-03
437	5.039734e-05	1.153487e-20	2.664410e-33	0.000000e+00	1.093877e-20
438	2.227816e-07	3.165488e-10	2.926277e-04	1.925000e-21	0.000000e+00

```
439  4.116869e-10  9.267271e-33  1.142607e-24  9.544452e-24  3.928001e-12

              439
0    5.814034e-14
1    2.322878e-22
2    2.392077e-30
3    2.801121e-24
4    1.378846e-30
..            …
435  2.970722e-35
436  9.635162e-28
437  3.604908e-27
438  2.610820e-16
439  0.000000e+00

[440 rows x 440 columns]
```

[52]:
```
'''
Summing over the relevances to check whether it adds upto the softmin
'''
relevance_matrix_by_customer_sum = relevance_matrix_by_customer.sum(axis=1)
relevance_matrix_by_customer_sum
```

[52]:
```
0      4.112392
1      2.753624
2      4.161826
3      3.259922
4      3.603235
         …
435    5.618648
436    4.829319
437    4.361749
438    3.374563
439    9.124005
Length: 440, dtype: float64
```

[53]:
```
"Check if there is a difference between softmin and relevance of each datapoint␣
 ↪"
max(round(relevance_matrix_by_customer_sum,3) - round(df_softmin,3))
```

[53]:
```
0.0
```

[54]:
```
'''
calculation of relevance by each feature,
i.e how much each feature contributes to each customer's relevance score
output will be a Nxd matrix where d is the number of features and N is the␣
 ↪number of data points
```

```
'''
relevance_per_customer_per_feature  = pd.DataFrame()
for feature in df_log.columns:
    feature_series = df_log[feature]
    for j in range(len(feature_series)):
        feature_squared = (feature_series - df_log.loc[j,feature])**2
        dist_j_k = dist_matrix.loc[:,j]
        feature_contri = feature_squared.div(dist_j_k)
        feature_contri = feature_contri.fillna(0)
        relevance_i_j = sum((relevance_matrix_by_customer.
 ↪iloc[j])*feature_contri)
        relevance_per_customer_per_feature.loc[j,feature] = relevance_i_j

relevance_per_customer_per_feature
```

[54]:

```
        Fresh      Milk   Grocery    Frozen  Detergents_Paper  Delicassen
0    0.904516  0.583854  0.250292  1.437025          0.468388    0.468318
1    0.644680  0.661569  0.185157  0.631423          0.350877    0.279917
2    0.589532  0.614826  0.175790  0.448208          0.342300    1.991170
3    0.464723  0.687186  0.493128  0.437054          0.460333    0.717498
4    0.472894  0.313277  0.237213  1.141064          0.595644    0.843142
..        ...       ...       ...       ...               ...         ...
435  0.202129  1.107492  1.623817  0.656590          1.101971    0.926649
436  0.600827  0.809939  1.203615  0.727913          1.038167    0.448858
437  0.324045  0.274384  0.377892  1.894288          0.638277    0.852863
438  0.708035  0.479280  0.290250  0.788470          0.299125    0.809403
439  1.141659  1.065617  1.690130  2.347667          0.499341    2.379591

[440 rows x 6 columns]
```

[55]:

```
"Check if there is a difference between softmin and relevance of each feature␣
 ↪contribution "
max(round(relevance_per_customer_per_feature.sum(axis=1),3) -␣
 ↪round(df_softmin,3))
```

[55]: 0.0

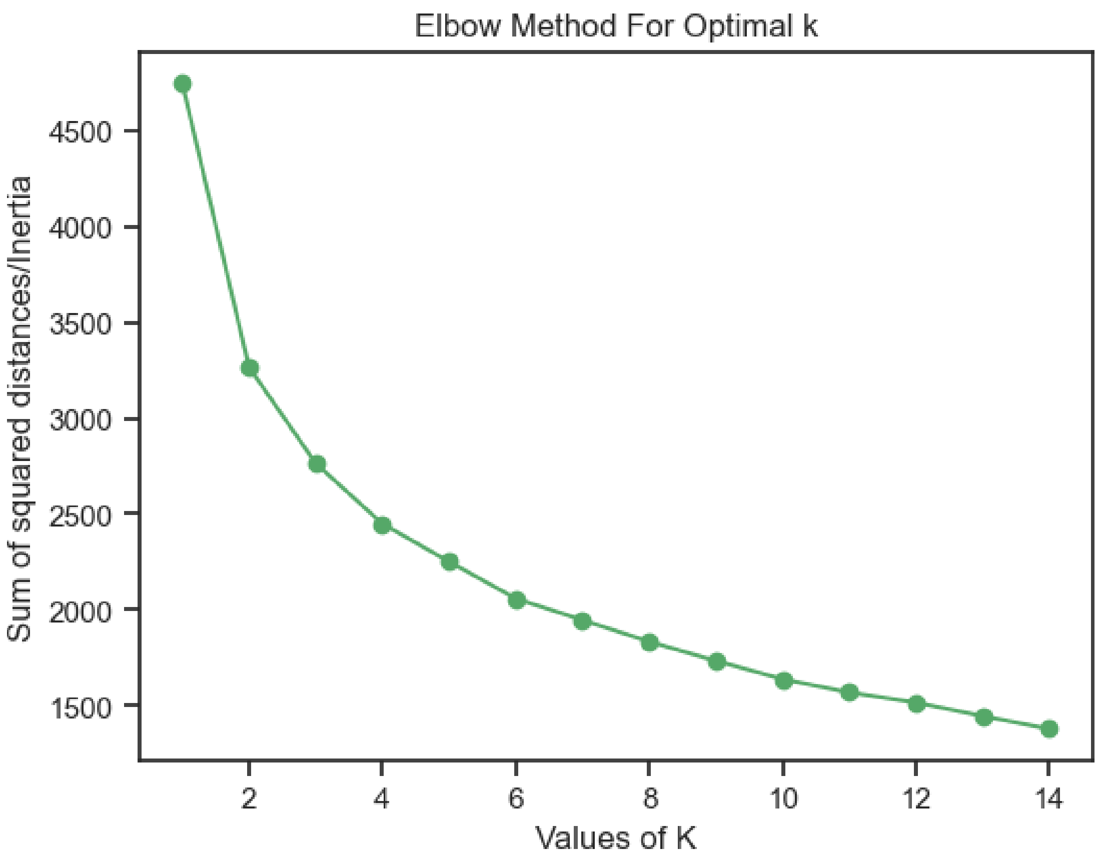
We have validated that the relevance by feature sums upto softminimum. This matrix can help us to determine which feature in each datapoint contributes to the anomaly score/softmin.

2.0.1 Part 4:

[56]:

```
'''
We apply k means to our data to our dataset to classify them in different␣
 ↪clusters.
```

```
Here, we experiment with different values of number of clusters, i.e., k and␣
↪choose the optimal k from the elbow plot.
'''

Sum_of_squared_distances = []
K = range(1,15)
for num_clusters in K :
        kmeans = KMeans(n_clusters=num_clusters,random_state=0)
        kmeans.fit(df_log)
        Sum_of_squared_distances.append(kmeans.inertia_)
plt.plot(K,Sum_of_squared_distances,'go-')
plt.xlabel('Values of K')
plt.ylabel('Sum of squared distances/Inertia')
plt.title('Elbow Method For Optimal k')
plt.show()
```



We choose 4 clusters to be optimal

[57]:
```
'''
```

```
Applying k-means with 4 clusters and displaying which cluster has how many␣
↪members.

'''

k=4
kmeans = KMeans(n_clusters=k, random_state=0)
kmeans.fit(df_log)
pred = kmeans.predict(df_log)

for i in range(0, k):
    x = len(pred[pred == i])
    print ('Cluster %d has %d members' % ((i + 1), x))
```

```
Cluster 1 has 144 members
Cluster 2 has 114 members
Cluster 3 has 121 members
Cluster 4 has 61 members
```

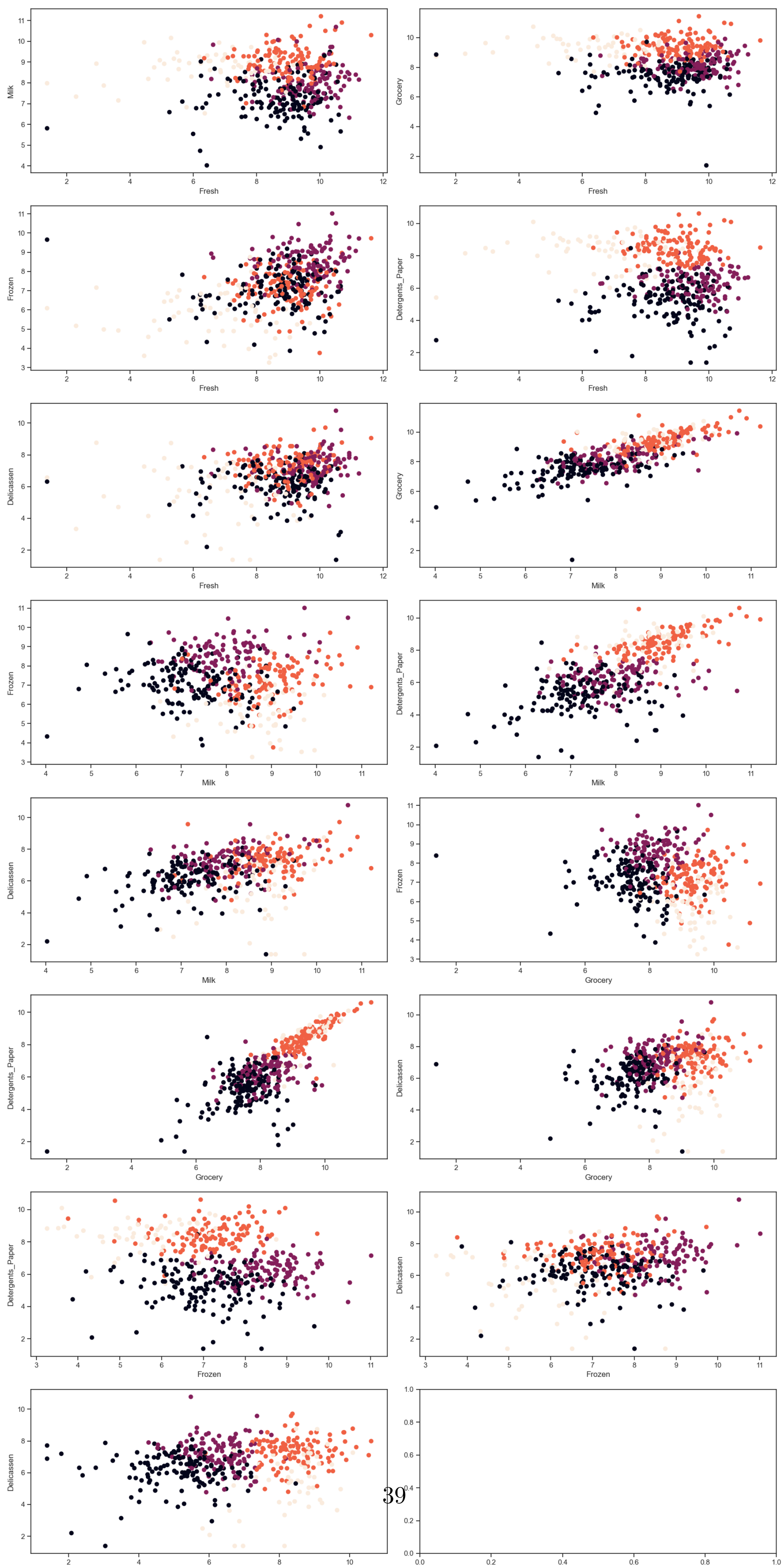
[58]:
```
df_log['cluster'] = kmeans.labels_
df_log
```

[58]:
```
         Fresh      Milk    Grocery    Frozen  Detergents_Paper  Delicassen  \
0     9.446992  9.175438   8.930891  5.370638          7.891705    7.199678
1     8.861917  9.191259   9.166284  7.474772          8.099858    7.482682
2     8.756840  9.083529   8.947026  7.785721          8.165364    8.967632
3     9.492960  7.087574   8.348064  8.764834          6.230481    7.489412
4    10.026413  8.596189   8.881697  8.272826          7.483244    8.553718
..         …         …          …         …                …           …
435  10.299037  9.396986   9.682092  9.483112          5.209486    7.698483
436  10.577172  7.266827   6.639876  8.414274          4.543295    7.760893
437   9.584108  9.647885  10.317053  6.082219          9.605216    7.532624
438   9.239025  7.591862   7.711101  6.946014          5.129899    7.661998
439   7.933080  7.437795   7.828436  4.189655          6.169611    3.970292

     cluster
0          2
1          2
2          2
3          1
4          1
..       …
435        1
436        1
437        2
438        0
439        0
```

```
[440 rows x 7 columns]
```

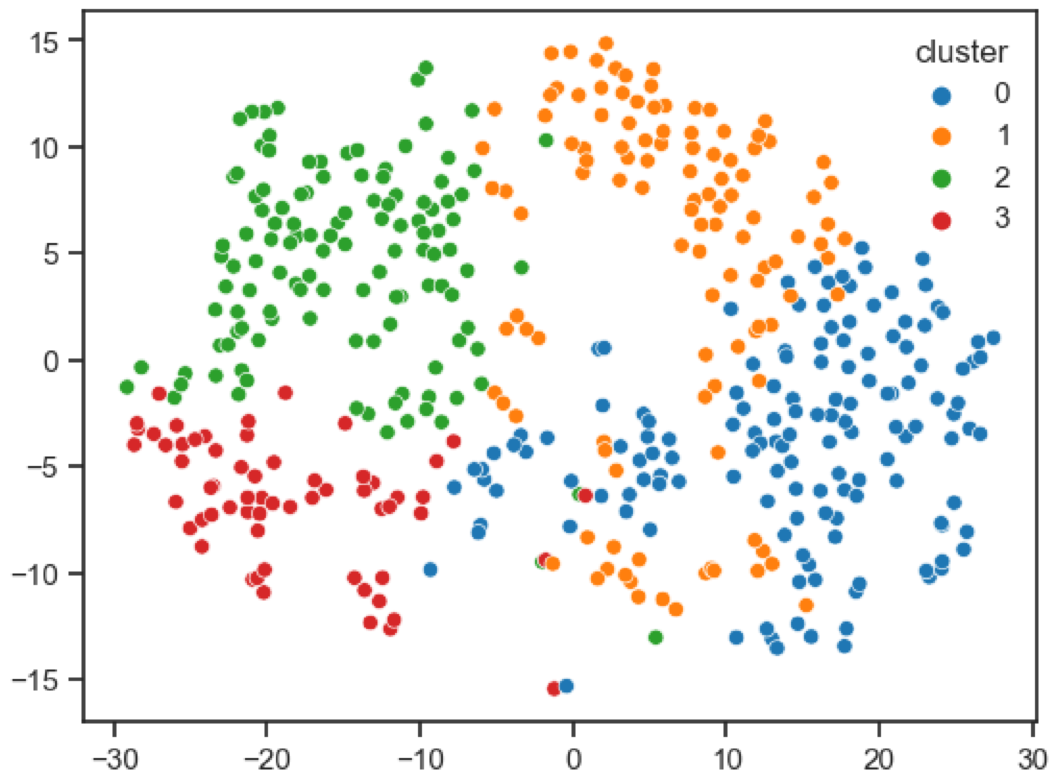
Plotting scatterplot of all possible combinations of columns categorized by their cluster membership

[59]:
```python
# Get all combinations of column pairs for x and y axes
col_pairs = [(x, y) for i, x in enumerate(df_log.columns) for j, y in␣
↪enumerate(df_log.columns) if i < j and "cluster" not in (x, y)]
# print(col_pairs)

# Determine the number of rows needed for the grid
num_rows = int(np.ceil(len(col_pairs) / 2))

# Create a grid of subplots with 2 columns and dynamic number of rows
fig, axs = plt.subplots(nrows=num_rows, ncols=2, figsize=(17, 34))

# Generate scatter plots for each combination of columns in x and y axes
for i, col_pair in enumerate(col_pairs):
    x_col, y_col = col_pair
    row_idx = i // 2
    col_idx = i % 2

    ax = axs[row_idx, col_idx]
#     print(row_idx, col_idx)
    ax.scatter(x=df_log[x_col], y=df_log[y_col], c=df_log['cluster'])
    ax.set_xlabel(x_col)
    ax.set_ylabel(y_col)


# Add a title for the whole plot
# plt.suptitle('Scatter plots of all column pairs')

plt.tight_layout()
plt.show()
```


Fresh
Milk
Grocery
Frozen
Detergents_Paper
Delicassen

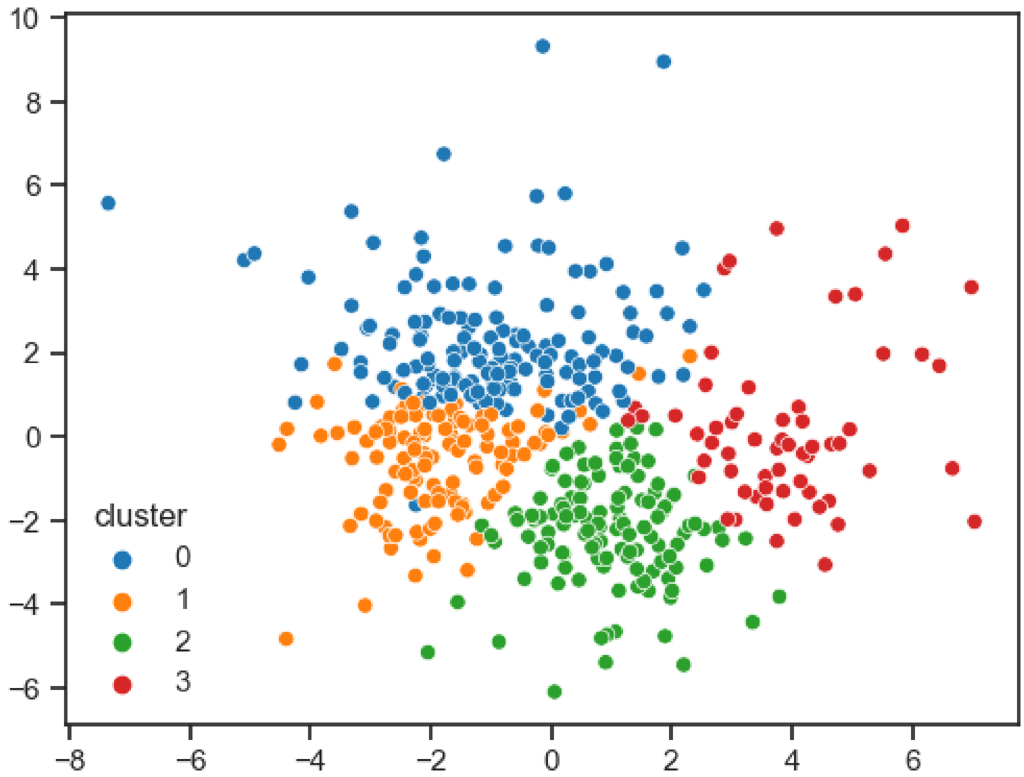
Visualizing K-Means performance in different methods of embedding, such as: T-SNE and MDS

[60]:
```
import warnings
warnings.filterwarnings('ignore')
import seaborn as sns
from sklearn.manifold import TSNE
tsne = TSNE(n_components=2, random_state=42)
embed = tsne.fit_transform(df_log.iloc[:,:-1])

sns.scatterplot(x=embed[:,0], y=embed[:,1], hue=df_log['cluster'],␣
 ↪palette='tab10')
plt.show()
```



[61]:
```
embedding = MDS(n_components=2)
df_mds = embedding.fit_transform(df_log.iloc[:,:-1])
sns.scatterplot(x=df_mds[:,0], y=df_mds[:,1], hue=df_log['cluster'],␣
 ↪palette='tab10')
```

[61]: <Axes: >



cluster
0
1
2
3
10
8
6
4
2
0
−2
−4
−6
−8
−6
−4
−2
0
2
4
6

[ ]: