Exercises for the course
Artifical Intelligence
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Exercise Sheet 5 (programming part)

In [1]:
```
import numpy, utils
```

Exercise 2: Inferences in a Bayesian Network (40 P)

In this exercise, we will consider the Bayesian network from the lecture, which can be used to infer the probability of a burglary from a few observables. The Bayesian network is shown below. Note that the variables indices are now between 0 and 4 (instead of 1 and 5) due to indexing starting at zero in Python.

The factors (matrices of conditional probability values) linking the different variables can be extracted from the module `utils`. They are already casted into 5-dimensional arrays in order to ease the subsequent computations.

In [2]:
```
P0   = utils.P0   # P(X0)
P1   = utils.P1   # P(X1)
P02  = utils.P02  # P(X2|X0)
P013 = utils.P013 # P(X3|X0,X1)
P34  = utils.P34  # P(X4|X3)
```

For example, $\Pr(X_3 = 1 | X_0 = 0, X_1 = 0)$, i.e. the probability of the door being damaged if there is no earthquake nor burglary can be obtained by usage of the function `numpy.take`, which selects indices in specific dimensions:

In [3]:
```
print('%.2f'%P013.take([0],0).take([0],1).take([1],3).item())
```

```
0.02
```

Reaction of Alarm to Burglary

We now would like to perform inferences in this Bayesian network. The first inference we consider is the probability that the alarm rings as a function of whether there is a burglary or not. This inference is achieved by the code below:

In [4]:
```
def A_given_B(val_burglary):

    Q = P0 * P1 * P02 * P013 * P34
    Q = Q.take([val_burglary],1)
    Q = Q.sum((0,2,3),keepdims=True)
    Q = Q/Q.sum()

    return Q
```

The following line executes the code above to known the probability that an alarm rings if a burglary doesn't take place, i.e. $\Pr(X_4 = 1|X_1 = 0)$, or if a burglary does take place, i.e. $\Pr(X_4 = 1|X_1 = 1)$:

In [5]:
```
print('%.2f'%A_given_B(0).take([1],4).item())
print('%.2f'%A_given_B(1).take([1],4).item())
```

```
0.04
0.97
```

Our computation indicates that the alarm rings with probability 0.04 if there is no burglary and 0.97 if there is one. Hence, according to our model, the alarm is quite responsive to the burglary event.

Inferring Burglary from News and Alarm

We now would like to infer the reverse, namely the probability of a burglary given an alarm. Furthermore, we also consider whether we have some earthquake news on the TV, in other words, we would like to implement a function that infers $P(X_1|X_2, X_4)$, using similar structures as the function `A_given_B` implemented above.

In [13]:
```
def B_given_NA(val_news,val_alarm):
    Q = P0 * P1 * P02 * P013 * P34
    Q = Q.take([val_alarm],4)
    Q = Q.take([val_news],2)
    Q = Q.sum((0, 3), keepdims=True)
    Q = Q/Q.sum() # normalize

    return Q
```

In [18]:
```
def B_given_A(val_alarm):
    Q = P0 * P1 * P02 * P013 * P34
    Q = Q.take([val_alarm],4)
    Q = Q.sum((0, 2, 3), keepdims=True)
    Q = Q/Q.sum() # normalize

    return Q

print('%.2f'%B_given_A(1).take([1],1).item())
```

```
0.53
```

Your function can be tested with the code below, which compute the probability of a burglary in presence of a news and an alarm.

In [14]:
```
print('%.2f'%B_given_NA(1,1).take([1],1).item())
```

```
0.09
```

Although the previous inference suggested that the alarm is responsive to burglary, the reverse does not appear to be true. The trigerring of the alarm (jointly with the TV news) only yields a probability of burglary of 0.09. Give an explanation to this paradox.

Firstly, the burglary has a low probability and the alarm can also be triggered by damage in the door caused by an earthquake. Because burglaries likely cause damage to the door, we have a high possibility of the alarm getting triggered during a burglary (the result from earlier). However, having news of an earthquake implies an earthquake has likely occurred, which in turn implies that the door is likely damaged. This means that the alarm is likely to be triggered even without a burglary. This is why the probability of a burglary given an alarm and news of an earthquake is low.