Airline Scheduling and Routing

ME 308 - Project Report

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Problem Solution

Problem tackled is such that given a connected graph with distances(a measure of time) between them, the demand across various cities and initial location of different airplanes, we maximize the profit of the airline and try to make sure that maximum passengers reach their desired location. The problem is built upon a solution which involved timesteps instead of time, wherein timestep is defined as a unit such that airplane reaches from one airport to another directly connected airport in this one timestep skip. Following is the optimization model for airplane routing and scheduling.

Objective & Constraints

$$\max \; W_{satisfy} \sum_{i \in P} pd_i \cdot (p_{i,pf_i,T_{max}} - 1) + W_{profit} \cdot profit$$

$$p_{i,ps_i,1} = 1 \qquad \forall i \in P \tag{1a}$$

$$\sum_{a \in N} p_{i,a,t} = 1 \qquad \forall i \in P, t \in 1, ..., T_{max} \tag{1b}$$

$$f_{i,fs_i,1} = 1 \qquad \forall i \in F - \{-1\} \tag{1c}$$

$$\sum_{a \in N} f_{i,a,t} = 1 \qquad \forall i \in F - \{-1\}, t \in 1, ..., T_{max} \tag{1d}$$

$$yp_{i,a,b,t} = p_{i,a,t} \cdot p_{i,b,t+1} \cdot c_{a,b} \qquad \forall i \in P, a, b \in N, t \in 1..T_{max} \tag{1e}$$

$$yf_{i,a,b,t} = f_{i,a,t} \cdot f_{i,b,t+1} \cdot c_{a,b} \qquad \forall i \in F - \{-1\}, a, b \in N, t \in 1..T_{max} \tag{1f}$$

$$\sum_{t \in t_1+1..t_2-1} p_{i,a,t_1} \cdot p_{i,b,t_2} \cdot p_{i,0,t} = t_2 - t_1 - 1 \implies t_2 - t_1 = d_{a,b} \qquad \forall i \in P, a, b \in N, t_1, t_2 \in 1..T_{max} \tag{1g}$$

$$\sum_{t \in t_1+1..t_2-1} f_{i,a,t_1} \cdot f_{i,b,t_2} \cdot f_{i,0,t} = t_2 - t_1 - 1 \implies t_2 - t_1 = d_{a,b} \qquad \forall i \in F - \{-1\}, a, b \in N, t_1, t_2 \in 1..T_{max} \tag{1h}$$

$$\sum_{a \in N - \{0\}} yp_{i,a,a,t} = \text{ticket}_{-1,i,t} \qquad \forall i \in P, t \in 1..T_{max} - 1 \tag{1i}$$

$$\sum_{j \in F} \text{ticket}_{j,i,t} = 1 \qquad \forall t \in 1..T_{max} - 1, i \in P \tag{1j}$$

$$0 \leq \sum_{i \in P} \text{ticket}_{j,i,t} \leq \text{Flight Capacity}; \qquad \forall j \in F - \{-1\}, t \in 1..T_{max} - 1 \tag{1k}$$

$$p_{i,0,t} = 1 \implies ticket_{j,i,t-1} = ticket_{j,i,t} \qquad \forall i \in P, j \in F - \{-1\}, t \in 2..T_{max} - 1) \tag{1l}$$

$$ticket_{j,i,t} = \sum_{a \in N, b \in N, \neg(a = b \in AN)} yp_{i,a,b,t} \cdot yf_{j,a,b,t} \qquad \forall i \in P, j \in F - \{-1\}, t \in 1..T_{max} - 1 \tag{1m}$$

Profit Definition

Profit is defined as the revenue minus the cost to company. Revenue is calculated as ticket cost for the completed trips with subtraction of the compensation we give to passengers which we have to leave as we wont be able to take them to their destination. Cost to company is the operating cost for an airplane per hour times the number of hours each plane is travelling across nodes.

Terminology

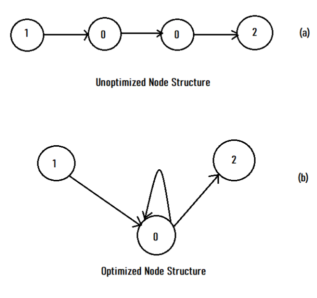
P is set of passenger IDs. **F** is set of flight IDs which includes flight ID *-1*. T_{max} is the maximum time in hours we wish to optimise for, generally set for a day hence 24. **N** is the set of all nodes in graph. $d_{a,b}$ is the distance between node a and b. $c_{a,b}$ is binary parameter with value 1 when node a and b are connected. Flight Capacity is the maximum passengers each flight is allowed to carry. $W_{satisfy}$ and W_{profit} are hyperparameters to set according to the weightage we wish to give to profit maximization vs customer satisfaction. pd_i is floating point value between 0 and 1 to measure the desire of a passenger to reach his destination as a businessman might be more willing to reach his destination than a student on a trip. $p_{i,a,t}$ is binary variable, 1 when passenger with ID i, at node a at time t. $f_{i,a,t}$ is binary variable, 1 when flight with ID i, at node a at time t. $yp_{i,a,bt}$ is binary variable, 1 when passenger with ID i, at node a at time t and at node b at time $t+1$. $yf_{i,a,bt}$ is binary variable, 1 when flight with ID i, at node a at time t and at node b at time $t+1$. $\text{ticket}_{j,i,t}$ binary variable, 1 if flight ID j carrying passenger ID i from time t to time $t+1$.

Explanation

(1a), (1c) states that start location of flight and passenger are fixed. *(1b), (1d)* constitute that the passenger or flight is at at most 1 node at given time instant. *(1e), (1f)* are not constraints, rather definition of the y variable which is 1 when a given connected transition occurs. *(1g), (1h)* are constraints to describe when an airline or passenger should land given that it is currently at node *0* or flying. We describe that landing can occur only if the time distance between previous airport node and current node is equal to distance between the previous airport node and the current airport it is landing at. This is one constraint which allows us to use the optimized node structure. *(1i)* describes that if a person stays at an airport node, it is associated with flight ID -1. *(1j)* describes that at all time steps each passenger is associated with a ticket or its motion wont be feasible. *(1k)* limits the number of tickets supplied by a flight at any given time. *(1l)* is the constraint that as long as a passenger is flying he has the same flight ID carrying him as flight change not feasible at intermediate nodes. *(1m)* defines when a ticket is valid, which is when there is at-least one flight flying along a node change along which the passenger given in the ticket is flying.

Key Ideas

Key Idea 1 The concept of a node *0* (node i corresponds to city i), node i represents nodes which serve as an airport, however node *0* corresponds to an intermediate node along a direct edge. Now connectedness between nodes is valid when nodes are 1 hour apart (1 unit distance apart). This formulation is designed such that the side of the problem still remains very small that is instead of placing this node at every point on the edge (1 hour apart), problem is designed such that a unique node *0* exist in the graph. For reference see figure.



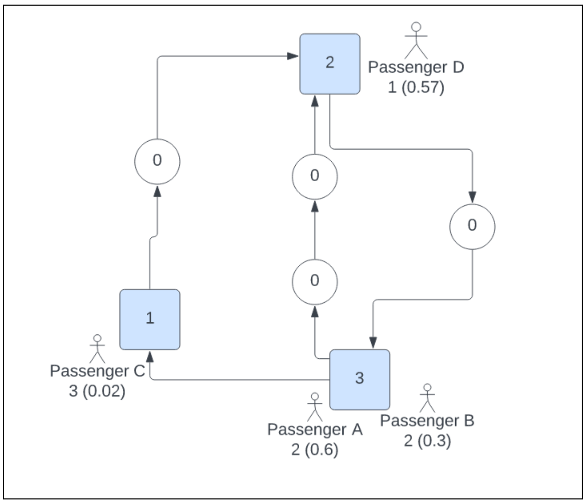


Figure 1: Input Graph to the model

Time	A	B	C	D	Flight 1	Flight 2
1	3	3	1	2	2	1
2	3	3	1	0	0	1
3	3	3	1	3	3	1
4	1	3	1	3	1	1
5	1	3	1	3	1	1
6	1	3	1	3	1	1
7	1	3	1	3	1	1
8	0	3	1	3	1	0
9	2	3	1	3	1	2
10	2	3	1	3	1	0
11	2	3	1	3	1	3
12	2	3	1	1	1	1

Figure 2: Solution Output from Model

Key Idea 2 Concept of a flight with flight ID *-1*. Flight ID *-1* is the flight responsible for transporting a passenger from one node to itself as long as it is an airport node and not an intermediate one. Though this might seem a trivial addition to the problem however this significantly simplifies the way constraints are written by making them more generalized instead of handling specific cases too much which slows the optimization process.
Key Idea 3 Concept of ticket variable. I have designed passenger constraints, and flight constraints such that their initial location is restricted and connectedness is ensured between nodes at consecutive time steps. The tying variable between flight and passenger would be the ticket variable (ticket[j, i, t] = 1 if flight ID j carries passenger i at time t. Of-course this would hold valid when the passenger is actually moving between two non equal nodes. This method of defining the constraints led to a lesser error prone approach and a more time efficient debugging method.

Results

What did you find? – a section which describes the data that was collected and the results of any statistical tests that were performed. It may also be prefaced by a description of the analysis procedure that was used. If there were multiple experiments, then each experiment may require a separate Results section.

Flight Scheduling and Routing

Most of the commercial airplanes in India are Airbus A320 which has the passenger capacity of around 150 passengers. Flight has an operating cost of 164000 rupees per hour which boils down to 1000 rupees per passenger per hour hence this is set as our flight per hour cost, which gets scaled according to passenger capacity. Cost of ticker per hour is set at 20 rupees as generally for a two hour flight, cost of ticket is 6000 rupees over a 150 passenger capacity airplane. Ticket compensation is the compensation cost to company when a willing passenger is not able to reach his destination with the given set of flights. This value is set to 70 by scaling a compensation of 10000 per passenger to a flights of size 150. Under these conditions our algorithms gives the path each of our airplane (with unique flight ID) should take along with which passengers it should pick up along the way. Over a period of 24 hours, we calculate the profit earned by the airline.
Following is the input graph to the solver and the output flight & passenger flow chart. Under each passenger in graph is his ID, his desired node destination and in decimal is his desire or need to reach his final destination. The problem is solved for 12 hours. On the right is location vs time chart wherein you see how each passengers position changes with time.

Discussion

Indeed as it turns out, the way flights operate, the do it on a loss as is seen from our simulation. Under almost all cases the profit turns out to be negative as is expected from an airline. However our models fails to analyse the case of overbooking which is a maneuver which airlines often undertake to increase revenue. Of course overbooking comes at the cost of compensating the cost of cancelling the ticket of some passenger which is more willing to let go off his ticket than someone who really needs his ticket, like a businessman.
