

1 Sum Of Two Angles

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

2 Difference Of Two Angles

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta}$$

3 Products To Sums

$$2 \sin \alpha \cdot \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$

$$2 \cos \alpha \cdot \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$$

$$2 \cos \alpha \cdot \cos \beta = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$

$$2 \sin \alpha \cdot \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$

4 Products Of Sums Of Two Angles

$$\sin(\alpha + \beta) \cdot \sin(\alpha - \beta) = \sin^2 \alpha - \sin^2 \beta$$

$$\cos(\alpha + \beta) \cdot \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$$

$$\tan(\alpha + \beta) \cdot \tan(\alpha - \beta) = \frac{\tan^2 \alpha - \tan^2 \beta}{1 - \tan^2 \alpha \cdot \tan^2 \beta}$$

5 Sums or Differences to Products

$$\sin C + \sin D = 2 \sin \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$$

$$\sin C - \sin D = 2 \cos \left(\frac{C + D}{2} \right) \sin \left(\frac{C - D}{2} \right)$$

$$\cos C + \cos D = 2 \cos \left(\frac{C + D}{2} \right) \cos \left(\frac{C - D}{2} \right)$$

$$\cos C - \cos D = 2 \sin \left(\frac{C + D}{2} \right) \sin \left(\frac{D - C}{2} \right)$$

6 Double Angles

$$\sin 2\theta = 2 \sin \theta \cdot \cos \theta$$

$$= \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 1 - 2 \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

7 Triple Angles

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

8 Miscellaneous

$$\sin \theta \cdot \sin(60 - \theta) \cdot \sin(60 + \theta) = \frac{\sin 3\theta}{4}$$

$$\cos \theta \cdot \cos(60 - \theta) \cdot \cos(60 + \theta) = \frac{\cos 3\theta}{4}$$

$$\cos^3 \theta + \cos^3(120 + \theta) + \cos^3(\theta - 120) = \frac{3}{4} \cdot \cos 3\theta$$

$$\tan \theta \cdot \tan(60 - \theta) \cdot \tan(60 + \theta) = \tan 3\theta$$

9 Sum Of Three Angles

$$\sin(\alpha + \beta + \gamma) = \sin \alpha \cdot \cos \beta \cdot \cos \gamma + \sin \beta \cdot \cos \alpha \cdot \cos \gamma + \sin \gamma \cdot \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \cdot \sin \gamma$$

$$\cos(\alpha + \beta + \gamma) = \cos \alpha \cdot \cos \beta \cdot \cos \gamma - \cos \alpha \cdot \sin \beta \cdot \sin \gamma - \cos \beta \cdot \sin \alpha \cdot \sin \gamma - \cos \gamma \cdot \sin \alpha \cdot \sin \beta$$

$$\tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \cdot \tan \beta \cdot \tan \gamma}{1 - \tan \alpha \cdot \tan \beta - \tan \beta \cdot \tan \gamma - \tan \alpha \cdot \tan \gamma}$$

10 AP, GP in Trigonometrical Ratios

$$\sin(A) + \sin(A + D) + \dots + \sin(A + (n - 1)D) = \frac{\sin(A + \frac{(n-1)D}{2}) \cdot \sin(\frac{nD}{2})}{\sin(\frac{D}{2})}$$

$$\cos(A) + \cos(A + D) + \dots + \cos(A + (n - 1)D) = \frac{\cos(A + \frac{(n-1)D}{2}) \cdot \sin(\frac{nD}{2})}{\sin(\frac{D}{2})}$$

$$\cos \theta \cdot \cos 2\theta \cdot \cos 2^2\theta \dots \cos 2^{(n-1)}\theta = \frac{\sin(2^n\theta)}{2^n \sin \theta}$$

11 Trigonometrical Equations

11.1 Value is 0

$$\sin \theta = 0, \theta = n\pi$$

$$\cos \theta = 0, \theta = (2n + 1)\frac{\pi}{2}$$

$$\tan \theta = 0, \theta = n\pi$$

11.2 Value is 1

$$\sin \theta = 1, \theta = (4n + 1)\frac{\pi}{2}$$

$$\cos \theta = 1, \theta = 2n\pi$$

11.3 Value is -1

$$\sin \theta = -1, \theta = (4n - 1)\frac{\pi}{2}$$

$$\cos \theta = -1, \theta = (2n + 1)\pi$$

11.4 $\theta = \alpha$

$$\sin \theta = \sin \alpha, \theta = n\pi + (-1)^n \alpha$$

$$\cos \theta = \cos \alpha, \theta = 2n\pi \pm \alpha$$

$$\tan \theta = \tan \alpha, \theta = n\pi + \alpha$$

$$\sin^2 \theta = \sin^2 \alpha, \cos^2 \theta = \cos^2 \alpha, \tan^2 \theta = \tan^2 \alpha, \theta = n\pi \pm \alpha$$