Nam Tu - 153076622 - Locality-Sensitive Hashing (LSH)

Task 2.1

1. Calculate the signature matrix by mapping hash values according to where a "1" appears in the original matrix. Put them all into a dataframe afterwards for easier visualization

In [1]:
```
import pandas as pd
import numpy as np

def h1(x):
    return (2*x+ 1)%6

def h2(x):
    return (3*x+2)%6

def h3(x):
    return (5*x+2)%6

#create the matrix
M = np.array([
    [1, 1, 0, 1],
    [0, 1, 0, 0],
    [0, 0, 0, 1],
    [1, 0, 1, 0],
    [0, 1, 1, 1],
    [0, 0, 1, 0]
])

#calculating the signature matrix
n_rows = M.shape[0]
columns = ["S1","S2","S3","S4"]
rows = ["h1","h2","h3"]
hash_functions = [h1,h2,h3]

signature_data = {}

for name, h in zip(rows, hash_functions):
    signature_row = []
    for j in range(len(columns)):
        rows_with_1 = np.where(M[:, j] == 1)[0]
        min_hash = min(h(r) for r in rows_with_1)
        signature_row.append(min_hash)
    signature_data[name] = signature_row

signature_df = pd.DataFrame(signature_data, index=columns).T

print("Signature Matrix:")
print(signature_df)
```

```
Signature Matrix:
    S1  S2  S3  S4
h1   1   1   1   1
h2   2   2   2   2
h3   2   1   3   0
```

2. Check if true permutation by checking if there are no repeats of the same value after calculating the hash value for all rows.

In [2]:
```python
print("Permutation check:")
for name, h in zip(rows, hash_functions):
    values = [h(x) for x in range(n_rows)]
    is_perm = len(set(values)) == n_rows
    print(f"{name}: {values} -> Permutation? {is_perm}")
print()
```

```
Permutation check:
h1: [1, 3, 5, 1, 3, 5] -> Permutation? False
h2: [2, 5, 2, 5, 2, 5] -> Permutation? False
h3: [2, 1, 0, 5, 4, 3] -> Permutation? True
```

3. Write functions for both true and estimated Jaccard Similarities, and then print them pair-wise. As per their names, true Jaccard Similarity should reflect the similarites better, and this can be seen clearly. The similarities after hashing are almost all the same.

In [3]:
```python
#Calculate true Jaccard similarity
def jaccard(col1, col2):
    set1 = set(np.where(M[:, col1] == 1)[0])
    set2 = set(np.where(M[:, col2] == 1)[0])
    return len(set1 & set2) / len(set1 | set2)

#Calculate estimated Jaccard similarity
def estimated_jaccard(c1, c2):
    matches = (signature_df.iloc[:, c1] == signature_df.iloc[:, c2]).sum()
    return matches / len(signature_df)

print("Pairwise Similarities:\n")
for c1 in range(len(columns)):
    for c2 in range(c1 + 1, len(columns)):

        true_sim = jaccard(c1, c2)
        est_sim = estimated_jaccard(c1, c2)

        print(f"{columns[c1]} vs {columns[c2]}")
        print(f"  True Jaccard      = {true_sim:.3f}")
        print(f"  Estimated Jaccard = {est_sim:.3f}\n")
```

```
Pairwise Similarities:

S1 vs S2
  True Jaccard      = 0.250
  Estimated Jaccard = 0.667

S1 vs S3
  True Jaccard      = 0.250
  Estimated Jaccard = 0.667

S1 vs S4
  True Jaccard      = 0.250
  Estimated Jaccard = 0.667

S2 vs S3
  True Jaccard      = 0.200
  Estimated Jaccard = 0.667

S2 vs S4
  True Jaccard      = 0.500
  Estimated Jaccard = 0.667

S3 vs S4
  True Jaccard      = 0.200
  Estimated Jaccard = 0.667
```

Task 2.2

1. Plotting the curves

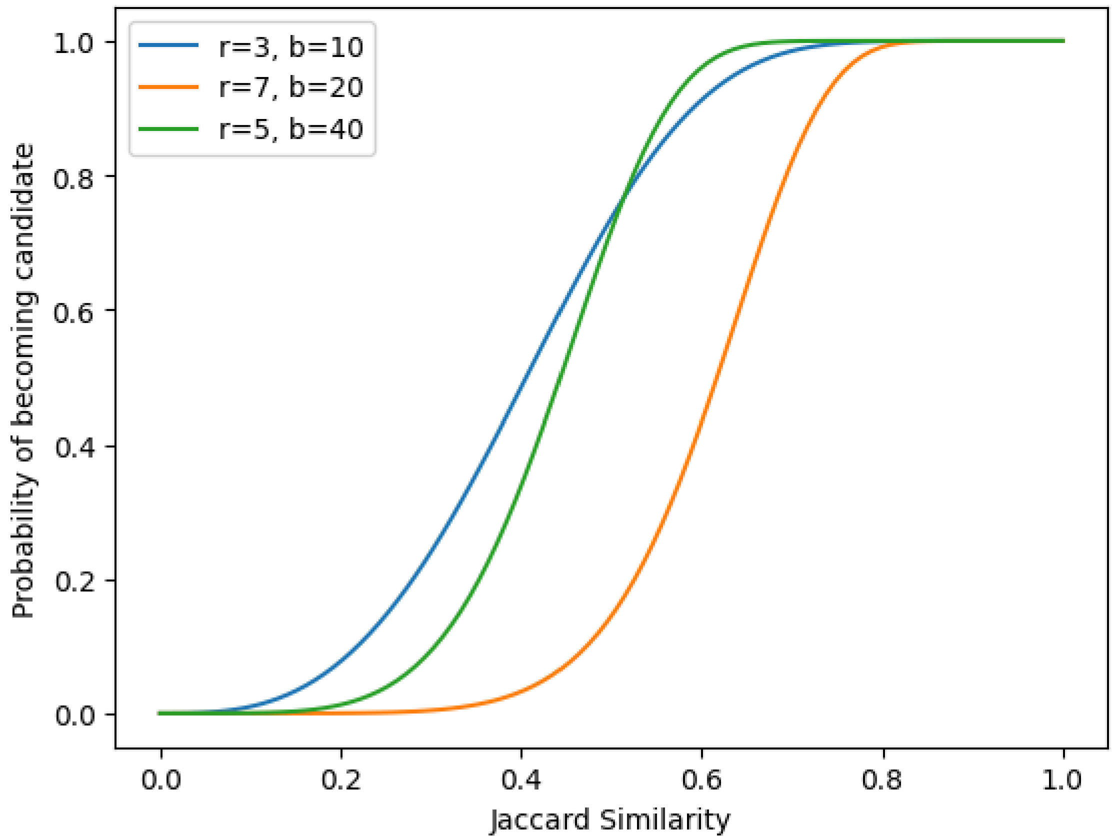
In [4]:

```
import matplotlib.pyplot as plt

rvals = [3,7,5]
bvals = [10,20,40]
s = np.linspace(0,1,100)

for r,b in zip(rvals,bvals):
    P = 1-(1-s**r)**b
    plt.plot(s, P,label=f"r={r}, b={b}")


plt.xlabel('Jaccard Similarity')
plt.ylabel('Probability of becoming candidate')
plt.legend()
plt.show()
```



2. Finding the values where P = 0.5 by finding the index of where P crosses the 0.5 threshold. Also calculate the estimate for comparison.

The results are extremely similar, validating the fact that it is an estimate.

In [5]:
```
# Find value of Jaccard similarity for which the probability of becoming a candi
print("Values of Jaccard similarity (s) where P ≈ 0.5 for each (r, b):")
for r, b in zip(rvals, bvals):
    P = 1 - (1 - s**r)**b
    est = (1/b)**(1/r)
    # Find index where P crosses 0.5
    idx = np.argmin(np.abs(P - 0.5))
    print(f"  r={r}, b={b}: s ≈ {s[idx]:.3f}, estimate ≈ {est:.3f}")
```

```
Values of Jaccard similarity (s) where P ≈ 0.5 for each (r, b):
  r=3, b=10: s ≈ 0.404, estimate ≈ 0.464
  r=7, b=20: s ≈ 0.616, estimate ≈ 0.652
  r=5, b=40: s ≈ 0.444, estimate ≈ 0.478
```

Task 2.3

Task 2.3.1 Shingling

According to the book, picking a k too small would make almost every document to seem similar to each other. "k

should be picked large enough that the probability of any given shingle appearing in any given document is low"

With the results of k = 3, one can see how many types of different words a single shingle can form, therefore not suitable for detecting similar documents

A good rule of thumb is k = 5 for emails, k = 9 for large documents

In [23]:
```
import pyspark
import re

sc = pyspark.SparkContext("local", "Similarity")

def make_shingles(text, shingle_len):
    text = text.lower()
    text = re.sub(r'[^a-z]+', '',text )
    for i in range( 0, len(text)-shingle_len-1 ):
        yield text[i:i+shingle_len]

shingle_length = 6

files = sc.wholeTextFiles('./E2_Data/*.txt')
files1 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files1.collect()

sample = files1.take(1)

for filename, shingles in sample:
    print("Shingle length 6:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 9

files2 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files2.collect()
sample = files2.take(1)

for filename, shingles in sample:
    print("\nShingle length 9:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 3

files3 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files3.collect()
sample = files3.take(1)

for filename, shingles in sample:
    print("\nShingle length 3:")
    print(filename)
    print(shingles[:5], "...")
```

```
Shingle length 6:
bank.txt
['abanki', 'bankis', 'ankisa', 'nkisaf', 'kisafi'] ...

Shingle length 9:
bank.txt
['abankisaf', 'bankisafi', 'ankisafin', 'nkisafina', 'kisafinan'] ...

Shingle length 3:
bank.txt
['aba', 'ban', 'ank', 'nki', 'kis'] ...
```

Test with preserving white spaces

It can be seen that preserving white space more than likely will cause documents to become even more likely, because every documents will have spaces in them (mostly), and taking white spaces into consideration will mean that the white spaces (and not the content alone) will skew the results.

In [24]:
```
def make_shingles(text, shingle_len):
    text = text.lower()
    text = re.sub(r'[^a-z]+\s+', '',text )
    for i in range( 0, len(text)-shingle_len-1 ):
        yield text[i:i+shingle_len]

shingle_length = 6

files = sc.wholeTextFiles('./E2_Data/*.txt')
files1 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files1.collect()

sample = files1.take(1)

for filename, shingles in sample:
    print("Shingle length 6:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 9

files2 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files2.collect()
sample = files2.take(1)

for filename, shingles in sample:
    print("\nShingle length 9:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 3

files3 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files3.collect()
sample = files3.take(1)
```

```
for filename, shingles in sample:
    print("\nShingle length 3:")
    print(filename)
    print(shingles[:5], "...")
```

```
Shingle length 6:
bank.txt
['a bank', ' bank ', 'bank i', 'ank is', 'nk is '] ...

Shingle length 9:
bank.txt
['a bank is', ' bank is ', 'bank is a', 'ank is a ', 'nk is a f'] ...

Shingle length 3:
bank.txt
['a b', ' ba', 'ban', 'ank', 'nk '] ...
```

Task 2.3.2 Calculating hashes

The code is hashing the usual shingles in ASCII letters into 32-bit integers hashes. Hashing compresses the data when the resulting hashes still has the same information but now in easier-to-work-with form (e.g. turning strings into 32-bit integers).

In [25]:

```
from numpy import uint32
from numpy import uint64
import random
import numpy

def generate_random_hash_params_A_P():
    A = random.getrandbits(64)
    P = random.getrandbits(64)
    while A>=P:
        A = random.getrandbits(64)
        P = random.getrandbits(64)
    return A , P

Ashingle, Pshingle = generate_random_hash_params_A_P()

def string_hash(shingle,slen,A,P):
    tmp = uint64(ord(shingle[0]))
    for i in range(1, slen):
        tmp = (tmp*uint64(A) + uint64(ord(shingle[i]))) % uint64(P)
    return uint32(tmp&uint64(0xFFFFFFFF))

def make_shingles(text, shingle_len):
    text = text.lower()
    text = re.sub(r'[^a-z]+', '',text )
    for i in range( 0, len(text)-shingle_len-1 ):
        yield string_hash(text[i:i+shingle_len],shingle_len, Ashingle, Pshingle)

shingle_length = 6

files = sc.wholeTextFiles('./E2_Data/*.txt')
files1 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files1.collect()
```

```
sample = files1.take(1)

for filename, shingles in sample:
    print("Shingle length 6:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 9

files2 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files2.collect()
sample = files2.take(1)

for filename, shingles in sample:
    print("\nShingle length 9:")
    print(filename)
    print(shingles[:5], "...")

shingle_length = 3

files3 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files3.collect()
sample = files3.take(1)

for filename, shingles in sample:
    print("\nShingle length 3:")
    print(filename)
    print(shingles[:5], "...")



sc.stop()
```

```
Shingle length 6:
bank.txt
[np.uint32(436660260), np.uint32(1596719775), np.uint32(2581751773), np.uint32(14
59804333), np.uint32(2913866958)] ...

Shingle length 9:
bank.txt
[np.uint32(1077587182), np.uint32(3796041808), np.uint32(1825871139), np.uint32(9
89618755), np.uint32(1815414769)] ...

Shingle length 3:
bank.txt
[np.uint32(625486320), np.uint32(1919798747), np.uint32(2849391294), np.uint32(13
90126676), np.uint32(2233985998)] ...
```

Task 2.3.3

The code produces the minhash signature matrix for every document. The documents now are each represented by 15 minhash numbers. (Printed results were way too long, limited to 5 files only and their 5 signature values)

In [31]:
```
from numpy import uint32
from numpy import uint64
```

```
import random
import numpy

import pyspark
import re

sc = pyspark.SparkContext("local", "Similarity")


def generate_random_hash_params_A_P():
    A = random.getrandbits(64)
    P = random.getrandbits(64)
    while A>=P:
        A = random.getrandbits(64)
        P = random.getrandbits(64)
    return A , P

Ashingle, Pshingle = generate_random_hash_params_A_P()

def string_hash(shingle,slen,A,P):
    tmp = uint64(ord(shingle[0]))
    for i in range(1, slen):
        tmp = (tmp*uint64(A) + uint64(ord(shingle[i]))) % uint64(P)
    return uint32(tmp&uint64(0xFFFFFFFF))

def make_shingles(text, shingle_len):
    text = text.lower()
    text = re.sub(r'[^a-z]+', '',text )
    for i in range( 0, len(text)-shingle_len-1 ):
        yield string_hash(text[i:i+shingle_len],shingle_len, Ashingle, Pshingle)

shingle_length = 6

files = sc.wholeTextFiles('./E2_Data/*.txt')
files1 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files1.collect()

def uint32_hash(stringhashes,A):
    for i in range(0, len(stringhashes)):
        stringhashes[i] = uint32((uint64(A)*uint64(stringhashes[i])) >> uint64(3
    return stringhashes

def min_hash(hashes,minhash_A):
    minhashes = []
    for i in range(0,len(minhash_A)):
        minhashes.append(min(uint32_hash(hashes,minhash_A[i])))
    return minhashes


num_minhashes = 15
minhash_A = []
for i in range(0,num_minhashes):
    minhash_A.append( random.getrandbits(64) )


minhashes1 = files1.map(lambda p: (p[0],min_hash(p[1],minhash_A)))
minhashes1.cache() #cache result for performance


sample = minhashes1.take(5)
```

```
for filename, signature in sample:
    print("File:", filename)
    print("First 5 minhash values:", signature[:5], "...")
    print()
```

```
File: bank.txt
First 5 minhash values: [np.uint32(53845), np.uint32(125334), np.uint32(94658), n
p.uint32(490632), np.uint32(150246)] ...

File: cat.txt
First 5 minhash values: [np.uint32(27048), np.uint32(103751), np.uint32(24815), n
p.uint32(54657), np.uint32(150246)] ...

File: catfood.txt
First 5 minhash values: [np.uint32(53845), np.uint32(21451), np.uint32(36572), n
p.uint32(474261), np.uint32(428511)] ...

File: hadoop.txt
First 5 minhash values: [np.uint32(217214), np.uint32(125334), np.uint32(178037),
np.uint32(347545), np.uint32(150246)] ...

File: hares.txt
First 5 minhash values: [np.uint32(53845), np.uint32(1054949), np.uint32(179206),
np.uint32(88730), np.uint32(150246)] ...
```

Task 2.3.4 Jaccard Similarity

Using the given formula, the corresponding number of minhashes for each error rate is:

0.3: 11;

0.2: 25;

0.1: 100;

0.05: 400;

Looking at the pairs, the accuracies seem to "swing" alot going from the first to third number. While a pair might increase or decrease rapidly in accuracies, some pairs see only decrease or increase. Only settling at a number with higher precision at 400 minhashes

In [19]:

```
from numpy import uint32
from numpy import uint64
import random
import numpy

import pyspark
```

```
import re

sc = pyspark.SparkContext("local", "Similarity")


def generate_random_hash_params_A_P():
    A = random.getrandbits(64)
    P = random.getrandbits(64)
    while A>=P:
        A = random.getrandbits(64)
        P = random.getrandbits(64)
    return A , P

Ashingle, Pshingle = generate_random_hash_params_A_P()

def string_hash(shingle,slen,A,P):
    tmp = uint64(ord(shingle[0]))
    for i in range(1, slen):
        tmp = (tmp*uint64(A) + uint64(ord(shingle[i]))) % uint64(P)
    return uint32(tmp&uint64(0xFFFFFFFF))

def make_shingles(text, shingle_len):
    text = text.lower()
    text = re.sub(r'[^a-z]+', '',text )
    for i in range( 0, len(text)-shingle_len-1 ):
        yield string_hash(text[i:i+shingle_len],shingle_len, Ashingle, Pshingle)

shingle_length = 6

files = sc.wholeTextFiles('./Jaccard_test/*.txt')
files1 = files.map(lambda p: (p[0].split("/")[-1], list(make_shingles( p[1], shi
res = files1.collect()

def uint32_hash(stringhashes,A):
    for i in range(0, len(stringhashes)):
        stringhashes[i] = uint32((uint64(A)*uint64(stringhashes[i])) >> uint64(3
    return stringhashes

def min_hash(hashes,minhash_A):
    minhashes = []
    for i in range(0,len(minhash_A)):
        minhashes.append(min(uint32_hash(hashes,minhash_A[i])))
    return minhashes


for num_minhashes in [11, 25, 100, 400]:

    minhash_A = [random.getrandbits(64) for _ in range(num_minhashes)]

    minhashes1 = files1.map(lambda p: (p[0], min_hash(p[1], minhash_A)))
    res = minhashes1.collect()

    print("\nNumber of Minhashes:", num_minhashes)

    count = 0
    for i in range(len(res)):
        for j in range(i+1, len(res)):  # avoid duplicates & self comparison
            sim = sum(
                numpy.array(res[i][1]) == numpy.array(res[j][1])
            ) / num_minhashes
```

```
            print(res[i][0], "vs", res[j][0], "similarity:", sim*100, "%")

            count += 1
            if count == 5:
                break
        if count == 5:
            break
```

```
Number of Minhashes: 11
bank.txt vs cat.txt similarity: 0.0 %
bank.txt vs hadoop.txt similarity: 9.090909090909092 %
bank.txt vs minhash.txt similarity: 0.0 %
bank.txt vs turtles.txt similarity: 0.0 %
cat.txt vs hadoop.txt similarity: 0.0 %

Number of Minhashes: 25
bank.txt vs cat.txt similarity: 12.0 %
bank.txt vs hadoop.txt similarity: 8.0 %
bank.txt vs minhash.txt similarity: 0.0 %
bank.txt vs turtles.txt similarity: 4.0 %
cat.txt vs hadoop.txt similarity: 4.0 %

Number of Minhashes: 100
bank.txt vs cat.txt similarity: 8.0 %
bank.txt vs hadoop.txt similarity: 6.0 %
bank.txt vs minhash.txt similarity: 3.0 %
bank.txt vs turtles.txt similarity: 8.0 %
cat.txt vs hadoop.txt similarity: 5.0 %

Number of Minhashes: 400
bank.txt vs cat.txt similarity: 7.75 %
bank.txt vs hadoop.txt similarity: 6.25 %
bank.txt vs minhash.txt similarity: 3.5000000000000004 %
bank.txt vs turtles.txt similarity: 6.75 %
cat.txt vs hadoop.txt similarity: 7.5 %
```

50 minhashes with 2 different single lengths with spaces/no spaces

Analysis: As predicted in the previous task, hashing with spaces brought the average similarity for a single file up. Also, a mere increase in 2 in terms of shingle length significantly reduces the similarities between 2 files. This was made clear in the book, having a small shingle length may increase the chances of 2 files being considered similar drastically.

(Disclaimer: the code below was edited by ChatGPT because mine was getting a bit too lengthy)

In [22]:
```
def make_shingles(text, shingle_len, keep_spaces):
    text = text.lower()
```

```
    if keep_spaces:
        text = re.sub(r'[^a-z ]+', '', text)
    else:
        text = re.sub(r'[^a-z]+', '', text)

    for i in range(0, len(text) - shingle_len + 1):
        yield string_hash(text[i:i+shingle_len],
                          shingle_len,
                          Ashingle,
                          Pshingle)

num_minhashes = 50
minhash_A = [random.getrandbits(64) for _ in range(num_minhashes)]

shingle_lengths = [4, 6]
space_options = [False, True]

for shingle_length in shingle_lengths:
    for keep_spaces in space_options:

        print("\n----------------------------------------")
        print("Shingle Length:", shingle_length,
              "| Keep Spaces:", keep_spaces)
        print("----------------------------------------")

        files1 = files.map(lambda p:
            (p[0].split("/")[-1],
             list(make_shingles(p[1], shingle_length, keep_spaces)))
        )

        minhashes = files1.map(lambda p:
            (p[0], min_hash(p[1], minhash_A))
        )

        res = minhashes.collect()

        # Select first 5 document pairs (unique pairs)
        count = 0
        for i in range(len(res)):
            for j in range(i+1, len(res)):

                sim = sum(
                    numpy.array(res[i][1]) ==
                    numpy.array(res[j][1])
                ) / num_minhashes

                print(res[i][0].ljust(20),
                      "vs.",
                      res[j][0].ljust(20),
                      "similarity:",
                      round(sim * 100, 2), "%")

                count += 1
                if count == 5:
                    break
            if count == 5:
                break

sc.stop()
```

```
----------------------------------------
Shingle Length: 4 | Keep Spaces: False
----------------------------------------
bank.txt            vs. cat.txt            similarity: 36.0 %
bank.txt            vs. hadoop.txt         similarity: 28.0 %
bank.txt            vs. minhash.txt        similarity: 20.0 %
bank.txt            vs. turtles.txt        similarity: 28.0 %
cat.txt             vs. hadoop.txt         similarity: 24.0 %

----------------------------------------
Shingle Length: 4 | Keep Spaces: True
----------------------------------------
bank.txt            vs. cat.txt            similarity: 42.0 %
bank.txt            vs. hadoop.txt         similarity: 30.0 %
bank.txt            vs. minhash.txt        similarity: 24.0 %
bank.txt            vs. turtles.txt        similarity: 40.0 %
cat.txt             vs. hadoop.txt         similarity: 30.0 %

----------------------------------------
Shingle Length: 6 | Keep Spaces: False
----------------------------------------
bank.txt            vs. cat.txt            similarity: 6.0 %
bank.txt            vs. hadoop.txt         similarity: 4.0 %
bank.txt            vs. minhash.txt        similarity: 2.0 %
bank.txt            vs. turtles.txt        similarity: 8.0 %
cat.txt             vs. hadoop.txt         similarity: 6.0 %

----------------------------------------
Shingle Length: 6 | Keep Spaces: True
----------------------------------------
bank.txt            vs. cat.txt            similarity: 14.0 %
bank.txt            vs. hadoop.txt         similarity: 14.0 %
bank.txt            vs. minhash.txt        similarity: 6.0 %
bank.txt            vs. turtles.txt        similarity: 8.0 %
cat.txt             vs. hadoop.txt         similarity: 8.0 %
```