**MATH 106: HISTORY OF MATHEMATICS**
**2024 WINTER – LECTURE 1**

Jukka Keränen



## 1. WELCOME!

The Math Department calls this course “History of Mathematics”, but a better title would be

*Mathematics: An Introduction Through Its History*

Our main goal is to come up with an understanding of what mathematics is all about by studying its historical development. That is: this is a *math* course, not a *history* course.

Historically, mathematics has grown and evolved in response to *problems* mathematicians were trying to solve. Our focus will therefore be on major problems and the new mathematical ideas that arose from attempts to solve those problems. We will also emphasize the importance of new *concepts* and the way they are used to acquire new mathematical knowledge and also to *reorganize* old knowledge.

One recurring theme in this course will be that mathematics is not exclusively, or even chiefly, about acquiring new mathematical knowledge. Rather, mathematics is aimed at acquiring more and better *understanding* of mathematical facts. I will expand on this theme throughout the course.

As with any math course, working on homework exercises is a vital part of your learning in this course.

We will follow the textbook fairly closely, but I will also provide some supplemental perspectives in the lessons, particularly towards the end of the course.

The main topics we will cover include Greek mathematics and the early development of calculus.

## 2. SUMMARY OF KEY INFORMATION ABOUT THE COURSE

| | |
|---|---|
| **Instructor** | Jukka Keränen |
| **Email** | math.106.winter.2024@gmail.com |
| **Office** | MS 6232 |
| **Student Hours** | M, W 2-3 PM (MS 6232); R 2-3 PM on Zoom; and *also* by appointment |
| **Class Meetings** | Lectures MWF 10:00 AM-10:50 AM in Franz Hall 1178 |
| **Prerequisites** | MATH 31A, 31B, 32A with a grade of "D-" or better |
| **Textbook** | J. Stillwell. *Mathematics and Its History*, 3rd edition |
| **Grading** | Your overall course grade will be determined by the following scheme:<br>5% Active Learning<br>20% Homework<br>20% Project<br>20% Midterm Exam<br>35% Final Exam |
| **Homework** | There will be **8 homework assignments**. Please see Section 5 for details. |
| **Exams** | **Midterm**: Monday, February 12 (usual lecture time; location TBD)<br>**Final Exam**: Friday, March 22, 11:30 AM-2:30 PM (location TBD)<br><br>All exams are closed-book. Per University regulations, you must take the Final Exam in order to pass the course. |

## 3. COURSE SCHEDULE

| Date | Lesson | Section (textbook) | Topic |
|---|---|---|---|
| **WEEK 1: PYTHAGOREAN THEOREM** | | | |
| 01/08 | | | Introduction to the Course! |
| 01/10 | **Lesson 1** | 1.1, 1.4 | Arithmetic and Geometry; Right-Angled Triangles |
| 01/12 | **Lesson 2** | 1.2, 1.3 | Pythagorean Triples; Rational Points on the Circle |
| **WEEK 2: GREEK GEOMETRY** | | | |
| 01/15 | | | **MARTIN LUTHER KING, JR. HOLIDAY** |
| 01/17 | **Lesson 3** | 1.5, 1.6 | Irrational Numbers; The Definition of Distance |
| 01/19 | **Lesson 4** | 2.1 | The Deductive Method |
| 01/21 | | | **Homework 1 due 11:59 PM** |
| **WEEK 3: GREEK GEOMETRY; EARLY NUMBER THEORY** | | | |
| 01/22 | **Lesson 5** | 2.2, 2.3 | The Regular Polyhedra; Ruler and Compass Constructions |
| 01/24 | **Lesson 6** | 2.4 | Conic Sections |
| 01/26 | **Lesson 7** | 3.2 | Polygonal, Prime, Perfect Numbers |
| 01/28 | | | **Homework 2 due 11:59 PM** |
| **WEEK 4: EARLY NUMBER THEORY** | | | |
| 01/29 | **Lesson 8** | 3.3 | The Euclidean Algorithm |
| 01/31 | **Lesson 9** | 5.2 | The Chinese Remainder Theorem |
| 02/02 | **Lesson 10** | 5.3 | Linear Diophantine Equations |
| 02/04 | | | **Homework 3 due 11:59 PM** |

**WEEK 5: POLYNOMIAL EQUATIONS**

| | | | |
|---|---|---|---|
| 02/05 | **Lesson 11** | 5.4 | Pell's Equation |
| 02/07 | **Lesson 12** | 6.3, 6.4 | Quadratic Equations; Quadratic Irrationals |
| 02/09 | **Lesson 13** | 6.5 | The Solution of the Cubic |

**WEEK 6: POLYNOMIAL EQUATIONS**

| | | | |
|---|---|---|---|
| 02/12 | | | **MIDTERM (during class meeting)** |
| 02/14 | **Lesson 14** | 6.6 | Angle Division |
| 02/16 | **Lesson 15** | 6.7 | Galois Theory: Angle Division Explained |
| 02/18 | | | **Homework 4 due 11:59 PM** |

**WEEK 7: ANALYTIC GEOMETRY**

| | | | |
|---|---|---|---|
| 02/19 | | | **PRESIDENTS' DAY HOLIDAY** |
| 02/21 | **Lesson 16** | 6.7 | Galois Theory: Unsolvability of the Quintic |
| 02/23 | **Lesson 17** | 7.1, 7.2 | Towards Analytic Geometry; Fermat and Descartes |
| 02/25 | | | **Homework 5 due 11:59 PM** |

**WEEK 8: CALCULUS**

| | | | |
|---|---|---|---|
| 02/26 | **Lesson 18** | 7.3,7.4 | Algebraic Curves; The Arithmetization of Geometry |
| 02/28 | **Lesson 19** | 9.1, 9.2 | What Is Calculus; Early Results on Areas and Volumes |
| 03/01 | **Lesson 20** | 9.3 | Maxima, Minima, Tangents |
| 03/03 | | | **Homework 6 due 11:59 PM** |

**WEEK 9: CALCULUS**

| | | | |
|---|---|---|---|
| 03/04 | **Lesson 21** | 9.5 | Newton's Calculus of Series |
| 03/06 | **Lesson 22** | 9.6 | The Calculus of Leibniz |
| 03/08 | **Lesson 23** | 10.4 | Summation of Series |
| 03/10 | | | **Homework 7 due 11:59 PM** |

**WEEK 10: NUMBER THEORY REVIVAL**

| | | | |
|---|---|---|---|
| 03/11 | **Lesson 24** | 10.6, 10. 7 | Generating Functions; The Zeta Function |
| 03/13 | **Lesson 25** | 11.2 | Fermat's Little Theorem |
| 03/15 | **Lesson 26** | 11.3 | Fermat's Last Theorem |
| 03/17 | | | **Homework 8 due 11:59 PM** |

**FINALS WEEK**

03/22 **FINAL EXAM (11:30 AM-2:30 PM, LOCATION TBD)**

## 4. INSTRUCTOR AND COURSE INFORMATION

Hello! My name is Jukka Keränen, and I am a mathematician.

My research is in *algebraic number theory*. Many of the main themes in my work can be traced, in part, to Andrew Wiles' epoch-making proof of *Fermat's Last Theorem* in 1995.

I enjoy all manner of pop culture, Lego, video games, running, and certain very specific automobiles.

### 4.1 Email

I have created a special-purpose email address just for this course:

math.106.winter.2024@gmail.com

### 4.2 Suggestion Box

If you want to send me an **anonymous message**, you can do so through my **Suggestion Box** located on the home page of our Canvas site. You are very welcome to use this method of communication. However, please keep in mind that I will have **no way of finding out who sent the message** and, so, I won't be able to respond to you directly.

**This feature will go live at noon on Monday, January 8.**

### 4.3 Official Mode of Communication

The official mode of communication for all practical matters pertaining to this course are email announcements sent from our Bruin Learn site. It is your responsibility to monitor your email for these announcements, and (just to be safe) occasionally check the announcements on our Bruin Learn site. All the announcements will be stored there.

### 4.4 Student Hours

My student hours will take place **Mondays 2:00-3:00 PM** and **Wednesdays 2:00-3:00 PM (MS 6232), Thursdays 2:00-3:00 PM on** Zoom here, and *also* **by appointment**. If you want to talk to me about the course material or, indeed, any aspect of this course, you are very welcome to make an individual appointment with me. It is **not** necessary to make an appointment if you just want to come to my regular student hours.

**4.5 Textbook**
The textbook for this course is

J. Stillwell. *Mathematics and Its History,* 3rd Edition.

**4.6 Course Website**
All the materials for this course will be stored on our Bruin Learn site.

**4.7 Campuswire**
I encourage you to use **Campuswire** to ask questions and connect with your classmates. You should already have received an invitation to our Campuswire class. You can also join through this link:

https://campuswire.com/p/G6CE8C9FD (code: **3747**)

The teaching team will be regularly monitoring the discussions and will answer your questions as needed. Please note that participation on Campuswire is **voluntary** and **will not be graded** in any way.

## 5. ACTIVITIES, ASSESSMENT, AND GRADING

I am committed to making sure the assessment of your learning in this course is comprehensive, fair, and equitable. Your grade in this class will be based on the number of points you earn out of the total number of points possible and is not based on your rank relative to other students. Furthermore, grades are assigned without strict limits on the proportion of each letter grade given in the course.

### 5.1 Activities and Assessment Overview

Your overall course score will be determined by the following scheme:

| LEARNING ACTIVITY | % of GRADE | MAX PTS | HOW TO EARN THEM |
|---|---|---|---|
| 1. Active Learning | 5% | 50 | 25 x 2 (drop lowest 2) |
| • in-class iClicker responses (1 point) + post-class reflection (1 point) | | | |
| 2. Homework | 20% | 200 | 7 x 29 (drop lowest 1) |
| 3. Project | 20% | 200 | 50 pts presentation, 150 pts report |
| 4. Midterm Exam | 20% | 200 | 200 |
| 5. Final Exam | 35% | 350 | 350 |
| **TOTAL** | **100%** | **1000** | |

**Late Work** Homework and post-class reflection assignments will be accepted late, **marked down 10% for each day past due**. This means you can submit up to 9 days late and still get a nonzero grade. If you have a pressing reason for submitting late (illness, family issues, etc.), please email me to ask for an extension. Extensions will be granted on a case-by-case basis, but don't be afraid to ask.

The individual assignments are **not curved in any way**, and **no letter grade is given** for them.

The percentage course score will be converted into a letter grade according to the usual cut-offs:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 97 and up | A+; | 93 to < 97 | A; | 90 to < 93 | A- | | |
| 87 to < 90 | B+; | 83 to < 87 | B; | 80 to < 83 | B- | | |
| 77 to < 80 | C+; | 73 to < 77 | C; | 70 to < 73 | C- | | |
| 67 to < 70 | D+; | 63 to < 67 | D; | 60 to < 63 | D-; | < 60 | F |

These percentage cutoffs indicate the *minimum letter grade* you will get for your overall course score. Depending on how the class did overall, you may well get a higher grade, *but never lower*, than what is indicated here.

**5.2 Learning Teams**

At your first TA section meeting, you will be assigned to a **4-member learning team**: your persistent micro-community within the larger Math 106 community. In the lectures and TA sections, you will sit next to your teammates and discuss the participation questions and homework problems. In the latter half of the quarter, you will collaborate with your teammates on a project the output from which will be an oral presentation in your TA section (Week 9 or Week 10) and a written report (please see below for details).

**5.3 Component Activities and Assessment**

**1. Active Learning** This course (like math in general) is *all* about active learning. Our active learning component will consist of two types of items: in-class participation and post-class reflections.

**1.1 In-Class Participation** There will be 27 lectures with in-class participation questions. I expect you to be present in every lecture and actively engage the course material by discussing with your teammates the questions I have prepared. You will earn these points by responding using iClicker.

Participating in each class meeting will be worth **1 point**.

In class, most of the participation questions will be completed after a small-group discussion. These discussions are an essential part of your learning process in this course, and you should not miss out on them, unless absolutely necessary. Folks, you came to UCLA to learn from each other!

**1.2 Post-Class Reflection** After each class meeting, you will answer the following question on Bruin Learn:

*What would you say is the most important thing you learned in class today?*

Answering this question will be worth **1 point**; any reasonable answer will be accepted.

Each post-class reflection will take you no more than **5 minutes**.

The post-class reflection is due at **11:59 PM** on the day of the corresponding class meeting.

For each lecture, the participation and post-class reflection scores are combined into a single active learning score. At the end of the course, you will have 27 active learning scores.

The **lowest 2 of these will be dropped** automatically when calculating your overall course score.

**2. Homework** This is a math course. *The only way* to learn math is to do a lot of exercises, and not only that, work on them steadily throughout the quarter. So, to help you learn, there will be **8 homework assignments, each one worth 29 points**. Each problem will be scored on completion only. In order to earn the completion marks for a given problem, however, a complete and reasonable attempt at a solution has to be presented.

- You will submit your homework on Gradescope.
- The due dates are:

    - **Homework 1** due 11:59 PM on Sunday, January 21
    - **Homework 2** due 11:59 PM on Sunday, January 28
    - **Homework 3** due 11:59 PM on Sunday, February 4
    - **Homework 4** due 11:59 PM on Sunday, February 18
    - **Homework 5** due 11:59 PM on Sunday, February 25
    - **Homework 6** due 11:59 PM on Sunday, March 3
    - **Homework 7** due 11:59 PM on Sunday, March 10
    - **Homework 8** due 11:59 PM on Sunday, March 17

The **lowest homework score will be dropped** automatically when calculating your course grade.

**3. Project** In Weeks 6-10, you will collaborate with your teammates on a project. The goal of the project is to explain and illustrate a topic from the history of mathematics. The details will be given in a separate document later on. For now, the main features of the project are as follows:

- In Week 6, your team will choose a topic from a list of possible topics, or propose a topic of your own (subject to instructor approval). For example, you might cover one of the sections from our textbook that was not covered in the lectures.

- TA sections in Weeks 7 and 8 will have time dedicated to working on the project. However, you are also expected to work on the project outside of class time.

- The output of the project will consists of a 10-minute **oral presentation** during your TA section in Week 9 or 10 and a **written report**. The report must be no more than 5 pages in length and will be due 11:59 PM on March 17 (Sunday of Week 10).

- The presentation is worth 50 points, the report 150 points.

**4. Midterm Exam** There will be one midterm exam:

- The Midterm Exam will take place **during the usual lecture time, 10:00-10:50 AM, on Monday, February 12 (Monday of Week 6)**.
- The location will be announced in Week 5.
- This will be an **individual exam**: you are not allowed to consult anyone else, or provide assistance to anyone else.
- The midterm exam will cover **Lessons 1-11**.

**5. Final Exam**

- The Final Exam will take place **11:30 AM-2:30 PM on Friday, March 22**.
- The location will be announced in Week 9.
- This will be **an individual exam**: you are not allowed to consult anyone else, or provide assistance to anyone else.
- The Final Exam will cover the entire course, but the emphasis will be on the material from **Lessons 12-26**.

More detailed information and ample prep materials will be provided closer to the date of each exam.

## 6. COURSE POLICIES

### 6.1 Our Inclusive Learning Environment

UCLA values diversity and inclusion. We expect everyone in this class to contribute to a respectful, welcoming, and inclusive environment to support the learning of all other members of the class. If there are aspects of the instruction or design of this course that result in barriers to your inclusion or accurate assessment or achievement, please notify us.

### 6.2 Academic Integrity – a Bruin's Code of Conduct

In short: You must not cheat, because cheating is morally wrong.

UCLA is a community of scholars committed to the values of integrity. In this community, all members including faculty, staff, and students alike are responsible for maintaining the highest standards of academic honesty and quality of academic work. As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. When accusations of academic dishonesty occur, the Office of the Dean of Students investigates and adjudicates suspected violations of this student code. Unacceptable behaviors include cheating, fabrication, plagiarism, multiple submissions without instructor permission, using unauthorized study aids, or facilitating academic misconduct.

Please review our campus' policy on academic integrity in the UCLA Student Conduct Code:

http://www.deanofstudents.ucla.edu/Student-Conduct-Code.

Once referred to the Office of the Dean of Students, allegations of academic dishonesty can lead to formal disciplinary proceedings. Being found responsible for violations of academic integrity can result in disciplinary actions such as the loss of course credit for an entire term, suspension for several terms, or dismissal from the University. Such negative marks on your academic record may become a major obstacle to admission to graduate, medical, or professional school.

We cannot make exceptions to our campus' policy on academic integrity, and as we hopefully have communicated effectively here, penalties for violations of this policy are harsh. Please do not believe it if you hear that "everyone does it". The truth is, you usually don't hear about imposed disciplinary actions because they are kept confidential. So our advice, just don't do it! Let's embrace what it means to be a true Bruin and together be committed to the values of integrity.

**Examples of academic dishonesty**

With respect to our course, examples of academic dishonesty include giving answers on assignments to someone else, receiving answers from someone else, turning in any written work that is not your own for points in our course, copying passages from websites, copying passages from your or any other textbook on any graded material in the course, or bringing a classmate's clicker to class to get participation points for them when they are absent. If you engage in these types of unacceptable behaviors, then you will receive a zero as your score for that assignment. If you are caught cheating on an exam, then you will receive a score of zero for the entire exam.

**Exams**
No cell phones, smart watches, or similar types of devices are allowed during exams. Accordingly, you may not use cell phones as a clock to keep time or as a calculator.

**6.3 Absences from Exams (required documentation policy)**
In the event that you must miss an exam, you should contact your instructor as soon as is reasonably possible. You will be asked for verification of the event (injury, death, etc) and the course instructor will determine whether a makeup exam is possible or to pro-rate your exam. Please note that only an exam can be pro-rated.

**6.4 Integrity of Course Contest**
Please protect the integrity of all course materials and content. By enrolling in this course, you agree to honor this request. Be mindful of the hard work and time that our instructors and TAs in the Math Department put into creating course materials such as exam and quiz questions, worksheets, lecture videos, and Bruincasts. Please do not upload course materials not created by you onto third-party websites or share content with anyone who is not enrolled in our course. We are grateful for your cooperation in honoring this important request.

**6.5 Regrading policy**
We make every effort to make sure that your homework and exam answers are graded accurately and fairly. Nevertheless, due to the large number of questions that your TAs will have to grade, mistakes do occasionally happen. If you believe that a mistake has been made in scoring your work, please submit a regrade request **within one week** of the assignment being returned to you.

For issues pertaining to questions you answered through Gradescope, please use the built-in functionality of Gradescope to submit a regrade request. For issues pertaining to questions you answered on Bruin Learn, please contact me through email.

## 7. STUDENT RESOURCES

### 7.1 Center for Accessible Education (CAE)

Students needing academic accommodations based on a disability should contact the Center for Accessible Education (CAE) at (310) 825-1501. CAE will assess all requested accommodations and communicate appropriately with us (your instructors). Any students with CAE approval for proctoring arrangements during exams will need to please inform us (or your TA) prior to the date of the exam. When possible, students should contact CAE within the first two weeks of the quarter to allow reasonable time to coordinate accommodations. For more information, please visit the CAE website: http://www.cae.ucla.edu.

### 7.2 Counseling and Psychological Services (CAPS)

Resources are available to foster the well-being of all UCLA students as they pursue their academic goals. Any student who finds themselves in immediate distress, please call Counseling and Psychological Services (CAPS) to speak directly with a counselor 24/7 at (310) 825-0768, or please call 911. For more information, pleases visit the CAPS website: http://www.counseling.ucla.edu.

### 7.3 Student Resources for Support and Learning

UCLA has a multitude of resources available to *all* students. Many of these resources are listed below (alphabetized by name), and we encourage students to explore them as needed.

- **Academic Achievement Program (AAP):** This program advocates and facilitates the access, academic success, and graduation of students who have been historically underrepresented in higher education; informs and prepares students for graduate and professional schools; and develops the academic, scientific, political, economic, and community leadership necessary to transform society: https://www.aap.ucla.edu
- **Academics in the UCLA Residential Community:** Free workshops on a wide variety of issues relating to academic & personal success; (310) 825-9315; https://reslife.ucla.edu (click on "academics")
- **Bruin Resource Center:** Includes services for transfer students, undocumented students, veterans, and students with dependents; http://www.brc.ucla.edu.
- **Career Center:** Don't wait until your senior year – visit the career center today! http://www.career.ucla.edu
- **Center for Accessible Education (Formerly Office for Students with Disabilities):** Located in A255 Murphy Hall: (310) 825-1501, TDD (310) 206-6083; http://www.cae.ucla.edu.
- **Counseling and Psychological Services (CAP):** Located in Wooden Center West; students in distress may call to speak directly with a counselor 24/7 at (310) 825-0768, or may call 911; http://www.counseling.ucla.edu
    - Commonly known as the "Red Folder", this tool is intended to provide you with quick access to important resources for assisting students in distress (see, say, do): https://ceils.ucla.edu/wp-content/uploads/sites/2/2016/08/911Guide.pdf
- **Dashew Center for International Students and Scholars:** Located in 106 Bradley Hall; (310) 825-1681; http://www.internationalcenter.ucla.edu
- **Dean of Students Office:** General resource for all Bruins. Learn about academic integrity issues and your first amendment rights. Get help if you've experienced rape or sexual assault. Report a bias

incident, and much more. Located in 1206 Murphy Hall; (310) 825-3871; http://www.deanofstudents.ucla.edu

- **LGBTQ Campus Resource Center:** Located in the Student Activities Center, B36; (310) 206-3628; http://www.lgbtq.ucla.edu
- **Letters & Science Academic Counseling Service:** Located in A316 Murphy Hall; (310) 825-1965; http://cac.ucla.edu.
- **Library**: Get help with your research, find study spaces, attend a workshop, rent a laptop, and more. Learn more at http://www.library.ucla.edu.
- **Student Legal Services:** Located in A239 Murphy Hall; (310) 825-9894; http://www.studentlegal.ucla.edu
- **Undergraduate Writing Center:** Peer learning facilitators (PLFs) are undergraduates who understand the challenges of writing at UCLA. Scheduled appointment and walk-in options are available, see http://wp.ucla.edu/wc for more information and to get assistance with your writing.
- **UCLA ONE**: This website (https://uclaone.com/) serves as UCLA's interactive, online gateway for mentorship, professional networking, peer driven career advice, and exclusive job leads. (Similar to LinkedIn but for the UCLA community).