Optimization-Basic Assignment

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Problem Statement **- Find the maximum profit that a company can make if the profit function is given by** $f(x) = 41 - 72x + 18x^2$

Solution

A function f(x) is said to be convex if following inequality is true for $\lambda \in [0, 1]$:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \tag{1}$$

Checking convexity of $f(x)$:

$$\begin{aligned} &\lambda \left(41 - 72x_1 - 18x_1^2\right) + (1 - \lambda)\left(41 - 72x_2 - 18x_2^2\right) \geq \\ &41 - 72\left(\lambda x_1 + (1 - \lambda)x_2\right) - 18\left(\lambda x_1 + (1 - \lambda)x_2\right)^2 \end{aligned} \tag{2}$$

$$18x_1^2\left(\lambda^2 - \lambda\right) + 18x_2^2\left(\lambda^2 - \lambda\right) + 36x_1x_2\left(\lambda^2 - \lambda\right) \geq 0 \tag{3}$$

$$x_1^2\left(\lambda^2 - \lambda\right) + x_2^2\left(\lambda^2 - \lambda\right) + 2x_1x_2\left(\lambda^2 - \lambda\right) \geq 0 \tag{4}$$

$$-\lambda\left(1 - \lambda\right)\left(x_1 - x_2\right)^2 \geq 0 \tag{5}$$

$$\implies \lambda\left(1 - \lambda\right)\left(x_1 - x_2\right)^2 \leq 0 \tag{6}$$

Equation (6) holds true for all $\lambda \in (0, 1)$. Hence the given function f(x) is concave For a general quadratic equation

$$f(x) = ax^2 + bx + c \tag{7}$$

1. For Maxima :
   Using gradient ascent method,

   $$x_n = x_{n-1} + \mu\frac{df(x)}{dx} \tag{8}$$

   $$\frac{df(x)}{dx} = -72 - 36x \tag{9}$$

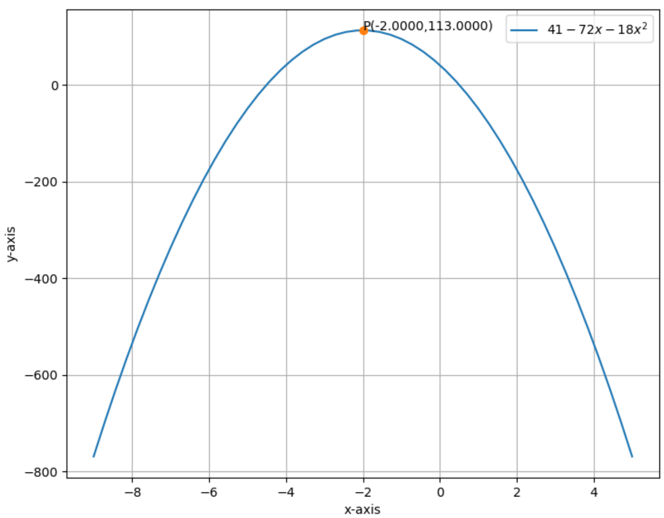
   After substituting 9 in 8 we get:

   $$x_n = x_{n-1} + \mu(-72 - 36x_{n-1}) \tag{10}$$

   Taking $x_0 = 1, \alpha = 0.001$ and $precision = 0.00000001$, values obtained using python are:

   $$\boxed{\text{Maxima} = 112.99999999999997 \approx 113} \tag{11}$$

   $$\boxed{\text{Maxima Point} = -1.9999999736912384 \approx -2.0} \tag{12}$$



Graph of f(x) $= 41 - 72x + 18x^2$