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Assignment-4

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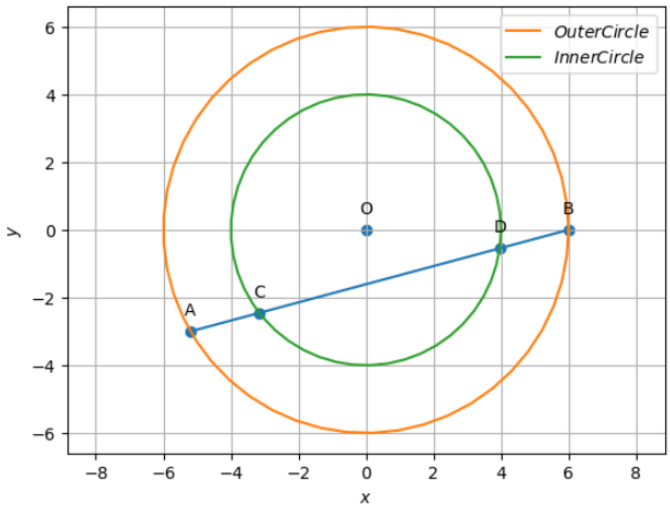
Problem Statement:

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AC = DB.



SOLUTION:

Let A and B be the end vertices of the chord intersecting two concentric circles, therefore C and D are points of the chord intersecting inner circle. We solve for the required points C and D as follows.

STEP-1

Calculating co-ordinates A and B :

Outer Circle has a radius, $r_o = 6$

Inner Circle has a radius, $r_i = 4$

We get coordinates of vertices A and B as follows :

$$A = r_o \begin{pmatrix} cos\theta_1 \\ sin\theta_1 \end{pmatrix} B = r_o \begin{pmatrix} cos\theta_2 \\ sin\theta_2 \end{pmatrix}$$

By taking $\theta_1 = 159^o$ and $\theta_2 = 0^o$,

We get,

$$\mathbf{A} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{1}$$

STEP-2

Finding the equation of line AB :

Using the parametric equation of the line

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \tag{2}$$

We know the equation of circle is,

$$x^2 + y^2 = r^2$$

The above equation can be expressed in vector form as

$$\mathbf{x}^\top \mathbf{x} = r^2 \tag{3}$$

Substituting **(2)** in above Eq.,

$$(\mathbf{A} + \lambda \mathbf{m})^\top (\mathbf{A} + \lambda \mathbf{m}) = r^2 \tag{4}$$

$$\implies \lambda^2 \|\mathbf{m}\|^2 + 2\lambda \mathbf{m}^\top \mathbf{A} + \|\mathbf{A}\|^2 - r^2 = 0 \tag{5}$$

yielding

$$\lambda = \frac{-\mathbf{m}^\top \mathbf{A} \pm \sqrt{(\mathbf{m}^\top \mathbf{A})^2 - \|\mathbf{m}\|^2 \left(\|\mathbf{A}\|^2 - r^2\right)}}{\|\mathbf{m}\|^2} \tag{6}$$

For this problem, the numerical values are

$$\mathbf{A} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix}, r = 4, \tag{7}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A}, \mathbf{m} = \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \tag{8}$$

Substituting above in **(6)**,

$$\lambda_1 = 0.18, \lambda_2 = 0.82 \tag{9}$$

Thus substituting λ_1 and λ_2 in **(2)** would give the desired points of intersection C and D.

For point C,

$$\mathbf{x} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix} + \lambda_1 \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} -3.16 \\ -2.45 \end{pmatrix} \tag{11}$$

For point D,

$$\mathbf{x} = \begin{pmatrix} -5.2 \\ -3 \end{pmatrix} + \lambda_2 \begin{pmatrix} 11.2 \\ 3 \end{pmatrix} \tag{12}$$

$$= \begin{pmatrix} 3.96 \\ -0.55 \end{pmatrix} \tag{13}$$

Thus, points C and D

$$\mathbf{C} = \begin{pmatrix} -3.16 \\ -2.45 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 3.96 \\ -0.55 \end{pmatrix} \tag{14}$$

Table

The input parameters are the lengths points A and C.

Symbol	**Value**	**Description**
r_o	6	Outer Radius
r_i	4	Inner Radius
$\mathbf{m}$	$\begin{pmatrix} 11.2 \\ 3 \end{pmatrix}$	Direction Vector of the Line

Construction

Vertex	**Co-ordinates**
O	(0,0)
A	(-5.2,-3)
B	(6,0)
C	(-3.16,-2.45)
D	(3.96,-0.55)