Dynamic Delegation with Adverse Selection*

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Abstract

I study a dynamic model of delegated decision making with adverse selection and imperfect monitoring. In each period, a principal may delegate to a biased agent who has better information. The quality of the agent's information depends on his ability. In the optimal mechanism where the agent's ability is publicly observable, the principal delegates to the agent at the beginning of their relationship and the agent behaves in the principal's interest. Depending on the history, the principal either commits to delegating forever or stops delegating eventually. When the agent's ability is private information, the optimal mechanism features pooling at the top. The principal offers the same mechanism to the agent if his ability is known to be above a cutoff.

1 Introduction

Delegated decision making is commonly observed in many economic activities. Within an organization, headquarters may delegate investment decisions to division managers. Division managers usually have a better knowledge of the prospect of a specific project, but at the same time, they may be willing to take more risks than headquarters. Similarly, universities

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delegate the hiring decision to departments which have better information on the quality of candidates but want to hire more people. The state government delegates to the local government the construction of local infrastructure, where the local government knows more about the benefits but the state government bears the cost. In all these examples, the agents have more information to make better decisions, but their interests are not perfectly aligned with the principals.

In most examples, the relationship is ongoing and the conflict of interests is persistent over time. As a result, the principal may use future delegation decisions to incentivize the agent to behave in her interest. Specifically, the principal would delegate more if the agent made more decisions in favor of the principal and delegate less otherwise. However, one important requirement to implement this policy is that the principal can monitor the agent's behavior sufficiently well, which may not be true in many situations. For example, when the division manager makes a poor investment, it may be hard to tell whether it is due to the manager's inappropriate risk management or to bad luck. The problem is further complicated if the precision of the division manager's information is unknown.

In this paper, I construct a dynamic model where a principal chooses whether or not to delegate to an agent at the beginning of each period. Upon delegation, the agent chooses to initiate a project or not. The project may be good or bad. The principal only wants good projects initiated, but the agent prefers to initiate both. Before making the decision, the agent observes a private signal on the quality of the project. Specifically, the agent observes a negative signal with some probability if the project is bad and observes nothing if the project is good. The probability of observing a negative signal conditional on the project being bad is referred to as the agent's ability (type).

The principal has full commitment power and perfectly observes which projects were initiated as well as the outcome of each initiated project. However, since the principal cannot observe the agent's private signal, she is unable to tell whether or not the agent took her preferred action given a bad outcome.

I first study the scenario where the agent's ability is publicly observed. In the optimal mechanism, the agent is assigned a quota for bad projects at the beginning. The principal delegates to the agent when the quota is positive and stops delegating when the quota drops to zero. The agent always behaves in the principal's interest upon delegation, i.e., to initiate a project if and only if he does not observe a negative signal. The quota decreases when the agent initiates a bad project and increases when the agent does not initiates a project. If the agent initiates a good project, the quota stays unchanged. In the long run, the principal either commits to delegating forever or stops delegating. The uncertainty comes from the imperfectness of the agent's private signals.

The dynamics of the relationship crucially depend on the agent's ability. When the agent's ability is higher, he initiates fewer bad projects on average and the principal will be better off. However, it is uncertain whether the agent will be better off or not. Intuitively, the agent's utility is determined by the quota for bad projects. When the agent is of higher ability, he is less likely to initiate a bad project if he behaves in the principal's interest. Therefore, the principal tends to offer a smaller quota. On the other hand, the relationship is more valuable for the principal when the agent is of higher ability. As a result, the principal should be more tolerant toward bad projects. In general, as the agent's ability increases, the agent gets better off when his ability is relatively low and worse off when his ability is relatively high.

Next, I turn to the model where the agent's ability is private information. In this scenario, the optimal mechanism features "pooling at the top". In other words, the principal optimally offers the same mechanism to the agent if his ability is known to be above a cutoff value. As mentioned above, when the agent's ability is relatively high, he gets worse off as his ability increases in the model where ability is publicly known. In other words, the high type has incentives to mimic the low type. But since the high type can always perfectly mimic the low type without a cost, it is not optimal for the principal to separate different types.

Related Literature. My paper broadly contributes to the literature on delegation. The delegation problem was first studied by Holmström (1978, 1980) and extended by many other papers such as Melumad and Shibano (1991), Martimort and Semenov (2006), Alonso and Matouschek (2008), and Kováč and Mylovanov (2009). In a static setting, a principal with commitment power faces an informed but biased agent who takes actions upon delegation. In the absence of monetary transfers, the objective is to characterize the optimal delegation set.

The problem is very different in a dynamic setting, where the principal can use future delegation decisions to incentivize the agent in earlier periods. Legros (1993) first studies the delegation in a dynamic setting. He considers a two-period game where the agent hides some information in the first period in order to be delegated again in the second period. Recently, many papers extend the setting to an infinite time horizon. Guo and Hörner (2020) consider a model where the principal chooses whether to delegate a project adoption choice to the agent. The agent privately observes the value of the project and there is an ongoing conflict of interests. They assume the value follows a Markov process, including the i.i.d. case as a benchmark. They characterize the optimal mechanism with full commitment power and show that the principal either stops delegating or commits to delegating forever eventually. Lipnowski and Ramos (2020) consider a very similar setting with the value of projects being i.i.d. But they study a model without commitment power and characterize the equilibrium payoff set in a repeated game. They show that the principal drifts toward conservatism in any Pareto efficient equilibrium. Li et al. (2017) also consider a model with no commitment power. However, they add an entry stage where the principal and the agent jointly decide to enter the game or not. This setup essentially gives the principal the commitment power because not entering the game is always an equilibrium at any point.

The key contribution of my paper is to include adverse selection in the model. Specifically, in addition to imperfect monitoring, the agent's type or ability is also private information. I focus on studying how the principal can separate different types of the agent and how the optimal mechanism changes accordingly. Aghion and Jackson (2016) study a similar setting where the agent's type is private information. But in their model, the principal can replace the agent at a cost and the agent's only objective is to stay in the relationship for as long as possible. The principal replaces the agent once her belief that the agent is a high type drops below a cutoff. They study the incentive provision problem with and without commitment power in this environment.

My paper is also related to the literature on dynamic contracting with both moral hazard and adverse selection. Some of the important works in this literature are Sung (2005), Sannikov (2007), Gershkov and Perry (2012), Cvitanić et al. (2013), Ulbricht (2016), and

Halac et al. (2016). One of the most important features in my paper, the non-monotonic relationship between the agent's utility in the model without adverse selection and the agent's type, is also present in Halac et al. (2016). The main difference is that I study a model where monetary transfers are not allowed.

2 The Model

A Principal (she) and an Agent (he) interact for infinitely many periods. Time is discrete and denoted by $t = 1, 2, \cdots$. In each period, the principal chooses to delegate a project adoption choice to the agent or not. The quality of a project can be good or bad, which is random at each period and independent over time. The ex ante common belief that a project is good is $p \in (0, 1)$ at each period. Upon delegation, the agent observes a private signal and chooses to initiate the project or not. As in Guo and Roesler (2016), I assume that the agent learns from bad news. Specifically, the agent observes nothing if a project is good and observes a bad signal with probability $\theta \in [0, 1]$ if a project is bad, where θ can be interpreted as the agent's ability (type). The distribution of θ, $F(\theta)$, is common knowledge.

The payoffs are as follows. If the principal doesn't delegate or the agent doesn't initiate the project, then both parties get 0. If the project is initiated, then the payoffs depend on the quality of the project. Both the principal and the agent get h if it's good and l if it's bad, where $h > l > 0$. In addition, the adoption of a project incurs a cost $c > 0$, which is borne solely by the principal. Assume $h > c > l$. Then there is a conflict of interests as the principal only wants to initiate good projects but the agent wants to initiate both. Both the principal and the agent are risk-neutral and impatient with discount factor $\delta \in (0, 1)$. Then the utility of the principal and the agent can be respectively expressed as

$$\pi = (1 - \delta) \sum_{t=0}^{\infty} \delta^t d_t a_t (v_t - c), \tag{1}$$

and

$$u = (1 - \delta) \sum_{t=0}^{\infty} \delta^t d_t a_t v_t, \tag{2}$$

where d_t is the probability of delegating by the principal, a_t is the probability of initiating a project conditional on delegation, and v_t is the value of the project at time t. As mentioned above, $v_t = h$ if the project is good and $v_t = l$ if the project is bad. Monetary transfers are not feasible.

The agent's actions are assumed to be perfectly observable. Specifically, the principal can observe if each project is initiated or not, and if initiated, whether its quality is good or bad. However, if a project is not initiated, then the principal never observes its quality. Despite the actions being perfectly observable, the monitoring is still imperfect in the sense that the agent's signals are imperfect and not observed by the principal. Consequently, if a bad project has been initiated, it could be the case that the agent observed a bad signal and initiated the project anyway, or that the agent has observed nothing. The first kind of behavior is undesirable for the principal but the second one is not. Being unable to distinguish these two kinds of behaviors, the principal cannot perfectly reward or punish the agent and so cannot achieve the first best.

The principal has full commitment power and offers a mechanism to the agent at time 0. When the agent's type is unknown, the mechanism may depend on the agent's report of his type. A mechanism specifies the probability of delegating and a recommended action at each period conditional on the public history, which includes the outcomes in all previous periods. Specifically, there are four possible outcomes in each period: a good project initiated, a bad project initiated, the agent does not initiate a project upon delegation, or the principal does not delegate. In the following, I first characterize the optimal mechanism in the benchmark where the agent's type is publicly known, and then explore the general model with private types.

3 The Benchmark

In this section, I study a baseline model where the agent's type, θ, is publicly observable. Let $\pi(u)$ be the maximum utility of the principal given the agent's utility is u. Following Spear and Srivastava (1987) and Thomas and Worrall (1990), I restrict attention to mechanisms which are functions of the agent's continuation utility. Specifically, given the agent's continuation utility at time t is u_t, the principal only needs to choose the probability of delegating at time t, $d_t(u_t)$, and the promised continuation utility at time $t+1$ conditional on the outcome at time t. Let $u_h(u_t)$, $u_l(u_t)$, $u_n(u_t)$, and $u_d(u_t)$ be the continuation utility conditional on a good project initiated, a bad project initiated, no projects initiated, and no delegation respectively. In the following, I first characterize $\pi(u)$ and then describe the optimal mechanism.

3.1 Payoff Frontier

Since the agent can never receive a negative payoff, the minimum of the agent's utility is 0, which is achieved when the principal never delegates. Similarly, the agent's maximum payoff is achieved when the principal always delegates and the agent always initiates a project. Let $\bar{v} := ph + (1-p)l$. Then $\pi(u)$ is defined on $[0, \bar{v}]$. Denote by $\pi_h(u)$ the continuation utility of the principal conditional on initiation of a good project, and define $\pi_l(u)$, $\pi_n(u)$, and $\pi_d(u)$ analogously. As in a standard moral hazard problem, I first determine which action of the agent should be induced. If the principal does not delegate, then the agent does not take any actions and there are no incentive problems. Let $\pi_0(u)$ be the optimal utility for the principal if she does not delegate in the current period. Then we have

$$\pi_0(u) = \sup_{u_d \in [0,\bar{v}]} \delta \pi_d(u_d) \tag{3}$$
$$s.t. \quad \delta u_d = u.$$

If the principal chooses to delegate, then the agent's actions can be characterized by a pair of numbers (a^g, a^b), where a^g is the probability of initiating a project when the agent observes nothing, and a^b is the probability when the agent observes a bad signal. Without loss of generality, I restrict attention to pure strategies[1]. Since the principal has the option to not delegate, there is no benefit for her to delegate and induce $(0,0)$. Then there are only three actions remaining, $(1,0)$, $(1,1)$, and $(0,1)$. The following result shows that we only need to consider the first two actions.

Lemma 1. *It is never optimal for the principal to induce* $(0,1)$.

[1]If it is optimal for the principal to induce a mixed strategy, then it must be optimal to induce at least one of the pure strategies in its support.

Intuitively, $(0,1)$ is worse for the principal than both $(0,0)$ and $(1,1)$. As for the agent, $(0,1)$ is better than $(0,0)$ but worse than $(1,1)$. To provide the agent with the promised utility, instead of inducing $(0,1)$, the principal can simply mix between $(0,0)$ and $(1,1)$ and be better off.

Denote by $\pi_1(u)$ the optimal utility for the principal if she induces $(1,0)$. Then

$$\pi_1(u) = \sup_{u_h,u_l,u_n} p[(1-\delta)(h-c)+\delta\pi_h(u_h)]+(1-p)\theta\delta\pi_n(u_n)+(1-p)(1-\theta)[(1-\delta)(l-c)+\delta\pi_l(u_l)] \tag{4}$$

subject to

$$\begin{aligned}
&(\mathrm{IC_G}) \qquad (1-\delta)(\hat{p_\theta}h+(1-\hat{p_\theta})l)+\delta(\hat{p_\theta}u_h+(1-\hat{p_\theta})u_l)\geq \delta u_n\\
&(\mathrm{IC_B}) \qquad (1-\delta)l+\delta u_l \leq \delta u_n\\
&(\mathrm{PK}) \qquad u=(1-\delta)(ph+(1-p)(1-\theta)l)+\delta(pu_h+(1-p)(1-\theta)u_l+(1-p)\theta u_n)\\
&(\mathrm{F}) \qquad u_h,u_l,u_n\in[0,\bar{v}]
\end{aligned}$$

where $\hat{p_\theta}=\frac{p}{p+(1-p)(1-\theta)}$ is the posterior belief of the agent conditional on no signals. The first incentive compatibility constraint states that the agent does not want to deviate to $(0,0)$. In other words, he is willing to initiate the project when he observes no signals. Similarly, the second constraint ensures that the agent does not want to initiate the project when he observes a bad signal. The Promise Keeping condition ensures that the agent gets exactly the promised utility. Finally, the Feasibility condition requires that the agent's continuation utility always lies in the interval $[0,\bar{v}]$.

Similarly, denote by $\pi_2(u)$ the optimal utility for the principal if she induces $(1,1)$. Then

$$\pi_2(u) = \sup_{u_h,u_l,u_n} (1-\delta)(\bar{v}-c)+\delta(p\pi(u_h)+(1-p)\pi(u_l)) \tag{5}$$

subject to

$$
\begin{array}{ll}
(\mathrm{IC_G}) & (1-\delta)(\hat{p_\theta} h + (1-\hat{p_\theta})l) + \delta(\hat{p_\theta} u_h + (1-\hat{p_\theta})u_l) \geq \delta u_n \\
(\mathrm{IC_B}) & (1-\delta)l + \delta u_l \geq \delta u_n \\
(\mathrm{PK}) & u = (1-\delta)\bar{v} + \delta(pu_h + (1-p)u_l) \\
(\mathrm{F}) & u_h, u_l, u_n \in [0, \bar{v}]
\end{array}
$$

Given the agent's utility u, the principal chooses the largest among $\pi_0(u)$, $\pi_1(u)$, and $\pi_2(u)$. Since the principal has access to randomized mechanisms, her maximization problem can be expressed as

$$\pi(u) = \max_{d_0,d_1,d_2,u_0,u_1,u_2} d_0\pi_0(u_0) + d_1\pi_1(u_1) + d_2\pi_2(u_2) \tag{6}$$

$$
\begin{aligned}
s.t. \quad & u = d_0u_0 + d_1u_1 + d_2u_2 \\
& d_0 + d_1 + d_2 = 1 \\
& d_0, d_1, d_2 \in [0,1]
\end{aligned}
$$

where d_i is the probability of choosing (π_i, u_i) $(i = 0,1,2)$. The following result characterizes $\pi(u)$.

Proposition 1. *Let $\underline{u} = ph$ and $\bar{u} = \bar{v} - \frac{(1-\delta)(1-p)}{1-\delta p}l$. There exists $\delta^* \in (0,1)$ such that when $\delta > \delta^*$, $\pi(u)$ is linear on $[0, \underline{u}]$ and $[\bar{u}, \bar{v}]$, and concave on $[\underline{u}, \bar{u}]$. Moreover, $\pi(u) = \pi_1(u)$ for $u \in [\underline{u}, \bar{u}]$.*

Specifically, it is optimal for the principal to induce $(1,0)$ whenever possible and randomize when u is very small or very large.

In the absence of monetary transfers, the principal can only use future delegation decisions to induce her preferred actions. This is feasible only when δ is large enough. Intuitively, when δ is small, the agent does not value future payoffs and would choose his most preferred actions in each period.

Figure 1 shows an example of $\pi(u)$ when $(1-\hat{p_\theta})(l-c)+\hat{p_\theta}(h-c)\geq 0$ and δ is large enough that, by Proposition 1, the principal optimally induces $(1,0)$ when the continuation utility u_t is in between $\underline{u}$ and $\bar{u}$.

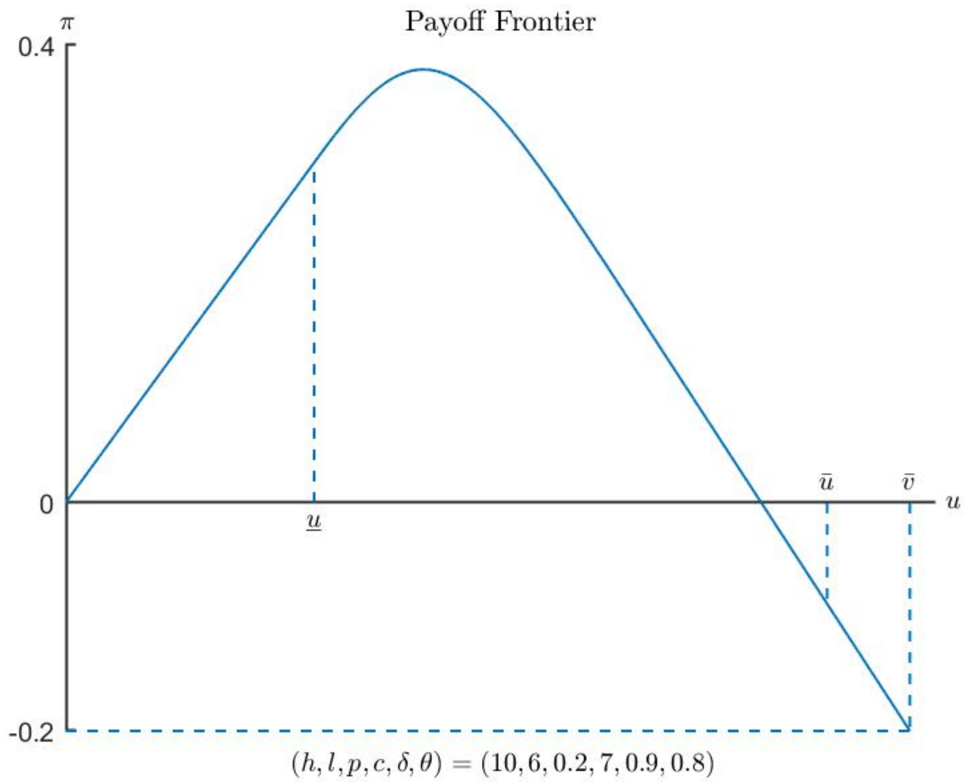

Figure 1: Payoff Frontier

3.2 Optimal Mechanism

By Proposition 1, the optimal mechanism has three possible forms. First, when $\bar{v}<c$ and θ is very small, it is not optimal for the principal to delegate even if the agent takes her preferred action. Therefore, the principal never delegates to the agent in the optimal mechanism. Secondly, when $\bar{v}>c$ and θ is very small, $\pi(u)$ may achieve the maximum at $\bar{v}$. In this case, the principal always delegates to the agent in the optimal mechanism and the agent always initiates the project. Finally, when the maximum is not achieved at either extreme, by concavity of $\pi(u)$, the maximum is achieved at some $u^*\in[\underline{u},\bar{u}]$. The principal delegates at first and induces $(1,0)$. The continuation utility decreases when the agent initiates a bad project and increases when the agent does not initiate a project. The continuation is randomly determined when $u < \underline{u}$ or $u > \bar{u}$. Eventually, the principal either stops delegating or commits to delegating forever.

The following theorem summarizes the above findings and fully characterizes the optimal mechanism.

Theorem 1. *The optimal mechanism takes one of the following three forms:*

(i) When $\bar{v} < c$, there exists $\hat{\theta} \in (0,1)$ such that the principal never delegates in the optimal mechanism if and only if $\theta \leq \hat{\theta}$;

(ii) When $\bar{v} \geq c$, there exists $\tilde{\theta} \in [0,1)$ such that the principal always delegates in the optimal mechanism if and only if $\theta \leq \tilde{\theta}$;

(iii) Otherwise, there exists a stochastic time $t^* > 0$ such that up to t^*, the principal delegates and the agent initiates a project if and only if he observes no signal. At t^*, the principal either stops delegating or commits to delegating forever, depending on the public history. t^* is also determined by the public history and is finite with probability one.

When $\bar{v} < c$, the cutoff $\hat{\theta}$ is determined by the equation $(1-\hat{p_\theta})(l-c)+\hat{p_\theta}(h-c) = 0$. This equation is equivalent to the statement that the principal gets a zero expected payoff when the agent always takes her preferred action, $(1,0)$. When θ is smaller, it is never optimal for the principal to delegate. The interesting feature is that the principal will delegate at the beginning as long as θ is large enough such that $(1-\hat{p_\theta})(l-c)+\hat{p_\theta}(h-c)$ is positive.

In the scenario where θ and δ are large enough, the maximum of $\pi(u)$ is achieved somewhere in between $\underline{u}$ and $\bar{u}$. The agent's continuation utility u_t evolves as follows: if a good project is initiated, the continuation utility is unchanged at $u_{t+1} = u_t$; if a bad project is initiated, the continuation utility decreases to $u_{t+1} = \frac{(1-\delta p)u_t-(1-\delta)\bar{v}}{\delta(1-p)}$; if the agent does not initiate the project, the continuation utility increases to $u_{t+1} = \frac{(1-\delta p)u_t-(1-\delta)ph}{\delta(1-p)}$. The evolution of u_t is calculated based on the promise keeping condition and the fact that (IC_B) is binding. When the agent observes no signal, there is no conflict of interests and the principal does not need to reward the agent for taking her desired action. Therefore, to induce $(1,0)$, the principal only needs to incentivize the agent to not initiate a project when he observes a bad signal. Since $\pi(u)$ is concave, it is not optimal for the principal to provide a larger reward or punishment. In other words, the agent should be induced to be indifferent between initiating the project or not when he observes a bad signal.

The implementation of the optimal mechanism is straightforward. The initial promised utility, u_0, can be viewed as a quota for bad projects. It decreases when the agent initiates a bad project and increases when the agent avoids a bad project. The principal stops delegating when the quota drops to zero and commits to delegating forever when the quota increases to some cutoff value.

3.3 Comparative Statics

In this section, I study how the principal's and the agent's payoff in the optimal mechanism depend on the agent's ability, θ. Let $\pi^*(\theta)$ be the maximum utility of the principal given the agent's ability. Let θ^* be the largest θ such that the principal never delegates or always delegates in the optimal mechanism. To avoid triviality, I focus on the case where $\theta > \theta^*$. The following result says that the principal is strictly better off when the agent's ability increases.

Proposition 2. *Assume $\theta > \theta^*$. Then $\pi^*(\theta)$ is strictly increasing in θ.*

Intuitively, when the agent is of higher ability, he gets more precise signals and will initiate fewer bad projects in expectation. At the same time, the principal can better monitor the agent's actions. Specifically, when the agent initiates a bad project, the principal is more confident that the agent has taken an undesirable action. As a result, the agent can extract less information rent from the principal. In the extreme case where $\theta = 1$, the principal stops delegating with one bad project initiated and never commits to delegating forever. Consequently, the agent only initiates good projects. In this case, the principal achieves the first best payoff.

The relationship between the agent's ability and his payoff in the optimal mechanism is more complicated. As noted above, the agent is induced to be indifferent between initiating a project or not when he observes a bad signal. Therefore, higher ability does not directly benefit the agent. In other words, whether the agent is better off or worse off is solely determined by the quota for bad projects in the optimal mechanism.

Let $u^*(\theta)$ be the agent's payoff in the optimal mechanism. $u^*(\theta)$ is chosen by the principal to maximize her expected payoff. There are two considerations for the principal in choosing $u^*(\theta)$. First, the principal wants to extend the duration before achieving one of the two absorbing states. Intuitively, the principal's payoff is largest when the agent takes action $(1, 0)$, which happens when u_t is between $\underline{u}$ and $\bar{u}$. Secondly, the principal wants to increase the probability of achieving $u_t = 0$ when $\bar{v} < c$ and increase the probability of achieving $u_t = \bar{v}$ when $\bar{v} > c$. Let $u^0(\theta)$ be the promised utility which maximizes the expected duration before achieving one of the two absorbing states. Then $u^*(\theta) < u^0(\theta)$ when $\bar{v} < c$ and $u^*(\theta) > u^0(\theta)$ when $\bar{v} > c$.

When θ increases, the agent is more likely to avoid a bad project so u_t is more likely to jump to the right. As a result, $u^0(\theta)$ decreases. At the same time, since the principal gets a larger payoff when the agent takes $(1, 0)$, the aforementioned first consideration becomes more important relative to the second. Therefore, the difference between $u^*(\theta)$ and $u^0(\theta)$ should be smaller. When $\bar{v} > c$, by $u^*(\theta) > u^0(\theta)$, the conjecture is that $u^*(\theta)$ decreases. But when $\bar{v} < c$, the direction of the change in $u^*(\theta)$ depends on which effect is larger. Simulations show that $u^*(\theta)$ increases when θ is small and decreases when θ is large. See Figure 2 for an illustration.

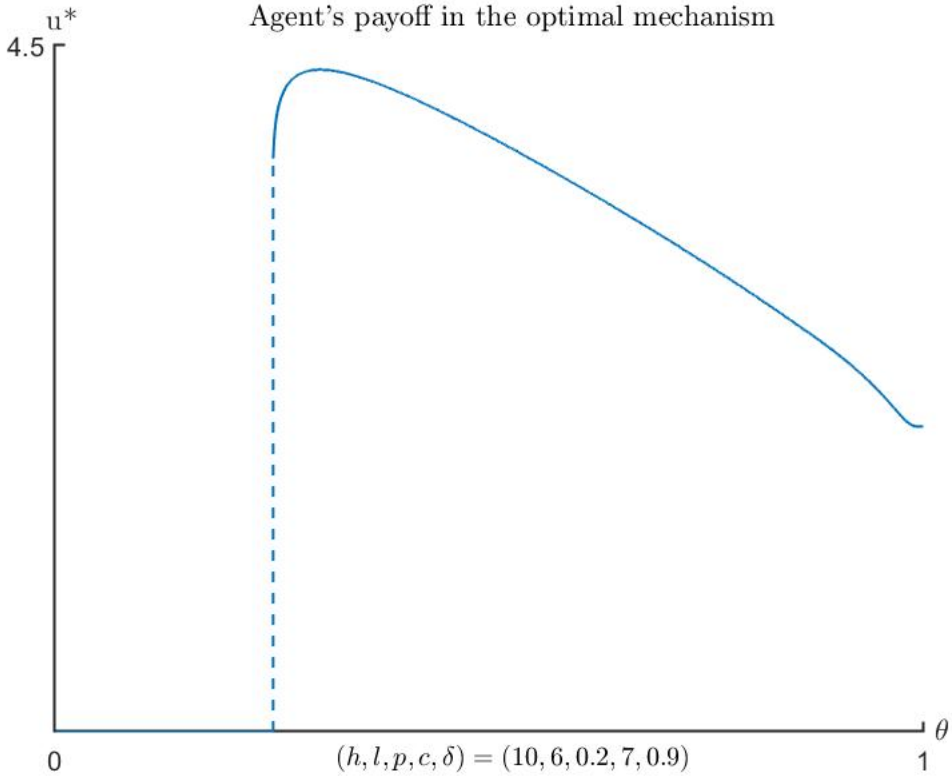

Figure 2: The Agent's Payoff As a Function of His Ability

4 Unknown Types

In this section, I consider the general model where the agent's type, θ, is unknown. By the revelation principle, I focus on truth-telling mechanisms where the agent reports his type at time 0. Before solving for the optimal mechanism, I first consider the agent's incentives to mimic other types.

By the argument in the previous section, given any optimal mechanism in the benchmark, the agent's utility does not depend on his ability. The reason is that the agent is made indifferent between initiating a bad project or not in the optimal mechanism. Therefore, all types of the agent can choose the action $(1, 1)$ and get the same utility. As a result, if the principal were to offer the optimal mechanism in the benchmark, whether one type has incentives to mimic the other type is determined by the relative value of $u^*(\theta)$.

For better intuition, consider the simple case with two types, $\theta_H > \theta_L$. As illustrated in

Figure 2, $u^*(\theta)$ is not monotone in θ. When θ is relatively high, the high type has incentives to mimic the low type. But when θ is relatively low and $\bar{v} < c$, the low type wants to mimic the high type. Intuitively, when the agent's type is high, the principal anticipates that he should initiate fewer bad projects in expectation. Therefore, the principal will be less tolerant toward bad projects. On the other hand, when the agent's type is low, the relationship is not very valuable to the principal. As a result, the principal does not have a strong incentive to maintain the relationship and avoid termination by mistake. Instead, once the agent's performance is not satisfactory, the principal would stop delegating immediately to avoid future loss.

I first consider the case where the low type has incentives to mimic the high type. Apparently, the high type can always mimic the low type without a cost, so he cannot be worse off in the optimal mechanism. On the other hand, since $u^*(\theta_H) < u^*(\theta_L)$, it is not optimal for the principal to make the high type better off. In general, we have the following result.

Theorem 2. *There exists $\bar{\theta} < 1$ such that when the agent's type is known to be above $\bar{\theta}$, the principal optimally offers the same mechanism to all types of the agent.*

$\bar{\theta}$ is defined such that u^{θ} decreases in θ when $\theta > \bar{\theta}$. Then the high type always has incentives to mimic the low type and it is not optimal for the principal to separate them. The optimal mechanism is the same as the one in the benchmark for some type θ in between $\bar{\theta}$ and 1. In spite of using the same mechanism for all types, the evolution of the principal-agent relationship is different for different types. High types are able to observe more bad signals and will thus initiate fewer bad projects in expectation. Therefore, it is more likely for the principal to have a long-time relationship with a high type.

This result crucially depends on the assumption of learning from bad news. With this setup, there is a conflict of interests only when the agent observes a bad signal. In other words, the agent only needs to be incentivized to take the principal's preferred action when he observes a bad signal. Since a bad signal is conclusive evidence for any type of the agent, the incentive compatibility constraint is the same for all types. As a result, given the agent's continuation utility, the optimal mechanism is the same for all types of the agent. In other words, the only difference between the optimal mechanism for different types is the initial promised utility. When the high type has incentives to mimic the low type, it is optimal to offer the same promised utility to both types of the agent. Therefore, the optimal mechanism is pooling.

Things are more complicated when the low type has incentives to mimic the high type. Since the low type has less information, he cannot always perfectly mimic the high type and may get a lower payoff than the high type given the same mechanism. However, as mentioned before, the low type gets the same payoff as the high type given the optimal mechanism for the high type in the benchmark. Therefore, in order to separate different types, the principal needs to increase the reward for a good action and increase the punishment for a bad action, so that the low type gets a smaller payoff. It is similar to the classic adverse selection model where the low type's optimal effort is smaller than the first best. The difference is that in this model, the low type has incentives to mimic the high type. By contrast, in the classic adverse selection model, the high type has incentives to mimic the low type.

Due to the dynamic feature of this model, it is difficult to characterize the exact form of the optimal mechanism in this scenario. However, it can be shown that the optimal mechanism is always separating when there are only two types and the low type has incentives to mimic the high type.

Proposition 3. *Suppose the agent has only two types $\theta_H > \theta_L$ and $u^*(\theta_H) > u^*(\theta_L)$. Then it is not optimal for the principal to offer the same mechanism to both types of the agent.*

Specifically, the optimal mechanism for the high type features a larger reward and punishment when the continuation utility u_t is between 0 and $\underline{u}$. Namely, the agent is promised a larger continuation utility when he does not initiate a project and a smaller continuation utility when he initiates a bad project. As a result, the low type will get a smaller payoff from this mechanism. At the same time, since $\pi(u)$ is linear on $[0, \underline{u}]$, the principal's payoff from the high type's mechanism does not change. Therefore, the principal is strictly better off by separating two types of the agent.

5 Concluding Remarks

In this paper, I study the optimal dynamic delegation policy in the presence of ongoing conflicts of interests and private information. The agent learns the quality of the project in each period from bad news and the probability of observing the bad news depends on the agent's type. In the benchmark where the agent's type is publicly known, the principal delegates at the beginning and the agent only initiates a project when he does not observe a bad news. Eventually, the principal either stops delegating or commits to delegating forever, depending on the history. In the general model where the agent's type is unknown, the principal optimally offers a pooling mechanism if the agent's type is known to be above a cutoff value. The mechanism is the same as the one in the benchmark for a specific type of the agent. If the possible types of the agent are relatively low, the principal may optimally separate different types of the agent. The mechanism offered to the high type features a larger reward and punishment.

References

Aghion, P. and M. O. Jackson (2016): "Inducing leaders to take risky decisions: dismissal, tenure, and term limits," *American Economic Journal: Microeconomics*, 8, 1–38.

Alonso, R. and N. Matouschek (2008): "Optimal delegation," *The Review of Economic Studies*, 75, 259–293.

Cvitanić, J., X. Wan, and H. Yang (2013): "Dynamics of contract design with screening," *Management science*, 59, 1229–1244.

Gershkov, A. and M. Perry (2012): "Dynamic contracts with moral hazard and adverse selection," *The Review of Economic Studies*, 79, 268–306.

Guo, Y. and J. Hörner (2020): "Dynamic allocation without money," .

Guo, Y. and A.-K. Roesler (2016): "Private learning and exit decisions in collaboration," *Northwestern WP*.

Halac, M., N. Kartik, and Q. Liu (2016): "Optimal contracts for experimentation," *The Review of Economic Studies*, 83, 1040–1091.

Holmström, B. (1978): *On Incentives and Control in Organizations.*, Stanford University.

——— (1980): "On the theory of delegation," Tech. rep., Discussion Paper.

Kováč, E. and T. Mylovanov (2009): "Stochastic mechanisms in settings without monetary transfers: The regular case," *Journal of Economic Theory*, 144, 1373–1395.

Legros, P. (1993): "Information revelation in repeated delegation," *Games and Economic Behavior*, 5, 98–117.

Li, J., N. Matouschek, and M. Powell (2017): "Power dynamics in organizations," *American Economic Journal: Microeconomics*, 9, 217–41.

Lipnowski, E. and J. Ramos (2020): "Repeated delegation," *Journal of Economic Theory*, 188, 105040.

Martimort, D. and A. Semenov (2006): "Continuity in mechanism design without transfers," *Economics Letters*, 93, 182–189.

Melumad, N. D. and T. Shibano (1991): "Communication in settings with no transfers," *The RAND Journal of Economics*, 173–198.

Sannikov, Y. (2007): "Agency problems, screening and increasing credit lines," *Unpublished Paper*.

Spear, S. E. and S. Srivastava (1987): "On repeated moral hazard with discounting," *The Review of Economic Studies*, 54, 599–617.

Sung, J. (2005): "Optimal contracts under adverse selection and moral hazard: a continuous-time approach," *The Review of Financial Studies*, 18, 1021–1073.

Thomas, J. and T. Worrall (1990): "Income fluctuation and asymmetric information: An example of a repeated principal-agent problem," *Journal of Economic Theory*, 51, 367–390.

Ulbricht, R. (2016): "Optimal delegated search with adverse selection and moral hazard," *Theoretical Economics*, 11, 253–278.

Appendix

Proof of Lemma 1. Suppose the principal wants to induce $(0,1)$. Denote by π_{01} his optimal utility. We want to show that $\pi_{01}(u) < \pi(u)$, for any u.

The principal's maximization problem is

$$\pi_{01}(u) = \sup_{u_h, u_l, u_n} (1-p)\theta[(1-\delta)(l-c) + \delta\pi_l(u_l)] + (1-(1-p)\theta)\delta\pi_n(u_n) \tag{7}$$

subject to

$$\begin{aligned}
&(\text{IC}_\text{G}) && (1-\delta)(\hat{p_\theta}h + (1-\hat{p_\theta})l) + \delta(\hat{p_\theta}u_h + (1-\hat{p_\theta})u_l) \leq \delta u_n \\
&(\text{IC}_\text{B}) && (1-\delta)l + \delta u_l \geq \delta u_n \\
&(\text{PK}) && u = (1-\delta)(1-p)\theta l + \delta((1-p)\theta u_l + (1-(1-p)\theta)u_n) \\
&(\text{F}) && u_h, u_l, u_n \in [0, \bar{v}]
\end{aligned}$$

where $\hat{p_\theta} = \frac{p}{p+(1-p)(1-\theta)}$ is the posterior belief of the agent conditional on no signals.

I first claim that $\pi_h(u) = \pi_l(u) = \pi_n(u) = \pi_d(u) = \pi(u)$ for all $u \in [0, \bar{v}]$. Suppose $\pi_h(u) < \pi(u)$ for some u. Consider an alternative strategy which promises the same continuation utility to the agent, but achieves a utility of $\pi(u)$ conditional on a good project. Since the agent's continuation utility is unchanged, the agent's incentives are unchanged and all constraints are still satisfied. Therefore, the principal gets a strictly larger payoff with this new strategy. The same argument applies to $\pi_l(u)$ and $\pi_n(u)$.

Let u_h^*, u_l^*, and u_n^* be the optimal choice variables. Define $u^{'} := (1-p)\theta u_l^* + (1-(1-p)\theta)u_n^*$.

Let $u_d = u^0$ in equation (3). Then $\pi_0(\delta u^0) = \delta\pi(u^0)$. Let $u_h = u_l = u^0$ and $u_n = 0$ in equation (5). It is easy to verify that both IC's are satisfied. Therefore, $\pi_2((1-\delta)\bar{v} + \delta u^0) \geq (1-\delta)(\bar{v} - c) + \delta\pi(u^0)$.

Since the principal can commit to a random mechanism, $\pi(u)$ is concave. Specifically, we can show that for all $\alpha \in [0,1]$ and $u', u'' \in [0, \bar{v}]$, $\pi(\alpha u' + (1-\alpha)u'') \geq \alpha\pi(u') + (1-\alpha)\pi(u'')$. To promise a continuation utility of $\alpha u' + (1-\alpha)u''$, one possible way is to randomize between u' and u''. In other words, the principal promises u' with probability α and promises u'' with probability $1-\alpha$. In this way, the principal can get $\alpha\pi(u') + (1-\alpha)\pi(u'')$. By definition,

$\pi(u)$ is the largest utility the principal can get given the agent's utility as u. Therefore, $\pi(\alpha u' + (1-\alpha)u'') \geq \alpha\pi(u') + (1-\alpha)\pi(u'')$.

By concavity of $\pi(u)$, $\pi(u^0) \geq (1-p)\theta\pi(u_l^*) + (1-(1-p)\theta)\pi(u_n^*)$. By $h > c > l$, we have $\pi_0(\delta u^0) > \pi_{01}(u)$ and $\pi_2((1-\delta)\bar{v} + \delta u^0) > \pi_{01}(u)$. By the promise keeping condition, $\delta u^0 < u < (1-\delta)\bar{v} + \delta u^0$. Therefore, there exists $d \in (0,1)$, such that $d \cdot \delta u^0 + (1-d) \cdot [(1-\delta)\bar{v} + \delta u^0] = u$. Let $d_0 = d, d_1 = 0, d_2 = 1-d, u_0 = \delta u^{'}$, and $u_2 = (1-\delta)\bar{v} + \delta u^0$ in equation (6). Then we have

$$\pi(u) \geq d \cdot \pi_0(\delta u^0) + (1-d) \cdot \pi_2((1-\delta)\bar{v} + \delta u^0) > \pi_{01}(u). \tag{8}$$

Therefore, it is never optimal to induce $(0,1)$. □

Proof of Proposition 1. As argued in the main text, the principal optimally induces $(1,0)$ or $(1,1)$, not delegate, or randomizes. If the principal does not delegate, by equation (3), it is equivalent to randomizing between 0 and $u_d = \frac{u}{\delta}$. If the principal induces $(1,1)$, consider the maximization problem given by (5). Since u_n does not enter the objective function, we can set $u_n = 0$ so that both IC constraints do not bind. By concavity of $\pi(\cdot)$, optimally $u_h = u_l$. By the promise keeping condition, we have

$$u_h(u) = u_l(u) = \frac{1}{\delta}(u - (1-\delta)\bar{v}). \tag{9}$$

As a result, inducing $(1,1)$ is equivalent to randomizing between $\bar{v}$ and $u_h(u)$. Therefore, the original problem reduces to the one in which the principal only induces $(1,0)$ or randomizes.

When the principal induces $(1,0)$, let u_h^*, u_l^*, and u_n^* be the optimal choices in the maximization problem given by (4). By $\mathrm{IC_B}$, $u_n^* > u_l^*$. I first show that $\mathrm{IC_B}$ is binding. Suppose $(1-\delta)l + \delta u_l^* < \delta u_n^*$. Then there exists $\epsilon > 0$, such that

$$(1-\delta)l + \delta(u_l^* + \theta\epsilon) = \delta(u_n^* - (1-\theta)\epsilon). \tag{10}$$

Let $u_l' = u_l^* + \theta\epsilon$ and $u_n' = u_n^* - (1-\theta)\epsilon$. Since u_n is decreased and u_l is increased, $\mathrm{IC_G}$ is still satisfied. By the expression for u_l' and u_n', the promise keeping condition is also satisfied. Finally, since u_l' and u_n' are in between of u_l^* and u_n^*, the feasibility condition is satisfied as well. Therefore, (u_h^*, u_l', u_n') is feasible. By concavity of $\pi(\cdot)$, the objective function weakly increases when we replace u_l^*, u_n^* by u_l', u_n'. As a result, $\mathrm{IC_B}$ is binding and we have $u_n(u) = u_l(u) + \frac{1-\delta}{\delta} l$.

Consider the maximization problem given by (4). Since $\mathrm{IC_B}$ is binding, the $\mathrm{IC_G}$ constraint can be reduced to

$$u_l - u_h \leq \frac{1-\delta}{\delta}(h-l). \tag{11}$$

Suppose $u_h < u_l$. Then we can increase u_h and decrease u_l and u_n such that the promise keeping constraint is satisfied. Then all the constraints are still satisfied and, by the concavity of $\pi(\cdot)$, the objective function weakly increases. Therefore, $u_h \geq u_l$ in the optimal mechanism and the $\mathrm{IC_G}$ constraint is not binding. Similarly, if $u_h > u_n$, we can decrease u_h and increase u_l and u_n such that all the constraints are satisfied and the objective function weakly increases. Therefore, $u_l \leq u_h \leq u_n$ in the optimal mechanism.

By $u_n(u) = u_l(u) + \frac{1-\delta}{\delta} l$, the promise keeping condition reduces to

$$pu_h + (1-p)u_l = \frac{1}{\delta}(u - (1-\delta)\bar{v}). \tag{12}$$

Apparently, $u \geq (1-\delta)\bar{v}$. Since $u_n \leq \bar{v}$ and $u_n = u_l + \frac{1-\delta}{\delta} l$, $u_l \leq \bar{v} - \frac{1-\delta}{\delta} l$. Then

$$\begin{aligned} u &\leq (1-\delta)\bar{v} + \delta(pu_h + (1-p)(\bar{v} - \frac{1-\delta}{\delta} l)) \\ &\leq (1-\delta)\bar{v} + \delta\bar{v} - (1-p)(1-\delta)l \quad (u_h \leq \bar{v}) \\ &= \bar{v} - (1-p)(1-\delta)l. \end{aligned} \tag{13}$$

When δ is large enough, $(1-\delta)\bar{v} \leq \bar{v} - (1-p)(1-\delta)l$ and $\pi_1(\cdot)$ is defined on the interval $[(1-\delta)\bar{v}, \bar{v} - (1-p)(1-\delta)l]$.

For $u < (1-\delta)\bar{v}$ and $u > \bar{v} - (1-p)(1-\delta)l$ where $\pi_1(\cdot)$ is not defined, the principal optimally randomizes and thus $\pi(u)$ is linear on these two intervals. To promise the agent utility of 0, the principal can never delegate and thus $\pi(0) = 0$. Similarly, to promise the agent utility of $\bar{v}$, the principal has to delegate forever and thus $\pi(\bar{v}) = \bar{v} - c$.

Let $\underline{u}'$ be the supremum of u such that $\pi_0(u') = \pi(u')$ for any $u' \in [0, u]$. Then for any $u' \in (0, \underline{u}')$, $\pi(u') = \delta\pi(u'/\delta)$. Therefore, $\pi'_+(u') = \pi'_+(\frac{u'}{\delta})$. By concavity of $\pi(\cdot)$, $\pi(\cdot)$ is linear on the interval $(u', \frac{u'}{\delta})$. Since u' is arbitrary, $\pi(\cdot)$ is a straight line on $[0, \frac{\underline{u}'}{\delta}]$. Let $\underline{u} = \frac{\underline{u}'}{\delta}$. By definition of $\underline{u}'$, $\underline{u}$ is the endpoint of the straight line starting from 0, and therefore, must be supported by a pure action. If $\underline{u} = \bar{v}$, then $\pi(u)$ is linear on the entire domain. Now consider the case where $\underline{u} < \bar{v}$. Then the principal must induce $(1, 0)$ at $\underline{u}$. Since $\pi_1(\cdot)$ is defined on the interval $[(1-\delta)\bar{v}, \bar{v} - (1-p)(1-\delta)l]$, we must have $\underline{u} \geq (1-\delta)\bar{v}$.

Denote by k the slope of the line between 0 and $\underline{u}$. When $u = (1-\delta)\bar{v}$, $u_h = u_l = 0$ and $u_n = \frac{1-\delta}{\delta} l$. When δ is large enough, $\frac{1-\delta}{\delta} l \leq (1-\delta)\bar{v}$. Then $\pi'_+((1-\delta)\bar{v}) = k$. As u increases, we can either increase u_h or u_n (together with u_l) such that the promise keeping condition is satisfied. As long as $u_h < u$ or $u_n < u$, by the expression for $\pi_1(\cdot)$, we have $\pi'_+(u) = \pi'_+(u_h) = k$ or $\pi'_+(u) = \pi'_+(u_n) = k$. By definition of $\underline{u}$, for any $\epsilon > 0$, $\pi'_+(\underline{u}+\epsilon) < k$. Therefore, it must be the case that $u_h(\underline{u}) = u_n(\underline{u}) = \underline{u}$. By $\mathrm{IC_B}$ binding, $u_l(\underline{u}) = \underline{u} - \frac{1-\delta}{\delta} l$. Finally, by the promise keeping condition, we have $\underline{u} = ph$.

By the expression for $\pi_1(\cdot)$,

$$\pi(\underline{u}) = (1-\delta)[p(h-c)+(1-p)(1-\theta)(l-c)]+\delta[p\pi(\underline{u})+(1-p)\theta\pi(\underline{u})+(1-p)(1-\theta)\pi(\underline{u}-\frac{1-\delta}{\delta}l)]. \tag{14}$$

Since $\pi(\cdot)$ is linear on $[0, \underline{u}]$, $\pi(\underline{u} - \frac{1-\delta}{\delta} l) = \frac{\underline{u} - \frac{1-\delta}{\delta} l}{\underline{u}} \pi(\underline{u})$. Then we can solve for $\pi(\underline{u})$ as

$$\pi(\underline{u}) = \frac{p(h-c) + (1-p)(1-\theta)(l-c)}{1 + \frac{(1-p)(1-\theta)l}{\underline{u}}}. \tag{15}$$

By $h > c > l > 0$, $\pi(\underline{u})$ is strictly increasing in θ. In one extreme case where $\theta = 1$, $\pi(\underline{u}) = p(h-c)$. Intuitively, in this scenario, the agent perfectly identifies the quality of each project and only initiates good projects. In the other extreme case where $\theta = 0$, $\pi(\underline{u}) = \frac{\bar{v}-c}{\bar{v}} \underline{u}$, which is the same as randomizing between 0 and $\bar{v}$. In other words, as long as $\theta > 0$, $\pi(u)$ is not linear on the whole domain.

Define $\bar{u}' = \sup_u \ \pi_2(\bar{v} - u) = \pi(\bar{v} - u)$. Then by the same logic, the principal optimally induces $(1, 0)$ at $\bar{v} - \frac{\bar{u}'}{\delta}$ and $\pi(\cdot)$ is linear on $[\bar{v} - \frac{\bar{u}'}{\delta}, \bar{v}]$. Denote by k_2 the slope of this straight line. Let $\bar{u} = \bar{v} - \frac{\bar{u}'}{\delta}$.

Since $\pi_1(\cdot)$ is defined on the interval $[(1-\delta)\bar{v}, \bar{v} - (1-p)(1-\delta)l]$, we must have $\bar{u} \leq \bar{v} - (1-p)(1-\delta)l$. When $u = \bar{v} - (1-p)(1-\delta)l$, we have $u_h = u_n = \bar{v}$ and $u_l = \bar{v} - \frac{1-\delta}{\delta} l$.

Note that $\bar{v} - \frac{1-\delta}{\delta} l < \bar{v} - (1-p)(1-\delta)l$. As u decreases, we can decrease u_h so $\pi'_-(u) = k_2$. I next show that $u_h(\bar{u}) = \bar{u}$. Suppose $u_h(\bar{u}) < \bar{u}$. By the expression for $\pi_1(\cdot)$, we have $\pi'_-(\bar{u}) = \pi'_-(u_h(\bar{u})) = k_2$. By definition of $\bar{u}$, for any $\epsilon > 0$, $\pi'_-(\bar{u} - \epsilon) < k_2$, which contradicts to the previous statement. Therefore, $u_h(\bar{u}) = \bar{u}$. Together with the promise keeping condition, we can solve for $\bar{u}$ as $\bar{u} = \bar{v} - \frac{(1-\delta)(1-p)}{1-\delta p} l$. It can be easily verified that $\bar{u} > \bar{v} - \frac{1-\delta}{\delta} l$ and $\bar{u} > \underline{u}$.

Finally, we need to show that $\pi_1(u) = \pi(u)$ for $u \in [\underline{u}, \bar{u}]$. Suppose $\pi_1(u) < \pi(u)$ for some $u \in [\underline{u}, \bar{u}]$. Then $\pi(u)$ must be achieved by randomization. In other words, there exists $u_a < u < u_b$ and $\lambda \in (0,1)$ such that $u = \lambda u_a + (1-\lambda) u_b$ and $\pi(u) = \lambda \pi(u_a) + (1-\lambda)\pi(u_b)$. by definition of $\underline{u}$ and $\bar{u}$, we have $u_a \geq \underline{u}$ and $u_b \leq \bar{u}$. Then both $\pi(u_a)$ and $\pi(u_b)$ cannot be achieved by randomization. Therefore, $\pi_1(u_a) = \pi(u_a)$ and $\pi_1(u_b) = \pi(u_b)$. Let $u_l(u_a)$, $u_h(u_a)$, and $u_n(u_a)$ be the optimal choice given u_a. Define $u_l(u_b)$, $u_h(u_b)$, and $u_n(u_b)$ similarly. Consider the optimization problem given u. Let $u_i(u) = \lambda u_i(u_a) + (1-\lambda) u_i(u_b)$, $i = h, l, n$. Then the choice variables $u_l(u)$, $u_h(u)$, and $u_n(u)$ satisfy the promise keeping condition. Since $\pi_h(\cdot) = \pi_l(\cdot) = \pi_n(\cdot) = \pi(\cdot)$, by equation (4) and concavity of $\pi(\cdot)$, $\pi_1(u) \geq \pi(u)$. Therefore, $\pi_1(u) = \pi(u)$ for any $u \in [\underline{u}, \bar{u}]$. $\square$

Proof of Proposition 2. (For convenience, the order of the proof of Proposition 2 and the proof of Theorem 1 is reversed.)

Assume $\theta_1 > \theta_2 > \theta^*$. Denote by Γ_2 the optimal mechanism when $\theta = \theta_2$. By the proof of Proposition 1, given Γ_2 and $u \in [\underline{u}, \bar{u}]$, the type-$\theta_2$ agent is always induced to be indifferent between initiating a bad project or not. Then the agent can get the same utility by always initiating a project, i.e., taking the action $(1,1)$. As a result, the agent of any type gets the same continuation utility given the mechanism Γ_2 and any public history. Specifically, the IC_B constraint is the same for the type-θ_1 agent. In other words, the type-θ_1 agent is always indifferent between initiating a bad project or not given Γ_2 and $u \in [\underline{u}, \bar{u}]$. Since $\pi_1(u) > \pi_2(u)$ for $u \in [\underline{u}, \bar{u}]$, the principal is strictly better off when the agent does not initiate a bad project at any point of time. Therefore, given Γ_2, the principal gets a strictly larger payoff when the agent is of a higher type. Then we must have $\pi^*(\theta_1) > \pi^*(\theta_2)$. $\square$

Proof of Theorem 1. When $\bar{v} < c$, whether the principal ever delegates to the agent depends on the sign of $\pi(\underline{u})$. By concavity of $\pi(\cdot)$ and $\pi(0) = 0$, when $\pi(\underline{u}) < 0$, $\pi(u) \leq 0$ for any

u. Therefore, it is optimal to never delegate to the agent. When $\pi(\underline{u}) > 0$, the maximum of $\pi(u)$ is achieved at some $u \in [\underline{u}, \bar{u}]$. By equation (15), $\pi(\underline{u}) > 0$ if and only if $(1-\hat{p_\theta})(l-c) + \hat{p_\theta}(h-c) > 0$. Since $\pi(\underline{u})$ is strictly increasing in θ, the cutoff $\hat{\theta}$ is determined by the equation $(1-\hat{p_\theta})(l-c) + \hat{p_\theta}(h-c) = 0$.

Similarly, when $\bar{v} \geq c$, the maximum of $\pi(u)$ is either achieved at $\bar{v}$ or somewhere between $\underline{u}$ and $\bar{u}$. By Proposition 2, $\pi(u)$ is increasing in θ for any $u \in (0, \bar{v})$. Therefore, the maximum of $\pi(u)$ is increasing in θ. When $\theta = 1$, by equation (15), $\pi(\underline{u}) = p(h-c) > \bar{v} - c$. In other words, the maximum of $\pi(u)$ is greater than $\pi(\bar{v})$ when $\theta = 1$. Then there exists $\tilde{\theta} < 1$ such that the maximum is achieved at $\bar{v}$ if and only if $\theta \leq \tilde{\theta}$. In this case, the principal always delegates to the agent in the optimal mechanism.

Finally, when the maximum of $\pi(u)$ is achieved somewhere between $\underline{u}$ and $\bar{u}$, the principal delegates at the beginning. By $\pi_1(u) = \pi(u)$ for $u \in [\underline{u}, \bar{u}]$, the agent is induced to take the action $(1, 0)$ as long as $u_t \in [\underline{u}, \bar{u}]$. Randomization takes place when $u_t \in (0, \underline{u})$ or $u_t \in (\bar{u}, \bar{v})$. The principal stops delegating once $u_t = 0$ and commits to delegating forever once $u_t = \bar{v}$.

The only thing left to show is that t^* is finite with probability one. First, there must exist a positive integer N such that for any initial $u_0 \in [\underline{u}, \bar{u}]$, the principal stops delegating after N consecutive bad projects. Otherwise it is impossible to induce the agent to not initiate a project when he observes a bad signal. Denote by $\epsilon > 0$ the probability that the agent initiates N consecutive bad projects. Then we can conclude that $\mathbb{P}(t^* \leq N \mid u_0 = u) \geq \epsilon$ for any $u \in [0, \bar{v}]$. As a result, for any positive integer k, $\mathbb{P}(t^* > kN \mid u_0 = u) \leq (1-\epsilon)^k$. Then for any $u \in [0, \bar{v}]$, $\mathbb{E}(t^* \mid u_0 = u) \leq \sum_{k=1}^{\infty} kN\epsilon(1-\epsilon)^{k-1}$ is finite. Therefore, t^* is finite with probability 1. □

Proof of Theorem 2. First I show that conditional on the agent's promised utility u, the optimal mechanism is the same for all types. By the proof of Proposition 1, the IC_B constraint is binding and $u_n(u) - u_l(u) = \frac{1-\delta}{\delta} l$. By equation (4), $\pi(u)$ is maximized when $\theta\pi'_+(u_n) + (1-\theta)\pi'_+(u_l) = \pi'_+(u_h)$, for any $u \in [\underline{u}, \bar{u}]$. By the envelope theorem, $\pi'_+(u) = p\pi'_+(u_h) + (1-p)\theta\pi'_+(u_n) + (1-p)(1-\theta)\pi'_+(u_l)$ for any $u \in [\underline{u}, \bar{u}]$. Therefore, $\pi'_+(u) = \pi'_+(u_h)$ for any $u \in [\underline{u}, \bar{u}]$. By concavity of $\pi(\cdot)$, it is easy to verify that it is optimal to choose $u_h = u$. Together with the promise keeping condition, we have $u_l(u) = \frac{(1-\delta p)u - (1-\delta)\bar{v}}{\delta(1-p)}$ and $u_n(u) = u_l(u) = \frac{(1-\delta p)u - (1-\delta)ph}{\delta(1-p)}$, which are independent of θ. Since $\underline{u}$ and $\bar{u}$ are also independent of θ, the optimal mechanism for any type of the agent is the same conditional on a given $u \in [0, \bar{v}]$.

Next, I show that there exists $\bar{\theta} < 1$ such that $u^*(\theta)$ is decreasing on $[\bar{\theta}, 1]$. Consider any $\theta_1 < \theta_2 \leq 1$ which are arbitrarily large. Let $\pi^1(u)$ and $\pi^2(u)$ be the corresponding payoff frontier. It is sufficient to show that for any $u \in [0, \bar{v})$, $(\pi^1)'_+(u) \leq 0$ implies $(\pi^2)'_+(u) \leq 0$. Consider $\pi(u + \epsilon) - \pi(u)$ where ϵ is arbitrarily small. When $u_t \in [\underline{u}, \bar{u}]$, the agent takes the same action $(1, 0)$ and the principal's payoff is not affected. When $u_t < \underline{u}$, there is a larger probability that it will end up with $\underline{u}$ when the initial u is larger. Similarly, when $u_t > \bar{u}$, there is a larger probability that it will end up with $\bar{v}$ when the initial u is larger. Then there exists $c_1, c_2 > 0$ such that $\pi'_+(u) = c_1 k_1 + c_2 k_2$, where k_1 and k_2 are the slopes of the linear parts of $\pi(u)$. Specifically, denote by τ_1 the smallest t such that $u_t < \underline{u}$ when u_t enters the region $[0, \underline{u})$ before $[\bar{u}, \bar{v}]$, and let $\tau_1 = \infty$ when u_t enters $[\bar{u}, \bar{v}]$ first. Define τ_2 in the symmetric way. Then by the expression for $u_l(u)$ and $u_n(u)$ derived above, $c_1 = \sum_{i=0}^{\infty} \sum_{j \leq i} \mathbb{P}(\tau_1 = i, J = j) \delta^i (\frac{1-\delta p}{\delta(1-p)})^j$, where J denotes the number of periods in which the agent initiates a project. Similarly, $c_2 = \sum_{i=0}^{\infty} \sum_{j \leq i} \mathbb{P}(\tau_2 = i, J = j) \delta^i (\frac{1-\delta p}{\delta(1-p)})^j$. By Proposition 2, as θ increases, k_1 increases and k_2 decreases. Therefore, we only need to show that $\frac{c_1(\theta)}{c_2(\theta)} k_1(\theta)$ decreases in θ when θ is large enough. Clearly, $c_2(\theta)$ increases in θ when θ is large enough. Then we just need to show $c_1(\theta) k_1(\theta)$ decreases in θ. By equation (15), $k_1(\theta) = \frac{p(h-c)+(1-p)(1-\theta)(l-c)}{ph+(1-p)(1-\theta)l}$. Let $\eta = 1 - \theta$. Then $c_1 = O(\eta)$. It is easy to verify that $k_1(\eta) \cdot \eta$ increases in η when η is small enough. Therefore, there exists $\bar{\eta} > 0$ such that $k_1(\eta) c_1(\eta)$ increases in η on $[0, \bar{\eta}]$. Consequently, there exists $\bar{\theta} < 1$ such that $\pi'_+(u)$ decreases in θ on $[\bar{\theta}, 1]$. In other words, for $\bar{\theta} \leq \theta_1 < \theta_2 \leq 1$, $(\pi^1)'_+(u) \leq 0$ implies $(\pi^2)'_+(u) \leq 0$. Then $u^*(\theta)$ is decreasing on $[\bar{\theta}, 1]$

Finally, I show that if the agent's type is known to be above $\bar{\theta}$, all types get the same payoff in the optimal mechanism. Consider any two types $\theta_L < \theta_H$ and let their payoffs in the optimal mechanism be u_L and u_H. Since the high type can always mimic the low type, we have $u_H \geq u_L$. Suppose $u_H > u_L$. First consider the case where $u_L < u^*(\theta_L)$. Since $\pi_L(u)$ is increasing in u on $[0, u^*(\theta_L)]$, $\pi_L(\min(u^*(\theta_L), u_H)) \geq \pi_L(u_L)$. Then the principal can promise the low type a utility of $\min(u^*(\theta_L), u_H)$ and offer the mechanism on the payoff frontier. Since IC_B is always binding, the high type can only get $\min(u^*(\theta_L), u_H)$ from this mechanism as well. Therefore, there is no incentive problem and the principal is better off.

Next consider the case where $u_L \geq u^*(\theta_L)$. Then $u_H > u_L \geq u^*(\theta_L) > u^*(\theta_H)$. Since $\pi_H(u)$ is decreasing in u on $[u^*(\theta_H), \bar{v}]$, $\pi_H(u_L) \geq \pi_H(u_H)$. By the same argument as before, there is no incentive problem when the high type is offered the optimal mechanism given u_L.

In conclusion, the principal optimally offers the same mechanism to all types of the agent. This mechanism is the same as the one offered to the agent whose type is known to be some $\theta \in [\bar{\theta}, 1]$. □

Proof of Proposition 3. Suppose the principal offers the same mechanism to both types of the agent. Denote it by Γ^*. Let u_h^* (u_l^*) be the high (low) type's payoff given Γ^*. Then $u_h^* \geq u_l^*$. First consider the case where $u_h^* > u_l^*$. By the proof of Proposition 1, the optimal mechanism for a certain type should induce that type of the agent to be indifferent between initiating a bad project or not when $u \in [\underline{u}, \bar{u}]$. As a result, all types of the agent should receive the same expected payoff given this mechanism. Since the high type gets a strictly larger payoff given Γ^*, Γ^* is not the optimal mechanism for the low type. In other words, there exists another mechanism Γ' such that the principal is strictly better off if she offers Γ' to the low type instead of Γ^*. Moreover, the high type gets $u_l^* < u_h^*$ given Γ'. Then the high type has no incentives to deviate. Therefore, it is strictly better off for the principal to offer Γ' to the low type and Γ^* to the high type.

Next I consider the case where $u_h^* = u_l^*$. By the same argument as above, the principal can achieve the payoff frontier with both types of the agent. Then to maximize the expected payoff, it is optimal to have $u^*(\theta_L) \leq u_l^* = u_h^* \leq u^*(\theta_H)$. Suppose $u^*(\theta_L) < u_l^*$. Consider the optimal mechanism for the high type. By the proof of Proposition 1, when $u \leq \underline{u}$ and the agent is induced to take $(1, 0)$, $u_l(u) \leq u_h(u) \leq u_n(u) \leq \underline{u}$. Since $\pi(u)$ is linear on $[0, \underline{u}]$, it does not matter what the values of $u_l(u)$, $u_h(u)$, and $u_n(u)$ are as long as the promise keeping condition is satisfied. Therefore, the principal can optimally choose these variables such that the high type strictly prefers to not initiate a bad project. As a result, the low type will get a smaller payoff than the high type given this mechanism. Then the principal can offer a different mechanism to the low type where the low type gets a smaller payoff. Since $u^*(\theta_L) < u_l^*$, the principal is better off with this new mechanism. Similarly, if $u_h^* < u^*(\theta_H)$, the principal can offer a different mechanism to the high type where the high type gets a larger payoff but the low type gets the same payoff as before. Then no incentives are violated and the principal gets a higher expected payoff with the high type. $\square$