# Supplemental Problems for Final Exam

Here are some problems involving material that we have discussed since Exam 3.

Fundamental Theorem of Calculus

1. Explain why the Fundamental Theorem of Calculus cannot be used to evaluate $\int_{-1}^{1} \frac{1}{x^2}\, dx$.

2. Consider the graph of the function g that is given below. Assume that the graph is built from line segments, semi-circles, and quarter-circles.
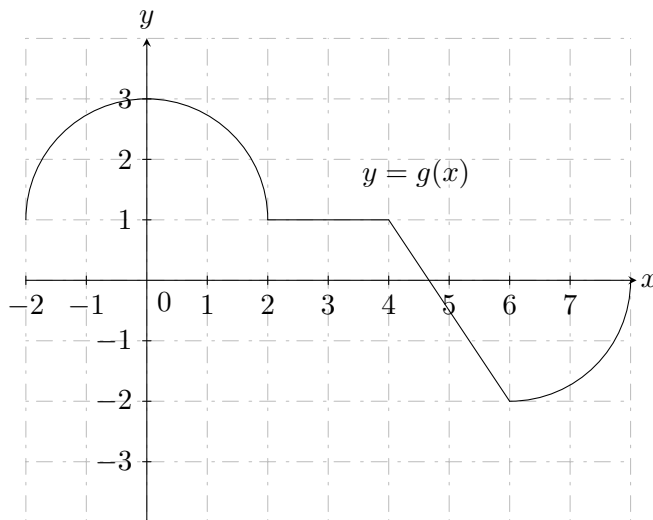
If G is the antiderivative of g satisfying $G(0) = 42$, what is $G(4)$?

3. Complete the following.

   (a) Let $A(x) = \int_0^x t^2 - t\, dt$. Find $A'(x)$.

   (b) Let $f(x) = \int_0^x \sqrt[3]{t^2 + 1}\, dt$. Find $f'(x)$.

   (c) Let $G(x) = \int_0^{x^2} t^3 \sin(t)\, dt$. Find $G'(x)$.

   (d) Let $C(x) = \int_x^{x^3} \cos(\cos(t))\, dt$. Find $C'(x)$.

4. Let $A(x) = \int_0^x \sin^2(t)\, dt$. Determine where A attains its absolute maximum value on $[0, \pi]$.

Indefinite and Definite Integrals

Compute each of the following integrals.

5. $\displaystyle\int \frac{1}{x^3} \, dx$

6. $\displaystyle\int \frac{x+5}{x^2} \, dx$

7. $\displaystyle\int \frac{\sin(x)}{\cos^2(x)} \, dx$

8. $\displaystyle\int_{-1}^{1} x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2 \, dx$

9. $\displaystyle\int_0^{\pi} \cos(2x) \, dx$

10. $\displaystyle\int_0^{\ln(2)} e^{x/3} \, dx$

11. $\displaystyle\int_1^{e^2} \frac{x+1}{x^2} \, dx$

12. $\displaystyle\int_1^2 \frac{x^3 - 2\sqrt{x}}{x} \, dx$

13. $\displaystyle\int_0^{1/2} \frac{4}{\sqrt{1-x^2}} \, dx$

14. $\displaystyle\int (3x-1)^{99} \, dx$

15. $\displaystyle\int 5x^2 \sqrt{x^3 - 2} \, dx$

16. $\displaystyle\int_0^2 xe^{x^2} \, dx$

17. $\displaystyle\int \sin^2(x)\cos(x) \, dx$

18. $\displaystyle\int_0^1 \frac{x}{x^2 + 1} \, dx$

19. $\displaystyle\int x^2 \sec^2(x^3) \, dx$

20. $\displaystyle\int \frac{x}{x^4 + 1} \, dx$

21. $\displaystyle\int x\sqrt{x-1} \, dx$

22. $\displaystyle\int \frac{e^x}{e^x + 1} \, dx$

23. $\displaystyle\int \frac{1}{\sqrt{1-9x^2}} \, dx$

Miscellaneous

24. Determine whether each of the following statements is true or false. Circle the correct answer.

(a) **True** or **False**: $\displaystyle\int_a^b f(x)g(x) \, dx = \int_a^b f(x) \, dx \cdot \int_a^b g(x) \, dx$

(b) **True** or **False**: $\displaystyle\int_a^b f(x) + g(x) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$

(c) **True** or **False**: If $f(x) \le g(x)$ on $[a,b]$, then $\displaystyle\int_a^b f(x) \, dx \le \int_a^b g(x) \, dx$.

(d) **True** or **False**: If $f'(x) = g'(x)$, then $f(x) = g(x)$.

(e) **True** or **False**: The formula $\displaystyle\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ works for *all* values of n.

25. Find f that satisfies $f'(x) = \sqrt{x}$ and $f(4) = 0$.

26. Find f that satisfies $f''(x) = x^2 + 4$, $f'(3) = 1$, and $f(1) = 6$.

27. Use basic properties of integrals to evaluate the following.

(a) $\displaystyle\int_1^{11} f(x) \, dx$ if $\displaystyle\int_0^1 f(x) \, dx = -7$ and $\displaystyle\int_0^{11} f(x) \, dx = 29$

(b) $\int_0^4 5f(x) + \sqrt{x}\ dx$ if $\int_0^{10} f(x)\ dx = 8$ and $\int_{10}^4 f(x)\ dx = -3$

28. Find a positive value of a such that $\int_a^{2a} \frac{3}{4}x(x^2 - a^2)^2\ dx = 1$.

29. A zombie moves in a straight line with velocity $v(t) = -t + 4$ mph after t hours of his start. How far is he from his original position after 6 hours?

30. A bungie jumper jumps off a bridge. Her downward velocity in feet per second, after t seconds of the fall, is $v(t) = 160(1 - e^{-t/5})$. This function is good for the first 5 seconds, after $t = 5$ the bungie cord slows her fall. How far did she fall in those 5 seconds?