

Groups of order ≤ 30

order	groups	order	groups	order	groups	order	groups
1	C_1	12 (cont.)	A_4	18 (cont.)	$D_3 \times C_2$	24 (cont.)	$Q_8 \times C_3$
2	C_2	13	C_{13}		$C_3 \rtimes D_3$		$D_3 \times C_4$
3	C_3	14	C_{14}	19	C_{19}		$D_3 \times C_2^2$
4	C_4		D_7	20	C_{20}		$C_3 \rtimes C_8$
	C_2^2	15	C_{15}		$C_{10} \times C_2$		$C_3 \rtimes D_4$
5	C_5	16	C_{16}		D_{10}		C_{25}
6	C_6	14	$C_8 \times C_2$		Dic_{10}		$C_5 \times C_5$
	D_3		C_4^2		Fr_{20}	26	C_{26}
7	C_7		$C_4 \times C_2^2$	21	C_{21}		D_{13}
8	C_8	16	C_2^4		$C_7 \rtimes C_3$	27	C_{27}
	$C_4 \times C_2$		D_8	22	C_{22}		$C_9 \times C_3$
	C_2^3		SD_8		D_{22}		C_3^3
	D_4		Mod_8	23	C_{23}		$C_9 \rtimes C_3$
	Q_8		Q_{16}	24	C_{24}		$C_3^2 \rtimes C_3$
9	C_9		$D_4 \times C_2$		$C_{12} \times C_2$	28	C_{28}
	$C_3 \times C_3$		$Q_8 \times C_2$		$C_6 \times C_2^2$		$C_{14} \times C_2$
10	C_{10}		$C_4 \rtimes C_4$		D_{12}		D_{14}
	$C_5 \times C_2$		$C_2^2 \rtimes C_4$		Dic_{12}		Dic_{14}
11	C_{11}		$C_4 \circ D_4$		S_4	29	C_{29}
12	C_{12}	17	C_{17}		$\text{SL}_2(\mathbb{Z}_3)$	30	C_{30}
	$C_6 \times C_2$	18	C_{18}		$A_4 \times C_2$		D_{15}
	D_6		$C_6 \times C_3$		$\text{Dic}_{12} \times C_2$		$D_5 \times C_3$
	Dic_6		D_9		$D_4 \times C_3$		$D_3 \times C_5$

The number of groups of order n is...

1009. 1

1010. 6

1011. 2

1012. 13

1013. 1

1014. 23

1015. 2

1016. 12

1017. 2

1018. 2

1019. 1

1020. 37

1021. 1

1022. 4

1023. 2

1024. 49,487,365,422

The number of p -groups, for $p = 2, 3, 5$ is. . .

2. 1	3. 1	5. 1
4. 2	9. 2	25. 2
8. 5	27. 5	125. 5
16. 14	81. 15	625. 15
32. 51	243. 67	3125. 77
64. 267	729. 504	15625. 684
128. 2,328	2187. 9,310	78125. 34,297
256. 56,092	6561. unknown	390625. unknown
512. 10,494,213		
1024. 49,487,365,422		
2048. unknown		

“The human race will never know the exact number of groups of order 2048.”
–John Conway (Princeton University)

Almost all finite groups are 2-groups

Amazing fact

There are 49,910,529,415 groups of order $|G| \leq 2000$.

Of these, 49,487,365,422 of them (99.2%!) have order 1024

Conjecture

Almost all finite groups are 2-groups. That is,

$$\lim_{n \rightarrow \infty} \frac{\# \text{ 2-groups groups of order } \leq n}{\# \text{ of groups of order } \leq n} = 1.$$

A few fun resources for exploring finite groups include:

- The interactive GroupExplorer website (only small groups):

<https://nathancarter.github.io/group-explorer/index.html>

- The noninteractive **GroupNames** website (comprehensive list):

people.maths.bris.ac.uk/~matyd/GroupNames/index.html