DAYSPRING INTERNATIONAL ACADEMY

Mathletics for Upper Secondary

Read the questions *carefully*. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Student's Name: _____

Instructor's Name: Nana Baah Akuffu

Time Allowed: 10 minutes

1. If $y = \sqrt{x^2 + 4}$, then write x in terms of y.

Solution:

$$\begin{aligned} y &= \sqrt{x^2 + 4} \\ y^2 &= x^2 + 4 \\ y^2 - 4 &= x^2 \\ x &= \sqrt{y^2 - 4} \end{aligned}$$

Answer: $\boxed{x = \sqrt{y^2 - 4}}$

2. The first term of a sequence is 2. If the common difference is -2, find the n^{th} term for the sequence.

Solution:

The first term a is 2, and common difference d is -2. The n^{th}, T_n, is given by

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 2 + (n-1) \times (-2) \\ &= 2 - 2n + 2 \\ &= 4 - 2n \end{aligned}$$

Answer: $\boxed{4 - 2n}$

3. Simplify
$$\frac{(x-1)x}{x^2}$$

Solution:
We simply cancel out the like terms since they are dividing.
$$\begin{aligned}\frac{(x-1)x}{x^2} &= \frac{(x-1)\times x}{x\times x},\\ &= \frac{(x-1)}{x}\\ &= 1-\frac{1}{x}\end{aligned}$$
Answer: $\boxed{1-\frac{1}{x} \quad \text{or} \quad \frac{x-1}{x}}$

4. Ashley has a box containing 5 pencils, 2 pens and 8 erasers. If x represents pencils, y pens and z erasers, write an expression for what is in the box.

Solution:
Answer: $\boxed{5x+2y+8z}$

5. Factorize the expression
$$3x^2-2x-5.$$

Solution:
$$\begin{aligned}3x^2-2x-5 &= 3x^2-5x+3x-5\\ &= x(3x-5)+1(3x-5)\\ &= (x+1)(3x-5)\end{aligned}$$

6. The longest side of a right-angled triangle is $5cm$. If the other two sides are the same, find their lengths.

Solution:
The longest side is the hypotenuse. Let x represent the other sides, then by using the Pythagoras theorem.

$$\begin{aligned} 5^2 &= x^2 + x^2 \\ 25 &= 2x^2 \\ x &= \sqrt{12.5}cm \end{aligned}$$

7. Solve the equation

$$5x - 100 = 5.$$

Leave your answer in a fraction in its lowest term.

Solution:

$$\begin{aligned} 5x - 100 &= 5 \\ 5x &= 105 \\ x &= 21. \end{aligned}$$

8. Simplify

$$\frac{x+2}{(x-2)(x+2)}.$$

Solution:

$$\frac{x+2}{(x-2)(x+2)} = \frac{1}{(x-2)}$$

9. Factorize $t^2 - 9$.

Solution:
$t^2 - 9 = t^2 - 3^2 = (t-3)(t+3)$

10. A rectangle has length $x-1$ and width $x+2$. Write an expression for the area of the rectangle.

Solution:
Let l and b be the length and breadth of the rectangle, then the area, A, will be

$$\begin{aligned} A &= l \times b \\ &= (x-1)(x+2) \\ &= x^2 - 2x - x - 2 \\ &= x^2 - 3x - 2. \end{aligned}$$

11. The value of

$$\frac{n(n+3)}{2} = \frac{3}{2}$$

when $n = 0$. True or False?

Solution:
When $n = 0$, $\dfrac{n(n+3)}{2} = \dfrac{0 \times (0+3)}{2} = \dfrac{0}{2} = 0$. So the answer is **False**.

12. If a and b are negative numbers, and $a < b$, then $b - a$ is negative. True or False?

Solution:
Let $b = -2$ and therefore a can be -3 since $a < b$. So $b - a = -2 - (-3) = -2 + 3 = 1 > 0$. Hence $b - a$ is positive not negative. So the answer is **False**.

13. Consider the fraction $\dfrac{p}{q}$. If $p > q$ then the value of $\dfrac{p}{q} > 1$. True or False?

Solution:
Since $p > q$, the fraction $\dfrac{p}{q}$ has the numerator bigger than the denominator and so in any case $\dfrac{p}{q} > 1$. The answer is **True**.

14. Expand and simplify $(x-1)(2-x)$.

Solution:
We start by opening the brackets and grouping like terms.

$$\begin{aligned}(x-1)(2-x) &= x(2-x) - 1(2-x)\\ &= 2x - x^2 - 2 + x\\ &= 3x - x^2 - 2.\end{aligned}$$

15. Solve the equation $x^2 - 2x - 6 = 2$.

Solution:
A quadratic equation is of the form $ax^2 + bx + c = 0$. The whole idea is make the equation equal to 0.

$$\begin{aligned}x^2 - 2x - 6 &= 2\\ x^2 - 2x - 8 &= 0, \quad \text{solving by factorization, we have,}\\ x^2 + 2x - 4x - 8 &= 0\\ x(x+2) - 4(x+2) &= 0\\ (x-4)(x+2) &= 0\end{aligned}$$

At this either $x - 4 = 0 \Rightarrow x = 4$ or $x + 2 = 0 \Rightarrow x = -2$.

16. Solve

$$\frac{y-1}{2} + \frac{y+1}{2}$$

Solution:
Since the denominators are the same,

$$\frac{y-1}{2} + \frac{y+1}{2} = \frac{y-1+y+1}{2} = \frac{2y}{2} = y$$

17. Find the value of x, if

$$5^{2-x} = \frac{1}{125}$$

Solution:
By the laws of indices,

$$5^{2-x} = \frac{1}{125}$$
$$5^{2-x} = \frac{1}{5^3}$$
$$5^{2-x} = 5^{-3}$$

Since the bases are the same, $2 - x = -3 \Rightarrow x = 5$

18. In a certain taxi ride, the cost is calculated by multiplying \$6 by the distance traveled, in *km*, and then adding \$3. If Stephan paid \$15 for the entire ride, how far did he go?

Solution:
Stephan paid \$15 and so,

$$15 - 3 = 12. \quad 12 \div 6 = 2.$$

So Stephan rode for $2km$.

19. A certain printer prints dots. The dots are in the form n^2 where n is the n^{th} paper. How many dots will print on the 12^{th} paper?

Solution:
The pattern on the sheet is of the form n^2 and so the 12^{th} paper will have $n = 12$ and so $12^2 = 144$.

20. *Challenge Problem.* A two digit integer is divisible by 5 but not by 10. If the digits are reversed, the number is 18 more then the original one. What is the integer?

Solution:
Let x be the first digit and y the second digit. Using the place value form, the number can be represented algebraically as $10x + y$. The number reversed is $10y + x$. Now since the number is divisible only by 5 and not 10 then it means the last digit is 5. Hence,

$$y = 5.$$

Mathematically the next statement of the question means;

$$\begin{aligned} 10y + x &= 10x + y + 18, \\ 9y - 9x &= 18 \quad \text{dividing through by 9 gives,} \\ y - x &= 2. \end{aligned}$$

Since $y = 5$, we have $5 - x = 2$ and so $x = 3$. Hence the number $10x + y = 10(3) + 5 = 35$. Indeed, $53 = 35 + 18$.