Formelzusammenfassung Seiten 131–135

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Determinante einer orthogonalen Matrix:
Für $A \in O(n)$, gilt $\det(A) \in \{-1, 1\}$.

$$\det(A) = \det(A^T) = \det(A^{-1}) = \frac{1}{\det(A)} \cdot \det(A)$$

$$(\det(A))^2 = 1 \tag{6.127}$$

$$\det(A) \in \{-1, 1\} \tag{6.128}$$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \tag{6.129}$$

$$\det(A) = \frac{1}{2} \cdot 2 - 0 \cdot 0 = 1 \tag{6.130}$$

$$A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = A^T \tag{6.131}$$

$$O_\pm(n) := \{A \in O(n) \mid \det(A) = \pm 1\} \tag{6.132}$$

$$SO(n) := O_+(n) \tag{6.133}$$

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$$G(v_1; \ldots; v_m) = \begin{bmatrix} \langle v_1, v_1 \rangle & \ldots & \langle v_1, v_m \rangle \\ \vdots & \ddots & \vdots \\ \langle v_m, v_1 \rangle & \ldots & \langle v_m, v_m \rangle \end{bmatrix} \tag{6.134}$$

$$G^T = G \tag{6.135}$$

$$\det(G) \geq 0 \tag{6.136}$$

$$G(\hat{e}_1; \ldots; \hat{e}_n) = 1 \tag{6.137}$$

$$A = \begin{bmatrix} v_1 & \ldots & v_m \end{bmatrix} \tag{6.138}$$

$$A^T \cdot A = G \tag{6.139}$$

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$$\mu(v_1; \ldots; v_m) = \sqrt{\det(G)} \tag{6.140}$$

$$\mu(v) = \sqrt{\langle v, v \rangle} = |v| \tag{6.141}$$

$$\mu(v_1; v_2) \text{ (Fläche des Parallelogramms)} \tag{6.142}$$

$$\mu(v_1; v_2; v_3) \text{ (Volumen des Spats)} \tag{6.143}$$

$$\mu(\hat{e}_1; \ldots; \hat{e}_n) = \sqrt{\det(1)} = 1 \tag{6.144}$$

$$\mu(v_1; \ldots; v_m) = 0 \text{ (für } m > n) \tag{6.145}$$

$$A = \begin{bmatrix} v_1 & \ldots & v_n \end{bmatrix} \tag{6.146}$$

$$\mu(v_1; \ldots; v_n) = |\det(A)| \tag{6.147}$$

$$\mu(v_1; \ldots; v_n) = \sqrt{\det(A^T \cdot A)} = \sqrt{\det(A)^2} = |\det(A)| \tag{6.148}$$

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$$A \cdot u = \begin{bmatrix} 7 & 2 \\ 3 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 14 \end{bmatrix} \tag{6.150}$$

$$A \cdot E_1 = \begin{bmatrix} 7 & 2 \\ 3 & 8 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix} = 5 \cdot E_1 \tag{6.151}$$

$$A \cdot E_2 = \begin{bmatrix} 7 & 2 \\ 3 & 8 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix} = 10 \cdot E_2 \tag{6.152}$$

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$$p_A(\lambda) = \det(\lambda I - A) \tag{6.156}$$

$$p_0(\lambda) = \lambda^n \tag{6.157}$$

$$p_1(\lambda) = (\lambda - 1)^n \tag{6.158}$$

$$p_A(\lambda) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0 \tag{6.159}$$

$$a_0 = (-1)^n \det(A) \tag{6.160}$$

$$p_A(\lambda) = \lambda \left(\lambda^{n-1} - \operatorname{tr}(A)\lambda^{n-2} + \ldots + a_1\right) \tag{6.161}$$

$$p_A(\lambda) = \lambda^2 - \operatorname{tr}(A)\lambda + \det(A) \tag{6.162}$$