1.

(a) $f(x)$ is convex, $\nabla^2 f(x) = Q$, $\lambda(Q) = \{\gamma, 1\}$, $L \geq \lambda_{max}(Q)$, so $L_{min} = \max\{\gamma, 1\}$.

(b) $m \leq \lambda_{min}(Q)$, so $m_{max} = \min\{\gamma, 1\}$.

(c) The result is shown in the table below.

Step Size	Number of Iterations
2.2	Do not converge
1	88
0.1	917
0.01	9206

(d) The result is shown in the table below.

γ	Number of Iterations
1	1
0.1	88
0.01	688
0.001	4603

The number of iterations increases as γ decreases.

2. The output is my implementation is

```
stepsize=0.1, number of iterations=58
optimal solution:
[[1.5       ]
 [1.99999521]]
```

The solution I have found in HW5 is $(1.5,2)^T$.

The output of np.linalg.solve is

```
solution from np.linalg.solve:
[[1.5]
 [2. ]]
```

Ignoring the numeric error, they agree with each other.

3. The output of the program is

```
stepsize=0.1, number of iterations=4189
optimal solution:
[[-1.4702005 ]
 [ 4.44377551]
 [-4.37548184]]
accuracy = 0.8666666666666667
```

4. $f(x)$ is differentiable and α-strongly convex, so

$$f(y) - f(x) - \nabla f(x)^T (y - x) \geq \frac{\alpha}{2} \|x - y\|^2$$

$g(x)$ is β-smooth, so

$$g(y) - g(x) - \nabla g(x)^T (y - x) \leq \frac{\beta}{2} \|x - y\|^2$$

Therefore,

$$h(y) - h(x) - \nabla h(x)^T (y - x) \geq \frac{\alpha - \beta}{2} \|x - y\|^2 \geq 0$$

which shows that $h(x)$ is convex.