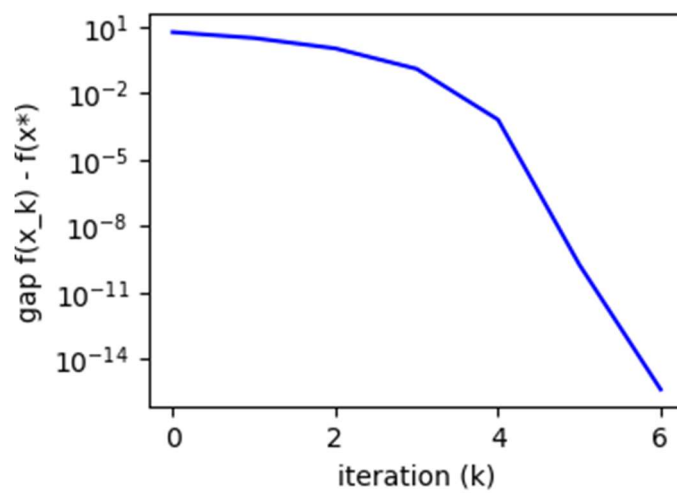
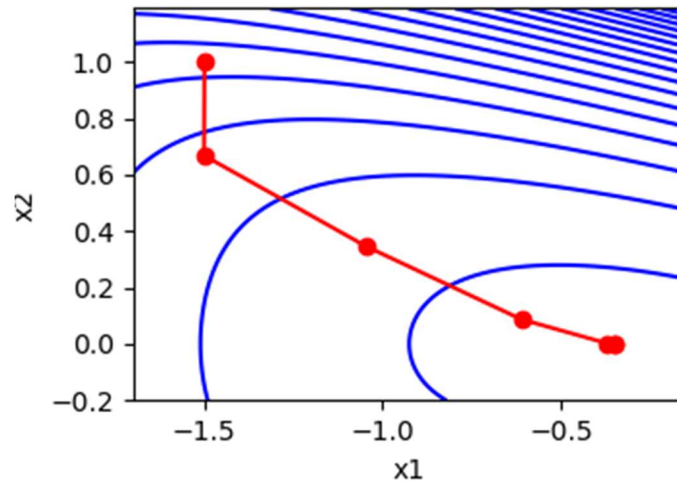


1.

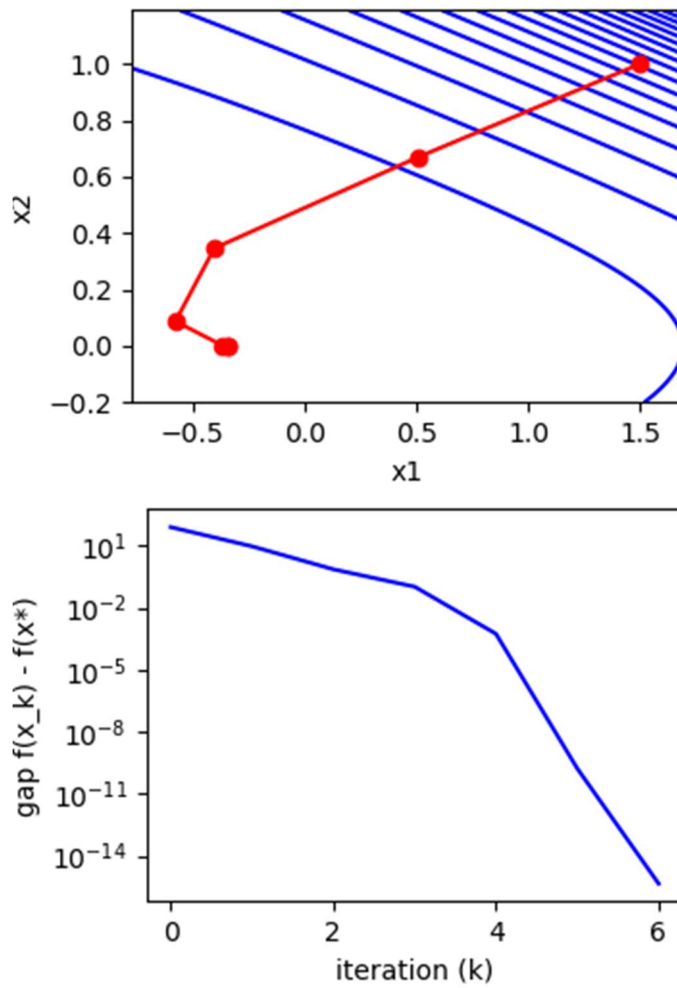
(a) Report:

```
Newton's method
number of iterations: 6
solution: [-3.46573590e-01 -8.40393334e-18]
value: 2.5592666966582156
```



(b) Report:

```
Newton's method
number of iterations: 6
solution: [-3.46573590e-01 -1.28431966e-17]
value: 2.5592666966582156
```



2.

(a)

$$\text{Let } f_i(w) = \log(1 + e^{-\eta_i x_i^T w})$$

$$\text{Then } f_i'(w) = [\sigma(\eta_i x_i^T w) - 1] \eta_i x_i^T$$

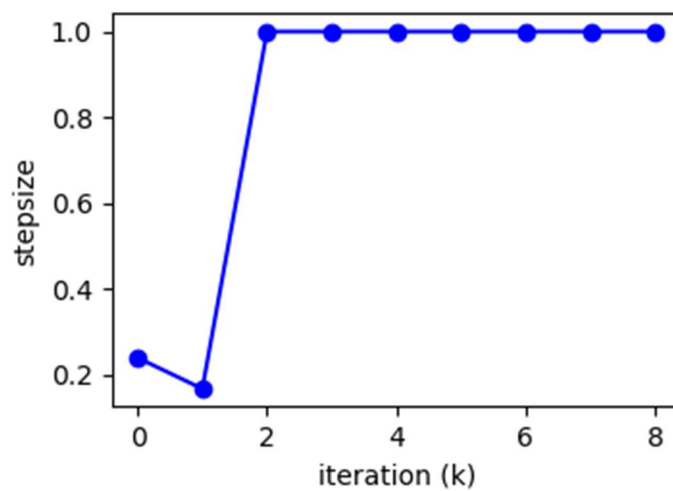
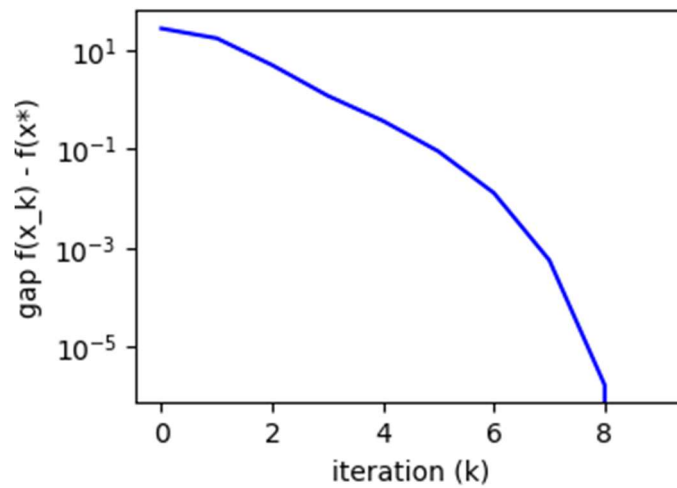
$$\begin{aligned} f_i''(w) &= \sigma'(\eta_i x_i^T w) (x_i \eta_i) \cdot \eta_i x_i^T \\ &= \sigma'(\eta_i x_i^T w) x_i x_i^T \quad (\eta_i^2 = 1) \end{aligned}$$

$$\nabla^2 f_i(w) = f_i''(w)^T = \sigma'(\eta_i x_i^T w) x_i x_i^T$$

$$\nabla^2 f(w) = \sum_{i=1}^m \nabla^2 f_i(w) = \sum_{i=1}^m \sigma'(\eta_i x_i^T w) x_i x_i^T$$

(b) Report:

```
Damped Newton's method  
number of iterations in outer loop: 9  
total number of iterations in inner loop: 9  
solution: [-1.47021306  4.44400878 -4.3758784 ]  
value: 2.876681099986131
```



(c) During the iteration, $\nabla^2 f(x_k)$ becomes singular.

$$3. (a) \quad x_{k+1} = x_k - \frac{f'(x_k)}{f''(x_k)} = x_k - \frac{1}{3}(x_k - a) = \frac{2}{3}x_k + \frac{1}{3}a$$

$$(b) \quad \text{If } x_k = a, \text{ then } x_{k+1} = a$$

$$\text{If } x_k > a, \text{ then } x_{k+1} > \frac{2}{3}a + \frac{1}{3}a = a$$

$$\text{If } x_k < a, \text{ then } x_{k+1} < \frac{2}{3}a + \frac{1}{3}a = a$$

$$\text{Thus, } \forall i, j, \quad \text{sgn}(\eta_i) = \text{sgn}(\eta_j)$$

$$\text{Obviously, } x_{k+1} - a = \frac{2}{3}(x_k - a)$$

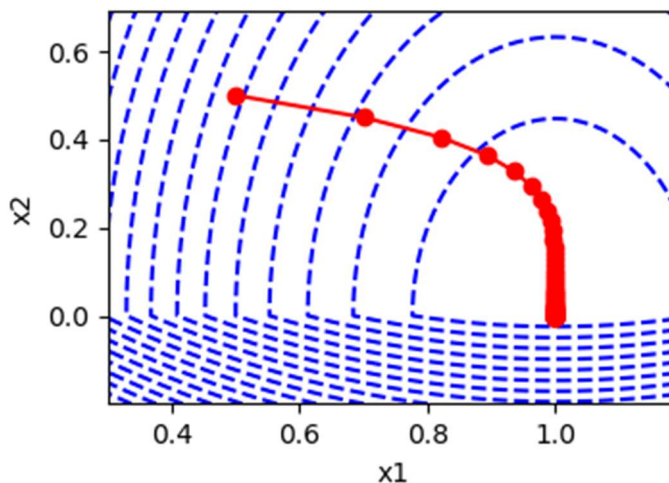
$$\text{so } \eta_{k+1} = \frac{2}{3}\eta_k$$

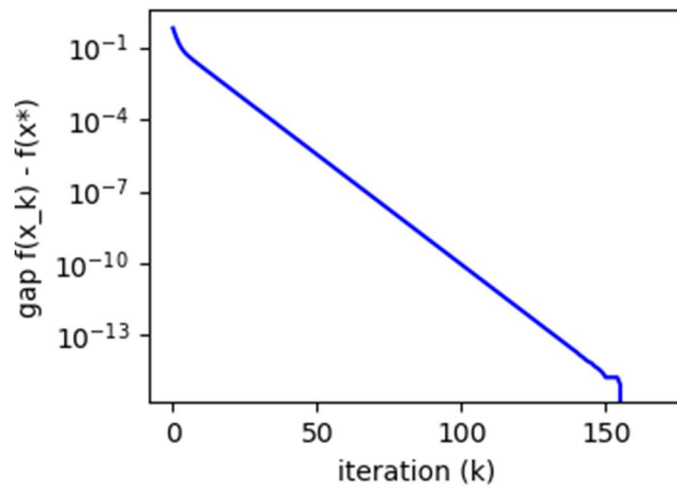
$$(c) \quad \eta_n = \left(\frac{2}{3}\right)^n \eta_0, \text{ so } |x_k - a| \text{ decays to zero exponentially}$$

4.

(a) Report:

```
lambda = 2
number of iterations: 169
solution: [1.00000000e+00 9.24600449e-09]
value: 6.5
```

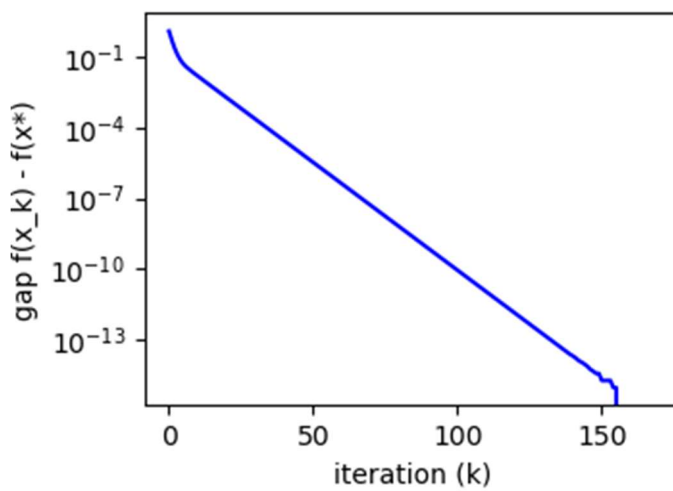
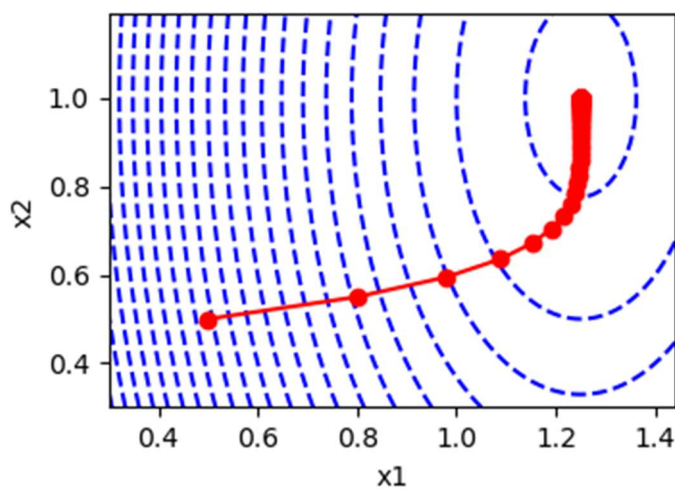




There is one zero.

(b) Report:

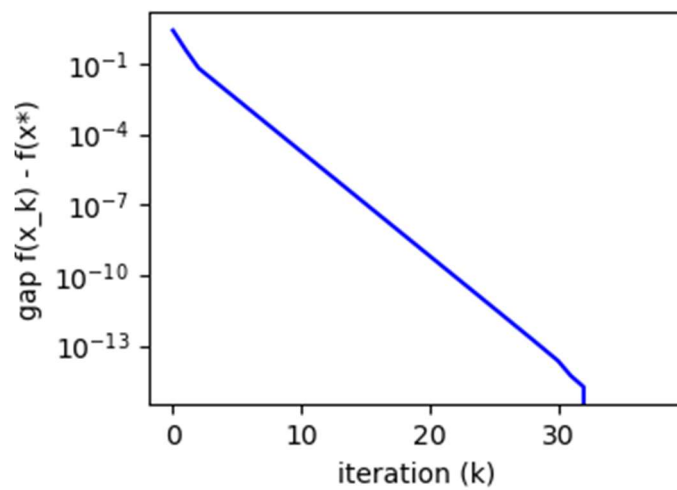
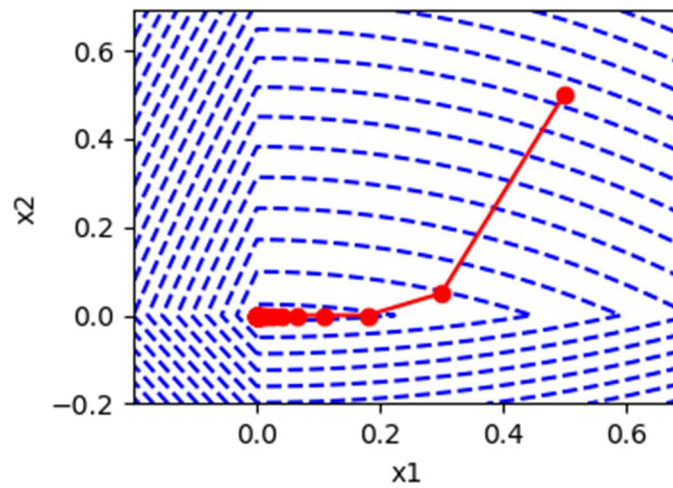
```
lambda = 1
number of iterations: 169
solution: [1.25      0.99999999]
value: 4.875
```



There are no zeros.

(c) Report:

```
lambda = 6  
number of iterations: 38  
solution: [1.85659632e-09 0.00000000e+00]  
value: 8.500000000000002
```



There are two zeros.