1. We need to verify that:

$$\langle \nabla f(\hat{x}_0), x - \hat{x}_0 \rangle = \langle \hat{x}_0 - x_0, x - \hat{x}_0 \rangle \geq 0, \ \forall x \in \overline{B}$$

$$\langle \hat{x}_0 - x_0, x - \hat{x}_0 \rangle = \frac{x_0^T x}{\|x_0\|} - \|\hat{x}_0\| - x_0^T x + \|x_0\|$$

$$= \left(\frac{1}{\|x_0\|} - 1\right) x_0^T x + \|x_0\| - 1$$

$x_0 \notin \overline{B}$, so $\|x_0\| > 1$, $\frac{1}{\|x_0\|} - 1 < 0$

Also, $x_0^T x \leq \|x_0\| \cdot \|x\| \leq \|x_0\|$

Thus, $\langle \hat{x}_0 - x_0, x - \hat{x}_0 \rangle \geq \left(\frac{1}{\|x_0\|} - 1\right) \cdot \|x_0\| + \|x_0\| - 1 = 0$