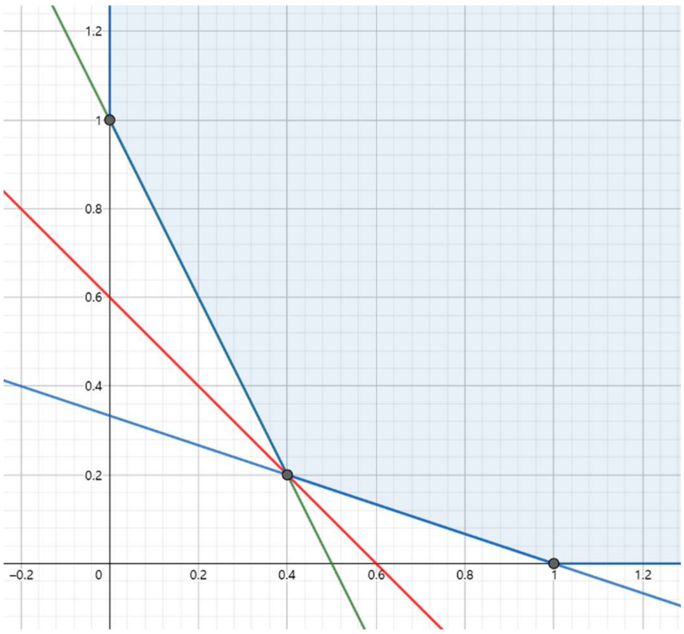
2. Feasible set is shown in the picture below. (The x-axis represents x_1, and the y-axis represents x_2.)

(a) From the picture below we know that the optimal point is the intersection of lines $2x_1 + x_2 \geq 1$ and $x_1 + 3x_2 \geq 1$, which is $(0.4, 0.2)$.



The output of the program using CVXPY is shown in the screenshot below.

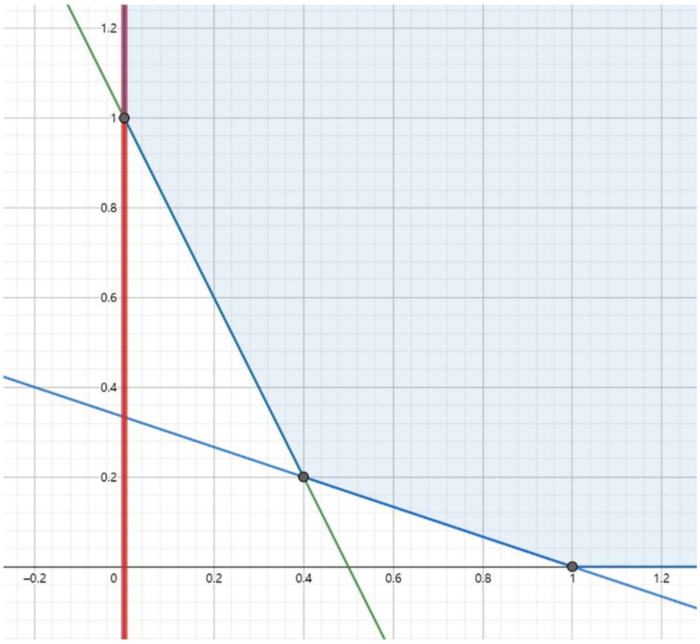
```
status: optimal
optimal value: 0.5999999999116253
optimal var: x1=0.3999999999724491, x2=0.1999999999391762
```

(b) The line $x_1 + x_2 = a$ can go up infinitively, so the optimal point does not exist.

The output of the program using CVXPY is shown in the screenshot below.

```
status: unbounded
optimal value: -inf
optimal var: x1=None, x2=None
```

(c) From the picture below we know that the optimal points are on the y-axis, so $x^* \in \{(0, x_2) \mid x_2 \geq 1\}$.



The output of the program using CVXPY is shown in the screenshot below.

```
status: optimal
optimal value: -2.2491441767693299e-10
optimal var: x1=-2.2491441767693299e-10, x2=1.5537158969947242
```

(d) The output of the program using CVXPY is shown in the screenshot below.

```
status: optimal
optimal value: 0.3333333334080862
optimal var: x1=0.3333333334080862, x2=0.33333333286259564
```

(e) The output of the program using CVXPY is shown in the screenshot below.

```
status: optimal
optimal value: 0.5000000000000002
optimal var: x1=0.5000000000000001, x2=0.1666666666666667
```

3.

(a) $\|\boldsymbol{x}\|_\infty \leq 1$ is equivalent to $-\mathbf{1} \leq \boldsymbol{x} \leq \mathbf{1}$. We introduce new variables $\boldsymbol{t} \in R^m$. Then the problem (1) is equivalent to

$$\min \mathbf{1}^T \boldsymbol{t}$$
$$\text{s.t.} -\boldsymbol{t} \leq \boldsymbol{Ax} - \boldsymbol{b} \leq \boldsymbol{t}$$
$$-\mathbf{1} \leq \boldsymbol{x} \leq \mathbf{1}$$

(b) The output of the program using CVXPY is shown in the screenshot below.

```
status: optimal
optimal value: 13.999999990735517
optimal var: [ 1. -1.]
```

(c) The output of the program using CVXPY is shown in the screenshot below.

```
status: optimal
optimal value: 13.999999998611603
optimal x: [ 1. -1.]
optimal t: [4. 6. 4.]
```

4.

(a) The normal equation is $X^T X w = X^T y$. $w^* = (X^T X)^{-1} X^T y = (1.5, 2)^T$.

(b) When $t = 1$, the output is

```
status: optimal
optimal value: 9.0000000633334
optimal w: [9.99962136e-01 3.78500362e-05]
```

The solution is different from that of (a), and it has one zero component.

When $t = 10$, the output is

```
status: optimal
optimal value: 4.000000000012024
optimal w: [1.49999883 1.99999744]
```

The solution is the same as that of (a), and it has no zero components.

(c) When $t = 1$, the output is

```
status: optimal
optimal value: 7.857489685034338
optimal w: [0.86270479 0.50570788]
```

The solution is different from that of (a), and it has no zero components.

When $t = 100$, the output is

```
status: optimal
optimal value: 4.0000000000000195
optimal w: [1.50000002 2.00000013]
```

The solution is the same as that of (a), and it has no zero components.