

Merge sort: 空间复杂度为  $O(n)$

算法: 可行性

$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) \dots$

$O(2^n) < O(n!) < O(n^n)$

$T_n =$   
 $\downarrow$   
 $\left\{ \begin{array}{l} \text{=} \\ \text{=} \end{array} \right.$

$w_1$  时间复杂度

1 2 3 4

$w_2$  算法时间复杂度:  
- searching  
- sorting

Divide and Conquer  
/ Recurrence  
Merge sort

$w_3$

$w_{3,1}$  Graph Theory  
图  $\left\{ \begin{array}{l} \text{adjacency} \\ \text{incident} \end{array} \right.$   
图论 - 相邻  $\leq$  邻接  
遍历: 无向图 = BFS  
DFS  
最小生成树  $\left\{ \begin{array}{l} \text{贪心} \\ \text{Prim} \\ \text{Kruskal} \end{array} \right.$   $O(|E| \log |V|)$

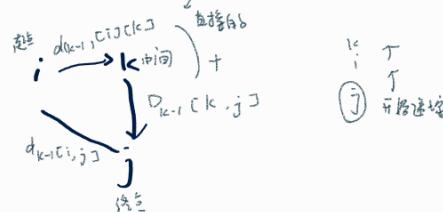
动态规划 - 递推子串出现: Horowitz 算法

圆最长公共子串  $\left\{ \begin{array}{l} \text{单源} \\ \text{多源} \end{array} \right.$  Dikks: 无关边  $O(|E| \times \log |V|)$

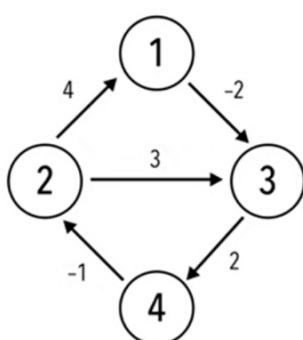
Bellman-Ford: 有向边  $O(|UE|)$

最长子序列 (LCS)

$O(n^3)$   $d_{k,i,j}$   
 $= \min(d_{k-1,i,j}, d_{k-1,i+1,j} + d_{k-1,i+1,j})$



$$d_{k,i,j} = d_{k-1,i,j} \quad \text{if } i = j \\ d_{k-1,i+1,j} + d_{k-1,i+1,j} \quad \text{if } i \neq j$$



	1	2	3	4
1	0	-1	-2	0
2	4	0	2	4
3	5	1	0	2
4	3	-1	1	0

# What we have learnt

methodology problems	Asymptotic idea	Brute force	Divide & Conquer	Dynamic Programming	Greedy	Space/Time	Branch & Bound	Backtracking	Complexity Theory
Efficiency	Big-O								
Sorting		Selection/Bubble/insertion ✓	Merge-sort ✓			Count sorting ✓			
Searching			Binary-searching						
String		searching ✓		Alignment/LCS ✓		Horspool algorithm ✓			
Graph/Combinatorics		DFS/BFS ✓		Floyd's Algorithm/Kruskal's/Assembly-line Knapsack	MST(Prim's/Kruskal's) Dijkstra's For Shortest path		Traveling salesman, Job assignment	n-Queens ✓ Sum of subset Hamiltonian Problem	Approximation: TSP problem: Nearest-Neighbor/twice round/fragment algorithm
Complexity									P/NP Circuit-SAT/3-SAT

PIP 雜記

$\text{NP} \neq \text{NP-C}$   
 + SAT  
 + vertex cover  
 + polygonal time  
 $A \rightarrow B$  reducible to  $B$   
 iff  $B$  有解  
 $A$  有解  
 $NP \rightarrow NP\text{-hard}$

Job	1	2	3	4
Person a	9	2	7	8
b	6	4	3	7
c	5	8	1	8
d	7	6	9	4

$\text{start} = 5 + 2 + 1 + 4$   
 $a-1 \quad a \geq 2$   
 $a-2, b \geq 1$   
 $b + 2 + 1 \geq 4$

$a \geq 2, b \geq 1, c \geq 3, d \geq 4$