Question 1:

(a) $f(n) = 6n + 3n^2 + n \log n + 3\sqrt{n}$

the order of $f(n)$ is $O(n^2)$

(b) prove: there is a constant c

such that $6n + 3n^2 + n \log n + 3\sqrt{n} \leq cn^2$, $\forall n \geq n_0$

$\because \log n \leq n \quad \therefore n \log n \leq n^2$

$3\sqrt{n} \leq 3n^2 \qquad 6n \leq 6n^2$

$\therefore 6n + 3n^2 + n \log n + 3\sqrt{n} \leq 6n^2 + 3n^2 + n^2 + 3n^2 = 13n^2$

$\therefore f(n) \leq 13n^2, \forall n \geq n_0$

Question 2:

(a) Procedure F(n)

if n == 0 or n == 1 or n == 2 or n == 3 then

return 1

else

return F(n-1) + F(n-2) + F(n-3)

(b) The time complexity of this algorithm is exponential $O(3^n)$

$\because$ for each value equal or greater than 4, the function make 3 recursive calls to n-1, n-2, n-3,

and each of these calls makes 3 more recursive calls and so on.

so the number of function calls grow expoentially with an exponential time complexity.

Question 3:

(a) the number of swapping operation is 3

(b) the number of key comparisons is 15

Question 4:

(a)

	a	b	c	d	e	f
a	0	1	0	1	0	0
b	1	0	1	0	1	0
c	0	1	0	0	1	1
d	1	0	0	0	1	0
e	0	1	1	1	0	1
f	0	0	1	0	1	0

a → b → d

b → a → c → e

c → b → e → f

d → a → e

e → b → d → c → f

f → e → c

(b)

	a	b	c	d	e	f
1	1	1	0	0	0	0
2	1	0	0	1	0	0
3	0	1	1	0	0	0
4	0	1	0	0	1	0
5	0	0	1	0	1	0
6	0	0	1	0	0	1
7	0	0	0	1	1	0
8	0	0	0	0	1	1

1 → a → b

2 → a → d

3 → b → c

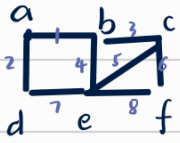
4 → b → e

5 → c → e

6 → c → f

7 → d → e

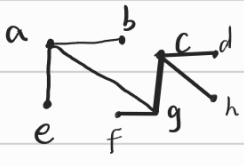
8 → e → f



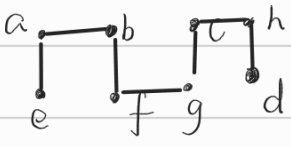
Question 5

(a)

a b e g f c h d



(b)

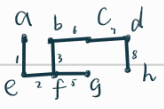


Question 6 → 连成一个整体，辐射一片

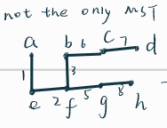
(a)

order selected	a(0,-)	b(-,∞)	c(-,∞)	d(-,∞)	e(-,∞)	f(-,∞)	g(-,∞)	h(-,∞)
a(0,-)		b(a,4)	c(-,∞)	d(-,∞)	e(a,1)	f(-,∞)	g(-,∞)	h(-,∞)
e(a,1)		b(a,4)	c(-,∞)	d(-,∞)		f(e,2)	g(-,∞)	h(-,∞)
f(e,2)		b(f,3)	c(f,10)	d(-,∞)			g(f,5)	h(-,∞)
b(f,3)			c(b,6)	d(-,∞)			g(f,5)	h(-,∞)
g(f,5)			c(g,12)	d(g,14)				h(g,8)
c(g,12)				d(c,7)				h(-,∞)
d(c,7)								h(d,8)
h(d,8)								

↳ 算边 weight



not the only MST

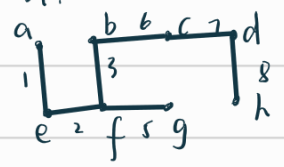


→ here is another

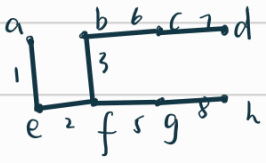
(b)

(a, e)	1
(e, f)	2
(f, b)	3
~~(a, b)~~	~~4~~
(f, g)	5
(b, c)	6
(c, d)	7
(d, h)	8
~~(g, h)~~	~~8~~
~~(f, c)~~	~~10~~
~~(g, c)~~	~~12~~
~~(g, d)~~	~~14~~

MST:

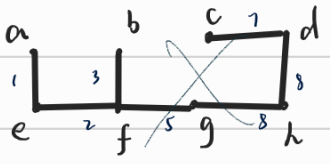


MST is not unique:

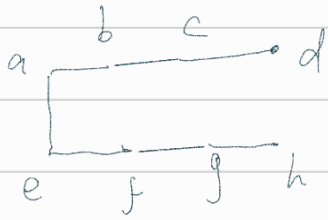


(c)

order selected	a(0,-)	b(-,∞)	c(-,∞)	d(-,∞)	e(-,∞)	f(-,∞)	g(-,∞)	h(-,∞)
a(0,-)		b(a,4)	c(-,∞)	d(-,∞)	e(a,1)	f(-,∞)	g(-,∞)	h(-,∞)
e(a,1)		b(a,4)	c(-,∞)	d(-,∞)		f(e,3)	g(-,∞)	h(-,∞)
f(e,3)		b(a,4)	c(f,3)	d(-,∞)			g(f,8)	h(-,∞)
b(a,4)			c(b,10)	d(-,∞)			g(f,8)	h(-,∞)
g(f,8)			~~c(g,20)~~ c(b,10)	d(g,22)				h(g,16)
h(g,16)			~~c(-,∞)~~	d(h,24) d(c,17)				h(g,16)
d(h,24)			~~c(d,31)~~	d(c,17)				
~~c(d,31)~~ d(c,17)								



每行选择之前一行最小值，重新计算各点距离



$\Rightarrow O(|E| \times \log|V|)$

$|E| \times \log|V|$

a→e→f→g→h

错误答案