Comparison of Loss Reserving Techniques Based on Run-Off Triangles

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Abstract

Four methods of loss reserving: Chain-Ladder, Loss Development, Cape Cod and Additive are used to calculate IBNR estimates for both amounts and counts for simulated claims data under different scenarios. Two types of stimation errors are calculated for each method. 200 runs of the simulation are performed and the methods are ranked after each run. The ranks are then summed up to obtain a final ranking. The results show that the Chain-Ladder and Loss development methods are inferior to the Cape Cod and Additive methods. The Additive method had the lowest estimation errors in most cases. IBNR reserves for claims with left-skewed delay distributions are observed to be difficult to estimate.

Keywords: liability, Incurred But Not Reported (IBNR), Chain-Ladder, Loss Development, Cape Cod, Additive, estimation

1 Introduction

In insurance, there is a delay between the time a risk event occurs and when it is actually reported to the insurance company. It therefore becomes necessary for the insurance company to set aside some monetary reserves for those claims that have arisen from events that have already occurred but not been reported. These reserves are known as Incurred But Not Reported (IBNR) reserves. Reserves can also be set aside for claims that have been reported but yet to be settled fully. These are referred to as case reserves (Boland, 2007). These fall under the umbrella term of loss reserving.

Tarbell (1934) proposed a method for loss reserving which has come to be known as the chain-ladder method. The paper presents a simple formula for calculating IBNR reserves based on number of reported claims per period, average incurred cost per reported claim and amount of incurred but not reported claims from the preceeding valuation period. This method has a drawback in that, random fluctuations caused by abnormally large claim amounts can distort the predicted reserves .Tarbell (1934) highlights this limitation by saying that the formula is unsuitable for insurance businesses with low claim frequency and large variations in average claim amounts. The chain-ladder method is a special case of the more general loss-development method (Schmidt, 2008).

Bornhuetter and Ferguson (1972) show that the chain ladder method is not suitable for loss reserving if business volumes are increasing year on year and that it is more appropriate to derive the IBNR estimate from the expected loss ratio. This method, now known as the Bornhuetter-Ferguson method assumes that abnormally high or low cumulative losses are not a reflection of future losses and compares the way liabilities develop to changes in the expected loss ratio.

The Cape Cod method (also known as the Stanard-Bühlmann method) was developed to address the shortcomings of the Chain Ladder Method. This method is an extension of the Bornhuetter-Ferguson method and is useful for insurers that lack the data to use the Bornhuetter-Ferguson method. Instead of using an arbitrary expected loss ratio, it estimates a loss ratio from exposures that are adjusted for trend to ensure that the de-trended loss ratios are expected to be the same for all periods (Bühlmann, 2016).

Stanard (1985) simulated thousands of loss data and applied four methods (chain ladder, additive, Bornhuetter-Ferguson and Cape Cod) in estimating loss reserves. The results were compared with actual simulated losses. (Stanard, 1985) concluded that the chain-ladder method showed bias and is the least suitable among the four methods and also noted that the additive method and the Cape Cod method produce results with significantly lower variation than the chain-ladder and Bornhuetter-Ferguson methods while the additive method may be totally bias-free.

IBNR reserves can also be estimated using some stochastic models like the over-dispersed Poisson model, the negative binomial model, log-normal model and the gamma model. Further information about stochastic loss reserving can be found in Mack (1993) and England and Verrall (2002).

The broad aim of this paper is to simulate claims data with different scenarios and compare the estimation errors that the Chain-Ladder, Loss Development, Cape Cod and Additive methods produce. This is an extension of the work done by Stanard (1985). The scenarios simulated are the nature of the book of insurance business (closed, growing or stable) and the distribution of claim delays (right-skewed, Uniform, Normal or left-skewed).

The paper consists of five main sections. Section 2 presents the theory behind run-off triangles and a summary of the four methods. This is followed by a description of the simulation procedure in section 3. The results from the simulation process are then presented in section 4. A discussion of the results follows in section 5.

2 Loss reserving techniques

The notation and equations used in this section are borrowed from Schmidt (2008). Suppose that a risk event leading to the ith claim, $C^l_{i,k}$, that occurred in calendar month i and is notified as a claim to the insurance company k months after the claim occurred. The month in which the claim occurred, i, is referred to as the accident month and the delay, k, is referred to as the development month. The total claims occurring in accident month i with development k is referred to as the incremental claims, $Z_{i,k}$, such that

$$Z_{i,k} = \sum_l C^l_{i,k}.$$

The total claims that occurred in accident month i that have been notified in or before development month k is referred to as the cumulative claims, $S_{i,k}$, such that

$$S_{i,k} = \sum_{j=0}^{k} Z_{i,j}.$$

The random variable $C^l_{i,k}$ can take on an amount equal to the total insurance claim for an analysis of claim amounts or the value 1 for an analysis of claim numbers. The monthly time period used can be replaced with days, quarters or years if needed.

Table 1: Incremental claims run-off triangle

	Development month (k)						
Accident month (i)	0	1	...	k	...	$n-1$	n
0	$Z_{0,0}$	$Z_{0,1}$	...	$Z_{0,k}$	...	$Z_{0,n-1}$	$Z_{0,n}$
1	$Z_{1,0}$	$Z_{1,1}$	...	$Z_{1,k}$	...	$Z_{1,n-1}$	
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...		
i	$Z_{i,0}$	$Z_{i,1}$	...	$Z_{i,k}$			
$\vdots$	$\vdots$	$\vdots$	...	$\vdots$			
$n-1$	$Z_{n-1,0}$	$Z_{n-1,1}$					
n	$Z_{n,0}$						

2.1 Basic triangle

Both incremental and cumulative claims can be presented in a table with the accident months on the vertical axis and development months on the horizontal axis. Table (1) illustrates how incremental claims can be represented in tabular form. Claims that have been reported so far form a triangle in the upper-left corner where $i + k \leq n$ and the blank space in the lower-right corner where $i + k \geq n + 1$ represents unobserved claims. The central aim of loss reserving is to estimate these unobserved claims.

2.2 The Bornhuetter Ferguson principle

A major assumption underlying the use of run-off triangles is that incremental claims from successive development months are proportional. This assumption can be represented mathematically by the multiplicative model which expresses the incremental losses from accident month i in development month k as

$$E[S_{i,k}] = \alpha_i \, \vartheta_k, \text{for all } i, k \in \{0, 1, ..., n\}, \tag{1}$$

where $\vartheta_k = E[Z_{i,k}]/E[S_{i,n}]$. $\vartheta_0, \vartheta_1, ..., \vartheta_n$ are referred to as a development pattern for incremental quotas.

The model also expresses cumulative losses from accident month i in development month k as

$$E[S_{i,k}] = \alpha_i \, \gamma_k, \text{for all } i, k \in \{0, 1, ..., n\}, \tag{2}$$

where α_i is the expected ultimate liability $E[S_{i,n}]$, $\gamma_k = \sum_{l=0}^{k} \vartheta_l$ is the proportion of ultimate liability that would have been reported by development year k and $\gamma_n = 1$. $\gamma_0, \gamma_1, ..., \gamma_n$ are referred to as a development pattern for cumulative quotas.

Schmidt (2008) illustrates how the Bornhuetter-Ferguson method can be adjusted to incorporate other methods of loss reserving that are based on run-off triangles by converting its parameters into a more general form. (Schmidt, 2008) defines the predictors of cumulative liabilities for the Bornhuetter-Ferguson method as

$$\hat{S}_{i,k} = S_{i,n-i} + \hat{\alpha}_i(\hat{\gamma}_k - \hat{\gamma}_{n-i}) \tag{3}$$

where $\hat{\alpha}$ is an estimator for expected ultimate liability $E[S_{i,n}]$ and $\hat{\gamma}_k$ is an estimator γ_k. Equation (3) provides a general framework within which all other methods can be defined.

2.2.1 Additive method

The use of the additive method requires the calculation of an incremental loss ratio, ζ_k^{AD}. The premiums earned or the total exposure for the ith month,

π_i is also needed. The predictor for cumulative liabilities,

$$\hat{S}_{i,k}^{AD} = S_{i,n-i} + \hat{\alpha}_i^{AD}(\hat{\gamma}_k^{AD} - \hat{\gamma}_{n-i}^{AD}). \tag{4}$$

$$\text{where, } \hat{\alpha}_i^{AD} = \pi_i \sum_{l=0}^{n} \hat{\zeta}_l^{AD}, \zeta_l^{AD} = \frac{\sum_{j=0}^{n-l} Z_{j,l}}{\sum_{j=0}^{n-l} \pi_j} \text{ and } \gamma_k^{\hat{A}D} = \frac{\sum_{l=0}^{k} \hat{\zeta}_l^{AD}}{\sum_{l=0}^{n} \hat{\zeta}_l^{AD}}.$$

2.2.2 Loss Development method

For the Loss Development method, the predictor for cumulative liabilities is given by

$$\hat{S}_{i,k}^{LD} = S_{i,n-i} + \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}}(\hat{\gamma}_k - \hat{\gamma}_{n-i}). \tag{5}$$

$\hat{\gamma}_k^{LD}$ can be calculated using any method of choice. For the purposes of this paper, $\hat{\gamma}_k^{LD} = \hat{\gamma}_k^{AD}$.

2.2.3 Chain-Ladder method

The chain ladder method is a special case of the Loss Development method where

$$\hat{S}_{i,k}^{CL} = S_{i,n-i} + \frac{S_{i,n-i}}{\hat{\gamma}_{n-i}^{CL}}(\hat{\gamma}_k^{CL} - \hat{\gamma}_{n-i}^{CL}). \tag{6}$$

$$\text{and } \hat{\gamma}_k^{CL} = \prod_{l=k+1}^{n} \frac{1}{\hat{\rho}_l} \text{ and } \hat{\rho}_k = \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}}.$$

2.2.4 Cape Cod method

To calculate the estimate of expected ultimate liability for the Cape Cod method, an estimator for the expected loss ratio is used along with the Chain-Ladder development factors. This estimator, (the Cape Cod loss ratio) is an estimator of the expected loss ratio and is of the form

$$\hat{\kappa} = \frac{\sum_{j=0}^{n} S_{j,n-j}}{\sum_{j=0}^{n} \hat{\gamma}_{n-j}\pi_j}..$$

The Cape Cod predictor for cumulative liability

$$\hat{S^{CC}_{i,k}} = S_{i,n-i} + \pi_i \, \hat{\kappa}^{CC} (\hat{\gamma}^{CL}_k - \hat{\gamma}^{CL}_{n-i}). \tag{7}$$

2.3 Accident month reserves

As mentioned earlier, the aim of loss reserving is to calculate the unobserved claim amounts or numbers. Incremental losses for accident month i and development month j are calculated as

$$\hat{Z}_{i,k} = \hat{S}_{i,k} - \hat{S}_{i,k-1} \, for \, i + k \geq n + 1.$$

Accident month reserves can the be calculated as

$$\hat{R}_i = \sum_{l=n-i+1}^{n} Z_{i,l} \, for \, i = 1, ..., n,$$

and the calender month reserves,

$$\hat{R}^{n+p} = \sum_{i=p}^{n} Z_{i,n-i+p} \, for \, p = 1, ..., n.$$

The total reserve,

$$\hat{R} = \sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}.$$

3 Simulation analysis

A decision was taken to simulate group life assurance data under three scenarios: a scheme whose membership grows at 2% per annum, a scheme whose membership remains stable and a scheme whose membership declines until every member has either died or exited the scheme.

3.1 Premium and claims information

3.1.1 Scheme membership

A set of 20,000 lives were generated with random ages between 25 and 55, and random genders. The ages were generated to correspond to the average distribution of ages in group life schemes from Schriek *et al.* (2013). Genders were generated so that the scheme consisted of approximately 65% males and the sums assured upon death were also generated to follow the salary distribution from Schriek *et al.* (2013).

The maximum age of new members was restricted to 55 to keep the simulation as realistic as possible. At age 65, each member is assumed to retire and leave the scheme.

3.1.2 Premium calculation and death simulation

At the beginning of each year, random numbers between 0 and 1 are generated for each member currently in the scheme and if that number is smaller than the mortality rate for the member at that age, the member is assumed to die within that year. The mortality rates used were obtained from Clur *et al.* (2013). The month of death is also generated as a random number between 1 and 12. The premiums payable per month for each member are also calculated for each member at the beginning of each year according to the formula,

$$Monthly\ premium = \frac{Sum\ assured\ \times\ Mortality\ rate}{12}$$

For simplicity, discounting was not used in the calculation of the premiums. Within each year, at the beginning of each month, the number of members still alive is aggregated along with the premiums paid by these members.

These numbers are recorded in a separate sheet along with the date as total lives exposed and total earned premiums respectively. If the members have a dead status and the simulated the process is currently in their generated death month, the member is assumed to die. The date of death is recorded and the member is removed from the scheme.

3.1.3 Changes over time

After each year, the ages of the members increase. The model was run from 01/01/1970 to 01/01/2010 for the stable and growing schemes. For the shrinking scheme, it was run until every member had either left the scheme or died.

3.2 Delay simulation

To analyse the effects of the distribution of delays on the IBNR estimates, four delay distributions were generated for each claim simulated. The four delays were generated to fit claims that have positively-skewed delays, uniformly distributed delays, normally distributed delays and negatively-skewed delays. The positively skewed delays were simulated with a gamma distribution with shape parameter of 1.2 and a scale parameter of 12.5 truncated at 36 months. The uniformly distributed reporting delays were simulated with a uniform distribution between 0 and 36. Normally distributed reporting delays were simulated using a normal distribution with mean of 18 and standard deviation of 4.2. Lastly, the negatively skewed delays were simulated by subtracting random numbers generated from the gamma distribution above from 36.

3.3 Reserve calculations

36 x 36 monthly run-off triangles were set up for the Chain-Ladder method, Loss Development method, Cape Cod method and the Additive method for both claim amounts and counts using the formulas described in the loss reserving techniques section of this paper. The run-offs were done on a cumulative basis. The claims list consisting of date of event and delays generated from the simulations above feed into tables that sort out the observed claims as at a specified reserving date from the unobserved ones. These tables are then run-off to calculate IBNR estimates for all unobserved claims using the four methods. The estimates are calculated on the 1st of January each year from three years after the beginning of the simulation to four years before the end of the simulation. To compare the prediction error of the four methods, two statistics were calculated. The first which is referred to as MSE^1 for the purposes of this paper, compares the total actual reserves to the total estimated reserves. It is calculated as:

$$MSE_j^1 = \frac{\sum_{i=0}^{36}(R_i - \hat{R}_i)^2}{36} \tag{8}$$

where R_i and $\hat{R}_i$ represent the actual and predicted accident month reserves respectively for accident month i. These are then summed over all j's, where j is the reserving date. MSE^1 calculates the total estimation error and does not take into account the variability of the results over all 36 periods. The second statistic MSE^2 addresses this limitation and is calculated as follows:

$$MSE_j^2 = (\sum_{i=0}^{36} R_i - \sum_{i=0}^{36} \hat{R}_i)^2 \tag{9}$$

3.4 Multiple simulations

To account for variability in the results, the simulations were run 200 times for each scenario. For each scenario, and for each delay distribution, a ranking between one and four was given to each method: one being the method with the lowest error statistic and 4 being the method with the highest error statistic. This was done for both MSE^1 and MSE^2.

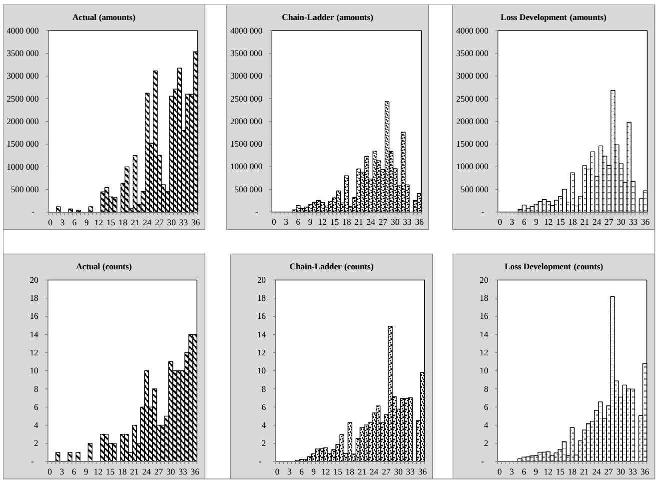
4 Results

4.1 One simulation

4.1.1 One reserving date

IBNR reserves were calculated at the beginning of each year from 1973 to 2006. The actual amounts realised are also recorded. Figure 1 shows the actual reserves (amounts and counts) as against those calculated using the chain-ladder and loss development methods for reserving date 01/01/1988. These results are from the scheme whose membership remains stable throughout the simulation.

Figure 1: Actual vs Chain-Ladder vs Loss development - right-skewed delay



The two methods produce similar results. They overstate the reserves in some periods and understate them in others. There is a pattern with respect to the direction of the size of the reserves as they try to match the pattern of actual reserves.

Figure 2: Actual vs Cape Cod vs Additive - right-skewed delay

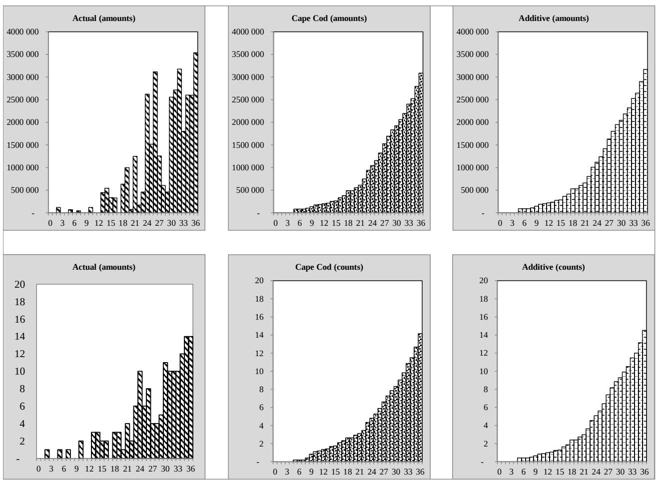
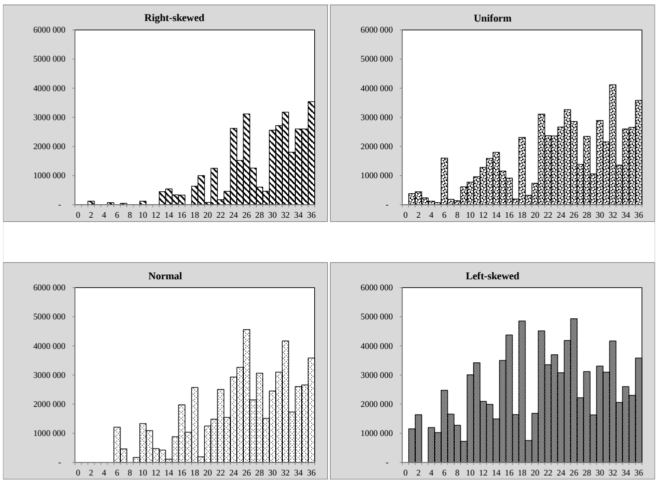


Figure 2 shows the actual reserves compared to the reserves calculated by the cape cod and additive methods with a right skewed delay. These two methods not only try to match the general shape of the actual reserves but also produce a smoothing effect over the 36 accident months. This leads to them understating and overstating reserves in different periods. For the right-skewed delay, the pattern of reserves starts low and rises steadily. The patterns for other delay distributions for actual reserves and using the additive method are shown in figures 3 and 4 respectively. Similar results were obtained with a growing scheme (see appendix)

Figure 3: Comparison of pattern of actual reserves - between delays



The right-skewed delay distribution shows a left-skewed pattern of reserves. The uniform and normal delay distributions also show this pattern but not as clear as the right skewed delay. For the left skewed delay distribution, the reserves rise slowly and drop off slightly at the end of the 36 accident months.

Figure 4: Comparison of pattern of additive reserves - between delays

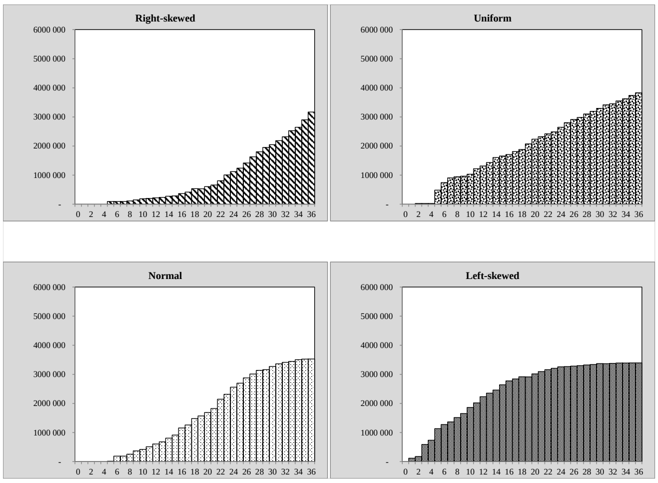
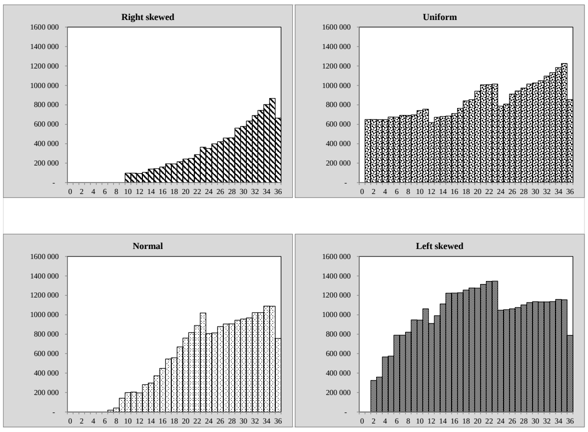


Figure 4 compares the pattern of reserves produced by the additive method between the four delay distribution. The additive method smooths out the patterns observed in figure 3 above. The Cape Cod method produced similar patterns. See appendix for the patterns produced by the Cape Cod method. Similar patterns are produced by the growing scheme (see appendix).

For a shrinking scheme, the smoothing effect is less pronounced as there are peaks and sudden drops in the estimated reserves.

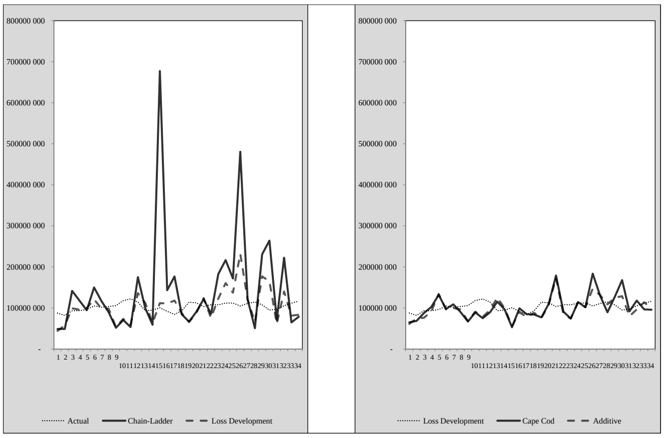
Figure 5: Comparison of pattern of reserves for a shrinking scheme



4.1.2 Total reserves

Since all the methods understated and overstated the actual results from one simulation over a 3 year period, the total reserves calculated for each reserving date were plotted against the total actual reserves for each method and delay distribution. This was done to see if any of the methods constantly understates the reserves, since overestimation is a more desirable outcome than underestimation. Figure 6 compares the total reserves for a left-skewed delay distribution. It can be seen that the reserves calculated using chain-ladder and loss development methods fluctuate around the actual reserves. The reserves calculated by the cape cod and additive methods also exhibit this characteristic but are more stable than the other two methods. For the other delay distributions, this effect is also noted (see appendix). None of the methods constantly overstate or understate the reserves but rather fluctuate around the actual reserves.

Figure 6: Comparison of total reserves for left-skewed delay



4.1.3 Estimation errors

The estimation errors(MSE^1 and MSE^2) calculated for one simulation of a stable scheme are shown in table 2. It can be seen that all the methods produced the highest errors when the delays followed a left-skewed distribution. The additive method produced the lowest errors among the four methods in this case. The cape cod method however produced the lowest errors when the delays were normally distributed.

Table 2: Estimation errors for a stable scheme

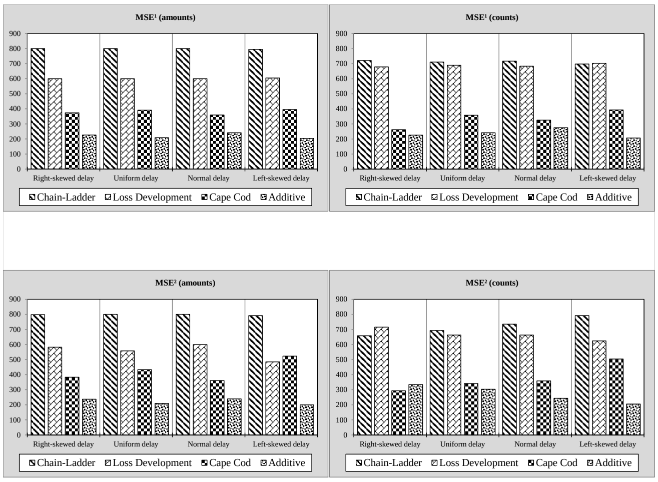
		Chain-Ladder	Loss Development	Cape Cod	Additive
		Amounts (billions)			
MSE¹	Right-skewed delay	62 559	55 088	17 301	**16796***
	Uniform delay	1 268 055	327 742	54 223	**36068***
	Normal delay	434 496	532 000	**32304***	32 712
	Left-skewed delay	14 703 557	1 281 201	70 394	**62180***
		Counts			
	Right-skewed delay	543	549	**159***	161
	Uniform delay	3 180	2 895	481	**366***
	Normal delay	10 437	5 653	**322***	326
	Left-skewed delay	18 471	25 521	1 172	**594***
		Amounts (billions)			
MSE²	Right-skewed delay	4 061 644	2 367 674	1 663 637	**1115893***
	Uniform delay	82 540 276	16 417 337	33 912 625	**10358783***
	Normal delay	24 776 673	29 128 186	**3343663***	3 713 146
	Left-skewed delay	598 082 637	55 491 384	32 842 166	**22758282***
		Counts			
	Right-skewed delay	43 336	45 903	**13487***	18 379
	Uniform delay	187 049	213 280	494 559	**125668***
	Normal delay	721 178	326 168	**44507***	46 034
	Left-skewed delay	976 210	1 242 459	868 978	**205015***

Slightly similar results were obtained for growing and shrinking schemes, with the cape cod and additive methods producing the lowest errors. (See appendix)

4.2 Multiple simulations

As mentioned earlier, the simulations were run 200 times for each scenario and the methods were ranked on each run. A ranking of 1 for the method with the lowest errors and 4 for the method with the highest errors. The rankings were summed up for all 200 simulations. Figure 7 compares the rankings for the methods for a stable scheme. The least errors were produced by the additive method in some cases and the cape cod method in other cases. Similar results were produced for a growing scheme and a shrinking scheme. See appendix for the tables showing the estimation errors for these schemes.

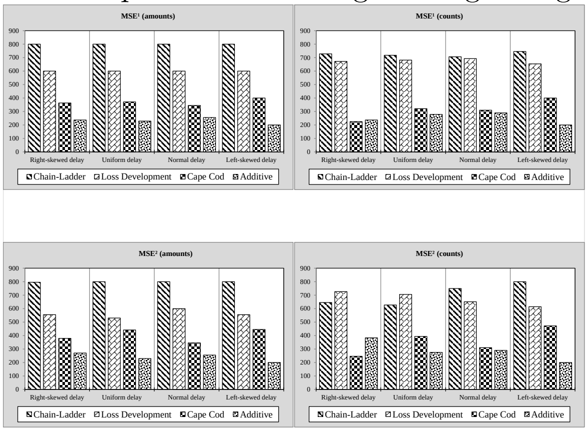
Figure 7: Comparison of rankings for a stable scheme



In all scenarios except for the MSE^2 errors for counts, the chain ladder had the lowest overall ranking. Most notably, the results for the chain-ladder and loss development methods are closer to each other for counts than for amounts. The results for the additive and cape-cod methods are also closer to each other for counts than for amounts. In all cases for claim amounts, except for MSE^1 error for the left-skewed delay where the loss development method ranked second, the order is very clear. The additive method with the highest ranking, followed by the cape cod, the loss development method and lastly the chain-ladder method.

Figure 8 compares the rankings for a growing scheme. Again, the order is clearer with amounts than for counts in most cases. However, the order for counts for claims with a left-skewed delay is is the same as for the amounts with the additive method ranking first and the chain-ladder method last. For MSE^2 errors for counts, the chain-ladder method performed better than the loss development method when the claims were right skewed or uniformly distributed. See appendix for the comparison of rankings for a shrinking scheme.

Figure 8: Comparison of rankings for a growing scheme



5 Discussion and conclusions

The results obtained from the simulations have shown that in most cases the cape cod and additive methods perform better than the chain ladder and loss development methods in estimating IBNR reserves. The chain-ladder method produced the highest estimation errors in most cases suggesting it is the weakest in terms of prediction. This highlights the difference between these two groups of methods. The chain-ladder being a subset of the more general loss development method and the cape cod and additive method being exposure based methods. Exposure based methods perform better due to the fact that they incorporate the insurer's exposure to loss which loss development methods ignore. The outperformance of the loss development method over the chain-ladder method can be attributed to the use of additive development patterns in this paper. The cape cod and additive methods, for one reserving period and over all accident months, smooth out the reserves. This effect is not seen with the chain-ladder and loss development methods.

The performance of the cape cod method is not much different from the additive method in terms of predicting claim counts. This is also noticed with the chain-ladder method and the loss development method. With claim amounts however, the order of performance is clear with the chain-ladder being the worst followed by the loss development method and the cape cod method. The results suggest that the additive method is the best at predicting reserves on an amounts basis.

All four methods struggle with predicting reserves for claim with delays that have a left-skewed distribution. The additive method, however produced the lowest errors for claims of this nature. A possible extension of this project is to simulate claims with exclusively left-skewed delay distributions and testing the performance of some stochastic reserving methods and comparing that to that of the additive method. The nature of the book of business however, does not seem to have a significant impact on the order of performance of the methods.

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A Appendices

A.1 Mortality rates and age distributions

This appendix shows the distribution of ages and mortality rates of group insured lives as presented in (Clur *et al.*, 2013). For the purposes of this paper, only age and gender were used to differentiate lives. Table 3 shows the distribution of ages among group insured lives calculated from (Schriek *et al.*, 2013). Table 4 shows the aggregated group insured lives table 2005-2009 (GL05-09 Aggregate).

Table 3: Distribution of ages

Age band	Proportion
25-34	0.362301
35-44	0.360331
45-54	0.277368

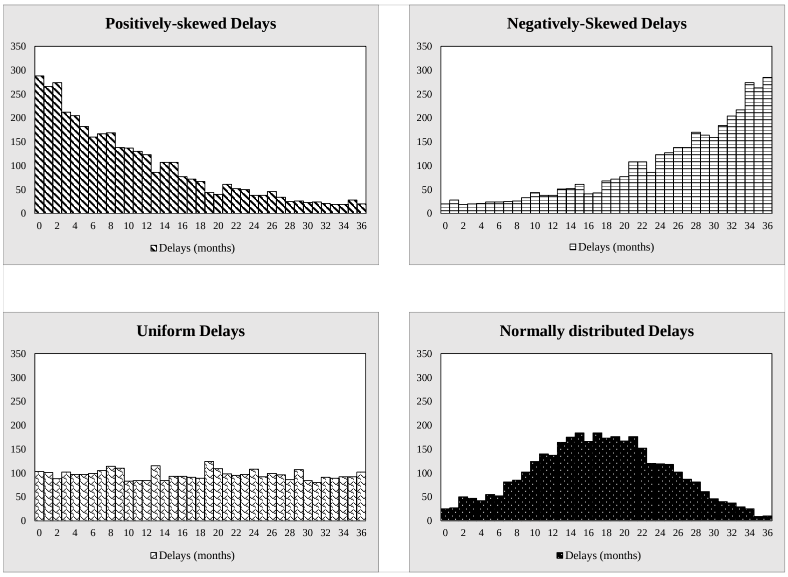
Table 4: Group life mortality rates

Age (x)	Male ($\mu_{x+0.5}$)	Female ($\mu_{x+0.5}$)
25	0.0030138	0.0020404
26	0.0035989	0.0024338
27	0.0042080	0.0028329
28	0.0048264	0.0032247
29	0.0054400	0.0035975
30	0.0060356	0.0039409
31	0.0066022	0.0042476
32	0.0071313	0.0045125
33	0.0076171	0.0047335
34	0.0080566	0.0049106
35	0.0084492	0.0050463
36	0.0087967	0.0051442
37	0.0091023	0.0052093
38	0.0093710	0.0052469
39	0.0096082	0.0052628
40	0.0098204	0.0052625
41	0.0100138	0.0052515
42	0.0101951	0.0052345
43	0.0103704	0.0052161
44	0.0105458	0.0052002
45	0.0107268	0.0051900
46	0.0109186	0.0051885
47	0.0111257	0.0051981
48	0.0113524	0.0052210
49	0.0116021	0.0052588
50	0.0118779	0.0053130
51	0.0121823	0.0053847
52	0.0125168	0.0054746
53	0.0128825	0.0055833
54	0.0132793	0.0057111
55	0.0137061	0.0058575
56	0.0141605	0.0060220
57	0.0146386	0.0062031
58	0.0151344	0.0063987
59	0.0156401	0.0066060
60	0.0161455	0.0068207
61	0.0166378	0.0070378
62	0.0171014	0.0072506
63	0.0175181	0.0074512
64	0.0178671	0.0076300
65	0.0181257	0.0077764

A.2 Delay distributions

Figure 9 shows the distributions of the delays generated for one simulation of a stable group scheme.

Figure 9: Distribution of delays for a stable membership scheme



A.3 One simulation

A.3.1 Stable scheme

This appendix presents the results for a stable scheme.

Figure 10: Comparison of patterns for Cape Cod method

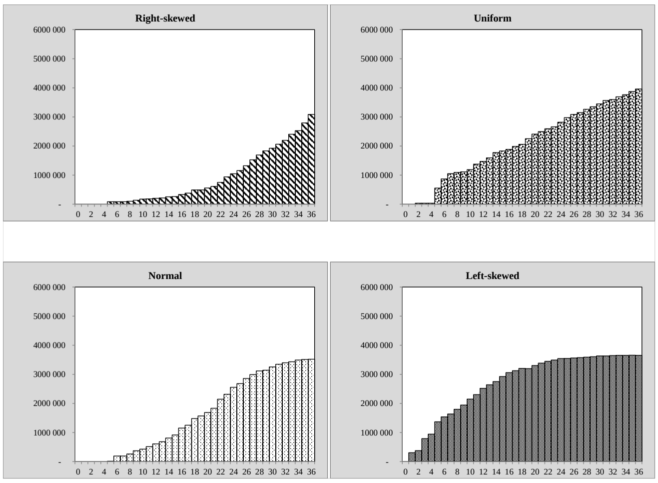


Figure 11: Comparison of total reserves for right-skewed delay

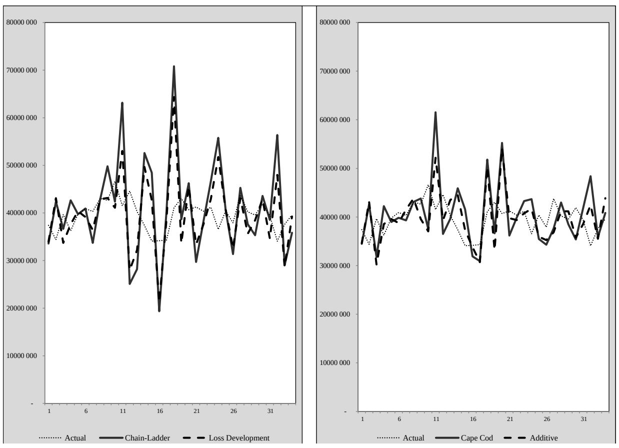
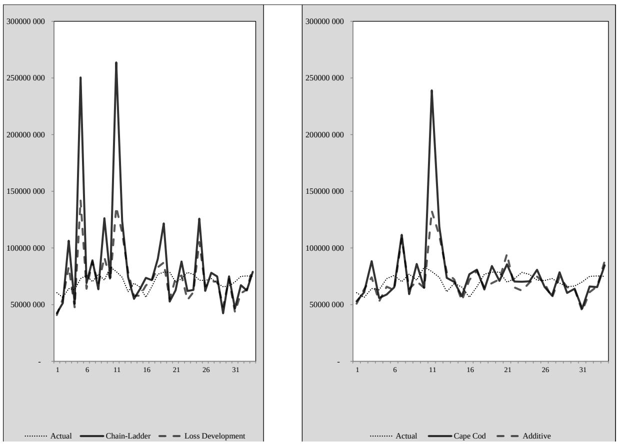


Figure 12: Comparison of total reserves for uniform delay



A.3.2 Growing scheme

This appendix shows the results obtained from one simulation of the scheme whose membership grows at 2% per annum. Figure 13 shows the actual reserves on 01/01/1988 as against the reserves calculated using the chain-ladder and loss development methods while figure 14 compares the actual reserves to the reserves calculated for the cape cod and additive methods.

Figure 13: Actual vs Chain-Ladder vs Loss development - right-skewed delay

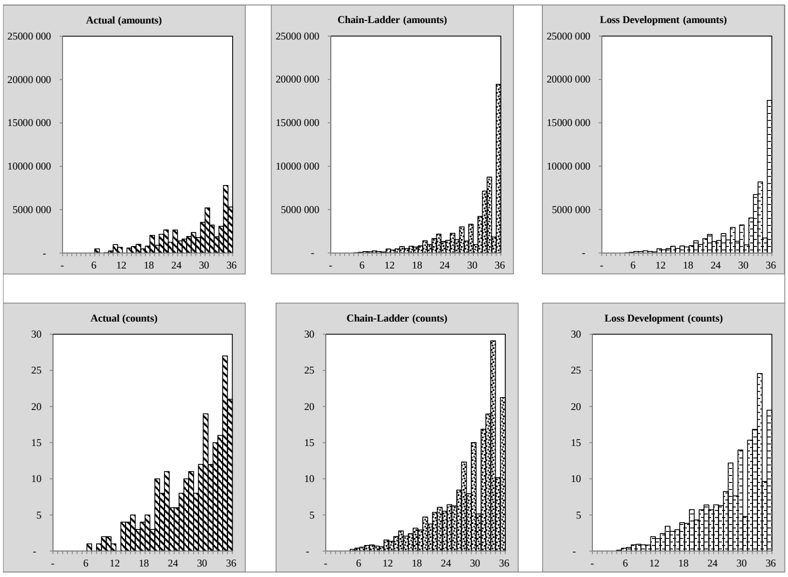


Figure 14: Actual vs Cape Cod vs Additive - right-skewed delay

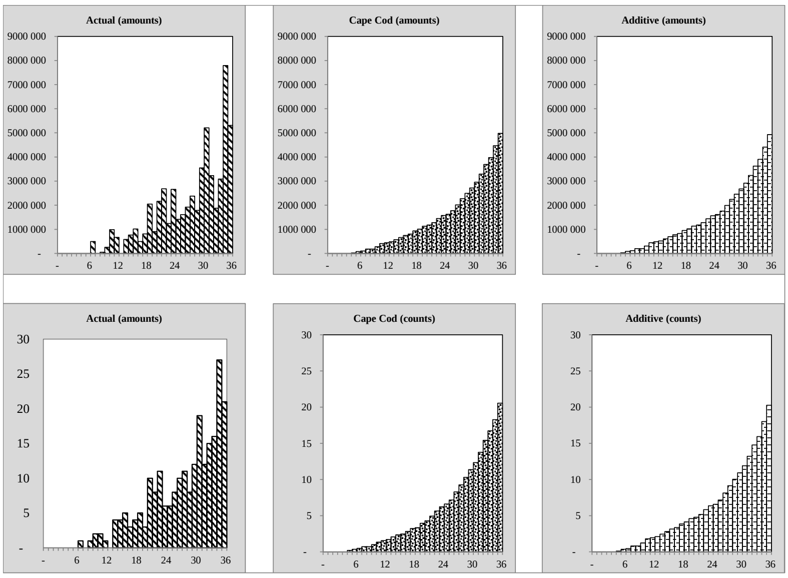


Figure 15: Estimation errors for a growing scheme

		Chain-Ladder	Loss Development	Cape Cod	Additive
		Amounts (billions)			
MSE¹	Right-skewed delay	81 429	76 327	24 184	**23981***
	Uniform delay	387 664	307 959	49 722	**46900***
	Normal delay	642 018	519 948	39 922	**39776***
	Left-skewed delay	2 789 587	2 129 601	92 750	**73345***
		Counts			
	Right-skewed delay	739	752	**248***	257
	Uniform delay	2 453	2 493	500	**484***
	Normal delay	9 623	9 069	416	**415***
	Left-skewed delay	43 734	53 232	1 042	**791***
		Amounts (billions)			
MSE²	Right-skewed delay	4 070 883	2 810 038	1 668 655	**1381231***
	Uniform delay	30 078 943	18 905 790	13 488 876	**10200036***
	Normal delay	23 775 428	15 039 826	4 543 470	**4247615***
	Left-skewed delay	227 179 301	102 669 699	48 307 939	**24776605***
		Counts			
	Right-skewed delay	31 824	47 340	**18238***	23 001
	Uniform delay	148 431	252 016	**99968***	119 646
	Normal delay	360 536	258 235	55 367	**48321***
	Left-skewed delay	2 195 299	2 891 123	596 606	**309380***

A.3.3 Shrinking scheme

This appendix shows the results obtained from one simulation of the scheme whose membership declines until every member is dead or has retired.

Figure 16: Estimation errors for a shrinking scheme

		Chain-Ladder	Loss Development	Cape Cod	Additive
		Amounts (billions)			
MSE¹	Right-skewed delay	25 378	23 244	7 289	**7185***
	Uniform delay	102 639	71 077	17 401	**14069***
	Normal delay	141 155	89 971	11 205	**10955***
	Left-skewed delay	810 871	338 017	65 554	**19626***
		Counts			
	Right-skewed delay	285	230	**67**	**67***
	Uniform delay	897	615	157	**143***
	Normal delay	2 478	1 918	101	**101***
	Left-skewed delay	26 057	2 800	953	**230***
		Amounts (billions)			
MSE²	Right-skewed delay	1 163 140	898 492	661 583	**563475***
	Uniform delay	12 545 284	5 099 848	7 659 016	**3468395***
	Normal delay	7 862 474	3 371 540	877 155	**658539***
	Left-skewed delay	38 399 695	6 946 492	57 265 923	**4342807***
		Counts			
	Right-skewed delay	21 455	17 639	**11063***	11 530
	Uniform delay	109 171	68 809	**40795***	49 292
	Normal delay	106 144	57 883	**5371***	6 027
	Left-skewed delay	877 135	**75035***	934 486	91 881

A.4 Multiple simulations

Figure 17 comapares the total of the rankings for a shrinking scheme.

Figure 17: Comparison of rankings for a shrinking scheme