Matlab for Motor Control

– Modeling DC Motor & Control

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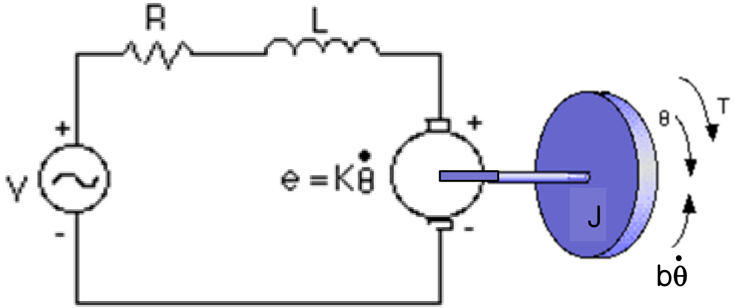
Physical Setup

- DC motor system
  - moment of inertia of the rotor (J) ($kg.m^2/s^2$)
  - damping ratio of the mechanical system (b) (Nms)
  - Back EMF & electromotive force constant (Ke & Kt) (Nm/Amp)
  - electric resistance (R) (ohm)
  - electric inductance (L) (H)
  - input : Voltage (V= k *d, where k: constant and d: PWM duty)
  - output : position of shaft (θ)
  - The rotor and shaft are assumed to be rigid



System Equations

- Exact model equations

$$J\ddot{\theta} + b\dot{\theta} = K_t i$$

$$L\frac{di}{dt} + Ri + K_e\dot{\theta} = V = kd$$

- Transfer function

$$s\left(Js + b\right)\Theta\left(s\right) - K_t I\left(s\right) = 0$$

$$K_e s\Theta\left(s\right) + \left(Ls + R\right)I\left(s\right) = kd$$

$$\frac{\Theta}{D} = \frac{kK_t}{s\left[\left(Js+b\right)\left(Ls+R\right)+K_eK_t\right]}$$

- State space

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -b/J & K_t/J\\ 0 & -K_e/L & -R/L\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix} + \begin{bmatrix}0\\ 0\\ a/L\end{bmatrix}d, \quad y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ i\end{bmatrix}$$

System Equations – with approximation

- Assumption: The motor system is **linear**. (It is not true.)
- Because $L << R$ for most motors, the simplified system model has only two system parameters. (This is reasonable.)
- Using approximate transfer function

$$\frac{\Theta}{D} \cong \frac{kK_t}{s\left[R\left(Js+b\right)+K_eK_t\right]} = \frac{K}{s\left(\tau s+1\right)}$$

- *If the moment of inertia J is negligible, the transfer function would be:*

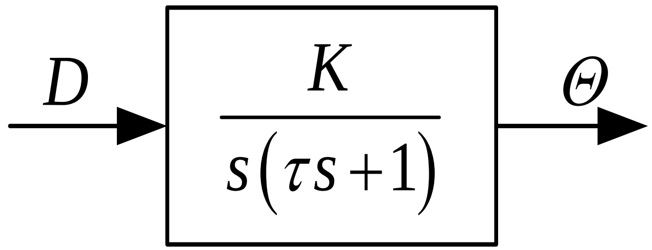
$$\frac{\Theta}{D} \cong \frac{K}{s}$$

- *State Space representation*

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & -1/\tau\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0 \\ K/\tau\end{bmatrix} d \qquad y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix}$$

P #1: Parameter identification by open-loop control

- Assumption: The motor system is **linear**. (It is not true.)



1. Calculate & plot angular velocity along time with the encoder reading of an open-loop response for a sufficient PWM duty step input, d = 200. (Be sure to get a smooth plot.)
2. Obtain the time constant τ of your motor system. Check the variation of τ for different values of d.
3. From the steady state angular speed, calculate the parameter K. Also check the variation of K for different values of d.

Open loop response

- Matlab program

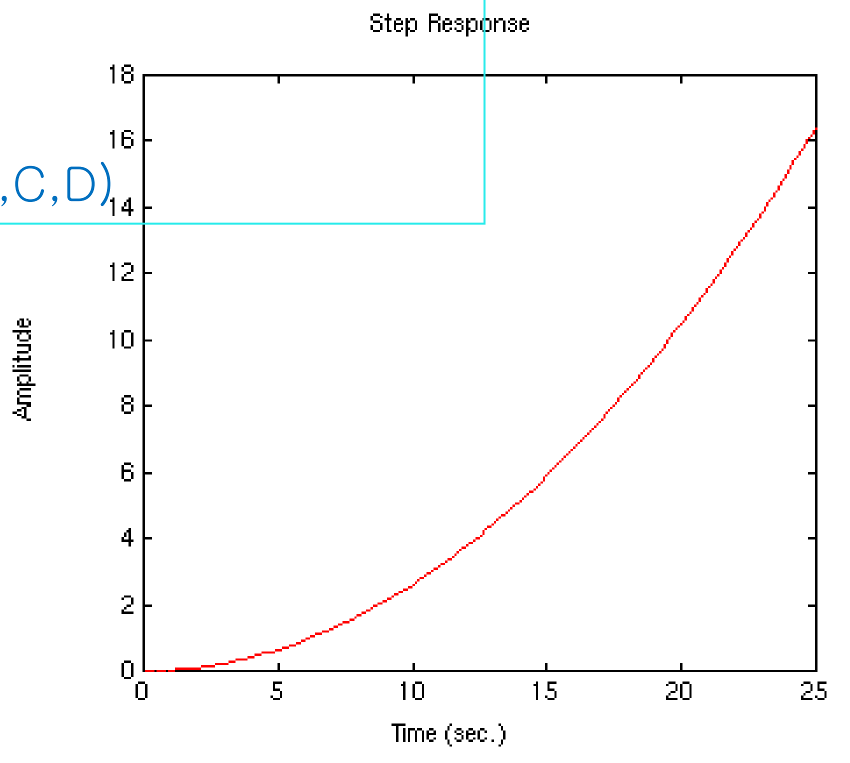
```
tau = (your obtained value);
K = (your obtained value);

A=[0 1; 0 -1/tau];
B=[0; K/tau];
C=[1 0];
D=[0];

motor=ss(A,B,C,D)
```

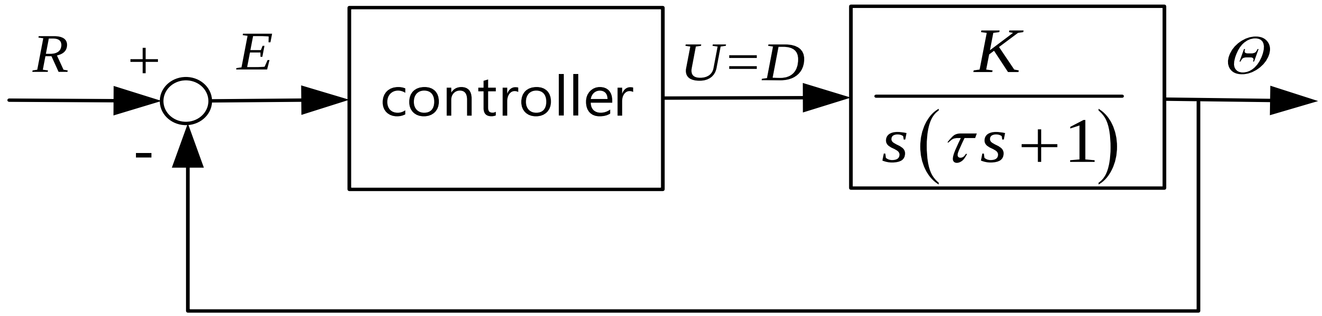
```
tau = (your obtained value);
K = (your obtained value);

%simplifies input
num = [K];
den = [1 0];
motor = tf(num, den)
step(200*motor)
```



Closed-loop Representation

- The block diagram for this example with a controller and unity feedback of the motor's position is shown below:



- The transfer function for a PID controller is:

$$K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_p s + K_I}{s}$$

PID Design

- General tips for designing a PID controller:

  1. Obtain an open-loop response and determine what needs to be improved.
  2. Add a proportional control to improve the rise time.
  3. Add a derivative control to improve the overshoot.
  4. Add an integral control to eliminate the steady-state error.
  5. Adjust each of K_P, K_I, and K_D until you obtain a desired overall response.

- You do not need to implement all three controllers (proportional, derivative, and integral) into a single system.
  - Keep the controller as simple as possible.

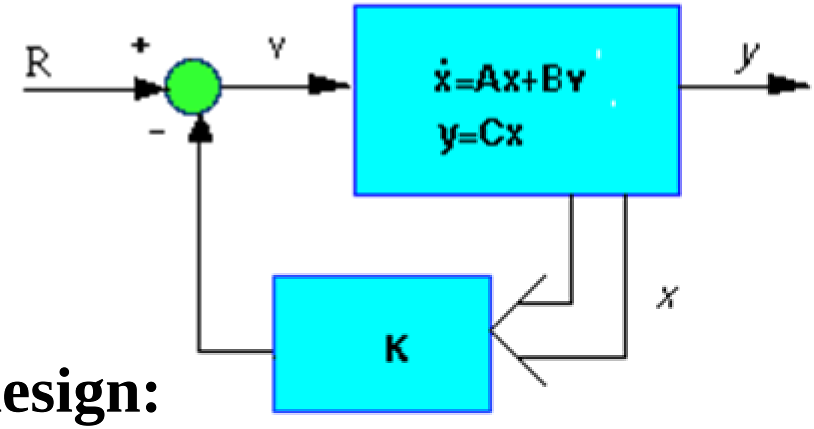
P #2: PID controller design

- For the given spec. of the step response
  - overshoot < 15%,
  - ts <= 0.5 s,
  - no steady-state error

1. Obtain Kp, Ki and Kd and draw the response to show the spec. of a reference step input (R = 200). Explain your controller design with the result.
   - There may exist a case that you cannot satisfy the given spec. In such case, obtain gains those give best performance close to the spec.
2. Check the step response (Mp, ts, ess) of the system for different values of *R*. (150 and 250)

Controller Design for a State Space representation

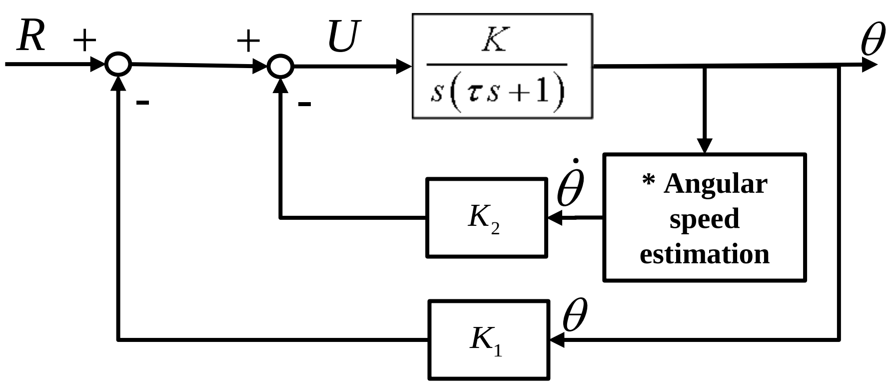
- **If the design criteria for a problem are given:**
  - Settling time less than 3 seconds
  - Overshoot less than 5%



- **Full-state feedback controller design:**
  - $|(s\mathbf{I}-(\mathbf{A}-\mathbf{BK}))| = 0$: char. eq.
  - overshoot $\leq 5\% \rightarrow \zeta = 0.7$
    (root locus : 45 degree line)
  - settling time $T_s \leq 3$ s $\rightarrow$
    $\sigma = 4.6/T_s = 4.6/3 = 1.53 \rightarrow$ poles at $-2 \pm 2i$.

State Space: Full-State Feedback Controller

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -1/\tau\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ K/\tau\end{bmatrix}d \qquad y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}$$



*** Angular speed estimation:**

- You should use the calculation method of 1 of P#1.
- Be sure to get smooth & proper speed $\dot{\theta}$.

P #3: Full-State Feedback Controller design

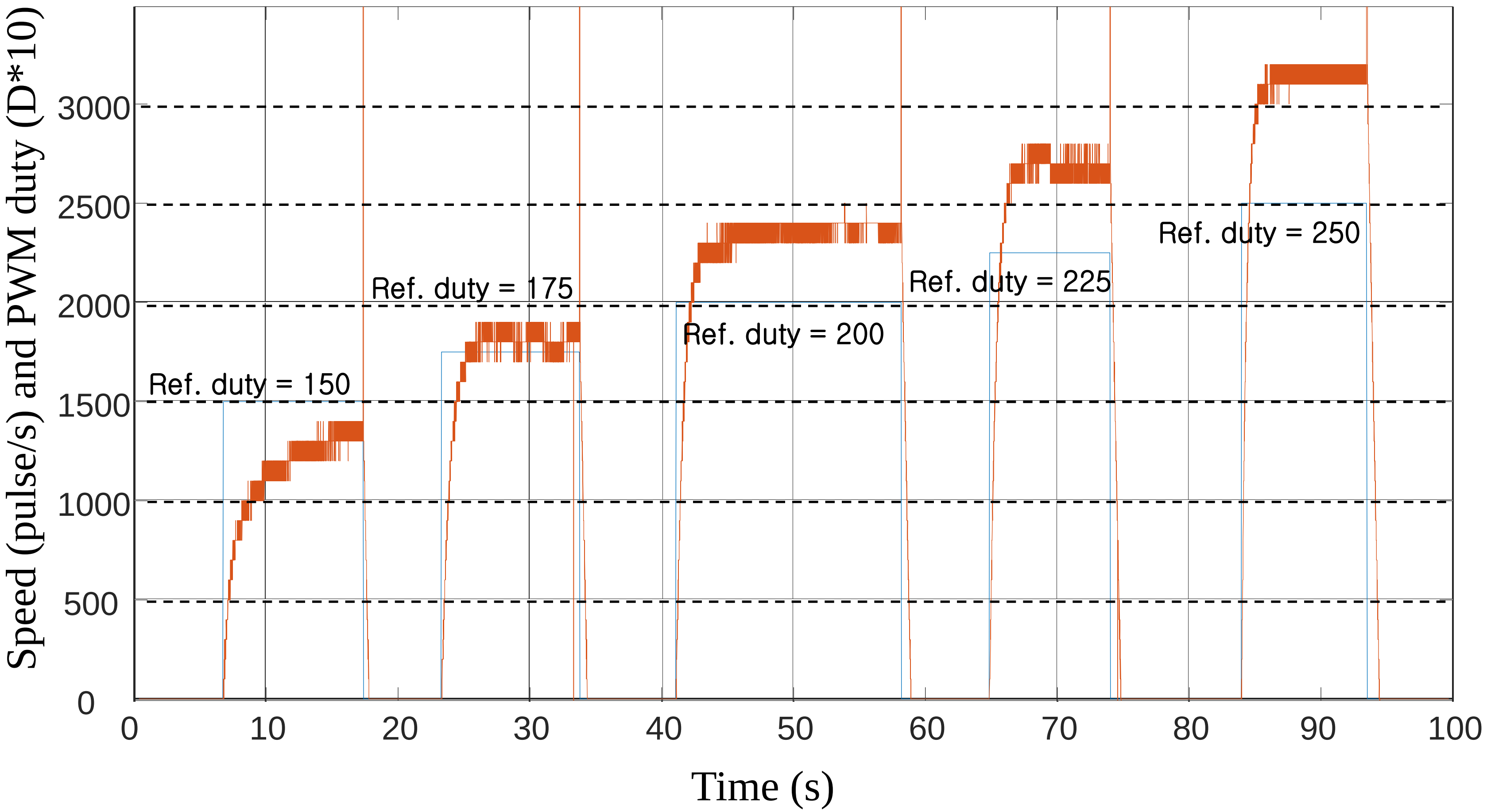
- For the given spec. of the step response
  - overshoot < 5%,
  - T_s <= 0.5 s,
  - no steady-state error

1. Obtain K_1 and K_2 and draw the response to show the spec. of a reference step input (R = 200).
   - There may exist a case that you cannot satisfy the given spec. In such case, obtain gains those give best performance close to the spec.
2. Check the step response (Mp, ts, ess) of the system for different values of R. (150 and 250)

Q & A

별첨

Open loop speed response for each D



Open loop speed response for D=250

- Approximately the DC gain is K=3100/250 = 12.4, time constant is t=0.4 (s)