2 Equivalent networks

2.2 Formulating equivalent networks

According to 2.1, we can get three equations to express Network 1.

$$\vec{a}^{(1)} = W^{(1)}\vec{a}^{(0)} + \vec{b}^{(1)} \qquad (1)$$

$$\vec{a}^{(2)} = W^{(2)}\vec{a}^{(1)} + \vec{b}^{(2)} \qquad (2)$$

$$\vec{a}^{(3)} = W^{(3)}\vec{a}^{(2)} + \vec{b}^{(3)} \qquad (3)$$

Organizing equation 1, 2, and 3, we can get the relation (eq.4) between the input ($\vec{a}^{(0)}$) and the output ($\vec{a}^{(3)}$).

$$\vec{a}^{(3)} = W^{(3)}W^{(2)}W^{(1)}\vec{a}^{(0)} + W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)} + \vec{b}^{(3)} \qquad (4)$$

In Network 2, we can get the equation 5.

$$\vec{a}^{(3)} = \widetilde{W}\vec{a}^{(0)} + \tilde{b} \qquad (5)$$

Network 1 and Network 2 are equivalent and Network 1's weigths and biases values are given, so we can get Network 2's weights ($\widetilde{W}$) and bias ($\tilde{b}$).

$$\widetilde{W} = W^{(3)}W^{(2)}W^{(1)}$$

$$\tilde{b} = W^{(3)}W^{(2)}\vec{b}^{(1)} + W^{(3)}\vec{b}^{(2)} + \vec{b}^{(3)}$$