Chapter 5

Graphs

https://www.scaler.com/topics/data-structures/graph-in-data-structure/

Graphs

- A Graph in Data Structure is a non-linear data structure that consists of nodes and edges which connects them.

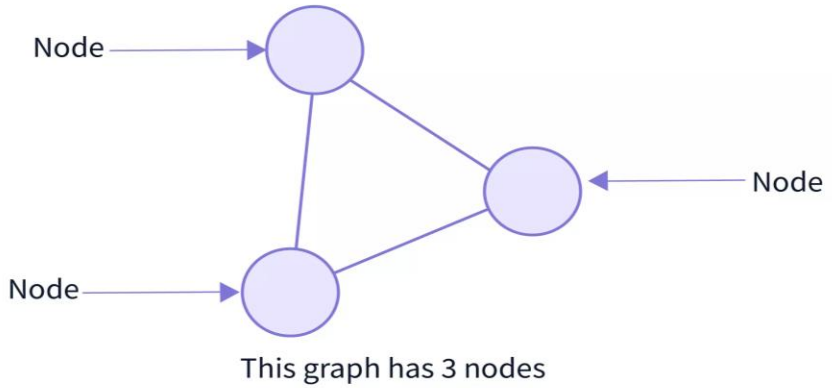
- A Graph in Data Structure is a collection of nodes that consists of data and are connected to other nodes of the graph.

Graph Terminologies

1. **Non-linear Data Structure:**
   - In a non-linear data structure, elements are not arranged linearly or sequentially.
   - Because the non-linear data structure does not involve a single level, an user cannot traverse all of its elements at once.

2. Nodes:

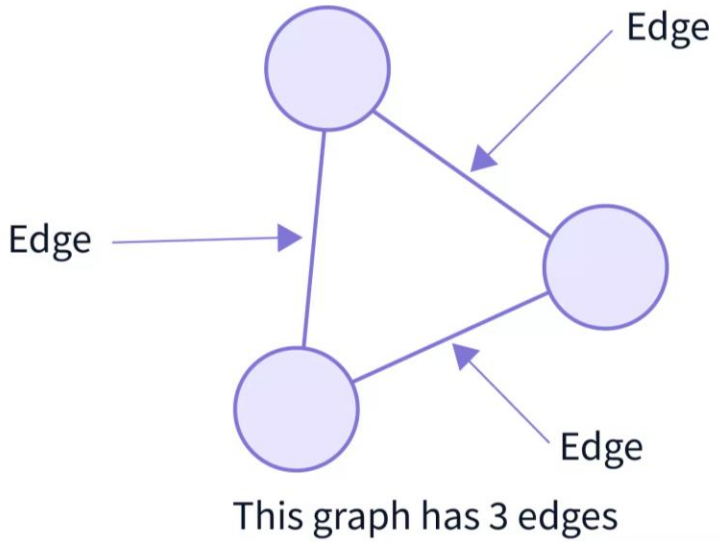
- Nodes create complete network in any graph.
- They are one of the building blocks of a Graph in Data Structure.
- They connect the edges and create the main network of a graph. They are also called **vertices**.
- A node can represent anything such as any location, port, houses, buildings, landmarks, etc.
- They basically are anything that you can represent to be connected to other similar things, and you can establish a relation between the them.



3. Edges:

- Edges basically connects the nodes in a Graph in Data Structure.
- They represent the relationships between various nodes in a graph.
- Edges are also called the path in a graph.

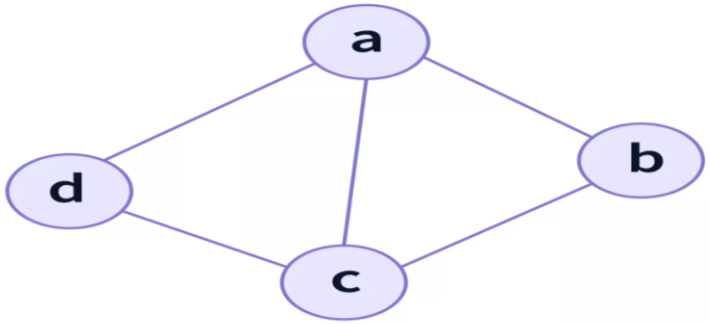
- The bellow image represents edges in a graph.
- A graph data structure (*V*,*E*) consists of:
  - A collection of vertices (*V*) or nodes.
  - A collection of edges (*E*) or path



Example:

The below image represents a set of edges and vertices:

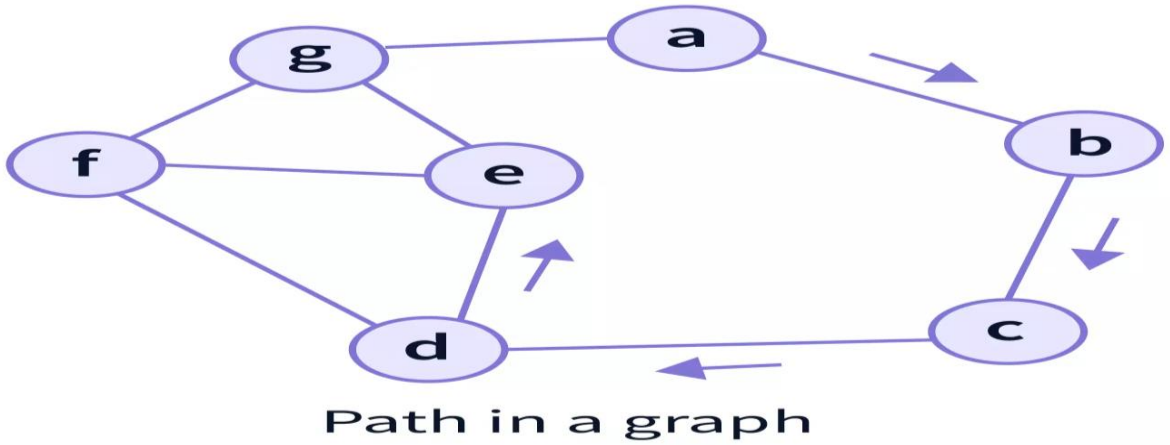
- A graph is a pair of sets (V, E), where V is the set of **vertices** and E is the set of **edges**, connecting the pairs of vertices.
- In the bellow graph:



- V = {a, b, c, d}
- E = {ab, ac, ad, bc, cd}
- In the above graph, **|V| = 4** because there are four nodes (vertices) and, **|E|=5** because there are five edges (lines).

1. Path

- A path in a graph is a **finite or infinite set of edges** which joins a set of vertices.
- It can connect to 2 or more nodes.
- Also, if the path **connects all the nodes** of a graph in Data Structure, then it is a **connected graph**, otherwise it is called a **disconnected graph**.
- There may or may not be path to each and every node of graph. In case, there is **no path to any node, then that node becomes an isolated node.**

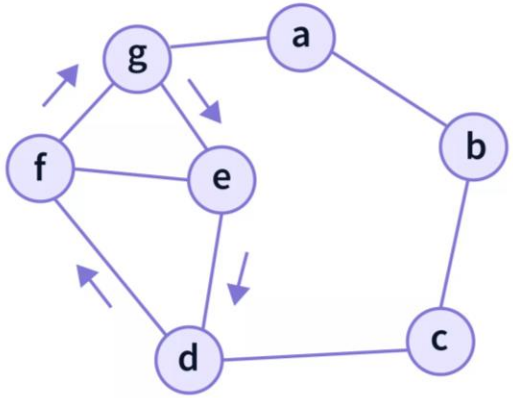


Path in a graph

- In the above graph, the path from '**a**' to '**e**' is = **{a,b,c,d,e}**

2. Closed Path:

- A path is called as **closed path if the initial node is same as terminal(end) node**.
- A path will be closed path if :*V*0 = *Vn*, where *V*0 is the starting node if the graph and *Vn* is the last node.
- So, the starting and the terminal nodes are same in a closed graph.

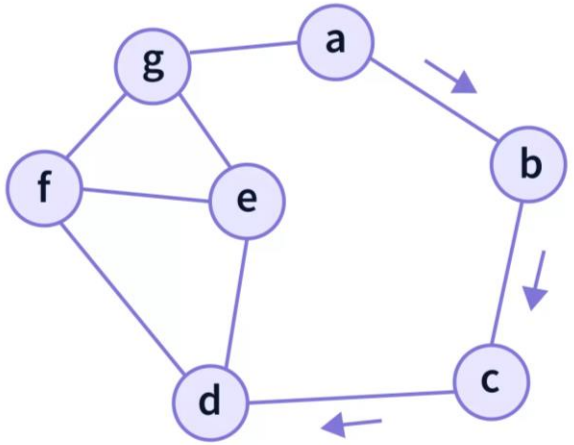


Closed Path in a graph

- The above graph have a closed path, where the **initial node = {e}** is same as the **final node = {e}**. So, the path becomes = {e,d,f,g,e}.

3. Simple Path

- A path that does **not repeat any nodes(vertices) is called a simple path**.
- A simple path in a graph exists if all the nodes of the graph are distinct, expect for the first and the last vertex, i.e. *V*0 = *Vn*, where *V*0 is the starting node if the graph and *Vn* is the last node.



Simple Path in a graph

- In this example, *a*,*b*,*c*,*d* is a simple path. Because, this **graph do not have any loop or cycle** and none of the paths point to themselves.

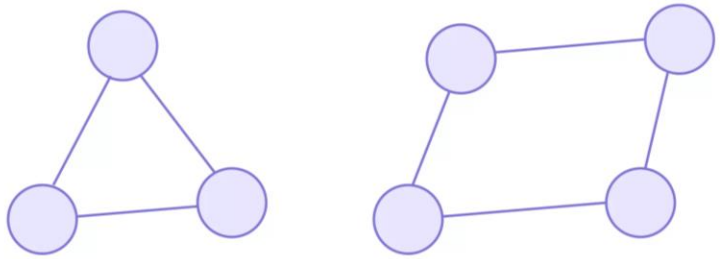
4. Degree of a Node:

- Degree of a node is the **number of edges connecting the node in the graph**.

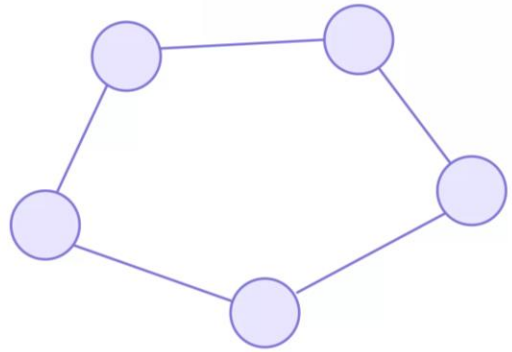
- A simple example would be, suppose in facebook, if you have 100 friends then the node that represents you has a degree of 100.

5. Cycle Graph:

- A simple graph of ‘n’ **nodes(vertices)** (n>=3) and n edges forming a cycle of length ‘n’ is called as a cycle graph.
- In a cycle graph, **all the vertices are of degree 2.**
- In the beside graphs,
  - Each vertex is having degree 2.
  - Therefore, they are cycle graphs.

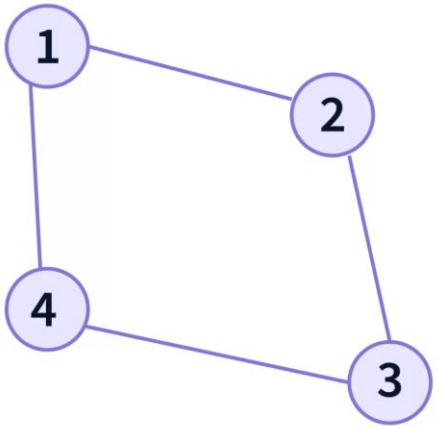


Cycle Graphs



5. Connected Graph:

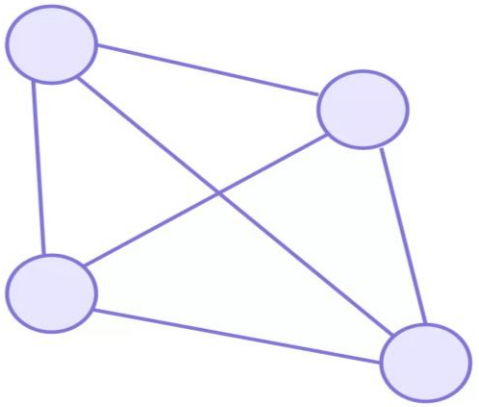
- Connected graph is a graph in which there is an **edge or path joining each pair of vertices.**
- So, in a connected graph, it is possible to get from one vertex to any other vertex in the graph through a series of edges.
- In this graph,
  - we can visit from any one vertex to any other vertex.
  - There **exists at least one path between every pair of vertices**.
  - There is **not a single vertex** in a connected graph, which is **unreachable(or isolated).**



Connected Graph

6. Complete Graph

- In a complete graph, **there is an edge between every single pair of node in the graph.**
- Here, **every vertex has an edge to all other vertices.** It is also known as a **full graph.**
- **Key Notes:**
  - A graph in which exactly one edge is present between every pair of vertices is called as a complete graph.
  - A complete graph of ‘n’ vertices contains exactly nC_2 edges.
  - A complete graph of ‘n’ vertices is represented as K_n.

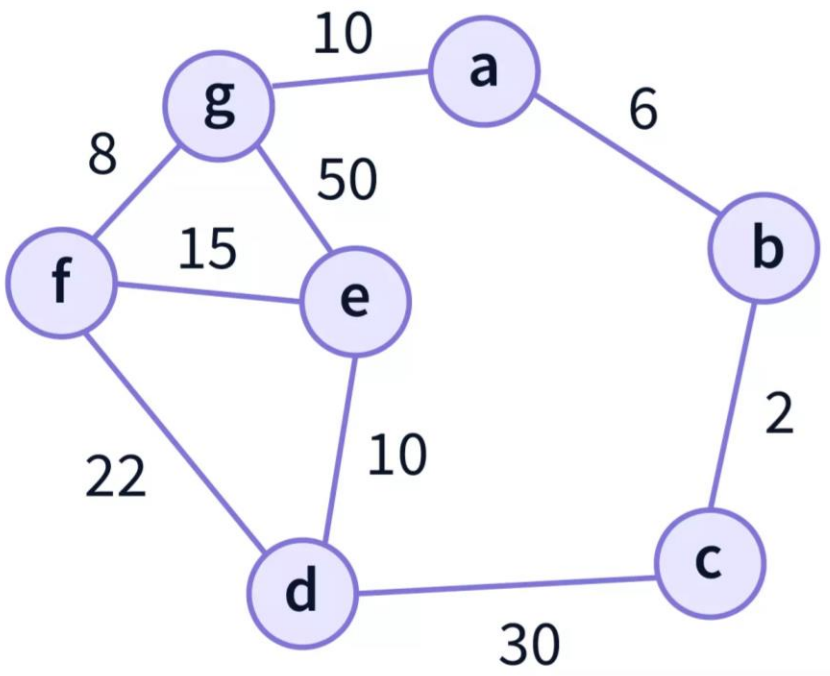


Complete Graph

- In the beside graph,
  - All the pair of nodes are connected by each other through an edge.
  - **Every complete graph is a connected graph, however, vice versa is not necessary.**
- **Note:**
  - **In a Complete graph, the degree of every node is n-1, where, n = number of nodes.**

7. Weighted Graph

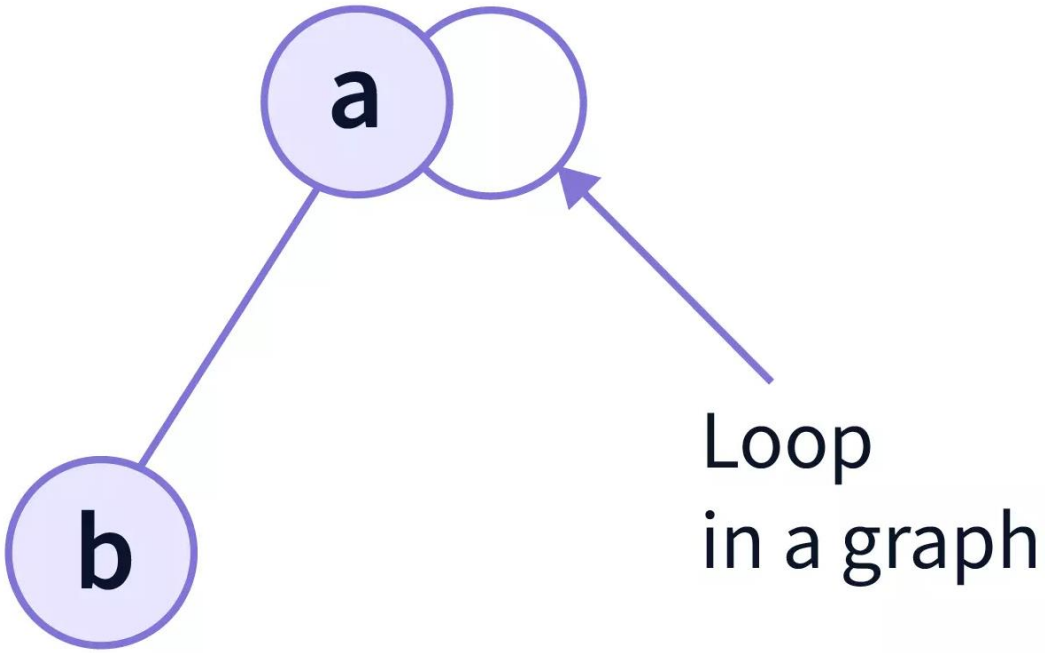
- In weighted graphs**, each edge has a value associated with them (called weight).**
- It refers to a **simple graph** that has weighted edges.
- The **weights are usually used to compute the shortest path** in the graph.
- Given graph is a weighted graph, where each edge is associated with a weight.
- The weights may represent for example, any distance, or time, or the number of connections shared between two users in a social network.
- It is **not mandatory in a weighted graph that all nodes have distinct weight**, i.e. some edges **may have same weights.**



Weighted graph

8. Loop:

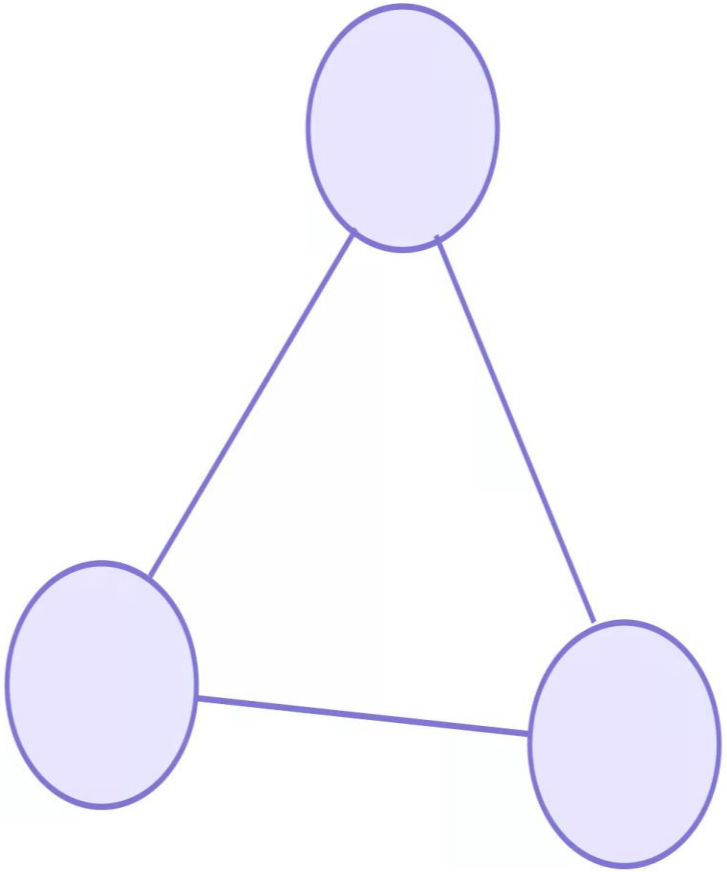
- A loop (also called a **self-loop**) is an **edge** that connects a vertex to itself.
- It is commonly defined as an **edge with both ends as the same vertex.**
- **Although all loops are cycles, not all cycles are loops.**
- Because, cycles do not repeat edges or vertices except for the starting and ending vertex.
- In the above graph,
  - A node 'a' has a loop in itself.
  - The staring and ending point of the edge in node 'a' is same.
  - Hence, there is a loop in the graph.



6. Simple Graph:

- A graph having **no self loops and no parallel edges** in it is called as a **simple graph.**

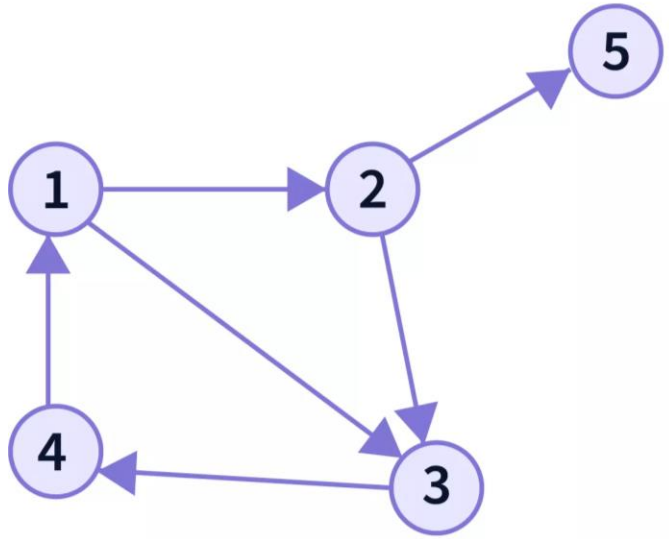
- Here,
  - This graph consists of three vertices and three edges.
  - There are neither self loops nor parallel edges.
  - Therefore, it is a simple graph.



Types of Graph:

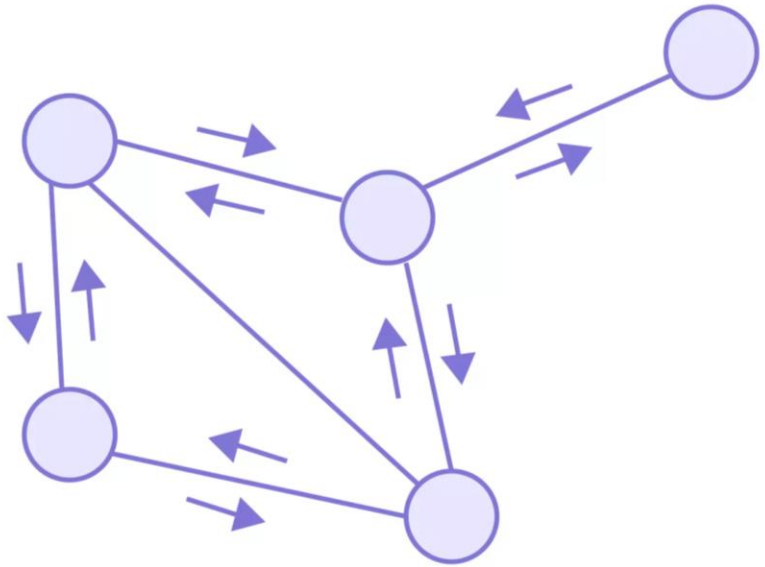
- Graphs are classified based on the characteristics of their edges.
- There are two types of graphs:

1. **Directed Graphs:**



Directed Graph

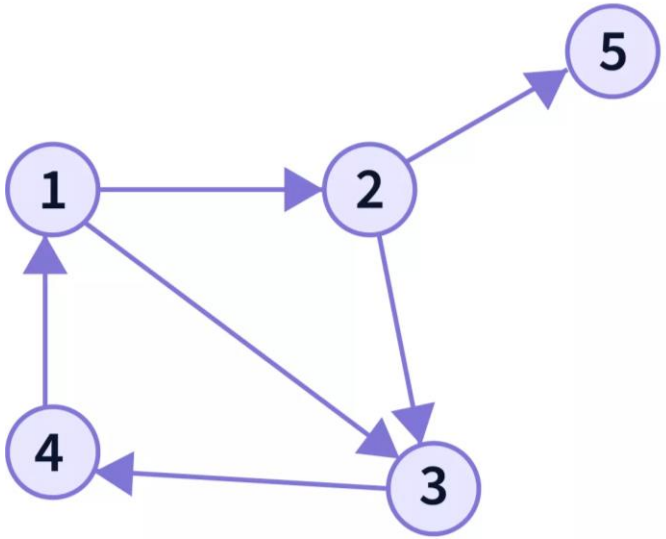
2.Undirected Graphs



Undirected Graph

1. Directed Graphs:

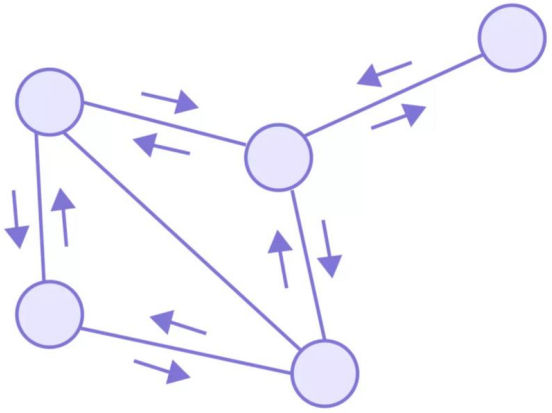
- Directed graphs in graph data structure are the graphs where the edges have **directions** from one node towards the other node.
- In Directed Graphs, we can only traverse from one node to another if the edge have a direction pointing to that node.
- In the given graph, you can see that the edges have arrows that point to a specific direction.
- So, **they are like a one-way street where you can only move from one node to another in the directed edge's direction, and not in the reverse direction.**
- Suppose, in the shown graph, we can go from **node 2** to **node 3**, but cannot go back to **node 2** via **node 3**.
- For going back to **node 2**, we have to find an alternative path like **3 -> 4 -> 1 -> 2** .
- Directed graphs are used in many areas. One of the usecase you may think of is a family tree, where there can be only the edge directed from parent to children.



Directed Graph

2. Undirected Graphs

- **Undirected graphs** have edges that **do not have a direction.**
- Hence, the graph can be **traversed in either direction.**
- The given graph is undirected. Here, the edges do not point to any direction.
  - **We can travel through both the directions**, so it is **bidirectional.**
  - In these graphs, we can reach to one node, from any other node.
  - You can think of undirected edges as **two-way** streets.
  - You can go from one node to another and return through that same "path".
  - Some areas where undirected graphs are very widely used may include the topology of digital social networks, where each friend of someone is that someone's friend; Suppose Steve is a friend of John, then John too is the friend of Steve.
  - Or, in computer networks, like if one device is connected to another, then the second one is also connected to the first.



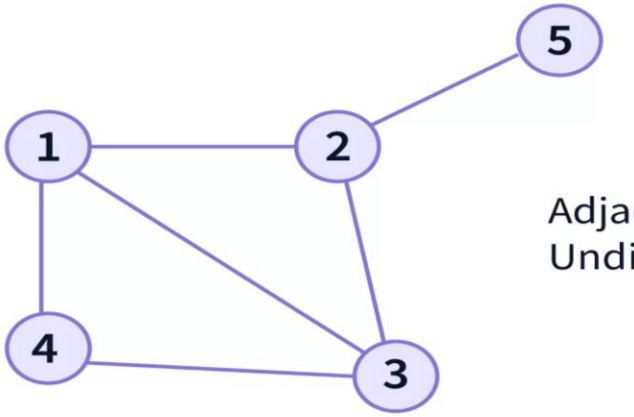
Undirected Graph

Graph Representation

- A graph representation is a technique **to store graph into the memory of computer**.
- We can represent a graph in many ways.
- The following two are the most commonly used representations of a graph.
  - **Adjacency Matrix**
  - **Adjacency Linked List**

Adjacency Matrix

- An Adjacency Matrix is a 2D array of size **V x V** where **V is the number of nodes(vertex)** in a graph.
- It is used to **represent a "finite graph", with 0's and 1's**.
- Since, it's size is **V x V**, it is a square matrix.
- The elements of the matrix indicates whether **pairs of vertices are adjacent or not** in the graph i.e. **is there any edge connecting a pair of nodes in the graph**.



Adjacency Matrix for Undirected Graph

Nodes →

Nodes ↓	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	0	1
3	1	1	0	1	0
4	1	0	1	0	0
5	0	1	0	0	0

The example shows the adjacency matrix for the **undirected graph**.
Let us note some important points:

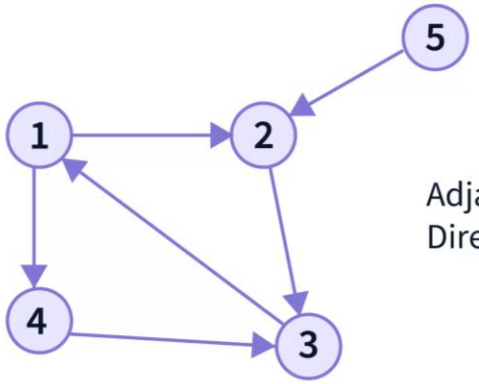
1.The adjacent matrix's row or column, consists of the nodes or vertices(that is numbered in red, in the above graph).

2. Each cell in the above matrix is represented as Aij, where i and j are nodes. The value of Aij is either 1 or 0 depending on whether there is an edge from vertex i to vertex j. If there is a node then, Aij= 1, otherwise Aij= 0
In the above graph, there is an edge between **node 1** & **node 2**, so in the matrix, we have A[1][2] = 1 and A[2][1] = 1.

3. If there is **no edge between 2 nodes**, then that cell in the **matrix will contain '0'**. For example, there is no edge from **node 1** to **node 5**, so, in the matrix, A[1][5] = 0 and A[5][1] = 0.

4. Adjacency matrix of an undirected graph is **symmetric**. Hence, the above graph is symmetric.

Adjacency Matrix for a Directed Graph



Adjacency Matrix for Directed Graph

Nodes →

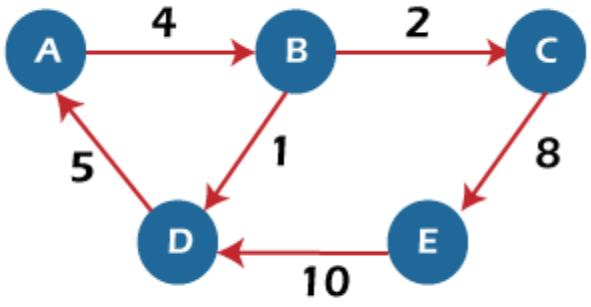
	1	2	3	4	5
1	0	1	0	1	0
2	0	0	1	0	0
3	1	0	0	0	0
4	0	0	1	0	0
5	0	1	0	0	0

Nodes →

- This example clearly shows that, for **node 1**, we have A[1][2] = 1 but A[2][1] = 0, because we have a directed edge from **node 1** to **node 2**, but there is no edge from **node 2** to **node 1**.
- Again, we have a node from **node 2** to **node 3**, so in the matrix, A[2][3] = 1, but A[3][2] = 0, because there is no node from **node 3** to **node 2**.
- **Adjacency Matrix for Weighted Graphs**
  - If the graph is weighted, then we usually call the matrix as the cost matrix. To store weighted graph using adjacency matrix form, we follow the following steps:
  - Here each cell at position A[i, j] holds the weight from edge i to j.
  - If the edge is not present, then it stores infinity or any largest value(which cannot be the weight of any node in the graph).
  - For same node, the value in the matrix is 0.
  - Apart from this, the rest of the steps are similar for the adjacency matrix of the graph.

Adjacency matrix for a weighted directed graph

- In the above image, we can see that the adjacency matrix representation of the weighted directed graph is different from other representations.
- It is because, in this representation, the non-zero values are replaced by the actual weight assigned to the edges.



weighted Directed Graph

	A	B	C	D	E
A	0	4	0	0	0
B	0	0	2	1	0
C	0	0	0	0	8
D	5	0	0	0	0
E	0	0	0	10	0

Adjacency Matrix

Pros of Adjacency Matrix

1. Adjacency Matrices **performs the basic operations** like adding an edge, **removing an edge, and checking** whether there is an edge from node i to node j very efficiently, and in constant time usually.

2. For **dense graphs,** where the number of edges are very large, **adjacency matrix** are the best choice.

3. Because, in big-O terms they don't take up more space, and operations are much faster.

4. Maximum of the cells of matrix are filled because of more number of edges, hence it is very space efficient. If the graph is sparse, then most of the cells are vacant, hence wasting more space.

Cons of Adjacency Matrix

1. The most notable disadvantage that comes with Adjacency Matrix is the usage of **V x V** space, where V is the number of vertices.

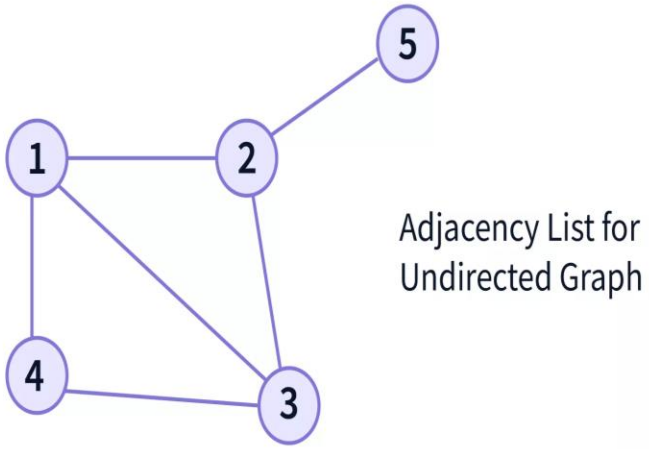
2. In real life, we do not have so dense connections ,where so many edges will be used.

3. So, the space mostly gets wasted, and hence for sparse graphs with less number of edges Adjacency Lists are always preferred.

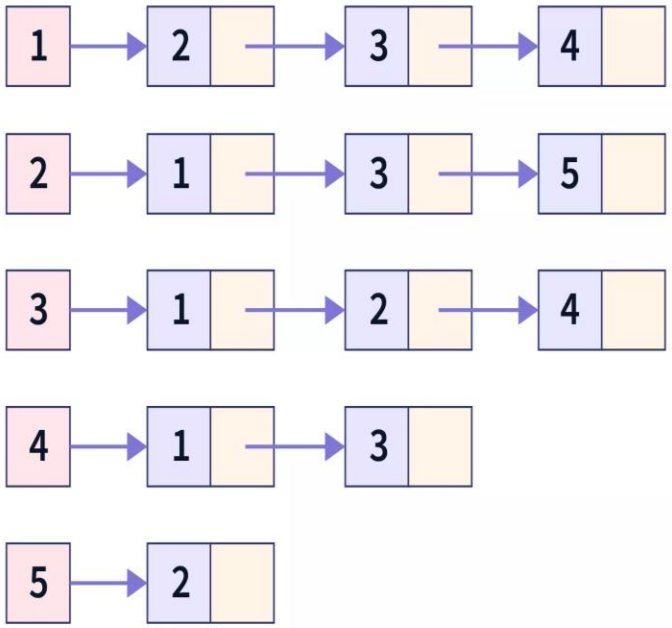
4. Operations like **inEdges**(to check whether there is an edge directing towards this node) and **outEdges**(to check whether there is an edge directing out from this node) are expensive in Adjacency Matrix (will study ahead).

Adjacency Linked List

- An adjacency list represents a graph as an **array of linked lists**.
- The **index** of the array **represents a node.**
- Each element in the **linked list** represents the nodes that are connected to that node by an edge.
- Let us take an example for easy visualization --



Adjacency List for
Undirected Graph



The graph in our example is undirected and we have represented it using the Adjacency List.

Let us look into some important points through this graph:

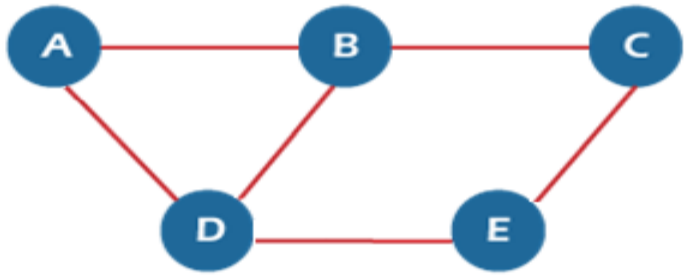
1. Here, **1, 2, 3, 4, 5** are the nodes(vertices) and each of them forms an **array of linked list** with all of its adjacent nodes(vertices).

2. In the graph, the **node 1** has 3 adjacent nodes namely -- **node 2**, **node 3**, **node 4**. So, in the list, the node is linked with 2, 3 & 4.

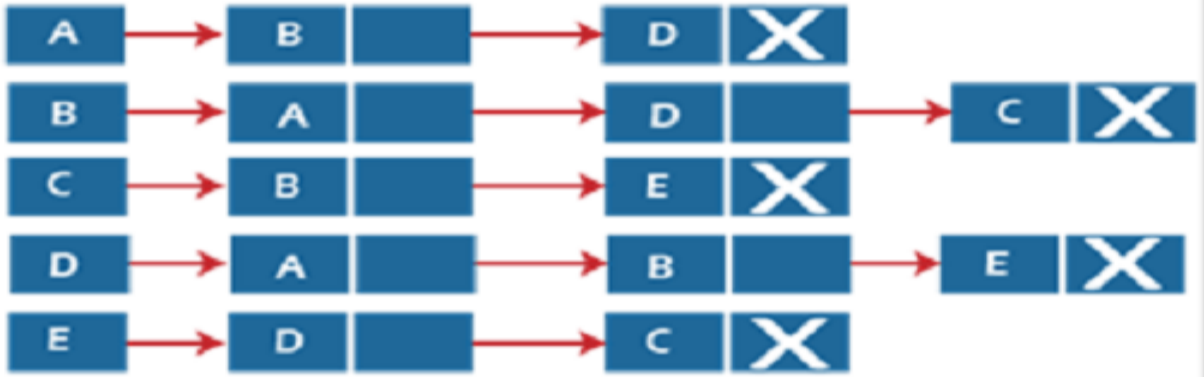
3. Node 4 has only 2 adjacent nodes, **node 1** & **node 3**, so it is linked to the nodes 1 and 3 only, in the array of linked list.

4. The last node in the linked list will point to **null**.

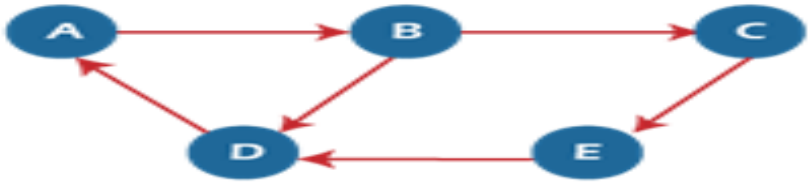
Adjacency List also follows the same rule in case of directed graph, where the nodes will only be linked to the nodes to whom they have a directed edge(or, to the nodes their outgoing edges are pointing to).



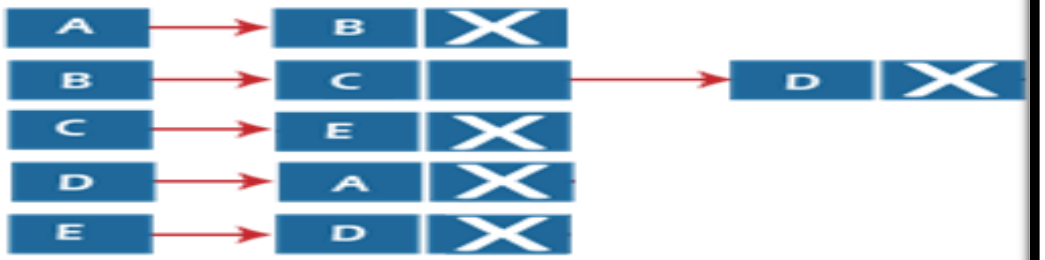
Undirected Graph



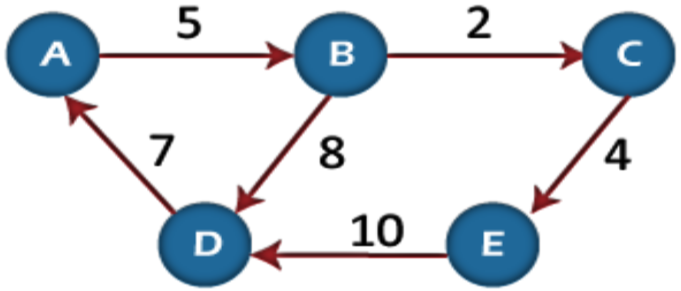
Adjacency List



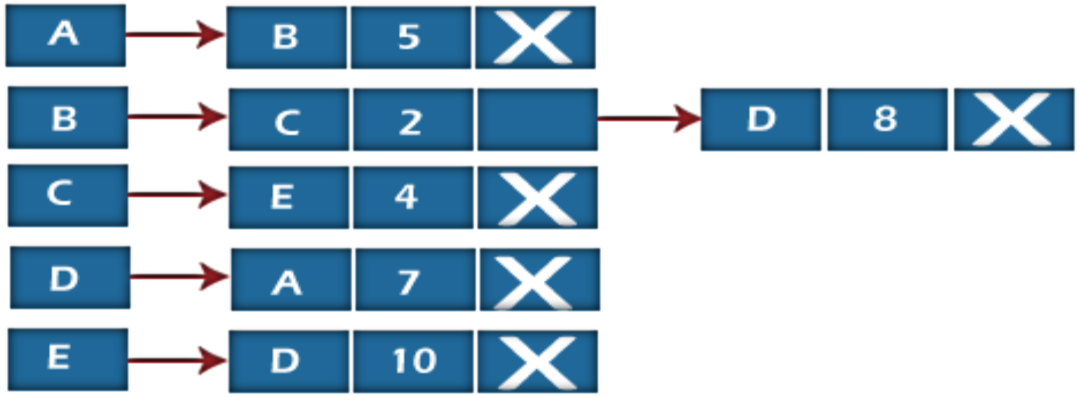
Directed Graph



Adjacency List



Weighted Directed Graph



Adjacency List

Pros of Adjacency Linked List

1. Since, we only store the value for the edges in the linked lists, the adjacency lists are efficient in terms of storage(for sparse graphs). So, if for some graph we have 1000 edges or 1 edge, we can represent them very efficiently without wasting any space, because we will use list for storage.
2. Since the adjacency lists are storage efficient, they are useful for storing sparse graphs. Sparse graphs are the graphs, which have the edges much lesser than the number of edges expected.
3. Adjacency list helps to find all the nodes next to any node easily. Because, a node, points to all the other nodes which are connected to it, hence it becomes very simple to find out all the adjacent nodes.

Cons of Adjacency Linked List

1. To find any particular node in an adjacency list, we need to explore all the connected nodes. So, usually it is slower as compared to finding a node in an adjacency matrix.
2. It is not a good option for **dense graphs** because, we will unnecessarily be storing so many edges in the linked list and hence it will neither be memory efficient, nor time efficient in terms of certain operations(like search or peek).

Operations of Graphs

- The primary operations of a graph include **creating a graph** with vertices and edges, and **displaying** the said graph.

- However, one of the **most common and popular operation** performed using graphs are **Traversal,** i.e. visiting every vertex of the graph in a specific order.

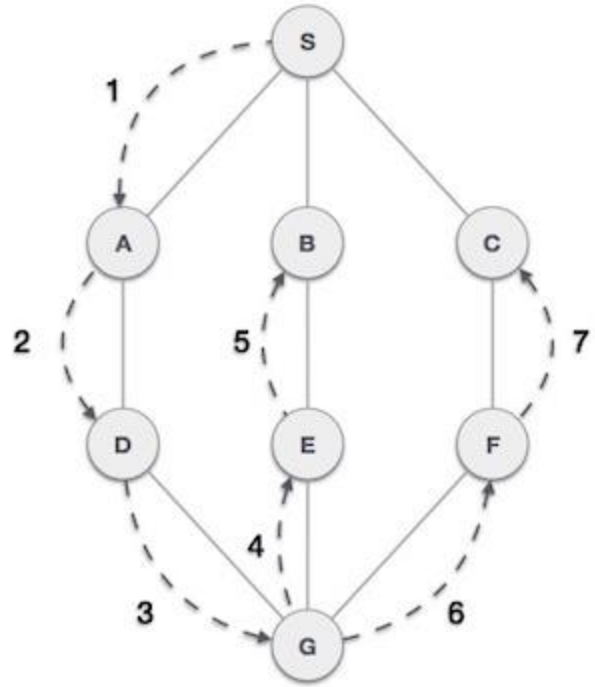
- There are two types of traversals in Graphs −
  1. **Depth First Search Traversal**
  2. **Breadth First Search Traversal**

Depth First Search Traversal

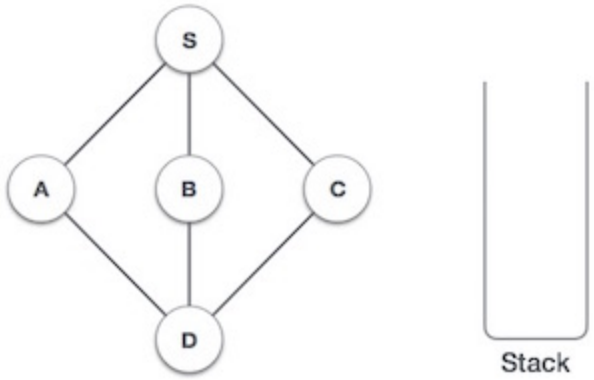
- Depth First Search (DFS) algorithm traverses a graph in a **Depthward motion** and **uses a stack to** remember to get the next vertex to start a search, when a dead end occurs in any iteration.

- As in the example given above, DFS algorithm traverses from S to A to D to G to E to B first, then to F and lastly to C.

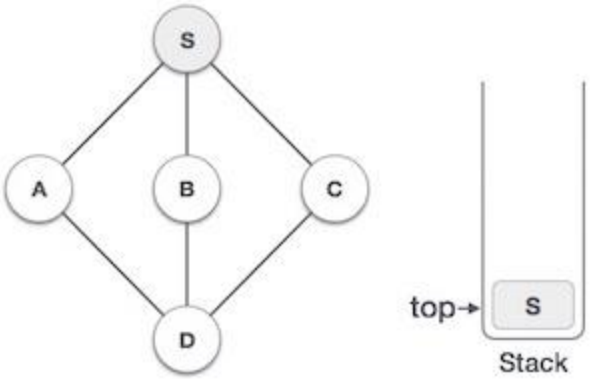
- It employs the following rules.
  - **Rule 1** − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Push it in a stack.
  - **Rule 2** − If no adjacent vertex is found, pop up a vertex from the stack.
  - (It will pop up all the vertices from the stack, which do not have adjacent vertices.)
  - **Rule 3** − Repeat Rule 1 and Rule 2 until the stack is empty.



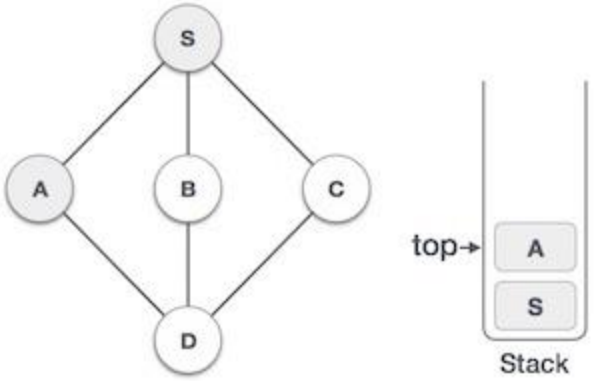
Step 1: Initialize the stack.



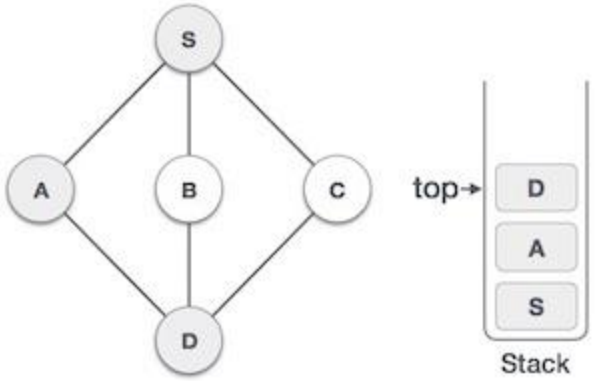
Step 2: Mark **S** as visited and put it onto the stack. Explore any unvisited adjacent node from **S**. We have three nodes and we can pick any of them. For this example, we shall take the node in an alphabetical order.



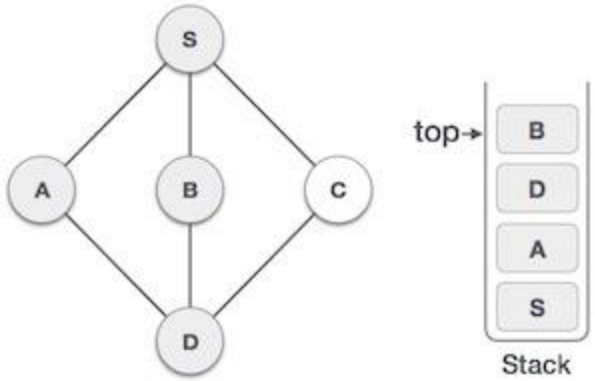
Step 3. Mark **A** as visited and put it onto the stack. Explore any unvisited adjacent node from A. Both **S** and **D** are adjacent to **A** but we are concerned for unvisited nodes only.



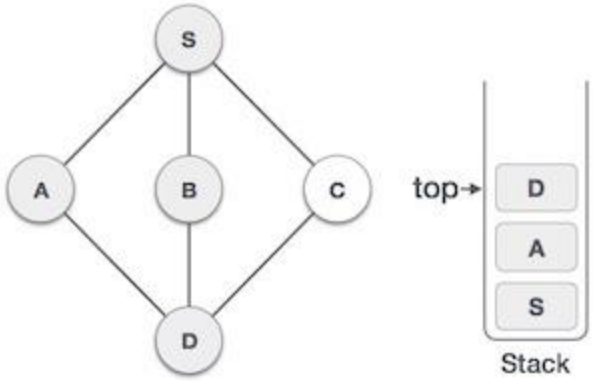
Step 4. Visit **D** and mark it as visited and put onto the stack. Here, we have **B** and **C** nodes, which are adjacent to **D** and both are unvisited. However, we shall again choose in an alphabetical order.



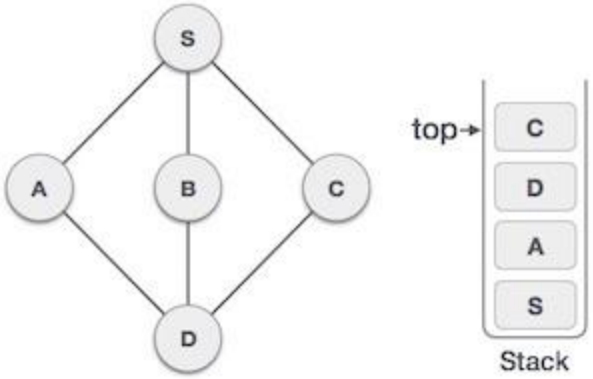
Step 5. We choose **B**, mark it as visited and put onto the stack. Here **B** does not have any unvisited adjacent node. So, we pop **B** from the stack.



Step 6. We check the stack top for return to the previous node and check if it has any unvisited nodes. Here, we find **D** to be on the top of the stack.



Step 7. Only unvisited adjacent node is from **D** is **C** now. So we visit **C**, mark it as visited and put it onto the stack.



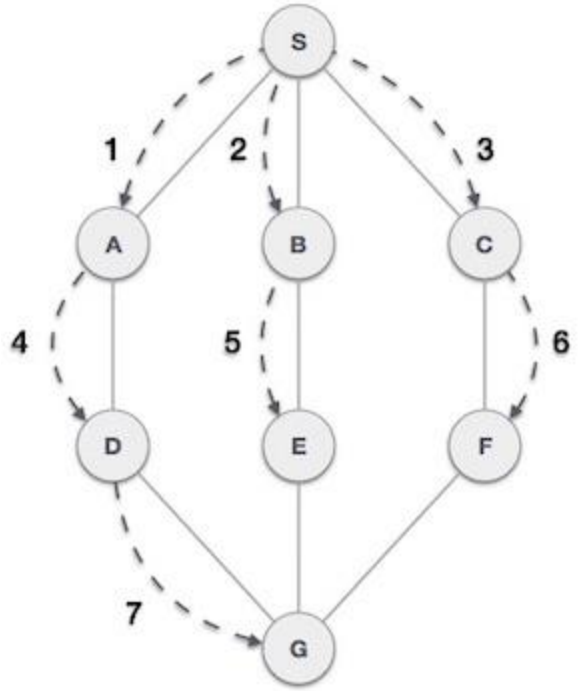
Step 8. As **C** does not have any unvisited adjacent node so we keep popping the stack until we find a node that has an unvisited adjacent node.

In this case, there's none and we keep popping until the stack is empty.

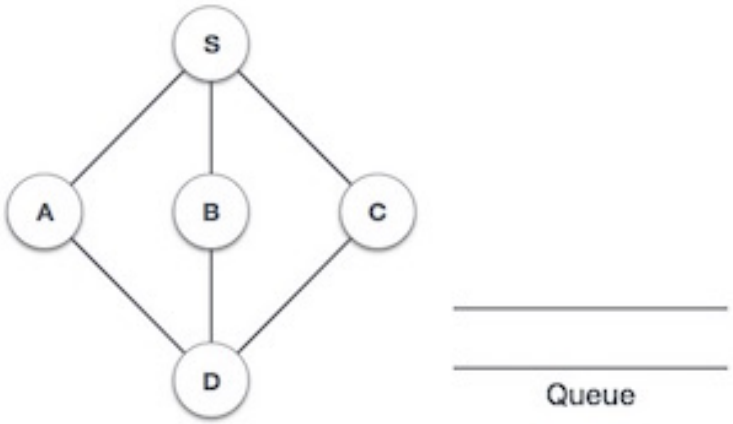
To know about the implementation of this algorithm in C programming language, https://www.scaler.com/topics/dfs-program-in-c/

Breadth First Search Traversal

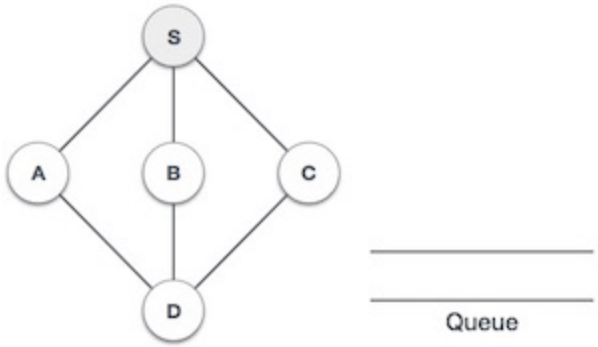
- Breadth First Search (BFS) algorithm traverses a graph in a **Breadthward motion** and **uses a queue** to remember to get the next vertex to start a search, when a dead end occurs in any iteration.
- As in the example given above, BFS algorithm traverses from A to B to E to F first then to C and G lastly to D. It employs the following rules.
  - **Rule 1** − Visit the adjacent unvisited vertex. Mark it as visited. Display it. Insert it in a queue.
  - **Rule 2** − If no adjacent vertex is found, remove the first vertex from the queue.
  - **Rule 3** − Repeat Rule 1 and Rule 2 until the queue is empty.



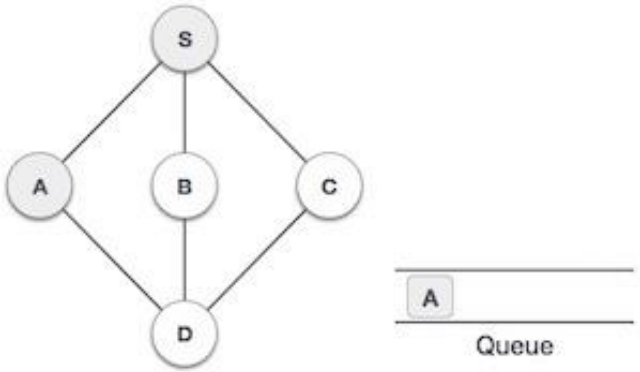
Step 1: Initialize the queue.



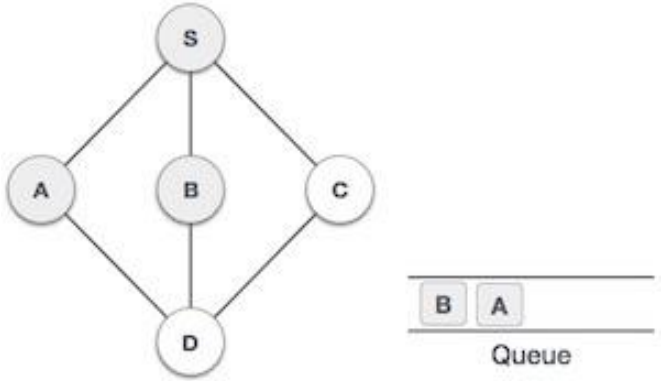
Step 2: We start from visiting **S** (starting node), and mark it as visited.



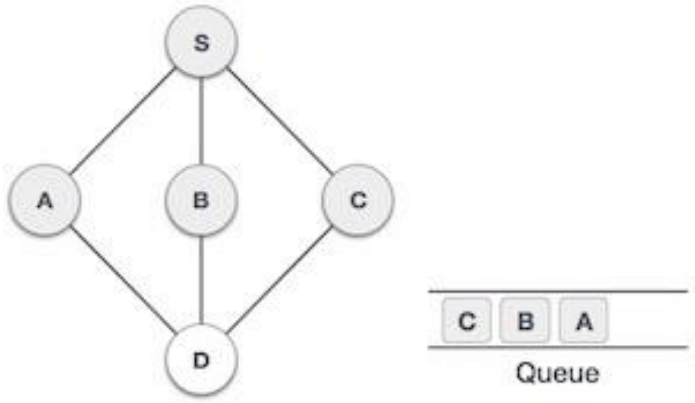
Step 3: We then see an unvisited adjacent node from **S**. In this example, we have three nodes but alphabetically we choose **A**, mark it as visited and enqueue it.



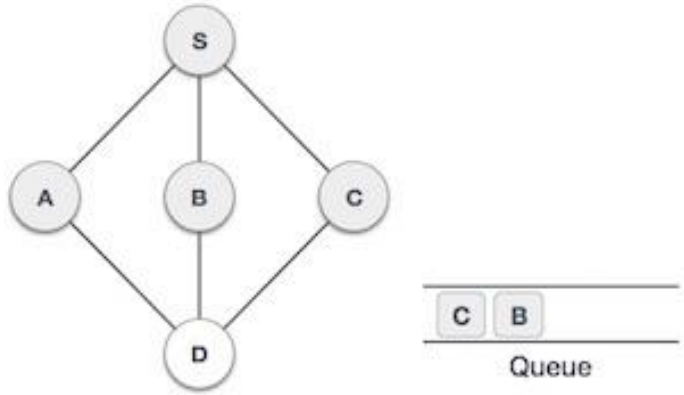
Step 4: Next, the unvisited adjacent node from **S** is **B**. We mark it as visited and enqueue it.



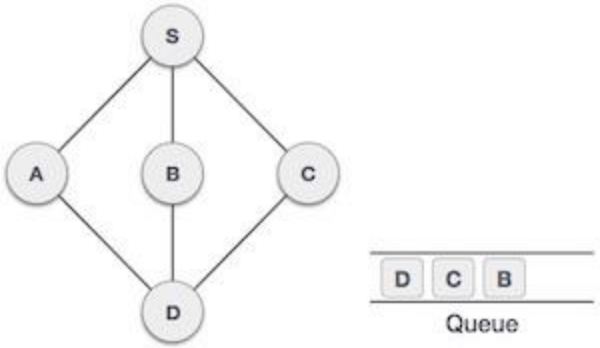
Step 5: Next, the unvisited adjacent node from **S** is **C**. We mark it as visited and enqueue it.



Step 6: Now, **S** is left with no unvisited adjacent nodes. So, we dequeue and find **A**.



Step 7: From **A** we have **D** as unvisited adjacent node. We mark it as visited and enqueue it.



- At this stage, we are left with no unmarked (unvisited) nodes. But as per the algorithm we keep on dequeuing in order to get all unvisited nodes.
- When the queue gets emptied, the program is over.
- The implementation of this algorithm in C programming language can be https://www.scaler.com/topics/bfs-program-in-c/

Practical Applications of Graph

1. GPS systems and Google Maps use graphs to find the shortest path from one destination to another.
2. The Google Search algorithm uses graphs to determine the relevance of search results.
3. World Wide Web is the biggest graph. All the links and hyperlinks are the nodes and their interconnection is the edges. This is why we can open one webpage from the other.
4. Social Networks like facebook, twitter, etc. use graphs to represent connections between users.
5. The nodes we represent in our graphs can be considered as the buildings, people, group, landmarks or anything in general , whereas the edges are the paths connecting them.

Conclusion

- A **Graph** data structure is a collection of nodes that have data and are connected to other nodes through edges.
- **Weighted graphs** are the graph in Data Structure in which the edges are given some weight or value based on the type of graph we are representing
- **Unweighted graphs** are the graph in Data Structure which are not associated with any weight or value. By default their weight or value is considered as 1.
- **Directed graphs** are the graph in Data Structure where the edges have some direction.
- **Undirected graphs** are the graph in Data Structure which do not have any particular direction.
- **Adjacency Matrix** representation of graphs are done through 2D arrays or matrices.
- **Adjacency List** are the array of linked list and are meant to store the edges connected to a node.
- We can perform various **operations** on graph in Data Structure like adding/ removing node, adding/ removing edge, graph traversals, displaying nodes, inedge, out-edge, etc.
- There are major **real-life implementations** of graph in Data Structures like they are used in social networks, google maps, google search engine, GPS, etc.